

# Real-time Production Performance Analysis Using Machine Degradation Signals: a Two-Machine Case

Yunyi Kang, Hao Yan and Feng Ju

**Abstract**—Machine degradation has significant impact on the production system performance. Its variation might lead to large deviation from the steady state performance of the whole system. In this work, we build up a model to estimate the long-term production performance of the two-machine-and-one-buffer production systems, given the real-time machine degradation signals. A phase-type distribution is generated to mimic the remaining life distribution of each machine given the degradation signal. Then a continuous Markovian model is formulated to predict long-term system throughput rate for a two-machine-and-one-buffer system. With the fluctuation of machine degradation signals, such a model can effectively estimate the expected system performance in real-time.

## I. INTRODUCTION

Performing real-time system output estimation and control of modern manufacturing systems is important while still challenging for many practitioners. The fluctuations of machine conditions, random disruption events, and maintenance activities can influence instant system performance and long-term predictions [1]. Especially as modern manufacturing systems are becoming more and more complex, system performance prediction based on real-time system information is highly demanded.

Moreover, with the development of sensor technology, massive sensors have been deployed to collect machine condition information, such as machine degradation signals, to monitor real-time system performance. However, a significant gap still exists between data collection and system level decision making. Therefore, a novel approach by integrating available sensor information and system physical properties is needed to evaluate system performance and conduct production control in real-time.

In this work, machine degradation, a gradual and accumulating process that deteriorates the machine operating conditions, is considered. It can influence the performance of a production system, such as production rate, product quality, energy consumption and production cost, which are discussed in many research works

[2]. Typically, a degradation signal is defined as a quantity computed from sensor information that captures the current state of the machine and provides information on how that condition is likely to evolve in the future [3]. Machines will fail when the degradation signal reaches a well-defined threshold.

To model the production systems with degrading machines, earlier research works develop queuing models to estimate system long-term performances [4], [5], [6]. In addition, Markovian analysis is also investigated based on the Bernoulli models, geometric models, multi-state models, and exponential models [7], [8], [9]. However, these models are difficult to incorporate real-time degradation information due to the strong requirement on operating distributions. Moreover, such a requirement is not widely met in the production practice, according to the empirical and analytical studies [10], [11]. In recent years the newly developed analytical models provide higher flexibility, with adjustable parameters on machine and thus less requirement on machine operating distribution [12], [13], [14], [15]. Nevertheless, these models do not provide clear guidance on incorporating real-time degradation signals to update system parameters and to fit for long-term production system performance estimation.

On the other hand, the reliability and prognostics research typically investigates real-time sensing data and degradation signals for degrading machines to predict performance based on machine health status from a single sensor [3], [16] and multiple sensors [17], [18]. However, such research mainly focuses on individual machine, seldom discussing how to integrate the study results to complex manufacturing systems and improve the system performance. In complex manufacturing systems, it is difficult to directly extend the study on individual machine to larger systems without a comprehensive study on system properties. Therefore, a model on system performance estimation using real-time degradation signals is pressingly needed.

To fill the research gap, we develop an analytical model to estimate the long-term performance of a two-machine-and-one-buffer production system given the degradation signal of each individual machine. To the best of our knowledge, this is the first work on system throughput modeling utilizing the real-time degradation

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signals for online remaining life prediction and real-time Bayesian updating. Such a model could provide a powerful tool to effectively estimate the system performance in real-time by incorporating the degradation process of individual machines and their interaction in the system level.

The rest of the paper is organized as follows: Section II describes the problem and provides model assumptions; Section III derives machine remaining life distributions and converts those to phase-type distributions using moment matching; Modeling procedures and Bayesian updating using real-time information are shown in Section IV with an illustrative case; Section V is dedicated to conclusions and future work.

## II. PROBLEM DESCRIPTION AND ASSUMPTIONS

An illustration of the two-machine-and-one-buffer system is shown in Figure 1. The circles and the rectangle represent the machines and buffer respectively. The flow of working parts within the manufacturing line is directed using arrows. For each machine, a specific degradation path is followed, with transforming probability depending on the operating states. The assumptions are addressed in the following with regards to the characteristics of machines, buffers, and their interactions.

