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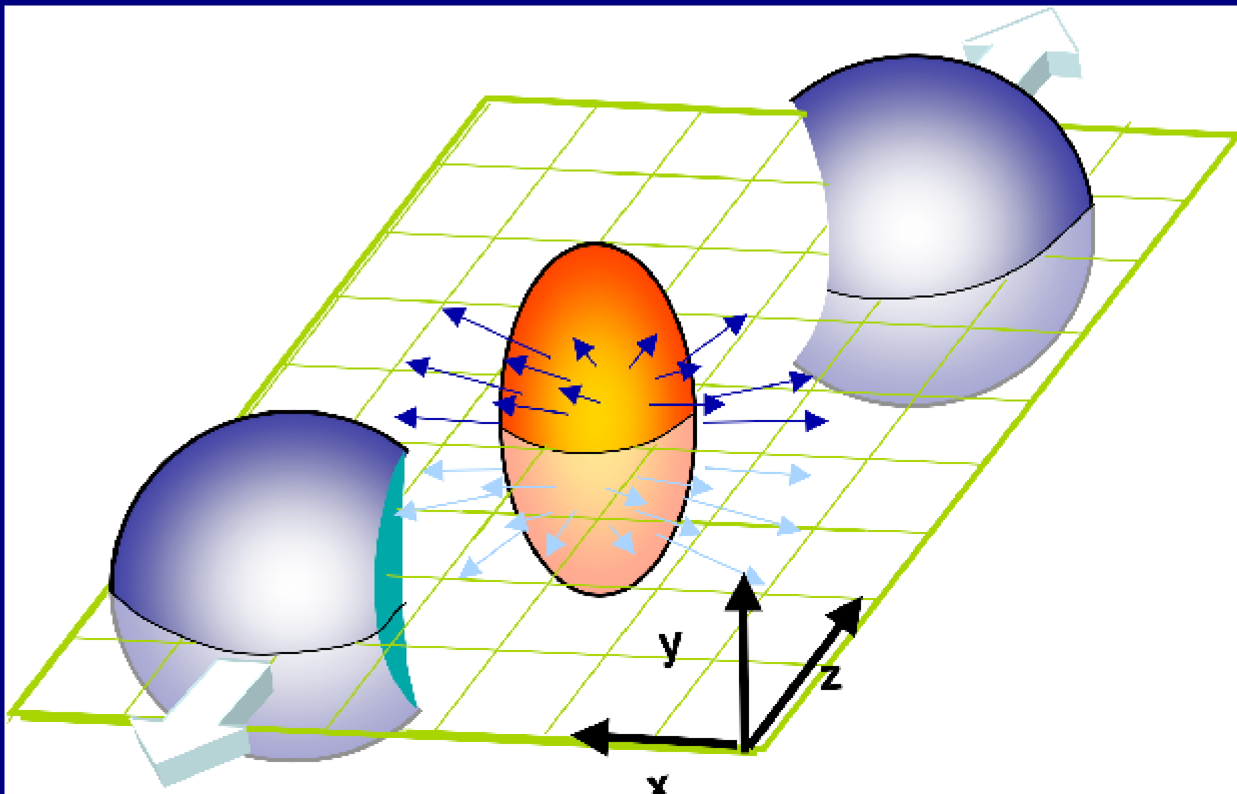
New developments of spin physics in relativistic heavy ion collisions

OUTLINE

- Introduction and local polarization puzzles
- Spin in relativistic fluids: quantum-relativistic theory
- Local thermodynamic equilibrium and its expansion
- Spin-thermal shear coupling and application to heavy ion physics
- Isothermal local equilibrium: solving the puzzle
- Spin and local parity violation

ASU Colloquium, June 16 2021

Introduction: relativistic nuclear collisions



$7 < \sqrt{s_{NN}} < 200$ GeV RHIC

$\sqrt{s_{NN}} \sim$ 2.76 – 5.5 TeV LHC

Goal: the production and study of the phases of QCD at finite T and μ_B

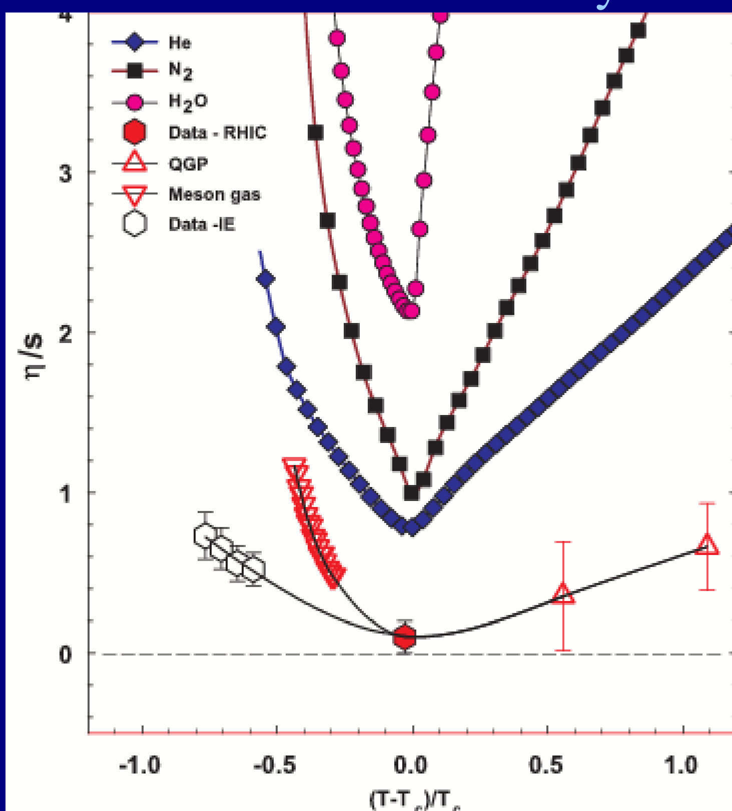
QGP is an extraordinary fluid

- It is the hottest ever made: $T \sim 5\ 10^{12}$ K

- It is the tiniest ever made: ~10 fm across

- It has the largest initial pressure, energy density and largest initial acceleration ($a \sim 10^{30}\ g$)   Surface gravity of a black hole ~ $3\ 10^{12}\ g/(M/M_s)$

