CSE 591: Theoretical Aspects of CPS

Signed based abstractions
References:
Tabuada Ch 7.4

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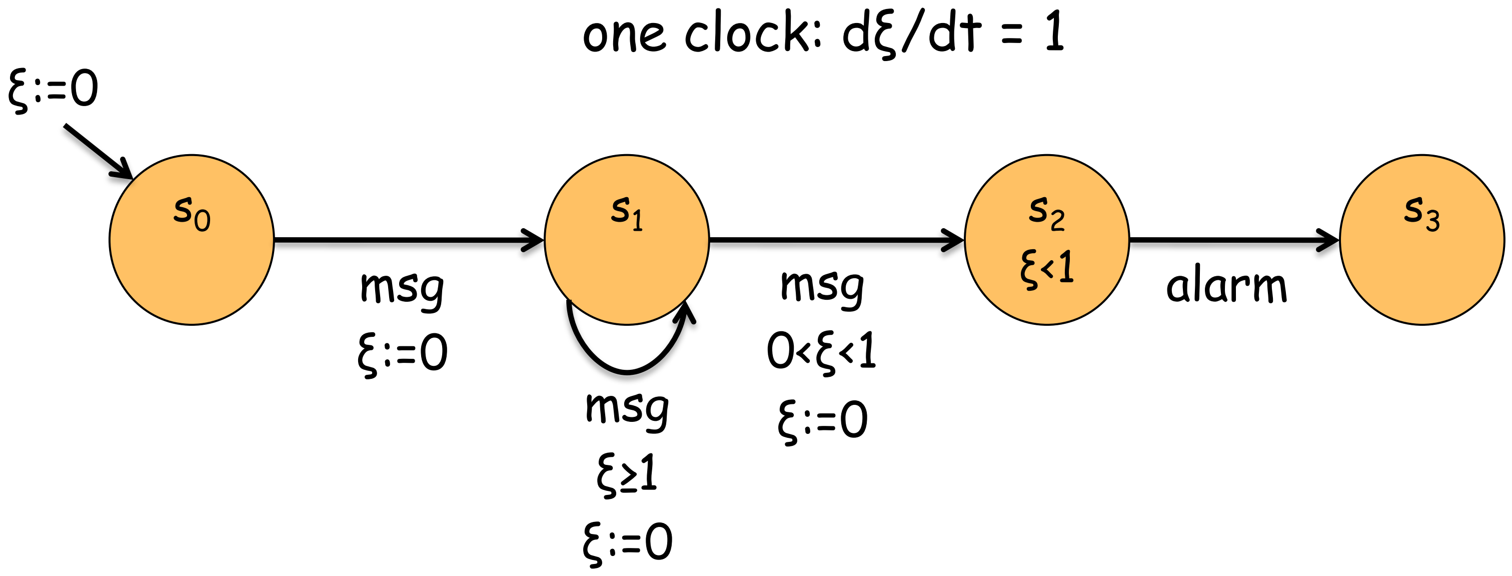
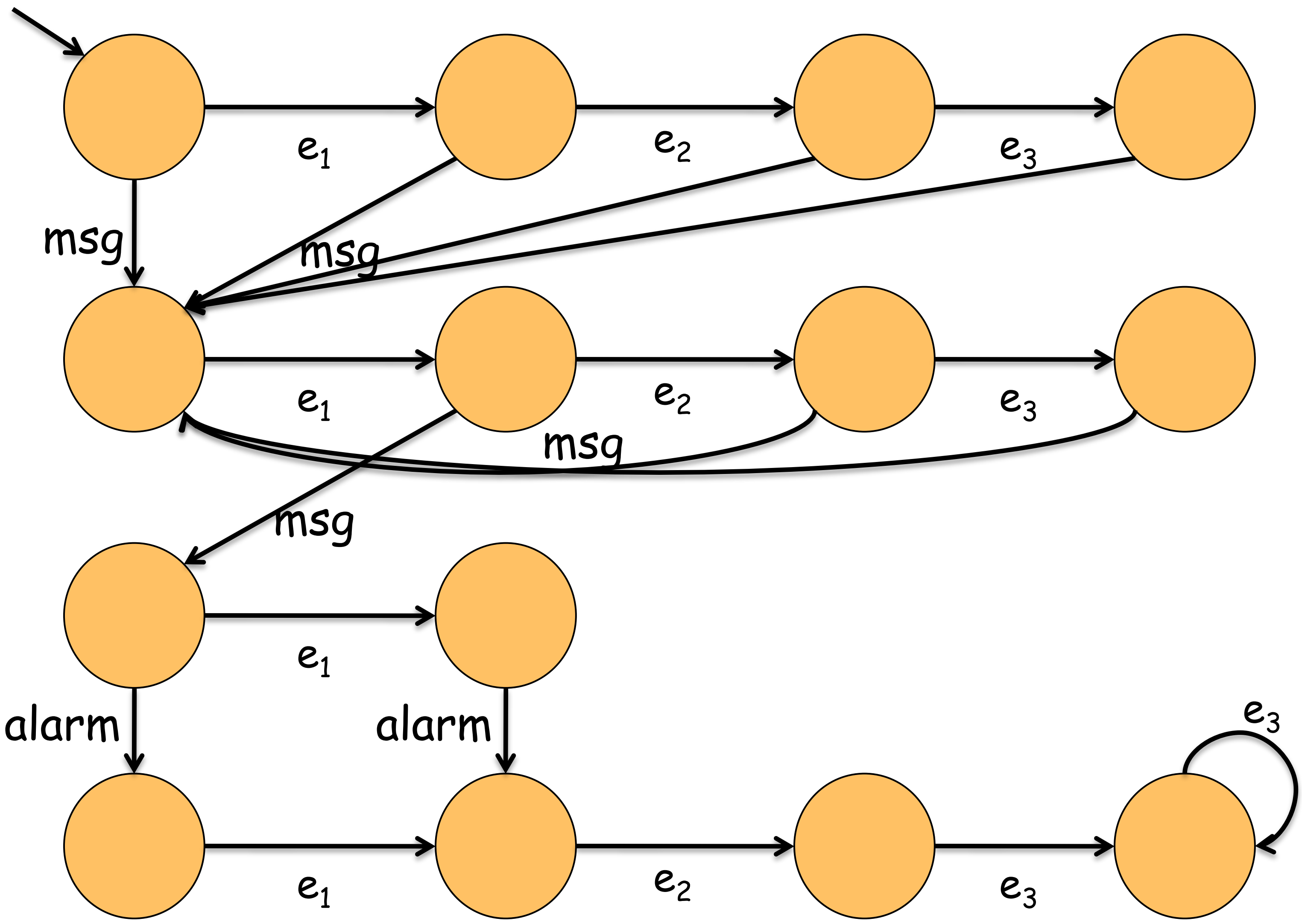
Review

