(Towards) Translating Temporal Logic to Controller Specifications

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Math free Presentation!

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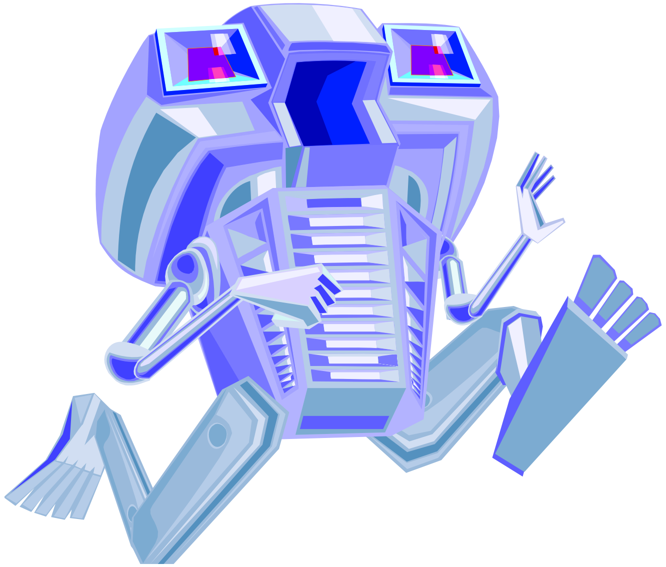
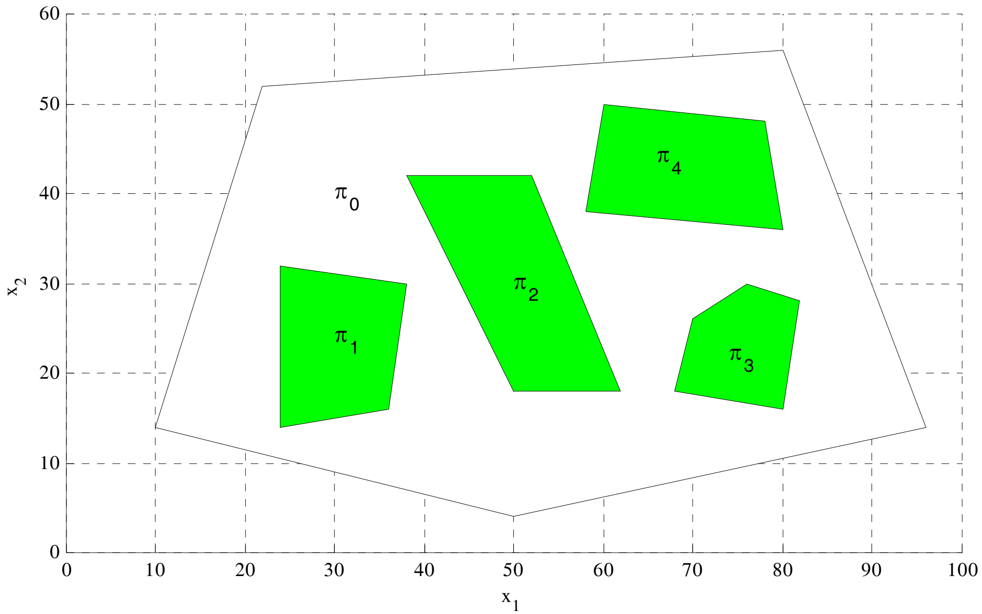
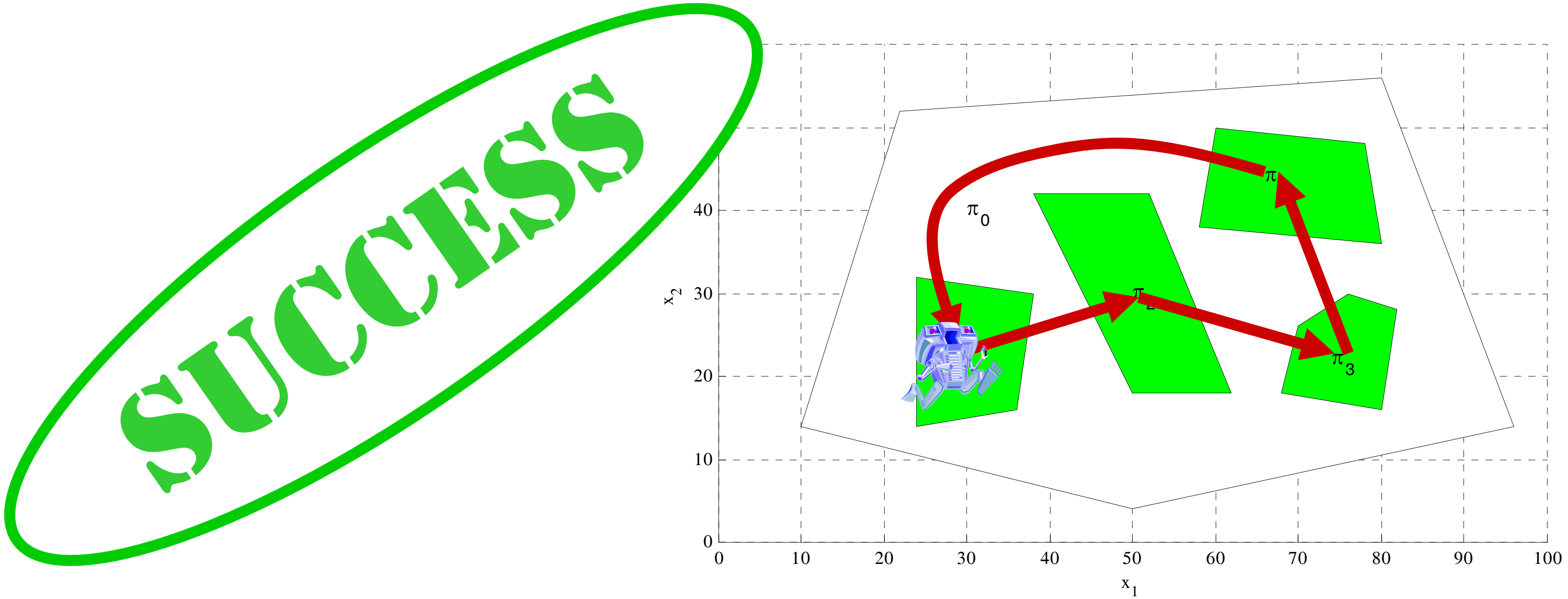
Motivation - Motion Planning





Motivation – Motion Planning



Task: "Stay always in π_0 and visit area π_2, then area π_3, then area π_4 and, finally, return to and stay in region π_1 while avoiding areas π_2 and π_3"

Good news

RTL: $\Box\pi_0 \wedge \Diamond(\pi_2 \wedge \Diamond(\pi_3 \wedge \Diamond(\pi_4 \wedge (\neg\pi_2 \wedge \neg\pi_3) U \pi_1)))$