- It has the lowest viscosity/entropy density ratio ever observed



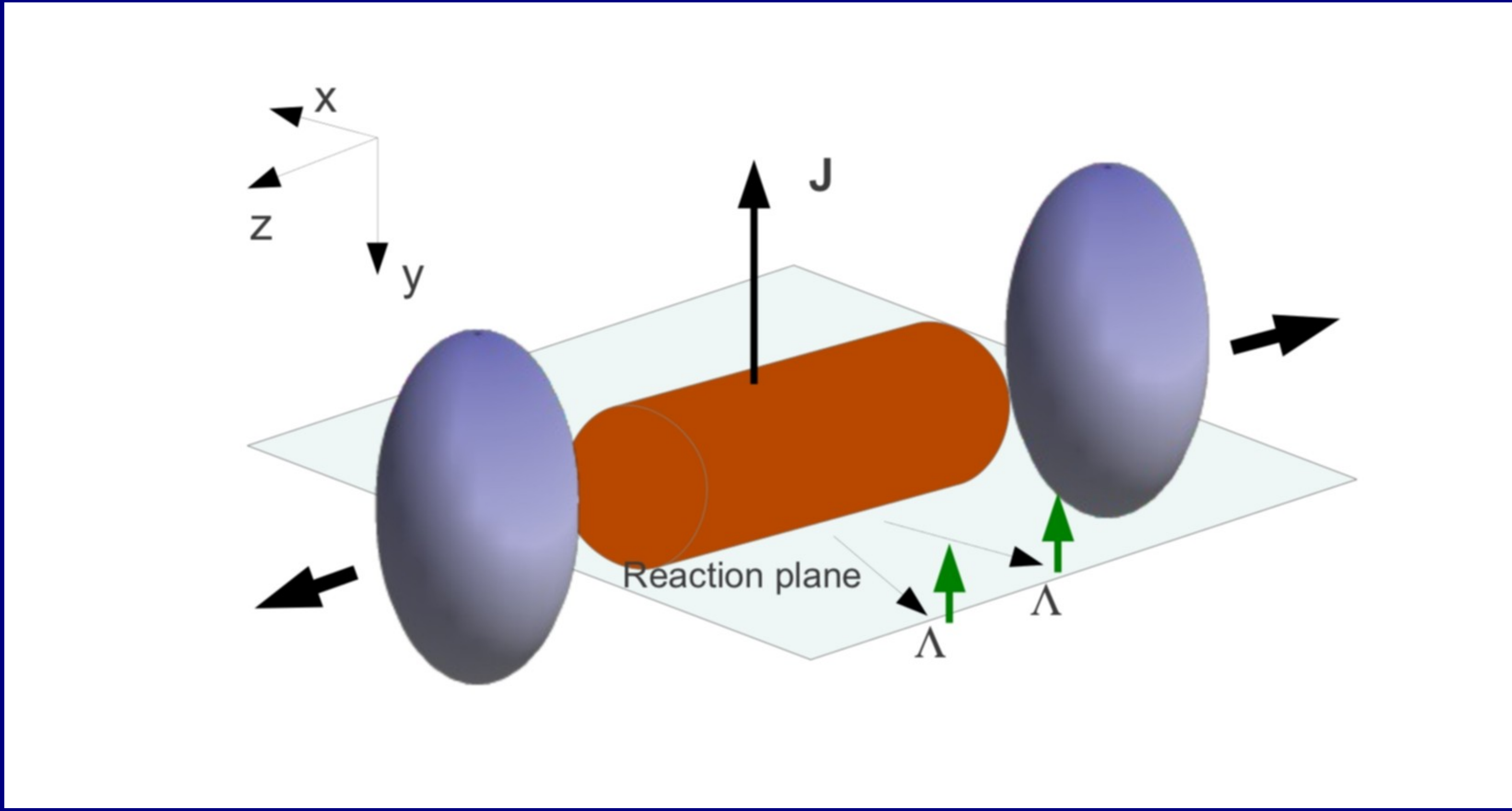
$$\eta/s \sim \lambda/\lambda_T$$

If the mean free path ~ mean wavelength one cannot even think of a “particle”. No kinetic description of the fluid is allowed.

QGP around T_c cannot be described in terms of colliding particles or quasiparticles and yet local thermodynamic equilibrium can be defined

Peripheral collisions: large angular momentum

Peripheral collisions ➡ Angular momentum ➡ Global polarization w.r.t reaction plane

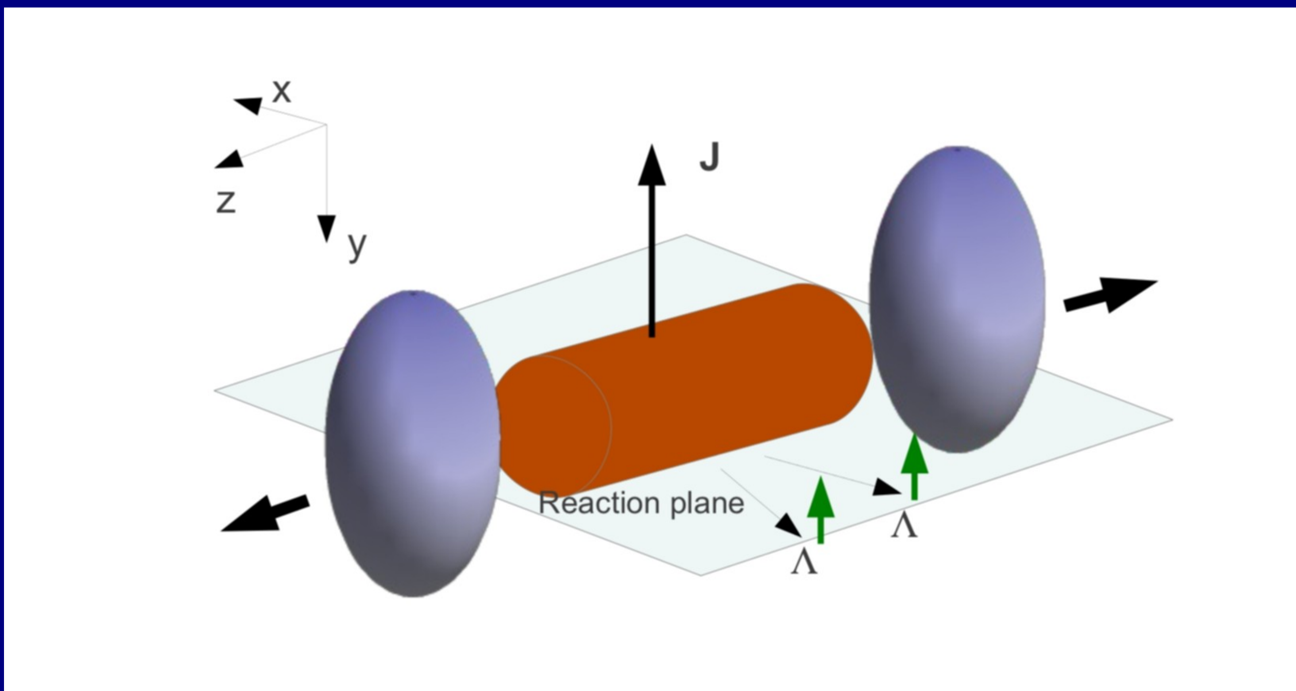


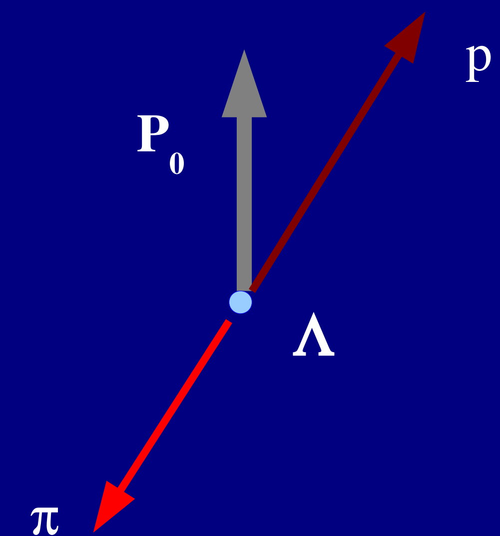
- Polarization estimated at quark level by spin-orbit coupling
  Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301
- By local thermodynamic equilibrium of the spin degrees of freedom
  F. B., F. Piccinini, Ann. Phys. 323 (2008) 2452; F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Spin µ (thermal) vorticity

