CSE 591: Theoretical Aspects of CPS

Exact System Relationships
Reference Textbook Ch 4.2 & 4.3

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Announcements

- Do not forget to take a look at HW0
  - If you feel comfortable that you know all the concepts, skip it
  - Otherwise, try to do it
- Project proposal deadline is approaching (17 Feb)
  - You must submit up to 2 page proposal
    - A small intro to the problem
      - Why it is interesting/challenging
      - Why it is related to CPS
      - What you are going to do and when
      - What you are going to deliver
  - Some project ideas from other courses: http://courses.ece.illinois.edu/ece598/sm/2008Fall/schedule.shtml

In the previous class

- Review of notation and definitions:
  - Alphabet: A non-empty finite set
    - Usually denoted by Σ (or Z in the textbook), eg Z = {0,1}, Z = {a,b,c}
  - Symbol: a member of the alphabet
    - Eg. 0 or 1 from Z = {0,1}
  - (finite or infinite) string: A (finite or infinite) sequence of symbols
    - Empty string: ε
    - Eg. If Z = {0,1}, then
      - Finite strings: 10, 0101, 1010111
      - Infinite strings: 11111…, 101010…
  - Concatenation: If α, β are strings, then the concatenation of α and β is written as αβ.
    - Eg. If α=11 and β=00, then αβ=1100
  - Language: A set a strings built over an alphabet

In the previous class

- Review of notation and definitions:
  - Star operator: $L^* = \{x_1x_2...x_k \mid k \geq 0 \text{ and each } x_i \in L\}$
    - Eg. $L = \{a,b,aa,aaa,...\}$, $L^* = \{aa,ab,ba,aaa,aab,aba,...\}$
  - Omega operator: $L^\omega = \{x_1x_2x_3... \mid \text{each } x_i \in L\}$
    - Eg. $L = \{a\}$, $L^\omega = \{aaa...\}$
  - If $L \subseteq Z^\omega$, then the set of all finite prefixes is
    - $\text{Fin } L = \{u \in Z^* \mid \exists v \in Z^\omega\ uv \in L\}$
  - If $L \subseteq Z^*$, then the limit of L is
    - $\text{Lim } L = \{u \in Z^\omega \mid \forall k \geq 0. u(0,k) \in L\}$
    - where
      - $u(m,n) = u_m u_{m+1} ... u_n$
      - $u(m,\omega) = u_m u_{m+1} ...$
      - $u(0,0) = \varepsilon$
    - Note: usually in the literature Lim L represents the set of infinite words that have infinitely many prefixes in the set L

