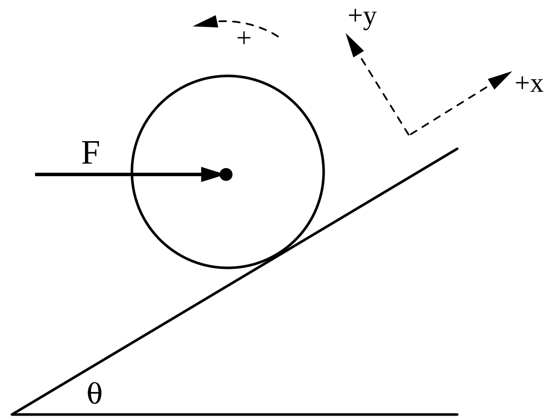
1. You are given a snapshot of a wheel on a ramp. You know nothing about the motion, except that, if the wheel is moving, it is rolling without slipping. There is a horizontal pushing force of known magnitude F acting on the axle of the wheel. Also, you know the angle θ, and the wheel mass M and radius R. Finally, you know that the moment of inertia of the wheel, about its axle, is cMR^2, with the value of c known.



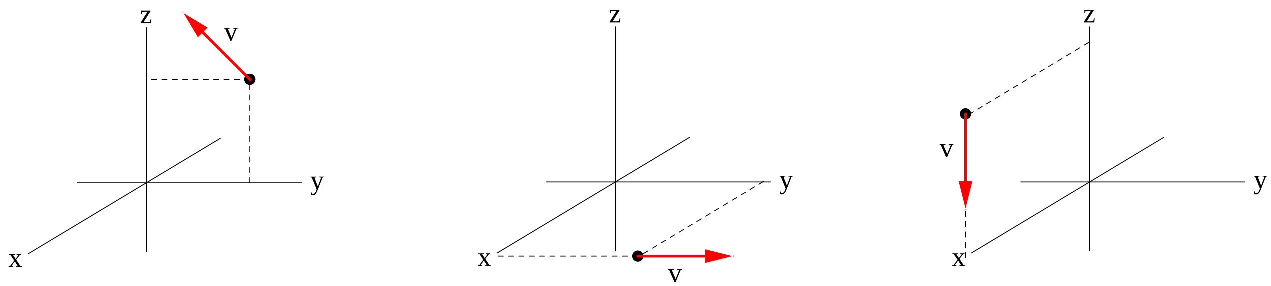
a. On the drawing, complete an extended free-body diagram for the wheel. For the purpose of your diagram, assume that friction points in the $+x$ direction. Write Newton's 2nd Law for the $+x$ direction; use f_x for the x component of friction and a_x for the x component of accleration, keeping in mind that components may either be positive or negative.

b. Write the rotational analog of Newton's 2nd Law for the wheel, with CCW as positive. Use f_x for the x component of friction and α_z for the z component of angular accleration, keeping in mind that components may either be positive or negative.

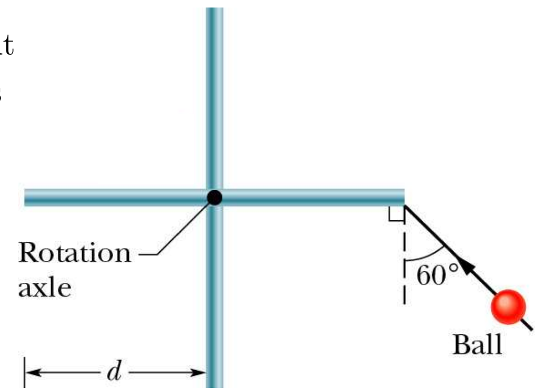
c. With these choices, the relation between α_z and a_x is $\alpha_z = -a_x/R$. (Why is the minus sign needed?) Solve three equations in three unknowns (symbolically) for f_x in terms of only known values. From your result, what will determine if friction actually points up or down the ramp in this case?

d. Assume that $\theta = 30°$, $M = 4.0$ kg, $R = 0.5$ m, $F = 20$ N, and $c = 1/2$. Find f_x and a_x. Do your results for the direction and size of friction and acceleration make sense? What can you say about the motion of the wheel now?

2. The fully correct definition of angular momentum is $\vec{L} \equiv \vec{r} \times \vec{p}$. The figures below show a particle with velocity $\vec{v}$. For each case, draw the arrow for the position vector of the particle (from the origin) in one color, and then draw the arrow for the particle's angular momentum about the origin in a second color. Place the tail of the arrow for $\vec{L}$ at the origin; draw it long and straight so that its direction is clear.



3. The figure shows a top view of a turnstile which is about to be struck by a ball of mud. The mud ball has a mass of 1.2 kg and is traveling at 8.0 m/s along the path shown, which may be assumed horizontal. By each of three methods, find the magnitude of the angular momentum of the mud ball, about the rotation axle of the turnstile, at the instant before the ball makes contact. The distance $d = 0.5$ m.



a. Use $L = rpsin(\phi)$. What is the angle ϕ between $\vec{r}$ and $\vec{p}$ at the instant just before collision?

b. Use $L = |p_\theta|r$. On the figure, draw the perpendicular-to-radial component of the momentum vector just before collision, and label it with its value.

c. Use $L = p\ell$, where ℓ is the lever arm. Draw the lever arm on the figure, and label it with its value. Was it necessary to specify the instant at which to calculate L?

d. Suppose the turnstile is locked in place, so that when the mud ball strikes the end of the arm, it simply hits and sticks, without moving the turnstile at all. What will have been the angular impulse on the mud ball about the axle as a result of the collision? HINT: Analogous to $\int \Sigma F_x dt = \Delta p_x$, the impulse-momentum theorem for impulsive torques is $\int \Sigma \tau_z dt = \Delta L_z$.