Plasma screening and the critical end point in the QCD phase diagram

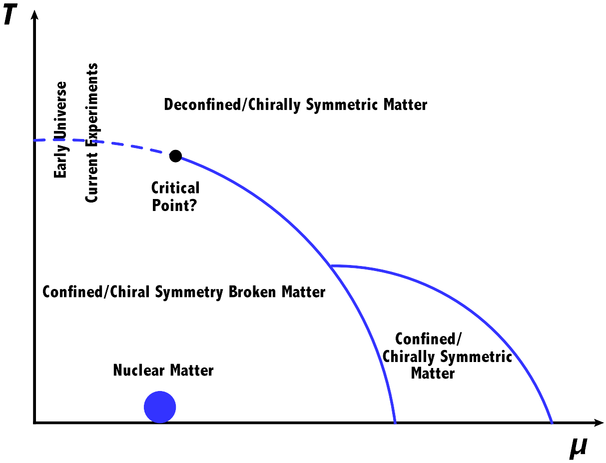
Alejandro Ayala
Instituto de Ciencias Nucleares,
Universidad Nacional Autónoma de México

Theoretical Physics Colloquium
College of Integrative Sciences and Arts
University of Arizona
October 2021

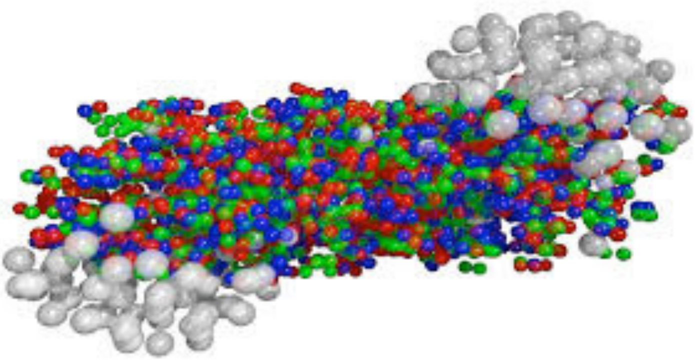
Contents

- Motivation: the QCD phase diagram
- QCD chiral symmetry
- QCD confinement and asymptotic freedom
- QCD at finite temperature and density: The phase diagram.
- Phase transitions.
- Search for the critical end point: event by event fluctuations
- The Linear Sigma Model with quarks: Explicit plasma screening in an effective model
- Summary.

The QCD phase diagram

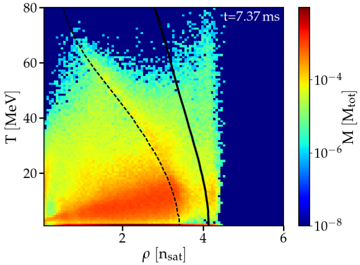


Heavy-Ion Physics



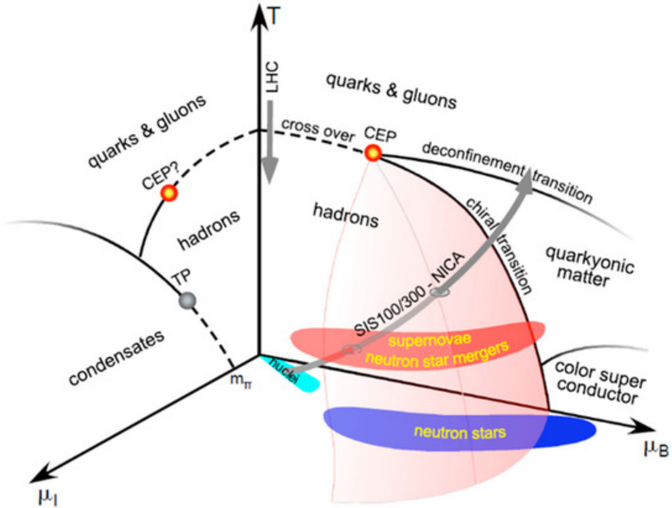
Neutron star mergers

Neutron star mergers



Population of the QCD phase diagram by a typical merger event of two neutron stars with 1.35 $M_\odot$ each, for $t = 7.37$ ms (left panel) and $t = 24.54$ ms (right panel) after the merging.