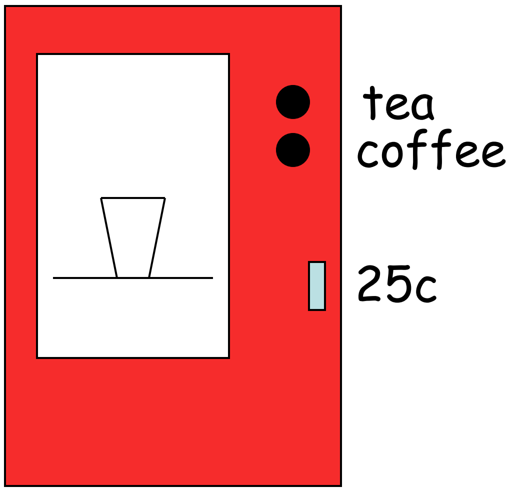
In the previous class

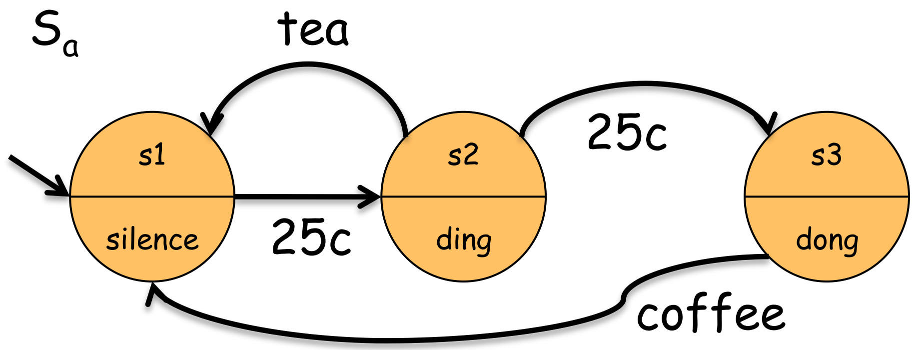
- Behavioral relationships
  - Given S, S':
    - how are B(S) and B(S') related?
    - how are Reach(S) and Reach(S') related?
- Similarity relationships
  - Let S_a, S_b with $Y_a=Y_b$. A relation $R \subseteq X_a x X_b$ is a **simulation relation** from S_a to S_b if
    1. $\forall x_{a0} \in X_{a0} \,.\, \exists x_{b0} \in X_{b0} \,.\, (x_{a0}, x_{b0}) \in R$
    2. $\forall (x_a, x_b) \in R \,.\, H_a(x_a) = H_b(x_b)$
    3. $\forall (x_a, x_b) \in R \,.$

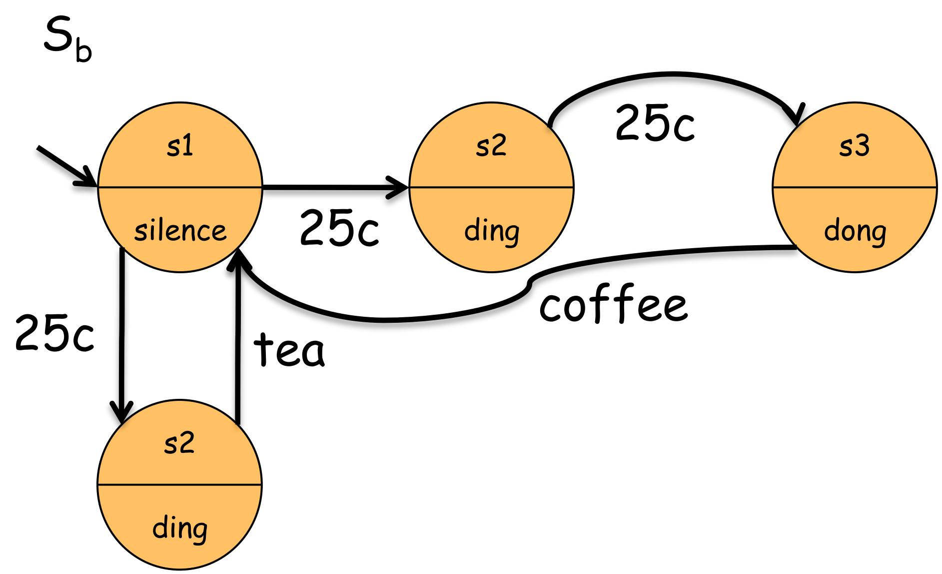
$$x_a \xrightarrow[a]{u_a} x_a' \quad \text{implies} \quad x_b \xrightarrow[b]{u_b} x_b' \quad \text{satisfying } (x_a', x_b') \in R$$

  - Given S_a, S_b with $Y_a=Y_b$, we say that S_a is simulated by S_b or that S_b simulates S_a, **written $S_a \preceq_S S_b$**, if there exists a simulation relation from S_a to S_b.

