CSE 591: Theoretical Aspects of CPS

Timed automata
References:
Tabuada Ch 7.2, Cassandras & Lafortune Ch 5.6

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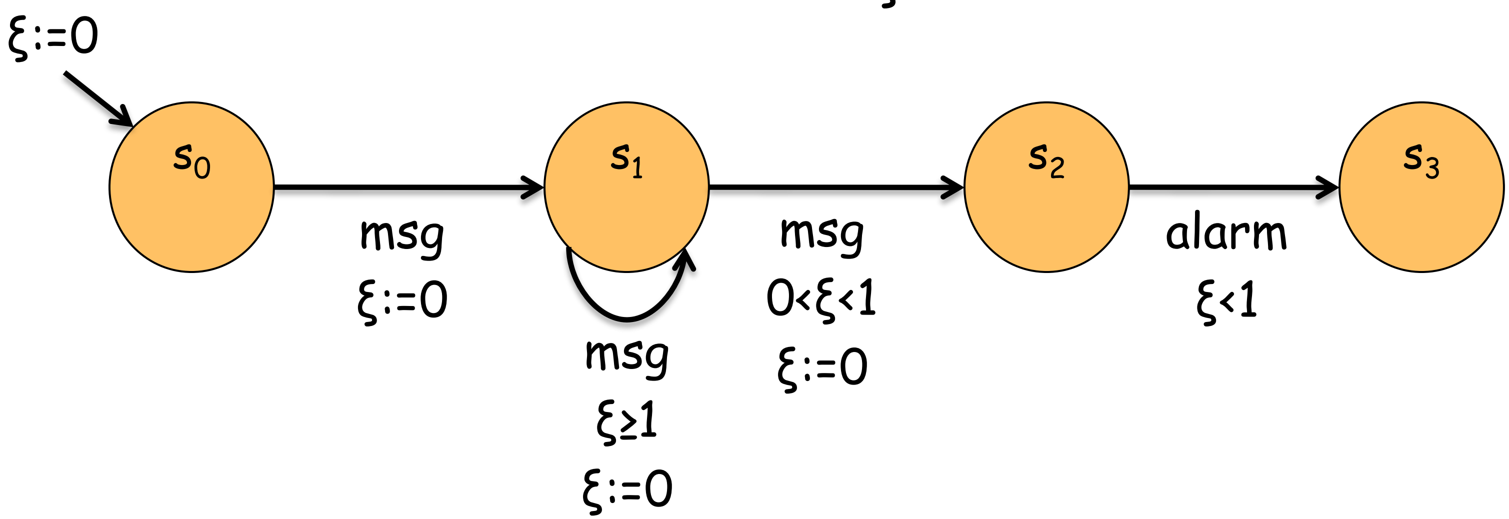
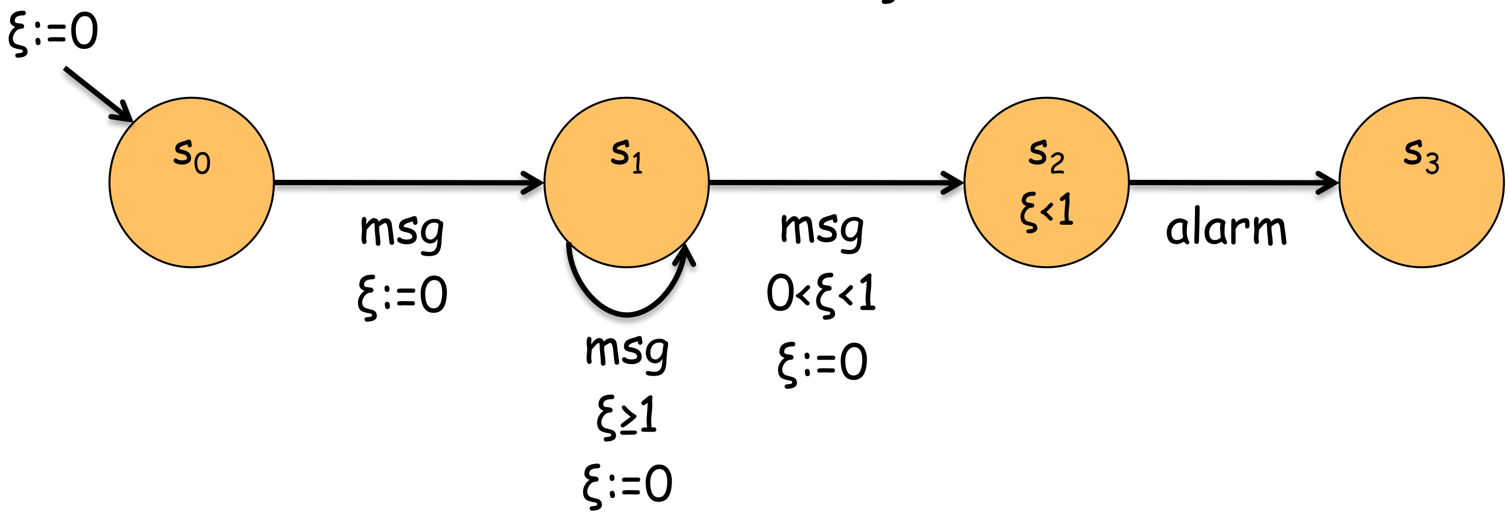
Example: Alarm 1

