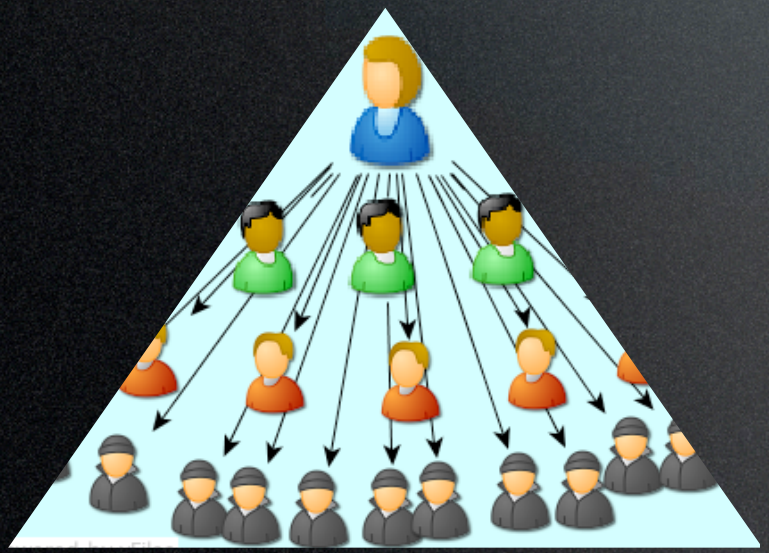
Community Detection using a Measure of Global Capacity to Influence



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Influence Based Definition of Community

- Claim : "Community comprises of individuals who have more capacity to influence individuals within the community than outsiders".

The *capacity* to have an effect on the character, development, or behavior of someone or something
or
the *effect* itself

Oxford English Dictionary

← Influence →

Influence in a large part is the ability to reach a crucial man through the right channels, the more the channels in reserve, the better

Pool & Kochen (1978)

Total Capacity to Influence

Influence Matrix → $P=\beta A + \beta\alpha A_1+\ldots+\beta\alpha^n A_n+\ldots= \beta A(I-\alpha A)^{-1}$

direct attenuation factor

adjacency matrix

indirect attenuation factor

$\underbrace{A x A x A \ldots\ldots A}_{n+1 \text{ times}}$

How Many

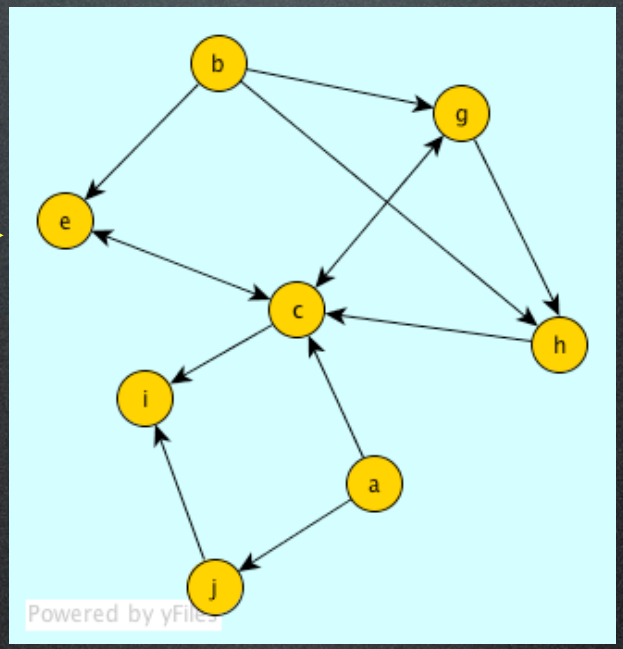
↓

The total influence of b on c thus depends on the number of (attenuated) channels from b to c or the sum of all weighted paths from b to c .

