We will now begin to study motions for which the ACCELERATION is CONSTANT. If the acceleration never changes, then there can be no difference between the instantaneous acceleration and the average acceleration. In symbols,

$$\mathbf{a}(t) = \bar{\mathbf{a}} = \mathbf{a}$$

where the last $\mathbf{a}$ stands for a constant value.

The simplest case of CONSTANT ACCELERATION is the case for which $\mathbf{a}$ and $\mathbf{v}$ are colinear, i.e. $\mathbf{a}$ and $\mathbf{v}$ are either parallel or antiparallel. In this simplest case, the motion is ONE DIMENSIONAL (1D).

Since we will be studying a 1D motion, we will choose to use 1D-VECTOR NOTATION. In this notation, we will indicate direction by a $+$ or $-$ sign, instead of using N, S, E, W, up, down, etc. We can only do this because the motion is 1D, and you must always remember to indicate the meaning of $+$ and $-$ (for example, if $+$ stands for E, then $-$ stands for W). The following table contrasts the symbols of FULL-VECTOR NOTATION, which we have used up to now, with the symbols for 1D-vector notation.

	Full-Vector Notation	1D-Vector Notation
position	$\mathbf{r}$	x or y
displacement	$\Delta\mathbf{r}$	Δx or Δy
velocity	$\mathbf{v}$	v
acceleration	$\mathbf{a}$	a

Note that the 1D-vector notation DOES NOT require the use of a vector symbol or boldfacing. That makes for convenience, but it creates a problem. The symbol v, with no vector symbol, when used in full-vector notation, stands for SPEED, not velocity. But in 1D-vector notation, v stands for velocity. This often creates confusion, but it is common practice. So in 1D-vector notation, to indicate speed we must use $|v|$.

Our task is to write down the 1D-vector equations for CONSTANT ACCELERATION. The first one is simply the definition of average velocity, written in 1D-vector notation:

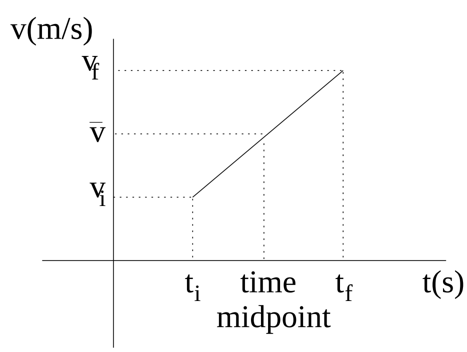
$$\bar{v} \equiv \frac{\Delta x}{\Delta t} \equiv \frac{x_f - x_i}{t_f - t_i} \tag{1}$$

The $(t_f - t_i)$ is usually just written as Δt. The second equation is another rewritten definition, the definition of (average) acceleration (I put the word "average" in parentheses because, for constant acceleration, there is no difference between "average" and "instantaneous"):

$$a \equiv \frac{\Delta v}{\Delta t} \equiv \frac{v_f - v_i}{\Delta t} \tag{2}$$

Note that I leave off the average bar because it is unnecessary for the case of constant acceleration.

We only need one more equation to solve any problem involving constant acceleration. This is a simple, and rather obvious, fact about average velocity for the case of constant a. Look at the following graph of v vs. t for a case in which the velocity is in the positive direction and the particle is speeding up:



Because the speed is increasing at a constant rate, it is easy to see from the graph that the average velocity $\bar{v}$ is halfway between the initial velocity v_i and the final velocity v_f. THIS WOULD NOT BE TRUE IF THE GRAPH OF v VS. t

WERE SOMETHING OTHER THAN A STRAIGHT LINE, i.e. if the acceleration were not constant. This simple fact gives us our third equation:

$$\bar{v} = \frac{v_i + v_f}{2} \tag{3}$$

Remember that Eq. (1) is the DEF of average velocity, and is always true. Our simple fact, Eq. (3), is only true for constant acceleration.

Look again at the graph of v vs. t shown above. At the beginning of the interval, t_i, the object's instantaneous velocity is v_i, and at the end of the interval, t_f, the object's instantaneous velocity is v_f. At the midpoint of the interval timewise (halfway between t_i and t_f), the value of the object's instantaneous velocity is equal to the object's average velocity for the interval. If you really understand the graph, this point is rather obvious, but it can be surprisingly useful when solving problems in constant acceleration. NOTE THAT THE MIDPOINT OF THE INTERVAL TIMEWISE DOES NOT MARK THE MIDPOINT OF THE INTERVAL SPACEWISE; the object must travel farther during the second half of the interval because it is traveling faster during that time.