one clock: $d\xi/dt = 1$

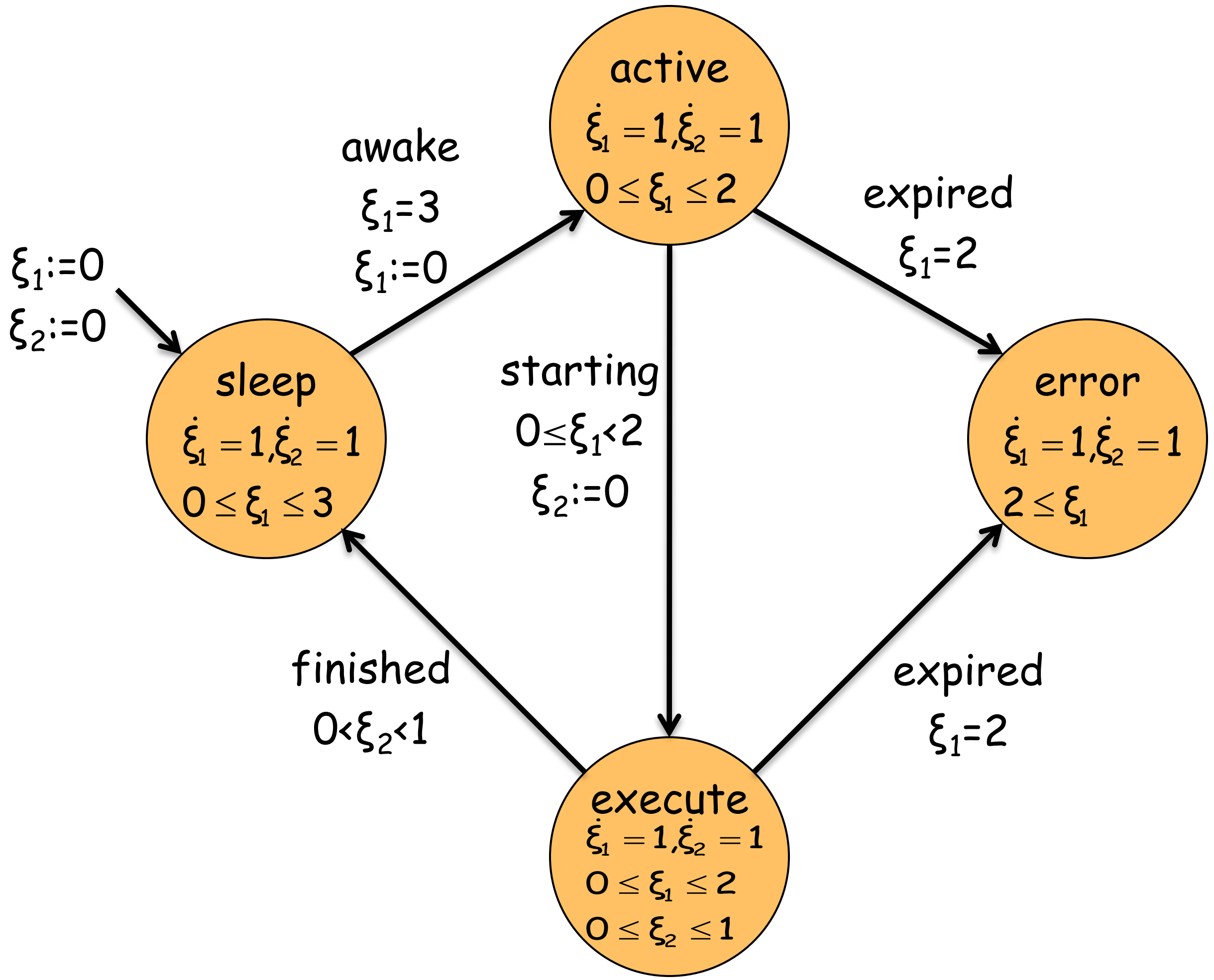


Example: Alarm 2

one clock: $d\xi/dt = 1$



Example: Scheduler



Question

- Subset sum
  - Model the subset sum problem as a reachability problem for a timed automaton
  - We are given 4 positive integers $K = \{k_1,k_2,k_3,k_4\}$ and a sum K_{sum}
  - Does $\Sigma k_i=K_{sum}$?

Review: Quotient System

- Let $S=\{X, X_0, U, \rightarrow, Y, H)$ be a system and Q be an equivalence relation on X where $(x,x')\in Q$ implies $H(x)=H(x')$. The quotient of S by Q - denoted $S_{/Q}$ - is the system $S_{/Q}=\{X_{/Q}, X_{/Q0}, U, \rightarrow_{/Q}, Y, H_{/Q})$, where
  1. $X_{/Q}=X/Q$
  2. $X_{/Q0}=\{x_{/Q}\in X_{/Q} \mid x_{/Q}\cap X_0\neq\emptyset\}$
  3. $(x_{/Q},u,x'_{/Q})\in\rightarrow_{/Q}$ if $\exists(x,u,x')\in\rightarrow$ with $x\in x_{/Q}$ and $x'\in x'_{/Q}$
  4. $H_{/Q}(x_{/Q})=H(x)$ for some $x\in x_{/Q}$

- **Theorem:** Let $S=\{X, X_0, U, \rightarrow, Y, H)$ be a system and Q be an equivalence relation on X where $(x,x')\in Q$ implies $H(x)=H(x')$. The relation
  $\Gamma(\pi_Q)=\{(x,x_{/Q})\in X\text{x}X_{/Q} \mid x_{/Q}=\pi_Q(x)\}$
  is a simulation relation form S to $S_{/Q}$.
- $\Gamma(\pi_Q)$ is a bisimulation relation between S and $S_{/Q}$ iff Q is bisimulation between S and S.

Review: Bisimulation

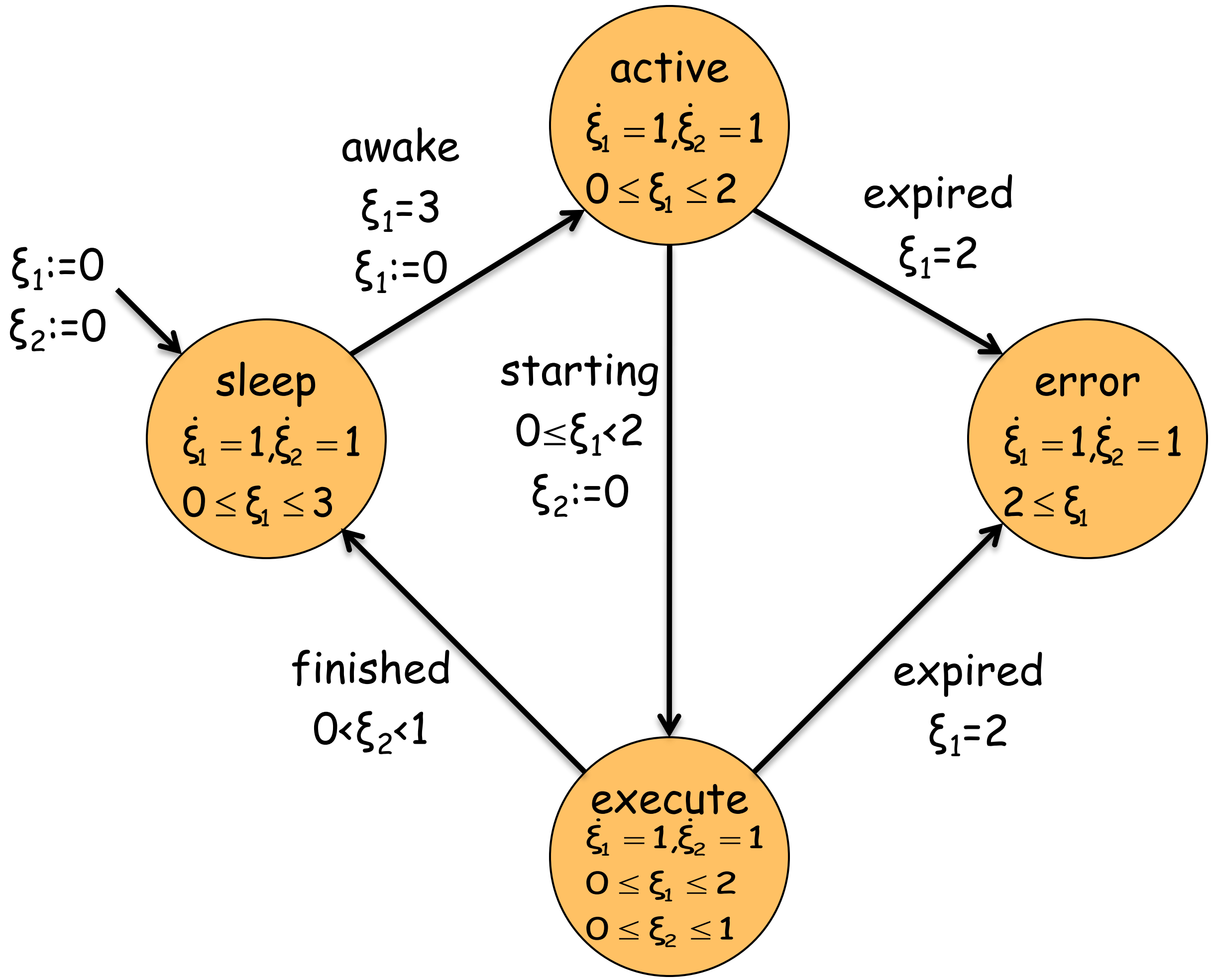
- Def. Given S_a, S_b with Y_a=Y_b we say that S_a is **bisimilar** to S_b, **denoted $S_a \cong_S S_b$**, if there exists a relation R satisfying:
  1. R is a simulation relation from S_a to S_b
  2. R^{-1} is a simulation relation from S_b to S_a