The extended QCD phase diagram



QCD: The theory of strong interactions

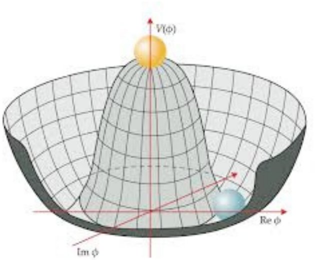
- Gauge theory with the **local** symmetry group $SU(N_c)$. (In the real world $N_c = 3$).
- The fundamental fields are the **quarks** (matter fields) and **gluons** gauge fields.
- Each one of the N_f **quark fields** belong to the **fundamental representation** of the color group which is (N_c)–dimensional, **antiquark fields** to the **complex conjugate of the fundamental representation**, also (N_c)–dimensional and **gluon fields** to the adjoint representation which is $(N_c^2 - 1)$–dimensional.

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{\psi}_i^a \left(i\gamma^\mu(\partial_\mu \delta^{ab} + ig_s A_\mu^{ab}) - m_i \delta^{ab} \right) \psi_i^b - \frac{1}{4} G_{\mu\nu}^\alpha G_\alpha^{\mu\nu};$$

$$G_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g_s f^{\alpha\beta\sigma} A_\mu^\beta A_\nu^\sigma; \qquad A_\mu^{ab} = A_\mu^\sigma (\tau_\sigma)^{ab}$$

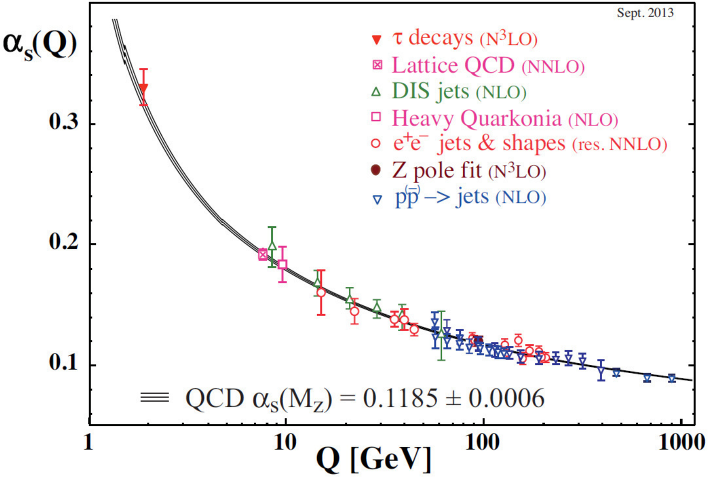
a, b run from 1 to N_c, α, β, σ run from 1 to $N_c^2 - 1$.

Spontaneous chiral symmetry breaking



Goldstone bosons correspond to the directions where the potential is flat

Running coupling



Running coupling

- Competition between color and flavor

$$\alpha_s(Q^2) = \frac{\alpha_s(\tilde{\mu}^2)}{\left[1 + b_1 \alpha_s(\tilde{\mu}^2) \ln\left(\frac{Q^2}{\tilde{\mu}^2}\right)\right]}$$

$$b_1 = \frac{11 N_c - 2 N_f}{12\pi}$$

$$N_c = 3, \quad N_f = 6 \implies b_1 > 0$$

α_s **decreases with** $Q = \sqrt{Q^2}$

Λ_{QCD}

- When the exchanged momentum in a given process is small, the coupling is so large that pertuerbative calculations become meaningless. This is the so called **non-perturbative regime**
- Processes where perturbation theory can be applied are usually those where the transferred momentum satisfies $Q^2 \gtrsim 1$ GeV^2.
- To quantify this statement, notice that we can define a transferred momentum value Λ_{QCD} small enough such that the denominator vanishes and thus the coupling blows up

$$1 + b_1\alpha_s(\tilde{\mu}^2)\ln(\Lambda^2_{QCD}/\tilde{\mu}^2) = 0, \quad \Lambda^2_{QCD} = \tilde{\mu}^2 e^{-\frac{1}{b_1\alpha_s(\tilde{\mu})}}$$

- Λ_{QCD} is a renormalization scheme dependent quantity. In the $\overline{\text{MS}}$ scheme and for three active flavors, its value is of order $\Lambda_{QCD} \sim 200 - 300$ MeV.

The phase diagram: QCD at finite T and μ_B.

- Two important parameters for QCD in equilibrium: The **temperature** T and the **baryon number density** n_B (or its conjugate variable $\mu_B = 3\mu_q$).

Since the intrinsic scale of QCD is $\Lambda_{QCD} \sim 200$ MeV, one expects a transition around $T \sim 200$ MeV, $n_B \sim \Lambda_{QCD}^3 \sim 1$ fm^{-3}.