Vending Machine Example

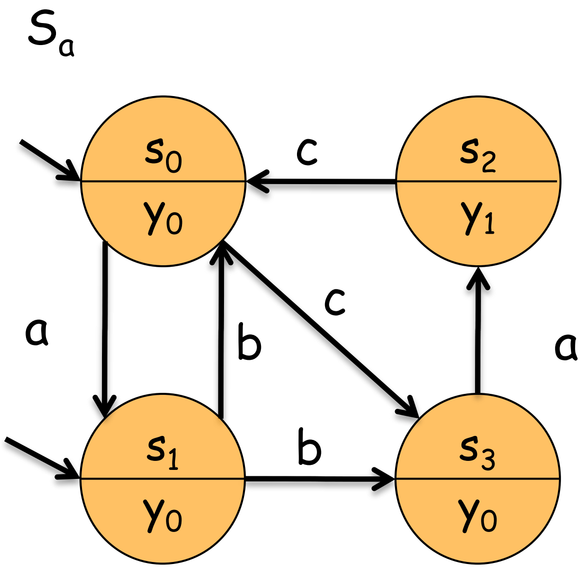


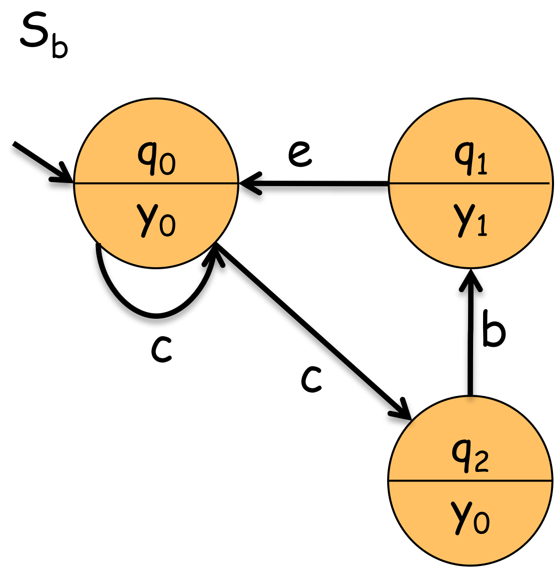




Example 4.15

Bisimulation $R = \{(s_0,q_0), (s_1,q_0), (s_2,q_1), (s_3,q_2)\}$





Bisimulation

- Def. Given S_a, S_b with $Y_a=Y_b$ we say that S_a is **bisimilar** to S_b, **denoted $S_a \cong_S S_b$**, if there exists a relation R satisfying:
  1. R is a simulation relation from S_a to S_b
  2. R^{-1} is a simulation relation from S_b to S_a

- Let S_a, S_b with $Y_a=Y_b$. A relation $R \subseteq X_a x X_b$ is a **bisimulation relation** between S_a and S_b if
  1. $\forall x_{a0} \in X_{a0} \,.\, \exists x_{b0} \in X_{b0} \,.\, (x_{a0}, x_{b0}) \in R$
  2. $\forall x_{b0} \in X_{b0} \,.\, \exists x_{a0} \in X_{a0} \,.\, (x_{a0}, x_{b0}) \in R$
  3. $\forall (x_a, x_b) \in R \,.\, H_a(x_a) = H_b(x_b)$
  4. $\forall (x_a, x_b) \in R \,.$

$$x_a \xrightarrow[a]{u_a} x_a' \text{ implies } x_b \xrightarrow[b]{u_b} x_b' \text{ satisfying } (x_a', x_b') \in R$$

$$x_b \xrightarrow[b]{u_b} x_b' \text{ implies } x_a \xrightarrow[a]{u_a} x_a' \text{ satisfying } (x_a', x_b') \in R$$

