MAE384 Fall 2012 Homework #4

In all problems, the argument of a sinusoidal function is always in radian.

1. Evaluate the first derivative of the function, \( f(x) = \cos(x^3) \), for the interval \( 0 \leq x \leq 3 \). First, find \( f''(x) \) analytically to prepare for later discussions.
   (a) Evaluate \( f'(x) \) at the discrete points of \( x = 0, 0.1, 0.2, ..., 2.9, 3.0 \) by setting \( h = 0.1 \) and using the following two formulas: (i) The 2-point forward difference scheme (1st formula in Table 6-1 in p. 259), and (ii) The 3-point forward difference scheme (2nd formula in Table 6-1 in p. 259). Plot the numerical results and analytic solution (total of 3 curves) in a single figure.
   (b) Repeat (a) but now set \( h = 0.01 \) and evaluate \( f'(x) \) at \( x = 0, 0.01, 0.02, ..., 2.99, 3.0 \).
   (c) Discuss the results in (a) and (b). [4 points]

2. Consider the non-uniform grid (shown in the diagram below) with \( x_i - x_{i-1} = 3h \) and \( x_{i+1} - x_i = h \). Derive a 3-point finite difference formula for the first derivative of \( f(x) \) at \( x = x_i \) that has a truncation error of \( O(h^2) \). Your formula should have the form:
   \[
   f'(x_i) = A f(x_{i-1}) + B f(x_i) + C f(x_{i+1}) + O(h^2) .
   \]
   Please clearly describe what your \( A, B, \) and \( C \) are in the final answer. [2 points]

3. All of the formula in Table 6-1 have a truncation error of \( O(h) \), \( O(h^2) \), or \( O(h^4) \). Try to derive a five-point finite difference formula for the second derivative of \( f(x) \) at \( x = x_i \) that has a truncation error of \( O(h^3) \). Moreover, the formula must have the following form:
   \[
   f''(x_i) = A f(x_{i-1}) + B f(x_i) + C f(x_{i+1}) + D f(x_{i+2}) + E f(x_{i+3}) + O(h^3) .
   \]
   In other words, the five points must include \( x_i \) itself, one point to its left, and three points to its right. The spacing between two adjacent grid points is \( h = \) constant. After solving the problem, clearly state what your \( A, B, C, D, \) and \( E \) are. [4 points]