Uniformly Accelerated Motion and Projectile Motion

Readings: Hay (1993), *The Biomechanics of Sports Techniques*, Prentice Hall, Englewood Cliffs, NJ, pp. 28-43 (posted on class web page).

Uniformly Accelerated Motion (UAM)

- Under certain circumstances, the human body (and/or other objects such as a ball or a shot) experience (approximately) *constant accelerations* resulting from (approximately) *constant resultant forces* applied to them.
- Example 1: A softball player sliding into second base. During a portion of the slide friction applies an approximately constant braking force.

UAM

- Example 2: During the flight phase of a long jump (high jump, shot put, diving, etc.) the body (or implement released by the body) experiences approximately constant acceleration (both in vertical and horizontal directions) when all other forces (e.g., air resistance) can be neglected compared to gravity. While this is a good assumption for some sports, it is a bad assumption for others like _______, ______, or _________.

Basic Equations of UAM

$$v_f = v_i + \qquad (1)$$

$$d = \tfrac{1}{2}(v_i + \qquad (2)$$

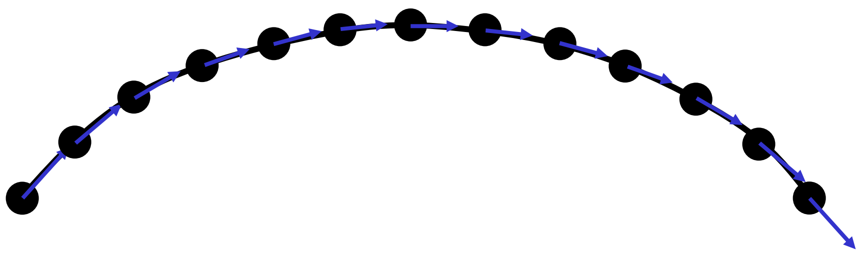
$$d = v_i t + \qquad (3)$$

$$v_f^2 = v_i^2 + \qquad (4)$$

Projectile Motion: A special case of uniformly accelerated motion

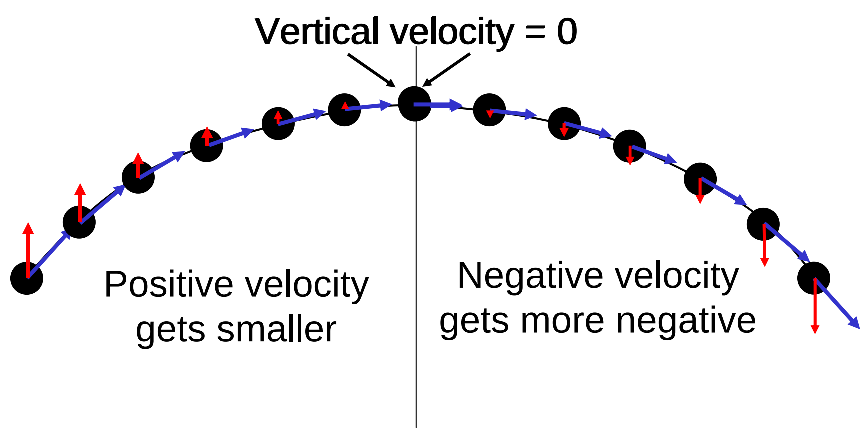
If air resistance is negligible then only gravity affects the path (or trajectory) of a projectile.

This path is a ________.

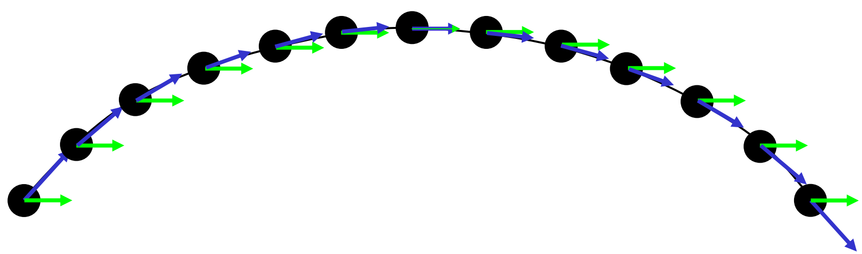


Horizontal and vertical components of velocity are _______________.