Who

↓

Do the two (1 hop) paths from node *a* to node *i* have equal capacity to influence node *i* ?



total n hop paths from i to j:

$i \xrightarrow{n} j = (A_n)_{ij}$

Probability of transmission from the immediate neighbors like e to c or g to c is β.

Probability of transmission over 1-hop paths such as b to c via e is $\beta\alpha$.

Probability of transmission along n-hop path is $\beta\alpha^{n-1}$

Influence Based Modularity

- Redefine Modularity
  Q= (connectivity within the community)-(expected connectivity within the community)

- Greater connectivity implies greater capacity to influence.

- Implies in best division of the network, influence of nodes within their community is more than their influence outside their community.

expected capacity of i to influence j

actual capacity of i to influence j

$$Q=\sum_{i,j}[P_{ij}-\overline{P_{ij}}]\delta(s_i,s_j)$$

index of the community node i belongs to

$\delta(s_i,s_j)=1$ if $s_i=s_j$,

otherwise $\delta(s_i,s_j)=0$

We have to maximize Q to obtain the communities the network is divided into.

Null Model

Given by the capacity to influence in an equivalent random graph which has the same number of nodes N as the original network

Expressed in terms of a NxN matrix $\overline{P}$ (where N is the total number of nodes in the network)

When all vertices are placed in a single group axiomatically Q=0

$$\sum_{i,j} [P_{ij}-\overline{P}_{ij}]=0$$

$$W=\sum_{i,j} P_{ij}=\sum_{i,j}\overline{P}_{ij}$$

Total expected capacity to influence (W)= Total actual capacity to influence

Expected influence on a node j(W_j^{in}) =Actual influence on node j in the original network

$$W_j^{in}=\sum_i P_{ij}=\sum_i \overline{P}_{ij}$$

Expected capacity of a node i to influence other (W_i^{out}) = Actual capacity of node i to influence of the node i in the original network

$$W_i^{out}=\sum_j P_{ij}=\sum_j \overline{P}_{ij}$$

Expected Capacity to Influence

Expected capacity of node i to influence node j

$$\bar{P}_{ij}=f_1(W_i^{out})f_2(W_j^{in})$$

where f_1 and f_2 are some functions.

$$W_i^{out}=\sum_j f_1(W_i^{out})f_2(W_j^{in})$$

$$=f_1(W_i^{out})\sum_j f_2(W_j^{in})$$

$$W_j^{in}=\sum_i f_1(W_i^{out})f_2(W_j^{in})$$

$$=f_2(W_j^{in})\sum_i f_1(W_i^{out})$$

$$f_1(W_i^{out})=C_1W_i^{out}$$

$$f_2(W_j^{in})=C_2W_j^{in}$$

For some constants C_1 and C_2

Detecting Community Structure

- Once modularity Q is derived , an algorithm is to be selected that divides the network into communities in a manner that optimizes Q.
- Decision version of modularity maximization is NP complete (Brandes et al.,2008)
- We use leading eigenvector method to obtain approximate solution.

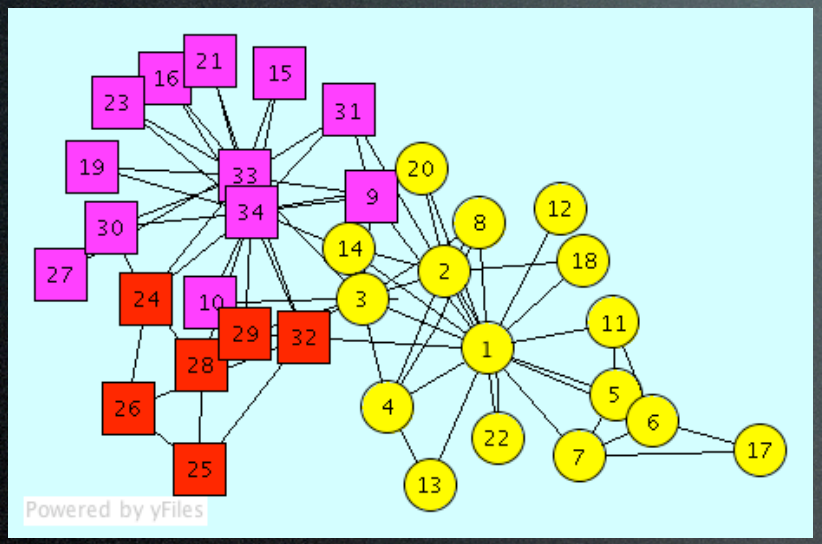
$$W=\sum_{i,j}\bar{P}_{ij}=\sum_{i,j}C_1C_2W_i^{out}W_j^{in}$$

