Multiple Sensor Data Fusion for Degradation Modeling and Prognostics Under Multiple Operational Conditions

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***Abstract*—Due to the rapid advances in sensing and computing technology, multiple sensors have been widely used to simultaneously monitor the health status of an operation unit. This creates a data-rich environment, enabling an unprecedented opportunity to make better understanding and inference about the current and future behavior of the unit in real time. Depending on specific task requirements, a unit is often required to run under multiple operational conditions, each of which may affect the degradation path of the unit differently. Thus, two fundamental challenges remain to be solved for effective degradation modeling and prognostic analysis: 1) how to leverage the dependent information among multiple sensor signals to better understand the health condition of the unit; and 2) how to model the effects of multiple conditions on the degradation characteristics of the unit. To address these two issues, this paper develops a data fusion methodology that integrates the information from multiple sensors to construct a health index when the monitored unit runs under multiple operational conditions. Our goal is that the developed health index provides a much better characterization of the health condition of the degraded unit, and, thus, leads to a better prediction of the remaining lifetime. Unlike other existing approaches, the developed data fusion model combines the fusion procedure and the degradation modeling under different operational conditions in a unified manner. The effectiveness of the proposed method is demonstrated in a case study, which involves a degradation dataset of aircraft gas turbine engines collected from 21 sensors under six different operational conditions.**

***Index Terms*—Data fusion, multiple operational conditions, multiple sensors, prognostics, remaining life prediction.**

NOMENCLATURE

HI	Health Index.
RLD	Remaining life distribution.
$x_{i,j}(t)$	Sensor measurement for unit i, sensor j, and time t.
$z_{i,j}(t)$	Data after log transformation of $x_{i,j}(t)$.
$\theta_{k,i,j}^{(l)}$	kth random effect parameter for unit i and sensor j under operational condition l.
$\theta_{i,j}$	Vector of random effect parameters for unit i and sensor j, $\theta_{i,j} = (\theta_{0,i,j}; \theta_{1,i,j}^{(1)}; \cdots; \theta_{1,i,j}^{(L)}; \cdots; \theta_{K,i,j}^{(1)}; \cdots; \theta_{K,i,j}^{(L)})$.
θ_i	Vector of random effect parameters of the health index for unit i.
$T_i^{(l)}(t)$	Total time (number of observations) that unit i runs under operational condition l up to time t.
$h_i(t)$	Constructed composite health index via the combination of multiple sensor signals.
$\boldsymbol{h}_i$	Vector of the health index for unit i, $h_i(t) = [h_i(1); h_i(2); \cdots; h_i(n_i)]$.
L	Total number of operational conditions.
m	Total number of historical offline units.
S	Total number of sensors.
n_i	Total number of available observations for unit i.
K	Order of polynomial degradation model.
Γ_i	Input matrix of the degradation model for unit i.
$\Delta z_{i,j}^{(l)}(t)$	Incremental value of the sensor measurement during the time interval $(t-1, t)$ for unit i and sensor j under operational condition l.
$\Delta z_i(t)$	Incremental vector of the sensor measurements during the time interval $(t-1, t)$ for unit i, $\Delta z_i(t) = [\Delta z_{i,1}^{(1)}(t), \cdots, \Delta z_{i,S}^{(1)}(t), \cdots, \Delta z_{i,1}^{(L)}(t), \cdots, \Delta z_{i,S}^{(L)}(t)]$.
$\boldsymbol{\Delta} z_i$	Matrix that records all the sensor increments of unit i, $\boldsymbol{\Delta} z_i = [\,\boldsymbol{\Delta} z_i(1); \ldots;\ \boldsymbol{\Delta} z_i(n_i)\,]$.
$\boldsymbol{w}^{(l)}$	Vector of fusion coefficients that combine S multiple sensor data under operational conditional l, $\boldsymbol{w}^{(l)} = [w_1^{(l)}; \ldots; w_j^{(l)}; \ldots, w_S^{(l)}]$.
$\boldsymbol{w}$	Vector of fusion coefficients under different operational conditions, $\boldsymbol{w} = [\boldsymbol{w}^{(1)}; \ldots;\ \boldsymbol{w}^{(L)}]$.
$\boldsymbol{B_i}$	Lower triangular matrix with all the nonzero elements equal to 1.
$\tilde{\epsilon}_i$	Estimated residual errors of the fitted degradation model for unit i.
$\boldsymbol{c}_i$	Weight coefficient matrix of the residual term $\tilde{\epsilon}_i$ for unit i.
$\boldsymbol{H}_i$	Projection matrix for unit i, $\boldsymbol{H}_i = \boldsymbol{c}_i\boldsymbol{\Gamma}_i(\boldsymbol{\Gamma}_i'\boldsymbol{c}_i^2\boldsymbol{\Gamma}_i)^{-1}\boldsymbol{\Gamma}_i'\boldsymbol{c}_i$.
$\boldsymbol{D}$	Matrix used to compute the unbiased variance of the failure threshold of the constructed health index.
$\boldsymbol{R}$	Matrix used to compute the monotonicity of the constructed health index.
r_1, r_2	Nonnegative tuning parameters that control the relative importance among the degradation model

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uncertainty, failure threshold uncertainty, and range information of the health index.

$\boldsymbol{M}_S^{(l)}$ Diagonal matrix denoting the trend information of each original sensor data under operational condition l.

$\boldsymbol{M}_{SL}$ Matrix that combines all sensor trend information under different operational conditions, $\boldsymbol{M}_{SL} = \mathrm{diag}(\boldsymbol{M}_S^{(1)}, \ldots, \boldsymbol{M}_S^{(l)})$.

$\boldsymbol{I}_i$ Identity matrix.

1_n Row vector of size $1 \times n$ with all elements equal to 1.

$\boldsymbol{A}$ Matrix used to construct the quadratic term of the objection function in the developed data fusion model.

0_n Column vector of size $n \times 1$ with all elements equal to 0.

e_i Relative difference between the predicted and the true failure time for unit i.

R_i True remaining lifetime for unit i.

$\tilde{R}_i$ Estimated remaining lifetime for unit i.

I. Introduction

Degradation modeling and prognostics have been important topics in both industrial and service applications. The essential goal is to utilize the useful information from sensor data, which are known as degradation signals [1], to accurately estimate the health status and make inference about the remaining lifetime of a unit. With such information available, a conditional-based maintenance strategy [2] can then be employed to effectively prevent unexpected failure, significantly improving safety issue, and reducing production cost [3], [4].

For degradation modeling and prognostics, most of the existing literature focuses on analyzing a single sensor signal when a unit runs under a single operational condition [4]–[7]. Specifically, a stochastic model is often first constructed based on historical observations in the training stage and then the model is calibrated based on the *in-situ* sensor measurements collected from a testing unit in real time [5], [7]. These approaches are effective under the assumption that the single sensor data is able to fully capture the stochastic nature of the degradation process. Unfortunately, as the system becomes more complex, relying on a single sensor signal may become insufficient to accurately characterize the underlying degradation mechanism, leading to inaccurate and unreliable remaining life prediction [8].

Due to the vast advances in sensing and computing technology, multiple sensors have been widely used to simultaneously monitor the health status of a unit. This raises two main challenges: 1) Some sensors may be more related to the underlying degradation mechanism, and, thus, they can show a strong degradation trend while others may not. As a result, efforts are needed to determine which sensor signals to use when prognostics is performed. 2) Signals collected from multiple sensors are often correlated and each signal only contains partial information of the degraded unit. In such a case, effective data fusion methods are desired to combine the dependent information from multiple sensors to make better characterization of the health status of the unit [9].

In addition to the issues from multiple sensors, the multiple operational conditions also pose additional challenges for effective degradation modeling and prognostics in practice. For example, when the degradation of Lithium-Ion batteries is studied, the repeated charging and discharging cycles exhibit two operational conditions [10], [11]. The second example is when studying the degradation of turbine engines, aircrafts during flights can experience multiple operational conditions, e.g., the altitude can range from 3000 to 18000 ft [12]. The third example is the degradation analysis of rotating bearings, in which multiple operational conditions, e.g., different rotation speeds, do exist [13]. It is not uncommon that a unit is required to run under different operational conditions to meet certain task requirements, while each operational condition may significantly accelerate or decelerate the degradation process. In the literature, efforts have been made that either analyze a single sensor signal collected under multiple operational conditions, or develop data fusion models that combine multiple sensor signals under a single operational condition for degradation modeling and prognostics. However, to the best of our knowledge, the existing literature still lacks an effective method that can simultaneously handle the challenges from both multiple sensors and multiple operational conditions.

To fill this literature gap, this paper aims to develop an effective data fusion model that constructs a composite health index (HI) via the combination of multiple sensor signals collected under multiple operational conditions. Our goal is that the developed HI provides a much better characterization of the health condition of the degraded unit, and, thus, leads to a better remaining life prediction. The rest of this paper is organized as follows: Section II reviews the existing data fusion methods and the related work on degradation modeling and prognostics under both single and multiple operational conditions. Section III develops a concrete formulation and a novel data fusion model that simultaneously addresses the two challenges resulted from both multiple sensors and multiple operational conditions. Section IV demonstrates and evaluates the proposed methodology based on the degradation dataset of aircraft gas turbine engines in [8] that contains 21 sensor signals collected under six operational conditions. Finally, Section V provides a conclusion and discussion of future directions.