Vertical velocity decreases at a constant rate due to the influence of gravity.



Horizontal velocity will ________ _________.



Vertical Motion of a Projectile

Consider:

A diver steps off a 10 m tower and falls into the water below.

Projectile kinematics are very predictable during the fall:

Time	Position	Velocity	Acceleration
0.0 s			
0.5 s			
1.0 s			
1.5 s			
2.0 s			
2.5 s			

How do we determine the exact time and speed at impact?

10 m tower? 30 m cliff?

What do we know?

What do we want to find?

Equation(s)?

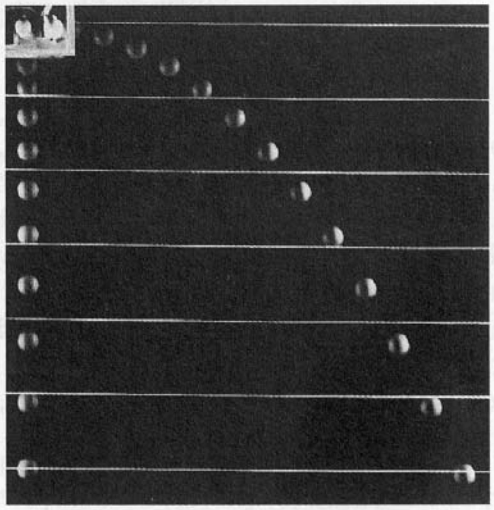


Fig. 3-6 One ball is released from rest, and at the same instant the other is given a horizontal initial velocity. Both balls are at the same elevation at any instant.

Horizontal Motion of a Projectile

Horizontal displacement or range of projectile is a main index of performance in many examples of projectile motion.

Examples?

IF air resistance is negligible, what forces affect horizontal motion of projectile?

If F_{horiz} = 0 horizontally, then a_{horiz} = 0

A special case of uniformly accelerated motion:

$$d =$$

If $a = 0$, then

$$d =$$

Rewriting for horizontal motion only…

$$d =$$

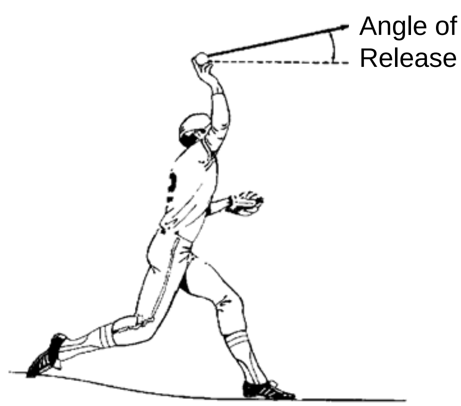
Provides good approximation of horizontal range when air resistance effects are negligible.

Three Primary Factors Affecting Trajectory

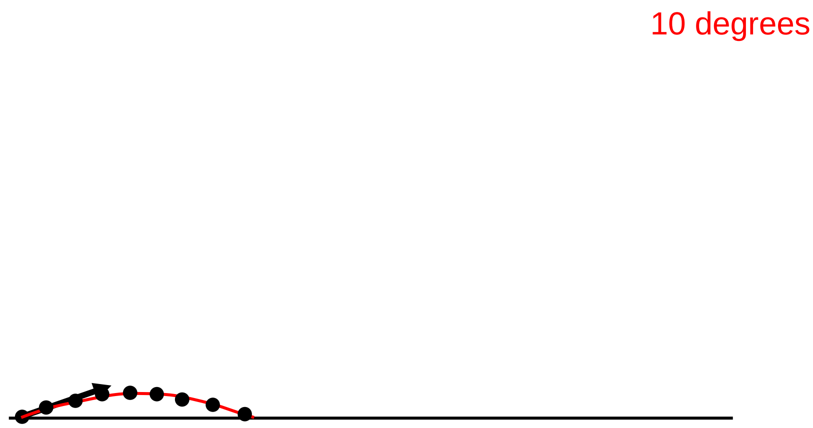
- Angle of Release
  - aka projection angle or take-off angle
- Relative Height of Release (RHR)
  - aka projection height
  - = release height – landing height
- Speed of Release
  - aka projection velocity or take-off velocity