How to observe it: global Λ polarization

Because of parity violation, the polarization vector of Λ can be measured in its decay
Into a proton and a pion





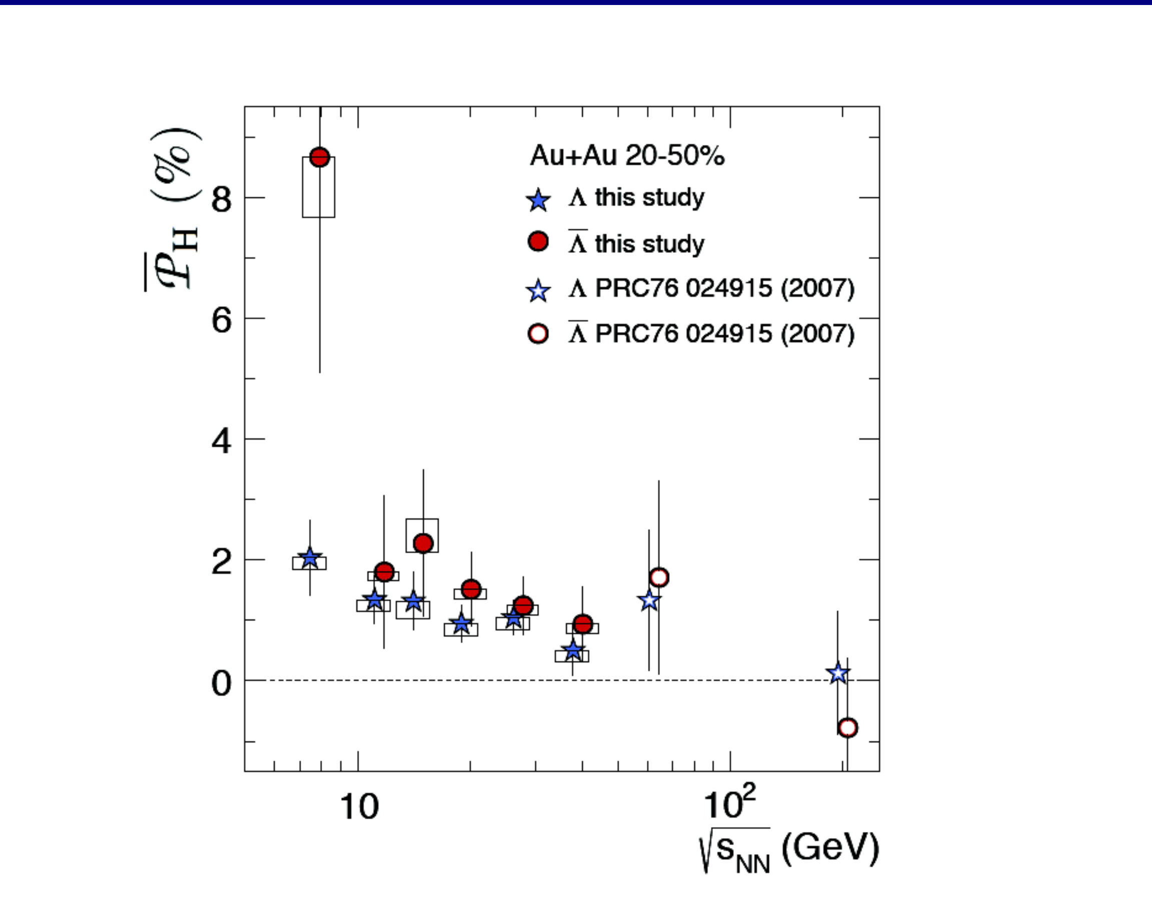
Distribution of protons in the Λ rest frame

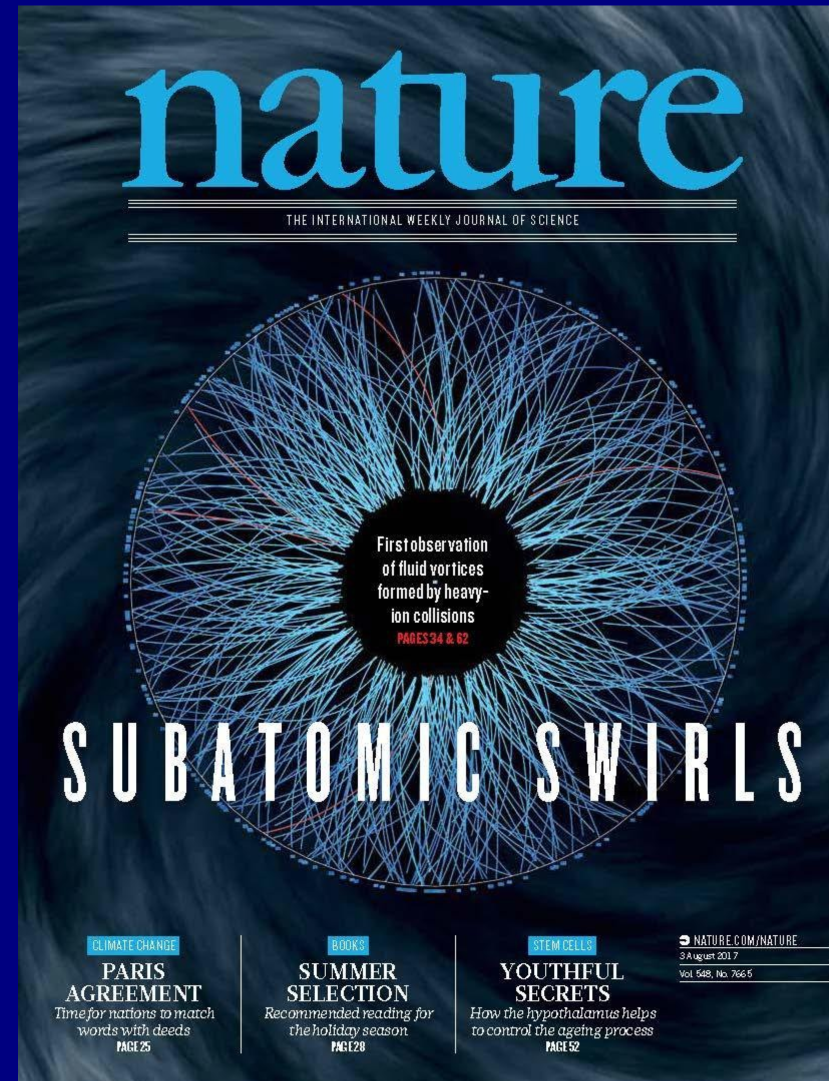
$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_0 \cdot \hat{\mathbf{p}}^*) \qquad \mathbf{P}_0(p) = \mathbf{P}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon + m)}\mathbf{P}(p) \cdot \mathbf{p}$$

