The Evolution of Primordial Neutrino Helicities in Cosmic Gravitational and Magnetic Fields and their Detection

with Jen-Chieh Peng
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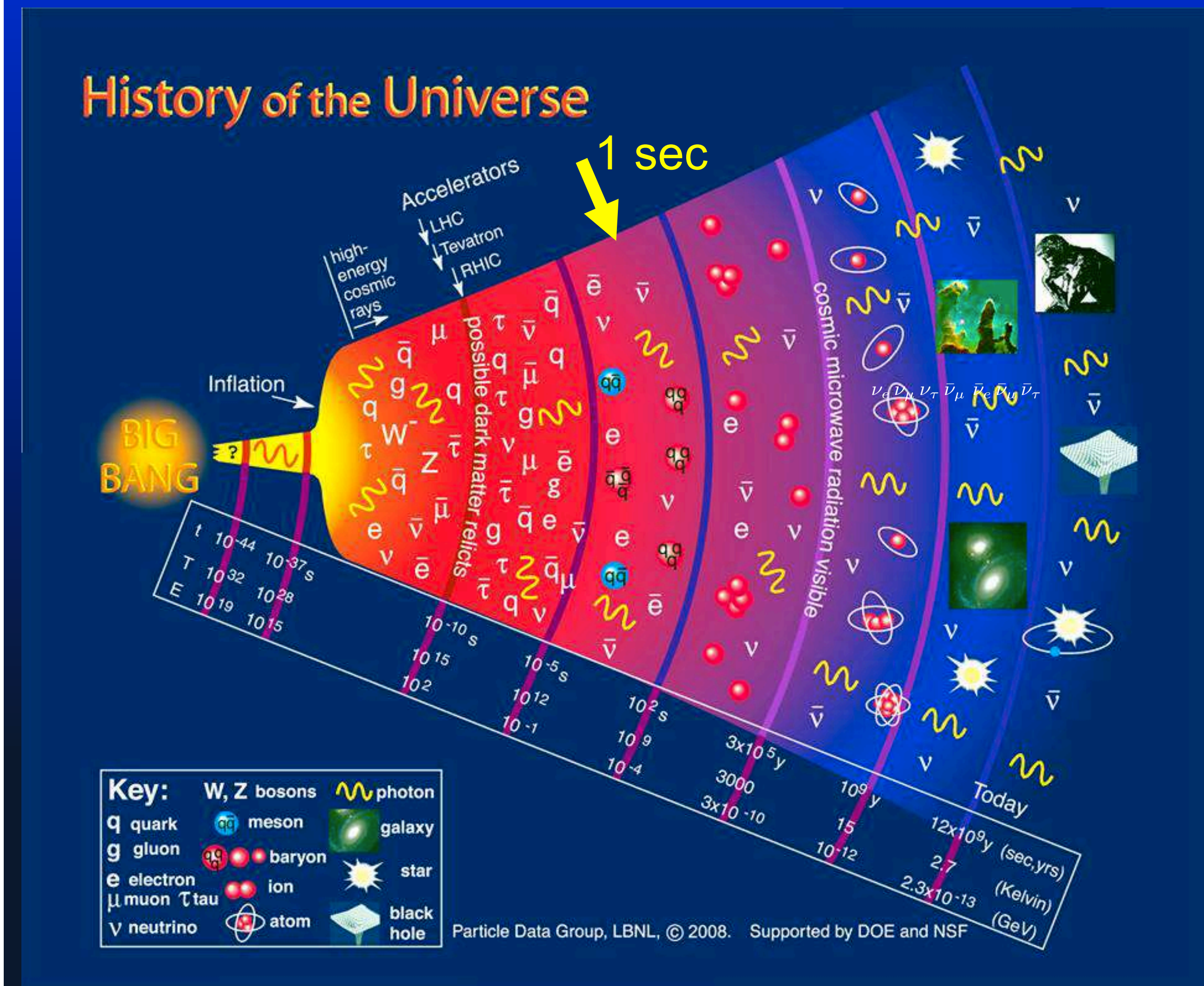
Prior to about one second after the Big Bang neutrinos (ν_e, ν_μ, ν_τ) and antineutrinos were in thermal equilibrium with electrons and quarks.



Since decoupling neutrinos have been "free streaming" through the cosmos.

Present density 56.25 /cm^3 of each

$$\nu_e\,\nu_\mu\,\nu_\tau\,\bar{\nu}_\mu\;\bar{\nu}_e\,\bar{\nu}_\mu\,\bar{\nu}_\tau$$

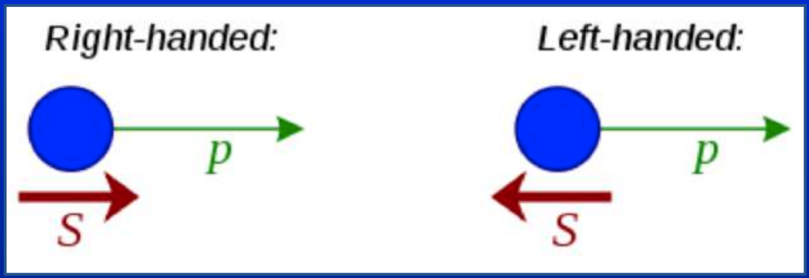
(~100 X solar ν_e)

Never detected!!

What happens to neutrinos between 1 sec and now, 13.8 billion years later?

Neutrinos have negative helicity: L handed

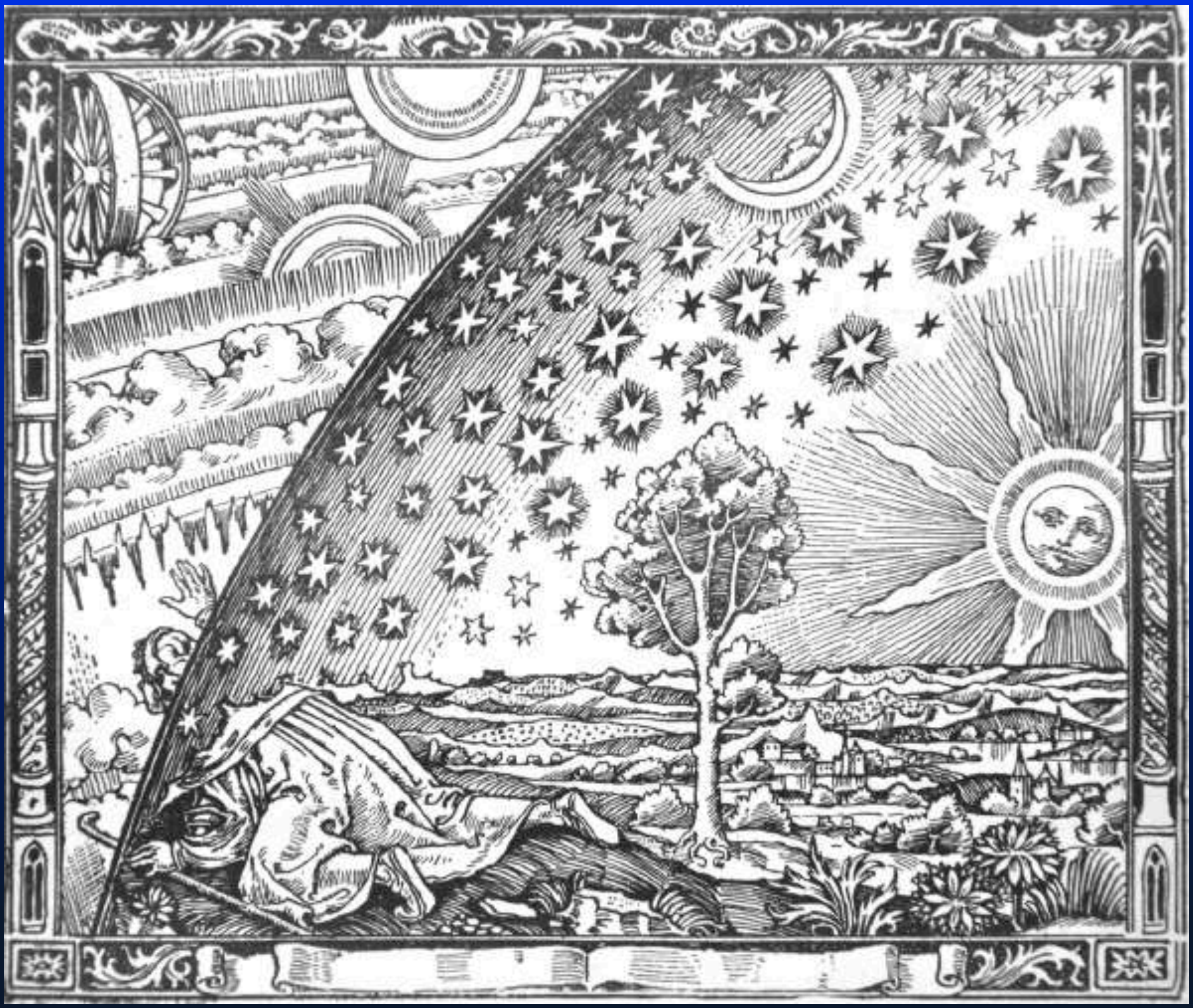
& antineutrinos positive helicity: R handed



A property of the weak interaction processes, not an intrinsic property of neutrinos

Both cosmic, and later galactic, magnetic fields as well as gravitational inhomogeneities can rotate the spins with respect to the momentum, and thus give neutrinos an amplitude to be right handed, and antineutrinos left handed!

The helicities of relic neutrinos are a new probe of cosmic gravitational and magnetic fields.



Flammarion 1888

Neutrinos 101

Neutrino magnetic moments & spin precession

Gravitational inhomogeneities & spin precession

Detection of relic neutrinos

Processes in equilibrium

$$\nu_\chi(\bar{\nu}_\chi) + e^\pm \leftrightarrow \nu_\chi(\bar{\nu}_\chi) + e^\pm$$

$$\nu_\chi(\bar{\nu}_\chi) + \nu_{\chi'}(\bar{\nu}_{\chi'}) \leftrightarrow \nu_\chi(\bar{\nu}_\chi) + \nu_{\chi'}(\bar{\nu}_{\chi'})$$

Scattering
$\chi = e, \mu, \tau$

$$\nu_\chi + \bar{\nu}_\chi \leftrightarrow e^- + e^+$$

Annihilation. For $T < m_\mu = 106$ MeV annihilations only to electrons

$$\nu_e + e^- \leftrightarrow e^- + \nu_e, \quad \bar{\nu}_e + e^- \leftrightarrow e^- + \bar{\nu}_e$$

Charge exchange, keeps ν_e in equilibrium longer

Decoupling, as densities decrease in expanding universe:

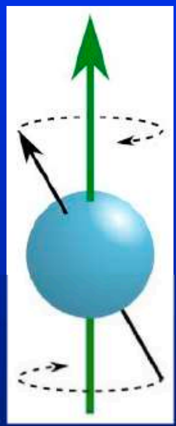
$$T(\nu_\mu) = T(\nu_\tau) \sim 1.5\,\mathrm{MeV}$$

$$T(\nu_e) \sim 1.3\,\mathrm{MeV}$$

$$t(\mathrm{sec}) \simeq \frac{1}{\sqrt{\mathrm{T(MeV)}}}$$

Magnetic field B rotates spins, but not momenta:

Since ν have non-zero mass they have magnetic moment



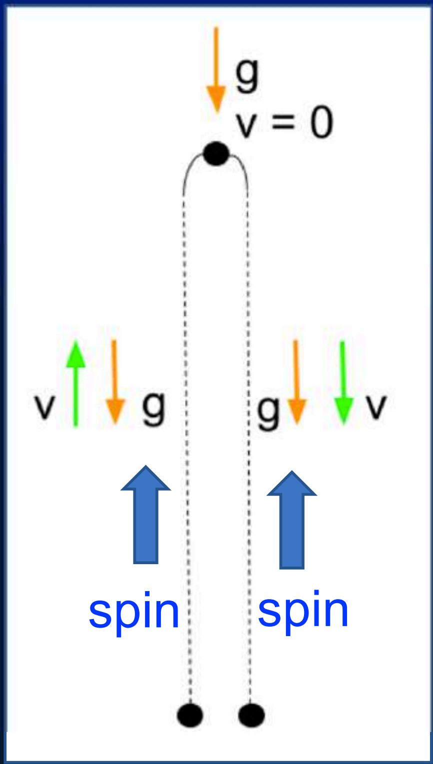
$$\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \left(\vec{S}_\parallel \times \vec{B}_\perp + \frac{1}{\gamma} \vec{S}_\perp \times \vec{B}_\parallel \right)$$

μ_ν = magnetic moment and $\gamma = 1/\sqrt{1-v^2}$ of neutrino

Gravitational potential Φ rotates momentum and spin:

$$\left.\frac{d\hat{p}}{dt}\right|_\perp = -\left(v + \frac{1}{v}\right) \vec{\nabla}_\perp \Phi \quad , \quad \left.\frac{d\vec{S}}{dt}\right|_\perp = -\frac{2\gamma+1}{\gamma+1} \vec{S} \cdot \vec{v}\, \vec{\nabla}_\perp \Phi$$

relativistic effect

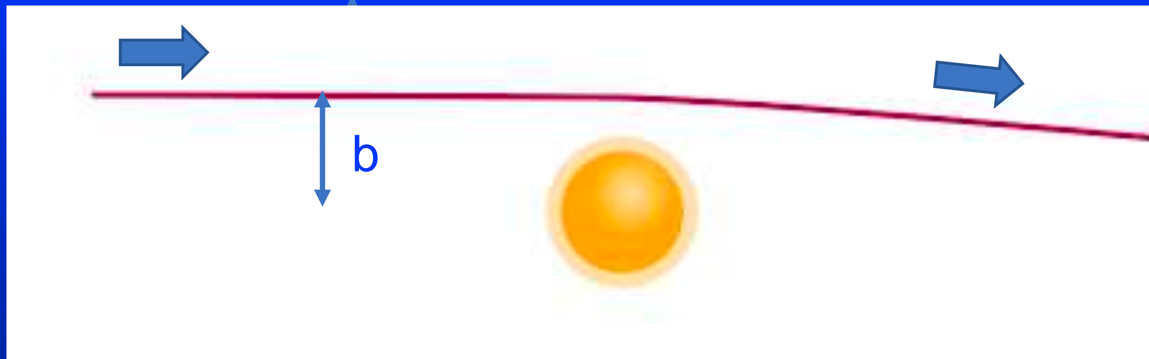


Spin bending lags momentum bending
(helicity = $h = \hat{p} \cdot \hat{S}$)

$$\boxed{\left(h \frac{d\hat{S}}{dt} - \frac{d\hat{p}}{dt} \right)_\perp = \frac{m}{p} \vec{\nabla}_\perp \Phi}$$