Angle of Release

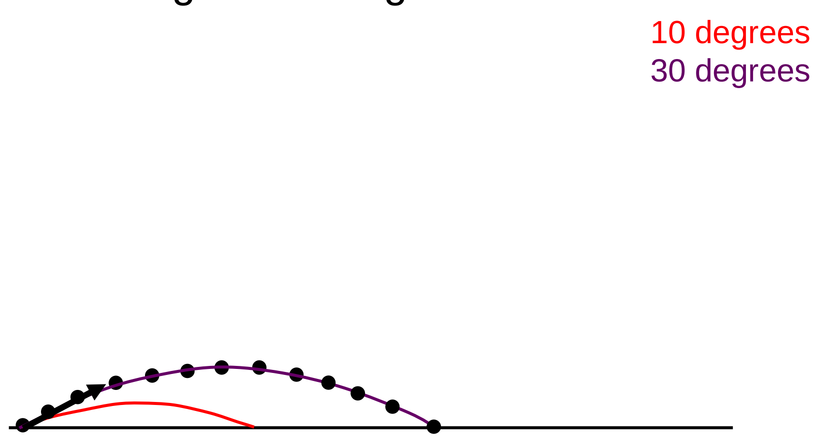
- The optimal angle of release is dependent on the goal of the activity.
- For maximal height the optimal angle is ___°.
- For maximal distance the optimal angle is ___° (if relative release height is zero).
- Optimal angle changes if relative release height is not equal to 0.