- Note: The textbook at the end of each chapters has references for additional reading
- Why we care if an infinite state system is bisimilar to a finite state system?

Example: Alarm



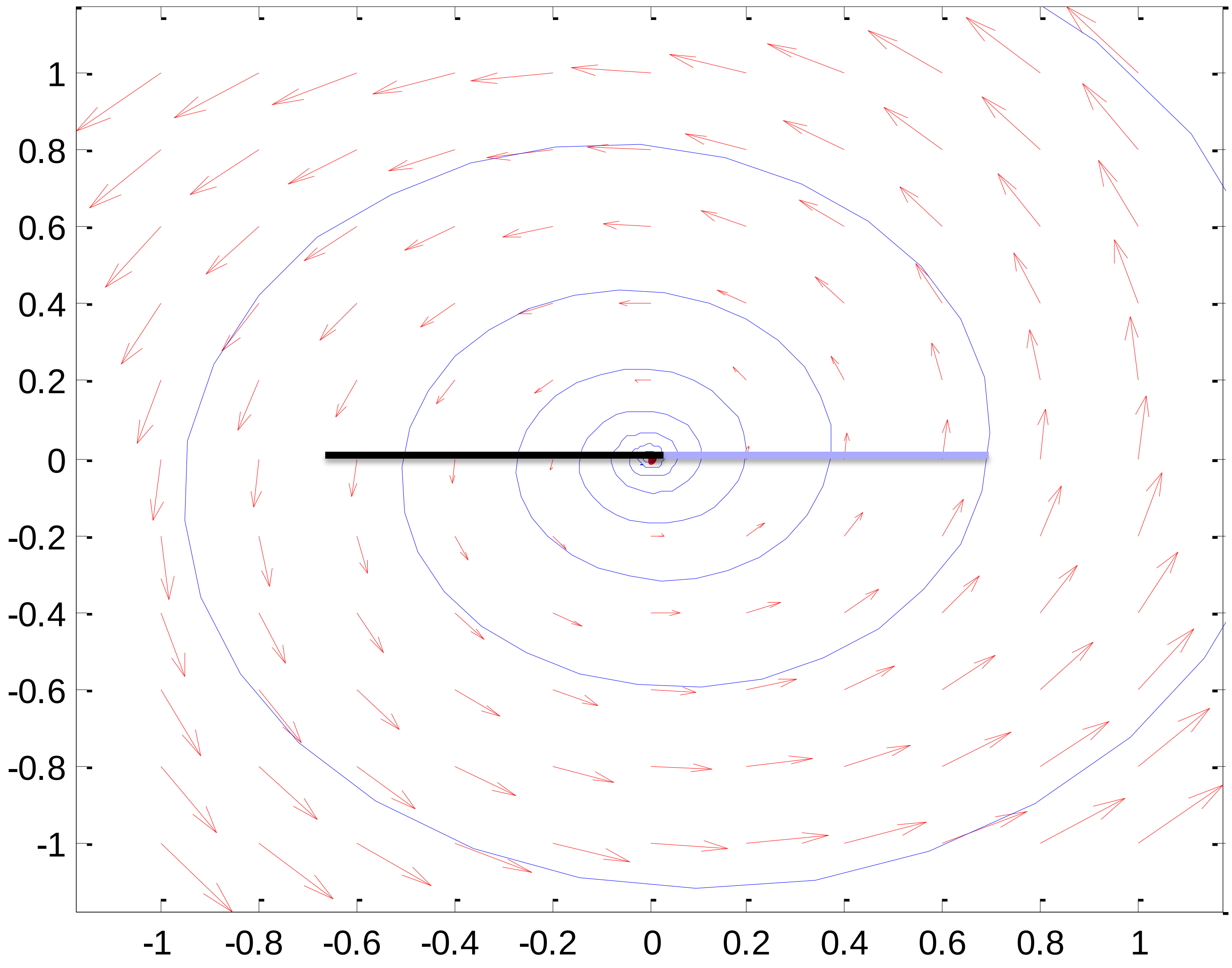
Example: Alarm



Review

- Does every infinite state system have a bisimilar finite state system?

