Transport and dissipation in neutron star mergers

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Alford, Bovard, Hanauske, Rezzolla, Schwenzer, arXiv:1707.09475

Alford, Harutyunyan, Sedrakian, arXiv:1907.04192, 2006.07975

Alford and Harris, arXiv:1803.00662, arXiv:1907.03795

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Outline

I Neutron star mergers as a probe of dense matter:
Different phases with similar EoS may be distinguishable by their Transport Properties

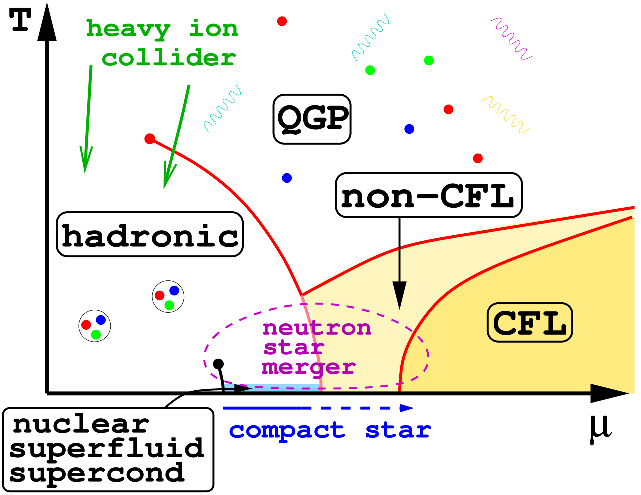
II Is thermal conductivity important in mergers?
Damping time for temperature inhomogeneities: is it fast enough to affect mergers?

III Is bulk viscosity important in mergers?
Damping time for density oscillations: is it fast enough to affect mergers?

IV Looking to the future

I. Mergers as a probe of dense matter

Conjectured QCD Phase diagram



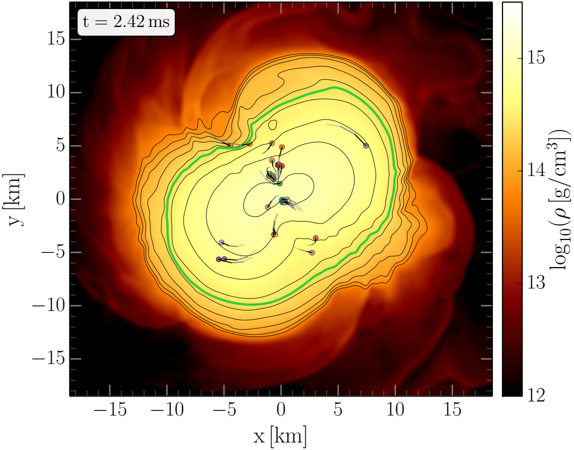
heavy ion collisions: deconfinement crossover and chiral critical point

neutron stars: quark matter core?

neutron star mergers: dynamics of warm and dense matter

Neutron star mergers

Mergers probe the properties of nuclear/quark matter at high density (up to $\sim 4n_{\text{sat}}$) and temperature (up to $\sim 60\,\text{MeV}$)

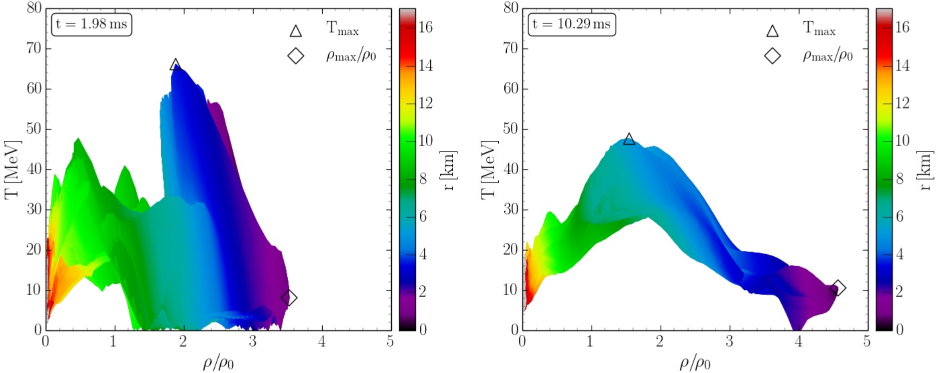


If we want to use mergers to learn about nuclear matter, we need to include all the relevant physics in our simulations.