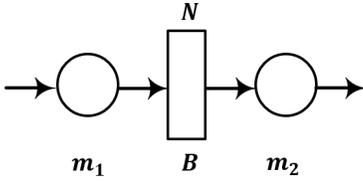


Fig. 1: A two-machine-and-one-buffer system

- 1) There are two machines in the system, denoted as  $m_1$  and  $m_2$ . The capacity of the buffer  $B$  is finite with an upper bound  $N$ .
- 2) For each machine  $m_k$ , it is collecting the real-time degradation signal  $z_k(t)$ , which follows a general path model with exponential degradation signal. We assume that the machine will fail when the degradation signal touches the failure threshold, denoted as  $D_k$ , of the machine,  $k = 1, 2$ .
- 3) The two machines are independent. The processing time (cycle time) for one part at machine  $k$  is  $\tau_k$ ,  $k = 1, 2$ . Respectively, the capacity (processing speed) for machine  $m_k$  is  $c_k$ ,  $k = 1, 2$ .
- 4) Machine  $m_1$  is blocked if it is up and the buffer is full at the beginning of a time slot. Machine  $m_2$  is never blocked.
- 5) Machine  $m_2$  is starved if it is up and the buffer is empty at the beginning of a time slot. Machine  $m_1$  is never starved.
- 6) The repair time for machine  $m_k$  follows exponential distribution with parameter  $\mu_k$ ,  $k = 1, 2$ .
- 7) The degradation signal or health index is observable or can be estimated in real-time to quantify machine health condition. The failure of a unit is often assumed to occur when the corresponding degradation signal first passes a predefined failure threshold. To model the evolution of a general degradation signal, we follow the general degradation path model as shown below.

$$z(t) = \eta(t; \theta) + \epsilon(t) \quad (1)$$

In this paper, we assume the degradation process  $\eta(t; \theta)$  can be modeled by a linear model  $\eta(t; \theta) = \theta_0 + \theta_1 t$ . The noise distribution of  $\epsilon(t)$  and the prior distribution of  $\theta$  is assumed to be normal.

The problem to be studied in this paper is to develop an analytical model to continuously predict long-term production performance given real-time machine degradation signals in a two-machine-and-one-buffer system.

## III. REMAINING USEFUL LIFE DISTRIBUTION FOR EACH MACHINE

### A. Machine operating distribution

We would like to derive the remaining life distribution of the machine health condition. Denote the remaining life of the machine  $k$  as  $R_k$ . Based on the assumption 2) and 7), the cumulative distribution function (CDF) of the estimated remaining lifetime  $R_k$  can be given by

$$\begin{aligned} P(R_k < t | z_k(t)) &= P(z_k(R_k + t) \geq D_k | z_k(t)) \\ &= \frac{E(z_k(R_k + t) - D_k)}{\sqrt{\text{Var}(z_k(R_k + t))}} = \Phi(g_k(t)), k = 1, 2, \end{aligned} \quad (2)$$

where  $g_k(t) = \frac{E(z_k(R_k + t) - D_k)}{\sqrt{\text{Var}(z_k(R_k + t))}}$  and  $\Phi(\cdot)$  is the CDF of the standard normal distribution. Given that the remaining life is nonnegative, i.e.  $R_k \geq 0$ , a truncated version of Equation (2) can be developed as follows:

$$\begin{aligned} P(R_k < D_k | z_k(t), R_k \geq 0) &= \\ &= \frac{\Phi(g_k(t)) - \Phi(g_k(0))}{1 - \Phi(g_k(0))}, k = 1, 2. \end{aligned} \quad (3)$$

Furthermore, the first two moments, i.e., the mean and variance of the remaining life, can also be calculated using numerical methods.