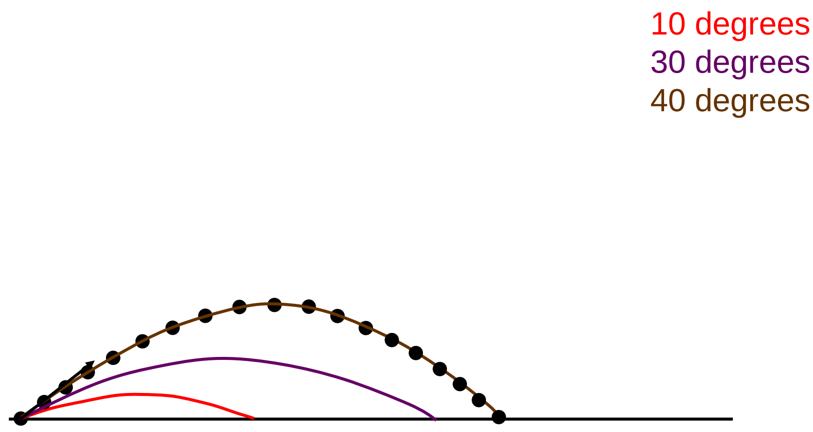
Release angle = 10 degrees



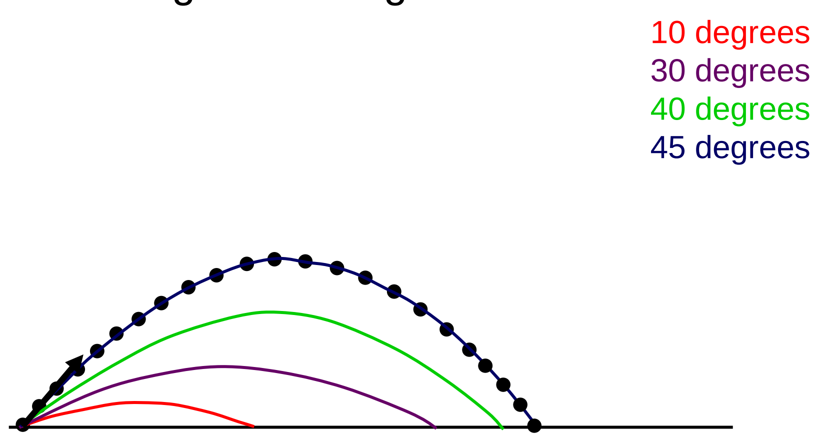
Release angle = 30 degrees



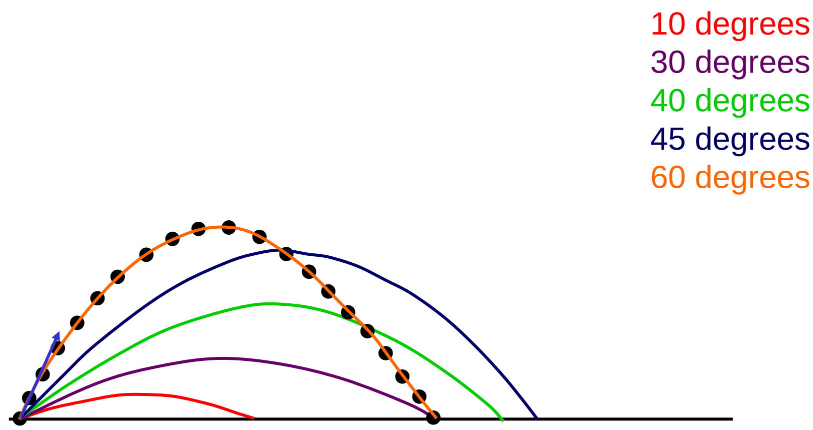
Release angle = 40 degrees



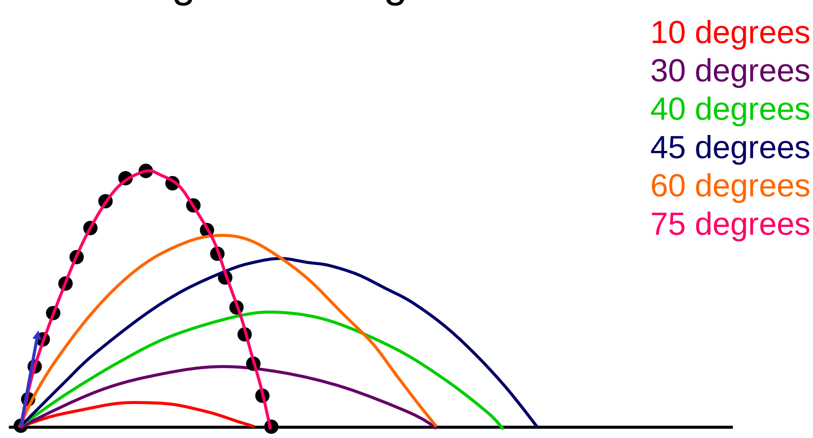
Release angle = 45 degrees



Release angle = 60 degrees



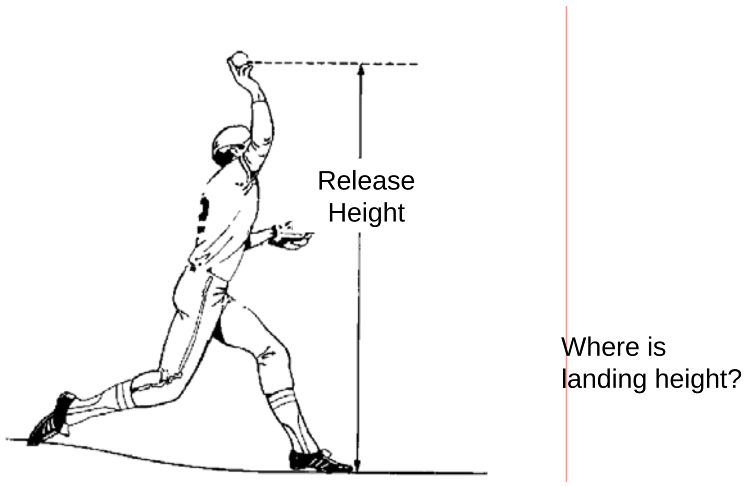
Release angle = 75 degrees