$\alpha = 0.642 \rightarrow 0.75$ (!) PDG 2020

Discovery of this phenomenon

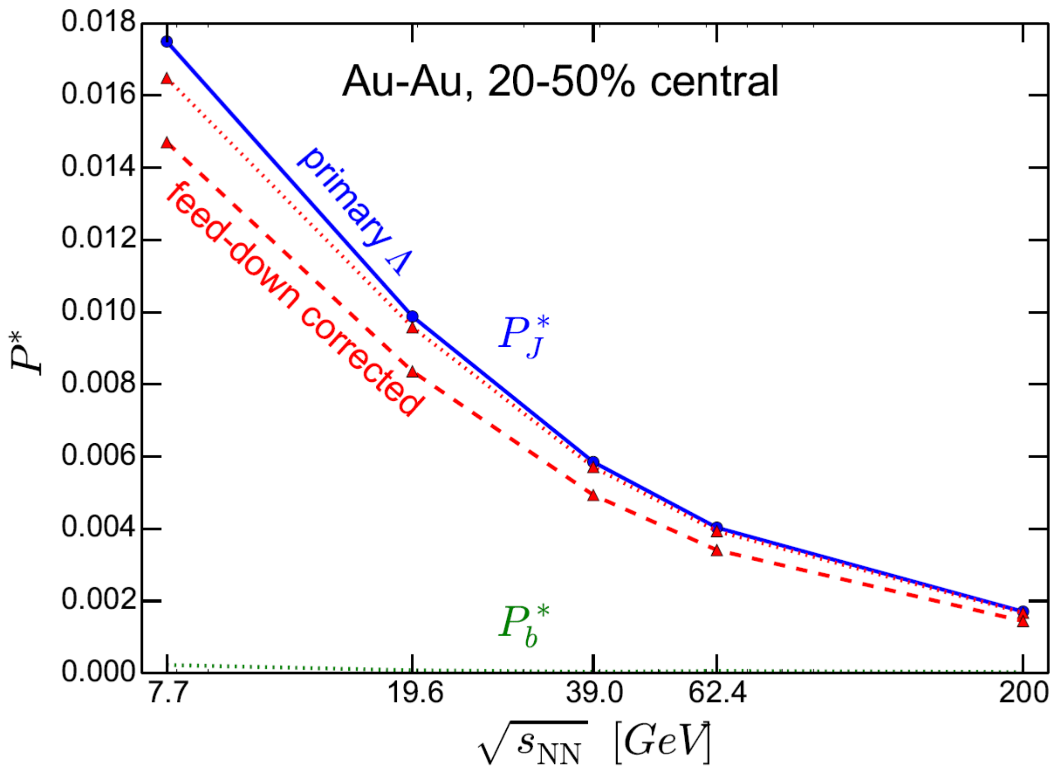
STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017





Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven
by a mean field (such as EM) whose coupling is *C-odd.*
Definitely favours the thermodynamic (equipartition) interpretation

Agreement between hydrodynamic predictions and the data



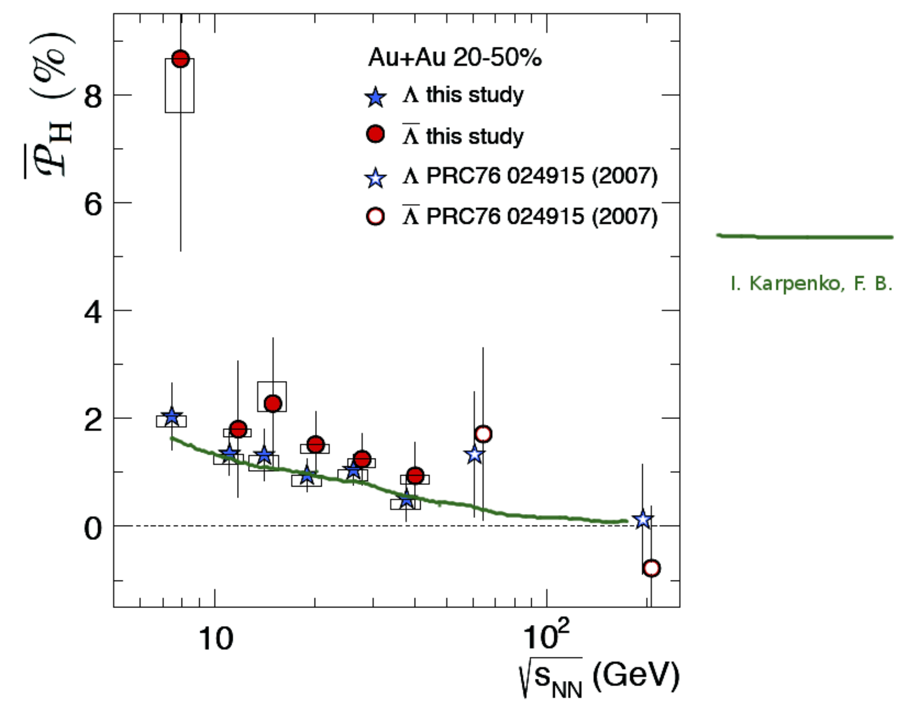
I. Karpenko and F. B., Eur. Phys. J. C 77 (2017) 213

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma d\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma d\Sigma_\tau p^\tau \, n_F}$$

$$\mathbf{S}^*_{\rm daughter} = C\mathbf{S}^*_{\rm parent}$$

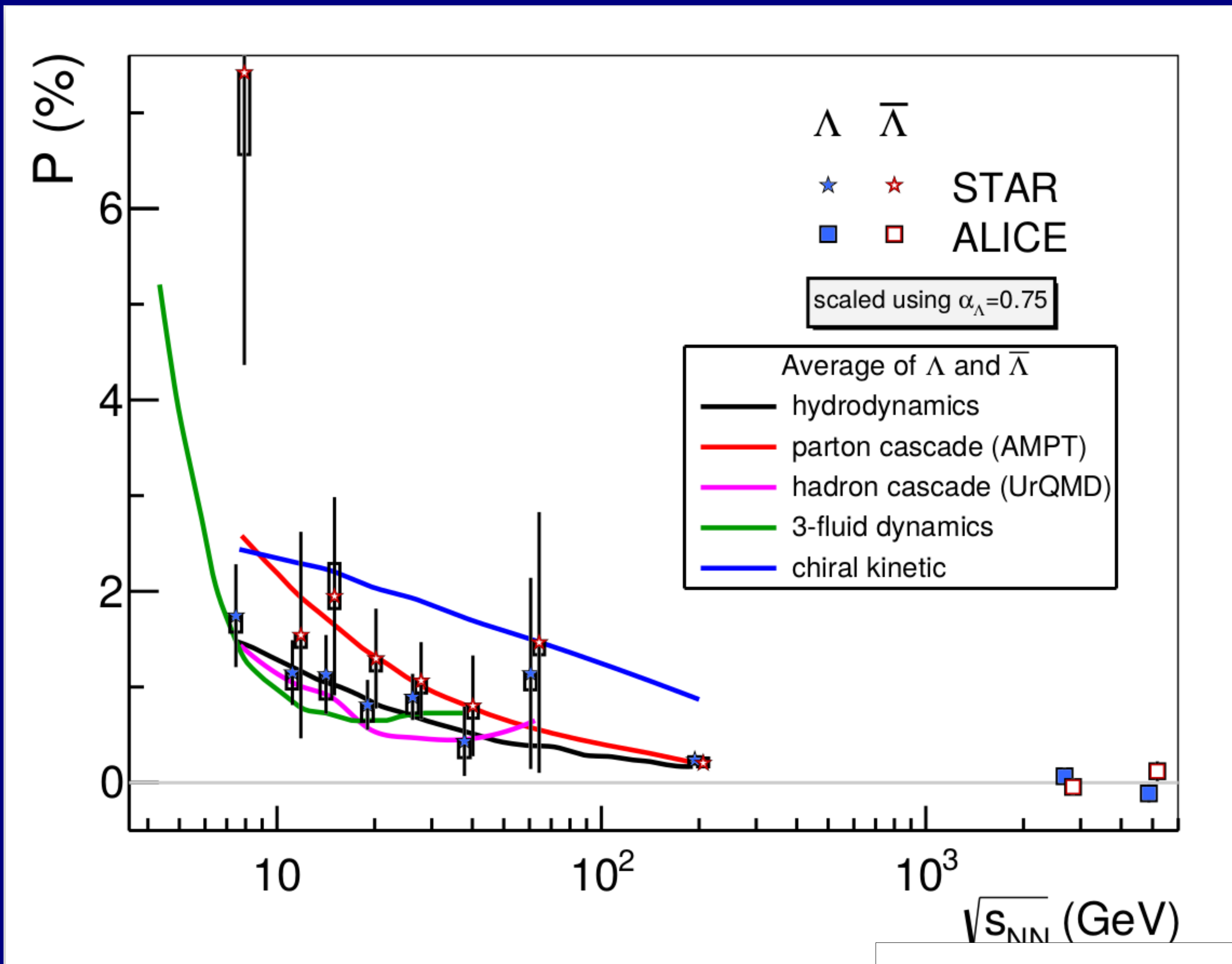
$$C = \sum_{\lambda_A,\lambda_B,\lambda'_A} T^J(\lambda_A,\lambda_B)T^J(\lambda'_A,\lambda_B)^* \sum_{n=-1}^{1} \langle\lambda'_A|\widehat{S}_{A,-n}|\lambda_A\rangle$$
$$\times \frac{c_n}{\sqrt{J(J+1)}}\langle J\lambda|J1|\lambda' n\rangle \left(\sum_{\lambda_A,\lambda_B} |T^J(\lambda_A,\lambda_B)|^2\right)^{-1}$$