- Exploration of a wider range of the phase diagram with n_B up to several times the normal nuclear matter density $n_0 \sim 0.16$ fm^{-3} can be carried out by the BES-RHIC and other facilities such as FAIR at GSI, NICA at JINR, J-PARC at JAEA and KEK.
- In nature, the interior of compact stellar objects is the relevant system where dense and low temperature QCD matter is realized.

Phase transitions

What is a phase transition?

- Transformation of a given substance from one state of matter to another.
- During the phase transition some quantities change, often in a discontinuous manner.
- Changes result in variations of external conditions such as pressure, temperature, etc.

Phase transitions

When does a phase transition happen?

- In technical terms, they occur when the **free energy is non-analytic (one of its derivatives diverges)** for some values of the thermodynamical variables.
- They result from the interaction of a **large number of particles** and in general it does not occur when the system is very small or has a small number of particles.
- On the phase transition lines **the free energies in both phases coincide.**
- Some times it is possible to change the state of a substance without crossing a phase transition line. Under these conditions one talks about a **crossover transition.**

Phase transitions

Classification, according to behavior of free energy as a function of a given thermodynamical variable (Ehrenfest). **They are named according to the derivative of lowest order that becomes discontinuous during the transition**

- **First order:** First derivative of free energy is discontinuous. **Example: boiling water.** Discontinuity in the density, i.e. derivative of free energy with respect to chemical potential.
- **Second order:** First derivative is continuous. Second derivative is discontinuous. **Example: Ferromagnetism.** The magnetization, i.e. the derivative of the free energy with respect to the external field is continuous. The susceptibility, i.e. the derivative of the magnetization with respect to the external field, is discontinuous.

Ideal gas hadron thermodynamics

- Consider an ideal gas of identical neutral scalar particles of mass m_0 contained in a box volume V. Assume Boltzmann statistics. The partition function is given by

$$
\begin{aligned}
\mathcal{Z}(T,V) &= \sum_N \frac{1}{N!}\left[\frac{V}{(2\pi)^3}\int d^3p\,\exp\left\{-\frac{\sqrt{p^2+m_0^2}}{T}\right\}\right]^N \\
\ln \mathcal{Z}(T,V) &= \frac{VTm_0^2}{2\pi^2}K_2(m_i/T) \\
\epsilon(T) &= -\frac{1}{V}\frac{\partial \ln \mathcal{Z}(T,V)}{\partial(1/T)} \xrightarrow{T\gg m_0} \frac{3}{\pi^2}T^4 \quad \text{energy density} \\
n(T) &= -\frac{1}{V}\frac{\partial \ln \mathcal{Z}(T,V)}{\partial(V)} \xrightarrow{T\gg m_0} \frac{1}{\pi^2}T^3 \quad \text{particle density} \\
\omega(T) &= \epsilon(T)/n(T) \simeq 3T \quad \text{average energy per particle}
\end{aligned}
$$

Chiral transition and hadronization

- Hadron multiplicities established very close to the phase boundary.

Statistical model (Hadron Resonance Gas model)

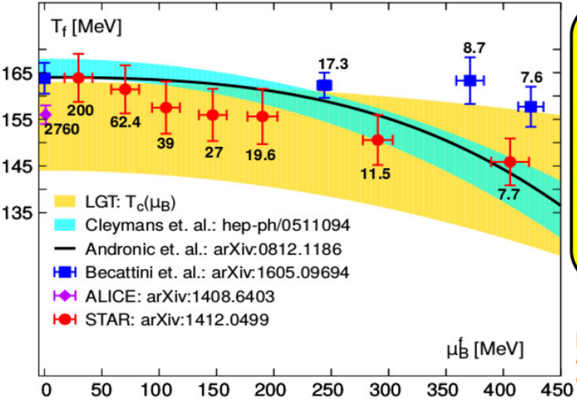
$$n_j = \frac{g_j}{2\pi^2}\int_0^\infty p^2 dp \left[\exp\left\{\sqrt{p^2+M_j^2}/T_{\rm ch}-\mu_{\rm ch}\right\}\pm 1\right]^{-1}$$

- From the hadron side, abundances due to multi-particle collisions whose importance is **enhanced due to high particle density in the phase transition region.** Collective phenomena play an important role.
- **Since the multi-particle scattering rates fall-off rapidly, the experimentally determined chemical freeze-out is a good measure of the phase transition temperature.**

Chiral transition and hadronization

Chiral transition, hadronization and freeze-out

LGT: $T_c(\mu_B) = 154(9)(1 - [0.006; 0.014](\mu_B/T)^2)\mathrm{MeV}$



phenomenological freeze-out / hadronization curve, QCD transition line and experimental data (obtained by assuming the validity of the HRG model) are consistent for