- Let S_a, S_b with Y_a=Y_b. A relation $R \subseteq X_a x X_b$ is a **bisimulation relation** between S_a and S_b if
  1. $\forall x_{a0} \in X_{a0} \,.\, \exists x_{b0} \in X_{b0} \,.\, (x_{a0}, x_{b0}) \in R$
  2. $\forall x_{b0} \in X_{b0} \,.\, \exists x_{a0} \in X_{a0} \,.\, (x_{a0}, x_{b0}) \in R$
  3. $\forall (x_a, x_b) \in R \,.\, H_a(x_a) = H_b(x_b)$
  4. $\forall (x_a, x_b) \in R \,.$

$$x_a \xrightarrow[a]{u_a} x_a' \text{ implies } x_b \xrightarrow[b]{u_b} x_b' \text{ satisfying } (x_a', x_b') \in R$$

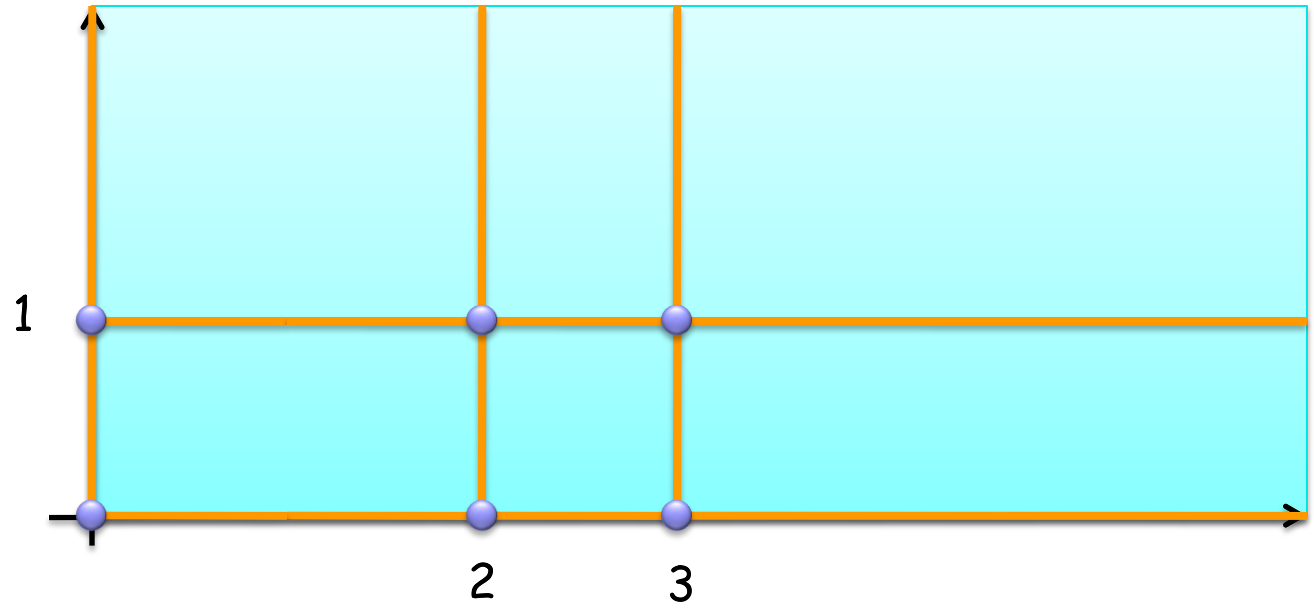
$$x_b \xrightarrow[b]{u_b} x_b' \text{ implies } x_a \xrightarrow[a]{u_a} x_a' \text{ satisfying } (x_a', x_b') \in R$$