$$=C_1C_2W^2$$

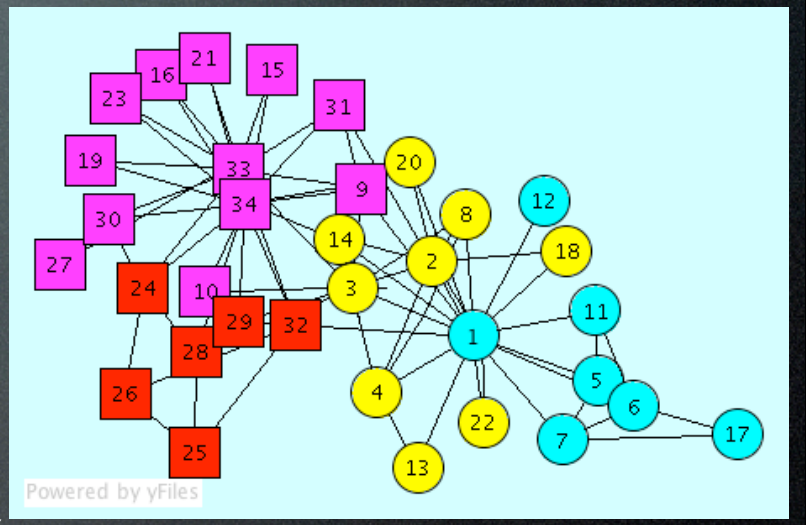
$$\bar{P}_{ij}=W_i^{out}W_j^{in}/W$$

$$Q=\sum\,[P_{ij}-(W_i^{out}W_j^{in}/W)]\delta(s_i,s_j)$$

Zachary's Karate Club



$\alpha=\beta=1/N$
$N=34$

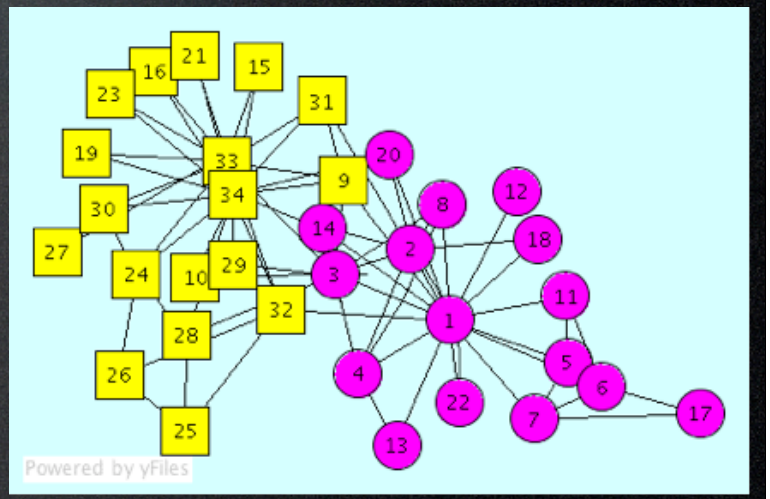


Communities found when our algorithm is run till termination

Communities found when Newman's (2004) algorithm is run till termination

Two factions the club got divided into are represented by circles and squares resp.Nodes predicted to belong to the same community are shown by the same color.