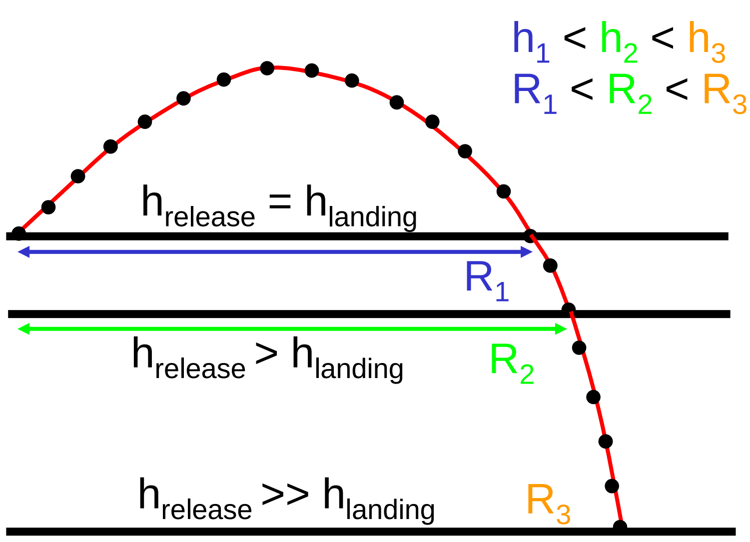
So angle that maximizes Range ($\theta_{optimal}$) = __ degrees (or so it appears)

Relative Height of Release (RHR)

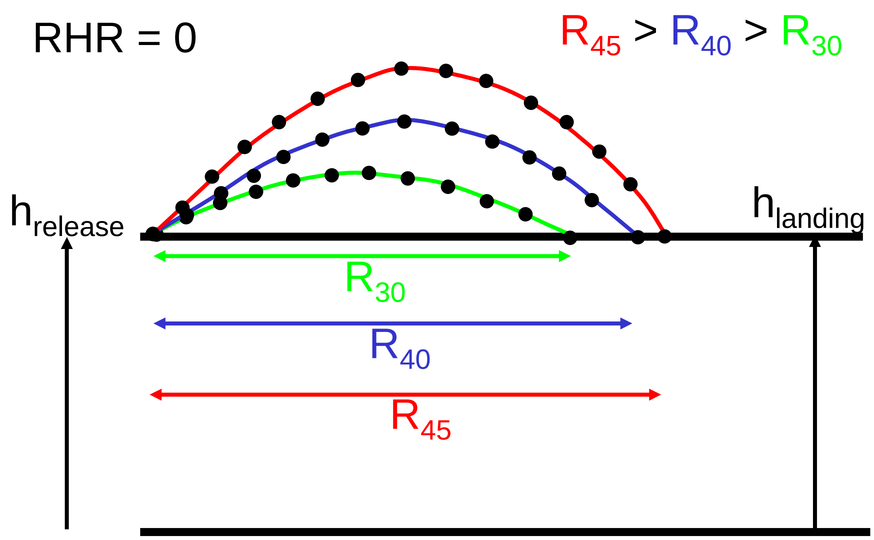
- RHR = release height - landing height



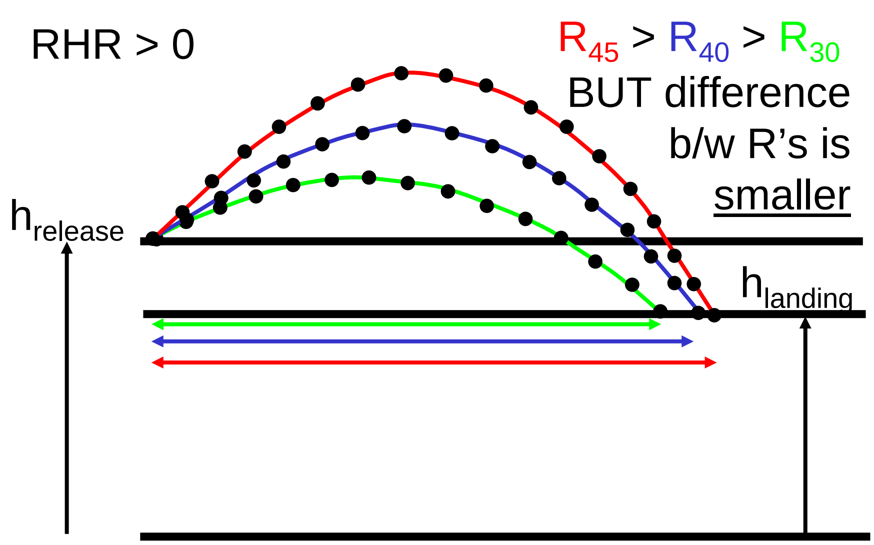
Effect of RHR on Horizontal Range (R) (when $\theta_{release}$ = 45 degrees)