### B. Machine operating-failure behavior estimation

Phase-type distributions are based on a method of stage technique defined as the time to absorption in a continuous time Markov chain (CTMC) with one absorbing state. It enables approximating the remaining life distribution of each machine using multiple operating states and therefore formulating a Markovian model to

estimate the system performance. In this work, we will adapt the given operating time distribution into a two-phase CTMC model as shown in Figure 2.

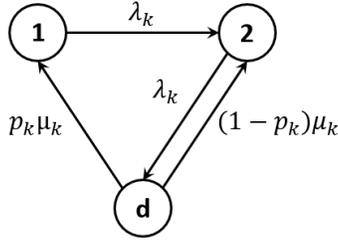


Fig. 2: Modeling single machine behavior using phase-type distribution

There are two virtual operating states and one failure state (denoted as  $d$ ) for each machine. When machine  $m_k$  fails, it can be recovered to either state 1 or state 2, with rate  $p_k \mu_k$  and  $(1 - p_k) \mu_k$  correspondingly. When the machine is in state 1, it can transfer to state 2 with rate  $\lambda_k$ . When the machine is in state 2, it can transfer to state  $d$  with rate  $\lambda_k$ .

### C. Moment matching

In order to find a phase-type distribution which can reflect the remaining life, we introduce a moment matching approach. The idea is to match the two moments of the remaining life distribution and the phase-type distribution, thus determining the unknown parameters in the phase-type distribution.

Let  $Y$  denote a random variable, representing the time between two failures in the machine as shown in Figure 2. Then the moment generating function of this variable,  $M_Y(z)$ , can be expressed as follows:

$$\begin{aligned} M_Y(z) &= \mathbf{E}(e^{zY}) = \int_{-\infty}^{\infty} e^{zy} f(y) dy \\ &= p \int_0^{\infty} e^{zy_1} e^{-\lambda_k y_1} dy_1 \int_0^{\infty} \lambda_k e^{-\lambda_k y_2} dy_2 \\ &\quad + (1-p) \int_0^{\infty} e^{zy_2} \lambda_k e^{-\lambda_k y_2} dy_2 \\ &= p \frac{\lambda_k}{\lambda_k - z} \frac{\lambda_k}{\lambda_k - z} + (1-p) \frac{\lambda_k}{\lambda_k - z}. \end{aligned} \quad (4)$$

Denote  $\eta_1$  and  $\eta_2$  as the first two moments of the phase-type distribution. Clearly,  $\eta_1$  and  $\eta_2$  can be obtained by taking the first and second order derivatives of  $M_Y(z)$ , and evaluating them at  $z = 0$ . Further, let  $\phi_1$  and  $\phi_2$  be the first two moment of the remaining life distribution. Note that the value of  $\phi_1$  and  $\phi_2$  can be approximated using Equation (3). The formulas of the

moment matching approach are shown as follows:

$$\begin{aligned} \frac{dM_Y(z)}{dz} \Big|_{z=0} &= \phi_1, \\ \frac{d^2 M_Y(z)}{dz^2} \Big|_{z=0} &= \phi_2. \end{aligned} \quad (5)$$

Therefore, we can solve the equations generated by the moment matching, thus constructing the phase-type distribution for machine's remaining useful life.

## IV. SYSTEM PERFORMANCE ANALYSIS

To estimate the performance of the two machines system, a Markovian model is formulated. Then the parametric updating scheme incorporating real-time degradation signals is provided using Bayesian methods.

### A. Modeling two-machine systems

In order to model the two-machine system with multiple machine operating states, we need to further specify the combination of machine states ( $\{1, 2, d\}$ ) for both machines and the buffer level in between.

First, we need to introduce the following notations:

- $X_{s_1 s_2}(h, t)$ : the probability density of machine  $m_1$  in state  $s_1$  and machine  $m_2$  in state  $s_2$ , with the buffer occupancy  $h$ ,  $0 < h < N$ , at time  $t$ .
- $Y_{s_1 s_2}(N, t)$ : the probability density of machine  $m_1$  in state  $s_1$  and machine  $m_2$  in state  $s_2$ , with full buffer at time  $t$ .
- $Y_{s_1 s_2}(0, t)$ : the probability density of machine  $m_1$  in state  $s_1$  and machine  $m_2$  in state  $s_2$ , with empty buffer at time  $t$ .