II. Literature Review

This section reviews the related literature on degradation modeling and prognostics that analyzes a single sensor signal collected under 1) a single operational condition; and 2) multiple operational conditions. In addition, existing data fusion approaches that combine multiple sensor signals under a single operational condition will also be discussed, which provides a foundation for our proposed model in Section III.

A. Degradation Modeling Based on a Single Sensor Signal Collected Under a Single Operational Condition

Most of the existing work in degradation modeling focuses on analyzing a single sensor signal collected under a single operational condition. For engineering systems, the failure of a unit is often assumed to occur when the corresponding degradation signal first passes a predefined failure threshold [5]. To describe the evolution of a degradation signal, a variety of degradation models have been developed in the literature, such as the general degradation path models [6], the random process models [14], [15], the time-series models [16], and the state-space models [17], [18]. In particular, here we focus on the degradation path model proposed by Lu and Meeker [6]:

$$z(t) = \eta(t; \phi, \boldsymbol{\theta}) + \epsilon(t) \tag{1}$$

where $z(t)$ represents the observed sensor measurement at time $t; \eta(\cdot)$ is the parametric form of the degradation model; ϕ is the vector of fixed effect parameters that represents common characteristics of the population and is assumed to be the same for all units; $\boldsymbol{\theta}$ is the vector of random effect parameters that is used to characterize the stochastic nature of the degradation process and is assumed to be different for each unit; and $\epsilon(t)$ refers to the measurement error, which is often assumed to be white noise, i.e., $\epsilon(t) \sim N(0, \sigma^2)$. Depending on the applications and the patterns of degradation signals, $\eta(\cdot)$ can be either linear, polynomial, exponential, or other types of functional forms.

As a specific example, Gebraeel [19] introduced the following exponential degradation model:

$$x_{i,j}(t) = \phi_j + \alpha_{i,j} e^{\sum_{k=1}^{K} \theta_{k,i,j} * t^k + \epsilon_{i,j}(t) - \frac{\sigma_j}{2}}.$$

Here, $x_{i,j}(t)$ is the sensor measurement for unit i, sensor j, and time t; ϕ_j is the fixed effect parameter for sensor j; $\theta_{k,i,j}$ is the kth random effect parameter for unit i and sensor j; and $\epsilon_{i,j}(t)$ follows $N(0, \sigma_j^2)$. Since $E(e^{\epsilon_{i,j}(t) - \frac{\sigma_j}{2}}) = 1$ and $E(x_{i,j}(t) | \alpha_{i,j}, \theta_{1,i,j}, \ldots \theta_{K,i,j}) = \phi_j + \alpha_{i,j} e^{\sum_{k=1}^{K} \theta_{k,i,j} * t^k}$, the author focused on modeling the data after log transformation

$$\begin{aligned} z_{i,j}(t) &= \ln(x_{i,j}(t) - \phi_j) \\ &= \theta_{0,i,j} + \sum_{k=1}^{K} \theta_{k,i,j} * t^k + \epsilon_{i,j}(t) \\ &= \boldsymbol{\Gamma}_i(t) \boldsymbol{\theta}_{i,j} + \epsilon_{i,j}(t) \end{aligned} \tag{2}$$

where
$\theta_{0,i,j} = \ln(\alpha_{i,j}) - \frac{\sigma_j}{2}$, $\boldsymbol{\Gamma}_i(t) = (1, t, \ldots, t^K) \in \Re^{1 \times (K+1)}$, and $\boldsymbol{\theta}_{i,j} = (\theta_{0,i,j}; \theta_{1,i,j}; \cdots; \theta_{K,i,j}) \in \Re^{(K+1) \times 1}$ is the vector of random effect parameters for unit i and sensor j.

B. Degradation Modeling Based on a Single Sensor Signal Collected Under Multiple Operational Conditions

Recently, research studies have also been extended to develop degradation modeling and prognostic techniques for addressing the challenge from time-varying operational conditions. For example, Gebraeel and Pan [13] developed a sensor-based degradation model based on the general path model under two operational conditions. Whitmore and Schenkelberg [20] considered a Wiener diffusion process with a time-scale transformation to address the challenge of multiple operational conditions. Similar ideas of using time-scale transformations can be also found in [21]–[25].

In addition to the above parametric approaches, another line of research focuses on investigating nonparametric functional data analysis techniques to model the effect of multiple operational conditions. For example, Zhou *et al.* [26] developed a nonparametric technique to model a single degradation signal based on sparse observations, in which the operational condition parameters were estimated by using the expectation-maximization (EM) algorithm. Shiau and Lin [27] applied a nonparametric regression to characterize the degradation signal of a light-emitting diode product under multiple stress levels.

Another unique challenge in developing the degradation modeling and prognostic techniques under multiple operational conditions is that the future operational conditions are often undetermined. For example, when the degradation of aircraft engines is studied, the future operational conditions may not be known in advance since the future flight schedule is subject to change due to weather conditions or other factors in practice. To address the challenge of unknown future operational conditions, Flory *et al.* [28] developed a switching diffusion model for predicting the remaining lifetime of a wind turbine component under random-varying operational conditions, in which the future conditions are modeled as a discrete time Markov chain and the condition parameter was estimated using a Markov chain Monte–Carlo (MCMC) procedure. However, this technique may not be effective in the case of multiple sensors and also the estimation procedure based on MCMC is normally slow.

C. Data Fusion Approaches to Degradation Modeling and Prognostics Under a Single Operational Condition

As different sensor data often contain partial and correlated information, data fusion methods have been commonly used to address the challenge of multiple sensors to improve the analytics results. Data fusion methods can be classified into two broad categories based on the implementation level of fusion techniques: data-level fusion and decision-level fusion. A detailed review on the related data fusion techniques can be found in [9].

For prognostics, the existing literature heavily relies on the decision-level fusion approach that integrates the results (e.g., voting) from separate analyses of each individual sensor data. However, this approach ignores the dependence of multiple sensor data and treats each sensor signal as equally important, thus often leading to biased results [19]. Moreover, decision-level fusion requires *repeated computations* based on individual sensor data, which may not be effectively used in practice when there are multiple sensor data, and the estimation procedure (e.g., based on MCMC or EM algorithm [26], [28]) is typically complex and slow. As a result, data-level fusion, which directly combines multiple sensor data or extracted features [29]–[31] into a 1-D HI, has shown promise as an effective solution for better characterizing the condition of a unit, and, thus, attracted much attention recently. One advantage of the data-level fusion method is that the constructed HI can be regarded as another degradation signal; thus, not only the existing degradation modeling and prognostic techniques that have been successfully demonstrated on a single sensor signal can be directly used, but also the HI can be readily integrated with other decision-level techniques to further improve the prognostic results if needed.

Along this direction, one of the research efforts was recently carried out by Liu *et al.* [32], who developed a nonparametric data-level fusion model to construct a composite HI, $h_i(t)$ via a linear combination of S multiple sensor signals, $\boldsymbol{z}_{i,\cdot}(t) = [z_{i,1}(t), \ldots,\ z_{i,S}(t)]$, i.e., $h_i(t)\ = \boldsymbol{z}_{i,\cdot}(t) * \boldsymbol{w}$. Here, $\boldsymbol{w} \in \Re^{S\times 1}$ is called the *fusion coefficient* for combining multiple sensor data. Specifically, the authors proposed to optimize the following two properties when constructing the HI:

Property 1: Once an initial fault occurs, the trend of the degradation signals should be monotonic.

Property 2: Given the same operational condition and failure mode, the variance in the estimated failure threshold of different units should be minimal.

For Property 2, please notice that in the reliability study, the failure threshold is normally a fix number by assuming the monitored signal truly characterizes the underlying physical transition of the degradation process. When such knowledge is unavailable, the data-driven approach can be applied as well, e.g., based on the last observation of sensor measurements before failure [32]. However, it should be mentioned that as shown in many real applications, the last observations of sensor signal in different units are quite different due to the stochastic nature of the degradation process. Therefore, the goal of Property 2 is to minimize the variance of the estimated failure threshold, so that less uncertainty will be involved in the residual life prediction.

While the data-level fusion approach in [32] showed a promising solution, one limitation of this technique is that it cannot guarantee the developed HI is suitable for the selected degradation model when prognostics are performed. To address this issue, Liu and Huang [33] developed a semiparametric data fusion model which considered both the fusion procedure and the degradation modeling in a unified manner. In particular, the authors proposed minimizing two types of uncertainties when constructing the HI: 1) degradation model uncertainty and 2) failure threshold uncertainty. Uncertainty 1 arises when a degradation model is used to fit the HI values; the authors thus proposed minimizing the weighted sum of square errors in the fitted degradation model: $\sum_{i=1}^{m} (\boldsymbol{c}_i \widetilde{\boldsymbol{\epsilon}_i})' \boldsymbol{c}_i \epsilon^{\sim}{}_i / m$, where $\epsilon^{\sim}{}_i = [\epsilon^{\sim}{}_i(1), \ldots,\ \epsilon^{\sim}{}_i(n_i)]'$ represents the residual errors in the fitted degradation model for the constructed HI of unit i, and $\boldsymbol{c}_i = \mathrm{diag}(c_i(1), \ldots, c_i(n_i)) \in \Re^{n_i \times n_i}$ is the weight coefficient matrix for the residual term $\epsilon^{\sim}{}_i$. Here, n_i is the total number of available observations for unit i, and m is the total number of units in historical offline records. Meanwhile, uncertainty 2 stems from the stochastic nature of the degradation process, such that different units fail at different points. The authors thus proposed minimizing variance in the estimated failure threshold of the developed HI: $\sum_{i=1}^{m} (h_i(n_i) - \bar{h}(n_i))^2/(m-1)$, where $\bar{h}(n_i) = \sum_{i=1}^{m} h_i(n_i)/m$ is the average of the last observed HI value before failure in all historical units.