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Problem Definition

Temporal Logic Controller Synthesis

Given a dynamical system Σ, a set of initial conditions X_0 and a flat RTL formula φ over Π, construct a closed-loop system in the form of a hybrid automaton H_φ such that the resulting system trajectories $x(t)$ starting at some point $x(0) \in X_0$ satisfy the formula φ.

$$\Sigma: \; \dot{x}(t) = f(x(t), u(t)) \quad x(t) \in X \subseteq \mathbb{R}^n \quad u(t) \in U \subseteq \mathbb{R}^n$$

In the past …

Approaches to synthesis of hybrid systems:

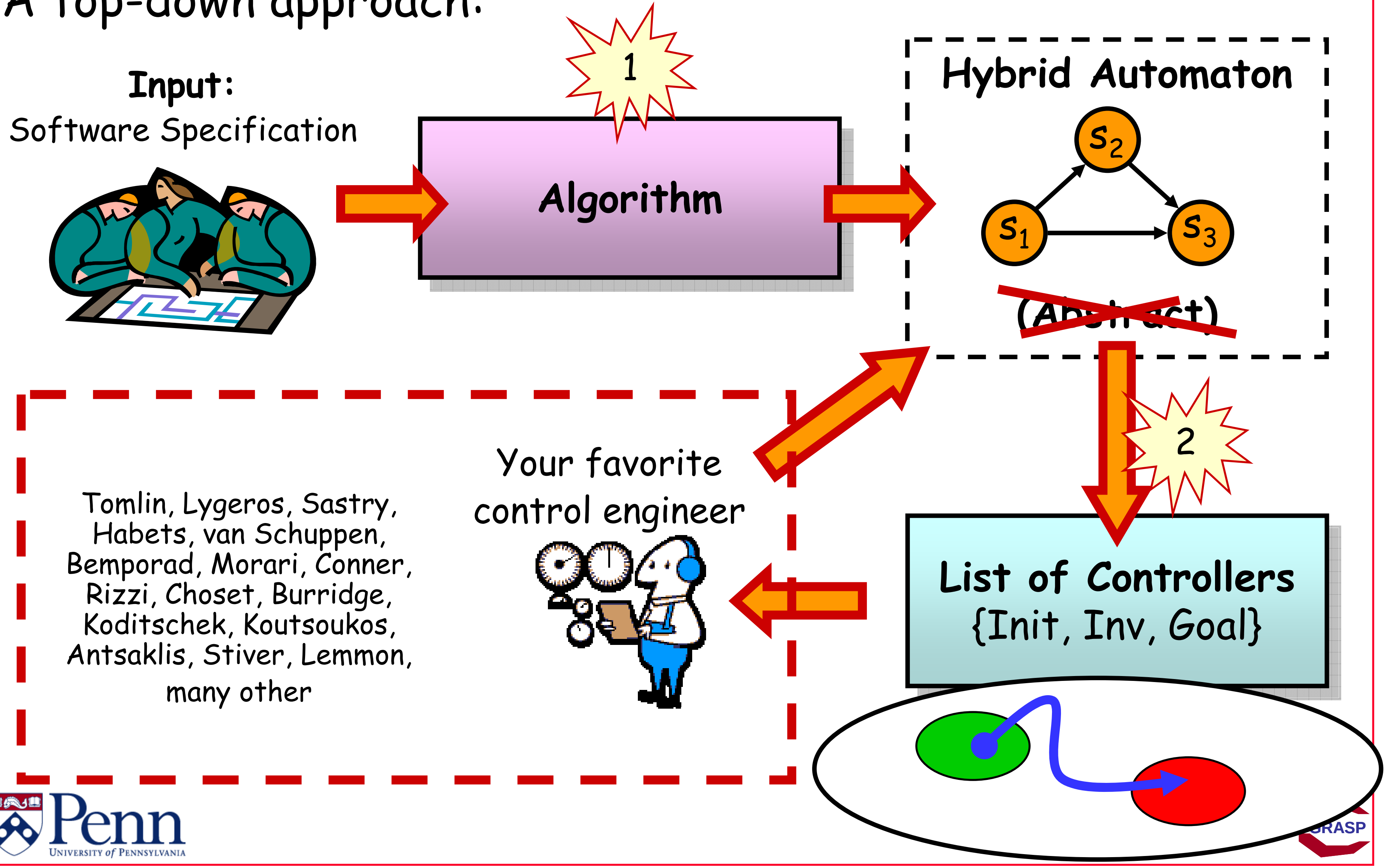
- Given controllers + switching rules $\Longrightarrow$ verify that HS satisfies spec
- Given controllers + spec $\Longrightarrow$ keep necessary controllers
- Given closed-loop dynamics + spec $\Longrightarrow$ design switching rules

Researchers: Alur, Henzinger, Tabuada, Davoren, Moor, Koutsoukos, Antsaklis, Stiver, Lemmon, Tomlin, Lygeros, Sastry, Habets, van Schuppen, Asarin, Bournez, Dang, Maler, Pnueli, Bemporad, Morari, Giorgetti, Lafferriere, Kloetzer, Belta, Kyriakopoulos, Kress-Gazit, Loizou, Pappas, Fainekos and many others …

- **Given spec $\Longrightarrow$ design controllers**

In this presentation …

A top-down approach:



Hybrid Automaton

A hybrid automaton is a tuple

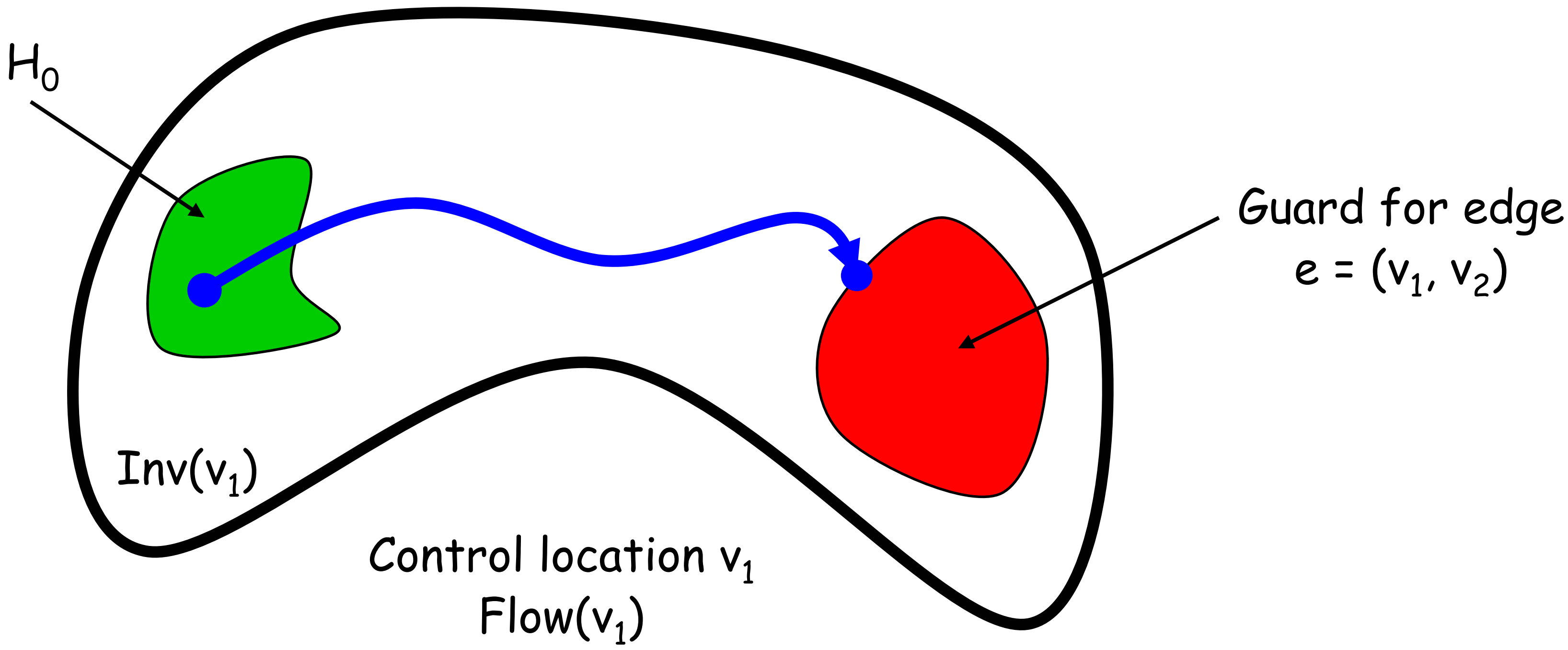
$$H = (X, V, E, Inv, Flow, Init, Guard, F)$$