Following the idea from Jacobs [7] and Li [19], with an extension to multiple machine operating states, the dynamic transition equations of density functions  $X_{s_1 s_2}(h, t)$  can be obtained through the integral equations. Taking  $X_{11}(\cdot)$  as an example, the probability density function at time  $t + \Delta t$  can be expressed as the summation of the following components:

- The system stays in the same state from time  $t$  to  $t + \Delta t$ , with probability  $X_1(h + (c_2 - c_1)\Delta t, t) e^{-(\lambda_1 + \lambda_2)\Delta t}$ .
- Machine  $m_2$  remains in the operating state 1 and machine  $m_1$  turns up from down state at time  $t + \tau$ . The probability can be expressed as:  $e^{-\lambda_2 \Delta t} \int_0^{\Delta t} X_{d1}(h + c_2 \Delta t - c_1(\Delta t - \tau), t) p_1 \mu_1 e^{-\mu_1 \tau} d\tau$ .
- Machine  $m_1$  remains in the operating state 1 and machine  $m_2$  turns up from down state at time  $t + \tau$ . The probability can be expressed as:  $e^{-\lambda_1 \Delta t} \int_0^{\Delta t} X_{1d}(h + c_2(\Delta t - \tau) - c_1 \Delta t, t) p_2 \mu_2 e^{-\mu_2 \tau} d\tau$ .
- A miscellaneous term representing the deviation of estimation with order  $O(\Delta t^2)$ .

When  $\Delta t$  is small enough, at most one state transition is available. Therefore,

$$\begin{aligned} X_{11}(h, t + \Delta t) &= X_{11}(h + (c_2 - c_1)\Delta t, t)e^{-(\lambda_1 + \lambda_2)\Delta t} \\ &+ e^{-\lambda_2\Delta t} \int_0^{\Delta t} X_{d1}(h + c_2\Delta t - c_1(\Delta t - \tau), t)p_1\mu_1 \\ &\times e^{-\mu_1\tau} d\tau \\ &+ e^{-\lambda_1\Delta t} \int_0^{\Delta t} X_{1d}(h + c_2(\Delta t - \tau) - c_1\Delta t, t) \\ &\times p_2\mu_2 e^{-\mu_2\tau} d\tau + O(\Delta t^2). \end{aligned} \quad (6)$$

Following the similar idea, we can express all the other  $X_{s_1 s_2}(h, t)$ 's. Further, simplifying the right hand side of Equation (6) using Taylor expansion with an accuracy of  $O(\Delta t)$ , we obtain

$$\begin{aligned} \frac{X_{11}(h, t + \Delta t) - X_{11}(h, t)}{\Delta t} &= \\ &-(\lambda_1 + \lambda_2)X_{11}(h, t) + (c_2 - c_1)\frac{\partial X_{11}(h, t)}{\partial h} \\ &+ p_1\mu_1 X_{d1} + p_2\mu_2 X_{1d} + O(\Delta t^2). \end{aligned} \quad (7)$$

Taking the limit of  $\Delta t \rightarrow 0$ , an differential equation can be obtained as:

$$\begin{aligned} \frac{\partial X_{11}(h, t)}{\partial t} + (c_1 - c_2)\frac{\partial X_{11}(h, t)}{\partial h} &= \\ &-(\lambda_1 + \lambda_2)X_{11}(h, t) + p_1\mu_1 X_{d1} + p_2\mu_2 X_{1d}. \end{aligned} \quad (8)$$

Furthermore, since the Markov process is irreducible, the limiting distributions with regards to  $t$  exist. Introducing the notations:

$$X_{s_1 s_2}(h) = \lim_{t \rightarrow \infty} X_{s_1 s_2}(h, t). \quad (9)$$

Then Equation (8) can be transformed into the steady state equation as shown below:

$$\begin{aligned} (c_1 - c_2)\frac{\partial X_{11}(h)}{\partial h} &= -(\lambda_1 + \lambda_2)X_{11}(h) \\ &+ p_1\mu_1 X_{d1} + p_2\mu_2 X_{1d}. \end{aligned} \quad (10)$$

Due to page limitation, only the transition equation for  $X_{11}(h, t)$  is listed. Following the similar idea, the steady state equations can be obtained for all the remaining system states. Then, a matrix form, denoted as  $\mathbf{A}$ , can be generated to express all the steady state differential equations, as shown in the next page.