Spin and momentum bent equally for massless particle (photon); no spin bending of non-relativistic particle

Ex: particle passing star of mass M at impact parameter b



$$\Delta\theta_p = \frac{2MG}{bv^2}(1+v^2)$$

momentum bending
v → 1 Einstein light bending

$$\Delta\theta_s = \frac{2MG}{b}\frac{2\gamma+1}{\gamma+1}$$

spin bending $\gamma = 1/\sqrt{1-v^2}$

$$\boxed{\theta \equiv \Delta\theta_s - \Delta\theta_p = -\frac{2MG}{b\gamma v^2}}$$

lag of spin with respect to momentum

Helicity flipping

Neutrino spin $|\downarrow\rangle \rightarrow \cos(\theta/2)|\downarrow\rangle + \sin(\theta/2)|\uparrow\rangle$

Probability of helicity flip = $\frac{1}{2}(1-\cos\theta) \rightarrow \frac{1}{4}\theta^2$ for small rotation

Evolution of primordial neutrinos from freezeout

Neutrinos produced in flavor eigenstates, linear superpositions of mass eigenstates 1,2,3,

$$\boxed{\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}}$$

Pontecorvo–Maki–Nakagawa–Sakata
PMNS mixing matrix

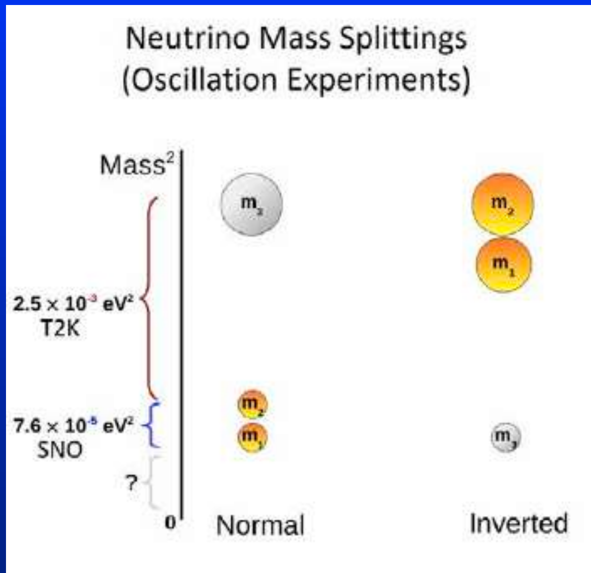
and in wave packets of size
~ electron mean free path $1/\alpha^2 T \sim 10^6 - 10^7 \mathrm{fm}$
and velocity $v = p/\sqrt{p^2 + m^2}$

Velocity dispersion of mass components $\delta v = (\Delta m/m) m^2/p^2$
>> velocity dispersion $(\delta p/p) m^2/p^2$ of given mass component

Flavor eigenstate (a) arrives at Earth in three well separated mass packets with relativistic thermal distributions:

$$f_a(p) = \sum_i \frac{|U_{ai}|^2}{e^{p/T_e} + 1}$$

Neutrino masses and thermal distributions



$$\Delta m^2_{21} = 7.50 \times 10^{-5} \mathrm{eV}^2$$

$$\Delta m^2_{31,N} = 2.52 \times 10^{-3} \mathrm{eV}^2$$

$$\Delta m^2_{31,I} = -2.51 \times 10^{-3}$$

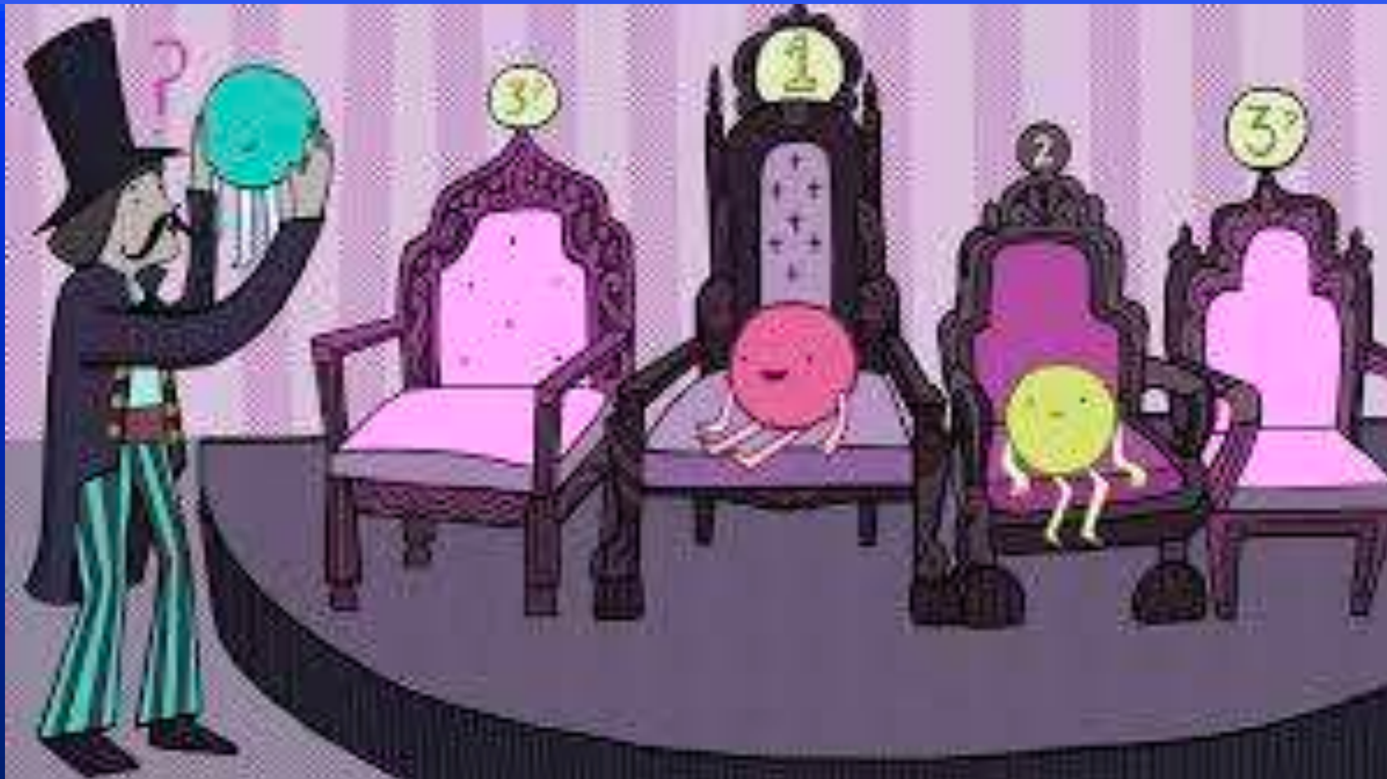
Distributions are fully relativistic even though at least two neutrino states (i=2,3 in N or 1,2 in I) are non-relativistic now:

$m_i \gg T_{\nu 0}$ = 1.945K = 1.676 x 10^{-4} eV

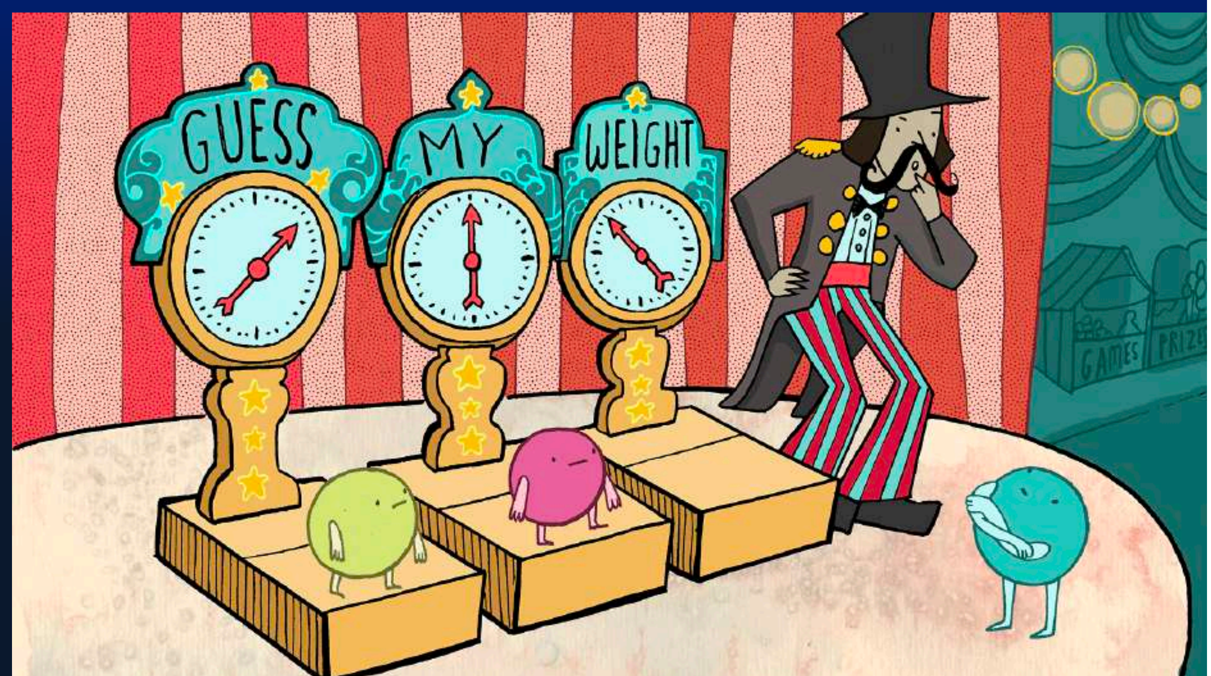
NH: $m_1 = 10^{-5}$ => $v_1 \sim 1$, $v_2 \sim 1/5$, $v_3 \sim 1/20$
IH: $m_1 = 10^{-5}$ => $v_3 \sim 1$, $v_1 \sim v_2 \sim 1/20$

Neutrino velocities v/c

(Neutrino temperature < T_{CMB} = 2.725K: neutrinos were not reheated!)



Is 3 heaviest
or lightest?



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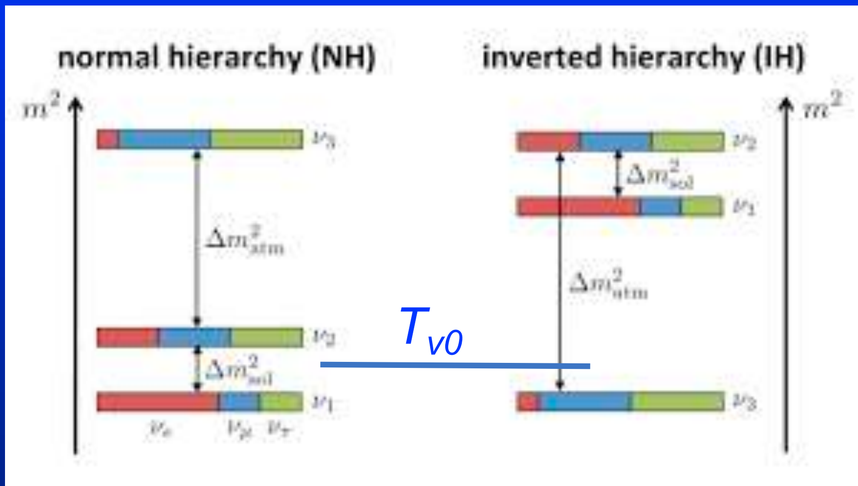

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Neutrino velocities v/c

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Electron neutrino distribution at decoupling:

$$f_e(p) = \sum_i \frac{|U_{ei}|^2}{e^{E_i/T_e} + 1} \qquad E_i = \sqrt{p^2 + m_i^2} \simeq p$$

Since p >>> m_i have p/T_e => relativistic distribution
Does not change since neutrinos are decoupled!

As universe expands:
scale factor *a* grows from ~10^{-10} at T =1 MeV to *a*=1 now

$$p \to p_0/a, \quad T \to T_0/a$$ (0 = now)

Were neutrinos to have remained in thermal equilibrium,

$$\frac{E_i}{T} \to \frac{\sqrt{p_0^2/a^2 + m_i^2}}{T_0/a} = \frac{\sqrt{p_0^2 + m_i^2\, a^2}}{T_0}$$

and distribution would be non-relativistic!