where

- X is the state space of the system Σ
- V is the set of control locations,
- $E \subseteq V \times V$ is the set of control switches
- $Inv : V \rightarrow P(X)$ assigns an invariant set to each location
- $Flow : V \times X \rightarrow P(\mathbb{R}^n)$ constraints the time derivative of the continuous part of the state
- $Init : V \rightarrow P(X)$ assigns to each control location a set of initial conditions,
- $Guard : E \rightarrow P(X)$ is the guard condition that enables a control switch e in E
- $F \subseteq V$ is the set of final locations

Trajectories of the Hybrid Automaton

Formally, the semantics of a hybrid automaton are given in terms of timed transition systems $T_H = (H, H_0, \rightarrow)$.



Note: the reset map is the identity, transitions are forced

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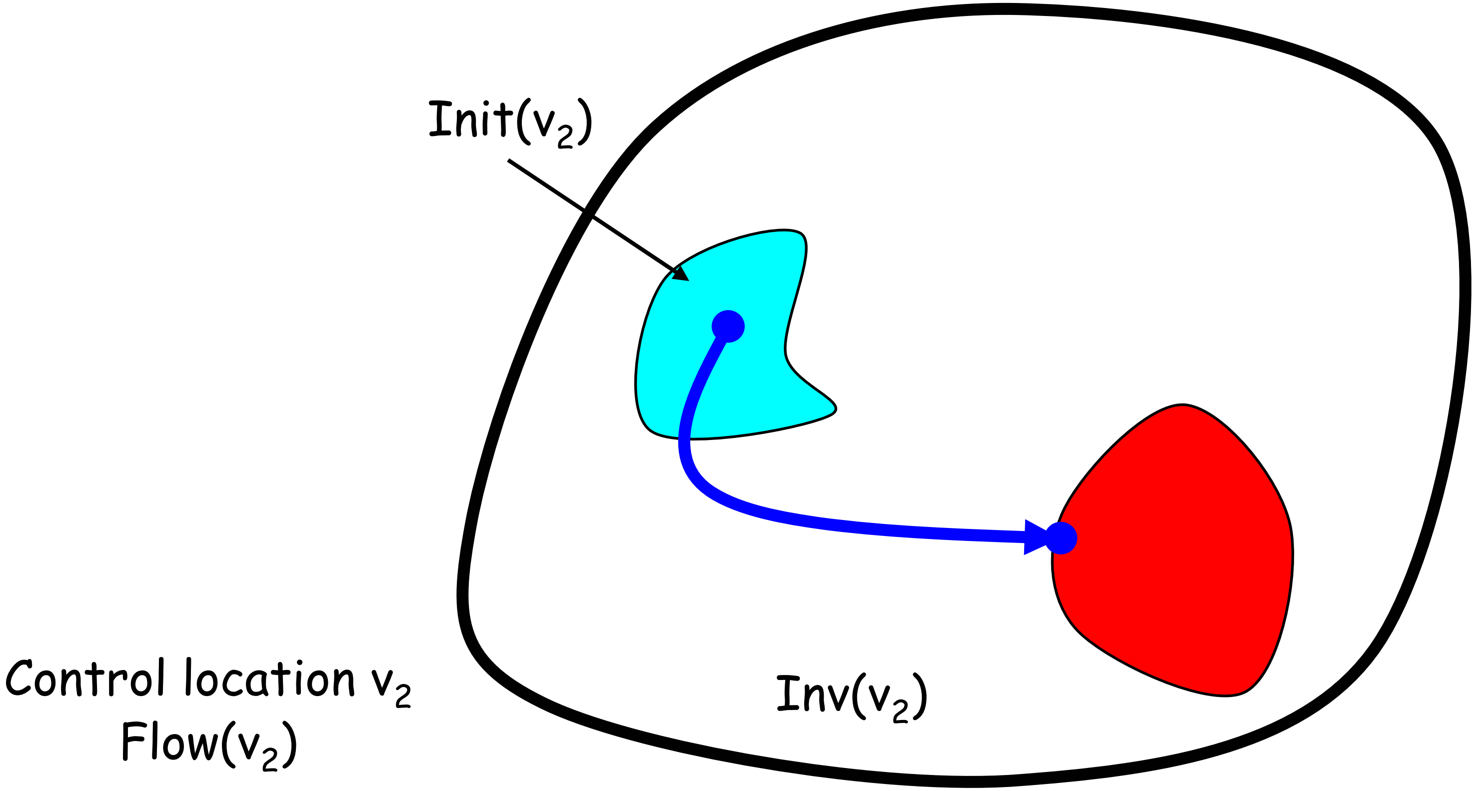
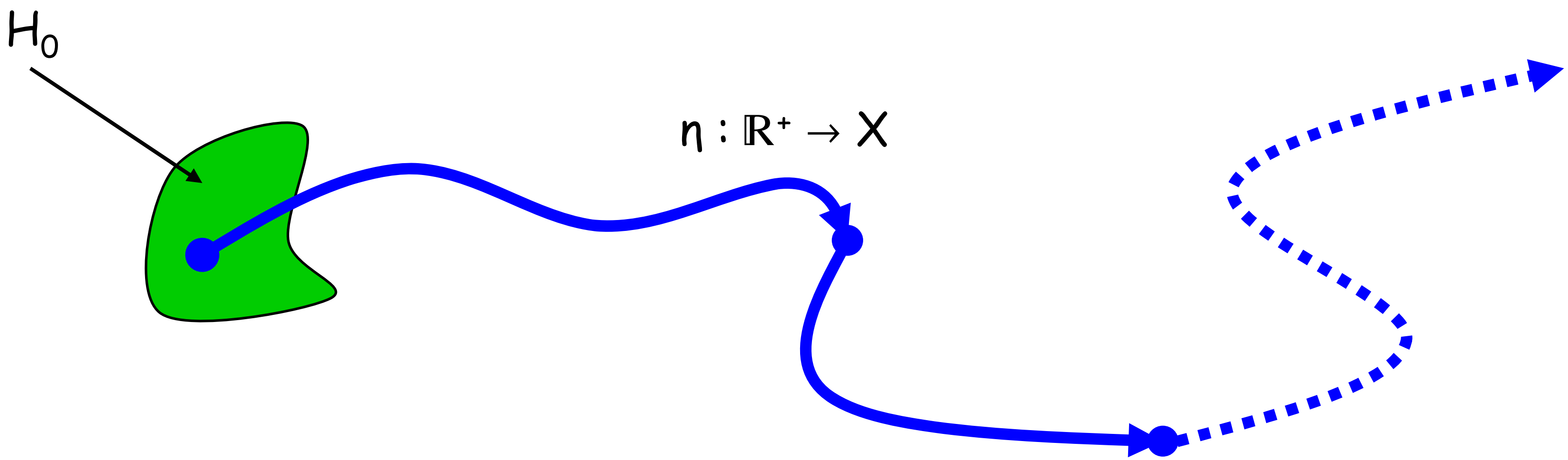