RHR and Angle of Release interact to affect R

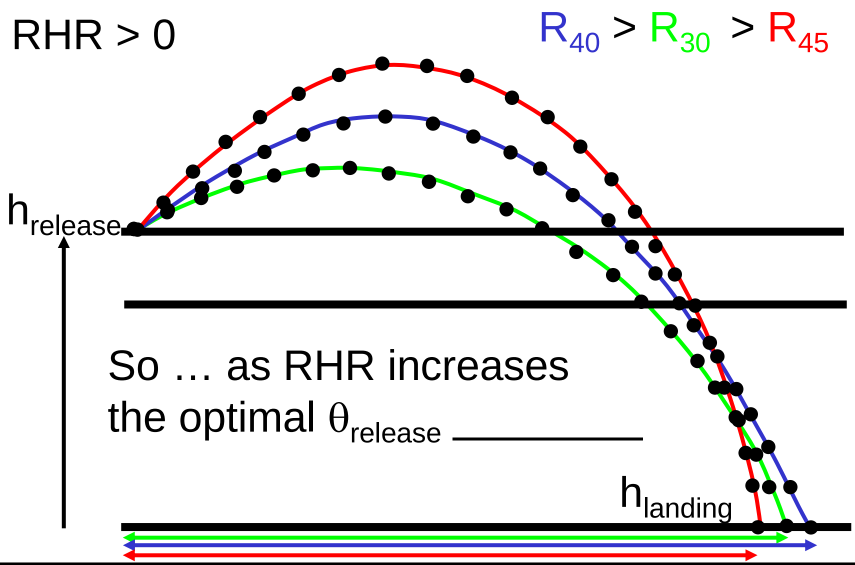


When $h_{landing} < h_{release}$



When $h_{landing} << h_{release}$

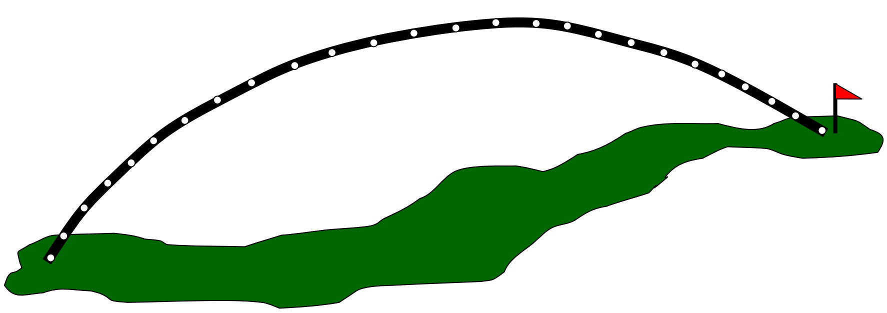
RHR > 0



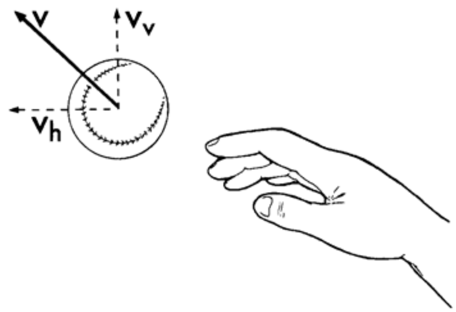
So … as RHR increases
the optimal $\theta_{release}$ ___________

It's possible to have a *negative* RHR ($h_{release} < h_{landing}$).

In this case the optimal $\theta_{release}$ is ___________ than 45 degrees.