F. B., I. Karpenko, M. Lisa, I. Upsal, S. Voloshin,
Phys. Rev. C 95 054902 (2017)



Updated comparison

F. B., M. Lisa, Polarization and vorticity in the QGP, Ann. Rev. Part, Nucl. Sc. 70, 395 (2020)

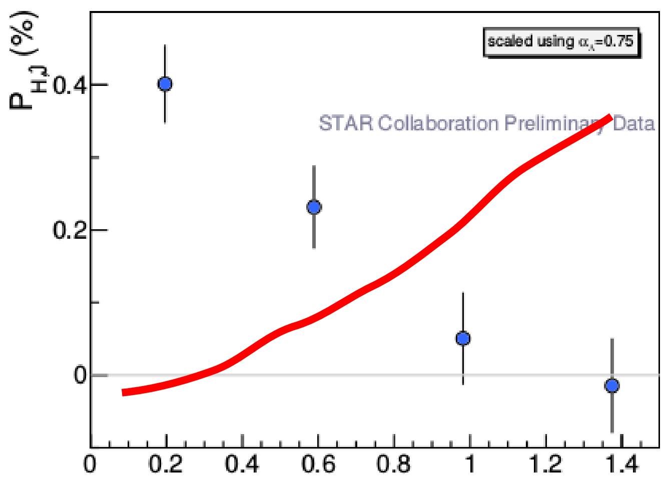


Different models of the collision, same formula for polarization

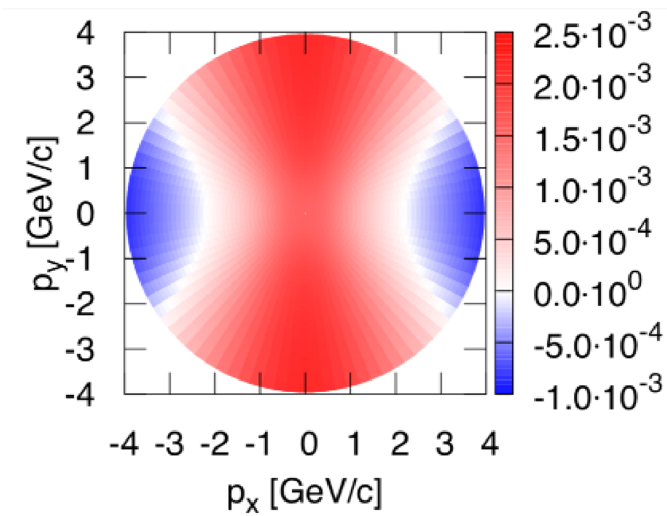
$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

Puzzles: momentum dependence of polarization

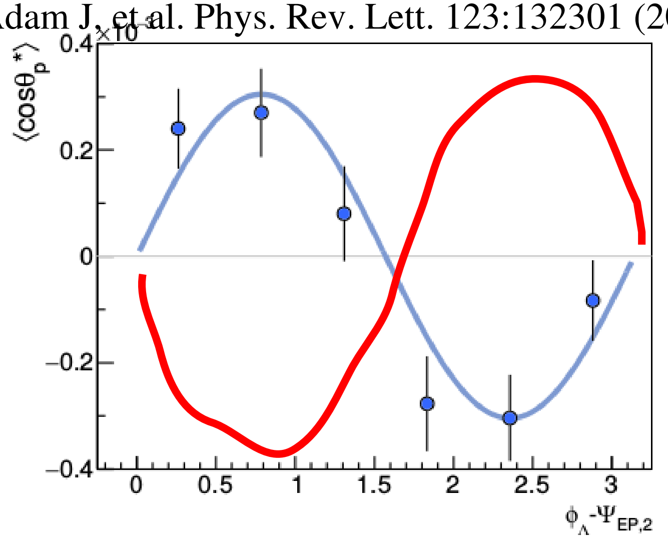
Theory prediction

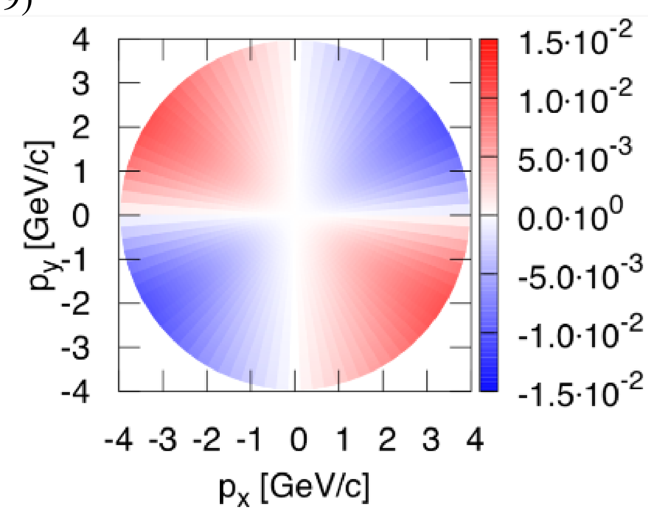


Niida T. Nucl. Phys. A982:511 (2019)



Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)





Not the effect of decays:

X. L. Xia, H. Li, X.G. Huang and H. Z. Huang, Phys. Rev. C 100 (2019), 014913

F. B., G. Cao and E. Speranza, Eur. Phys. J. C 79 (2019) 741

How to solve the problem?