Note: the reset map is the identity, transitions are forced

Language of the timed transition system

The set of all trajectories η of T_H starting from a state in H_0 is the language $L(T_H)$ of the timed transition system T_H.

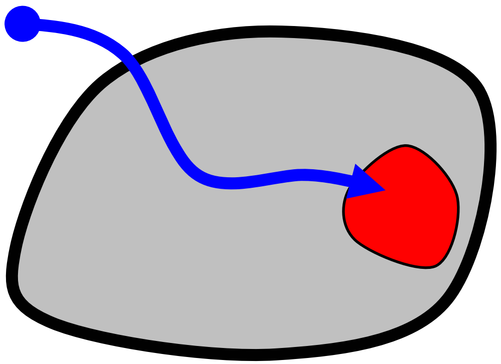


Linear Temporal Logic: Continuous Time

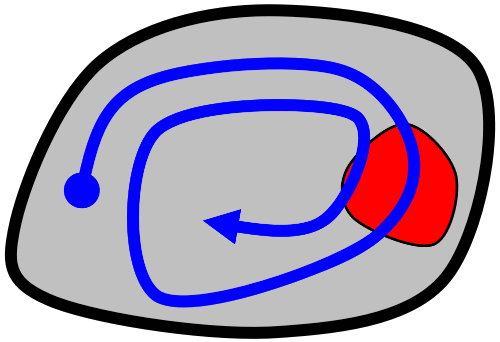
Like propositional logic, but also reasoning wrt time …

Basic operators:
Propositional: ∧, ∨, ¬
Temporal: ◊, □, U, R

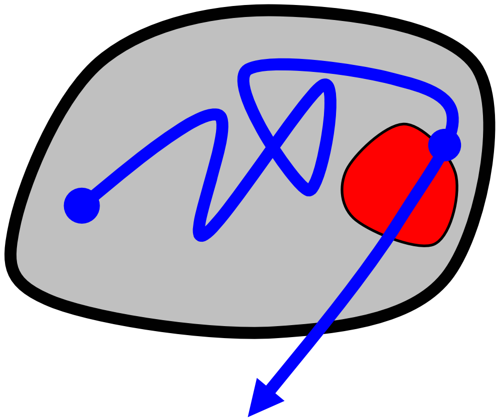
◊(red)
Eventually red



□(gray)
Always gray



(gray) U (red)
gray Until red



Flat LTL with continuous time semantics

Let Π be a set of atomic propositions and Π^* be the set of Boolean combinations of atomic propositions, i.e. $(\pi_1 \wedge \pi_2) \vee \neg \pi_3$.

Define the denotation $[[.]] : \Pi \rightarrow P(X)$ which extends naturally over Π^* as:

$$[[\pi_1 \wedge \pi_2]] = [[\pi_1]] \cap [[\pi_2]] \text{ and } [[\neg\pi]] = [[\pi]]^c$$

Syntax in NNF: $\varphi ::= \pi^* \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \pi^* U \varphi_2 \mid \pi^* R \varphi_2$

Semantics:

$\eta \vDash \pi^*$ iff $\eta(0) \in [[\pi^*]]$

$\eta \vDash \varphi_1 \wedge \varphi_2$ iff $\eta \vDash \varphi_1$ and $\eta \vDash \varphi_2$

$\eta \vDash \varphi_1 \vee \varphi_2$ iff $\eta \vDash \varphi_1$ or $\eta \vDash \varphi_2$

$\eta \vDash \pi^* U \varphi$ iff $\exists t \geq 0$ s.t. $\eta|_t\ \eta \vDash \varphi_2$ and $\forall s \in [0,t]\ \eta(s) \in [[\pi^*]]$

$\eta \vDash \pi^* R \varphi$ iff $\forall t \geq 0\ \eta|_t\ \eta \vDash \varphi_2$ or $\exists s \in [0,t]$ s.t. $\eta(s) \in [[\pi^*]]$

Some notation: $\eta|_t(s) = \eta(t+s)$ $\quad$ $LTL(op_1, \ldots, op_n)$

$\Diamond\varphi = T U \varphi$ $\quad$ $\Box\varphi = F R \varphi$

LTL(U,$\vee$,$\wedge$) to HA

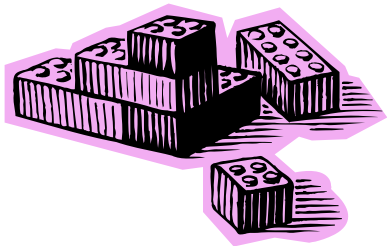
1 **Algorithm**

Modular construction using the structure of φ.
($\equiv$ proof by induction)

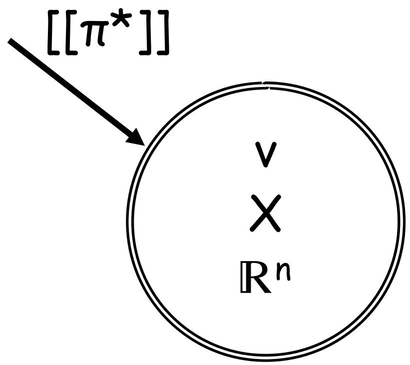
Syntax in NNF: $\varphi ::= \pi^* \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \pi^* U \varphi$

Ex. $\Diamond(\pi_2 \wedge \Diamond(\pi_3 \wedge \Diamond(\pi_4 \wedge (\neg\pi_2 \wedge \neg\pi_3) U \pi_1)))$