Example: Scheduler

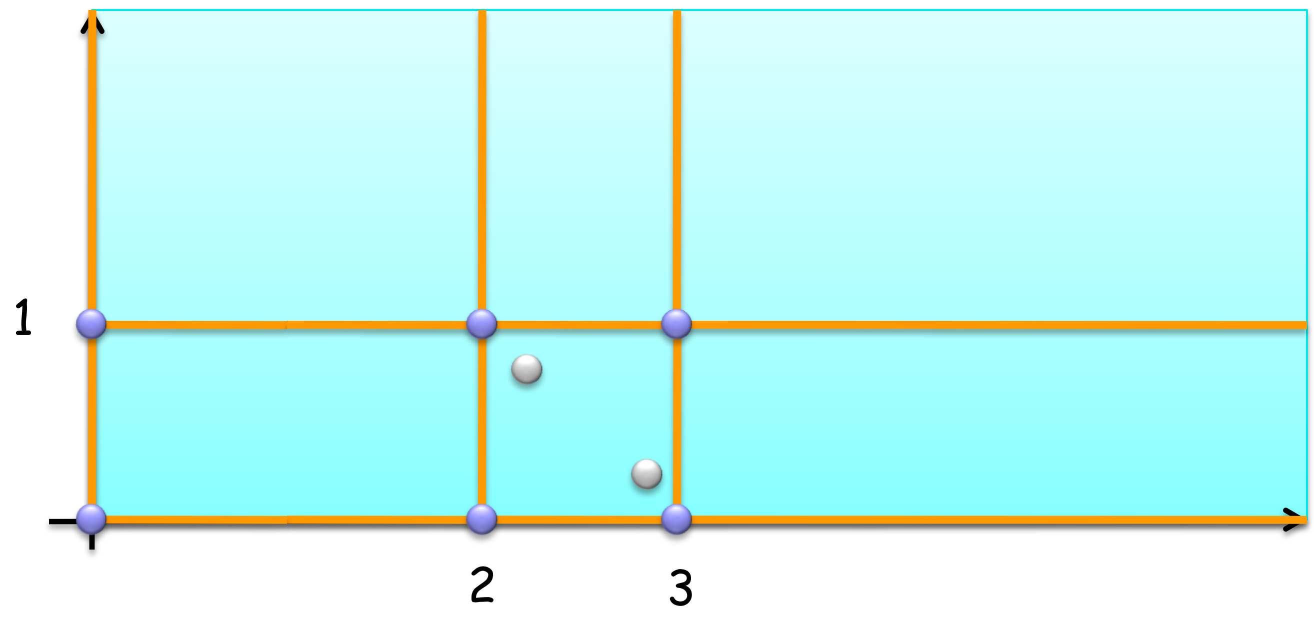


2D Continuous state space

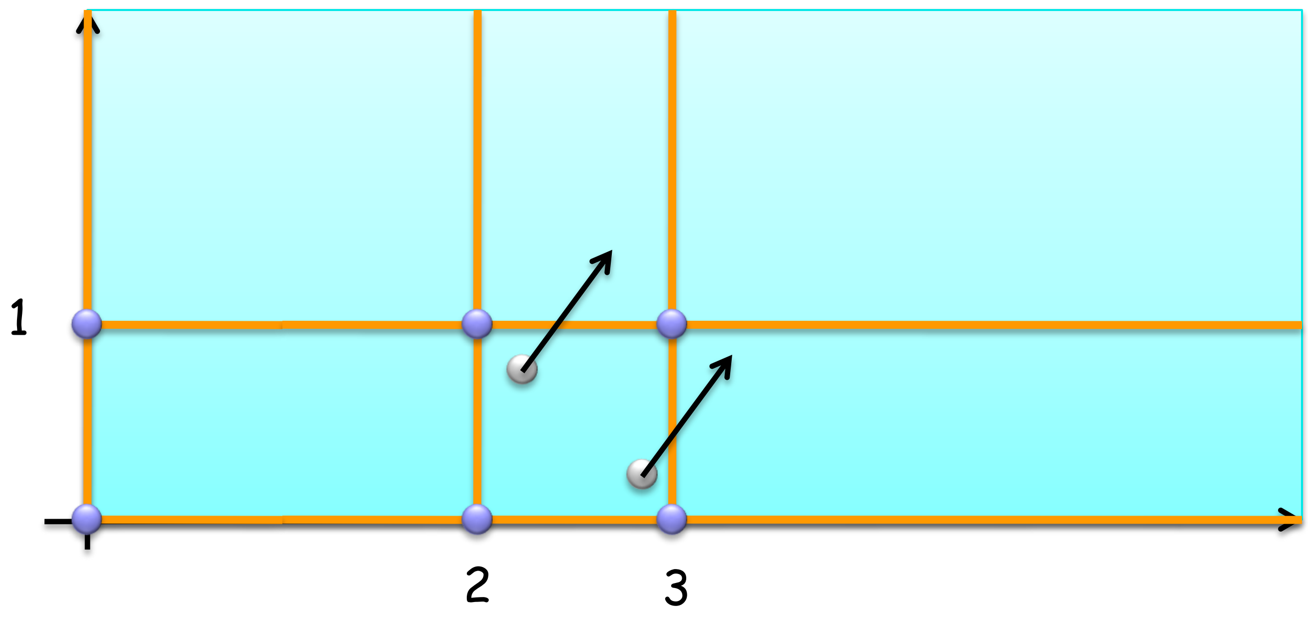
Equivalence classes Q



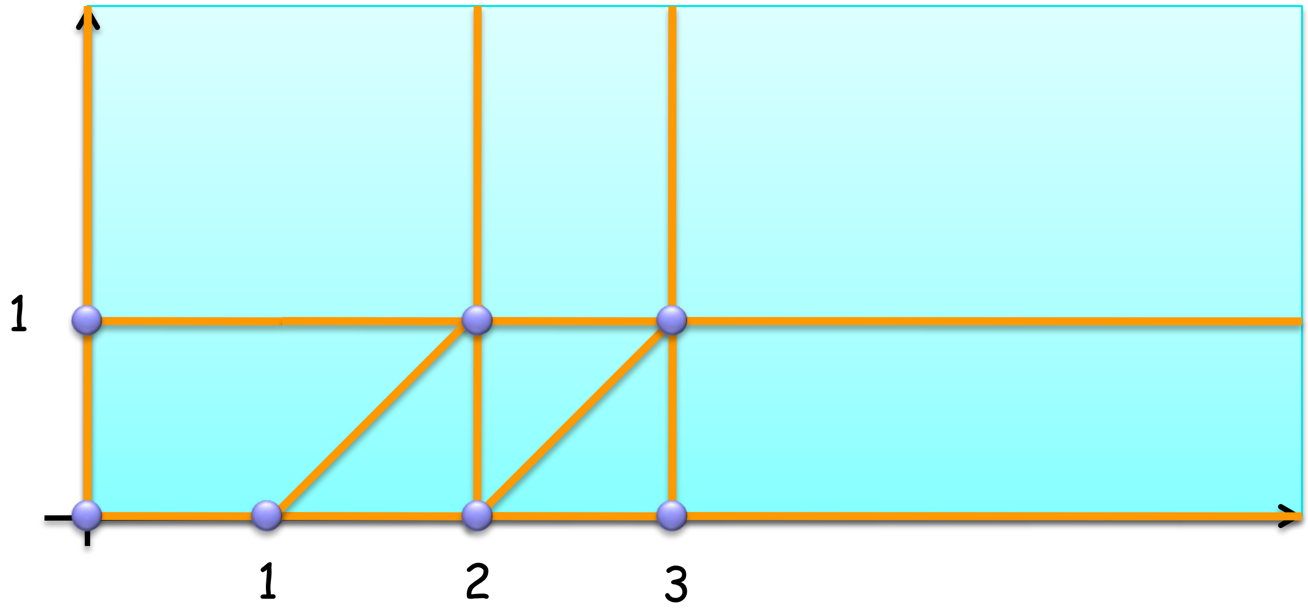
