We will now apply the concepts learned in Chapters 1-7 to the topic of FLUIDS. Fluids are materials that can flow, i.e. liquids and gases. We need to begin with the definitions of DENSITY and of PRESSURE.

DEF The (MASS) DENSITY (ρ) of any material is the ratio of its mass m to its volume V. In symbols,

$$\rho \equiv \frac{m}{V} \qquad \text{units are kg/m}^3$$

Density is a positive-only scalar. The symbol ρ is the Greek letter "rho". The word "mass" is usually understood.

The standard unit for volume is the cubic meter (m^3);

$$1.00 \text{ m}^3 = 1.00 \times 10^6 \text{ cm}^3 = 1000 \text{ liters, i.e. } 1.00 \text{ liter} = 1000 \text{ cm}^3.$$

A table of the densities of common substances is available on page 301 of our text. But there is one substance for which you should know the density, because it is so simple. The density of fresh water (ρ_{H_2O}), at a temperature of 3.98 °C, is 1000 kg/m^3 = 1.00 kg/liter = 1.00 g/cm^3. The kg unit was originally defined by this relationship, i.e. the mass of one liter of fresh water. The density of salt water is a bit larger; for salt water, at 3.98 °C, ρ is about 1025 kg/m^3 (but of course this value depends on the salt concentration). Notice that the density of water does depend on the temperature; the density of any gas is even more sensitive to changes in temperature. However, we will ignore these temperature dependences for now -- we begin thermal physics in about two weeks.

Next, we must define PRESSURE. Here are two points of emphasis before I give you the formal definition. First, pressure exists at ALL points in a fluid (like tension exists at all points in a rope). Second, a fluid PUSHES in all directions with equal strength (unless you consider surface tension, fluids cannot pull). Here is the formal definition:

DEF The PRESSURE (P) at a particular location $\mathbf{r}$ is a fluid is

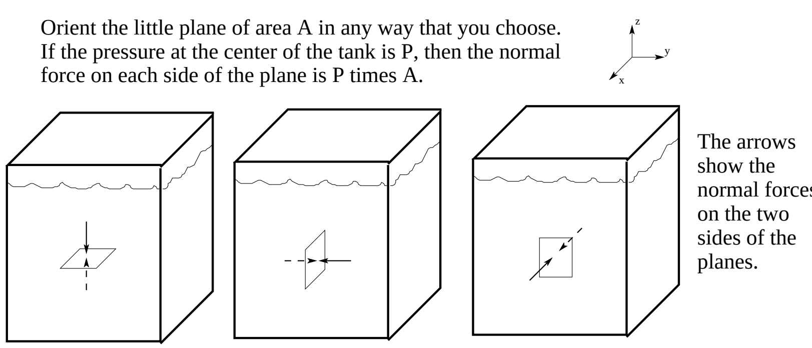
$$P \equiv \frac{F_N \text{ at } \mathbf{r}}{A} \qquad \text{units are N/m}^2 \equiv \text{ Pascals (Pa)}$$

where F_N is the magnitude of the normal force that would be exerted by the fluid on either side of a plane surface placed at $\mathbf{r}$, and

A is the area of that surface (assumed to be small).

Pressure is a positive-only scalar (NOT a vector).

My drawing which goes with this DEF is shown below (this one challenged my computer drawing skills). It shows a rectangular tank nearly filled with fluid. I will test the pressure with an imaginary, infinitely thin, plane surface of area A, assumed to be small. I have shown the plane oriented in three $\perp$ directions, but you may imagine the plane oriented in any way that you choose; it makes no difference. We can measure the magnitude of the normal force on either side of the plane; call it F_N. The pressure P at the location of the plane (remember the area is small, so that means the location of the center of the plane is not in question) is then given by $P \equiv F_N/A$.



As an example for pressure, let's consider the pressure of our atmosphere. Air is not really massless; a cubic meter of air at sea level has a mass of about 1.25 kg and thus a weight of about 12.3 N. A column of air with a cross-sectional area of 1.0 square meter which reaches from sea level to the top of our atmosphere weighs about 101,300 N. You can determine the air pressure at sea level by making a FBD for such a column of air and applying the 2nd Law.

you draw FBD

$$\Sigma F_y = 0$$

$$+F_N - F_G = 0$$

$$\Rightarrow \quad F_N = 101,300 \text{ N}$$

$$\Rightarrow \quad P \equiv \frac{F_N}{A}$$

$$= 101,300 \text{ N/m}^2$$

$$= 101,300 \text{ Pa}$$

This value of 101,300 Pa is thus the average pressure of the Earth's atmosphere at sea level and is assigned to a special unit, the "atmosphere" (atm); 1.0 atm ≡ 101,300 Pa. You can thus see that the normal force on the ground by a square meter of air is huge, over 100,000 N! In lecture, I will demonstrate the strength of this normal force.

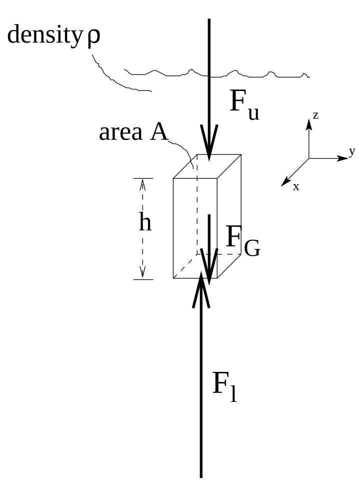
Now that we have the two basic DEFs, we can begin to learn some principles of fluid behavior. We will cover four main principles in our study of fluids, Pascal's Principle, Archimedes' Principle, the Equation of Continuity, and Bernoulli's Principle; the first two are principles for the behavior of STATIC fluids, and the second two are principles for the behavior of FLUIDS IN MOTION. Today's required reading will cover Pascal's Principle:

PASCAL'S PRINCIPLE: (massless fluid)

In a static massless fluid, the pressure is the same at all locations.