Neutrinos propagation in an expanding universe

Metric of expanding universe with weak gravitational inhomogeneities:

$$\boxed{ds^2 = a(u)^2[-(1+2\Phi)du^2 + (1-2\Phi)d\vec{x}^{\,2}]}$$

a = scale factor grows from ~10^{-10} at T =1 MeV to a=1 now
u = conformal time, dt = a du
x = comoving spatial coordinates

Φ = weak potential, driven by density and pressure fluctuations

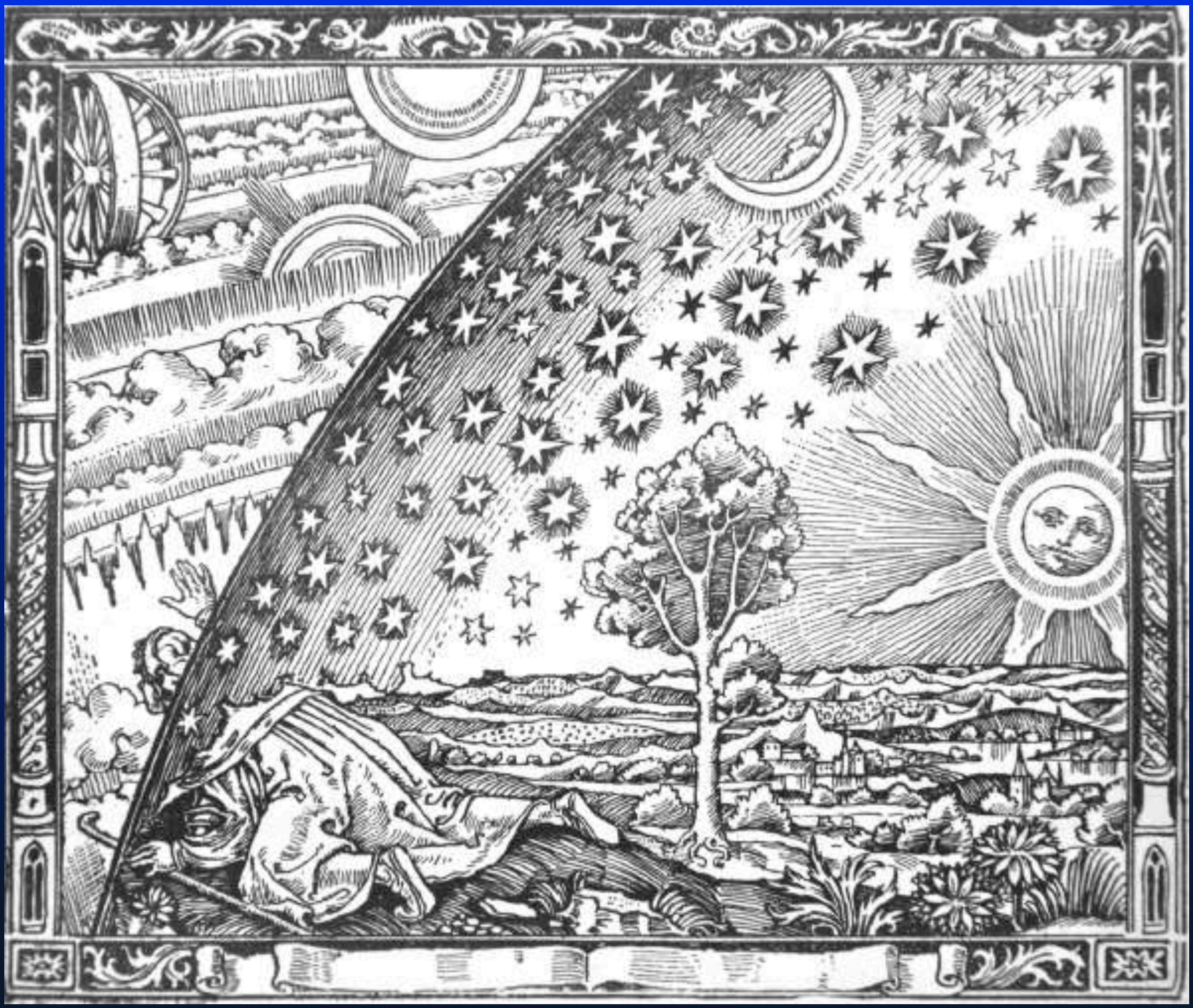
$$\nabla_x^2\Phi = 4\pi G\,(\delta\rho(\vec{x}\,) + 3\delta P(\vec{x}\,))\,a(u)^2$$

Φ(x) independent of *a*, at long wavelengths $\delta\rho\ \ a^2 \propto a^0$

Radiation dominated era ($P = \rho/3$) , down to redshift ~ 10^4 :

$$\delta\rho/\bar{\rho} \sim a^2, \quad \bar{\rho} \sim a^4$$

Matter dominated era, from redshift ~ 10^4 to now, $\delta\rho/\bar{\rho} \sim a, \quad \bar{\rho} \sim a^2$



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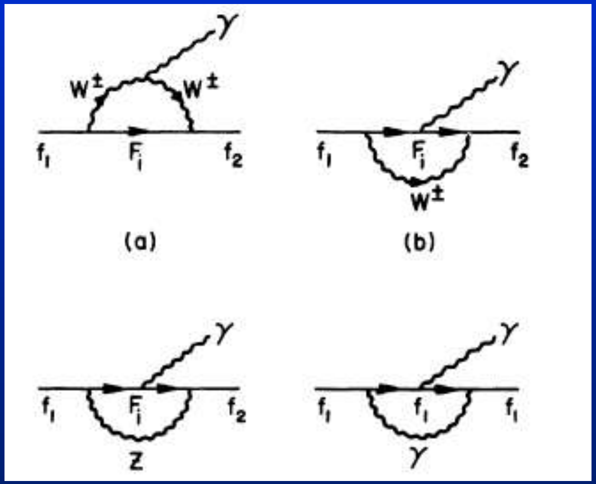
Neutrinos 101

Neutrino magnetic moments & spin precession

Gravitational inhomogeneities & spin precession

Detection of relic neutrinos

Rotation of neutrino spins in magnetic fields via neutrino magnetic moment



Standard model processes lead to a non-zero neutrino magnetic moment

$$\mu_\nu^{\rm SM} \simeq \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu \simeq 3\times 10^{-21} m_{-2}\,\mu_B$$

Fujikawa-Schrock PRL 1980

μ_B = Bohr magneton $= e/2m_e$

$m_{-2} = m_\nu/10^{-2}{\rm eV}$

But the magnetic moment could be much larger (BSM physics!)

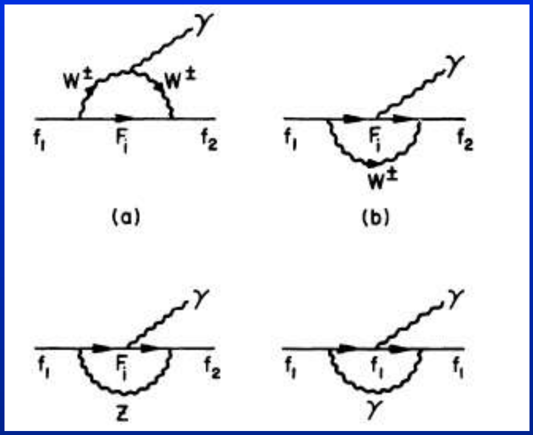
Upper bounds: $\mu_\nu < 2.9\times 10^{-11}\mu_B$ GEMMA Kalinin reactor expt (2010) $\bar\nu + e^-$

$\mu_{\nu_e} < 2.8\times 10^{-11}\mu_B$ Borexino (2017, solar $\nu + e^-$)

Theoretical "naturalness" bound: $\mu_\nu \lesssim 10^{-16} m_{-2}\mu_B$

Bell et al. PRL 2005

Diagonal vs. transition magnetic moments



Diagonal: interaction with magnetic field between equal mass states (neutrino $m_1 = m_2$)

Transition: interaction only between different mass states ($m_1 \neq m_2$)

Are neutrinos Dirac or Majorana fermions?

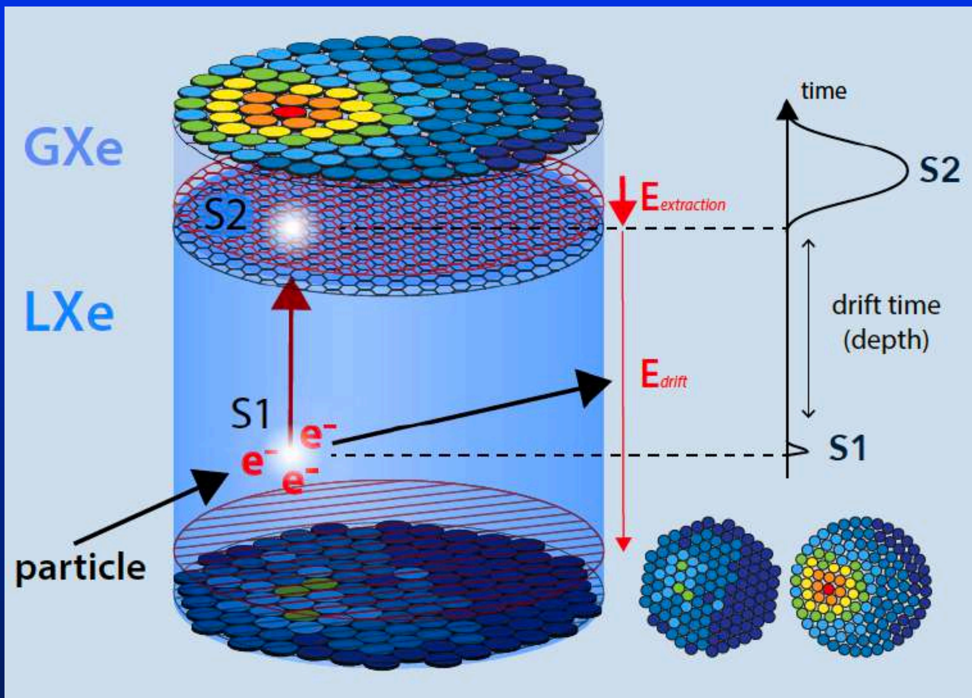
Dirac neutrinos can have both diagonal and transition moments.

Diagonal moments of Majorana neutrinos identically zero; only transition moments: $\mathrm{CPT} => \langle i|\mu_\nu \vec{S}|j\rangle = -\langle \bar{j}\,|\mu_\nu \vec{S}|\bar{i}\,\rangle$

Propagation through cosmic and galactic magnetic fields cannot change neutrino mass state.

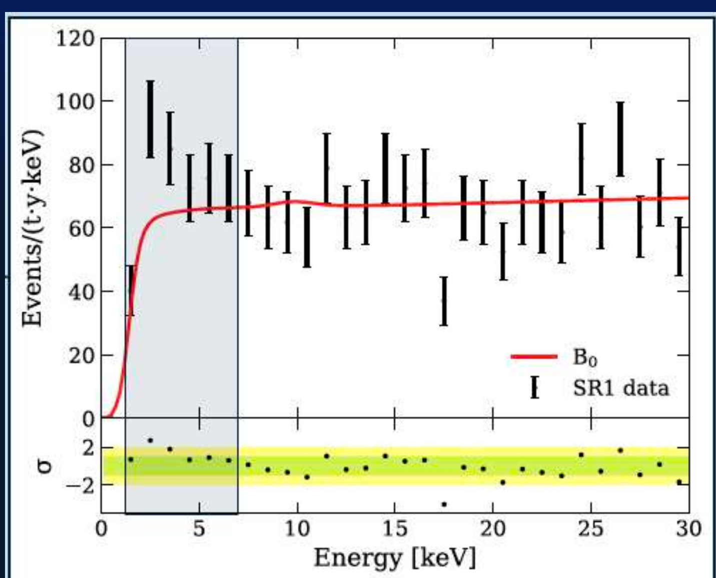
Only Dirac neutrinos can have helicities changed by magnetic fields.

XENON1T experiment (in Gran Sasso)



One ton TPC of liquid Xe
Sees both electron
& nuclear recoils

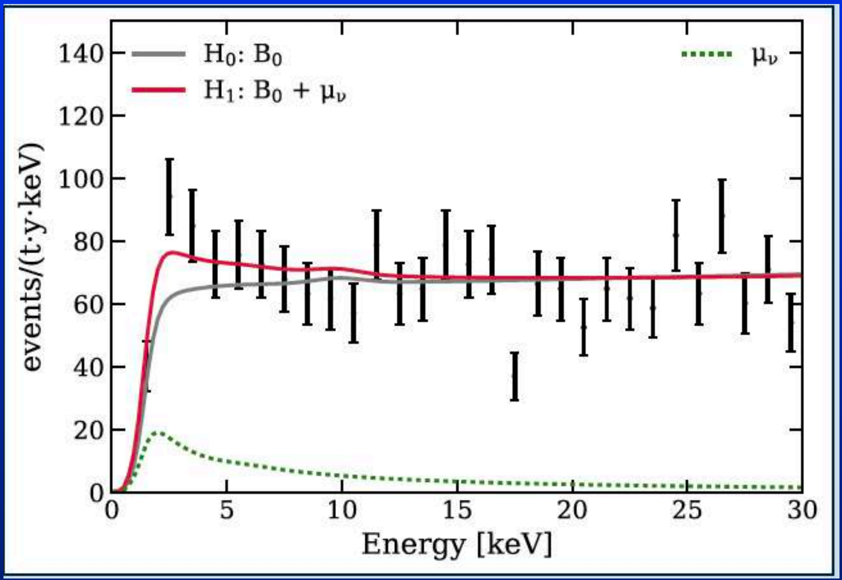
Search for WIMPS (weakly interacting massive particles) & other dark matter.
Sensitive to physics beyond standard model: solar axions bosonic dark matter,
magnetic moments of solar neutrinos



Excess of low energy electron events
1-7 keV over expected background

Aprile et al. PR D 102, 072004 (2020)

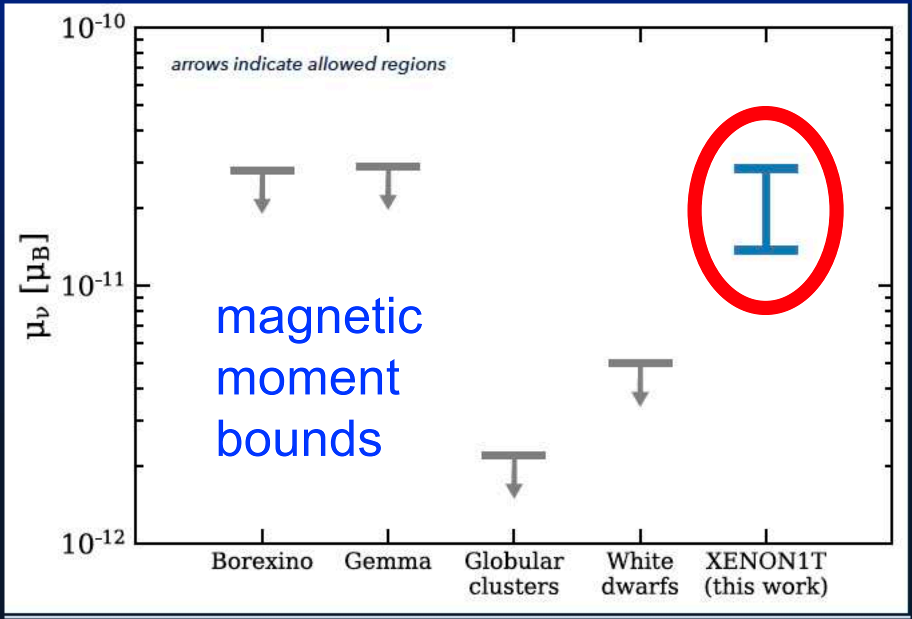
XENON1T low energy electron event excess



Possible explanations:

Large neutrino mag. moment (3.2σ)
Solar axions (3.5σ)
Tritium (in Xe) beta decays

Excess consistent with neutrino magnetic moment
$\mu_{\nu,1T} \sim 1.4 - 2.9 \times 10^{-11} \mu_B$



Beyond Standard Model physics??