Alternating Similarity Relationships

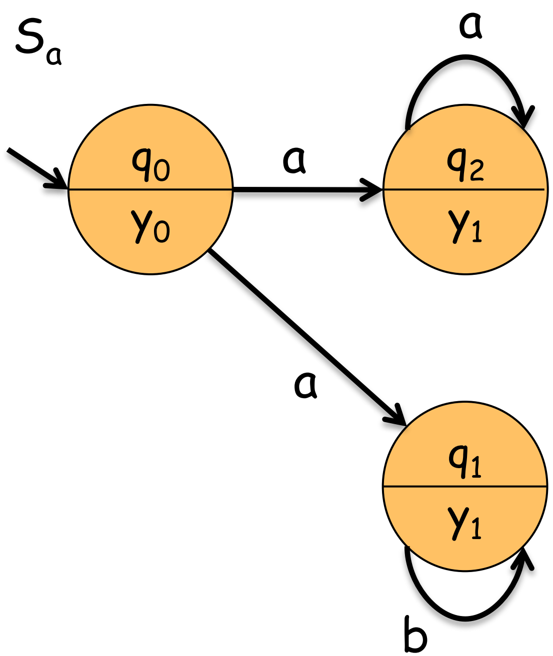
- Def. Let S_a, S_b with $Y_a=Y_b$. A relation $R \subseteq X_a x X_b$ is an **alternating simulation relation** from S_a to S_b if
  1. $\forall x_{a0} \in X_{a0} \,.\, \exists x_{b0} \in X_{b0} \,.\, (x_{a0}, x_{b0}) \in R$
  2. $\forall (x_a, x_b) \in R \,.\, H_a(x_a) = H_b(x_b)$
  3. $\forall (x_a, x_b) \in R \,.$
     $\forall u_a \in U_a(x_a) \,.\, \exists u_b \in U_b(x_b) \,.$
     $\forall x_b' \in Post_{ub}(x_b) \,.\, \exists x_a' \in Post_{ua}(x_a) \,.\, (x_a', x_b') \in R$

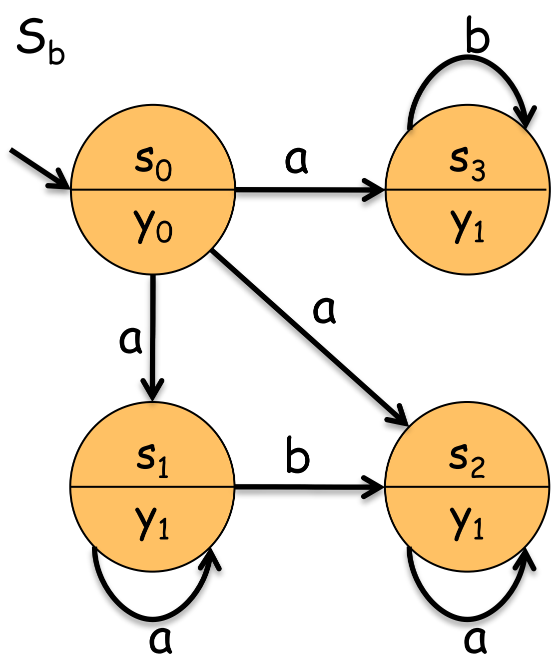
- Given S_a, S_b with $Y_a=Y_b$, we say that S_a is alternatingly simulated by S_b or that S_b alternatingly simulates S_a, **written $S_a \preceq_{AS} S_b$**, if there exists an alternating simulation relation from S_a to S_b.

Example 4.21

Simulation R = $\{(q_0,s_0), (q_1,s_1), (q_2,s_2)\}$

Alternating Simulation R' = $\{(q_0,s_0), (q_1,s_1), (q_1,s_2), (q_1,s_3)\}$





Alternating Similarity Relationships

- Def. Let R be an alternating simulation relation between S_a and S_b. The **extended alternating simulation relation** $R^e \subseteq X_a \times X_b \times U_a \times U_b$ associated with R is defined by all $(x_a, x_b, u_a, u_b) \in X_a \times X_b \times U_a \times U_b$ such that
  1. $(x_a, x_b) \in R$
  2. $u_a \in U_a(x_a)$
  3. $u_b \in U_b(x_b)$ and $\forall x_b' \in Post_{ub}(x_b) \,.\, \exists x_a' \in Post_{ua}(x_a) \,.\, (x_a', x_b') \in R$

- Def. Given S_a, S_b with $Y_a = Y_b$ we say that S_a is **alternatingly bisimilar** to S_b, **denoted $S_a \cong_{AS} S_b$**, if there exists a relation R satisfying:
  1. R is an alternating simulation relation from S_a to S_b
  2. R^{-1} is an alternating simulation relation from S_b to S_a