Rezzolla group, Frankfurt

Video

Nuclear material in a neutron star merger



M. Hanauske, Rezzolla group, Frankfurt

Significant spatial/temporal variation in:

- temperature
- fluid flow velocity
- density

so we need to allow for

- thermal conductivity
- shear viscosity
- bulk viscosity

Role of transport/dissipation in mergers

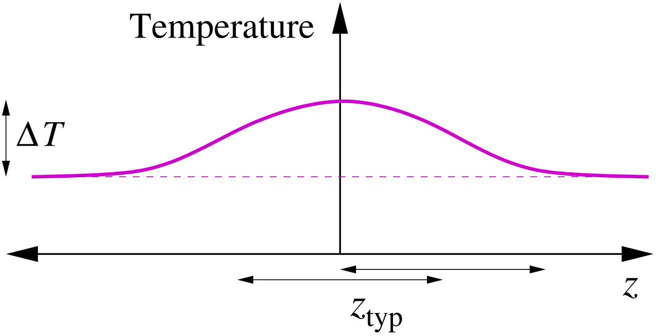
The important dissipation mechanisms are the ones whose equilibration time is $\lesssim 20\,\text{ms}$

Executive Summary:

Thermal equilibration:	might be fast enough to play a role, if • neutrinos are trapped • there are short-distance temperature gradients
Shear viscosity:	similar conclusion.
Bulk viscosity:	could damp density oscillations on the same timescale as the merger. Include bulk viscosity in merger simulations.

II. Thermal equilibration under merger conditions

Thermal equilibration time



Volume
$V \sim z_{\rm typ}^3$

Surface area
$A \sim 6z_{\rm typ}^2$

Time to equilibrate: $\tau_\kappa = \dfrac{\text{extra heat in region}}{\text{rate of heat outflow}} = \dfrac{E_{\rm therm}}{W_{\rm therm}}$

Thermal diffusion is important if $\tau_\kappa \lesssim 20\,\text{ms}$

Estimating thermal equilibration time

Extra heat in region: $E_{\rm therm} = c_V V \Delta T \approx c_V z_{\rm typ}^3 \Delta T$

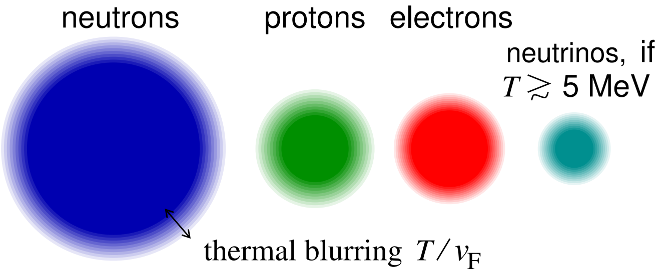
Rate of heat outflow: $W_{\rm therm} = \kappa \dfrac{dT}{dz} A \approx \kappa \dfrac{\Delta T}{z_{\rm typ}} 6 z_{\rm typ}^2$

Time to equilibrate: $\tau_\kappa = \dfrac{E_{\rm therm}}{W_{\rm therm}} \approx \dfrac{c_V z_{\rm typ}^2}{6\kappa}$

To calculate the thermal equilibration time τ_κ, we need
- specific heat capacity c_V
- thermal conductivity κ

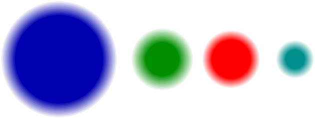
Nuclear material constituents

Fermi surfaces:



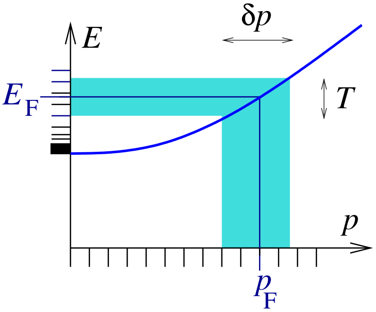
neutrons:	$\sim 90\%$ of baryons	$p_{Fn} \sim 350\,\text{MeV}$
protons:	$\sim 10\%$ of baryons	$p_{Fp} \sim 150\,\text{MeV}$
electrons:	same density as protons	$p_{Fe} = p_{Fp}$
neutrinos:	only present if mfp $\ll 10\,\text{km}$	i.e. when $T \gtrsim 5\,\text{MeV}$

