In this exercise, we will use Excel (or something similar) to compute the motion of a mass (treated as a particle) moving in one dimension and subject to a net force which is NOT constant. If we are able to compute the net force (either based on the elapsed time, or else on the particle's position), then the 2nd Law will provide the acceleration. If we choose an appropriately small Δt (during which the net force is almost constant), we can calculate the change in velocity and position for that tiny Δt. We then update the net force, and calculate the change in velocity and position for the next Δt. In the limit that $\Delta t \to 0$, this method must give us the correct motion of our particle.

As a first example, assume that the net force on the mass is always directed towards the origin, with a magnitude proportional to distance from the origin, *i.e.* $\Sigma F_x = -Kx$, where K is a proportionality constant in units of N/m. For any time t, the 2nd Law then gives $a_x(t) = -(K/m)x(t)$. We will begin with the mass at rest and located somewhere away from the origin, and then compute and graph the subsequent motion.

1. Open a blank spreadsheet. The parameters for our calculation will be Δt, K, and m; enter these parameter names in cells H1-3 respectively (*i.e.* enter deltaT= in cell H1, etc). In cells I1-3, enter our selected values for these parameters. Begin with $\Delta t = 0.1$ s, $K = 4$ N/m, and $m = 1.0$ kg (enter only the numbers, not the units). You will have the freedom to change these parameters in the completed spreadsheet.
2. Make the column for time. Enter the label t in cell A1. In cell A2, enter a zero (our initial value of time). In cell A3, enter a formula which will be applied to all subsequent times, namely that the next time value is equal to the previous time value plus Δt; so enter =A2+I1 as the formula for cell A3 (the dollar signs indicate that the cell I1 is the fixed location of the value of Δt), then click in cell A3 and drag the dot at the lower right-hand corner of the cell down to cell A100.
3. Now make the columns for x, v_x, and a_x. Enter the appropriate labels in cells B1, C1, and D1, respectively, then enter the initial conditions in the second row cells for x and v_x, namely that at time zero the mass is at location $x = 1$ m with zero x velocity. For a_x, use a formula, namely $= -(K/m)x$ (in the formula bar for cell D2, enter $= (-1) * (\$I\$2/\$I\$3)*$B2, and then apply to cells D2-D100).
4. We next need formulas for x and v_x for each subsequent time. For infinitesimal Δt's, the correct formula for x would be $x(t + \Delta t) = x(t) + v_x(t) * \Delta t$. For larger values of Δt (such as 0.1 s), our calculation is improved by using an estimate of the average velocity during the time interval t to $t + \Delta t$, namely $v_{x,avg} = v_x(t) + a_x(t) * 0.5 * \Delta t$; enter this formula in cell C3 (*i.e.* =C2+D2*0.5*I1). In cell C3, we now have an estimated average velocity for the interval from t=0 to t=0.1 s instead of the computed x-velocity at time t=0.1 s. Alternatively, we can think of

this entry as the estimated x-velocity at time t=0.05 s. Therefore, for the remainder of our velocity column, we will use $v_x(t + 0.5\Delta t) = v_x(t - 0.5\Delta t) + a_x(t) * \Delta t$ (notice that, in this formula, we are using $a_x(t)$ as an estimate of the average value of a_x in the interval from $t - 0.5\Delta t$ to $t + 0.5\Delta t$). Enter this formula in cell C4 (*i.e.* =C3+D3*I1) and apply it to cells C4-C100. Finally, we can enter a formula for $x(t)$; in cell B3, enter =B2+C3*I1, and apply to cells B3-B100.

5. Check your work. If everything is correct, you should have the following results in row 100: t=9.8 s, x=0.708618 m, v_x=−1.26239 m/s, and a_x=−2.83447 m/s^2.
6. Graph your results. Choose INSERT Chart > Scatter with Smooth Lines and Markers, then right click on the chart and Select Data. Remove any data automatically placed on the chart by Excel, then Add data. Click on cell B1 for the Series name, then select cells A2-A100 (time) for Series X values, and cells B2-B100 (position) for Series Y values (for the Y values, you must first delete anything automatically placed there by Excel). Now add a second curve to your graph. Right click to Select Data > Add data then click on cell D1 for the Series name, once again select cells A2-A100 for Series X values, and this time select cells D2-D100 (a_x) for Series Y values. Enable Legend under Chart Elements to label the curves.
7. Your graph should show a simple oscillatory motion for the mass, with slightly more than three cycles in the 9.8-s duration of our calculation. You should observe that a_x is always in the opposite direction of x, as is required by $a_x = -(K/m)x$.