Example of eq (7.9)



Review: o-minimal hybrid systems

- For what classes of infinite-state systems there exist bisimilar finite state systems?
- An answer can be provided by the theory of o-minimal hybrid systems
- Flows and sets definable in o-minimal structures have “nice” intersection properties

Model theory

- Studies structures through properties of their definable sets

- Every structure L has an associated language $\mathfrak{L}$ of formulas
  - Eg wff built using constants, operations, logical connectives, quantifiers, relations etc

- Def. The theory of $\mathfrak{L}$ is order-minimal if every definable subset of $\mathbb{R}$ is a finite union of points and intervals.

Interesting o-minimal structures

- $(\mathbb{R},+,-,\times,<,0,1)$
  - Admits quantifier elimination

- $(\mathbb{R},+,-,\times,<,0,1,\exp)$
  - No quantifier elimination

- $(\mathbb{R},+,-,\times,<,0,1,\exp,\{f\})$
  - $f : [-1,1]^n \to \mathbb{R}$ is a real analytic function

Classes of o-minimal hybrid systems

- $(\mathbb{R},+,-,<,0,1)$
  - Polyhedral sets and linear flows
  - Initialized timed automata, multirate automata, rectangular automata

- $(\mathbb{R},+,-,x,<,0,1)$
  - Semi-algebraic sets and polynomial flows

- $(\mathbb{R},+,-,x,<,0,1,\exp,\{f\})$
  - Subanalytic sets and exponential flows
  - Eg systems $dx/dt = Ax$ where the eigenvalues of A are real or A is diagonalizable and its eigenvalues are imaginary

Signed based abstractions

Let:

- $\Sigma=(\mathbb{R}^n,f)$ be a dynamical system
- $P=\{p_i\}_{i\in I}$ be a collection of smooth real valued functions on $\mathbb{R}^n$ (induces a partition $\mathbb{P}$)
- L be the set of initial states

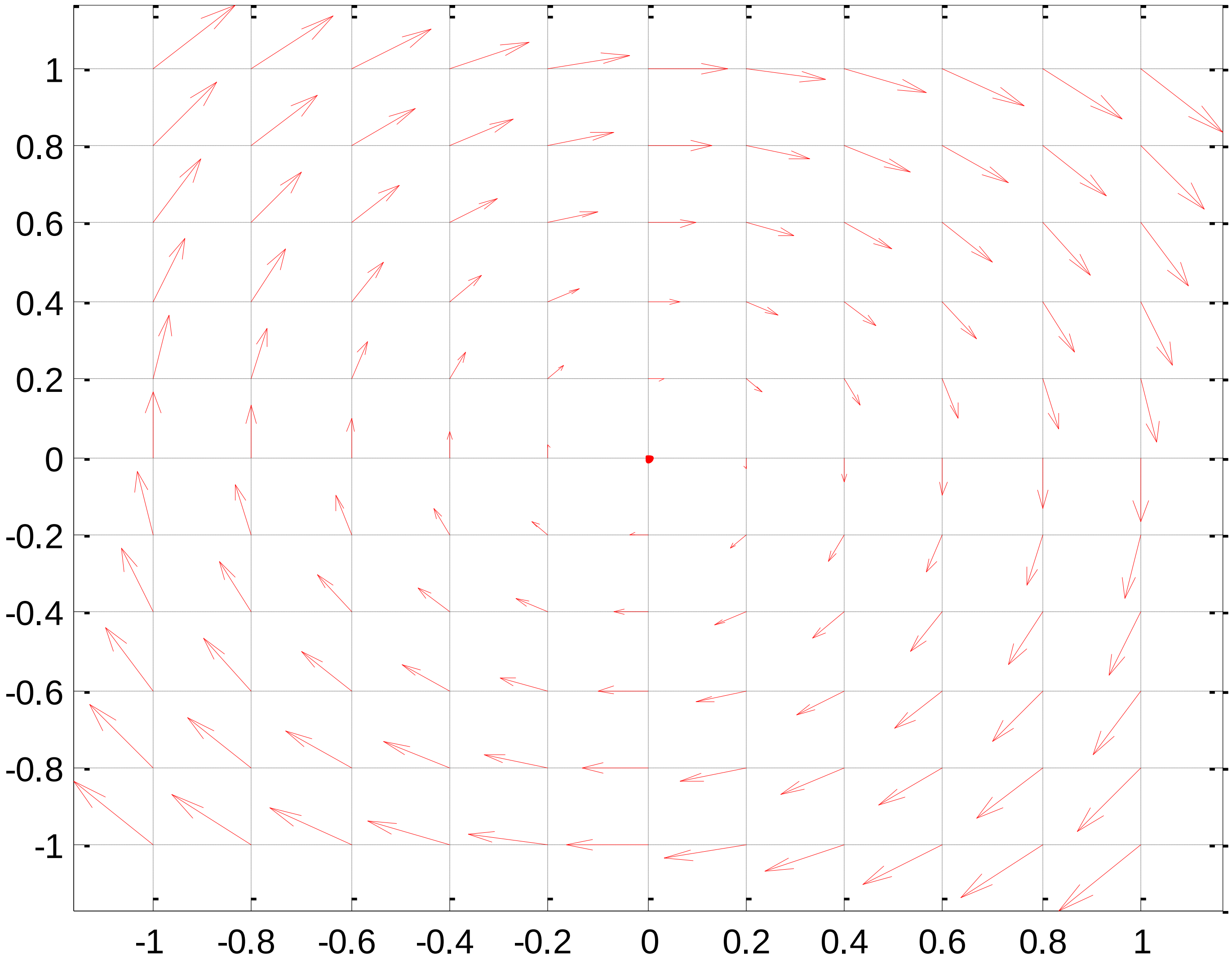
Then we can construct a finite state system $S_{PL}(\Sigma)$

