

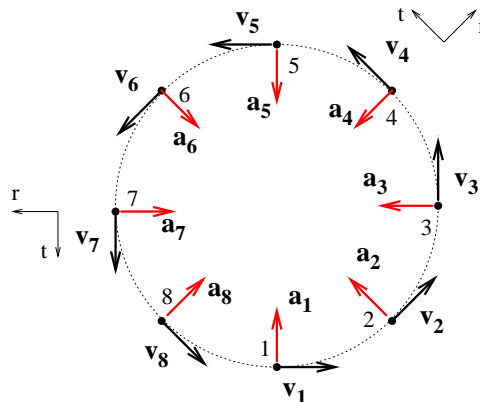
In Chapter 5, you will learn the "right way" (which for us means the Newtonian way) of looking at the

DYNAMICS OF CIRCULAR MOTION

Before we begin to discuss dynamics, be sure that you recall the variables of circular motion by reviewing the required reading for lecture 5. The important result that we got in lecture that day was that an object in Uniform Circular Motion (UCM) is always being accelerated towards the center of the circle. Once you are confident in your knowledge of the kinematics of circular motion, you are ready to consider the dynamics.

When we studied elevators in lecture 8, we learned that accelerated frames of reference are NOT VALID for the application of Newton's Laws. That idea is the key to looking at the dynamics of circular motion in the "right way". Anything going in a circle is being accelerated towards the center. Therefore, you must always put yourself in the position of the stationary observer in order to apply Newton's Laws; if you put yourself in the frame of reference of the object going around the circle, either you will get wrong answers, or you will have to invent imaginary forces. Now that you have the basic idea in mind, let's get down to the details.

Consider an object in UCM with speed v and going around in the CCW direction. The motion diagram for the object is drawn below.



For this motion, v is constant but \mathbf{v} is not constant, and a is constant but \mathbf{a} is not constant. The vector quantities are not constant because the direction keeps changing. If the radius of the circle is r , then the magnitude of \mathbf{a} is v^2/r and \mathbf{a} always points toward the center of the circle. The acceleration vectors are drawn in red.

We want to define a new coordinate system which is useful for discussing circular motion problems. Define the radial (r) direction as being along a radius, with AWAY FROM CENTER usually chosen as the positive r direction. Define the tangential (t) direction as being tangent to the circle, with CCW usually chosen as the positive t direction. This radial-and-tangential (rt) coordinate system is drawn for points 4 and 7 in my motion diagram. You can see that the rt coordinate system is quite different from the xy coordinate system in one important way; the xy coordinate system doesn't change orientation as the particle moves, while the rt coordinate system rotates as the particle goes around the circle. (You should be able to draw the rt coordinate system for any of the points in my motion diagram.) But other than this difference, the two systems work in the same way -- we will divide problems into r and t parts instead of dividing them into x and y parts. For UCM, the acceleration of the moving object always points in the centripetal direction (towards the center); in our new rt coordinate system, we would say that the acceleration always points in the negative r direction (just another way of saying "towards the center").

Now for the Dynamics. As always, there are only two kinds of forces, gravity and contact forces. Our method for solving dynamics problems is to first make a FBD and then to "write the 2nd Law". For a particle in UCM, writing the r part of the 2nd Law will always look like

$$\begin{aligned}
 r \text{ part : } \Sigma F_r &= ma_r \\
 \Sigma F_r &= m\left(-\frac{v^2}{r}\right)
 \end{aligned}
 \tag{1}$$

There may be any number of gravitational and/or contact forces in the ΣF_r sum, except there must always be at least one force because ΣF_r cannot be zero. I have put the minus sign in front of $\frac{v^2}{r}$ because I have assumed that the positive radial direction is AWAY FROM THE CENTER. Writing the t part of the 2nd Law will always look like

$$\begin{aligned}
 t \text{ part : } \Sigma F_t &= ma_t \\
 \Sigma F_t &= 0
 \end{aligned}$$

In this case, there may be no tangential forces, or else whatever tangential forces do exist add up to zero, because there is no tangential acceleration (i.e. the particle is not speeding up as it goes around the circle). Remember, this result is for UCM; if the speed of the particle is changing, then ΣF_t is not necessarily equal to zero.

One thing to watch out for when doing problems with forces and circular motion is the symbol T . T can stand for tension if a string or a rope is involved, but it can also stand for the period, i.e. the time to go around once. You must choose the correct meaning from the context in which the symbol is used.

Finally, a point of special emphasis. The quantity $\frac{mv^2}{r}$ is often called the "centripetal force". I hesitate to do this, because the use of this name is a point of confusion for many students. Students often think that the centripetal force is some new kind of force. IT IS NOT. There are only two kinds of forces, gravity and contact forces, and this won't change until you learn about electromagnetic forces in PHY 112. The centripetal force is not a kind of force at all, no more than the "net force" is a kind of force.

The correct idea about centripetal force is expressed by equation (1) in this required reading. The quantity $\frac{mv^2}{r}$ is the magnitude of the net radial force on an object of mass m moving in a circle of radius r with speed v . The minus sign in equation (1) tells us that this net radial force points in the minus r direction, i.e. the centripetal direction. Therefore, the "centripetal force" is in fact just a net force; it is the net radial force acting on the object going around our circle. Our book uses the symbol F_C for the magnitude of the centripetal force. No symbol is really necessary, because the centripetal force, being a net force, NEVER GOES ON A FBD -- the centripetal force is always the result of adding together whatever radial forces DO appear in your FBD. However, if you ever do need a symbol for the magnitude of the centripetal force, I strongly suggest that you use $|\Sigma F_r|$ instead. This will always remind you that the centripetal force is a net force and not a kind of force.

In lecture, we will do many examples involving the dynamics of circular motion. After lecture, but before starting your HW for this lecture, I suggest that you go through Self-Assessment Tests 5.1 and 5.2 available at the "C&J 6th Ed Web Site" link on our course web page. These questions will check your understanding of the concepts involved in the dynamics of circular motion.