We will now continue our study of CONSTANT ACCELERATION by looking at an important case for which **a** and **v** are NOT COLINEAR. Since the acceleration can now be "sideways" to the velocity, the object can now change direction and move out of 1D.

The case we will examine most closely is the case of FREE FALL in 2D, also known as PROJECTILE MOTION. We will assume that the Earth is flat and we will ignore air resistance. The contant acceleration for this case is

$$\mathbf{a} = 9.8 \text{ m/s}^2 \quad \text{DOWN}$$

Our two dimensions are VERTICAL and HORIZONTAL. We'll use x for horizontal and y for vertical. What we have is two 1D problems, only connected by the elapsed time, which is the same for both dimensions. In other words, THE x MOTION HAS NO EFFECT ON THE y MOTION. If you don't believe this, I will do a demonstration during lecture that I hope will convince you.

The x MOTION is dreadfully simple. Since the acceleration is DOWN, it has no x component, and since $a_x = 0$, v_x is constant. So there is no difference between the average x velocity and the instantaneous x velocity. The only equation we have is the DEF of average velocity (and we don't need the average bar):

$$v_x \equiv \frac{\Delta x}{\Delta t} \tag{1x}$$

The y MOTION is constant acceleration in 1D. The acceleration is $a_y = 9.8$ m/s^2 DOWN. YOU choose whether DOWN is $+$ or $-$. Our equations for the y motion are the EQUATIONS OF KINEMATICS FOR CONSTANT ACCELERATION IN 1D:

$$\bar{v}_y \equiv \frac{\Delta y}{\Delta t} \tag{1y}$$

$$a_y \equiv \frac{\Delta v_y}{\Delta t} \equiv \frac{v_{yf} - v_{yi}}{\Delta t} \tag{2y}$$

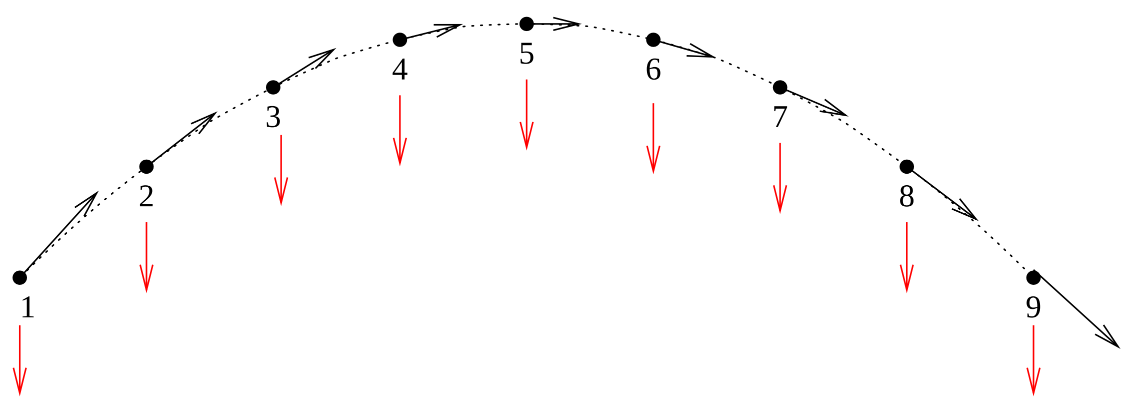
$$\bar{v}_y = \frac{v_{yi} + v_{yf}}{2} \tag{3y}$$

$$\Delta y = v_{yi}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2 \tag{4y}$$

$$v_{yf}^2 - v_{yi}^2 = 2a_y\Delta y \tag{5y}$$

Some of the variables now have double subscripts. The order of these subscripts does not matter. For example, you can write v_{yi} or v_{iy}; both mean the initial y velocity. Any vector answers that you need can always be gotten from the component answers, i.e. $\Delta\mathbf{x}$ + $\Delta\mathbf{y}$ = $\Delta\mathbf{r}$, and $\mathbf{v}_x$ + $\mathbf{v}_y$ = $\mathbf{v}$. Which means that you will need the vector analysis skills you learned in Chapter 1 (for example, $|\mathbf{v}|$ = $\sqrt{v_x^2 + v_y^2}$).

Let's look at a simple example. Imagine a ball fired from ground level with v_y = 19.6 m/s UP and v_x = 10 m/s EAST. Here is a motion diagram of the resulting motion, assuming there is no hill or valley at the landing point. I have arbitrarily chosen to use nine points in my motion diagram.



I have drawn a dotted line through the points of the motion diagram. This line traces the path of the ball, and is called the TRAJECTORY; it is also the graph of Δy vs Δx. Since Δy is parabolic in time (by Eq. (4y)), and since Δt = $\Delta x/v_x$ (by Eq. (1x)), we know that the trajectory is a parabola. I have drawn the velocity vectors in black and the acceleration vectors in red. The acceleration is of course the same for all 9 points, 9.8 m/s^2 DOWN. The speed (the length of the velocity vectors) is greatest at points 1 and 9, and smallest at the top (point 5). Recall that the ball was fired with v_y = 19.6 m/s UP and v_x = 10 m/s EAST, and check your understanding by answering the following questions. You should be able to answer all six questions without the use of a calculator and without writing down any equations.

1. What is the speed at the top of the motion?
2. What is the velocity at the top of the motion?
3. How long does it take to go up? (HINT: What is the change in y velocity during the first half of the motion?)
4. How long does the whole trip take?
5. How far away from the launch point does it land?
6. How far up does it go? (HINT: What is the average y velocity for the up trip?)

The answers can be found at the ANSWERS TO EVEN-NUMBERED PROBLEMS link on the course web page.

Finish up by doing Check Your Understanding (CYU) 3 on C&J page 65, and Self-Assessment Test (SAT) 3.1 at the Student Companion Site for the C&J 6th Edition available at

www.wiley.com/college/cutnell