Table I summarizes the previous research efforts reviewed above in this section. To the best of our knowledge, the existing literature still lacks an effective data fusion model that can handle the challenges from both multiple sensor signals and multiple operational conditions. The main contribution of this paper is to fill this literature gap to achieve an effective degradation modeling and deliver a better prognostic result.

TABLE I
LITERATURE REVIEW SUMMARY FOR DEGRADATION MODELING AND PROGNOSTICS WITH SINGLE/MULTIPLE SENSOR SIGNALS UNDER SINGLE/MULTIPLE OPERATIONAL CONDITIONS

	Single operational condition	Multiple operational conditions
Single sensor signal	General degradation path models [6], [19], random process models [14], [15], time-series models [16], some state-space based models [17], [18], etc.	Parametric/nonparametric degradation models under either known or unknown future conditions [13], [20], [27], etc.
Multiple sensor signals	Decision-level fusion, non-parametric/semiparametric data fusion models [32], [33], etc.	Our proposed data fusion method

III. Methodology Development

In this section, we will introduce our proposed data-level fusion model in details. Similar to the approach in [33], our proposed method also integrates the fusion procedure and the degradation modeling in a unified manner. However, our method is different from the one in [33] as our model is able to address the challenge of multiple operational conditions. To begin with, we will first introduce our developed degradation model for a single sensor signal under multiple operational conditions. Based on the developed degradation model, we will then propose a novel data-level fusion approach to handle the challenges from both multiple sensors and multiple operational conditions.

A. Proposed Degradation Model for a Single Sensor Signal Under Multiple Operational Conditions

Recall that a degradation signal is a time series collected from a sensor, which measures a physical degradation process. The existing model in (1), however, does not consider the distinct effect of each operational condition on the degradation path of a unit. To address this issue, we propose extending the general path model in (1) by allowing the parameters $\boldsymbol{\theta}$ and ϕ to be dependent on the operational condition: $z(t) = \eta(t; \phi^{(l)}, \boldsymbol{\theta}^{(l)}) + \epsilon(t)$. Here, $l = 1, \ldots, L$ denotes the index of the operational condition, and we assume that there are in total L number of operational conditions. Considering the model in (2), then the degradation model with multiple operational conditions becomes

$$\begin{aligned} z_{i,j}(t) &= \ln\left(x_{i,j}(t) - \phi_j^{(l)}\right) \\ &= \theta_{0,i,j} + \sum_{k=1}^{K}\sum_{l=1}^{L} \theta_{k,i,j}^{(l)} \times \left(T_i^{(l)}(t)\right)^k + \epsilon_{i,j}(t) \\ &= \boldsymbol{\Gamma}_i(t)\boldsymbol{\theta}_{i,j} + \epsilon_{i,j}(t). \end{aligned} \tag{3}$$

Here, $\phi_j^{(l)}$ denotes the baseline value of sensor j under operational condition l; $\theta_{0,i,j}$ refers to the initial degradation state and is assumed to be independent of operational conditions; $\theta_{k,i,j}^{(l)}$ is the corresponding degradation rate of unit i and sensor j under operational condition l; $T_i^{(l)}(t)$ records the total time (number of observations) that unit i runs under operational condition l up to time t; and K is the order of the degradation model. Similarly to the model in (1), $\boldsymbol{\theta}_{i,j} = (\theta_{0,i,j}; \theta_{1,i,j}^{(1)}; \cdots; \theta_{1,i,j}^{(L)}; \cdots; \theta_{K,i,j}^{(1)}; \cdots; \theta_{K,i,j}^{(L)}) \in \Re^{(KL+1)\times 1}$ refers to the random effect parameters for unit i and sensor j, and $\boldsymbol{\Gamma}_i(t) = (1, T_i^{(1)}(t) \cdots, T_i^{(L)}(t), \cdots, (T_i^{(1)}(t))^K, \cdots, (T_i^{(L)}(t))^K) \in \Re^{1\times(KL+1)}$. This generalized degradation path model is expected to be able to characterize the effects of multiple operational conditions on the degradation signal through the condition-dependent random effect $\theta_{k,i,j}^{(l)}$.

B. Data-Level Fusion Model for Multiple Sensors Under Multiple Operational Conditions

In this section, we will develop a data-level fusion model for multiple sensors under multiple operational conditions. In particular, we propose to extend the one in [33] by taking the effects of multiple operational conditions into consideration. Similarly to the previous approaches [32], [33], we also consider the linear fusion model, i.e., $h_i(t) = \boldsymbol{z}_{i,\cdot}(t) * \boldsymbol{w}$, to highlight our main idea. We will elaborate the proposed model in details through the following two steps: 1) construction of the HI; and 2) estimation of the fusion coefficient.

1) Construction of the HI: Here, the main challenge for constructing the HI is that the degradation rate depends on not only the sensor information, but also the operational condition. Considering that some sensor signals are only sensitive to certain operational conditions, consequently, we propose a condition-specific *fusion coefficient* in the developed data fusion model. This means that the weights for combining multiple sensor data are different when the unit runs under different operational conditions. In particular, we denote the fusion coefficient as $\boldsymbol{w}^{(l_t)} = [w_1^{(l_t)}; \ldots; w_j^{(l_t)}; \ldots, w_S^{(l_t)}] \in \Re^{S\times 1}$, which is used to combine the incremental values from all S sensor data at time $t-1$ when the operational conditional is l_t. In this paper, we assume that the operational condition does not change during the time interval $(t-1, t)$, which is denoted as l_t.

The increment of the constructed HI of unit i during the time interval $(t-1, t)$, $\Delta h_i(t)$, can be then computed as $\Delta h_i(t) = \sum_{J=1}^{S} w_j^{(l_t)} \Delta z_{i,j}^{(l_t)}(t) = \Delta z_i(t)\boldsymbol{w}$, where $\Delta \boldsymbol{z}_i(t) = [\Delta z_{i,1}^{(1)}(t), \cdots, \Delta z_{i,S}^{(1)}(t), \cdots, \Delta z_{i,1}^{(L)}(t), \cdots, \Delta z_{i,S}^{(L)}(t)] \in \Re^{1\times SL}$ is the increment vector of sensor measurements for unit i during the time interval $(t-1, t)$, $\Delta z_{i,j}^{(l_t)}(t) = z_{i,j}(t) - z_{i,j}(t-1)$, $\Delta z_{i,j}^{(l)}(t) = \{ \begin{matrix} \Delta z_{i,j}^{(l_t)}(t), if\ \ l = l_t \\ 0,\ if\ \ l \neq l_t \end{matrix}$ which implies only S elements of $\Delta \boldsymbol{z}_i(t)$ are non-zero and equal to $\Delta z_{i,j}^{(l_t)}(t), j = 1, \cdots, S$, and $\boldsymbol{w} = [\boldsymbol{w}^{(1)}; \ldots;\ \boldsymbol{w}^{(L)}] \in \Re^{SL\times 1}$. Please note that $\Delta z_{i,j}^{(l_1)}(1) = z_{i,j}(1)$. Next, the HI $h_i(t)$ at time t can be constructed by summing up all the increments of the constructed HI up to time t: $h_i(t) = \sum_{t'=1}^{t} \Delta h_i(t') = \sum_{t'=1}^{t} \Delta z_i(t')\boldsymbol{w}$. To simplify, the constructed 1-D HI of unit i can be written in the matrix form: $\boldsymbol{h}_i = [h_i(1); h_i(2); \cdots; h_i(n_i)] = \boldsymbol{B_i}\boldsymbol{\Delta z}_i\boldsymbol{w} \in \Re^{n_i\times 1}$, where

$$\boldsymbol{B_i} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 1 & \cdots & \cdots & 1 \end{pmatrix} \in \Re^{n_i\times n_i}$$

is the lower triangular matrix with all the nonzero elements equal to 1, n_i is the total number of available observations for unit i, and $\boldsymbol{\Delta z}_i = [\,\boldsymbol{\Delta z}_i(1); \ldots;\ \boldsymbol{\Delta z}_i(n_i)] \in \Re^{n_i\times SL}$ is the matrix recording all the sensor increments of unit i. It should be noted that here the only unknown parameter is the proposed fusion coefficient $\boldsymbol{w}$. Below, we will discuss in details on how to estimate $\boldsymbol{w}$.