Equivalence classes Q



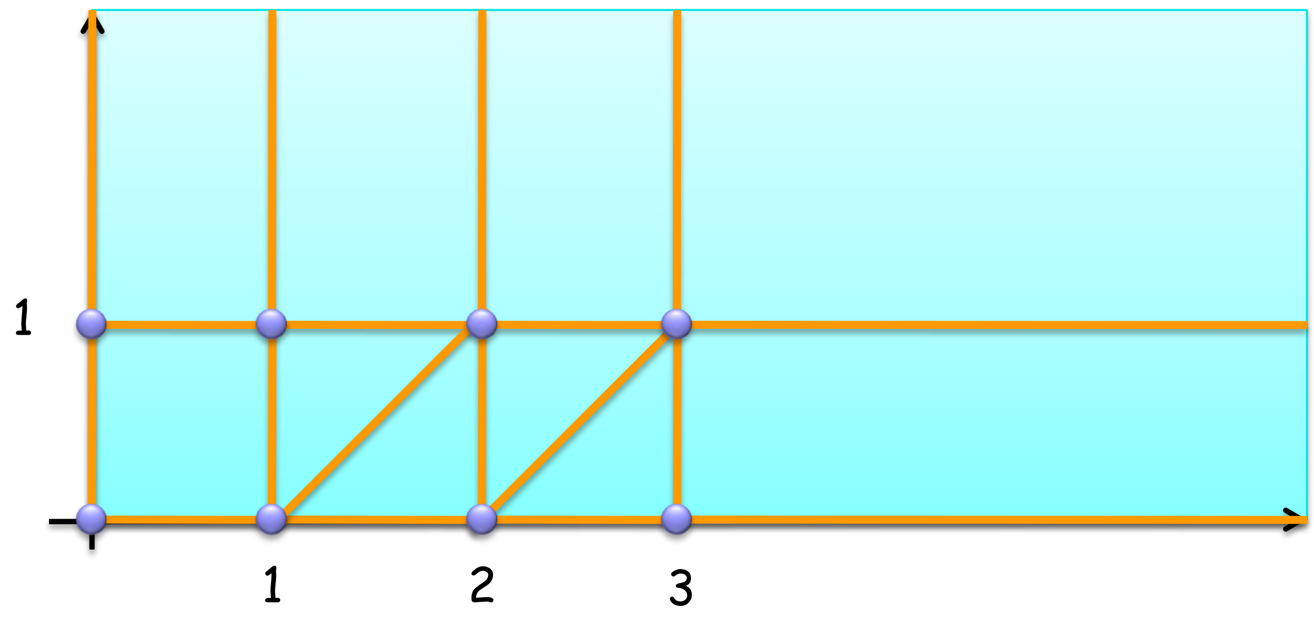
Equivalence classes Q



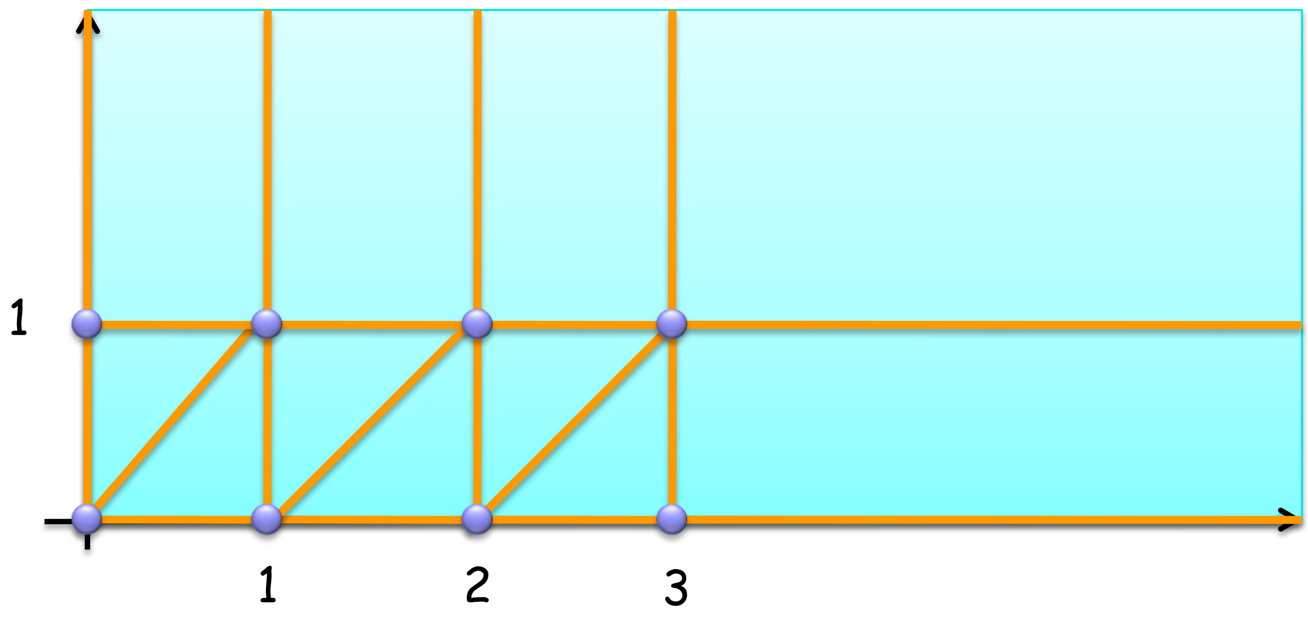
Equivalence classes Q'