No information on whether diagonal or transition moment

Spin precesses in magnetic field, but momentum does not

(neutrinos are electrically neutral)

Thus magnetic fields change neutrino helicity: $h = \hat{S} \cdot \hat{p}$

Spin rotation by angle θ => helicity reversal probability $\sin^2(\theta/2)$

Define spin in rest frame of neutrino.

Rest frame precession

$$\frac{d\vec{S}}{d\tau} = 2\mu_\nu \vec{S} \times \vec{B}_R$$

B_R = magnetic field in rest frame

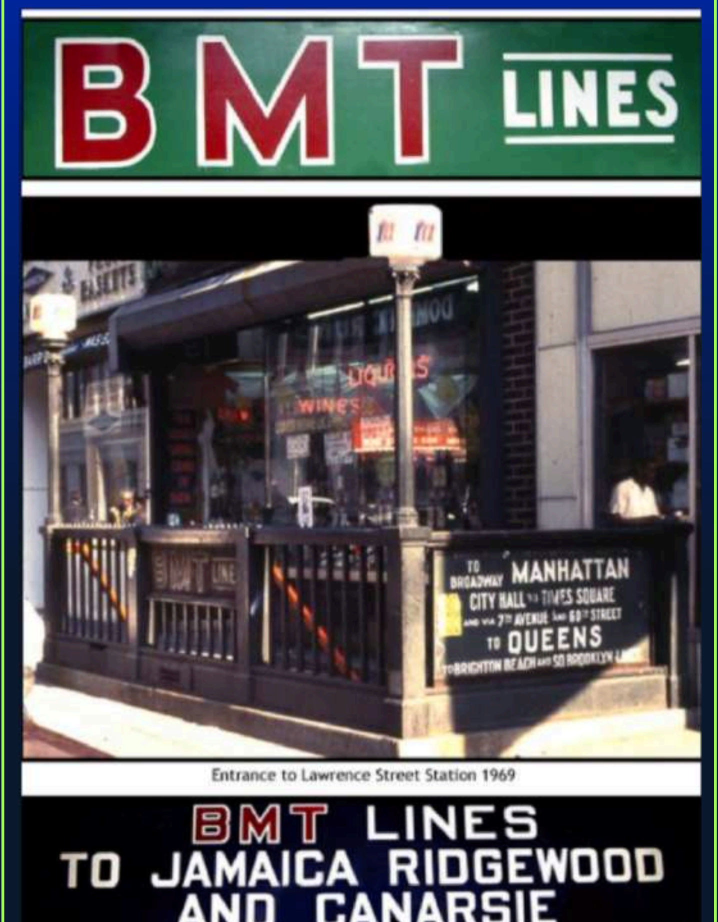
In terms of "lab" frame magnetic field $B_{\parallel R} = B_{\parallel}, \quad B_{\perp R} = \gamma B_{\perp}$

$$\gamma = 1/\sqrt{1 - v^2}$$

Bargmann-Michel-Telegdi (BMT) equations of motion:

$$\boxed{\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \left(\vec{S}_\parallel \times \vec{B}_\perp + \underbrace{\frac{1}{\gamma}\vec{S}_\perp \times \vec{B}_\parallel}_{\text{negligible for small rotation from longitudinal}} \right)}, \quad \frac{dS_\parallel}{dt} = 2\mu_\nu (\vec{S} \times \vec{B})_\parallel$$

Brooklyn-Manhattan Transit



Cumulative spin rotation along ν trajectory:

$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t)$$

for small angular changes.

Apply to galaxies, and
to cosmic magnetic fields

Magnetic field lines in M51- Whirlpool Galaxy

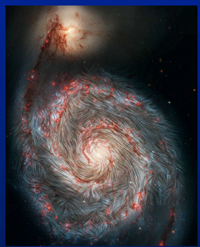
SOFIA (on a 747) IR superimposed on Hubble image

Neutrino spin rotation by galactic magnetic field

For uniform galactic magnetic field: $\theta_g \sim 2\mu_\nu B_g \frac{\ell_g}{v}$

ℓ_g = mean crossing distance of the galaxy

But galactic fields are uniform only over coherence length Λ_g~ kpc
so spin direction does a random walk in magnetic field.



$$\langle\theta^2\rangle_g \simeq \left(2\mu_\nu B_g \frac{\Lambda_g}{v}\right)^2 \frac{\ell_g}{\Lambda_g}$$

ex., Milky Way with characteristic parameters (spherical cow approx):

$$B_g \sim 10\,\mu\mathrm{G}, \ell_g \sim 16\mathrm{kpc}, \Lambda_g \sim \mathrm{kpc}$$

$$\boxed{\langle\theta^2\rangle_{\mathrm{MW}} \sim\ 4 \times 10^{29} m^2_{-2} \left(\frac{\Lambda_g}{1\,\mathrm{kpc}}\right)\left(\frac{B_g}{10\mu\mathrm{G}}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2}$$

$\mu_\nu \sim 1.5 \times 10^{-15}\mu_B \sim 10^{-4}\mu_{1T} => \sqrt{\langle\theta^2\rangle} \sim 1$: helicity randomizes

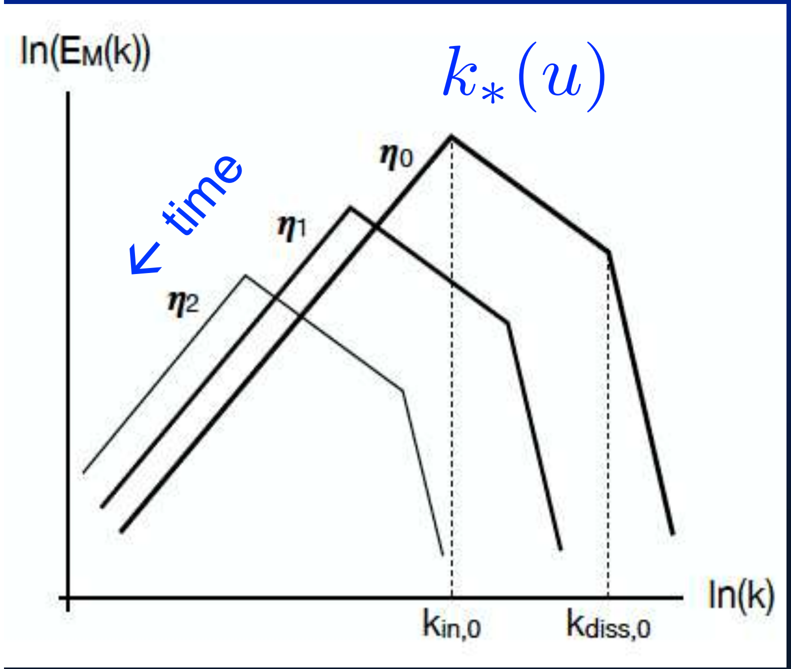
Spin rotation as neutrino propagates through cosmic magnetic field

$$\frac{\vec{S}_\perp}{|\vec{S}\,|} = \pm 2\mu_\nu \int dt\, \hat{v} \times \vec{B}(t) \quad \Rightarrow \quad \langle \theta^2 \rangle_c = 4\mu_\nu^2 \,\langle \Big(\int du\, a(u) \vec{B}_\perp(u) \Big)^2 \rangle_c$$

dt   perp to v

Magnetic field correlation function:

$$\langle B_i(\vec{x}) B_j(\vec{x}\,') \rangle = (-\delta_{ij}\nabla^2 + \nabla_i \nabla_j) F(r) + \epsilon_{ijk} \nabla_k G(r)$$



F =normal and G = helical correlation
(G does not contribute to rotation
since $\nabla_z G(r)$ odd in x-x'.)

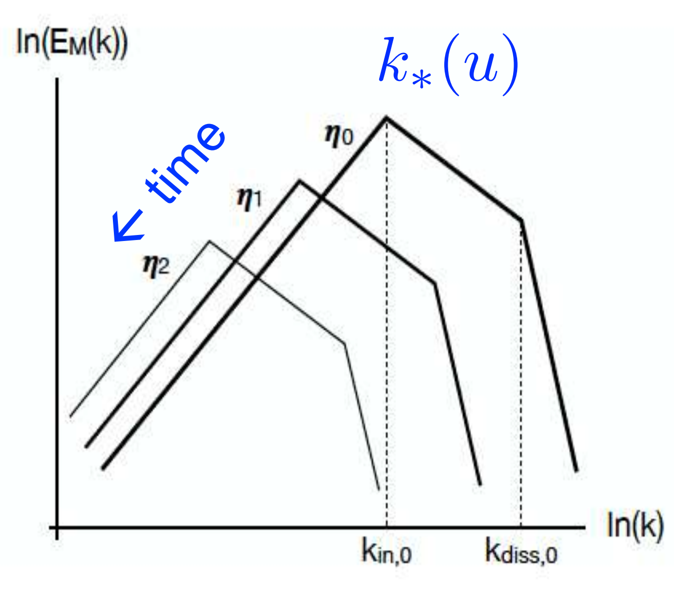
Normal correlation function

$$\langle B_i(\vec{x}) B_j(\vec{x}\,') \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{2} P_B(k) e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}$$

Schematic of $P_B(k)$ with increasing conformal time (η =u): $\eta_0 < \eta_1 < \eta_2$

T. Vachaspati, arXiv:2010.10525 (ROPP)

$$P_B(k) = (2\pi)^2 E_M(k)/k^2$$



Structure of $P_B(k)$:

$$k < k_* : P_B \sim k^s, s \sim 2$$

$$k > k_* : P_B \sim k^{-q}, q \sim 2 + 5/3$$

sum rule: $\int \frac{d^3k}{(2\pi)^3} P_B(k) = \langle \vec{B}^{\,2} \rangle$

Integrating the magnetic field along the neutrino trajectory, x_3= u:

$$\langle \theta^2 \rangle_c = 4\mu_\nu^2 \int du du' a(u) a(u') \int \frac{d^3k}{(2\pi)^3} e^{ik_z(u-u')} \frac{1 - k_z^2/k^2}{2} P_B(k)$$

The u integrals large only for $k_z \sim 0$. Neglect k_z in $(1 - k_z^2/k^2)P(k)$

=> $\int dk_z e^{ik_z(u-u')} = 2\pi\delta(u - u')$

=> $\boxed{\langle \theta^2 \rangle_c \simeq \frac{\mu_\nu^2}{\pi} \int_{u_d}^{u_0} du a(u)^2 \int_0^\infty dk_\perp k_\perp P_B(k_\perp) \simeq \mu_\nu^2 \pi \int_{u_d}^{u_0} du a(u)^2 \frac{\langle \vec{B}^2 \rangle}{k_*}}$

$$\langle \theta^2 \rangle_c \simeq \mu_\nu^2 \pi \int_{u_d}^{u_0} du\, a(u)^2 \frac{\langle \vec{B}^2 \rangle (u)}{k_*(u)}$$

Conservation of flux: a^2B~ const. => $\langle \vec{B}^2(u) \rangle \simeq B_0^2 / a(u)^4$ (0= now)

$$k_*(u) \sim \frac{2\pi}{\Lambda_0 a(u)^{1/2}}$$ (Λ_0 = coherence length of cosmic B field)

$$\boxed{\langle \theta^2 \rangle_c = \frac{1}{2} \mu_\nu^2 B_0^2 \Lambda_0 \int_{u_d}^{u_0} \frac{du}{a(u)^{3/2}}}$$

Main contribution is from radiation-dominated era (a ~ u):
from neutrino decoupling, u_d ($a_d \sim 10^{-10}$)
to matter-radiation equality, u_{eq} ($a_{eq} \sim 0.8 \times 10^{-4}$)

$$\langle \theta^2 \rangle_c \simeq 9 \left(\frac{\Lambda_0}{R_u} \right) \frac{(\mu_\nu t_0 B_0)^2}{(a_{eq} a_d)^{1/2}}$$