2) Estimation of the Fusion Coefficient: To estimate the fusion coefficient $\boldsymbol{w}$, we will consider two scenarios, depending on whether the future operational condition is known or not.

a) Future Operational Condition is Known: When the future operational condition is known, we employ the degradation model introduced in Section III-A to model the evolution of the HI as the constructed HI can be regarded as a single sensor signal. To be specific, if using the model in (3) (we have deleted the subscript notation j in θ to emphasize that this is the random effect parameter for the HI):

$$h_i(t) = \theta_{0,i} + \sum_{k=1}^{K}\sum_{l=1}^{L} \theta_{k,i}^{(l)} * \left(T_i^{(l)}(t)\right)^k + \epsilon_i(t). \tag{4}$$

Equivalently, this can be rewritten in the matrix form as

$$\boldsymbol{h}_i = \boldsymbol{\Gamma}_i\boldsymbol{\theta}_i +\ \epsilon_i$$

where
$\boldsymbol{\theta}_i = (\theta_{0,i}; \theta_{1,i}^{(1)}; \cdots, \theta_{1,i}^{(L)}; \cdots, \theta_{K,i}^{(1)}; \cdots, \theta_{K,i}^{(L)}) \in \Re^{(KL+1)\times 1}$
and

$$\boldsymbol{\Gamma}_i = \begin{bmatrix} 1 & T_i^{(1)}(1) & \cdots & T_i^{(L)}(1) & \cdots & (T_i^{(1)}(1))^K & \cdots & (T_i^{(L)}(1))^K \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ 1 & T_i^{(1)}(n_i) & \cdots & T_i^{(L)}(n_i) & \cdots & (T_i^{(1)}(n_i))^K & \cdots & (T_i^{(L)}(n_i))^K \end{bmatrix} \in \Re^{n_i\times(KL+1)}.$$

Inspired by the approaches in [32] and [33], here we consider the following three desired properties, including 1) minimizing degradation model uncertainty, 2i) minimizing failure threshold uncertainty, and 3) maximizing the range information in the constructed HI:

1) To minimize the degradation model uncertainty for the constructed HI, we also propose to minimize the weighted residual sum of square $\sum_{i=1}^{m} ||\boldsymbol{c}_i\widetilde{\epsilon}_i||^2$ in the fitted degradation model as in [33], where $\epsilon^{\sim}{}_i = \boldsymbol{h}_i - \boldsymbol{\Gamma}_i\boldsymbol{\theta}_i$. Here, the random effect parameter $\boldsymbol{\theta}_i$ can be solved via the weighted least square method, i.e., $\boldsymbol{\theta}_i = (\boldsymbol{\Gamma}'_i\boldsymbol{c}_i^2\boldsymbol{\Gamma}_i)^{-1}(\boldsymbol{\Gamma}'_i\boldsymbol{c}_i^2\boldsymbol{h}_i)$. Thus, the weighted residual term can be calculated by $\boldsymbol{c}_i\epsilon^{\sim}{}_i = \boldsymbol{c}_i\boldsymbol{h}_i - \boldsymbol{c}_i\boldsymbol{\Gamma}_i\boldsymbol{\theta}_i = (\boldsymbol{I}_i - \boldsymbol{H}_i)\boldsymbol{c}_i\boldsymbol{h}_i = (\boldsymbol{I}_i - \boldsymbol{H}_i)\boldsymbol{c}_i\boldsymbol{B}_i\boldsymbol{\Delta z}_i\boldsymbol{w}$, where $\boldsymbol{H}_i =$

$\boldsymbol{c}_i\boldsymbol{\Gamma}_i(\boldsymbol{\Gamma}_i'\boldsymbol{c}_i^2\boldsymbol{\Gamma}_i)^{-1}\boldsymbol{\Gamma}_i'\boldsymbol{c}_i \in \Re^{n_i\times n_i}$ is known as the projection matrix and $\boldsymbol{I}_i \in \Re^{n_i\times n_i}$ is the identity matrix. Here, recall that c_i is the corresponding weight coefficient for the residual term $\epsilon^{\sim}{}_i$. According to [32], as a unit degrades, the accuracy of remaining life prediction becomes increasingly sensitive to the model fitting result, and, thus, the weight coefficient needs to be nondecreasing $c_i(t+1) \geq c_i(t), i = 1,\ldots,m, t = 1,\ldots,n_i - 1$. In addition, considering different units are equally important in the data fusion model, the following constraint needs to be satisfied as well $\sum_{t=1}^{n_i} c_i(t) = 1$. Based on these two constraints, the Liu *et al.* [32] further provided two options by setting $\{c_{i,t}\}$ to be either an arithmetic series or a geometric series. Specifically, if we assume that $c_i(t)$ follows a geometric series, i.e., $c_i(t) = c_i(1)q^{t-1}$, then q can be determined by solving $c_i(1)q^{n_i} - c_i(1) - q + 1 = 0$.

2) The mean of failure threshold of the HI, u can be estimated from the last observations of all units before failure in the training dataset, i.e., $u = \bar{h}(n_i) = \frac{1}{m}\sum_{i=1}^m h_i(n_i) = \frac{1}{m}\sum_{i=1}^m \sum_{t=1}^{n_i} \boldsymbol{\Delta z}_i(t)\boldsymbol{w} = \sum_{i=1}^m 1_{n_i}\boldsymbol{\Delta z}_i\boldsymbol{w}$, where $1_{n_i} \in \Re^{1\times n_i}$ is a row vector with all elements equal to 1. Then, the variance of the estimated failure threshold $v = \frac{1}{m-1}\sum_{i=1}^m (h_i(n_i) - u)^2$ can be written as $v = \boldsymbol{w}'\boldsymbol{D}\boldsymbol{w}$, in which $\boldsymbol{D} = \frac{1}{m-1}(\sum_{i=1}^m \boldsymbol{\Delta z}_i' 1_{n_i}' 1_{n_i}\boldsymbol{\Delta z}_i - \frac{1}{m}\boldsymbol{G}'\boldsymbol{G})$ and $\boldsymbol{G} = \sum_{i=1}^m 1_{n_i}\boldsymbol{\Delta z}_i$. See the Appendix for the detailed proof.

3) In addition to these two uncertainties 1 and 2 as mentioned in [33], Liu *et al.* [34] have also demonstrated that the range information of a degradation signal is another important criterion to ensure a successful prognostic result, and, thus, here we adopt this criterion in our developed data fusion model as well. In particular, we propose to maximize the range information of the constructed HI: $\frac{1}{m}\sum_{i=1}^m (h_i(n_i) - h_i(1)) = \frac{1}{m}\sum_{i=1}^m (\sum_{t=1}^{n_i}\boldsymbol{\Delta z}_i(t)\boldsymbol{w} - \boldsymbol{\Delta z}_i(1)\boldsymbol{w}) = \boldsymbol{R}\boldsymbol{w}$, where $\boldsymbol{R} = \frac{1}{m}\sum_{i=1}^m(\sum_{t=2}^{n_i}\boldsymbol{\Delta z}_i(t)) \in \Re^{1\times SL}$. This optimizing effort is intended to assign higher weights during the fusion procedure to the sensor signal with larger range information under the corresponding operational condition.

By combining these three desired properties 1–3, we develop a data-level fusion model under multiple operational conditions

$$\begin{aligned}\text{obj} &= \min_{\boldsymbol{w}} \frac{1}{m}\sum_{i=1}^m ||\boldsymbol{c}_i\widetilde{\epsilon}_i||^2 + r_1\boldsymbol{w}'\boldsymbol{D}\boldsymbol{w} - r_2\boldsymbol{R}\boldsymbol{w}\\ \text{s.t.} &1_S\boldsymbol{M}_S^{(l)}\boldsymbol{w}^{(l)} = 1, l = 1,\ldots,L,\ \boldsymbol{M}_{SL}\boldsymbol{w} \geqslant 0_{SL}\end{aligned} \tag{5}$$

where r_1 and r_2 are the nonnegative tuning parameters that control the relative importance among the degradation model uncertainty, failure threshold uncertainty, and range information of the HI. Their values can be determined by cross validation. In the constraints, the first set of constraints $1_S\boldsymbol{M}_S^{(l)}\boldsymbol{w}^{(l)} = 1, l = 1,\ldots,L$ means that the HI is a weighted average of multiple sensor data via the fusion coefficient $\boldsymbol{w}^{(l)}$ under each condition l. Here, $1_S \in \Re^{1\times S}$ is a row vector with all elements equal to 1, and $\boldsymbol{M}_S^{(l)} \in \Re^{S\times S}$ is a diagonal matrix denoting the trend information of each original sensor data with 1 (or -1) on the diagonal entry if the corresponding sensor signal shows an increasing (or decreasing) trend under operational condition l. Here, we assume that the trend information of each sensor should be consistent under each operational condition. The second constraint $\boldsymbol{M}_{SL}\boldsymbol{w} \geq 0_{SL}$ ensures that the sign of fusion coefficient $\boldsymbol{w}$ is consistent with the trend information of the corresponding sensor in each operational condition, where $\boldsymbol{M}_{SL} = \text{diag}(\boldsymbol{M}_S^{(1)},\ldots,\boldsymbol{M}_S^{(l)}) \in \Re^{SL\times SL}$ and $0_{SL} \in \Re^{SL\times 1}$ is a column vector with all elements equal to 0.