Speed of Release



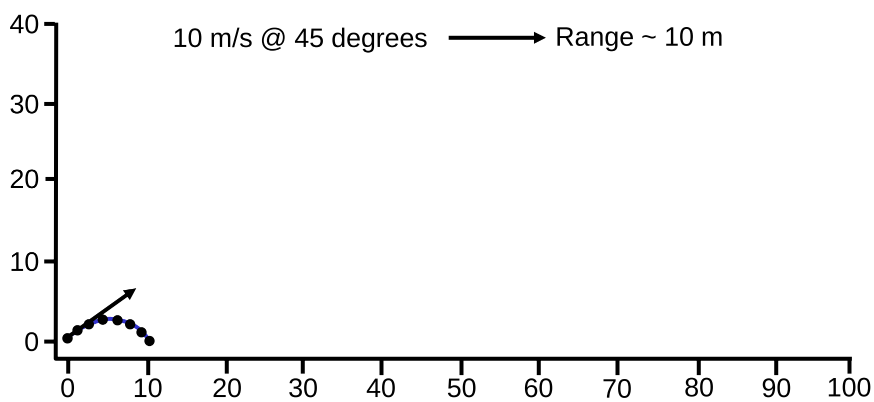
•Horizontal velocity does not change while the object is in the air.

•Vertical velocity changes by -9.8 m/s for every second the object is in the air.

$$R=[v^2\sin\theta\cos\theta+v\cos\theta((v\sin\theta)^2+2gh)^{1/2}]/g$$

•Because R α v^2, it has the greatest influence on the horizontal range of the projectile

The effect of Speed of Release on the horizontal range of a projectile



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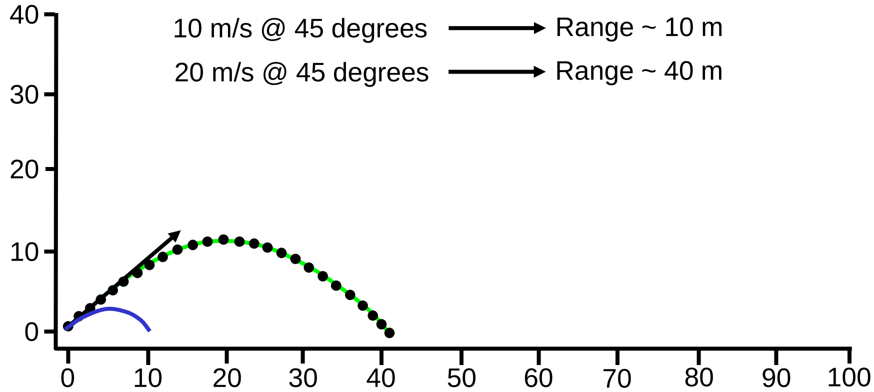
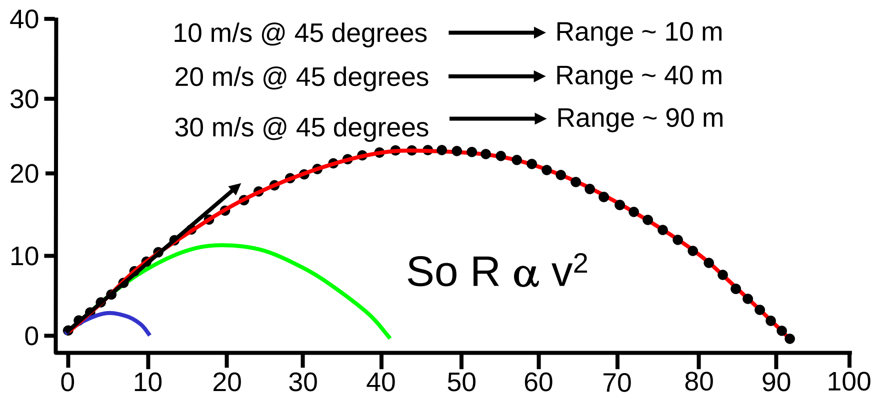
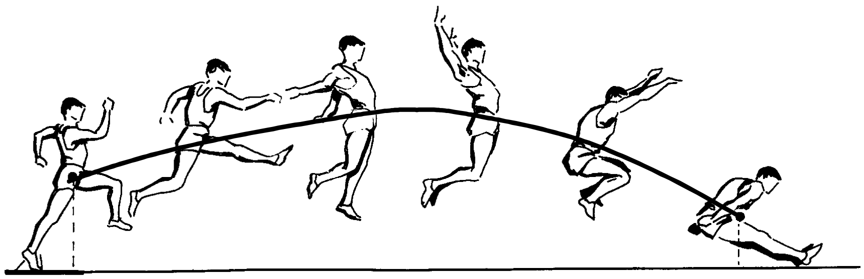