- $X = \{-1,0,1\}^P$
- $X_0 = \{g\in X \mid \langle g\rangle \cap L \neq \emptyset\}$
- $U = \{*\}$
- $Y = \mathbb{R}^n/ \mathbb{P}$
- $H(g) = \langle g\rangle$

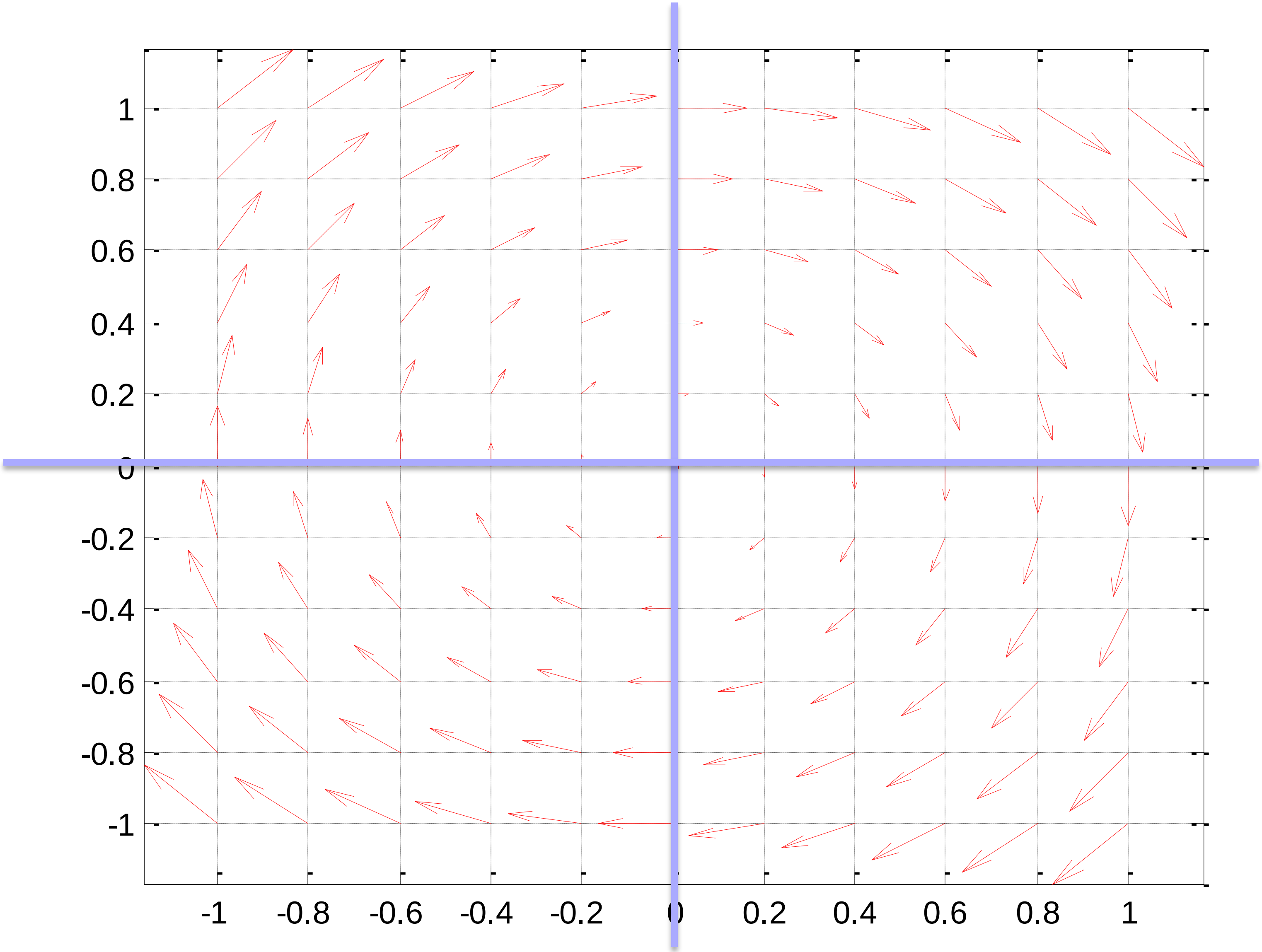
Signed based abstractions

- $g \rightarrow g'$ if for every $i \in I$ any of the following holds:
  1. $g(p_i)=1$ implies any of the following
     1. $sign(L_f p_i(\langle g \rangle)) \subseteq \{1,0\}$ and $g'(p_i)=1$
     2. $sign(L_f p_i(\langle g \rangle)) \supseteq \{1,0\}$ and $g'(p_i) \in \{1,0\}$
  2. $g(p_i)=0$ implies any of the following
     1. $sign(L_f p_i(\langle g \rangle)) =\{1\}$ and $g'(p_i)=1$
     2. $sign(L_f p_i(\langle g \rangle)) =\{-1\}$ and $g'(p_i)=-1$
     3. $sign(L_f p_i(\langle g \rangle)) =\{-1,1\}$ and $g'(p_i) \in \{-1,1\}$
     4. $sign(L_f p_i(\langle g \rangle)) \supseteq \{0\}$ and $g'(p_i) \in \{-1,0,1\}$
  3. $g(p_i)=-1$ implies any of the following
     1. $sign(L_f p_i(\langle g \rangle)) \supseteq \{1\}$ and $g'(p_i) \in \{0,-1\}$
     2. $sign(L_f p_i(\langle g \rangle)) \subseteq \{-1,0\}$ and $g'(p_i)=-1$