Specific heat capacity



Dominated by neutrons

$$c_V \sim \text{number of states available to carry energy} \lesssim T$$
$$\sim \text{vol of mom space with states available to carry energy} \lesssim T$$
$$\sim {p_{Fn}}^2 \delta p$$



$$\delta p = \frac{T}{v_{Fn}} = T \times \frac{m_n^*}{p_{Fn}}$$

$$c_V \sim {p_{Fn}}^2 \delta p \sim {p_{Fn}}^2 \frac{m_n^*}{p_{Fn}} T \sim m_n^* p_{Fn} T$$

(Note: neutron density $n_n \sim {p_{Fn}}^3$)

$$\boxed{c_V \approx 1.0\, m_n^*\, {n_n}^{1/3}\, T}$$

Thermal conductivity

Thermal conductivity $\kappa \propto n\, v\, \lambda$

Dominated by the species with the right combination of
- high density
- weak interactions $\Rightarrow$ long mean free path (mfp) λ

neutrons: high density, but strongly interacting (short mfp) ✗

protons: low density, strongly interacting (short mfp) ✗

electrons: low density, only EM interactions (long mfp) ✓

neutrinos:
- $T \lesssim 5\,\mathrm{MeV}$: $\lambda >$ size of merged stars, so they all escape, density $= 0$ ✗
- $T \gtrsim 5\,\mathrm{MeV}$: $\lambda <$ size of merged stars, but still very long mfp! ✓✓

Neutrino-dominated thermal equilibration

Neutrino-trapped regime, $T \gtrsim 5\,\mathrm{MeV}$

equilibration time $\tau_\kappa \approx \dfrac{c_V z_{\rm typ}^2}{6\kappa}$ $\qquad \kappa^{(\nu)} \approx 0.33 \dfrac{n_\nu^{2/3}}{G_F^2 m_n^{*2} n_e^{1/3} T}$

$$\tau_\kappa^{(\nu)} \approx \boxed{700\,\mathrm{ms}} \left(\frac{z_{\rm typ}}{1\,\mathrm{km}}\right)^2 \left(\frac{T}{10\,\mathrm{MeV}}\right)^2 \left(\frac{\mu_e}{2\mu_\nu}\right)^2 \left(\frac{0.1}{x_p}\right)^{1/3} \left(\frac{m_n^*}{0.8\,m_n}\right)^3$$

Neutrino thermal transport may be important if there are thermal gradients on $\lesssim 0.1\,\mathrm{km}$ scale

That can lead to $\tau_\kappa \lesssim 20\,\mathrm{ms}$

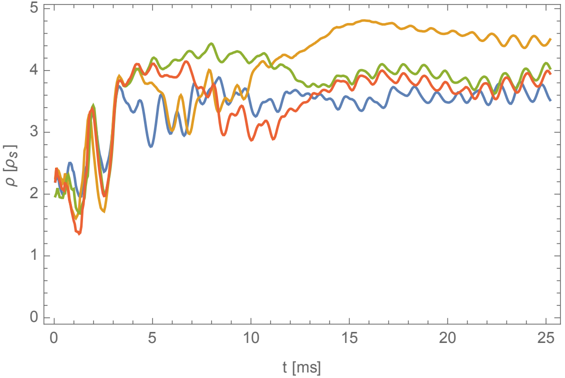
III. Damping of density oscillations in mergers

Equivalently: sound attenuation via bulk viscosity

Density oscillations in mergers

Density vs time for tracers in merger

(Bulk viscosity neglected)



Tracers (co-moving fluid elements) show dramatic density oscillations, especially in the first 5 ms.

Amplitude: up to 50%

Period: 1–2 ms

How long does it take for bulk viscosity to dissipate a sizeable fraction of the energy of a density oscillation?

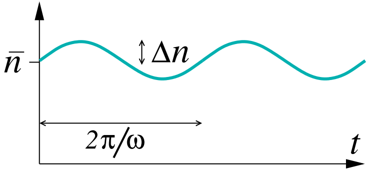
What is the damping time τ_ζ?