- *Final hadronic interactions/rescattering*

L. Csernai, J. Kapusta. Y. Xie, C. Barros, ...

- *Dissipative corrections*

K. Hattori, M. Hongo, S. Bhadury, W. Florkowski, A. Jaiswal, S. Shi, A. Kumar, R. Sing, D. Hou, J. Liao, …..

- *Lack of local thermodynamic equilibrium in the spin sector: kinetics*

Q. Wang, X. L. Sheng, X. N. Wang. Z. T. Liao, N. Weickgennant, D. Rischke, J. H. Gao, E. Speranza, X. G. Huang, P. Zhuang, C. M. Ko, Y. Sun, H. U. Yee, J. Kapusta, ….

- *Role of the spin tensor: additional spin potential required*

W. Florkowski, R. Ryblewski, E. Speranza, ...

Polarization in a relativistic fluid: theory

F. Becattini, arXiv:2004.04050, to appear in Springer Lecture Notes in Physics.

The covariant Wigner function of the free Dirac field:

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) :\rangle$$
$$= \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle : \overline{\Psi}_B(x+y/2)\Psi_A(x-y/2) :\rangle$$

$$\langle \widehat{X}\rangle = \mathrm{tr}(\widehat{\rho}\widehat{X})$$

It allows to calculate the spin density matrix for spin ½:

$$\Theta(p)_{rs} = \frac{\int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_r(p) W_+(x,p) u_s(p)}{\sum_t \int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_t(p) W_+(x,p) u_t(p)}$$

And the mean spin vector in these three equivalent forms:

$$S^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\beta\gamma\delta} p_\delta \frac{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4(\Sigma_{\beta\gamma} W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\, \mathrm{tr}_4(\gamma^\mu\gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma\cdot p\, \mathrm{tr}_4 W_+(x,p)}$$

$$S^\mu(p) = -\frac{1}{4}\epsilon^{\mu\beta\gamma\delta} p_\delta \frac{\int \mathrm{d}\Sigma_\lambda \mathrm{tr}_4(\{\gamma^\lambda, \Sigma_{\beta\gamma}\} W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

Density operator of quantum relativistic fluid

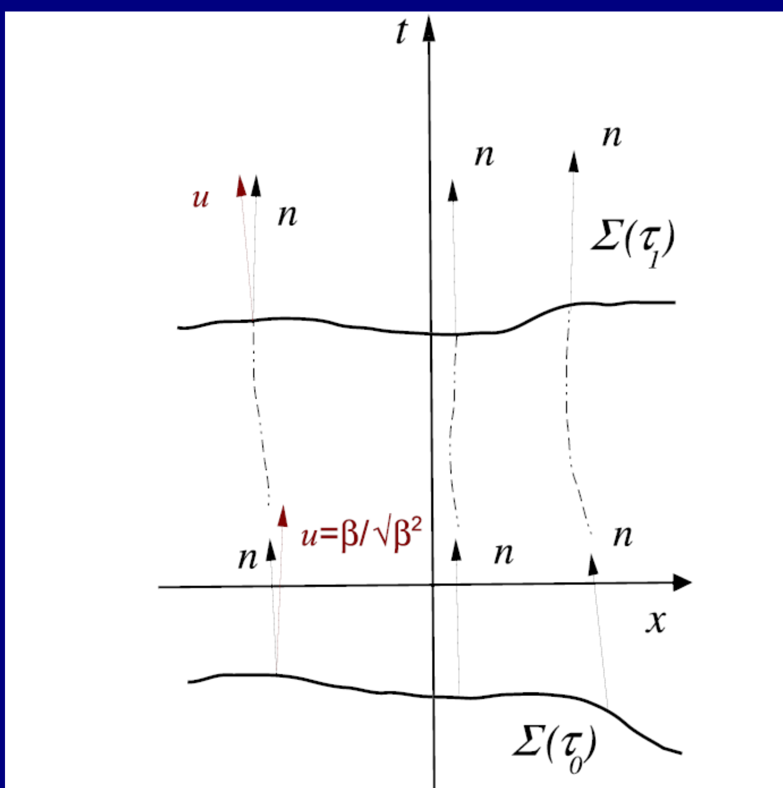
Needed to calculate the Wigner function! $$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

General covariant
Local thermodynamic
Equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

$$\beta = \frac{1}{T} u$$

$$\zeta = \frac{\mu}{T}$$



The operator is obtained by maximizing the entropy

$$S = -\mathrm{tr}(\widehat{\rho} \log \widehat{\rho})$$

with the constraints of fixed energy-momentum density

Zubarev, 1979, Ch, Van Weert 1982

See also:
F. B., L. Bucciantini, E. Grossi, L. Tinti,
Eur. Phys. J. C 75 (2015) 191

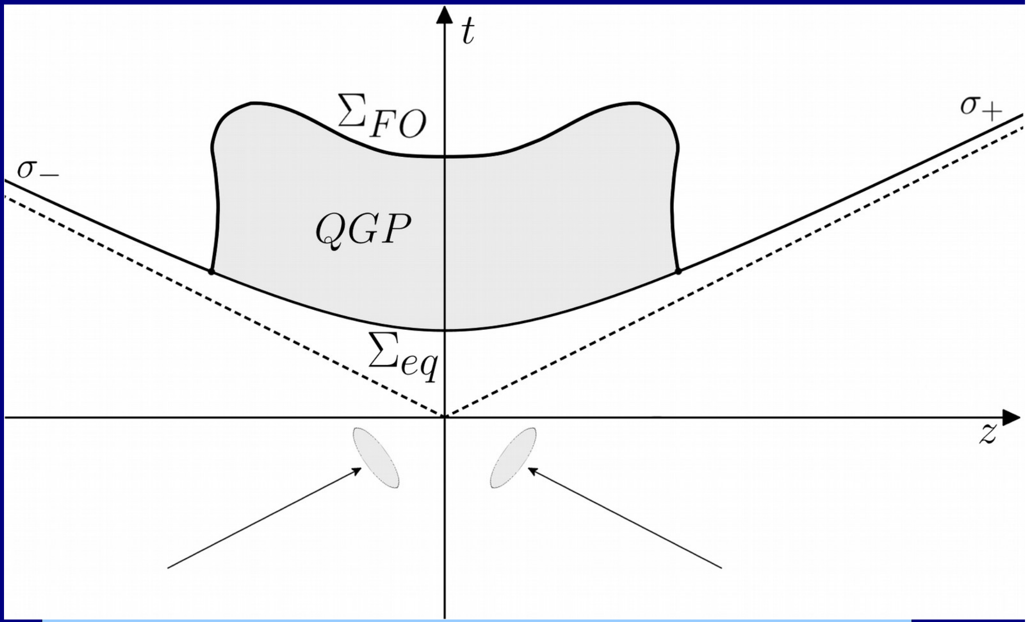
T. Hayata, Y. Hidaka, T. Noumi, M. Hongo,
Phys. Rev. D 92 (2015) 065008

The actual statistical operator (Zubarev theory)

The above density operatot is "time" dependent, cannot be the actual one!