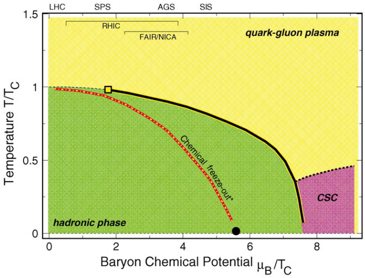
$$\mu_B/T \lesssim 3$$

HOWEVER

physics is quite different at lower and upper end of the current error bar on Tc

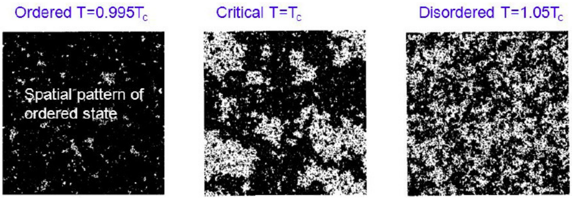
→ **probed with net-charge correlations&fluctuations**

Is there a Critical End Point?



- Most of the **effective models** suggest the existence of a QCD critical point (μ_{CEP}, T_{CEP}) somewhere in the middle of the phase diagram **where the crossover line becomes a first order transition line.**
- Signals are and will be looked for in current and future facilities.

Critical point and critical phenomena



2D-Ising model simulation from ISNB4-563-02435-X C33421

Critical Phenomena :

- Density fluctuations and cluster formations.
- Divergence of Correlation length (ξ).
  Susceptibilities (χ), heat capacity (C_V) ,
  Compressibility (κ) etc.
  Critical opalescence.
- Universality and critical exponents determined by the symmetry and dimensions of underlying system.

First CP is discovered in 1869 for CO_2

T_c = 31°C

Can we discovery the Critical Point of Quark Matter ? (**Put a permanent mark in the QCD phase diagram in text book.**)

T_c ~ Trillion (10^{12}) °C

Xiaofeng Luo ISMD 2016, Jeju, Korea, Aug. 29-Sept. 2, 2016 4 / 18

Analysis tools: Fluctuations of conserved quantities

- A powerful tool to experimentally locate the CEP is the study of **event-by-event fluctuations** in relativistic heavy-ion collisions

Fluctuations are sensitive to the early thermal properties of the created medium. In particular, the possibility to detect **non Gaussian fluctuations** in conserved charges is one of the central topics in this field

- Let $n(x)$ be the density of a given charge Q in the phase space described by the set of variables x. These quantities are related by

$$Q = \int_V dx\, n(x)$$

- where V is the total phase space volume available. When the measurement of Q is performed over the volume V in a thermal system, we speak of a thermal fluctuation

Analysis tools: Fluctuations of conserved quantities

- For example, the variance of Q is given

$$\langle \delta Q^2 \rangle_V = \langle (Q - \langle Q \rangle_V)^2 \rangle_V = \int_V dx_1 dx_2 \langle \delta n(x_1) \delta n(x_2) \rangle$$

- The integrand on the right-hand side is called a **correlation function**, whereas the left-hand side is called a (second order) **fluctuation**

We see that fluctuations are closely related to correlation functions

In relativistic heavy-ion collisions, fluctuations are measured on an event-by-event basis in which the number of some charge or particle species is counted in each event

Analysis tools: Fluctuations of conserved quantities

- For a probability distribution function $\mathcal{P}(x)$ of an stochastic variable x, the moments are defined as

$$\langle x^n \rangle = \int dx\, x^n \mathcal{P}(x)$$

- We can define the **moment generating function** $G(\theta)$ as

$$G(\theta) = \int dx\, e^{x\theta} \mathcal{P}(x)$$

- from where

$$\langle x^n \rangle = \frac{d^n}{d\theta^n} G(\theta)\Big|_{\theta=0}$$

Analysis tools: Cumulant generating function

$$K(\theta) = \ln G(\theta)$$

- The cumulants of $\mathcal{P}(x)$ are defined by

$$\begin{aligned}
\langle x^n \rangle_c &= \frac{d^n}{d\theta^n} K(\theta)\bigg|_{\theta=0}, \\
\langle x \rangle_c &= \langle x \rangle, \\
\langle x^2 \rangle_c &= \langle x^2 \rangle - \langle x \rangle^2 = \langle \delta x^2 \rangle, \\
\langle x^3 \rangle_c &= \langle \delta x^3 \rangle, \\
\langle x^4 \rangle_c &= \langle \delta x^4 \rangle - 3\langle \delta x^2 \rangle^2.
\end{aligned}$$