LTL(U,$\vee$,$\wedge$) to HA

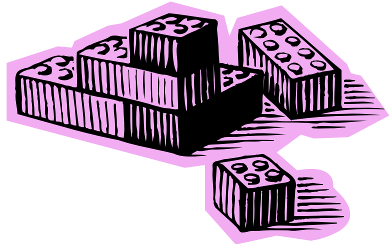


Proposition 2 (base case): Let $\varphi = \pi$, then
$H_{\pi^*} = (X, \{v\}, \emptyset, X, \mathbb{R}^n, [[\pi^*]], \emptyset, \{v\})$



proof immediate from definition: $\eta \models \pi^*$ iff $\eta(0) \in [[\pi^*]]$

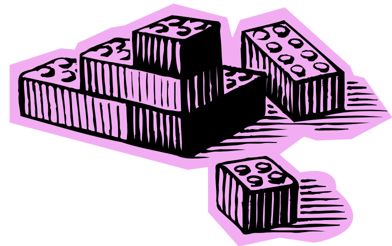
Basic Building Blocks



$\varphi = \varphi_1 \wedge \varphi_2$ or $\varphi = \varphi_1 \vee \varphi_2$

Proposition 1 [Henzinger 95]:
Let H_1 and H_2 be hybrid automata.
If $H = H_1 \cup H_2$, then $L(T_H) = L(T_{H1}) \cup L(T_{H2})$.
If $H = H_1 \cap H_2$, then $L(T_H) = L(T_{H1}) \cap L(T_{H2})$.

LTL(U,∨,∧) to HA

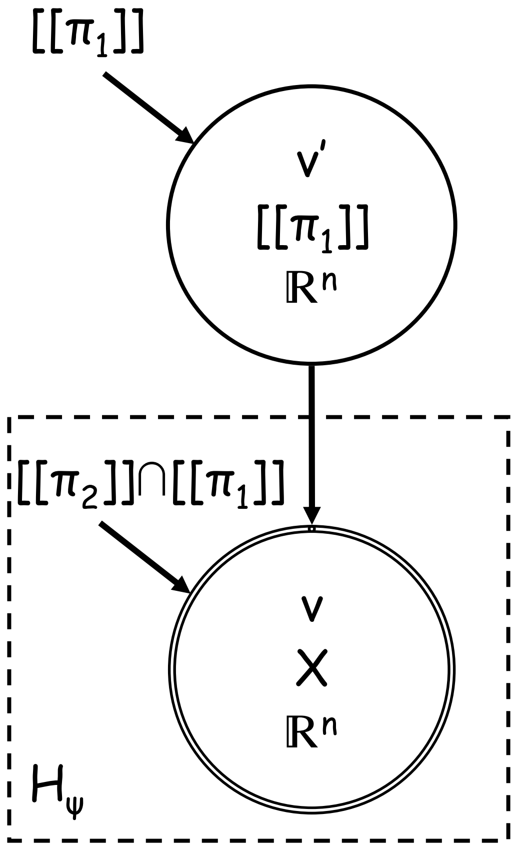


Proposition 3: Let $\varphi = \pi^* U \psi$, then
$H_\varphi = (X, V \cup \{v'\}, E', Inv', Flow', Init', Guard', F)$
Let $H_\psi = (X, V, E, Inv, Flow, Init, Guard, F)$. Then:

- $Inv'(v') = Init'(v') = [[\pi^*]]$ and $Flow(v') = \mathbb{R}^n$
- $Inv'(v) = Inv(v)$ for all $v \in V$
- $Flow'(v) = Flow(v)$ for all $v \in V$
- $Init'(v) = Init(v) \cap [[\pi^*]]$ for all $v \in V$
- $E' = E \cup \{(v', v) \mid v \in V_{in}\}$
- $Guard'(v', v) = Init'(v)$ for all $v \in V_{in}$
- $Guard'(e) = Guard(e)$ for all $e \in E$

where $V_{in} = \{v \in V \mid Init(v) \cap [[\pi^*]] \neq \emptyset\}$

Ex. $\pi_1 U \pi_2$



proof immediate from definition:
$\eta \vDash \pi^* U \psi$ iff $\exists t \geq 0$ s.t. $\eta|_t\ \eta \vDash \psi$ and $\forall s \in [0,t]\ \eta(s) \in [[\pi^*]]$

LTL(U,$\vee$,$\wedge$) to HA

Algorithm 1 The LTL($\underline{\mathcal{U}}, \wedge, \vee$) Fragment

Input: A formula $\phi \in$ LTL($\underline{\mathcal{U}}, \wedge, \vee$)
Output: The hybrid automaton $\mathcal{H}_\phi$

procedure $\mathrm{LTL}_{\mathcal{U}}\mathrm{TOHA}(\phi)$
  if $\phi = \overline{\pi}$ **then**
    return $\mathcal{H}_{\overline{\pi}}$ ▷ Proposition 2
  else if $\phi = \phi_1 \wedge \phi_2$ **then**
    return $\mathrm{LTL}_{\mathcal{U}}\mathrm{TOHA}(\phi_1) \cap \mathrm{LTL}_{\mathcal{U}}\mathrm{TOHA}(\phi_2)$
  else if $\phi = \phi_1 \vee \phi_2$ **then**
    return $\mathrm{LTL}_{\mathcal{U}}\mathrm{TOHA}(\phi_1) \cup \mathrm{LTL}_{\mathcal{U}}\mathrm{TOHA}(\phi_2)$
  else if $\phi = \overline{\pi}\mathcal{U}\psi$ **then**
    $\mathcal{H}_\psi \leftarrow \mathrm{LTL}_{\mathcal{U}}\mathrm{TOHA}(\psi)$
    return $\mathcal{H}_\phi$ ▷ Proposition 3
  end if
end procedure

LTL(U,∨,∧) to HA

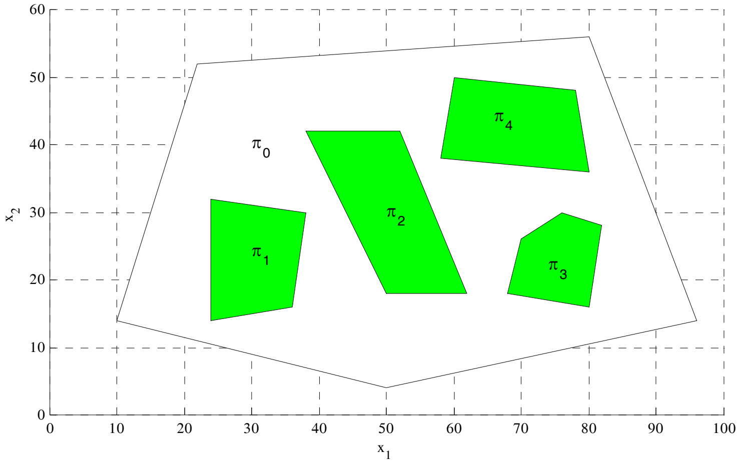
Remarks:

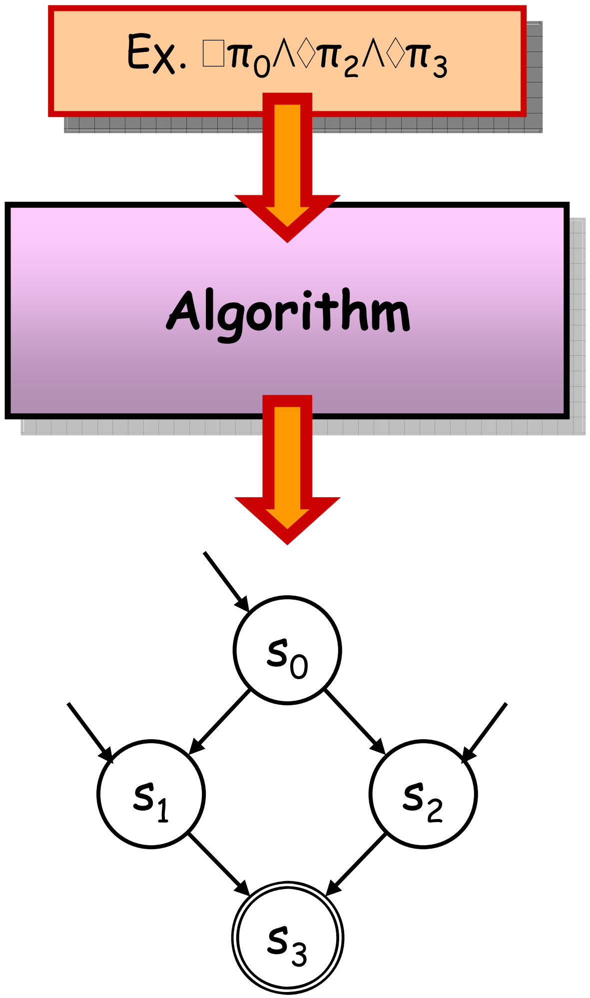
- "Fairness conditions"
- Computational complexity
  - Size of H1∩H2: $O(n_1 n_2)$
  - Size of final automaton $O(\exp(|\varphi|))$
  - Set intersections and complementation
    - set theoretic operations, not reachability etc.
- Note: graph is acyclic
- Final hybrid automaton might have empty guards and invariant sets
  - Then formula might be unsatisfiable

$LTL_{U,\Box}$ fragment

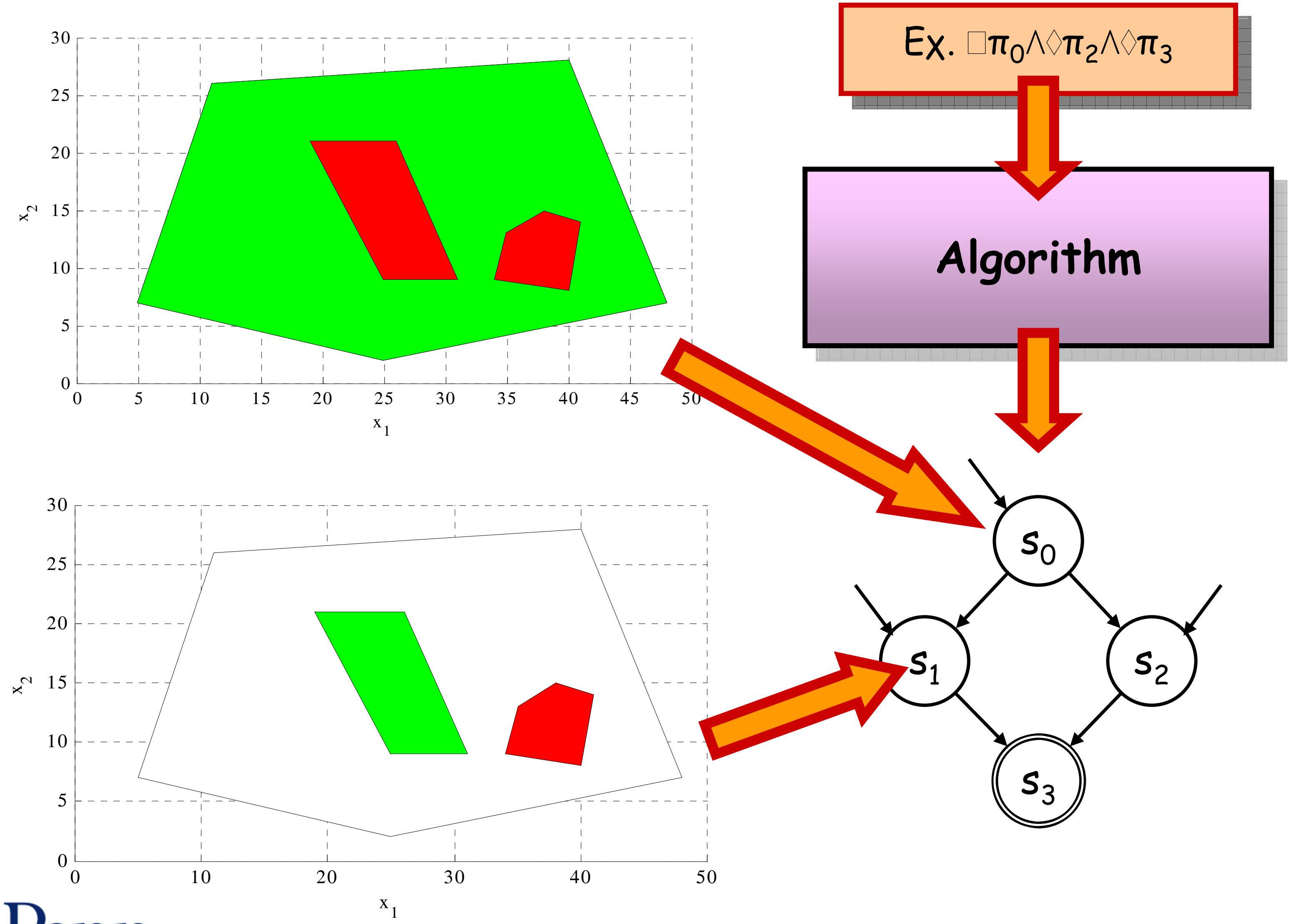
- Similar to $LTL(U,\vee,\wedge)$ we can get an algorithm for $LTL(\Box,\vee,\wedge)$
- General solution for $\varphi_1 \wedge \varphi_2$ or $\varphi_1 \vee \varphi_2$
  - with $\varphi_1 \in LTL(U,\vee,\wedge)$ and $\varphi_2 \in LTL(\Box,\vee,\wedge)$
- More general solution for any formula:
  - $\varphi ::= \pi^* \mid \varphi_{\Box} \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \pi^* U \varphi$
  - $\varphi_{\Box} \in LTL(\Box,\vee,\wedge)$
  - Ex. $\Box\pi_0 \wedge \Diamond(\pi_2 \wedge \Diamond(\pi_3 \wedge \Diamond(\pi_4 \wedge (\neg\pi_2 \wedge \neg\pi_3) U \pi_1)))$