In the Zubarev's theory, this is the LTE at some initial "time":

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right] .$$



NOTE: T_B stands for the symmetrized Belinfante stress-energy tensor

With the Gauss theorem

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \, \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right] ,$$

Local equilibrium, non-dissipative terms

Dissipative terms

Incidentally: global thermodynamic equilibrium

$$\widehat{\rho} = \frac{1}{Z} \exp \left[- \int_\Sigma \mathrm{d}\Sigma_\mu \, \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Independent of the 3D hypersurface Σ if

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \qquad \partial_\mu \zeta = 0$$

Killing equation

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

The density operator becomes

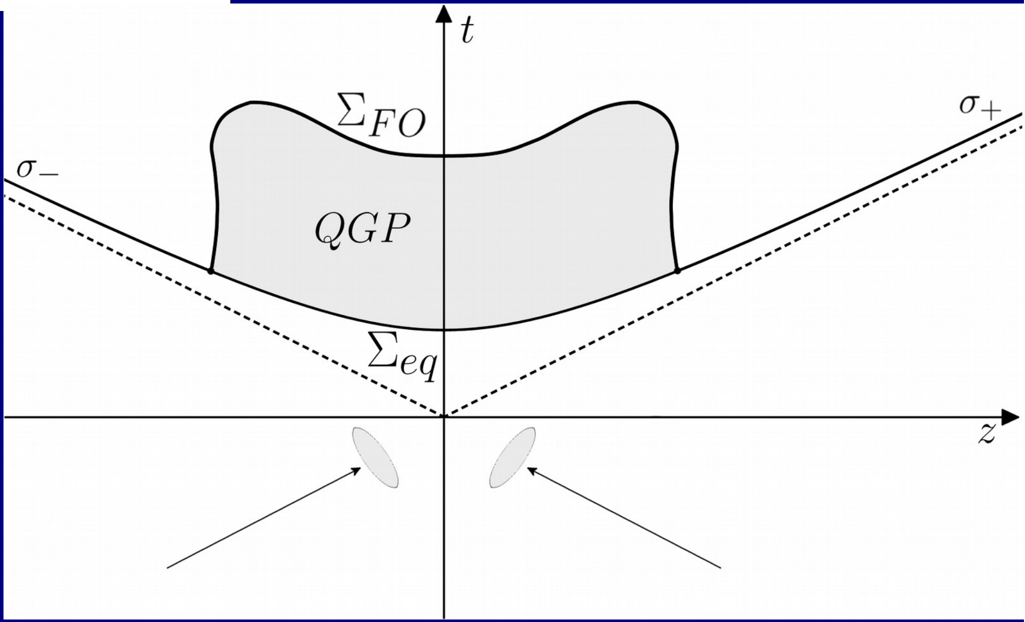
$$\widehat{\rho} = \frac{1}{Z} \exp \left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

Local thermodynamic equilibrium approximation

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp \left[- \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \, \widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right]$$

$$= \frac{1}{Z} \exp \left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \, \widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right]$$

Corresponding to the ideal fluid: Neglecting dissipative term in the exponent of the density operator



$$W(x,k) \simeq W(x,k)_{\rm LE} = {\rm Tr}(\widehat{\rho}_{\rm LE} \widehat{W}(x,k))$$

Mean value of a local operator: Taylor expansion

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}_B^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y)\beta_\nu(x) = \beta_\nu(x)\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y) = \beta_\nu(x)\widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp\left[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}_x^{\mu\nu} - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}_x^{\mu\nu} + \ldots\right]$$

$$\widehat{J}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}_x^{\mu\nu} - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}_x^{\mu\nu} + \ldots]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Linear response theory

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z \; \mathrm{e}^{z(\widehat{A}+\widehat{B})}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}} \simeq \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z \; \mathrm{e}^{z\widehat{A}}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}}$$

$$\widehat{A} = -\beta_\mu(x)\widehat{P}^\mu$$

$$\widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$W(x,k) \simeq \frac{1}{Z}\mathrm{Tr}(\mathrm{e}^{\widehat{A}+\widehat{B}}\widehat{W}(x,k)) \simeq \ldots$$

CORRELATORS

$$\langle \widehat{Q}^{\mu\nu}_x \widehat{W}(x,p) \rangle \qquad \langle \widehat{J}^{\mu\nu}_x \widehat{W}(x,p) \rangle$$

Spin mean vector at leading order in thermal vorticity

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

Neglected by "prejudice" because of the formula below...

$$S^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\beta\gamma\delta}p_\delta \frac{\int {\rm d}\Sigma_\lambda p^\lambda {\rm tr}_4(\Sigma_{\beta\gamma}W_+(x,p))}{\int {\rm d}\Sigma_\lambda p^\lambda {\rm tr}_4 W_+(x,p)}$$

+ Linear response theory

$$n_F = ({\rm e}^{\beta\cdot p - \xi} + 1)^{-1}$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma {\rm d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma {\rm d}\Sigma_\tau p^\tau\, n_F}$$

See also

R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904
W. Florkowski, A. Kumar and R. Ryblewski, Phys. Rev. C 98 (2018) 044906
Y. C. Liu, L. L. Gao, K. Mameda and X. G. Huang, Phys. Rev. D 99 (2019) 085014
N. Weickgenannt, X. L. Sheng, E. Speranza, Q. Wang and D. H. Rischke, Phys. Rev. D 100 (2019) 056018

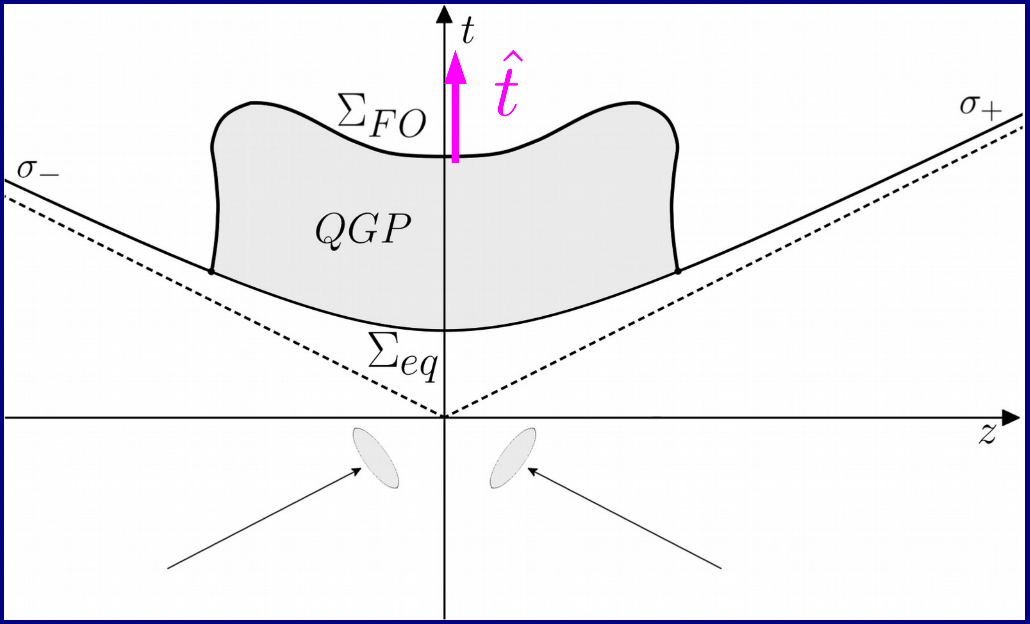
Surprise: thermal shear does contribute!