As $\frac{1}{m}\sum_{i=1}^m ||\boldsymbol{c}_i\widetilde{\epsilon}_i||^2 = \frac{1}{m}\sum_{i=1}^m \boldsymbol{w}'\boldsymbol{\Delta z}_i'\boldsymbol{B}_i'\boldsymbol{c}_i'(\boldsymbol{I}_i - \boldsymbol{H}_i)\boldsymbol{c}_i\boldsymbol{B}_i\boldsymbol{\Delta z}_i\boldsymbol{w}$, (5) can be further simplified into a linear-constrained convex quadratic programming

$$\begin{aligned}&\text{obj} = \min_{\boldsymbol{w}}\boldsymbol{w}'\boldsymbol{A}\boldsymbol{w} - r_2\boldsymbol{R}\boldsymbol{w}\\ &\text{s.t.}1_S\boldsymbol{M}_S\boldsymbol{w}^{(l)} = 1, l = 1,\ldots,L,\ \boldsymbol{M}_{SL}\boldsymbol{w} \geq 0_{SL}\end{aligned} \tag{6}$$

where $\boldsymbol{A} = \frac{1}{m}\sum_{i=1}^m \boldsymbol{\Delta z}_i'\boldsymbol{B}_i'\boldsymbol{c}_i'(\boldsymbol{I}_i - \boldsymbol{H}_i)\boldsymbol{c}_i\boldsymbol{B}_i\boldsymbol{\Delta z}_i + r_1\boldsymbol{D}$. It can be shown in the Appendix that $\boldsymbol{A}$ is positive semidefinite (P.S.D.) matrix, and, thus, (6) can be solved efficiently by existing quadratic programming solvers.

b) Future Operational Condition is Unknown: When the future operational condition is unknown, the degradation model in (4) cannot be used anymore to describe the evolution of the HI since it requires the knowledge of the operational condition at future time t. To address this issue, one intuitive approach is to assume that the operational condition follows a multinomial distribution, and then apply the Monte–Carlo simulation method to generate the future conditions from the multinomial distribution. While this approach is theoretically sound, it is computationally inefficient to conduct the simulation at each time point. As a result, here we consider another approach by constructing a condition-robust HI such that the derived HI is insensitive to the operational conditions.

Definition: The HI is called *condition robust* if the coefficients in the degradation model of the HI are independent of the operational condition.

In this way, the following degradation model in (7) is considered for the condition-robust HI:

$$h_i(t) = \theta_{0,i} + \sum_{k=1}^K \theta_{k,i} * t^k + \epsilon_i(t). \tag{7}$$

Compared with the case when the future operational is known in (4), here, the coefficients $\theta_{0,i}, \theta_{k,i}$ in (7) are independent of the operational condition l.

Again, this can be written in the matrix form as: $\boldsymbol{h}_i = \boldsymbol{\Gamma}_i\boldsymbol{\theta}_i + \epsilon_i$, in which $\boldsymbol{\theta}_i = [\theta_{0,i};\theta_{1,i};\cdots;\theta_{K,i}] \in \Re^{(K+1)\times 1}$ and $\boldsymbol{\Gamma}_i = \begin{bmatrix}1 & 1 & \cdots & 1\\ \vdots & \vdots & & \vdots\\ 1 & j & \cdots & j^K\\ \vdots & \vdots & & \vdots\\ 1 & n_i & \cdots & n_i^K\end{bmatrix} \in \Re^{n_i\times(K+1)}$. Notice that the HI in (4) is not condition robust since the degradation coefficient $\theta_{k,i}^{(l)}$ is dependent on the operational condition l.

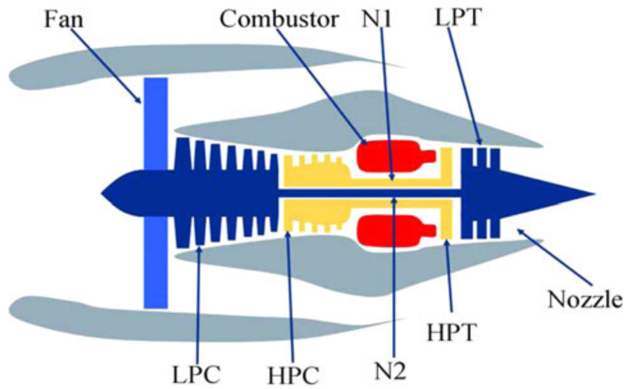

Fig. 1. Simplified engine diagram simulated by C-MAPSS [36].

To achieve the condition-robust HI, we propose to relax the L number of constraints in (5), i.e., $1_S \boldsymbol{M}_S^{(l)} \boldsymbol{w}^{(l)} = 1, l = 1, \cdots, L$ with one normalization constraint: $\frac{1}{L} 1_{SL} \boldsymbol{M}_{SL} \boldsymbol{w} = 1$, where $1_{SL} \in \Re^{1 \times SL}$ is a row vector with all elements equal to 1. The intuition behind this approach is that each operational condition may result in different levels of uncertainties when prognostics are performed. As a result, we relax the L number of constraints that require each operational condition to be equally important when the future operational condition is unknown. This allows more flexibility in the fusion coefficient $\boldsymbol{w}$ and automatically adjust different levels of uncertainties due to multiple operational conditions when constructing the condition-robust HI. It should be noticed that this problem can also be regarded as similar to the ones in [32] and [33] with only one operational condition since the HI is condition robust. In this way, when the future operational condition is unknown, our proposed data-level fusion model becomes

$$\begin{array}{c} \text{obj} = \min\limits_{\boldsymbol{w}} \ \boldsymbol{w}' \boldsymbol{A} \boldsymbol{w} - r_2 \boldsymbol{R} \boldsymbol{w} \\ \text{s.t. } \frac{1}{L} 1_{SL} \boldsymbol{M}_{SL} \boldsymbol{w} = 1, \ \boldsymbol{M}_{SL} \boldsymbol{w} \geq 0_{SL} \end{array}. \tag{8}$$

IV. Case Study

A. Dataset Description

In this case study, we will implement and evaluate the proposed data fusion model based on the degradation dataset of the turbofan engine provided in [35]. The dataset is generated from commercial modular aero-propulsion system simulation (C-MAPSS), which has been widely used to simulate the degradation performance in engines due to wear and tear according to the usage pattern under various operational conditions [36]. The schematic diagram of the commercial aircraft gas turbine engine is shown in Fig. 1.

The degradation dataset [35] used in this case study includes six operational conditions with different combinations of Altitude, throttle resolver angle (TRA), and Mach number, which are listed in Table II. In particular, the change of the operational conditions in each unit is relatively random and the pattern of operational conditions for different units is quite different as well.

TABLE II
Detailed Description of Different Operational Conditions [8]

Operation	Altitude (Kft)	Mach number	TRA
1	35	0.8400	100
2	42	0.8408	100
3	25	0.6218	60
4	25	0.7002	100
5	20	0.2516	100
6	10	0.7002	100

TABLE III
Detailed Description of the 21 Sensors [8]

Symbol	Description	Units
T2	Total temperature at fan inlet	°R
T24	Total temperature at LPC outlet	°R
T30	Total temperature at HPC outlet	°R
T50	Total temperature at LPT outlet	°R
P2	Pressure at fan inlet	psia
P15	Total pressure in bypass-duct	psia
P30	Total pressure at HPC outlet	psia
Nf	Physical fan speed	r/min
Nc	Physical core speed	r/min
epr	Engine pressure ratio (P50/P2)	–
Ps30	Static pressure at HPC outlet	psia
phi	Ratio of fuel flow to Ps30	pps/psi
NRf	Corrected fan speed	r/min
NRc	Corrected core speed	r/min
BPR	Bypass Ratio	–
farB	Burner fuel-air ratio	–
htBleed	Bleed Enthalpy	–
Nf_dmd	Demanded fan speed	r/min
PCNfR_dmd	Demanded corrected fan speed	r/min
W31	HPT coolant bleed	lbm/s
W32	LPT coolant bleed	lbm/s

The dataset consists of 260 units (i.e. $m = 260$) that include complete run-to-failure data with a total of 53 759 observations (i.e., $\sum_{i=1}^{m} n_i = 53759$). At each observation time, a total of 21 sensor measurements that include comprehensive information at different locations of the engine and the operational conditions are collected. The detailed information of the 21 sensors is shown in Table III.

In this study, we select 80% of the units (i.e., $260 \times 0.8 = 208$) as training units and the remaining 20% of the units ($260 \times 0.2 = 52$) as testing units. For training units, all the observations are assumed to be available and will be used to learn the developed data fusion model, while for testing units, observations of each testing unit are assumed to be only available until some random points prior to its failure.

The detailed simulation models, such as initial wear of units, parameters of the degradation model, and failure threshold are not given to users. Thus, the underlying premise here is that users have to rely on the available dataset described above to infer the health status and the remaining lifetime of the testing units.