Can we get $\tau_\zeta \lesssim 20\,\text{ms}$?

Density oscillation damping time τ_ζ

Density oscillation of amplitude Δn at angular freq ω:

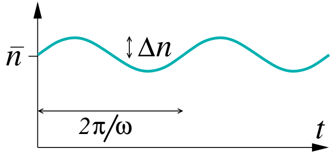
$$n(t) = \bar{n} + \Delta n \cos(\omega t)$$



Damping Time: $\tau_\zeta = \dfrac{\text{energy stored in oscillation}}{\text{rate of energy loss}} = \dfrac{E_{\rm comp}}{W_{\rm comp}}$

Bulk viscous damping is important if $\tau_\zeta \lesssim 20\,\text{ms}$

Calculating damping time



Energy of density oscillation:
(K = nuclear incompressibility)

$$E_{\rm comp} = \frac{K}{18}\bar{n}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

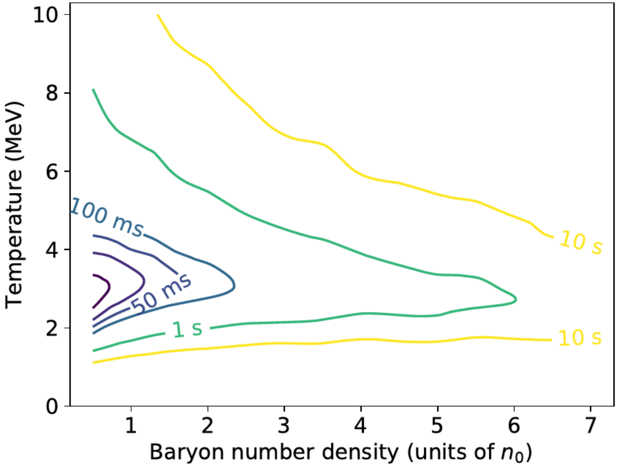
Compression dissipation rate:
(ζ = bulk viscosity)

$$W_{\rm comp} = \zeta\frac{\omega^2}{2}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

Damping Time: $\tau_\zeta = \dfrac{E_{\rm comp}}{W_{\rm comp}} = \dfrac{K\bar{n}}{9\omega^2\,\zeta}$

To calculate the density oscillation damping time τ_ζ, we need
- nuclear incompressibility K (from EoS)
- bulk viscosity ζ (from beta-equilibration of proton fraction)

Damping time results (ν-transparent)

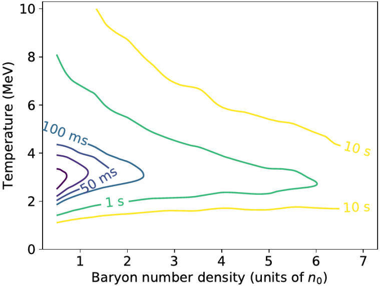


EoS: HS(DD2)
$M_{\max}$: $2.42\,M_\odot$
$R_{1.4\,M_\odot}$: $13.3\,\mathrm{km}$

Oscillation freq:
$f = 1\,\mathrm{kHz}$

Fast damping at $T \sim 3\,\mathrm{MeV}$, $n \lesssim 2n_{\mathrm{sat}}$

Damping time behavior



Damping Time $\quad \tau_\zeta = \dfrac{K\bar{n}}{9\omega^2\,\zeta}$

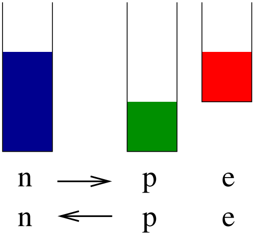
Characteristics of the damping time plot:

- Damping gets slower at higher density.
  Baryon density $\bar{n}$ and incompressibility K are both increasing.
  Oscillations carry more energy $\Rightarrow$ slower to damp
- Non-monotonic T-dependence: damping is fastest at $T \sim 3\,\text{MeV}$.
  Damping is slow at very low or very high temperature.
  Non-monotonic dependence of bulk viscosity on temperature

Bulk viscosity and beta equilibration

When you compress nuclear matter, the proton fraction wants to change.