$R_u = cu_0$ = radius of universe
$u_0 = 3t_0$

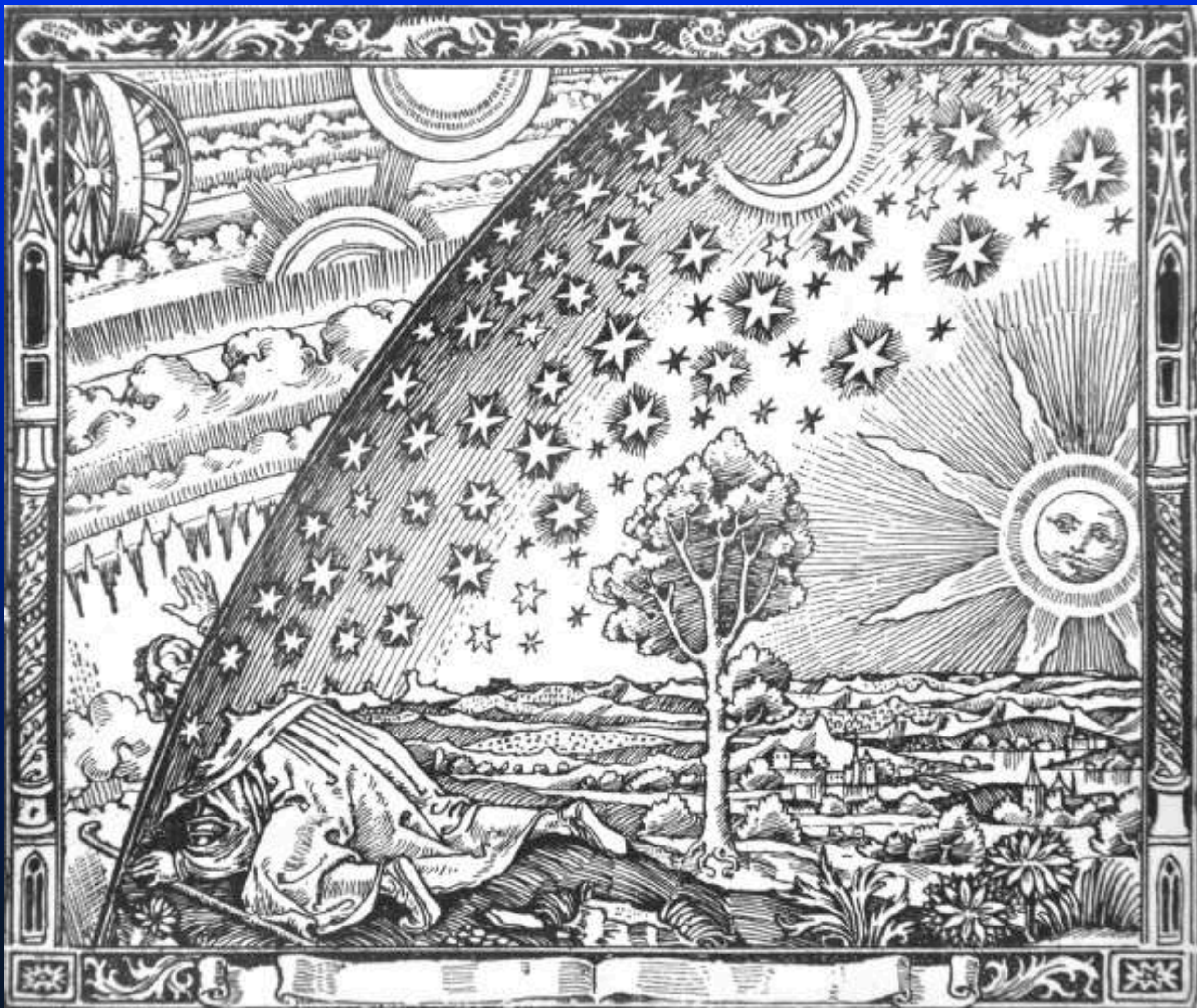
$$\simeq 2 \times 10^{27} \left(\frac{\Lambda_0}{1\,\mathrm{Mpc}} \right) \left(\frac{B_0}{10^{-12}\mathrm{G}} \right)^2 \left(\frac{\mu_\nu}{\mu_B} \right)^2$$

$$\langle\theta^2\rangle_c \simeq 2\times 10^{27}\left(\frac{\Lambda_0}{1\,\mathrm{Mpc}}\right)\left(\frac{B_0}{10^{-12}\mathrm{G}}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

If low-energy electron-event excess found in the XENON1T experiment does arise from a neutrino moment, it could lead to a significant spin rotation. Even for $\mu_\nu \sim 10^{-3}\mu_{1T} \sim 10^{-14}\mu_B$ would have near randomization of final helicities.

If the neutrino is a Majorana particle, the XENON 1T excess would occur entirely from transition magnetic moments. No helicity changes from magnetic fields.

To within uncertainties in magnetic fields, correlation lengths, and neutrino masses, the estimated spin rotation in cosmic magnetic fields is basically comparable to that in galaxies.



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Rotation of neutrino spins by gravitational inhomogeneities

Gravitational potential Φ rotates momentum and spin:

$$\frac{d\hat{p}}{dt}\Big|_\perp = -\left(v + \frac{1}{v}\right)\vec{\nabla}_\perp \Phi \qquad , \qquad \frac{d\vec{S}}{dt}\Big|_\perp = -\frac{2\gamma+1}{\gamma+1}\vec{S}\cdot\vec{v}\,\vec{\nabla}_\perp \Phi$$

Spin bending lags momentum bending $\left(h\frac{d\hat{S}}{dt} - \frac{d\hat{p}}{dt}\right)_\perp = \frac{m}{p}\vec{\nabla}_\perp \Phi$

Again, neutrino undergoes a random walk through the inhomogeneities.

For massless neutrino momentum bending angle:

$$\langle(\Delta\theta_p)^2\rangle = 4\int dx_3 dx_3' \nabla_{x\perp}\cdot\nabla_{x'\perp}\langle\Phi(x_3)\Phi(x_3')\rangle$$

In terms of gravitational fluctuation power spectrum,

$$\langle\Phi(x)\Phi(x')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\Psi(k)$$

$$\langle(\Delta\theta_p)^2\rangle = 4\int dx_3 dx_3' \nabla_{x\perp}\cdot\nabla_{x'\perp}\langle\Phi(x_3)\Phi(x_3')\rangle \quad , \quad \langle\Phi(x)\Phi(x')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\Psi(k)$$

Thus $\langle(\Delta\theta_p)^2\rangle = 4\int dx_3 dx_3' \int \frac{d^3k}{(2\pi)^3} e^{ik_3(x_3-x_3')} k_\perp^2 \Psi(k)$

x_3' integral => $2\pi\delta(k_z)$

$$\boxed{\langle(\Delta\theta_p)^2\rangle = \frac{2}{\pi}\int du \int \frac{dk_\perp}{k_\perp}\Psi(k_\perp)}$$

Relate field fluctuations to density fluctuations $\delta(\vec{x}\,) \equiv \delta\rho(\vec{x}\,)/\bar{\rho}$

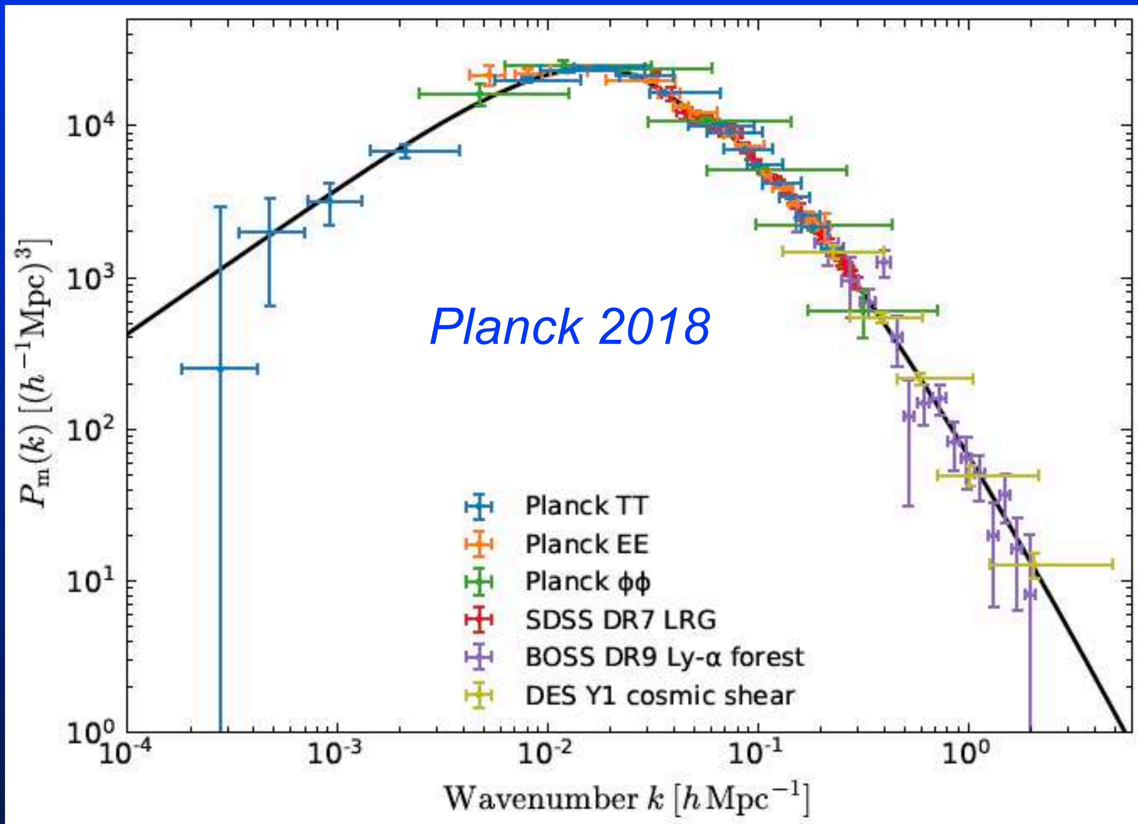
$$\langle\delta(\vec{x}\,)\delta(\vec{x}\,')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')} P(k)$$

$$\nabla_x^2\Phi = 4\pi G\left(\delta\rho(\vec{x}\,) + 3\delta P(\vec{x}\,)\right) a(u)^2$$

$$\boxed{\Psi(k) = (4\pi G\bar{\rho}a^2)^2\zeta\frac{P(k)}{k^4}}$$

In radiation dominated era
$P = \rho/3$, $\zeta = 4$
In matter dominated era
neglect P, $\zeta = 1$

Density fluctuation spectrum)



P(k) ~ k for k <k_{max}
(Harrison-Zel'dovich)
P(k) ~ $k^{-\nu}$ for k > k_{max}
Scales as
a^2 in matter dom. era
a^4 in rad. dom. era, k <k_{max}

$$\langle(\Delta\theta_p)^2\rangle = \frac{2}{\pi}\int du \int \frac{dk_\perp}{k_\perp}\Psi(k_\perp)$$

$$\Psi(k) = (4\pi G\bar{\rho}a^2)^2\zeta\frac{P(k)}{k^4}$$

$$\langle(\Delta\theta_p)^2\rangle = 32\pi\zeta\int du(G\bar{\rho}a^2)^2\int\frac{dk}{k}P(k) \quad \simeq \frac{72}{\pi}\frac{1}{u_0^3}\int_0^\infty \frac{dk}{k}P_0(k)$$

$$G\bar{\rho}a^3 = 3/(2\pi u_0^2)$$

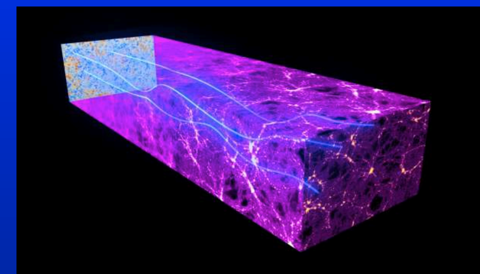
$$\int(dk/k)P_0(k) \simeq 7.25\times 10^4(\mathrm{Mpc/h})^3$$

h = Hubble parameter ~ 0.7

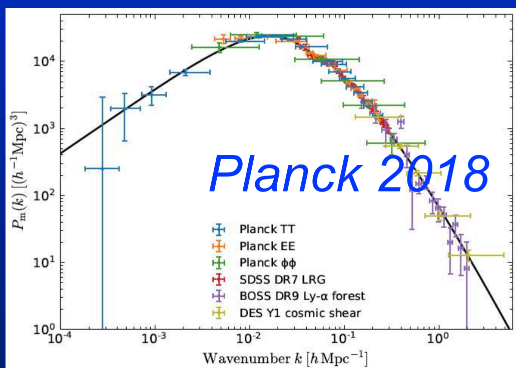
Gravitational lensing of cosmic neutrino background

(G. Holder)

$$\langle(\Delta\theta_p)^2\rangle \simeq \frac{72}{\pi}\frac{1}{u_0^3}\int_0^\infty \frac{dk}{k}P_0(k)$$