TABLE IV
MEAN AND STD (IN PARENTHESIS) OF THE ESTIMATED σ_j^2 OF ALL SELECTED SENSORS AND THE HEALTH INDICES BASED ON TWO DATA FUSION MODELS

Name	T24	T30	T50	P30	Nf	Phi	NRf
Value	0.0461	0.0437	0.0248	0.0267	0.0674	0.0728	0.0840
(STD)	(0.0006)	(0.0006)	(0.0008)	(0.0005)	(0.0022)	(0.0003)	(0.0028)
Name	NRc	htBleed	W31	W32	HI-single	HI-multiple	
Value	0.0195	0.0558	0.0341	0.0181	0.0191	0.0099	
(STD)	(0.0011)	(0.009)	(0.0030)	(0.0035)	(0.0021)	(0.0007)	

TABLE V
MEAN AND STD (IN PARENTHESIS) OF THE ESTIMATED VARIANCE IN THE FAILURE THRESHOLD OF ALL SELECTED SENSORS AND THE HEALTH INDICES BASED ON TWO DATA FUSION MODELS

Name	T24	T30	T50	P30	Nf	Phi	NRf
Value	0.0164	0.0186	0.0086	0.1100	0.2521	0.1118	0.2460
(STD)	(0.001)	(0.0011)	(0.0006)	(0.0155)	(0.0423)	(0.0118)	(0.0158)
Name	NRc	htBleed	W31	W32	HI-single	HI-multiple	
Value	0.7046	0.0213	0.0679	0.0526	0.0040	0.0035	
(STD)	(0.0660)	(0.0009)	(0.0030)	(0.0035)	(0.0007)	(0.0002)	

B. Sensor Preselection

One step that needs to be done before implementing the proposed data fusion model is to screen out potential useful information from the 21 sensors. Liu and Huang [33] considered a preliminary sensor selection criterion based on whether the sensor signal demonstrates a consistent increasing or decreasing trend in all the training units. Here, we adopt the same criterion, i.e., only selecting the sensor signal if it shows a consistent trend under each operational condition in all the training units. In this way, 11 out of 21 sensors are selected, including T24, T30, T50, P30, Nf, Phi, NRf, NRc, htBleed, W31, and W32. Without loss of generality, we further standardize the dataset before the data fusion as in [32] and [33].

C. Proposed Data-Level Fusion Model Under Multiple Operational Conditions

In this section, we will evaluate the performance of the proposed data-level fusion model under multiple operational conditions (we denote the constructed HI as “HI-multiple”). As a comparison, we will also consider two benchmark approaches based on: 1) each original single sensor data considering multiple operational conditions; and 2) the HI constructed by the semiparametric data-fusion model in [33] that ignores the effects of multiple operational conditions (denoted as “HI-single”).

For degradation modeling, we employ the exponential degradation model introduced in Section III-A to fit each original sensor data under multiple operational conditions. Similarly to the approach in [19], we also focus on fitting the log-transformed data as given in (3) with a second-order polynomial model ($K = 2$). Please note that we also tried different polynomial orders $K = 1,\ 2,\ 3$ and computed the corresponding mean-square error as $4.9967,\ \ 3.5088,\ \ 3.4068$. Then, we conducted a model adequacy test, which suggested $K = 2$ is sufficient for modeling the degradation signal.

In this case study, we consider a more general scenario that the future operational condition is unknown, and, therefore, the condition-robust HI is constructed by (7) and the parameters w and $\theta_{k,i}$ are estimated through training units by (8). Fig. 2 shows an illustration of the model fitting result in each original sensor and the constructed conditional-robust HI under multiple operational conditions. From Fig. 2, we can conclude that the constructed condition-robust HI provides a much better model fitting result and is not condition-dependent anymore.

Table IV compares the mean and the standard (STD) deviation of the estimated residual variance σ_j^2 in the fitted degradation models for all selected sensors, HI-single, and HI-multiple. It clearly shows that the HI constructed by the proposed data-level fusion model (HI-multiple) provides the best model fitting result.

Table V further computes the mean and the STD of the estimated variance in the failure threshold of all selected sensors, HI-single, and HI-multiple. Similarly, Table V clearly shows that the variance of the failure threshold in the constructed HI by the proposed data-level fusion model is smaller than the others.

D. Estimation of Remaining Life Distribution (RLD)

From Tables IV and V, we can see that the proposed data-level fusion model can construct an HI with the smallest degradation modeling errors and variance in the failure threshold. In this section, we will further numerically evaluate the performance of our constructed HI when it is used for real-time prognostics. One commonly used method is proposed by Gebraeel [19], who considered a Bayesian approach to online update the RLD by combining the prior information from historical records and the real-time sensor measurements from a testing unit.

In particular, the algorithm first estimates the degradation coefficient $\boldsymbol{\theta}_i$ by fitting the constructed condition-robust HI for each historical unit (e.g., see Fig. 2) using (7). Second, similar to the approach in [33], we also assume that $\boldsymbol{\theta}_i$ follows a multivariate normal distribution $N(\boldsymbol{\mu}^0, \boldsymbol{\Sigma}^0)$, where the mean $\boldsymbol{\mu}^0$ and the covariance $\boldsymbol{\Sigma}^0$ can be estimated based on the fitted degradation coefficient $\boldsymbol{\theta}_i$: $\boldsymbol{\mu}^0 = \frac{1}{m}\sum_{i=1}^m \boldsymbol{\theta}_i$ and $\boldsymbol{\Sigma}^0 = \frac{1}{m-1}\sum_{i=1}^m (\boldsymbol{\theta}_i - \boldsymbol{\mu}^0)(\boldsymbol{\theta}_i - \boldsymbol{\mu}^0)^T$. Then, once new sensor measurements are observed during the online monitoring, we calculate the condition-robust HI with the derived optimal fusion coefficient $\boldsymbol{w}^*$. Next, degradation coefficients will be further calibrated based on the calculated HI in real time. To be specific, recall that $\boldsymbol{h}_i$ refers to the sequence of HI values up to the current observation time n_i for testing unit i. Then, the posterior distribution of the degradation coefficient $\boldsymbol{\theta}_i$ will still follow a multivariate normal distribution: $\boldsymbol{\theta}_i|\boldsymbol{h}_i \sim N(\boldsymbol{\mu}^1, \boldsymbol{\Sigma}^1)$, where the posterior mean $\boldsymbol{\mu}^1 = (\frac{\boldsymbol{\Gamma}_i'\boldsymbol{\Gamma}_i}{\sigma^2} + (\boldsymbol{\Sigma}^0)^{-1})^{-1}(\frac{\boldsymbol{\Gamma}_i'\boldsymbol{h}_i}{\sigma^2} + (\boldsymbol{\Sigma}^0)^{-1}\boldsymbol{\mu}^0)$ and the posterior variance $\boldsymbol{\Sigma}^1 = (\frac{\boldsymbol{\Gamma}_i'\boldsymbol{\Gamma}_i}{\boldsymbol{\sigma}^2} + (\boldsymbol{\Sigma}^0)^{-1})^{-1}$.

Since $h_i(n_i+t) = \theta_{0,i} + \sum_{k=1}^2 \theta_{k,i} * (n_i + t)^k + \epsilon_i(n_i + t)$, $h_i(n_i + t)$ is normally distributed with mean $E(h_i(n_i + t)) = [1, n_i + t, (n_i + t)^2]\boldsymbol{\mu}^1$ and variance $V(h_i(n_i + t)) =$

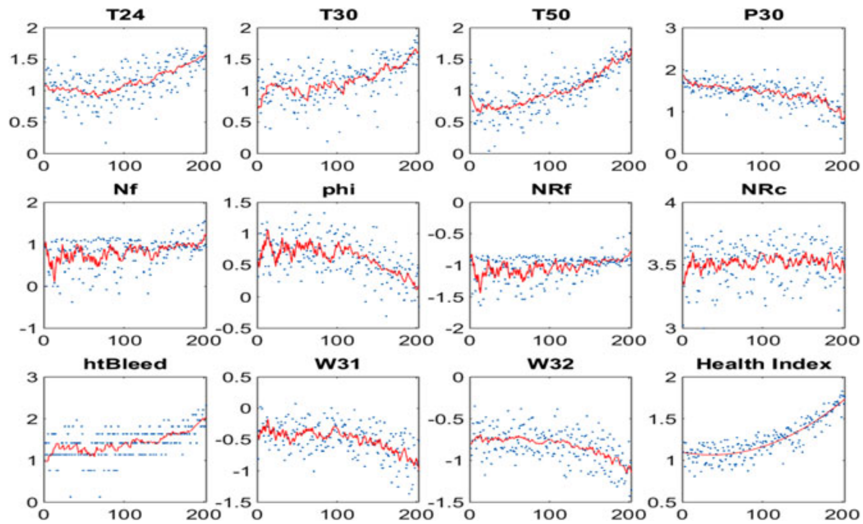

Fig. 2. Illustration of the model fitting results in all selected sensor data and the constructed HI with respect to the number of cycle measurements.