Next, make sure that you understand the graph by drawing a similar graph for the case in which the velocity is in the negative direction and the speed is increasing. You will be asked to identify such a graph in one of the PRS questions for this reading.

We can use equations (1)-(3) to solve any and all problems involving constant acceleration. To check your understanding, try the following simple questions. If you understand the three equations, you should be able to do all of these questions in your head, without writing down any equations or use of a calculator. You can find the answers at the "Answers to Even-Numbered Problems" link on the class web page.

1. (a) If you accelerate from 0-6 m/s in 4 s, what is your average acceleration?

   (b) Assuming a is constant, what is your average velocity during the 4 s?

   (c) How far did you travel in the 4 s?

   (d) How fast were you traveling at t = 2 s?

   (e) How far had you traveled at t = 2 s? (HINT: What was your average velocity during the first 2 s?)

2. Starting from rest, I speed up by 5 m/s every second for 4 s.

   (a) How fast am I traveling at the end of the 4 s?

   (b) What was my acceleration during the 4 s?

3. A truck is traveling at 30 m/s. It decelerates at 3 m/s every sec. How long will it take to stop?

4. You are accelerating at 4 m/s^2. At t = 2 s you are traveling at 14 m/s.

   (a) How fast are you traveling 3 s later?

   (b) How far did you travel in these 3 sec? (HINT: What was your average velocity during those 3 s?)

Even though Eqs. (1)-(3) are sufficient to solve any problem involving constant acceleration, in some cases considerable algebra would be involved. We can save ourselves much work by using (1)-(3) to create two more equations which are frequently useful. I will do the work for the first one:

Solve (3) for v_f:

$$v_f = 2\bar{v} - v_i$$

Use (1) to plug in for $\bar{v}$:

$$v_f = 2\frac{\Delta x}{\Delta t} - v_i$$

Plug in for v_f in Eq. (2):

$$a = \frac{2\frac{\Delta x}{\Delta t} - v_i - v_i}{\Delta t}$$

Solve for Δx:

$$\Delta x = v_i \Delta t + \frac{1}{2}a(\Delta t)^2 \tag{4}$$

Now, limber up your algebra muscles by doing the work for the second useful equation.

Solve (1) for Δx:

$$\Delta x =$$

Use (3) to plug in for $\bar{v}$:

$$\Delta x = \qquad (i)$$

Solve (2) for Δt in terms of a and $v_f - v_i$:

$$\Delta t =$$

Plug this result for Δt into Eq. (i):

$$\Delta x =$$

Do all the multiplications on the Right-Hand side and then simplify:

$$\Delta x =$$

Your result should easily transform into this equation:

$$v_f^2 - v_i^2 = 2a\Delta x \qquad (5)$$

Eqs. (4) and (5) are often useful, and will appear on the Equation Sheet which accompanies your exam. You should find them valuable in solving your HW problems. You have to know the two definitions and the simple fact (Eqs (1)-(3)).

In writing these 5 equations, it is customary to let $t_i \rightarrow 0$ and $t_f \rightarrow t$, so that $\Delta t \rightarrow t$. With $t_i = 0$, it is then usual to let $x_i \rightarrow x_0$ and $v_i \rightarrow v_0$. And with $t_f = t$, it is usual to let $x_f \rightarrow x(t)$ or just plain x, and $v_f \rightarrow v(t)$ or just v. With these assumptions, the five equations look like this:

$$x = x_0 + \bar{v}t \qquad (1)$$

$$v = v_0 + at \qquad (2)$$

$$\bar{v} = \tfrac{1}{2}(v_0 + v) \qquad (3)$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad (4)$$

$$v^2 - v_0^2 = 2a(x - x_0) \qquad (5)$$

I like the original style better, but I will use both versions. Our book further assumes that $x_0 = 0$.

NOTE: If you have gone through this carefully, it should not be necessary to read Sections 2.4 and 2.5 of the text.