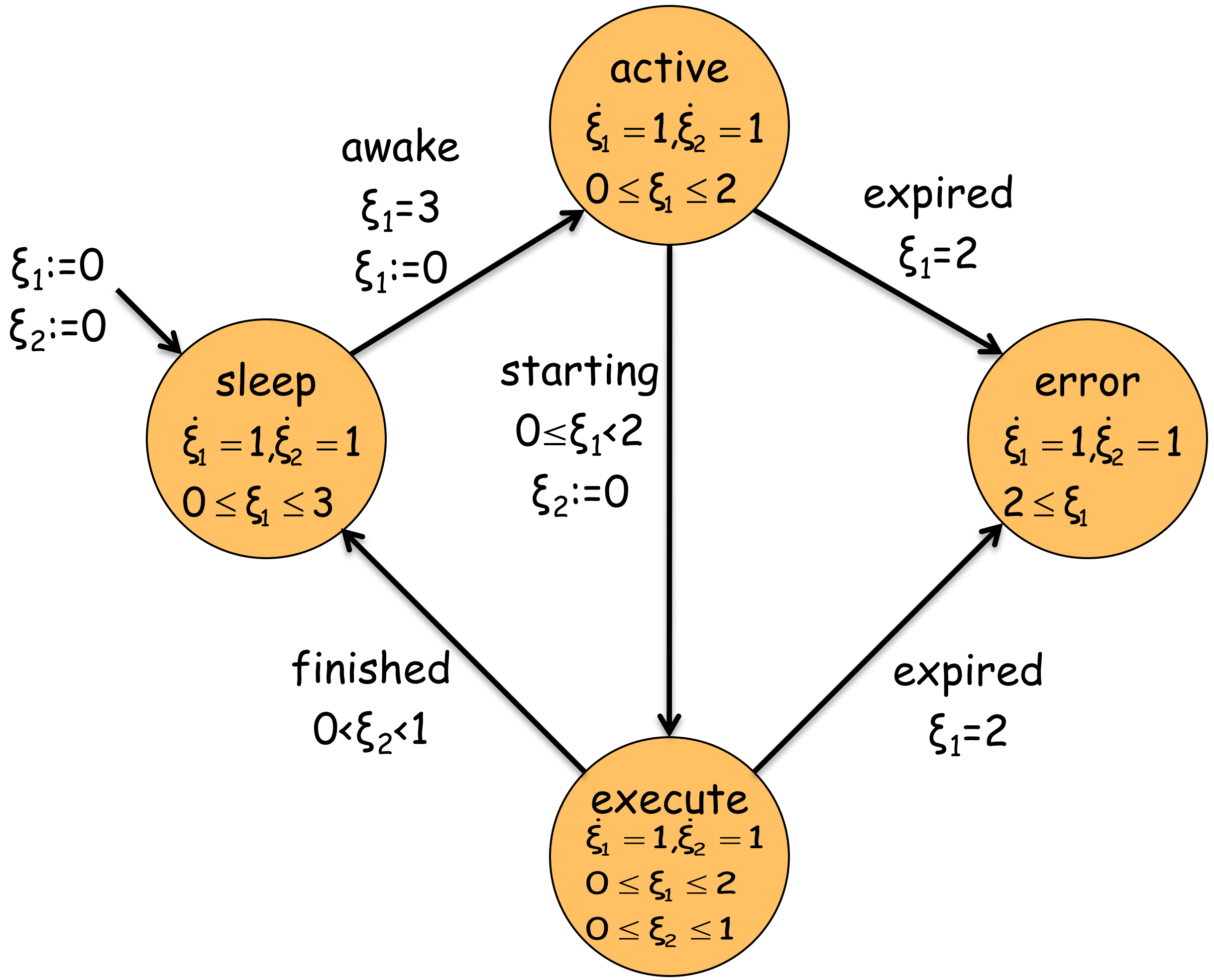
Equivalence classes Q''



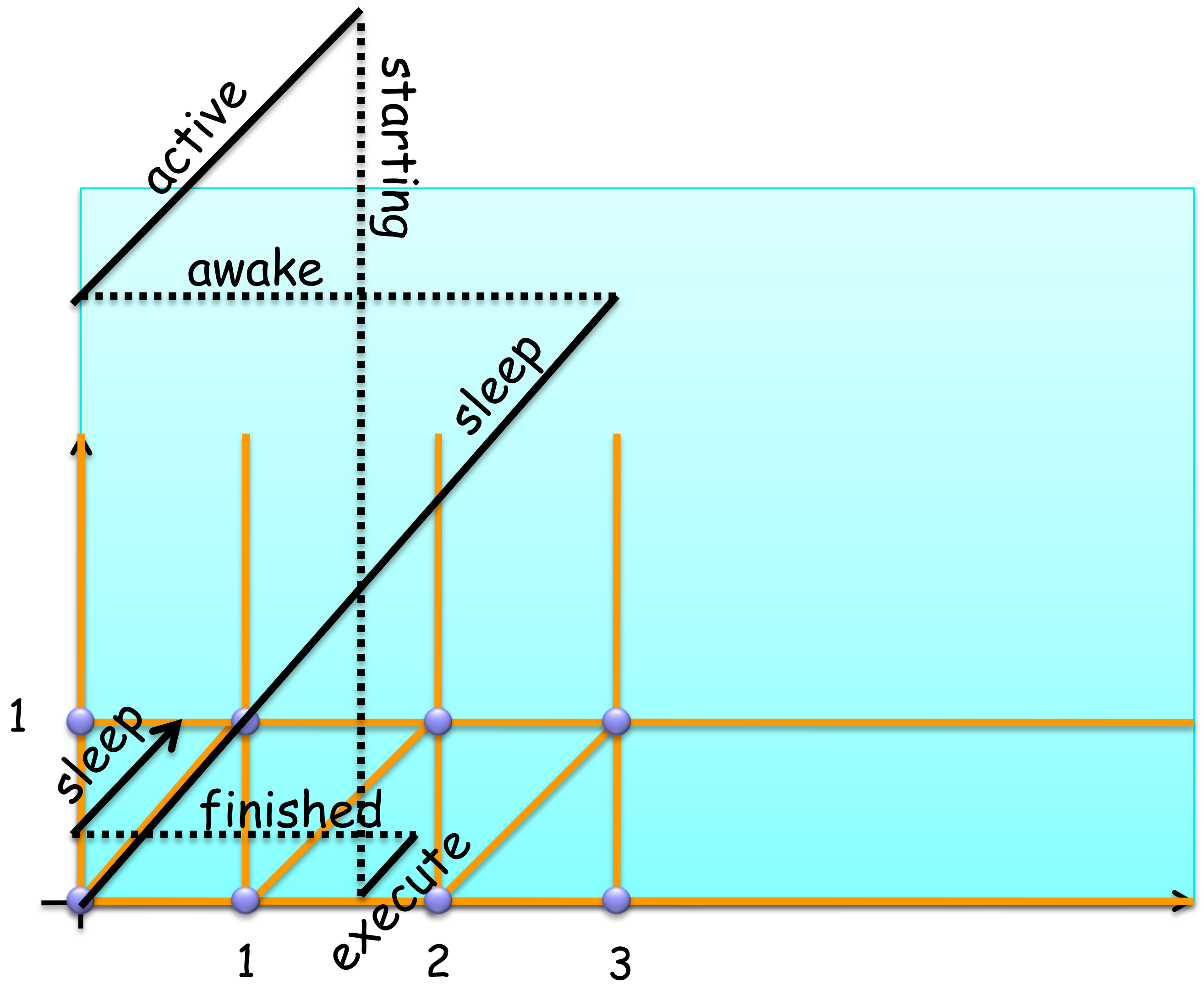
Equivalence classes Q'''