Analysis tools: Cumulant generating function

- The relation with thermodynamics comes through the partition function $\mathcal{Z}$, which is the fundamental object

> The partition function is also the moment generating function and therefore **the cumulant generating function is given by** $\ln \mathcal{Z}$

- Cumulants are extensive quantities. Consider the number N of a conserved quantity in a volume V in a grand canonical ensemble. It can be shown that its cumulant of order n can be written as

$$\langle N^n \rangle_{c,V} = \chi_n V$$

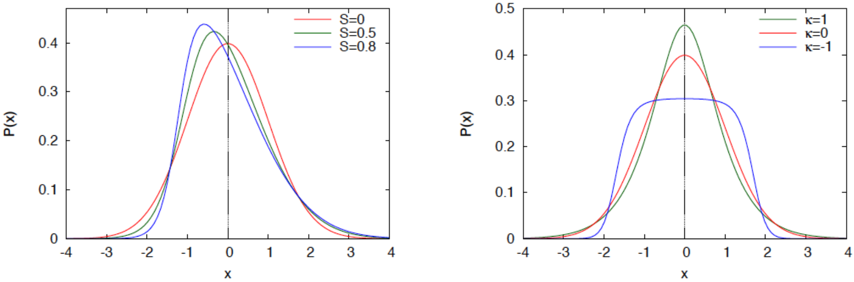
> χ_n are called the **generalized susceptibilities**

Analysis tools: Cumulant generating function

Cumulants higher than second order vanish for a Gaussian probability distribution, non-Gaussian fluctuations are signaled by non-vanishing higher order cumulants

Two important higher order moments are the **skewness** S and the **curtosis** κ. The former measures the asymmetry of the distribution function whereas the latter measures its sharpness

Fluctuations of conserved quantities



Analysis tools: Cumulant generating function

When the stochastic variable x is normalized to the square root of the variance, σ, such that $x \to \tilde{x} = x/\sigma$, the skewness and the kurtosis are given as the third and fourth-order cumulants

$$S = \langle \tilde{x}^3 \rangle_c, \qquad \kappa = \langle \tilde{x}^4 \rangle_c$$

Analysis tool: Fluctuations of conserved quantities

- Experimentally it is easier to measure the **central moments** M: $M^{ijk}_{BQS} = \langle (B - \langle B \rangle)^i (Q - \langle Q \rangle)^j (S - \langle S \rangle)^k \rangle$.
- On the other hand, derivatives of $\ln \mathcal{Z}$ with respect to the chemical potentials give the susceptibilities χ:

$$\chi^{ijk}_{BQS} = \frac{\partial^{i+k+j}(P/T^4)}{\partial^i(\mu_B/T)\partial^j(\mu_Q/T)\partial^k(\mu_S/T)}; \quad P = \frac{T}{V} \ln \mathcal{Z}.$$

$$\Longrightarrow \chi_{XY} = \frac{1}{V} T^3 M^{11}_{XY}$$

Analysis tools: Cumulant generating function

When fluctuations of conserved charges in relativistic heavy-ion collisions are well described by hadron degrees of freedom in equilibrium, their cumulants should be consistent with models that describe these degrees of freedom, such as the Hadron Resonance Gas (HRG) model

On the other hand, when fluctuations deviate from those in the HRG model, they can be used as experimental signals of non-hadron and/or non-thermal physics

Near the **CEP**, higher order cumulants behave anomalously, in particular, they **change sign** in the vicinity of the critical point. They are also sensitive to the increase of correlation lengths

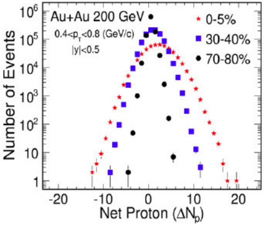
Fluctuations of conserved quantities

1. Higher sensitivity to correlation length (ξ) and probe non-gaussian fluctuations.

$$C_{1,x} = \langle x \rangle, C_{2,x} = \langle (\delta x)^2 \rangle,$$
$$C_{3,x} = \langle (\delta x)^3 \rangle, C_{4,x} = \langle (\delta x)^4 \rangle - 3\langle (\delta x)^2 \rangle^2$$

$$\langle (\delta N)^3 \rangle_c \approx \xi^{4.5}, \quad \langle (\delta N)^4 \rangle_c \approx \xi^7$$

M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).
M.Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103, 262301 (2009).
Y. Hatta, M. Stephanov, Phys. Rev. Lett. 91, 102003 (2003).