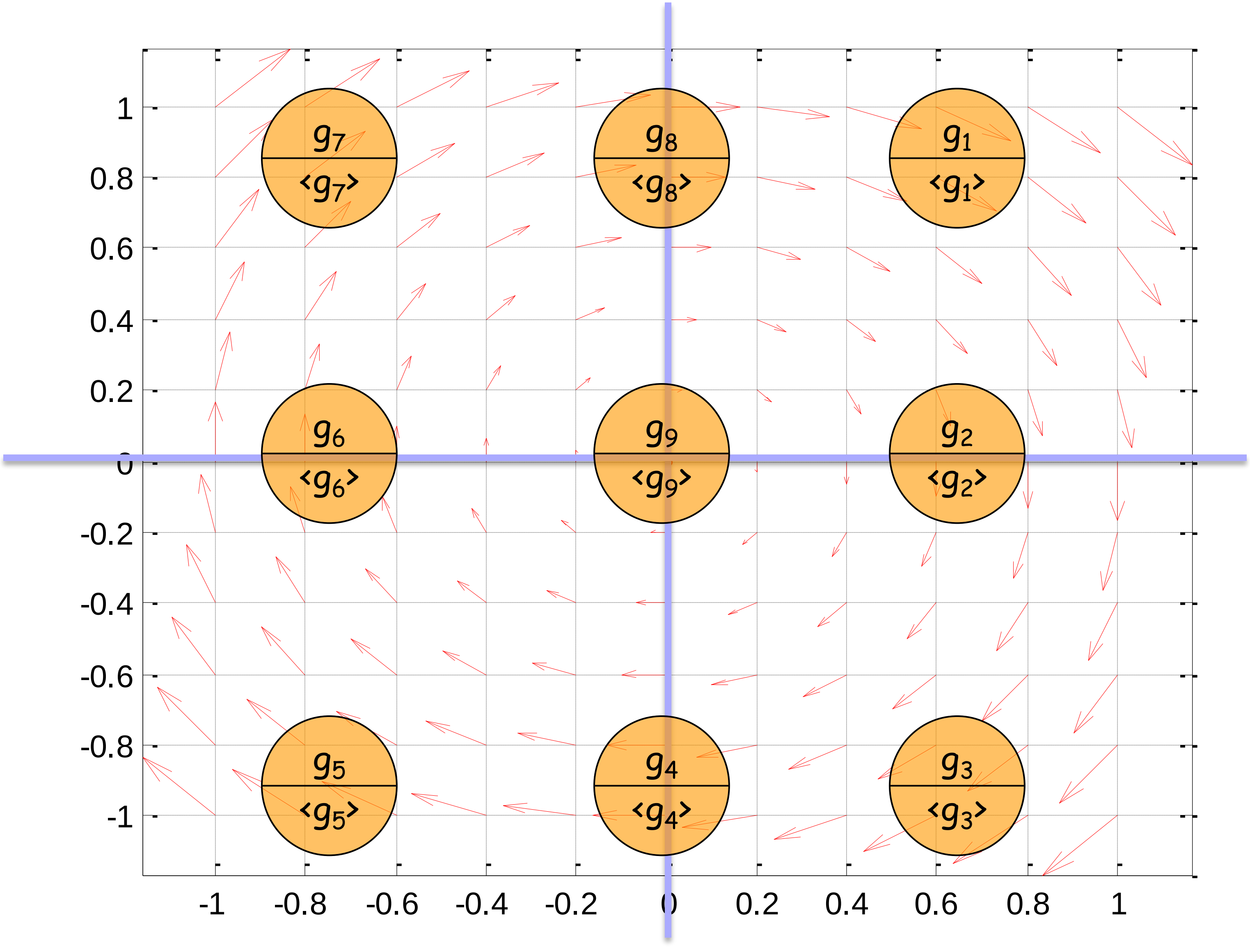
Example 7.20



Example 7.20



Example 7.20