Single Iteration of Newman's and our algorithm



College Football

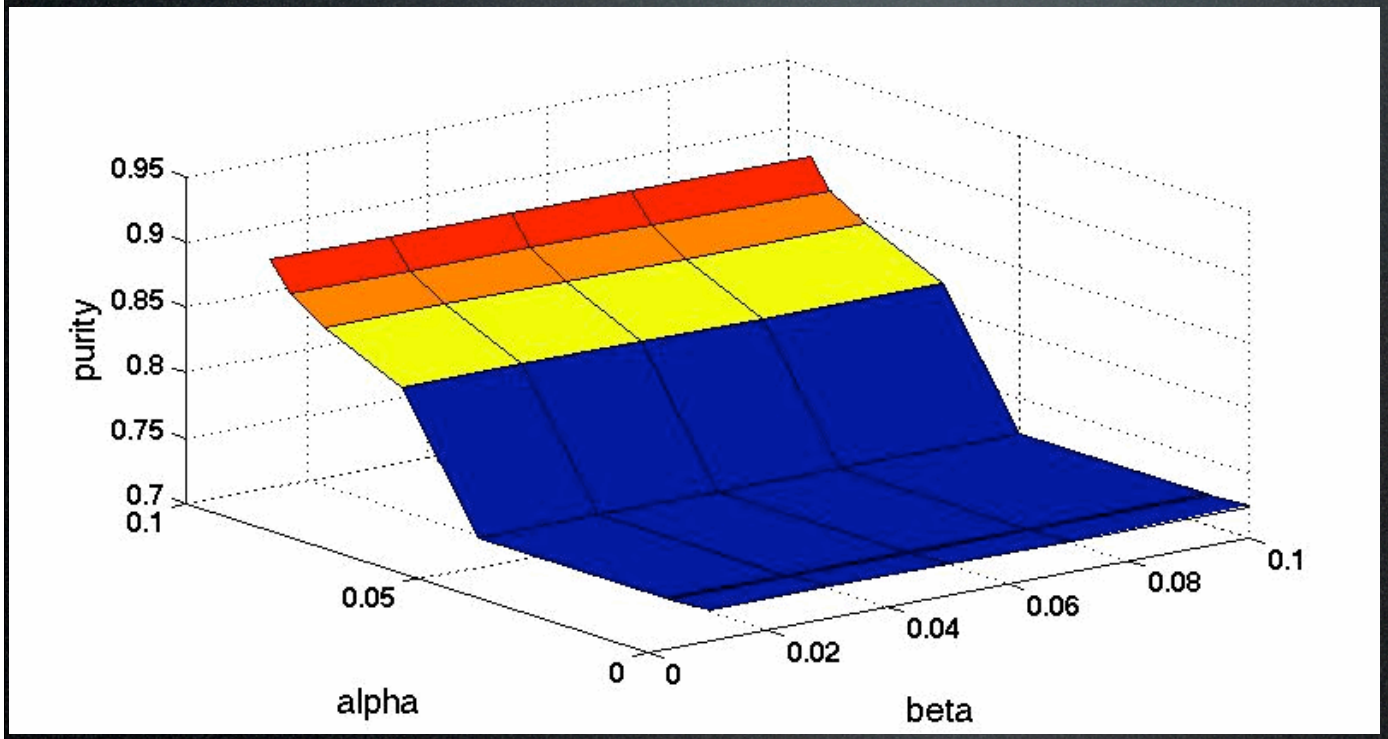
Schedule of Division 1 games for 2000 season (Girvan et al., 2002)

nodes: teams

edges: regular season games between 2 teams they connect

inter-conferences and intra-conferences games not equally distributed → natural communities may be bigger than the conferences

number natural communities predicted changes from 8 at α=0 to 4 at α=0.1



Purity

Similarity of a node pair =1 if the two nodes actually belong to the same community(observed), 0 otherwise.