The effect of Speed of Release on the horizontal range of a projectile



Long Jump

- What is the optimum angle of takeoff for long jumpers?



- RHR > 0 (take-off height > landing height)
- Optimum Angle should be slightly less than ___________
- Research shows that it should be _______ degrees

Athlete	*Distance of Jump Analyzed (m)*	*Speed of Takeoff (m/s)*	*Optimum Angle of Takeoff for Given Speed (deg)*	*Actual Angle of Takeoff (deg)*
Mike Powell (USA)	**8.95**	**9.8**	**43.3**	**23.2**
Bob Beamon (USA)	**8.90**	**9.6**	**43.3**	**24.0**
Carl Lewis (USA)	**8.79**	**10.0**	**43.4**	**18.7**
Ralph Boston (USA)	**8.28**	**9.5**	**43.2**	**19.8**
Igor Ter-Ovanesian (USSR)	**8.19**	**9.3**	**43.2**	**21.2**
Jesse Owens (USA)	**8.13**	**9.2**	**43.1**	**22.0**
Elena Belevskaya (USSR)	**7.14**	**8.9**	**43.0**	**19.6**
Heike Dreschler (GDR)	**7.13**	**9.4**	**43.2**	**15.6**
Jackie Joyner-Kersee (USA)	**7.12**	**8.5**	**42.8**	**22.1**
Anisoara Stanciu (Rom)	**6.96**	**8.6**	**42.9**	**20.6**
Vali Ionescu (Rom)	**6.81**	**8.9**	**43.0**	**18.9**
Sue Hearnshaw (GB)	**6.75**	**8.6**	**42.9**	**18.9**

Actual Angle of Takeoff ~ _______ degrees

Long Jump

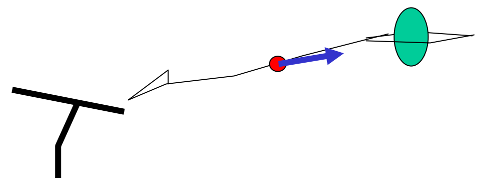
- when a jumper is moving at 10 m/s
  - the foot is not on the ground long enough to generate a large takeoff angle
  - so jumpers maintain speed and live with a low takeoff angle
- **v is the most important factor in projectile motion**

VALUES FOR HYPOTHETICAL JUMPS UNDER DIFFERENT CONDITIONS

Variable	Values for Actual Jump (1)	Speed of Takeoff Increased 5% (2)	Angle of Takeoff Increased 5% (3)	Relative Height of Takeoff Increased 5% (4)
Speed of Takeoff	8.90 m/s	9.35 m/s	8.90 m/s	8.90 m/s
Angle of Takeoff	20	20	21	20
Relative Ht of Takeoff	0.45 m	0.45 m	0.45 m	0.47 m
Horizontal Range	6.23 m	6.77 m	6.39 m	6.27 m
Change in Horiz Range	--	**0.54 m**	**0.16 m**	**0.04 m**
Distance of Jump	7.00 m	7.54 m	7.16 m	7.04 m

A Final Example of Optimizing Projectile Motion

Swim Starts



McLean et al. (*J. Appl. Biomech.*, 2000)

Similar to Long Jump

• maximize speed of takeoff
• small but positive projection height
• optimal angle should be slightly less than 45 degrees

Takeoff Velocity	Typical values
Speed	4-5 m/s
Angle	3-7 degrees

Why do you think the swimmers use such small angles of takeoff ? (<7° rather than 45°)