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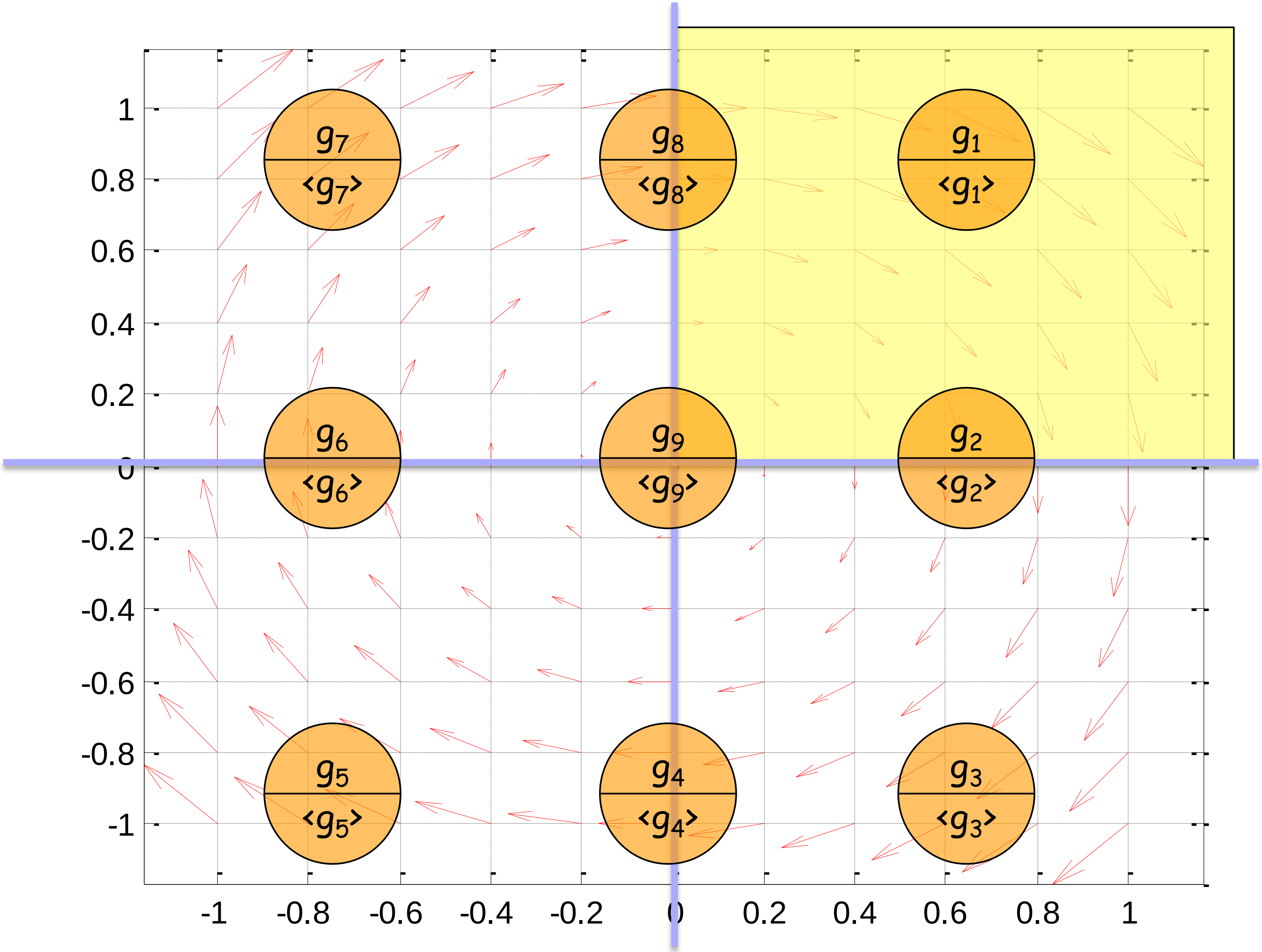
Example 7.20