Therefore, we have to solve the equation to find the analytical results of  $\mathbf{X}(\mathbf{h})$ :

$$\mathbf{c}\mathbf{X}(\mathbf{h})' = \mathbf{A}\mathbf{X}(\mathbf{h}), \quad (11)$$

where

$$\begin{aligned} \mathbf{c} &= \begin{bmatrix} c_1 - c_2 & c_1 - c_2 & c_1 & c_1 - c_2 & \\ & c_1 - c_2 & c_1 & -c_2 & -c_2 & 0 \end{bmatrix}, \\ \mathbf{X}(\mathbf{h}) &= \begin{bmatrix} X_{11}(h) & X_{12}(h) & X_{1d}(h) & X_{21}(h) & X_{22}(h) \\ X_{2d}(h) & X_{d1}(h) & X_{d2}(h) & X_{dd}(h) \end{bmatrix}. \end{aligned} \quad (12)$$

The general solution of the differential equation systems has the following format:

$$\mathbf{X}(\mathbf{h}) = \sum_{i=1}^9 k_i \nu_i e^{\gamma_i h}, \quad (13)$$

where  $\gamma_i$  is the eigenvalue,  $\nu_i$  is the eigenvector and  $k_i$  is a constant that needs to be determined. All the  $\gamma_i$  and  $\nu_i$  can be found through the singular value decomposition approach on matrix  $\mathbf{A}$ .

The boundary state transition probabilities in steady state,  $Y.(0)$  and  $Y.(N)$ , can be determined following the similarly ideas. Furthermore, in order to find the values of  $k_i$  in Equation (13), the particular solutions at some bound conditions should be found.

First of all, notice that the summation of all the probabilities should be equal to one.

$$\int_0^N X.(h)dh + Y.(0) + Y.(N) = 1. \quad (14)$$

Besides, when the first machine is faster ( $c_1 > c_2$ ), and when both machines are operating, the buffer level cannot be zero; on the other hand, if the second machine is faster, the buffer level cannot be full. Therefore, we have the following conditions:

- If  $c_1 > c_2$

$$Y_{11}(0) = 0, Y_{12}(0) = 0, Y_{21}(0) = 0, Y_{22}(0) = 0. \quad (15)$$

- If  $c_1 < c_2$

$$Y_{11}(N) = 0, Y_{12}(N) = 0, Y_{21}(N) = 0, Y_{22}(N) = 0. \quad (16)$$

Following the procedures mentioned above, the probabilities of all the system states can be obtained, and thus the system performance can be determined as follows:

$$\begin{aligned} TP &= \sum_{s_1 \in \{1,2\}} \sum_{s_2 \in \{1,2\}} \left( \int_0^N (X_{s_1 s_2}(h) + X_{ds_2}(h))dh + \right. \\ &\quad \left. c_1 Y_{s_1 s_2}(0) + c_2 Y_{s_1 s_2}(N) \right). \end{aligned} \quad (17)$$

Notice that the system throughput is expressed as the summation of three components categorized using the buffer level: the time when the buffer is between 0 and  $h$ , the period when machine  $m_1$  is blocked and when machine  $m_2$  is starved.