Congratulations. You now have a spreadsheet that can be used to determine the motion of particles moving in 1D and subject to varying forces (as, for example, in the HW2 problem entitled "1D Motion with Variable Acceleration"). You can add extra parameters if needed; for example, you could add a parameter for a non-zero initial x-velocity. At our next recitation, we will expand the spreadsheet to handle 2D motions.

8. Try out other possible values for the parameters. For example, it is easy to verify that the amplitude of the acceleration depends **linearly** on the value of K. What about frequency? Determine the frequency of the oscillations for K values of 1.0, 4.0, and 16.0 N/m. How does the frequency depend on the value of K? Be specific.

9. In Column F, add a cosine function of time, with $\sqrt{K/m}$ as the conversion factor between time in seconds and angle in radians. Put a label in cell F1, and put an appropriate formula in cell F2; apply that formula to cells F2-F100. Add this cosine function to the graph; for several cycles of the motion, you should find little difference between your graph of $x(t)$ and the graph of the cosine function.

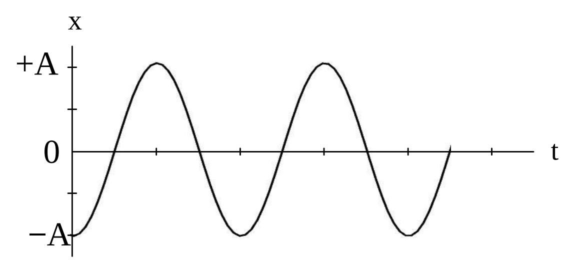
10. Follow the steps below to verify that the function $x(t) = Acos(\omega t + \phi)$ is, in fact, a solution of the differential equation $-Cx(t) = a_x(t)$, provided that $\omega = \sqrt{C}$.

a. Write the first time derivative of $x(t)$ (i.e. the x-velocity equation).

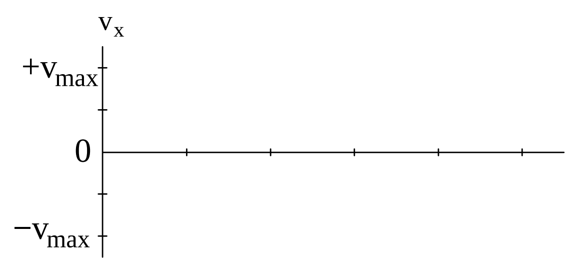
b. Write the second time derivative of $x(t)$ (i.e. the x-acceleration equation).

c. Plug the position and acceleration equations into the differential equation, and simplify as much as possible. Is the claim correct?

11. The position-versus-time graph shown is for a particle in simple harmonic motion. Assume $x(t)$ is written in terms of a cosine function with a phase constant. Each tick on the time axis is one-half a period.

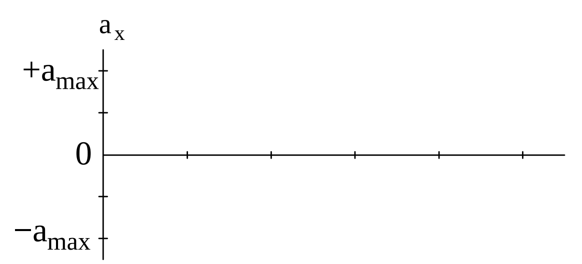


a. What is the phase constant ϕ? Choose a number with the smallest possible absolute value. Explain how you determined it.



b. Draw the corresponding velocity and acceleration graphs.

c. When x is less than zero, is a_x ever less than zero? If so, at which points in the cycle?



d. Can you make a general conclusion about the relationship between the signs of x and of a_x?

e. When x is greater than zero, is v_x ever greater than zero? Describe the motion of the oscillator while x is greater than zero.