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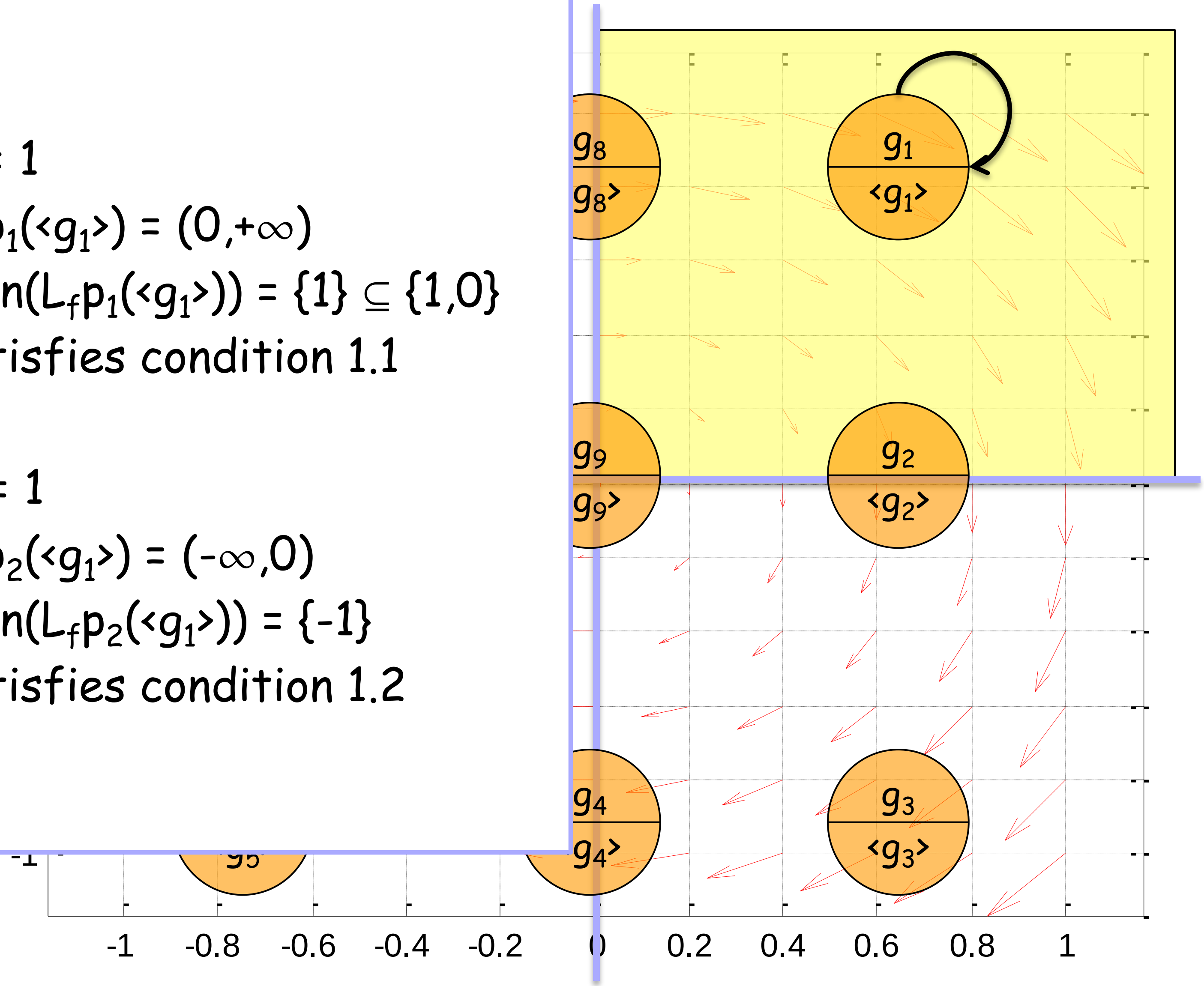
Example 7.20

- $g_1 \rightarrow g_1$

- $g_1(p_1) = 1$
  - $L_f p_1(\langle g_1 \rangle) = (0,+\infty)$
  - $\mathrm{sign}(L_f p_1(\langle g_1 \rangle)) = \{1\} \subseteq \{1,0\}$
  - satisfies condition 1.1

- $g_1(p_2) = 1$
  - $L_f p_2(\langle g_1 \rangle) = (-\infty,0)$
  - $\mathrm{sign}(L_f p_2(\langle g_1 \rangle)) = \{-1\}$
  - satisfies condition 1.2