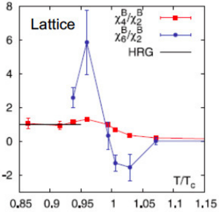


2. Connection to the susceptibility of the system.

$$\frac{\chi_q^4}{\chi_q^2} = \kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} \qquad \frac{\chi_q^3}{\chi_q^2} = S\sigma = \frac{C_{3,q}}{C_{2,q}},$$

$$\chi_q^{(n)} = \frac{1}{VT^3} \times C_{n,q} = \frac{\partial^n (p/T^{\wedge}4)}{\partial(\mu_q)^n}, q = B, Q, S$$

S. Ejiri et al, Phys.Lett. B 633 (2006) 275. Cheng et al, PRD (2009) 074505. B. Friman et al., EPJC 71 (2011) 1694. F. Karsch and K. Redlich , PLB 695, 136 (2011). S. Gupta, et al., Science, 332, 1525(2012). A. Bazavov et al., PRL109, 192302(12) // S. Borsanyi et al., PRL111, 062005(13) // P. Alba et al., arXiv:1403.4903



Xiaofeng Luo — ISMD 2016, Jeju, Korea, Aug. 29-Sept. 2, 2016 — 5 / 18

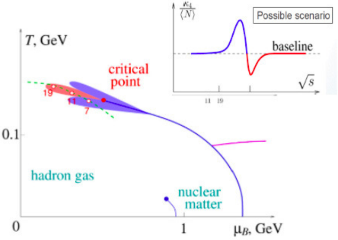
Higher moments, larger sensitivity to correlation length ξ

- In HIC's, the simplest measurements of fluctuations are event-by-event variances in observables such as multiplicities or mean transverse momenta of particles.
- At the CEP, these variances diverge approximately as ξ^2. **They manifest as a non-monotonic behavior as the CEP is passed by during a beam energy scan.**
- In a realistic HIC, the divergence of ξ is tamed by the effects of *critical slow down* (the phenomenon describing a finite and possibly large relaxation time near criticality).
- However, higher, non-Gaussian moments of the fluctuations depend much more sensitively on ξ.
- **Important to look at the Kurtosis κ (proportional to the fourth-order cumulant C_4), which grows as ξ^7.**

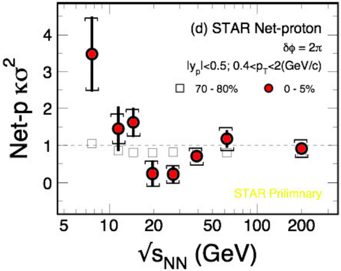
Fourth order fluctuations: Net proton

$\kappa\sigma^2 = C_4/C_2$

Model



STAR BES Data



M.A. Stephanov, PRL107, 052301 (2011).
Schaefer&Wanger,PRD 85, 034027 (2012)
Vovchenko et al., PRC92, 054901 (2015)
JW Chen et al., PRD93, 034037 (2016)
arXiv: 1603.05198.

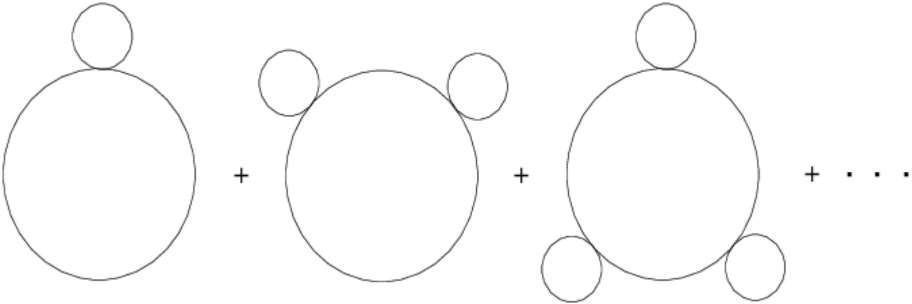
Non-monotonic energy dependence is observed for 4th order net-proton fluctuations in most central Au+Au collisions.