“Abstract” Hybrid Automaton





"Abstract" Hybrid Automaton

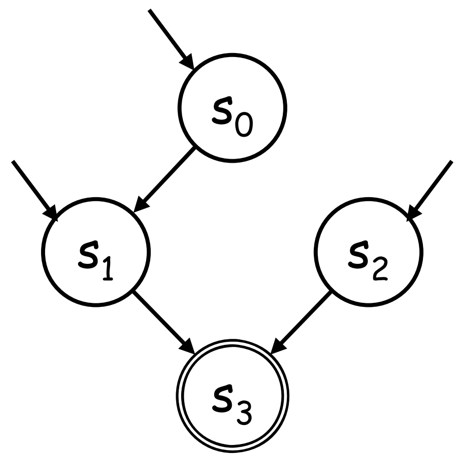


A practical solution

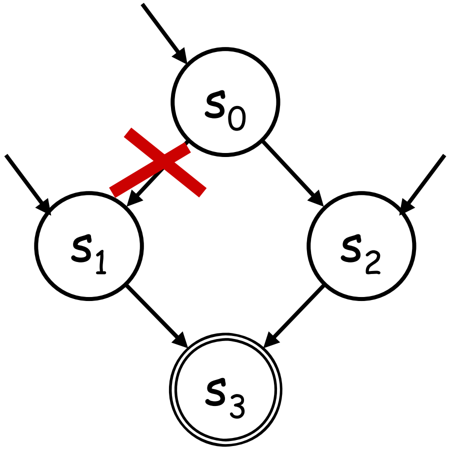
Backward reachability using
Breadth First Search:

✓ fix initial conditions

✓ one outgoing edge from each control location



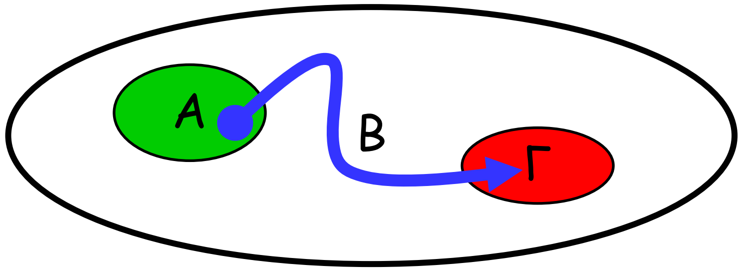
2 Derive controller constraints



Feedback control law $g_c : X \longrightarrow U$

Controller constraints $c = \{A, B, \Gamma\}$

- A initial conditions, i.e. $x(0) \in A$
- Γ final conditions, i.e. $x(t_g) \in \Gamma$
- B invariant, $\forall t \in [0, t_g].x(t) \in B$



$c_3 = \{\cup_{e \in I3} Guard(e), Inv(s_3), \emptyset\} \implies g_{c3}$

$c_1 = \{Guard(e_{I1}) \cup Init(s_1), Inv(s_1), Guard(e_{O1})\} \implies g_{c1}$

$c_2 = \{Init(s_2), Inv(s_2), Guard(e_{O2})\} \implies g_{c2}$

$\cancel{c_0 = \{Init(s_0), Inv(s_0), Guard(e_{O1})\} \implies g_{c0}}$

$c_0 = \{Init(s_0), Inv(s_0), Guard(e_{O1})\} \implies g_{c0}$

Main result

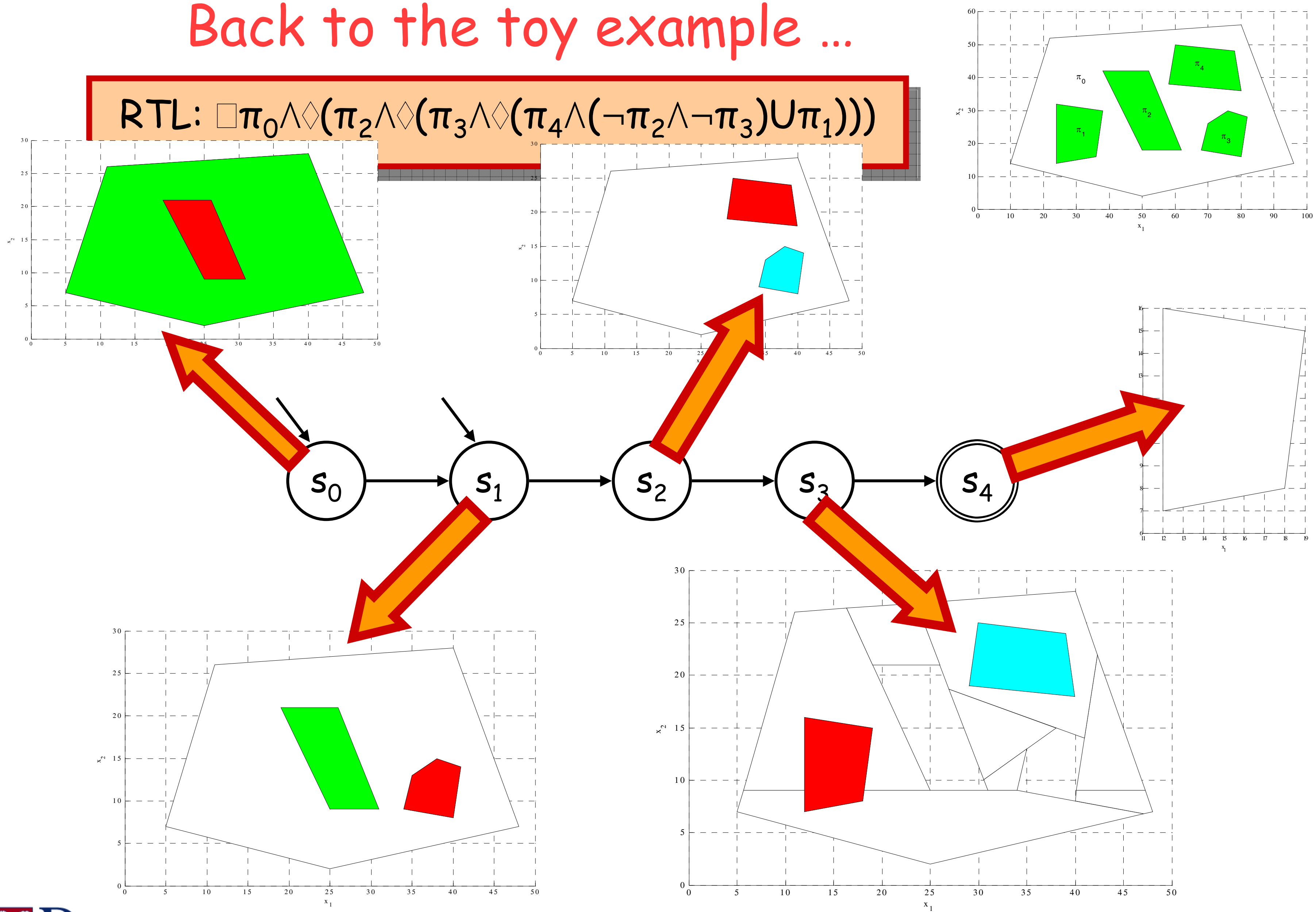
Let $\varphi \in LTL_{U,\square}$ be satisfiable wrt to our model of computation. Then:

(i) φ can be converted into list of controller specifications.