Age of the universe t_0 = 13.8 Gy => cu_0 = 12.7 Gpc



$$\int (dk/k)P_0(k) \simeq 7.25\times 10^4(\mathrm{Mpc/h})^3$$

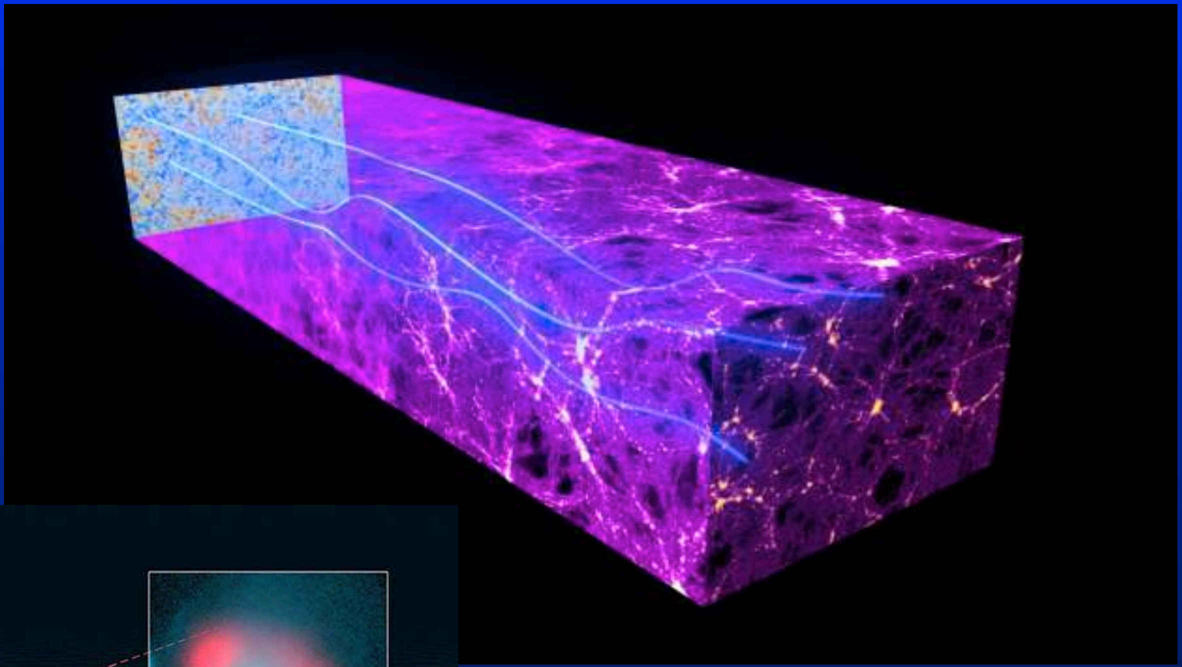
h = Hubble parameter ~ 0.7

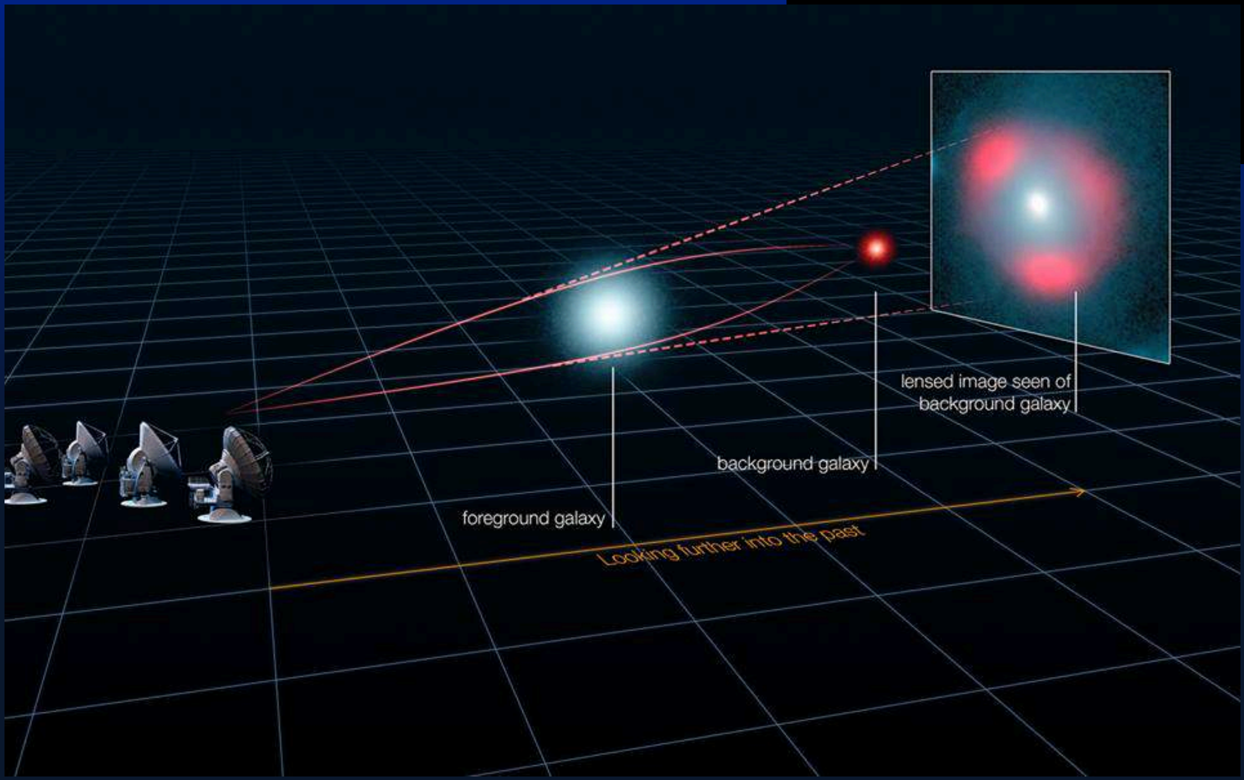
RMS momentum bending = lensing of cosmic neutrino background ~ 5.3 arcmin

Lensing of CMB ~ 2.7 arcmin. Most efficient at smaller z (≲10). Reionization of intergalactic H => photon-e scattering.

(Weak electron-neutrino scattering after reionization insignificant)

Gravitational lensing of cosmic neutrino background





Gravitational spin rotation with respect to momentum, Θ

Main effect in matter dominated era from redshift ~ 10^4 to now

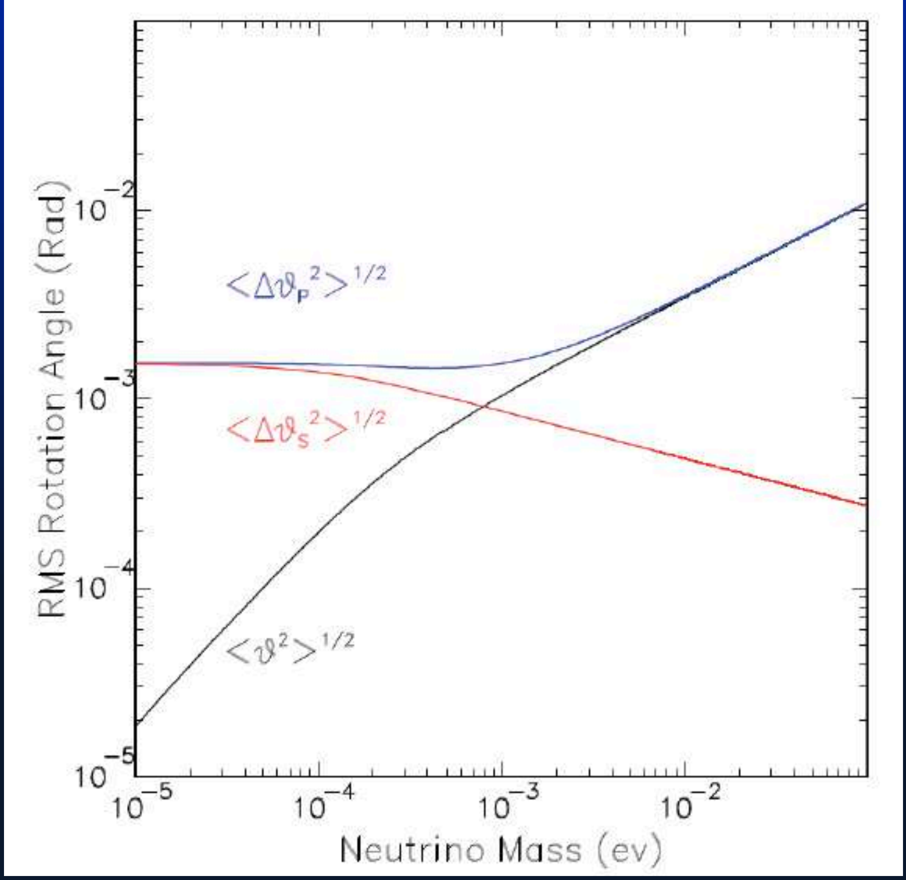
Momentum rotation with finite neutrino mass:

$$\langle(\Delta\theta_p)^2\rangle = \frac{18Y}{\pi u_0}\int_{u_{eq}}^{u_0} du\, v\left(v+\frac{1}{v}\right)^2$$

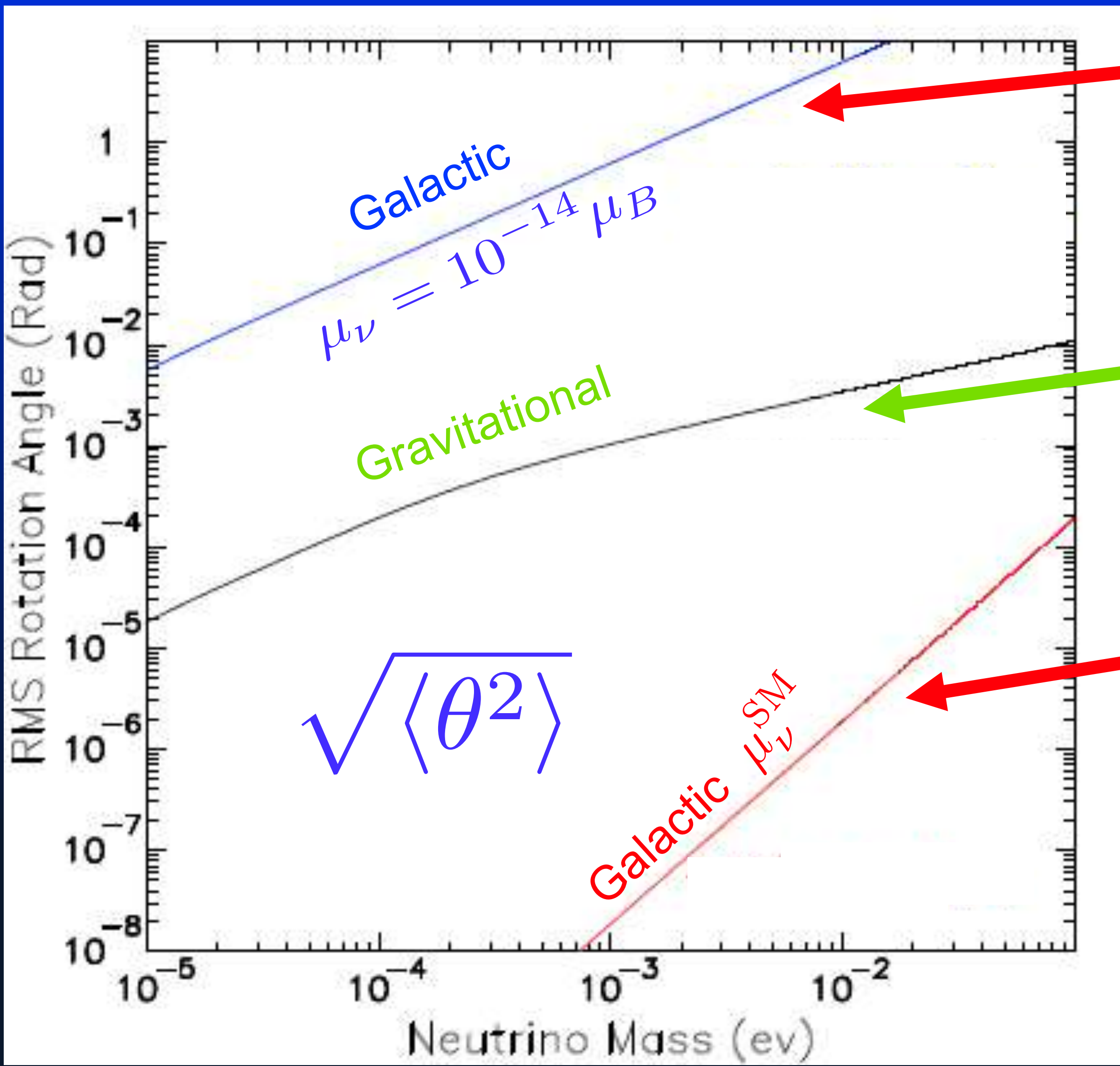
$$Y \equiv \frac{1}{u_0^3}\int_0^\infty \frac{dk}{k}P_0(k) \quad \simeq 10^{-7}$$

Spin rotation with respect to momentum.

$$\langle\theta^2\rangle = \frac{18Y}{\pi u_0}\int_{u_{eq}}^{u_0} du\left(v-\frac{1}{v}\right)^2$$



Spin rotation from gravitational vs. magnetic fields



Rotation in Milky Way

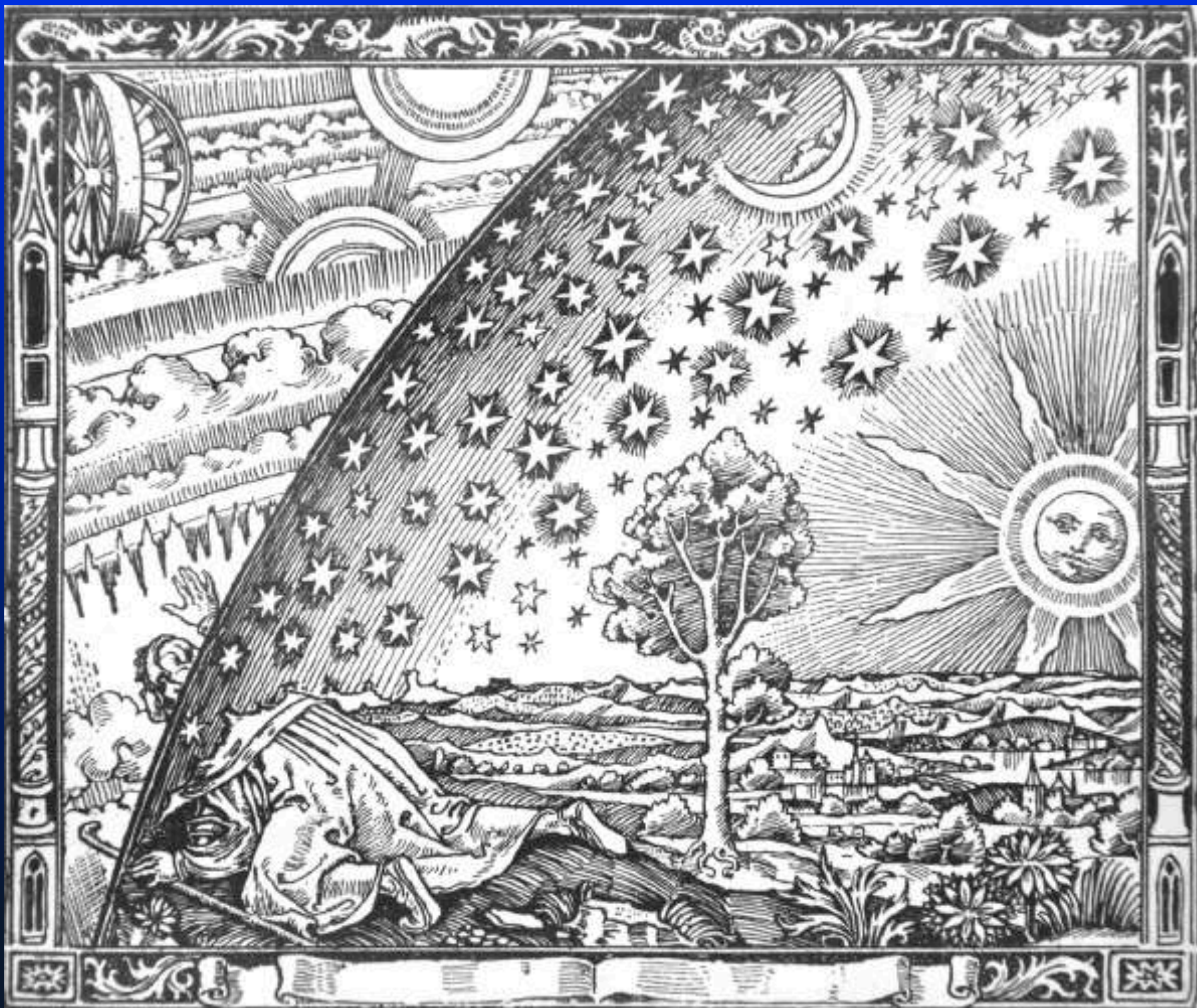
$$B_g = 10\,\mu\mathrm{G},\ \Lambda_g = 1\,\mathrm{kpc}$$

$$\mu_\nu = 10^{-14}\mu_B$$

Gravitational rotation

Rotation in Milky Way with standard model magnetic moment

$$\mu_\nu^{\rm SM} \simeq 3\times 10^{-23} m_{\rm eV}\,\mu_B$$



Flammarion 1888

Neutrinos 101

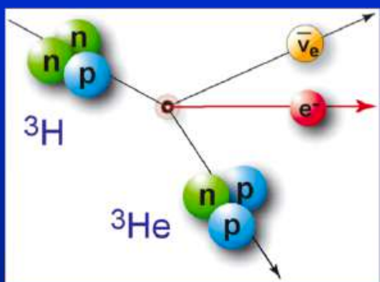
Neutrino magnetic moments & spin precession

