In lecture 20, we learned the definitions of DENSITY and of PRESSURE. In addition, we learned PASCAL'S PRINCIPLE for static fluids -- if an external force is used to apply pressure to an enclosed fluid at rest, the resulting increase in pressure applies to every portion of the fluid and to the walls of the containing vessel. We used Pascal's Principle, combined with the variation of pressure with depth in a massive static fluid, i.e. $P_l = P_u + \rho g h$, to do examples with a hydraulic lift. In this reading, you will learn another principle of static fluids, ARCHIMEDES' PRINCIPLE. Then, in lecture, I will explain two useful principles of fluids in motion, the EQUATION OF CONTINUITY and BERNOULLI'S PRINCIPLE.

ARCHIMEDES' PRINCIPLE deals with the BUOYANT FORCE exerted on an object by a fluid; we begin with the definition of buoyant force:

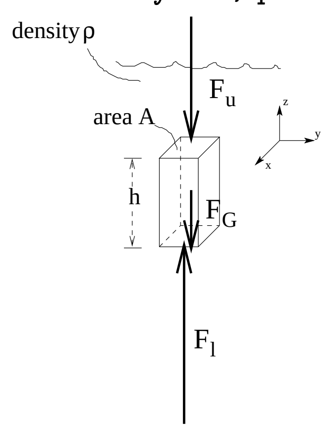
DEF For an object submerged, or partially submerged, in a fluid (in the presence of gravity), the net upward force on the object due to contact with the fluid is called the BUOYANT FORCE ($\mathbf{F}_B$).

The buoyant force is NOT a new force; it is simply the total contact force on the object by the fluid. If the object is submerged, then the fluid is of course pushing normally on every square mm of the object; i.e. the object is being pushed from all directions. Therefore, you might initially think that the net contact force on the object by the fluid must be zero. However, since all real fluids have mass, and since the pressure in a massive fluid always increases with depth, the forces on each little square mm must be bigger at the bottom of the object. Therefore, the net force on the object due to contact with the fluid must be upward. As an example using this DEF, we will prove ARCHIMEDES' PRINCIPLE, which is stated at the top of the next page.

ARCHIMEDES' PRINCIPLE

The magnitude of the buoyant force on an object which is submerged, or partially submerged, in a fluid is equal to W_{fl}, the weight of the fluid displaced by the object. In symbols, $F_B = W_{fl}$.

To prove ARCHIMEDES' PRINCIPLE, I will use the same drawing I used for explaining the variation of pressure with depth. However, there are important differences. This time the column is the submerged object; the column does NOT contain the fluid, but instead a different material or even some combination of materials. I have chosen the columnar shape to make our work easy, but in fact the object can have any shape at all. In the earlier case, $\Sigma F_y = 0$; but, in this case, we know nothing about the object's acceleration -- it may be upward, downward, or zero, but that won't matter for our work. Once again, I have omitted the x and y forces from my FBD; please supply them.



All of the x and y forces, as well as $\mathbf{F}_u$ and $\mathbf{F}_l$, are the contact forces on the object by the fluid; $\mathbf{F}_B$ is the sum of these six forces. The x and y forces all cancel, but F_l is bigger than F_u because the pressure is higher at the lower level. So, $F_B = F_l - F_u$. The weight of fluid displaced by the object is equal to the density OF THE FLUID times the volume OF THE OBJECT (because the object is entirely submerged) times the acceleration due to gravity; i.e. $W_{fl} = \rho(Ah)g$. We now start with the variation of pressure with depth and prove Archimedes' Principle:

pressure with depth		$P_l = P_u + \rho g h$
DEF of pressure, $P \equiv \dfrac{F_N}{A}$		$\dfrac{F_l}{A} = \dfrac{F_u}{A} + \rho g h$
algebra	$\Rightarrow$	$F_l = F_u + \rho g h A$
algebra	$\Rightarrow$	$F_l - F_u = \rho (A h) g$
from paragraph above	$\Rightarrow$	$F_B = W_{fl}$

I have three points of emphasis concerning Archimedes' Principle. As I wrote before, our result depends in no way on the shape of the object; I just chose this shape to make our work easy. Second, for a submerged object, the result is independent of the depth of the object -- for an object submerged in water, F_B is the same whether it is 1.0 m under the water or 10.0 m under the water. And third, for an object floating on the surface of a fluid, the volume of fluid displaced is NOT the volume of the object, but rather the volume of that part of the object which is below the surface.

Problems with Archimedes' Principle are straightforward applications of FBD's and Newton's 2nd Law; the only thing new is that $F_B = W_{fl}$. We will begin lecture 21 with an Archimedes' Principle example and demonstration. The demo is called the CARTESIAN DIVER; you can see a simulation of this demo at

http://lectureonline.cl.msu.edu/~mmp/applist/f/f.htm

The key to this demo is that the gas in the diver is compressible.

To test your understanding of Archimedes' Principle, as well as your understanding of the DEFs of density and pressure, and of the variation of pressure with depth and Pascal's Principle, please do Self-Assessment Test (SAT) 11.1 available at the "C&J 6th Ed Web Site" link from our class web page. This is one of the better SATs in our textbook, and should provide a good test of your understanding of Sections 1-6 in Chapter 11.