Only weak interactions can change proton fraction, and they are rather slow...



	neutrino-transparent ($T \lesssim 5\,\text{MeV}$)*	neutrino-trapped ($T \gtrsim 5\,\text{MeV}$)*
neutron decay	$n \to p + e^- + \bar{\nu}_e$	$\nu_e + n \to p + e^-$
electron capture	$p + e^- \to n + \nu_e$	$p + e^- \to n + \nu_e$
	forward $\neq$ backward	$A + B \leftrightarrow C + D$

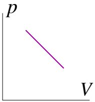
* Neutrino transparency is a finite volume effect, which occurs when the neutrino mean free path is greater than the size of the system. Our system is a neutron star, $R \sim 10\,\text{km}$

Bulk viscosity: phase lag in system response

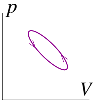
Some property of the material (proton fraction) takes time to equilibrate.
Baryon density n and hence fluid element volume V gets out of phase with applied pressure p:

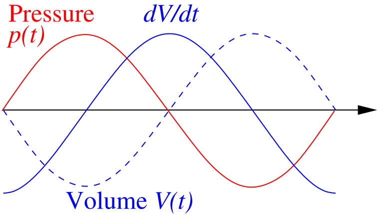
$$\text{Dissipation} = -\int p\, dV = -\int p\, \frac{dV}{dt} dt$$

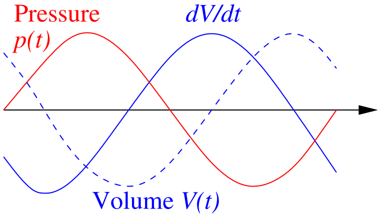
No phase lag.
Dissipation $= 0$



Some phase lag.
Dissipation > 0





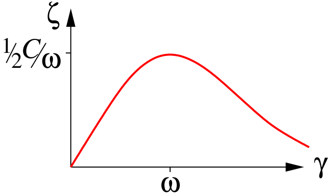


Bulk viscosity: a resonant phenomenon

Bulk viscosity is maximum when

$$\underset{\gamma}{\text{(internal equilibration rate)}} = \underset{\omega}{\text{(freq of density oscillation)}}$$

$$\zeta = C \frac{\gamma}{\gamma^2 + \omega^2}$$



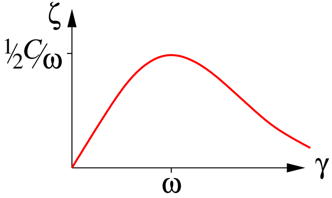
C is a combination of susceptibilities

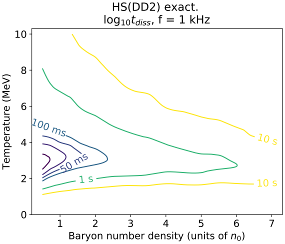
- Fast equilibration: $\gamma \to \infty \;\Rightarrow\; \zeta \to 0$
  System is always in equilibrium. No pressure-density phase lag.
- Slow equilibration: $\gamma \to 0 \;\Rightarrow\; \zeta \to 0$.
  System does not try to equilibrate: proton number and neutron number are both conserved. Proton fraction fixed.
- Maximum phase lag when $\omega = \gamma$.

Resonant peak in bulk viscosity

We now see why bulk visc is a non-monotonic fn of temperature.

$$\zeta = C\frac{\gamma}{\gamma^2+\omega^2}$$





Beta equilibration rate γ is sensitive to temperature (phase space at Fermi surface)

Maximum bulk viscosity in a neutron star merger will be when equilibration rate matches typical compression frequency $f \approx 1\,\text{kHz}$.

I.e. when $\gamma \sim 2\pi \times 1\,\text{kHz}$

IV. Summary

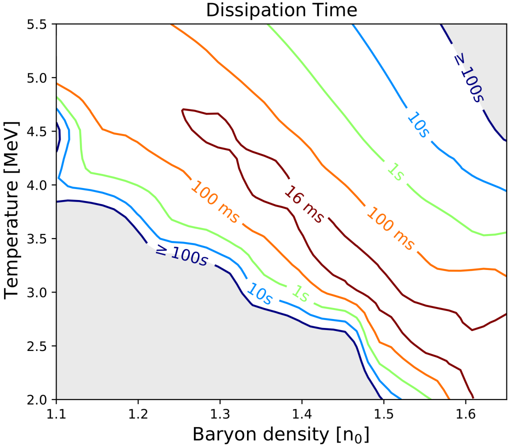
- Some forms of dissipation are probably physically important for neutron star mergers.
- Thermal conductivity and shear viscosity may become significant in the neutrino-trapped regime ($T \gtrsim 5\,\mathrm{MeV}$) if there are fine-scale gradients ($z \lesssim 100\,\mathrm{m}$).
- In neutrino-transparent nuclear matter (at low density and $T \sim 3\,\mathrm{MeV}$) bulk viscosity significantly damps density oscillations.