Gravitational inhomogeneities & spin precession

Detection of relic neutrinos

Detection of relic neutrinos

Tritium beta decay

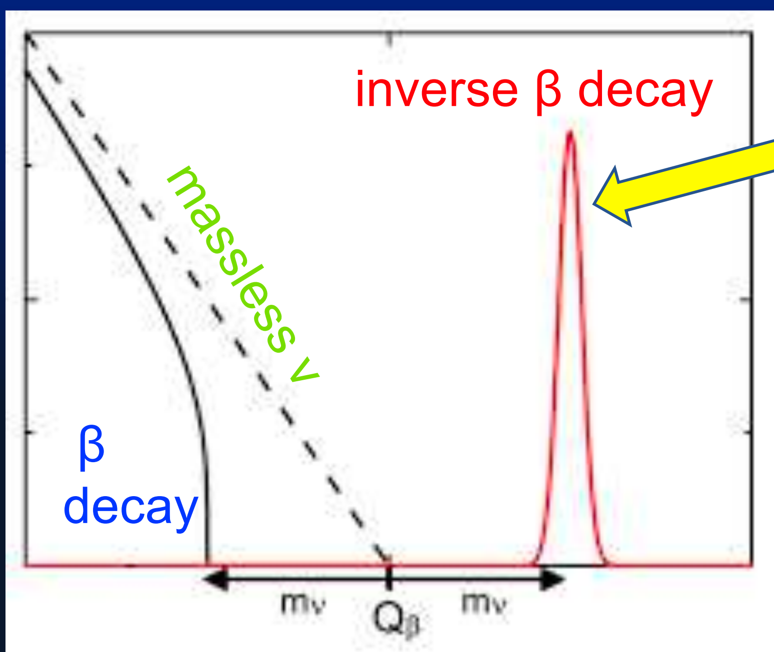


KATRIN

Detect electron neutrinos via
inverse tritium beta decay (never observed)

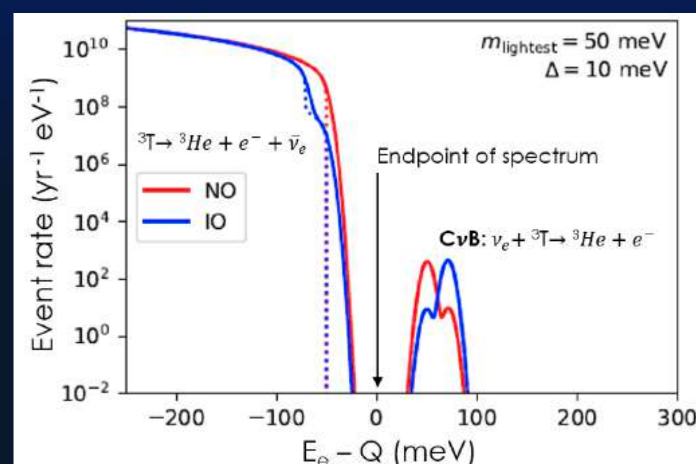
Weinberg PR 1962

PTOLEMY experiment
➔ 100 g ^3H



events vs. electron energy

in principal, have 3 peaks at the
3 neutrino masses, weighted by $|U_{ei}|^2$



PTOLEMY, JCAP 2019

Cross section for ITBD (p = neutrino and p_e = electron momena)

$$\sigma_i^h(p,p_e) = \frac{G_F^2}{2\pi v_i}|V_{ud}|^2|U_{ei}|^2 F(\eta)\frac{m_{^3\mathrm{He}}}{m_{^3\mathrm{H}}} E_e p_e A_i^h(\bar{f}^2 + 3\bar{g}^2)$$

(F = f = Fermi)

V_{ud} = CKM matrix element, U_{ei} = PMNS matrix element

$F = 2\pi\eta/(1-e^{-2\pi\eta})$ = e - ^{3}He Coulomb correction, $\eta = e^2/v_e$

$\bar{f}^2$ = Fermi and $3\bar{g}^2$ = Gamow-Teller nuclear form factors

$$A_i^{helicity=\pm} = 1 \mp v_i$$: only helicity dependence in cross section

Total ITBD rate: $$\Gamma_{ITBD} = \sum_{massesi,h=\pm}\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T_{\nu 0}}+1}\sigma_i^h v_i$$

$$\sigma_i^h(p,p_e) = \frac{G_F^2}{2\pi v_i}|V_{ud}|^2|U_{ei}|^2 F(\eta)\frac{m_{^3\mathrm{He}}}{m_{^3\mathrm{H}}} E_e p_e A_i^h(\bar{f}^2 + 3\bar{g}^2)$$

Neutrino dependent part of rate = A_{eff} :

Dirac: neutrinos only, no antineutrinos

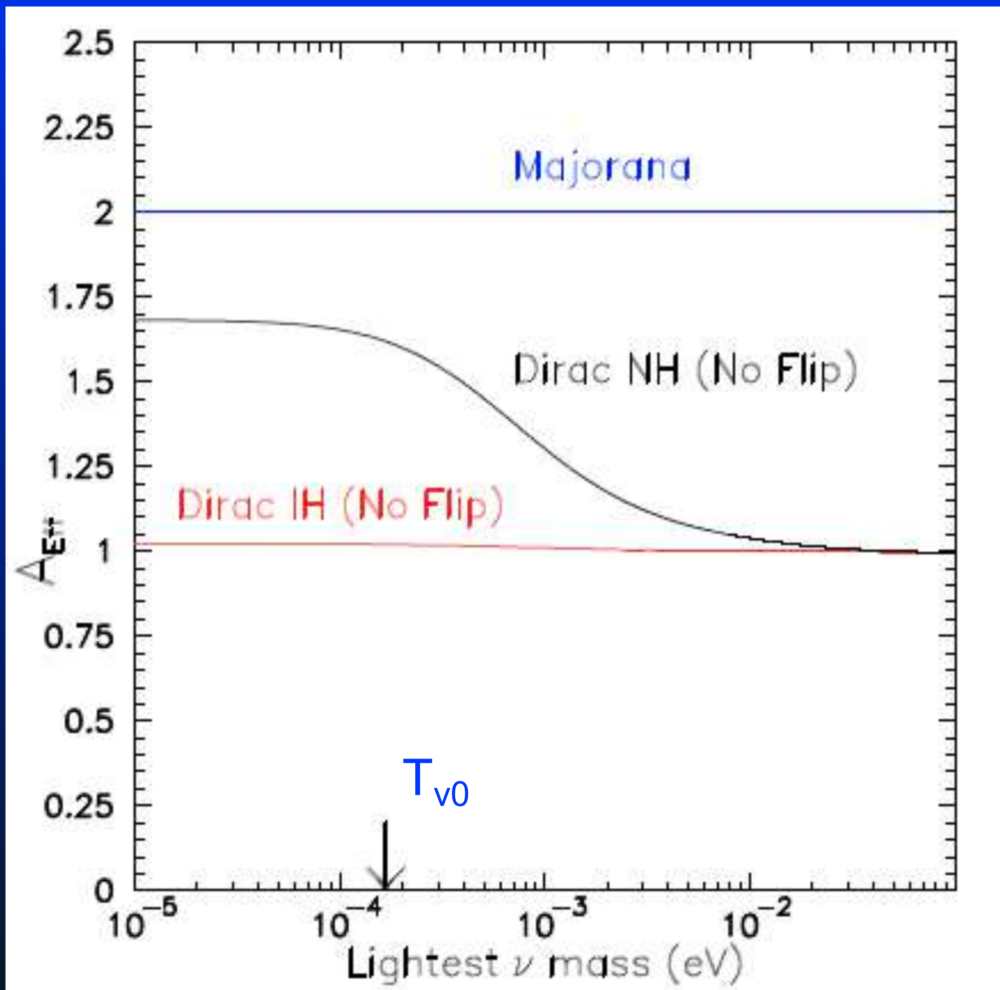
$$A_{\mathrm{eff,D}} = \sum_{i,h=\pm} |U_{ei}|^2 \Big\langle (1 \mp v_i)\langle \frac{1}{2}(1 \mp \cos\theta_i)\Big\rangle_T = \sum_i \big(|U_{ei}|^2(1 + \langle v_i \cos\theta_i\rangle_T\big)$$

T = thermal average plus average of spin rotation in neutrino's history.

Majorana: both neutrinos and antineutrinos contribute:

$$A_{\mathrm{eff,M}} = \big(1 + \sum_i |U_{ei}|^2\langle v_i\rangle_T\big) + \big(1 - \sum_i |U_{ei}|^2\langle v_i\rangle_T\big) \equiv 2$$

Neutrino mass and hierarchy dependence in ITBD capture

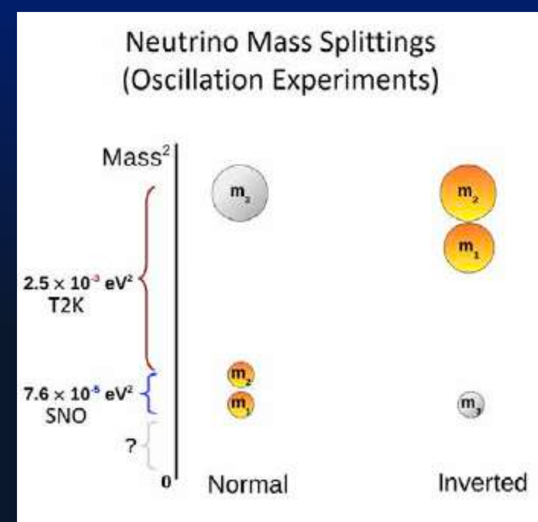


no helicity flipping

$$A_{\mathrm{eff,M}} = 2$$

$$A_{\mathrm{eff,D}} = \left(1 + \sum_i |U_{ei}|^2 \langle v_i \rangle_T\right)$$

Normal	Inverted
$\lvert U_{e1}\rvert^2 = 0.6794$	$\lvert U_{e1}\rvert^2 = 0.6793$
$\lvert U_{e2}\rvert^2 = 0.2990$	$\lvert U_{e2}\rvert^2 = 0.2989$
$\lvert U_{e3}\rvert^2 = 0.0216$	$\lvert U_{e3}\rvert^2 = 0.0218$

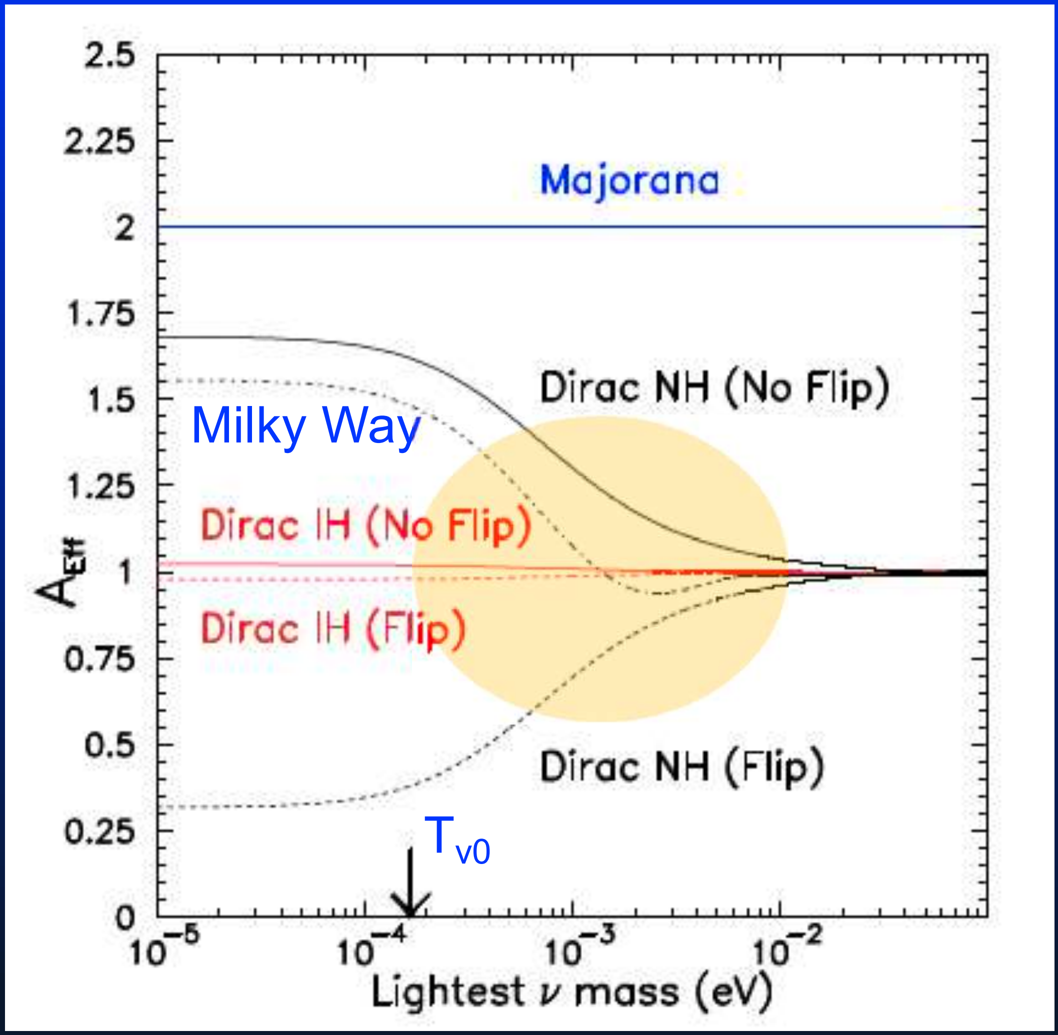


NH: $m_1=10^{-5}$ => $v_1 \sim 1$, $v_2 \sim 1/5$, $v_3 \sim 1/20$

IH: $m_1=10^{-5}$ => $v_3 \sim 1$, $v_1 \sim v_2 \sim 1/20$

But 3 couples most weakly => small mass dependence in IH

With helicity flipping



$$A_{\mathrm{eff,M}} = 2$$

$$A_{\mathrm{eff,D}} = \left(1 + \sum_i |U_{ei}|^2 \langle v_i \cos\theta_i \rangle_T \right)$$