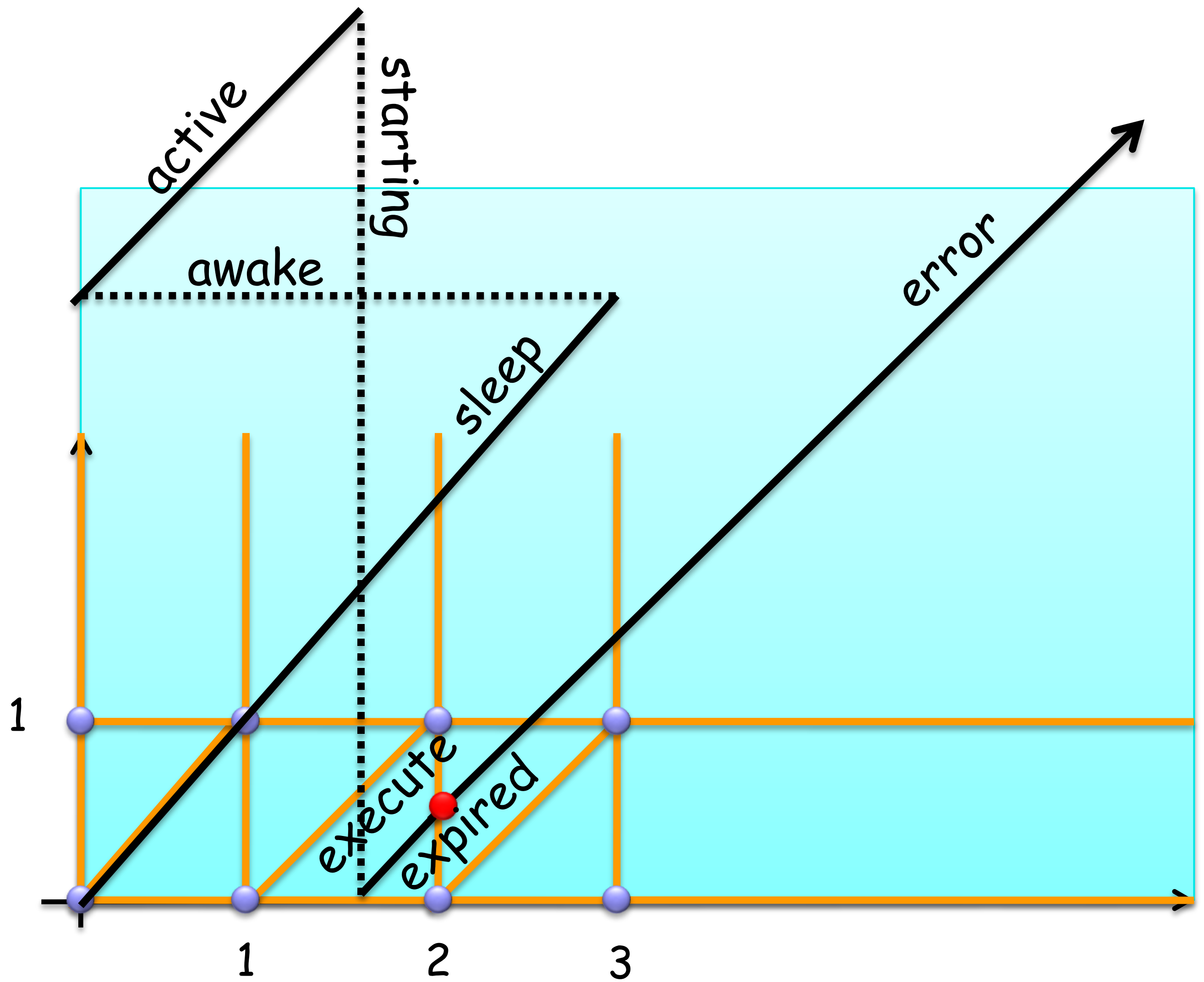
Example: Scheduler



System trajectories

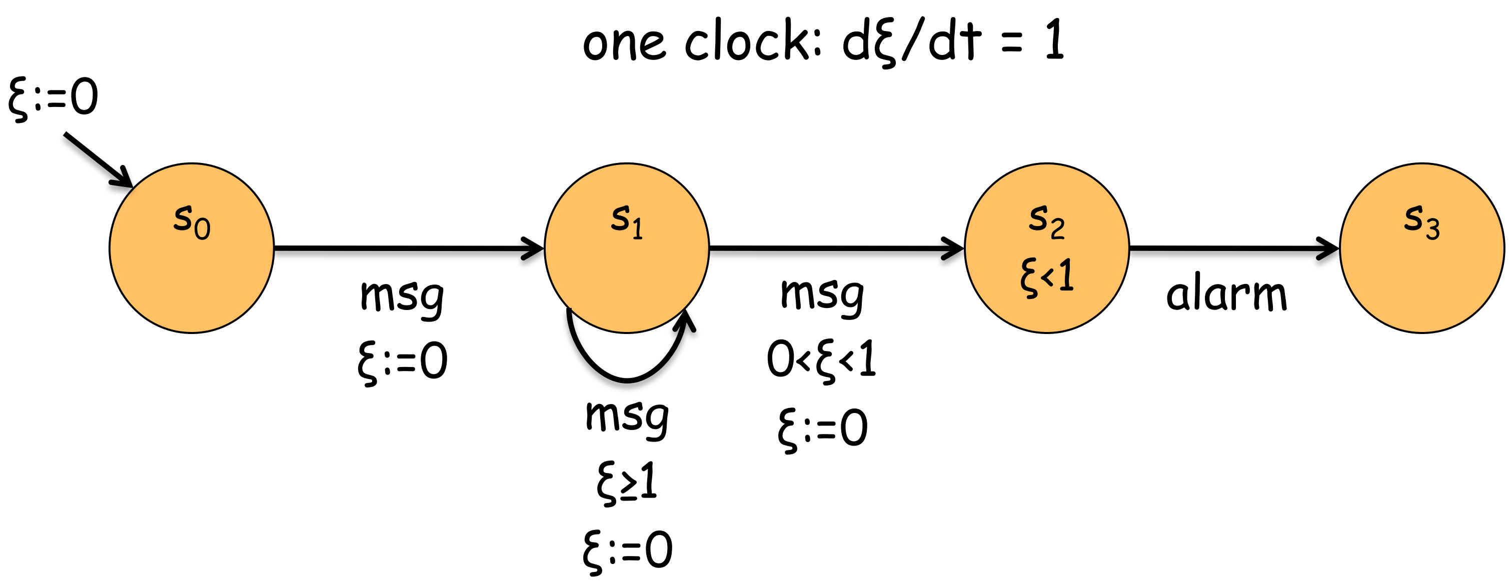


System trajectories



Example: Alarm 2

What are the regions of interest?