Of course, there is no such thing as a massless fluid, but many times it will be appropriate to ignore the mass of a fluid. For example, say you have a tank of compressed air. Since the air is "compressed", we expect that the pressure inside the tank is bigger than 101,300 Pa (1 atm). Even if the tank is a big tank, say 1.0 m^3, the weight of the air in the tank is likely to be 50 N or less, and thus would have an insignificant effect on the normal forces at the tank walls. For all practical purposes we can then consider the air to be "massless". So, in such a case, it makes sense to ask the question "What is the pressure in the tank?", and expect a single number as a reply. HOWEVER, if the mass of the fluid is significant, then the pressure in the fluid varies with depth. We will work out the variation with depth by using a FBD and the 2nd Law.

Consider a STATIC (not moving) massive fluid of density ρ. Draw an imaginary box around a column of the fluid that has cross-sectional area A and height h, and make a FBD for that column. In this case, I have drawn the FBD, but I have drawn only the z forces; please supply the x and y forces that I have left out. The normal force exerted on the upper surface of the column by the fluid above it is labeled $\mathbf{F}_u$, the normal force exerted on the lower surface of the column by the fluid below it is labeled $\mathbf{F}_l$, and the weight of the column of fluid is labeled $\mathbf{F}_G$. Let's use P_u for the pressure at the top of the column and P_l for the pressure at the bottom of the column; then by the DEF of pressure F_u = $P_u \cdot A$ and F_l = $P_l \cdot A$. We can get the mass of the column from the DEF of density, i.e. $m \equiv \rho V$ = $\rho A h$. Therefore,



$$\Sigma F_y = 0$$

$$+F_l - F_u - F_G = 0$$

$$\Rightarrow \qquad F_l = F_u + F_G$$

$$\Rightarrow \qquad P_l A = P_u A + (\rho A h) g$$

$$\Rightarrow \qquad P_l = P_u + \rho g h$$

This equation gives the variation of pressure with depth in a massive fluid and will appear on your equation sheet.

Here's a simple example. The pressure on top of the water in a fresh water lake at sea level is about 1.0 atm. At what depth is the pressure equal to 2.0 atm?

variation of pressure with depth $$P_l = P_u + \rho g h$$

P_u is 1.0 atm, P_l is 2.0 atm $$2.0\ \text{atm} = 1.0\ \text{atm} + \rho g h$$

$$\Rightarrow \quad \rho g h = 1.0\ \text{atm}$$

$$\Rightarrow \quad h = \frac{101,300\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 10.3\ \text{m}$$

So as a diver descends beneath the surface of fresh water, every 10.3 m of depth means an increase in pressure of 1.0 atm.

Now we are ready for the full version of Pascal's Principle:

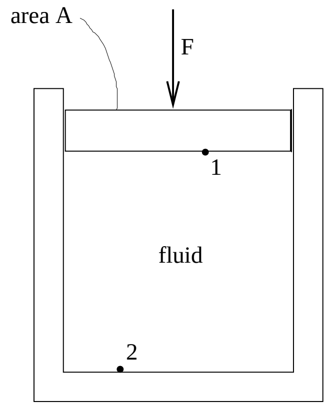
PASCAL'S PRINCIPLE

Pressure applied to an enclosed fluid at rest is transmitted undiminshed to every portion of the fluid and to the walls of the containing vessel.

In other words, given $P_l = P_u + \rho g h$, then any increase in P_u causes the same increase in P_l. This version of the principle is good for any static fluid, "massless" or not. We will finish this reading with a simple example illustrating Pascal's Principle.

Consider a pot with a moveable piston as drawn below. Our fluid is contained by the pot underneath the piston. The piston is completely "fluid-tight" (i.e. air-tight or water-tight, etc.) and can move without friction. The area of the piston is A and the mass of the piston is negligible. By applying force to the top of the piston (we can stack weights on it or simply push down on it) we can increase the pressure at any location within the fluid.

Let F stand for the magnitude of the total force pushing down on the piston. The points 1 and 2 simply indicate two locations within the fluid; let P_1 and P_2 be the pressures at those locations.



If we increase F by 100 N, then we increase P_1 by the amount (100 N/A). Pascal's principle says simply that we also increase P_2 by that same amount. If the fluid is massive, then P_2 is of course larger than P_1, because it is at a lower depth, but the INCREASE in both pressures is (100 N/A).

Finally, in the above example, will the piston move down when the pressure is increased? If the fluid is a GAS, then the answer to our question is "yes"; when the pressure is increased, the piston will move down and the gas will be compressed. Because pressure in a gas is sensitive to temperature, thermodynamics is required to calculate the amount of compression -- this comes later. If the fluid is a LIQUID, then the answer to our question is "no" (a better answer is "only a little bit"); liquids are practically incompressible -- a huge change in pressure is required for even a little compression. We will ignore any compressibility of a liquid; in our text, compressibility of liquids (Bulk Modulus) is discussed on page 285.

During lecture, I will explain the workings or two or three kinds of pressure gauges, and we will do an example using a hydraulic lift, which is an application of Pascal's Principle.