Next steps:

- Include bulk viscosity in merger simulations.
- Calculate bulk viscous damping for other forms of matter: hyperonic, pion condensed, nuclear pasta, quark matter, etc
- Damping of f-modes in inspiral?
- Beyond Standard Model physics...?

Hyperonic matter

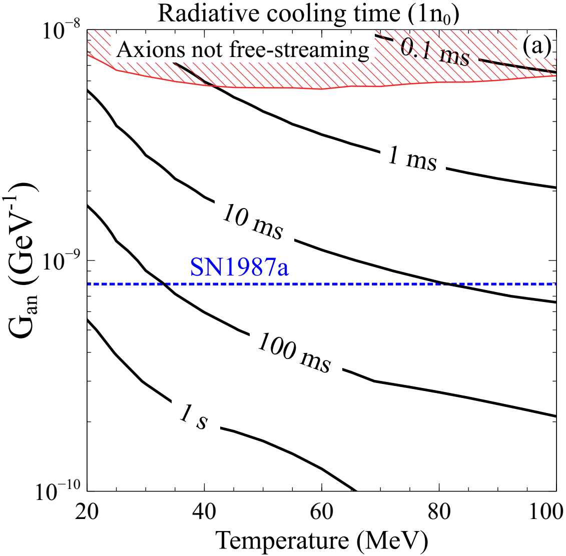
Bulk viscous damping time for density oscillations



Alford and Haber, in preparation

Cooling by axion emission

Time for a hot region to cool to half its original temperature



Harris, Fortin, Sinha, Alford
`arXiv:2003.09768`

Extra slides

Why is resonance with $1\,$kHz at $T \sim$ MeV?

Let's estimate $\gamma(T)$ and see when it is $2\pi \times 1\,$kHz.

$$\frac{dn_a}{dt} = -\gamma\left(n_a - n_{a,\mathrm{equil}}\right)$$
$$\Gamma_{n\to p} - \Gamma_{p\to n} \sim -\gamma \frac{\partial n_a}{\partial \mu_a}\mu_a$$

In FS approx, at β-equilibrium,

$$\Gamma_{n\to p} = \Gamma_{p\to n} \sim G_F^2 \times (p_{Fn}^2 T) \times (p_{Fp} T) \times T^3$$

If we push it away from β equilibrium by adding μ_a, the leading correction is to replace one power of T with μ_a

$$\Gamma_{n\to p} - \Gamma_{p\to n} \sim G_F^2 (p_{Fn}^2 T) \times (p_{Fp} T) \times T^2 \mu_a$$

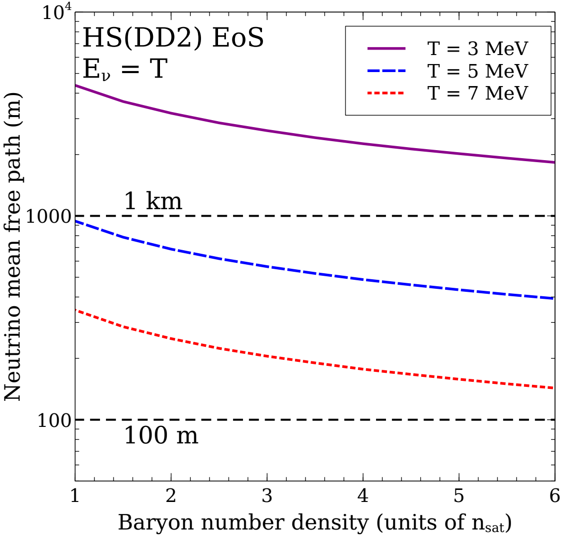
So

$$\gamma \sim \frac{\partial \mu_a}{\partial n_a} G_F^2\, p_{Fn}^2\, p_{Fp}\, T^4 \sim \frac{1}{(30\,\mathrm{MeV})^2} \frac{(350\,\mathrm{MeV})^2 (150\,\mathrm{MeV})}{(290\,\mathrm{GeV})^4} T^4$$