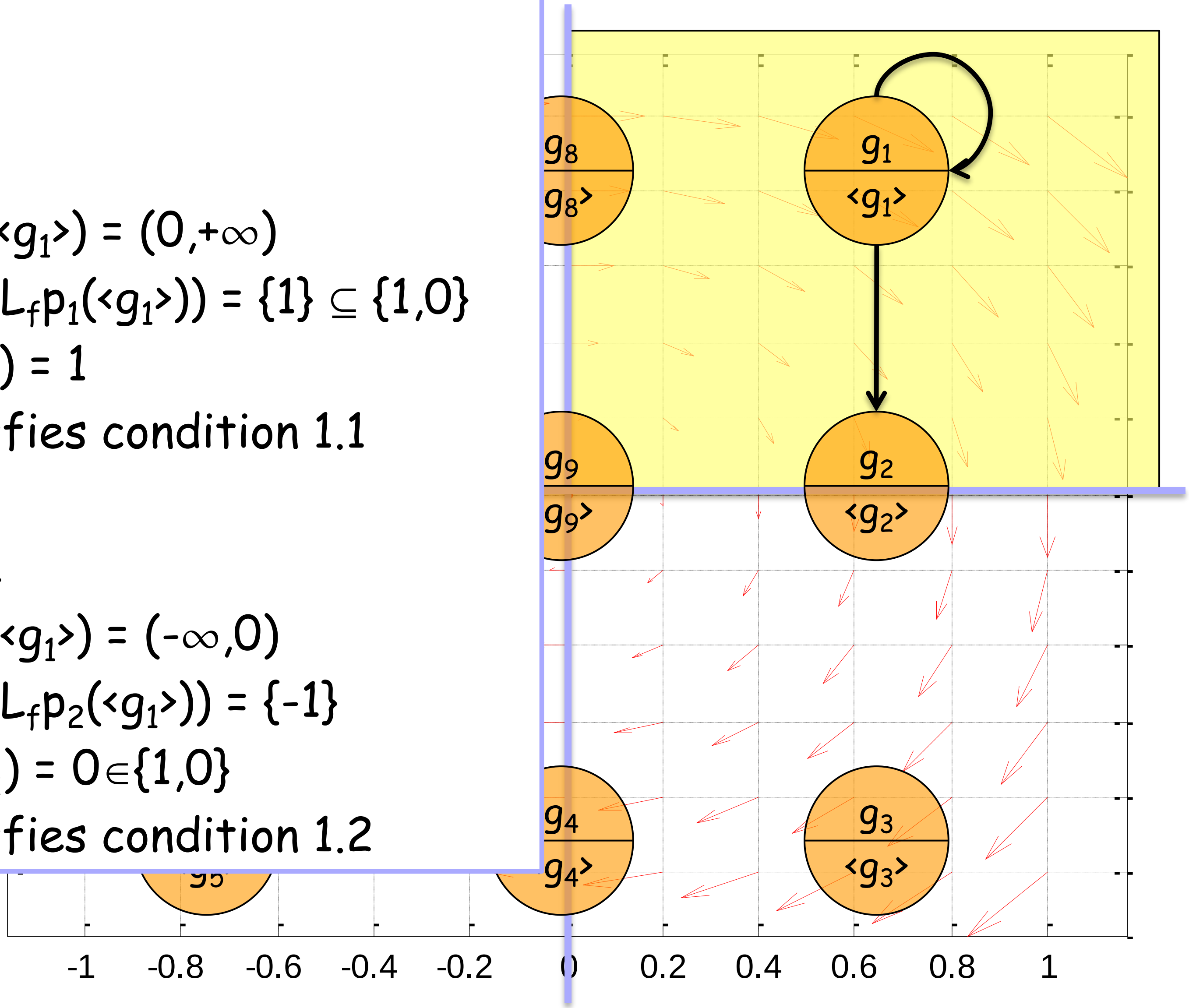


Example 7.20

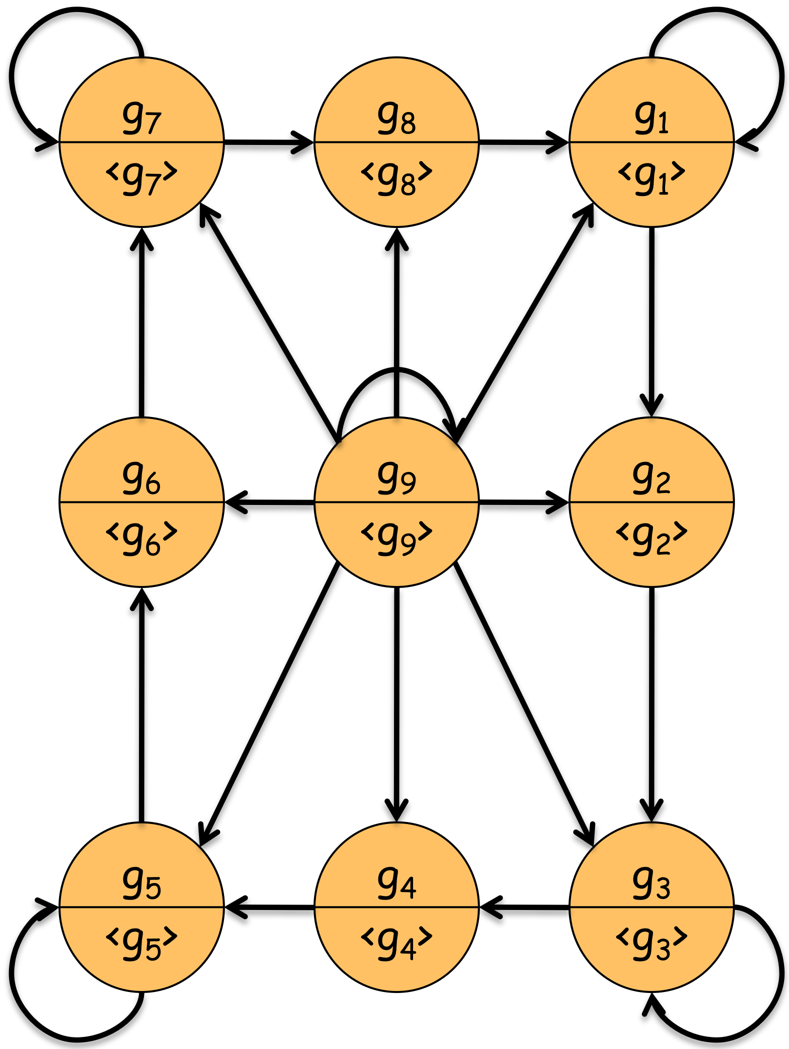
- $g_1 \rightarrow g_2$

- $g_1(p_1) = 1$
  - $L_f p_1(\langle g_1 \rangle) = (0,+\infty)$
  - $\text{sign}(L_f p_1(\langle g_1 \rangle)) = \{1\} \subseteq \{1,0\}$
  - $g_2(p_1) = 1$
  - satisfies condition 1.1

- $g_1(p_2) = 1$
  - $L_f p_2(\langle g_1 \rangle) = (-\infty,0)$
  - $\text{sign}(L_f p_2(\langle g_1 \rangle)) = \{-1\}$
  - $g_2(p_2) = 0 \in \{1,0\}$
  - satisfies condition 1.2



Example 7.20: $S_{PL}(\Sigma)$
$S_{PL}(\Sigma)$ is not bisimilar to $S_{\mathbb{P}L}(\Sigma)$!



Example 7.20: $S_{\mathbb{P}L}(\Sigma)$