$[1, n_i + t, (n_i + t)^2]\boldsymbol{\Sigma}^1[1, n_i + t, (n_i + t)^2]' + \sigma^2$. Then, the cumulative distribution function (CDF) of the estimated remaining lifetime $\tilde{R}_i$ given the updated degradation coefficient $\boldsymbol{\theta}_i|\boldsymbol{h}_i$ can be calculated by considering both the uncertainties involved in the constructed HI and failure threshold

$$F_{\tilde{R}_i|\boldsymbol{h}_i}(t) = P\left(\tilde{R}_i \le t|\boldsymbol{h}_i\right) = P\left(h_i\left(n_i+t\right) \ge u|\boldsymbol{h}_i\right) = 1 - P\left(h_i\left(n_i+t\right) < u|\boldsymbol{h}_i\right) = \Phi\left(\frac{E\left(h_i\left(n_i+t\right)\right) - u}{\sqrt{V\left(h_i\left(n_i+t\right)\right) + v}}\right) = \Phi\left(g\left(t\right)\right). \tag{9}$$

Recall that here u and v refer to the mean and the variance of failure threshold of the constructed HI, respectively. $\Phi(\cdot)$ is the CDF of the standard normal distribution. Given that the remaining lifetime is nonnegative, i.e., $\tilde{R}_i \ge 0$, a truncated CDF conditioning on $\tilde{R}_i \ge 0$ is used

$$P\left(\tilde{R}_i \le t|\boldsymbol{h}_i, \tilde{R}_i \ge 0\right) = \frac{\Phi\left(g\left(t\right)\right) - \Phi\left(g\left(0\right)\right)}{1 - \Phi(g\left(0\right))}. \tag{10}$$

Since the RLD is skewed, it is preferred to utilize the median as the point estimator for remaining life prediction. This can be achieved by setting $P(\tilde{R}_i \le t|\boldsymbol{h}_i, \tilde{R}_i \ge 0) = 0.5$.

Define the remaining life prediction error e_i as the relative difference between the predicted and the true failure time for unit i

$$e_i = \left|\frac{\left(n_i + \tilde{R}_i\right) - \left(n_i + R_i\right)}{n_i + R_i}\right| = \left|\frac{\tilde{R}_i - R_i}{n_i + R_i}\right| \tag{11}$$

where n_i is the number of available observations in testing unit i, R_i is the true remaining lifetime for testing unit i, and $\tilde{R}_i$ is the estimated remaining lifetime for testing unit i.

We implement the above prognostic procedure based on each original sensor data, HI-single, and HI-multiple. Fig. 3 shows the comparison results in the remaining life prediction errors at different percentage of available observations. For example, the "90%" label refers to the prediction errors when testing units have observed 90% of data toward the end of life. From Fig. 3, we can see that the HI constructed by the proposed method has a better performance than 1) the best single sensor data from all 11 sensors with considerations of multiple operational conditions, and also 2) the HI constructed by the data-level fusion approach in [33] without considering the effects of multiple operational conditions. One possible reason is that our proposed method inherits the advantage of both approaches in 1 and 2, i.e., our method not only considers the effect of multiple operational conditions, but also takes advantage of the rich and dependent information from multiple sensor data when constructing the HI.

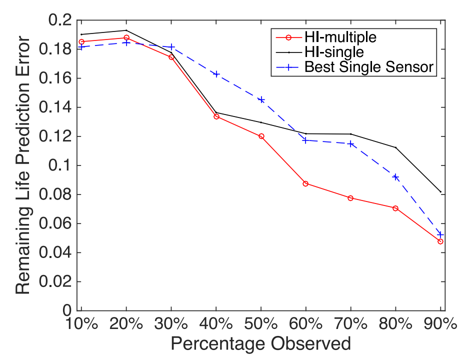

Fig. 3. Comparison results of the remaining life prediction errors at different percentages of observations for the best single sensor data by considering multiple operational conditions, the HI constructed by the existing data-level fusion model without considering multiple operational conditions, and the HI derived from the proposed data-level fusion model.

Fig. 3 also shows that the remaining life prediction becomes more accurate when more percentage of observations is available. This is mainly because of the following two reasons. First, as more data are observed, we are more confident about the calibrated degradation model when making predictions. In the meantime, as the percentage of data observed increases, we are only required to make a prediction in a shorter future time period, and, thus, less uncertainty is involved in the prognostics. Second, in this study, we also adopt a similar approach as in [33] that assigns $c_{i,t}$ as an increasing arithmetic series. This allows more penalties given to the model fitting errors as the

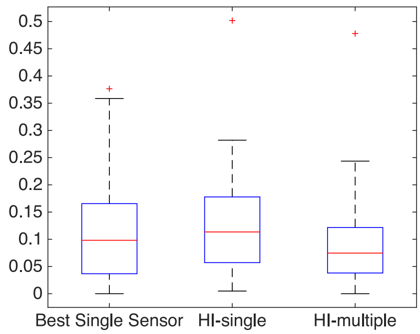

Fig. 4. Comparison results of the remaining life prediction errors at 60% of observations for the best single sensor, the HI-single, and the HI-multiple.

unit approaches to failure. Thus, the remaining life predictions by using the constructed HI become more accurate as more data are observed. To show the range of the prediction errors for the testing units, in Fig. 4, we further provide the boxplot of the residual life prediction errors of all sensors with 60 percentage of lifetime observed. Again, we focus on the performance of the best single sensor, HI-single, and HI-multiple. This figure shows that the prediction errors for the testing units by using the proposed method have not only the smallest mean but also the smallest STD.

V. Conclusion

The massive data collected from multiple sensors have enabled an unprecedented opportunity to better understand the current health status and make inferences about the future behavior of the operation unit. Conventionally, degradation modeling and prognostics have been focused on a single sensor data for analysis. While advanced data fusion models have been developed to take advantage of the rich and dependent information from multiple sensors, they often assume the unit runs under a single operational condition, which may not be effectively used in practice. Thus, a critical question that remains to be resolved is how to simultaneously tackle the challenges from both multiple sensors and multiple operational conditions.

The main contribution of this paper is to fill this gap by developing a systematic data-level fusion methodology via the construction of a composite HI. The derived HI not only combines the information from multiple sensors, but also considers the effects of multiple operational conditions to achieve a better understanding of the health status of the unit over the lifecycle. In particular, the HI is constructed through optimizing three derived properties, including minimizing both the model fitting errors and variance of the failure threshold, and maximizing the range information. Two practical scenarios are also discussed in the proposed data fusion model, depending on whether the future conditions are known or not. The developed method was then tested and validated by using the dataset of aircraft gas turbine engine that was generated by C-MAPSS. The case study has shown that the constructed HI outperforms each original single sensor data that considers multiple operational conditions, and also the HI constructed by the existing data-level fusion method without considering the effects of multiple operational conditions in the context of degradation modeling and prognostics.

There are several important topics for future studies. First, this paper assumes that the operational condition only changes at the discrete time. Future research is needed to address the challenge when the condition is changing continuously. Second, this paper focuses on the linear fusion function when combining multiple sensor data. More studies are needed to extend this fusion function into nonlinear cases. Third, it is worth investigating how to extend the current degradation model into a more generic form when developing the data fusion model. Finally, this paper assumes that the fusion coefficient $\boldsymbol{w}^*$ that characterizes the relationships among sensor measurements does not change over time and can be estimated based on training units. It would be an interesting topic to study how to online update the fusion coefficient in our future work.

Appendix

Proof of the variance of the estimated *failure threshold can be written as* $\boldsymbol{w}'\boldsymbol{D}\boldsymbol{w}$.

Proof: Recall that the last observation the HI of unit i can be represented as: $\frac{1}{m-1}(\sum_{i=1}^m (h_i(n_i))^2 - m(\frac{1}{m}\sum_{i=1}^m h_i(n_i))^2) = \frac{1}{m-1}(\sum_{i=1}^m \boldsymbol{w}'\boldsymbol{\Delta z}_i' 1_{n_i}' 1_{n_i} \boldsymbol{\Delta z}_i \boldsymbol{w} - m(\frac{1}{m}\sum_{i=1}^m 1_{n_i} \boldsymbol{\Delta z}_i \boldsymbol{w})^2) = \boldsymbol{w}'\boldsymbol{D}\boldsymbol{w}$, in which $\boldsymbol{D} = \frac{1}{m-1}(\sum_{i=1}^m \boldsymbol{\Delta z}_i' 1_{n_i}' 1_{n_i} \boldsymbol{\Delta z}_i - \frac{1}{m}\boldsymbol{G}'\boldsymbol{G})$ and $\boldsymbol{G} = \sum_{i=1}^m 1_{n_i} \boldsymbol{\Delta z}_i$.