Solve for when $\gamma = 2\pi \times 1\,\mathrm{kHz} = 4 \times 10^{-18}\,\mathrm{MeV}$:

$$T \sim 1\,\mathrm{MeV}$$

Neutrino mean free path



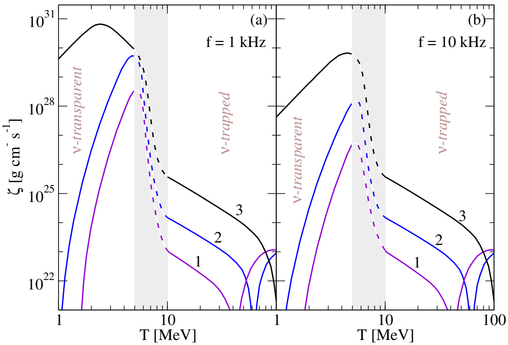
When does neutrino trapping begin?

mfp $\sim 3\,$km: $T = 2$-$3\,$MeV

mfp $\sim 1\,$km: $T = 4$-$5\,$MeV

mfp $\sim 0.3\,$km: $T = 6$-$7\,$MeV

The neutrino-trapped regime



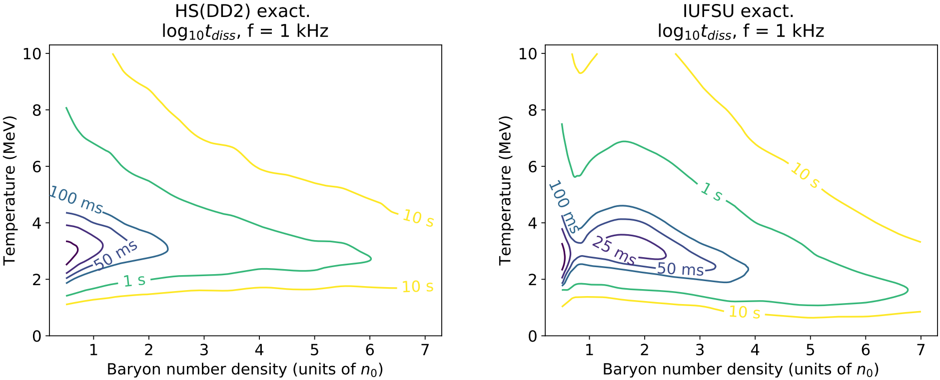
Bulk viscosity is lower in hot matter ($T \gtrsim 5\,\text{MeV}$).

- β equilibration is too fast, above resonant temperature.
- The relevant susceptibilities are smaller, so the peak bulk visc is smaller

Damping time results (ν-transparent)

Results for two eqns of state:

name	type	$M_{\rm max}$	$R_{1.4\,M_\odot}$	d-Urca threshold
HS(DD2)	stiffer	$2.42\,M_\odot$	13.3 km	none
IUFSU	softer	$1.96\,M_\odot$	12.8 km	$4n_{\rm sat}$



No direct Urca

d-Urca threshold at $4n_{\rm sat}$

At $T \sim 3\,\text{MeV}$, some EoS give $\tau_\zeta \lesssim 20\,\text{ms}$

Compression $\Rightarrow$ β-equilibration

Density oscillations in cold ($T \lesssim 1\,\text{MeV}$) nuclear matter

- Does compression/rarefaction drive nuclear matter out of β-equilibrium? Yes
- Why?
  Neutron are semi-relativistic so under compression their E_F rises quite a bit, but protons are very nonrelativistic so their E_F doesn't change much, so the neutrons can decay into protons.
- What process re-establishes β equilibrium? Urca process.

$$
\begin{aligned}
n &\rightarrow p + e^- + \bar{\nu}_e \\
p + e^- &\rightarrow n + \nu_e
\end{aligned}
$$

- At what temperature does the resultant equilibration rate match the frequency of density oscillations in mergers, $\sim 1\,\text{kHz}$?
  How do we calculate the rate of these processes?