Example: Alarm 2

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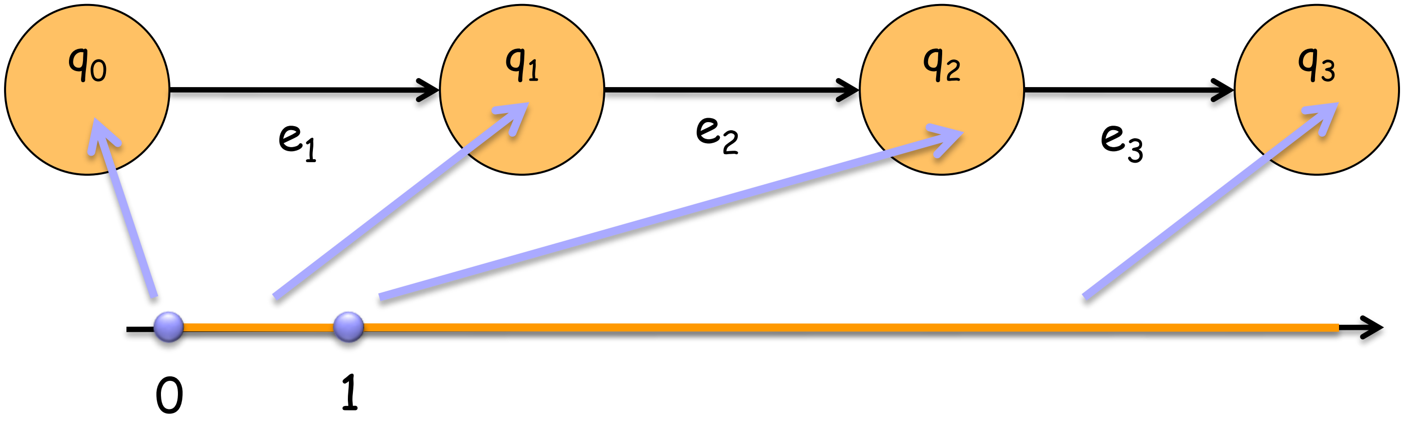
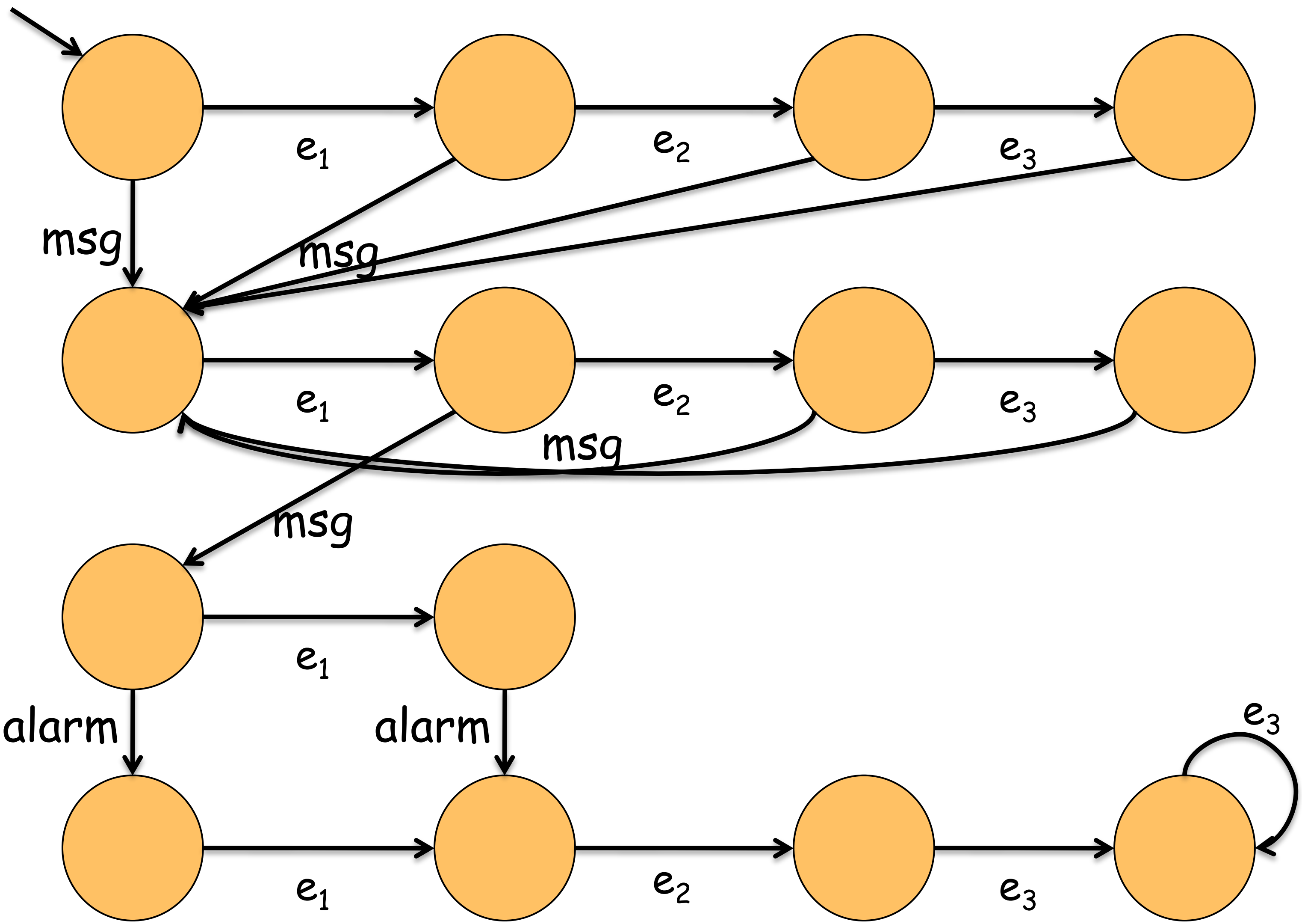


Example: Alarm 2



Timed Automata in Practice

- UPPAAL (http://www.uppaal.com/)
  - Verification
  - Planning and Scheduling
  - Testing real time systems
  - Timed games
  - Probabilistic timed automata
  - Times tool (http://www.timestool.com/)
    - modeling,
    - schedulability analysis
    - synthesis of schedules and executable code
    - worst case reaction time (WCRT) *analysis*