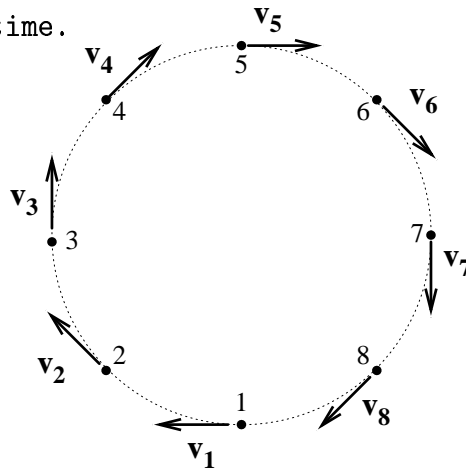


We will now study a very important case for which the ACCELERATION is NOT CONSTANT. This is the case of

UNIFORM CIRCULAR MOTION

(UCM)

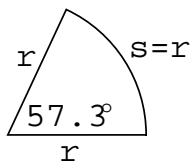
UCM is just circular motion at constant speed. A motion diagram for a particle (or object) in UCM is drawn below. I have chosen to draw eight dots in this case, so that the time to go around the circle once is divided up into eight equal intervals of time.



From the way I have chosen to number the dots, you can see that this particle is going around in the CLOCKWISE (CW) direction. For a particle going around in the COUNTERCLOCKWISE (CCW) direction, the numbering would go around the other way. I have arbitrarily chosen to put dot #1 at the bottom of the circle; you could choose to start the motion anywhere you like. All eight velocity vectors have the same length, indicating that the SPEED never changes. But, of course, the direction is changing constantly, so the VELOCITY is not constant, but is always changing. Therefore, there is always an acceleration, which we are going to figure out in lecture. For now, I haven't drawn in the acceleration vectors for each dot. Your task in this required reading is to learn the variables and units we will use to describe CIRCULAR MOTION (both UCM and non-UCM, which will be introduced in Chapter 8).

We will use r for the radius of the circle, and we will use s for the distance traveled along the circular arc. Both r and s are in units of m, and both are positive-only scalars. Since the object is going around a circle, it makes sense to talk about the object's ANGULAR DISPLACEMENT (though we can still ask questions about displacement Δr as well). For the ANGULAR DISPLACEMENT, we will use the symbol $\Delta\theta$, and we will measure in units of radians. For now, we will let $\Delta\theta$ be a positive-only scalar for convenience (but we will make it a 1D vector in Chapter 8).

I will stop the introduction of variables now to say a few words about the radian unit. One radian, by definition, is just that fraction of a circle for which $s = r$. As shown in the drawing below, one radian turns out to be equal to an angle of about 57.3° . If we redrew the picture with an angle of two radians, the arclength would then be $2r$. In other words, the arclength can always be gotten by multiplying the angle in radians times the radius. The little table below shows the arclengths for a variety of angles in radians.



1 rad

$\Delta\theta$	2	π	2π
s	$2r$	πr	$2\pi r$

You can see from the table that radian measure just COUNTS the number of radii in s . So radians is a DIMENSIONLESS unit; since $\Delta\theta \cdot r = s$, therefore $\text{rad} \cdot \text{m} = \text{m}$. You must remember this when you are treating units as algebraic quantities.

Now that you understand the radian unit, we can make a formal definition of the angular displacement.

DEF The ANGULAR DISPLACEMENT ($\Delta\theta$) for a particle undergoing circular motion is the arclength traveled divided by the radius of the circular path. In symbols,

$$\Delta\theta \equiv \frac{s}{r}$$

Angular displacement is a positive-only scalar (for now). The units are radians (rad).

Back to the introduction of variables. One revolution (rev) or one cycle means once around the circle. Therefore

$$1 \text{ rev} = 2\pi \text{ rad} \quad \text{and} \quad \Rightarrow \frac{2\pi \text{ rad}}{\text{rev}} \quad \text{is a useful conversion factor.}$$

The PERIOD (T) is just the time to go around once in units of s/rev or just s (since rev, like rad, is DIMENSIONLESS). The FREQUENCY (f) is the number of revolutions per second (rev/s or just s^{-1}). So $f = (1/T)$.

There are two more variables, one that you already know, followed by a new definition. As usual, v is the speed. Since the speed is constant for UCM,

$$v = \bar{u} \equiv \frac{d}{\Delta t} = \frac{s}{\Delta t}$$

So, for once around the circle

$$v = \frac{2\pi r}{T}$$

DEF The ANGULAR SPEED (ω) for a particle undergoing circular motion is

$$\omega \equiv \frac{\Delta\theta}{\Delta t}$$

ω is a positive-only scalar (for now -- in Chapter 8 we will call it angular velocity and change it to a 1D vector). The units are rad/s or just s^{-1} . The symbol " ω " is the Greek letter omega -- please don't call it double-u! (NOTE: This definition is technically for the average angular speed, but we don't need that distinction now because the angular speed, like the SPEED v , is constant.)

The relationship between ω and f is a bit unusual. Both measure the same thing, the rate at which the particle is going around the circle. Only the units are different, ω is in rad/s while f is in rev/s. So changing from ω to f is just a matter of converting, with the conversion factor 2π rad/rev. We don't usually use two different symbols in such a case; for example whether I measure the speed in m/s or km/h, I still use the symbol v . But it is useful to have two different symbols in this case, because both rad and rev are dimensionless units; so the two symbols help us to keep track of the difference.

So how do we change the angular speed ω into the speed v ? Just put in the DEF of $\Delta\theta$, namely $\Delta\theta \equiv \frac{s}{r}$.

$$\omega \equiv \frac{\Delta\theta}{\Delta t} \Rightarrow \omega = \frac{s}{r\Delta t}$$

and since

$$v = \frac{s}{\Delta t} \Rightarrow \omega = \frac{v}{r}$$

This last equation will appear on your equation sheet.

To check your understanding of the variables we have defined, try the questions below. The answers are available at the ANSWERS TO EVEN-NUMBERED PROBLEMS web page. Try not to look back at the reading.

Answer the following questions if the frequency is 5 rev/s and the radius is 2.0 m:

1. What is the angular speed?
2. What is the speed?
3. What is the period?
4. What is the angular displacement in 3 sec?
5. What arclength is covered in 3 sec?

During lecture, we will figure out an equation that will give us the acceleration of the object.