Xiaofeng Luo ISMD 2016, Jeju, Korea, Aug. 29-Sept. 2, 2016 11 / 18

Linear sigma model with quarks

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{a^2}{2}(\sigma^2+\vec{\pi}^2) \\
&- \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu\partial_\mu\psi - g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi, \\
\sigma &\to \sigma + v \\
\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2 \\
&- \frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2 \\
&+ \frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4 + i\bar{\psi}\gamma^\mu\partial_\mu\psi - gv\bar{\psi}\psi + \mathcal{L}_I^b + \mathcal{L}_I^f \\
\mathcal{L}_I^b &= -\frac{\lambda}{4}\Big[(\sigma^2+(\pi^0)^2)^2 + 4\pi^+\pi^-(\sigma^2+(\pi^0)^2+\pi^+\pi^-)\Big], \\
\mathcal{L}_I^f &= -g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi
\end{aligned}
$$

Plasma screening in TFT: Ring diagrams



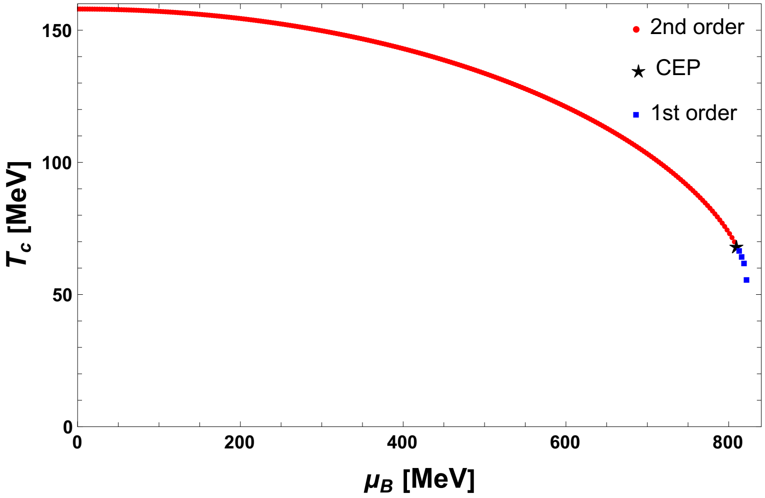
Effective potential

$$
\begin{aligned}
V^{\text{eff}} &= V^{\text{tree}} + V^{\text{b}} + V^{\text{f}} + V^{\text{Ring}} \\
V^{\text{tree}}(v) &= -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 \\
V^{\text{b}}(v,T) &= T\sum_n \int \frac{d^3k}{(2\pi)^3} \ln D_{\text{b}}(\omega_n,\vec{k})^{1/2} \\
V^{\text{f}}(v,T,\mu) &= -T\sum_n \int \frac{d^3k}{(2\pi)^3} \text{Tr}[\ln S_{\text{f}}(\tilde{\omega}_n,\vec{k})^{-1}] \\
V^{\text{Ring}}(v,T,\mu) &= \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln[1+\Pi_{\text{b}} D(\omega_n,\vec{k})] \\
\Pi_{\text{b}} &\equiv \Pi_\sigma = \Pi_{\pi^\pm} = \Pi_{\pi^0} \\
&= \lambda\frac{T^2}{2} - N_{\text{f}}N_{\text{c}}g^2\frac{T^2}{\pi^2}\left[\text{Li}_2\left(-e^{-\frac{\mu}{T}}\right) + \text{Li}_2\left(-e^{\frac{\mu}{T}}\right)\right]
\end{aligned}
$$

Effective potential

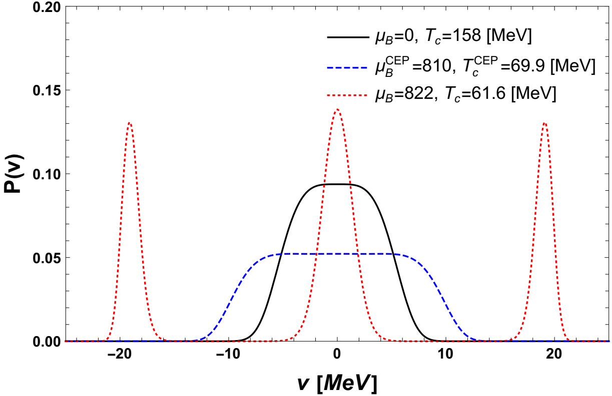
$$
\begin{aligned}
V^{\text{eff}}(v) &= -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 \\
&+ \sum_{\text{b}=\pi^\pm,\pi^0,\sigma} \left\{ -\frac{T^4\pi^2}{90} + \frac{T^2 m_{\text{b}}^2}{24} - \frac{T(m_{\text{b}}^2+\Pi_{\text{b}})^{3/2}}{12\pi} \right. \\
&- \left. \frac{m_{\text{b}}^4}{64\pi^2}\left[\ln\left(\frac{\tilde{\mu}^2}{(4\pi T)^2}\right) + 2\gamma_E\right]\right\} \\
&+ N_c N_f \left\{ \frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{\tilde{\mu}^2}{T^2}\right) - \psi^0\left(\frac{1}{2}+\frac{i\mu}{2\pi T}\right) - \psi^0\left(\frac{1}{2}-\frac{i\mu}{2\pi T}\right)\right.\right. \\
&+ \left. \psi^0\left(\frac{3}{2}\right) - 2\left(1+\ln(2\pi)\right) + \gamma_E\right] \\
&- \frac{m_f^2 T^2}{2\pi^2}\left[\text{Li}_2\left(-e^{-\frac{\mu}{T}}\right) + \text{Li}_2\left(-e^{\frac{\mu}{T}}\right)\right] \\
&+ \left. \frac{T^4}{\pi^2}\left[\text{Li}_4\left(-e^{-\frac{\mu}{T}}\right) + \text{Li}_4\left(-e^{\frac{\mu}{T}}\right)\right]\right\}
\end{aligned}
$$