Relativistic neutrino dominates

Bounded by Dirac NH (no flip) and Dirac NH (flip) curves

Helicity rotation in Milky Way

$$B_g = 10\,\mu\mathrm{G},\ \Lambda_g = 1\,\mathrm{kpc}$$

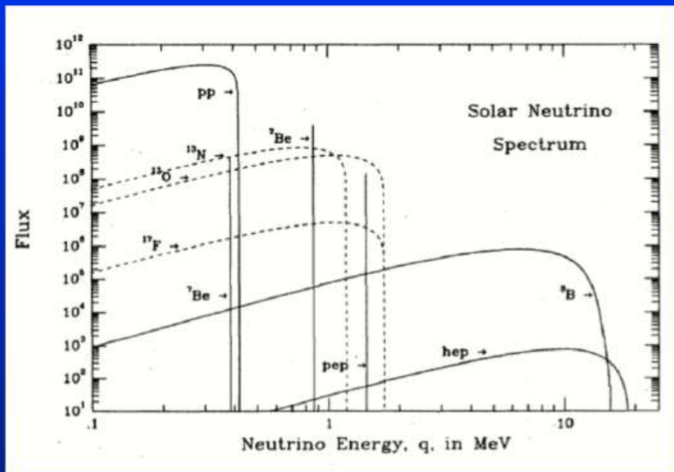
$$\mu_\nu = 5 \times 10^{-14} \mu_B$$

IH: spin rotation makes tiny difference

NH: spin rotation makes noticeable difference for $m_1 \lesssim 10^{-2}$ eV

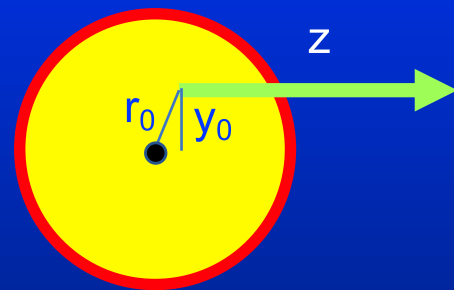
Peaks from small mass neutrinos hard to resolve with present technology

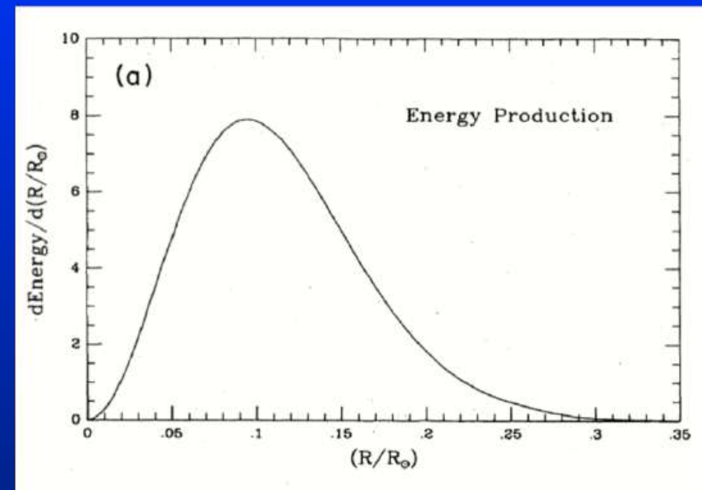
Gravitational helicity modification of solar neutrinos



dominated by pp neutrinos

$$p + p \to {}^2\mathrm{H} + e^+ + \nu_e$$

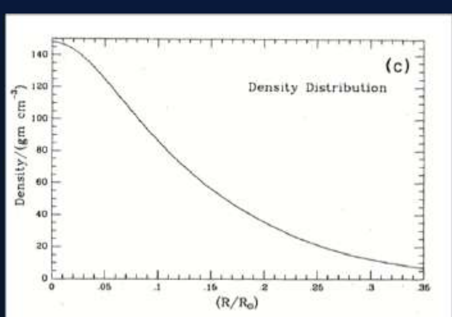




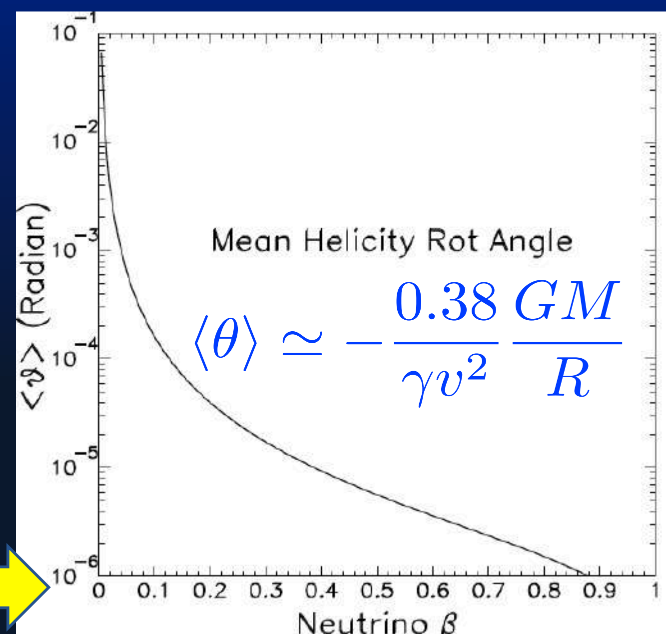
emitted at about $R_{sun}/10$.

$$\theta(y_0, r_0)) = -\frac{1}{\gamma v^2}\int_{z_0}^{\infty} dz\, \nabla_y \Phi(r)$$

$$= -\frac{1}{\gamma v^2}\int_{z_0}^{\infty} dz\, \frac{GM(r) y_0}{r^3}$$



Average over spatial emission and density distributions in Sun



Significant helicity modification of heavy particles with spin, e.g., dark photons, from Sun.

Neutrinos from neutron stars and diffuse supernova background

Spin rotation of MeV neutrinos from the diffuse supernova background and neutron stars potentially detectable. Ex., via $\bar{\nu} + p \to e^+ + n$, using Gd-doped Super-K detector to detect n.

From neutron stars and SN:

--Magnetic rotation comparable to that in galaxies:

$$\boxed{\theta \sim \mu_\nu BR/c \sim 5 \times 10^{13}\,(R/10\,\mathrm{km})\left(B/10^{12}\mathrm{G}\right)(\mu_\nu/\mu_B)} \propto 1/R$$

$B \propto 1/R^2 =>$ neutron stars rotate spins more than SN

Galaxies: $\sqrt{\langle\theta_g^2\rangle} \sim 5 \times 10^{14}(\mu_\nu/\mu_B)$ for $B_g \sim 10\,\mu\mathrm{G}, \ell_g \sim 16\mathrm{kpc}, \Lambda_g \sim \mathrm{kpc}$

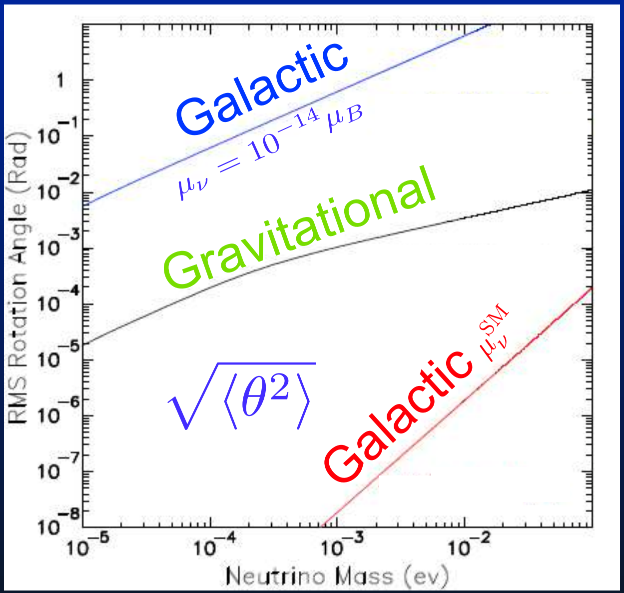
--Gravitational rotation of spins $\sim GM/\gamma R$, negligible since $\gamma \sim 10^{8-9}$

Conclusions

Relic neutrino helicities new probe of cosmic gravitational and magnetic fields

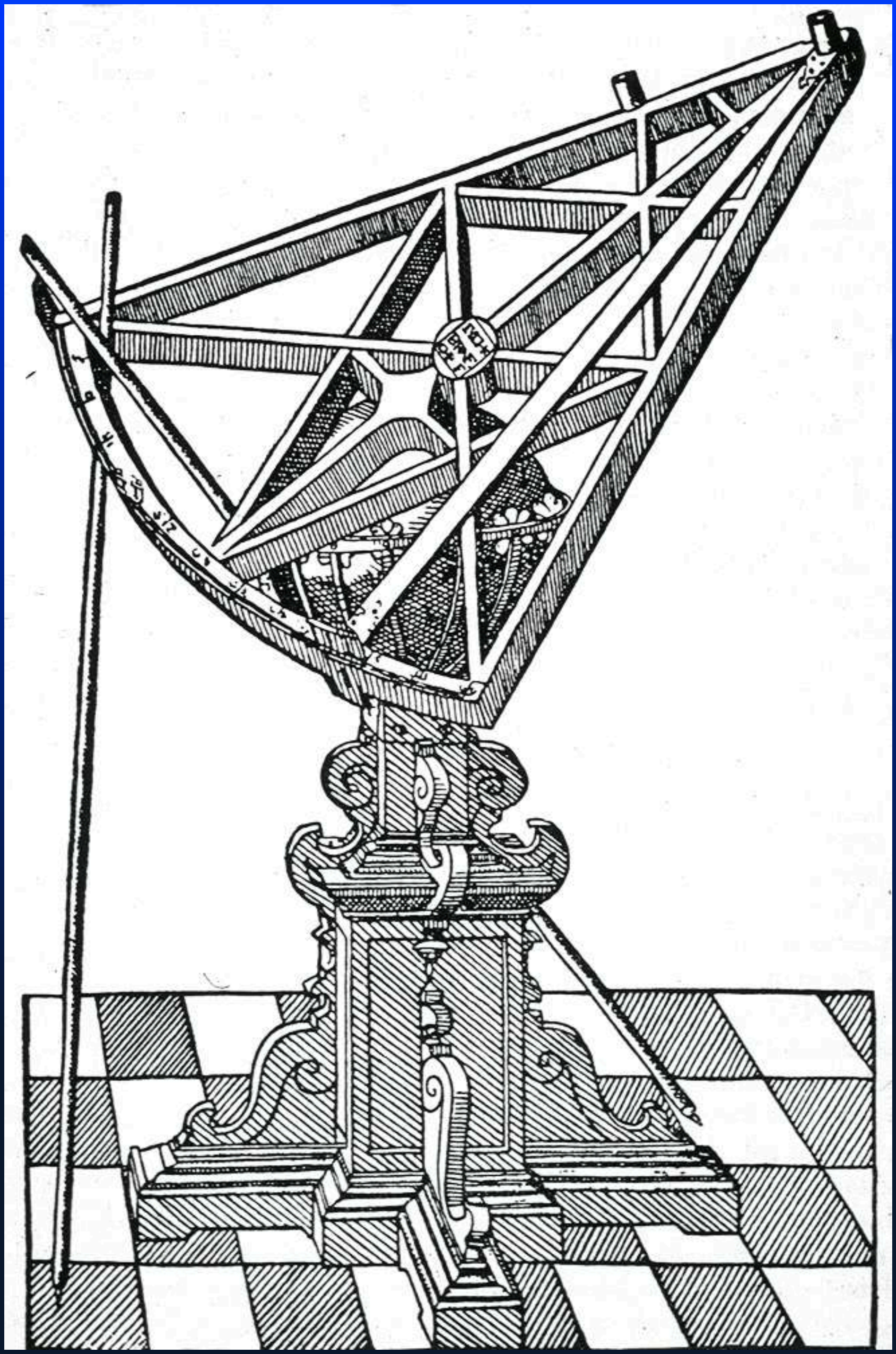
Significant helicity changes of relic neutrinos for neutrino magnetic moment μ_ν even three-four orders of magnitude smaller than suggested by XENON1T.

Gravitational helicity changes few orders of magnitude smaller cf. large μ_ν rotations, but much larger than for μ_ν in Standard Model



Need significant improvement in electron energy resolution in ITBD to resolve helicity modifications.

Thank you



Tycho Brahe’s sextant, ca 1580