# Definitions for Current and Resistance

Here is a reasonable definition of current for beginning students:

DEF: The (AVERAGE) CURRENT ($I_{(av)}$ or $i$) through any area A is

In Symbols: $$I_{(av)} \equiv \frac{\Delta Q}{\Delta t}$$ in Units of C/s $\equiv$ Amperes (A)

where $\Delta Q$ is the absolute value of the net charge that flows through area $A$ in time $\Delta t$. The "average" is usually understood. For this definition, current is a positive-only scalar; it is most useful for steady currents (i.e. currents that don't change with time). If currents can change with time (as in AC circuits), we need another definition:

DEF: The INSTANTANEOUS CURRENT ($I(t)$) is the limit as $\Delta t$ goes to zero of the average current.

In Symbols: $$I(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$ in Units of C/s $\equiv$ Amperes (A)

But these two definitions don't quite agree, because in this definition $Q$, and $I(t)$, must be signed scalars, so $\Delta Q$ cannot be an absolute value (we are doing calculus). To understand the source of the sign in $I(t)$, you need to see the true definition of current, which begins with CURRENT DENSITY.

DEF: The CURRENT DENSITY ($\vec{J}(\vec{r})$) at location $\vec{r}$ is the time rate of charge flow per unit area through an infinitesimal area, at right angles to the flow, located at $\vec{r}$; the direction of current density is the direction of positive charge flow. $\vec{J}(\vec{r})$ has units of A/m$^2$. Current density is a VECTOR.

Now we can finally give a precise definition of current.

DEF: The CURRENT ($I_A$) THROUGH AREA $A$ is:

In Symbols: $$I_A \equiv \int_A \vec{J}(\vec{r}) \cdot d\vec{A}$$ in Units of C/s $\equiv$ Amperes (A)

*i.e.*, the current is the FLUX of the current density. Thus, current is a signed scalar, but the only significance of the sign is OUR CHOICE of direction for the $d\vec{A}$ vectors. If the angle between $\vec{J}$ and $d\vec{A}$ is bigger than 90°, then $I_A$ is negative. **IF** (and only if) $\vec{J}(\vec{r})$ is constant over some area $A$ (at a right angle to the flow), then the magnitude of $\vec{J}$ is simple:

$$|\vec{J}| = \frac{|I|}{A}$$ in Units of A/m$^2$

## RESISTANCE

For an object with a constant resistance, the definition is rather simple:

The RESISTANCE ($R$) between any two points on a conducting object is

$$R = \frac{V}{I} \qquad \text{in Units of V/A} \equiv \text{Ohms } (\Omega)$$

where $V$ is the absolute value of the applied potential difference (the voltage) between those two points, and

$I$ is the absolute value of the current that results from this applied potential difference.

However, if the resistance of the object depends on the amount of current (this is true for all objects to some degree), then the definition becomes:

$$R(I) = \lim_{\Delta I \to 0} \frac{\Delta V}{\Delta I} = \frac{dV}{dI} \qquad \text{in Units of V/A} \equiv \text{Ohms } (\Omega)$$

where $\Delta V$ is the small change in voltage which produces the small change in current $\Delta I$, when a current $I$ is already passing between the two points on the object.

Both definitions assume the absence of induced electric fields within the conducting object. If induced electric fields are present, then the equation $V$=$IR$ is not generally true; for this reason the resistance definitions are not written with the usual three-bar equal sign.

## CONDUCTIVITY

The CONDUCTIVITY ($\sigma$) of a material is the ratio of the magnitude of the current density at some point in the material to the electric field strength that produces that current density.

In Symbols: $$\sigma = \frac{J}{E} \qquad \text{in Units of (A/m}^2\text{)/(V/m) = 1/}\Omega\cdot\text{m}$$

or in vector form: $$\vec{J} = \sigma \vec{E} \qquad (1)$$

## RESISTIVITY

The RESISTIVITY ($\rho$) of a material is the reciprocal of the conductivity.

In Symbols: $$\rho \equiv \frac{1}{\sigma} \qquad \text{in Units of } \Omega\cdot\text{m}$$

which allows us to write (1) as $\vec{E} = \rho \vec{J}$.