Effective phase diagram



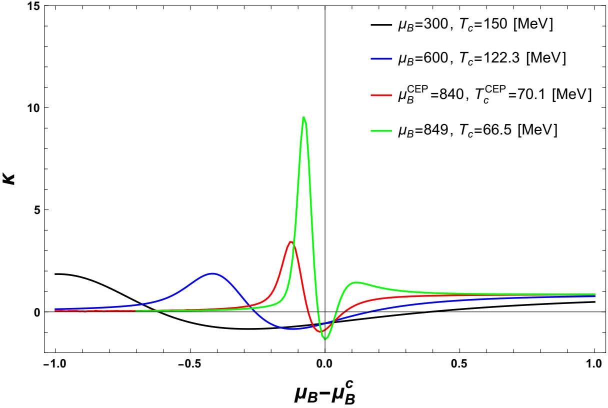
Baryon number fluctuations in the LSMq

$$\mathcal{P}(v) = \exp\left\{-\Omega V^{\text{eff}}(v)/T\right\}$$

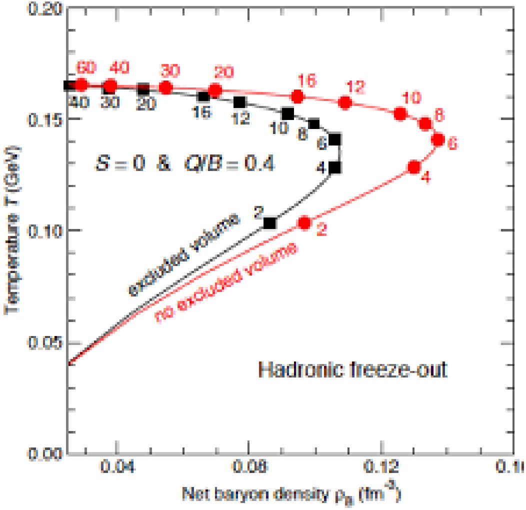


Baryon number fluctuations in the LSMq

$$\mathcal{P}(v) = \exp\left\{-\Omega V^{\text{eff}}(v)/T\right\}$$

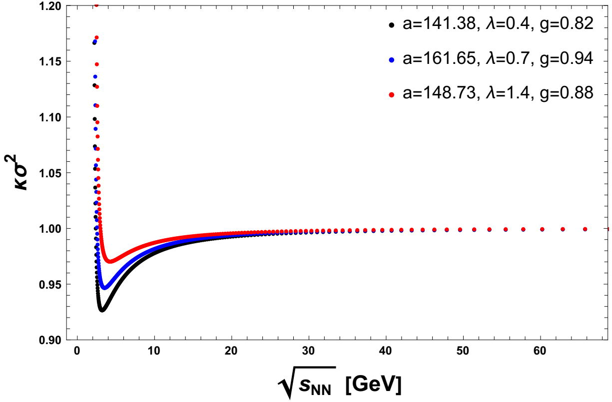


Freeze-out line Randrup & Cleymans, PRC **74**, 047901 (2006)



Baryon number fluctuations in the LSMq

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}} \quad d = 1.308\text{GeV}, \; e = 0.273\text{GeV}^{-1}$$



Summary

- Deviations from HRG behavior when using LSMq as an effective QCD model up to **ring diagrams contribution.**
- Ring diagrams inclusion is equivalent to introducing screening effects at finite T and μ_B.
- CEP signaled by divergence of $\kappa\sigma^2$
- CEP found at low T and high μ_B (NICA, HADES?)

¡Muchas Gracias!