Purity of a detected community= total pairwise similarity of nodes belonging to that community

Maximum total similarity obtained when all teams belonging to same community end up in same predicted communities

Purity of community detection= total pairwise similarity of all detected communities /maximum total similarity

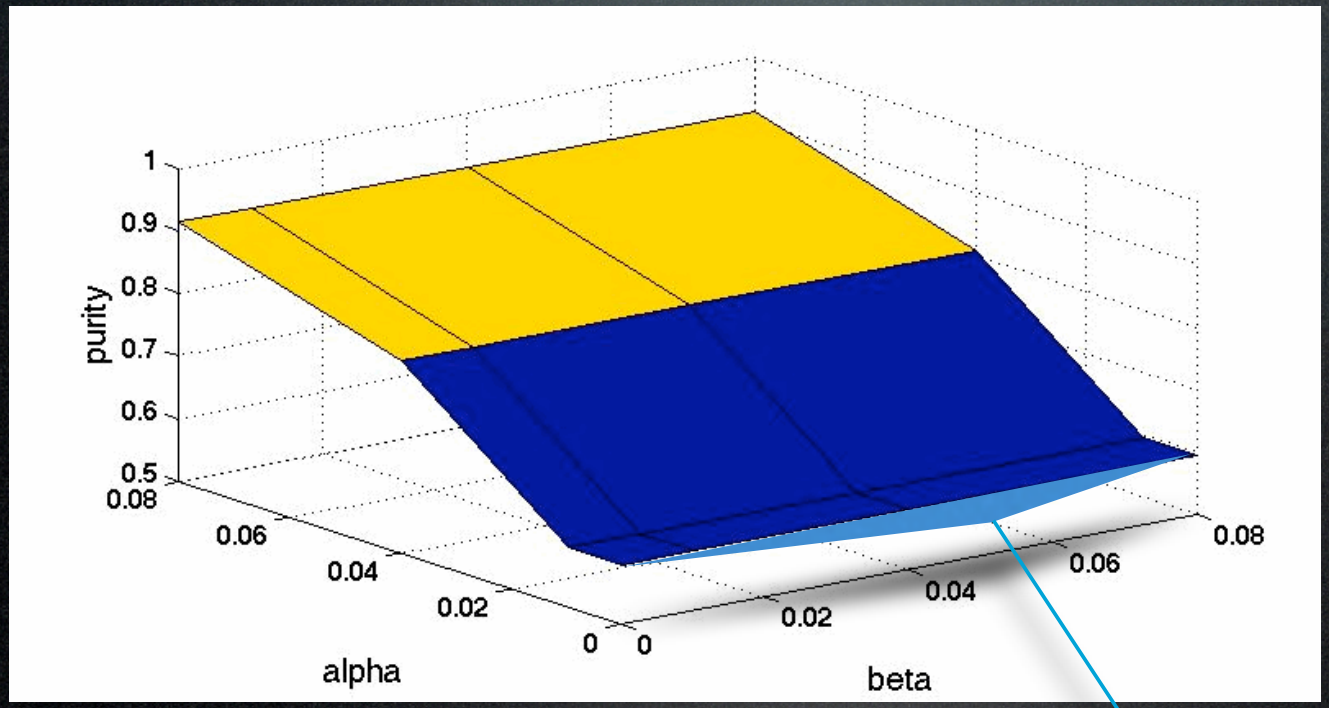
Political Books

Political books data compiled by V.Krebbs

nodes:books about politics sold by online bookseller Amazon

edges:co-purchasing of books by the same buyers

49 marked as *conservative*, 43 as *liberal* and 13 as *neutral*



reduces to community detection using edge based modularity (Newman,2006) when alpha=0

α=0.08 leads to 2 groups with 6 of the *neutral* books in one group containing 52 books (46 *conservative*) and 7 in the other group containing 53 books (43 *liberal*)
Indicates that 6 of the *neutral* books maybe *conservatively* inclined and 7 of them be *liberally* inclined.