(ii) The hybrid automaton H resulting from the closed-loop dynamics satisfies: $\forall \eta \in L(H) \,.\, \eta \models \varphi$

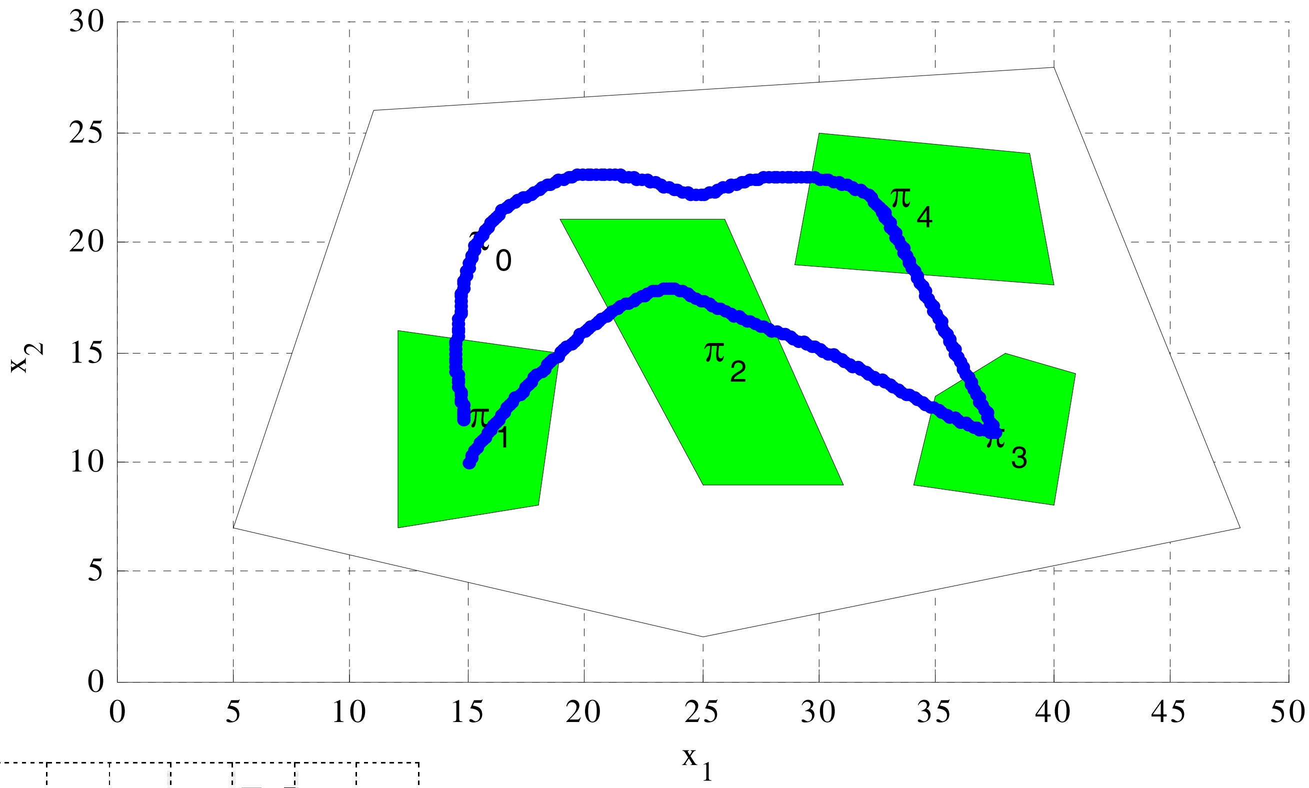
Back to the toy example …

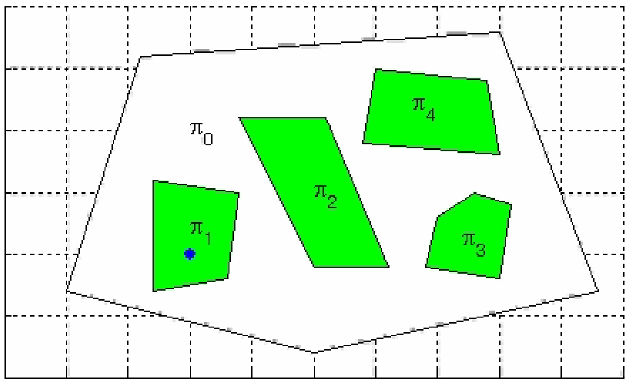
RTL: $\square \pi_0 \wedge \Diamond(\pi_2 \wedge \Diamond(\pi_3 \wedge \Diamond(\pi_4 \wedge (\neg \pi_2 \wedge \neg \pi_3) U \pi_1)))$

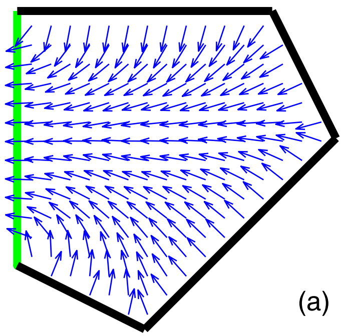


13 Controllers

Back to the toy example ...







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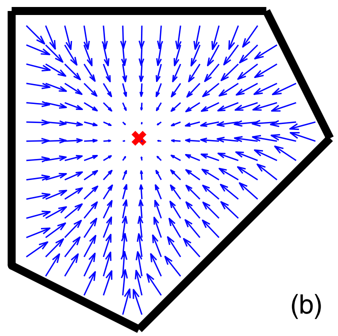

(b)

Conner et. al. IROS 2003

Conclusions

- ☑ Top-down approach for the synthesis of HA from temporal logic
  - ☑ identified a fragment of LTL that provides a modular construction
  - ☑ algorithm is based on the closure properties of HA
- ☑ Advantages of the proposed framework:
  - ☑ Clean separation between software specification and controller design
  - ☑ Potentially smaller number of required controllers
  - ☑ The closed-loop dynamics can be designed using different methodology in each control location
  - ☑ "Real" continuous time semantics
    - ☑ Ability to distinguish between events that must hold at a point in time or for an interval

Future work

- ☑ Introduce robustness into the system
  - ☑ Fainekos, Girard, Pappas: *Hierarchical Synthesis of Hybrid Controllers from Temporal Logic Specifications*, HSCC 2007 (to appear)
- ❑ Complete the framework for the full LTL
  - ❑ i.e. add "liveness"
- ❑ Design for distributed specifications
- ❑ Built a complete toolbox with libraries of controllers

Thank You!

Any Question(s) ?