Proof of the matrix $\boldsymbol{A}$ *is P.S.D.*

Proof: First, we prove that $\boldsymbol{D}$ is a P.S.D. matrix. This can be done by showing that $\boldsymbol{D} = \frac{1}{m-1}(\sum_{i=1}^m \boldsymbol{\Delta z}_i' 1_{n_i}' 1_{n_i} \boldsymbol{\Delta z}_i - \frac{1}{m}\boldsymbol{G}'\boldsymbol{G}) = \boldsymbol{U}'\boldsymbol{U}$, in which

$$\boldsymbol{U} = \sqrt{\frac{1}{m-1}} \begin{bmatrix} 1_{n_1} \boldsymbol{\Delta z}_1 - \frac{1}{m}\boldsymbol{G} \\ \vdots \\ 1_{n_m} \boldsymbol{\Delta z}_m - \frac{1}{m}\boldsymbol{G} \end{bmatrix}.$$

Thus, $\boldsymbol{D}$ is a P.S.D. matrix. Second, since $\boldsymbol{\Delta z}_i' \boldsymbol{B}_i' \boldsymbol{c}_i' (\boldsymbol{I}_i - \boldsymbol{H}_i) \boldsymbol{c}_i \boldsymbol{B}_i \boldsymbol{\Delta z}_i = \boldsymbol{\Delta z}_i' \boldsymbol{B}_i' \boldsymbol{c}_i' (\boldsymbol{I}_i - \boldsymbol{H}_i)' (\boldsymbol{I}_i - \boldsymbol{H}_i) \boldsymbol{c}_i \boldsymbol{B}_i \boldsymbol{\Delta z}_i$ is a P.S.D. matrix, and the sum of P.S.D. matrices is also P.S.D., we finish the proof that matrix $\boldsymbol{A} = \frac{1}{m}\sum_{i=1}^m \boldsymbol{\Delta z}_i' \boldsymbol{B}_i' \boldsymbol{c}_i' (\boldsymbol{I}_i - \boldsymbol{H}_i) \boldsymbol{c}_i \boldsymbol{B}_i \boldsymbol{\Delta z}_i + r_1 \boldsymbol{D}$ is P.S.D.

References

[1] W. B. Nelson, *Accelerated Testing: Statistical Models, Test Plans, and Data Analysis*. New York, NY, USA: Wiley, 2009.
[2] R. K. Mobley, *An Introduction to Predictive Maintenance*. Oxford, U.K.: Butterworth-Heinemann, 2002.
[3] D. A. Tobon-Mejia, K. Medjaher, N. Zerhouni, and G. Tripot, "A data-driven failure prognostics method based on mixture of Gaussians hidden Markov models," *IEEE Trans. Rel.*, vol. 61, no. 2, pp. 491–503, Jun. 2012.
[4] W. Caesarendra, A. Widodo, P. H. Thom, B.-S. Yang, and J. D. Setiawan, "Combined probability approach and indirect data-driven method for bearing degradation prognostics," *IEEE Trans. Rel.*, vol. 60, no. 1, pp. 14–20, Mar. 2011.
[5] N. Z. Gebraeel, M. A. Lawley, R. Li, and J. K. Ryan, "Residual-life distributions from component degradation signals: A Bayesian approach," *IIE Trans.*, vol. 37, no. 6, pp. 543–557, 2005.
[6] C. J. Lu and W. O. Meeker, "Using degradation measures to estimate a time-to-failure distribution," *Technometrics*, vol. 35, no. 2, pp. 161–174, 1993.
[7] N. Gebraeel, A. Elwany, and J. Pan, "Residual life predictions in the absence of prior degradation knowledge," *IEEE Trans. Rel.*, vol. 58, no. 1, pp. 106–117, Mar. 2009.

[8] A. Saxena, K. Goebel, D. Simon, and N. Eklund, "Damage propagation modeling for aircraft engine run-to-failure simulation," in *Proc. Int. Conf. Progn. Health Manag.*, 2008, pp. 1–9.
[9] D. L. Hall and J. Llinas, "An introduction to multisensor data fusion," *Proc. IEEE*, vol. 85, no. 1, pp. 6–23, Jan. 1997.
[10] D. Liu, H. Wang, Y. Peng, W. Xie, and H. Liao, "Satellite lithium-ion battery remaining cycle life prediction with novel indirect health indicator extraction," *Energies*, vol. 6, no. 8, pp. 3654–3668, 2013.
[11] D. Liu, W. Xie, H. Liao, and Y. Peng, "An integrated probabilistic approach to Lithium-Ion battery remaining useful life estimation," *IEEE Trans. Instrum. Meas*, vol. 64, no. 3, pp. 660–670, Mar. 2015.
[12] A. J. Chipperfield, B. Bica, and P. J. Fleming, "Fuzzy scheduling control of a gas turbine aero-engine: A multiobjective approach," *IEEE Trans. Ind. Electron.*, vol. 49, no. 3, pp. 536–548, Jun. 2002.
[13] N. Gebraeel and J. Pan, "Prognostic degradation models for computing and updating residual life distributions in a time-varying environment," *IEEE Trans. Rel.*, vol. 57, no. 4, pp. 539–550, Dec. 2008.
[14] K. Yang and J. Xue, "Continuous state reliability analysis," in *Proc. Int. Symp. Prod. Qual. Integr.*, 1996, pp. 251–257.
[15] M. J. Zuo, R. Jiang, and R. Yam, "Approaches for reliability modeling of continuous-state devices," *IEEE Trans. Rel.*, vol. 48, no. 1, pp. 9–18, Mar. 1999.
[16] S. Lu, H. Lu, and W. J. Kolarik, "Multivariate performance reliability prediction in real-time," *Rel. Eng. Syst. Safety*, vol. 72, no. 1, pp. 39–45, 2001.
[17] E. Zio and G. Peloni, "Particle filtering prognostic estimation of the remaining useful life of nonlinear components," *Rel. Eng. Syst. Safety*, vol. 96, no. 3, pp. 403–409, 2011.
[18] M. E. Orchard and G. J. Vachtsevanos, "A particle-filtering approach for on-line fault diagnosis and failure prognosis," *Trans. Inst. Meas. Control*, vol. 31, pp. 221–246, 2009.
[19] N. Gebraeel, "Sensory-updated residual life distributions for components with exponential degradation patterns," *IEEE Trans. Autom. Sci. Eng.*, vol. 3, no. 4, pp. 382–393, Oct. 2006.
[20] G. Whitmore and F. Schenkelberg, "Modelling accelerated degradation data using Wiener diffusion with a time scale transformation," *Lifetime Data Anal.*, vol. 3, no. 1, pp. 27–45, 1997.
[21] K. Doksum and A. Hoyland, "Models for variable-stress accelerated life testing experiments based on wiener processes and the inverse gaussian distribution," *Theory Probab. Appl.*, vol. 37, no. 1, pp. 137–139, 1993.
[22] C.-M. Liao and S.-T. Tseng, "Optimal design for step-stress accelerated degradation tests," *IEEE Trans. Rel.*, vol. 55, no. 1, pp. 59–66, Mar. 2006.
[23] C. Park and W. J. Padgett, "Stochastic degradation models with several accelerating variables," *IEEE Trans. Rel.*, vol. 55, no. 2, pp. 379–390, Jun. 2006.
[24] S.-T. Tseng, N. Balakrishnan, and C.-C. Tsai, "Optimal step-stress accelerated degradation test plan for gamma degradation processes," *IEEE Trans. Rel.*, vol. 58, no. 4, pp. 611–618, Dec. 2009.
[25] L. Bian and N. Gebraeel, "Stochastic methodology for prognostics under continuously varying environmental profiles," *Stat. Anal. Data Mining*, vol. 6, no. 3, pp. 260–270, 2013.
[26] R. Zhou, N. Gebraeel, and N. Serban, “Degradation-based residual life prediction under different environments using sparse signals,” *Ann. Appl. Stat.*, vol. 8, no. 3, pp. 1671–1689, 2014.
[27] J.-J. H. Shiau and H.-H. Lin, "Analyzing accelerated degradation data by nonparametric regression," *IEEE Trans. Rel.*, vol. 48, no. 2, pp. 149–158, Jun. 1999.
[28] J. A. Flory, J. P. Kharoufeh, and N. Z. Gebraeel, “A switching diffusion model for lifetime estimation in randomly-varying environments,” *IIE Trans.*, vol. 46, no. 11, pp. 1227–1241, 2014.
[29] T. Heger and M. Pandit, "Optical wear assessment system for grinding tools," *J. Electron. Imag.*, vol. 13, no. 3, pp. 450–461, 2004.
[30] D. Simon and D. L. Simon, "Aircraft turbofan engine health estimation using constrained Kalman filtering," *J. Eng. Gas Turbines Power*, vol. 127, no. 2, pp. 323–328, 2005.
[31] K. Salahshoor, M. Mosallaei, and M. Bayat, “Centralized and decentralized process and sensor fault monitoring using data fusion based on adaptive extended Kalman filter algorithm,” *Measurement*, vol. 41, no. 10, pp. 1059–1076, 2008.
[32] K. Liu, N. Z. Gebraeel, and J. Shi, "A data-level fusion model for developing composite health indices for degradation modeling and prognostic analysis," *IEEE Trans. Autom. Sci. Eng.*, vol. 10, no. 3, pp. 652–664, Jul. 2013.
[33] K. Liu and S. Huang, "Integration of data fusion methodology and degradation modeling process to improve prognostics," *IEEE Trans. Autom. Sci. Eng.*, vol. 13, no. 1, pp. 344–354, Jan. 2016.
[34] K. Liu, A. Chehade, and C. Song, “Optimize the signal quality of the composite health index via data fusion for degradation modeling and prognostic analysis,” *IEEE Trans. Autom. Sci. Eng.*, 2015, doi: 10.1109/TASE.2015.2446752.
[35] A. Saxena and K. Goebel, “C-MAPSS data set,” *NASA Ames Prognostics Data Repository*, 2008.
[36] D. K. Frederick, J. A. DeCastro, and J. S. Litt, “User’s guide for the commercial modular aero-propulsion system simulation (C-MAPSS),” NASA, Technical Memorandum No. TM-2007-215026, 2007.

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