### B. Bayesian updating

After the remaining life distribution and the throughput rate is estimated, we need to perform the real-time update on the machine remaining life prediction and the system throughput  $TP$  with more data. To do this, we define  $\eta_k(t; \theta) = \Gamma_k(t)\theta_k$  in the general path model (1)

$$\mathbf{A} = \begin{bmatrix} -(\lambda_1 + \lambda_2) & 0 & p_2\mu_2 & 0 & 0 & 0 & p_1\mu_1 & 0 & 0 \\ \lambda_2 & -(\lambda_1 + \lambda_2) & (1-p_2)\mu_2 & 0 & 0 & 0 & 0 & p_1\mu_1 & 0 \\ 0 & \lambda_2 & -(\lambda_1 + \mu_2) & 0 & 0 & 0 & 0 & 0 & p_1\mu_1 \\ \lambda_1 & 0 & 0 & -(\lambda_1 + \lambda_2) & 0 & p_2\mu_2 & (1-p_1)\mu_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & \lambda_2 & -(\lambda_1 + \lambda_2) & (1-p_2)\mu_2 & 0 & (1-p_1)\mu_1 & 0 \\ 0 & 0 & \lambda_1 & 0 & \lambda_2 & -(\lambda_1 + \mu_2) & 0 & 0 & (1-p_1)\mu_1 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & -(\mu_1 + \lambda_2) & 0 & p_2\mu_2 \\ 0 & 0 & 0 & 0 & \lambda_1 & 0 & \lambda_2 & -(\mu_1 + \lambda_2) & (1-p_2)\mu_2 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & \lambda_2 & -(\mu_1 + \mu_2) \end{bmatrix}$$

with  $\Gamma_k(t) = [1 \ t]$ ,  $\theta_k = [\theta_{k,0} \ \theta_{k,1}]^T$ . We can also write this in the vector format as follow  $\mathbf{z}_k = \Gamma_k \theta_k$ , where  $\mathbf{z}_k = [z(1), \dots, z(t)]$  and  $\Gamma_k = \begin{bmatrix} 1 & \dots & 1 \\ 1 & \dots & t \end{bmatrix}$ .  $t$  is the time to now from the last time machine fails.

Before the online Bayesian updating, we need to conduct the offline analysis. To do so, we need to collect the degradation signals over the time period  $i = 1, \dots, n$ , and then estimate the mean and covariance of the degradation coefficient  $\theta_k$  based on the  $n$  time points as sample mean  $\mu_{k,0} = \frac{1}{n} \sum_{i=1}^n \theta_{k,i}$  and sample covariance matrix  $\Sigma_{k,0} = \frac{1}{n-1} \sum_{i=1}^n (\theta_{k,i} - \mu_0)(\theta_{k,i} - \mu_0)^T$ . This mean and covariance can be used as the prior distribution. When collecting the real-time sensing data for machine condition, we perform the Bayesian updating by computing the posterior distribution of  $\theta_k$ , which is also normal distributed with mean and covariance updated as  $\mu_{k,1} = \Sigma_1^{-1} (\frac{\Gamma_k' \mathbf{z}_k}{\sigma_k^2} + \Sigma_{k,0}^{-1} \mu_{k,0})$ ,  $\Sigma_{k,1} = (\frac{\Gamma_k' \Gamma_k}{\sigma_k^2} + \Sigma_{k,0}^{-1})^{-1}$ . The posterior distribution can then be used to update Equation (3) as described in the previous sections.

### C. Validation

To evaluate the performance, 50 sets of simulation experiments are conducted. The system parameters are randomly generated using the following procedure:

*Procedure 1:*

- 1) Set up machine capacity  $c_1 = 1.1$ ,  $c_2 = 0.9$  and buffer level  $N = 3$ .
- 2) Set up machine repair rate  $r_1 = 0.55$ ,  $r_2 = 0.56$ .
- 3) Randomly select  $\theta_1$  and  $\theta_2$ , the two parameter in the degradation signals for the single machine.
- 4) Randomly select the time  $t$  that a machine has been operating on.

The performance measurement adopted is the relative error of the results from the analytical model and the corresponding simulation experiment, denoted as  $\delta_{acc}$ :

$$\delta_{acc} = \frac{TP_{sim} - \widehat{TP}}{\widehat{TP}} \times 100\%,$$

where  $TP_{sim}$  is the simulation results, and  $\widehat{TP}$  is the analytical solution to the model. The results are shown in Figure 3.

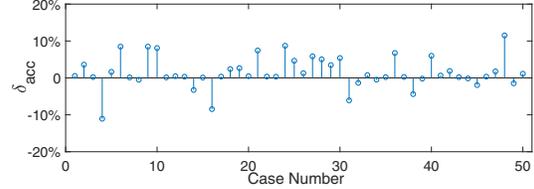


Fig. 3: Model validation for general serial production systems

It can be found that on average, the deviation of the production performance from the model is typically within 10% of that from the simulation results, which shows that the proposed model could deliver relatively high accuracy.

### D. An illustrative example

At the initial stage (when the system operates for around 75 cycles), both machines are in good operating states, with low degradation level. The mean remaining time before failure for both machines is long, according to the remaining life distributions. Therefore, the throughput rate is high. Theoretically, over the long run, the throughput rate cannot be larger than the minimum capacity in the system (which is commonly defined as the bottleneck). In this case, the minimum capacity is 0.9 for machine  $m_2$ . Since the machine is less likely to fail, the long-term estimated system throughput is very close to the theoretical upper bound, or 0.9.

Later, when either machine approaches to the threshold of breaking down, the estimated system performance drops significantly. Since the single machine degradation process is not monotone, it can be observed that the estimation increases when the status of a machine recovers in a short period of time. For example, between time 320 and 325, there is a decrease on the degradation level of machine  $m_1$ . The throughput rate, similarly, shows an increase between time 320 and 325 due to the recovery of machine status. However, when either of the machine fails, over the long run, the estimated production rate reaches to zero; even though for the short time, there might be output due to the remaining parts in the buffer. The estimated throughput will remain zero when the

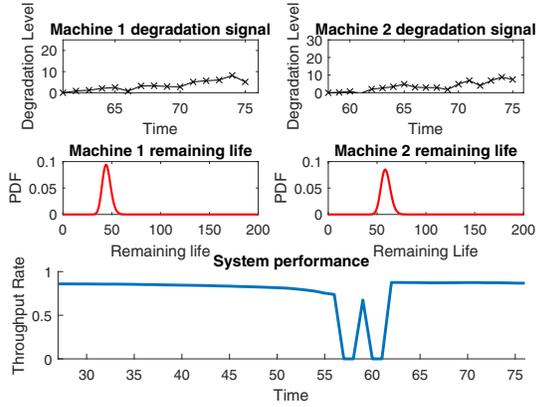


Fig. 4: An example when two machines are in low degradation levels

failed machine is under repair and then jump up when both machines are under operations.

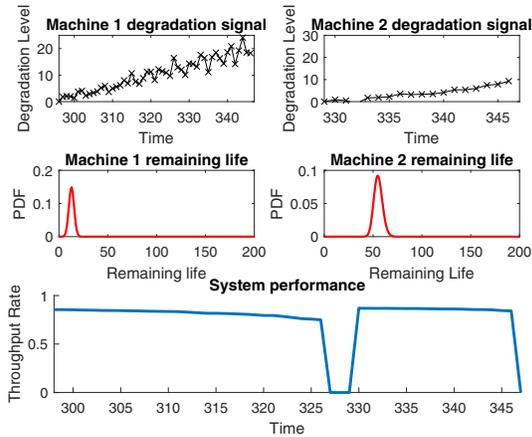


Fig. 5: An example when the degradation levels of  $m_1$  is high

## V. CONCLUSIONS

In this work, we develop an analytical model to predict the long-term production performance of a two-machine-and-one-buffer production system given real-time machine degradation signals. We generate the phase-type distribution to mimic the remaining life distribution of each machine and formulate a continuous time Markovian model to estimate the long-term system throughput rate. In the future, such a model can be extended to more complicated manufacturing systems, such as assembly lines. Furthermore, different types of degradation signals, such as Brownian motions, can be applied to enrich the applications of the model in real practice.

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