F. B., M. Buzzegoli, A. Palermo, arXiv:2103.10917

Including the neglected term and applying linear response theory, another contribution is found out

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F},$$

Same (not precisely the same) formula obtained by Liu and Yin with a different method: S. Liu, Y. Yin, arXiv:2103.09200



Dependence on a specific vector is not surprising as this term arises from the correlator

$$\langle \widehat{Q}^{\mu\nu}_x \widehat{W}(x,p)\rangle$$

But Q is not a tensor and, unlike J, it does depend on the hypersurface

Why do we have a dependence on Σ ?

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{\mu\nu}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{\mu\nu}$ does not vanish, therefore it does depend on the integration hypersurface and

$$\widehat{\Lambda} \widehat{Q}^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{Q}^{\alpha\beta}$$

What is this new term?

Does it have a non-relativistic limit?
Let us decompose it

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

All terms are relativistic (they vanish in the infinite *c* limit) EXCEPT grad T terms, which give rise to:

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v} \times \frac{\int \mathrm{d}^3\mathrm{x}\, n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\, n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Application to relativistic heavy ion collisions

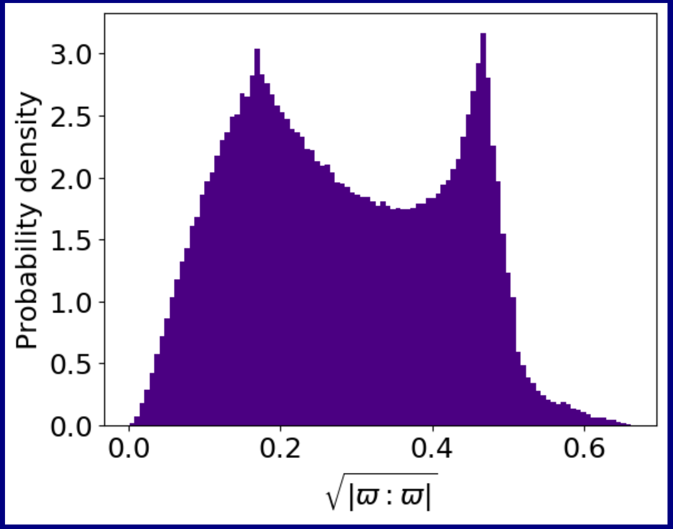
F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko, arXiv:2103.14621

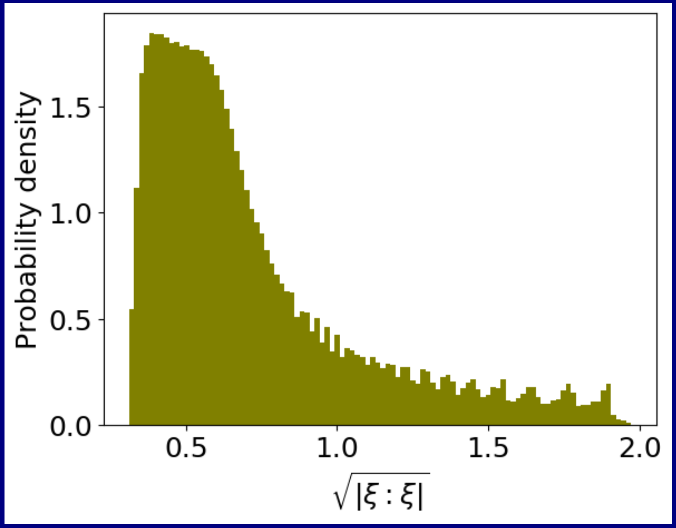
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau} \frac{p_\tau p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1-n_F)\hat{t}_\rho \xi_{\sigma\lambda}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

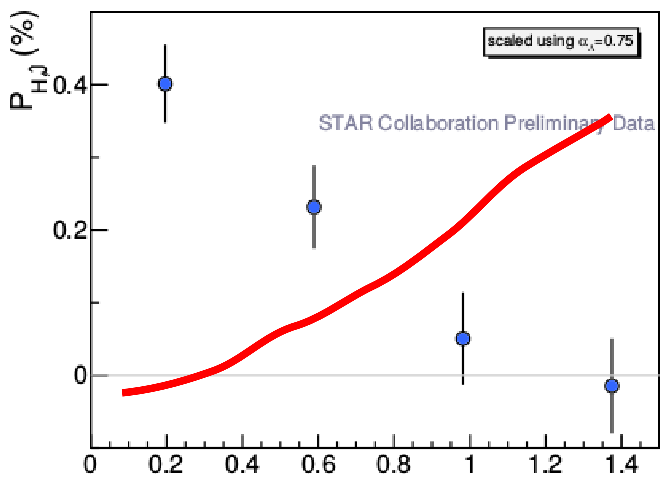
Is linear response theory adequate?



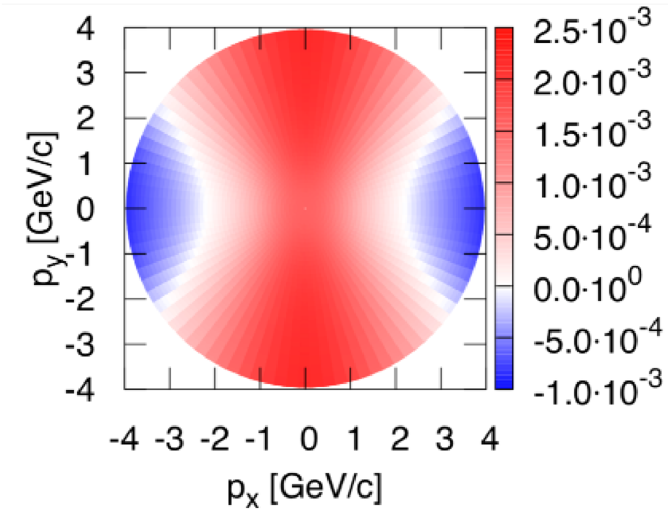


Puzzles: momentum dependence of polarization

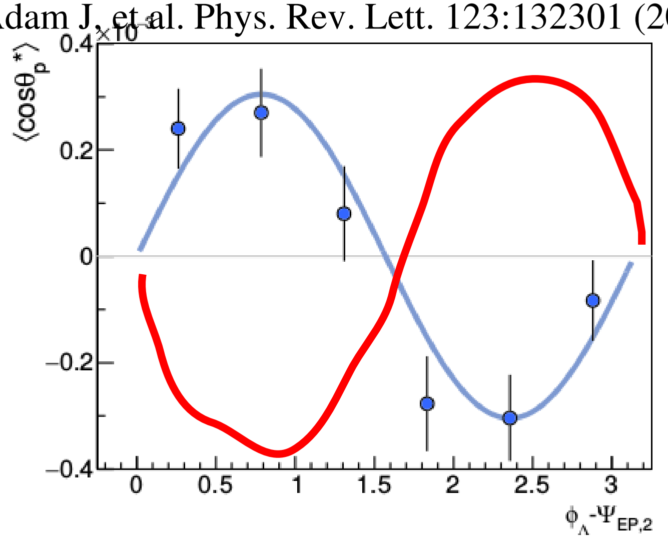
Theory prediction

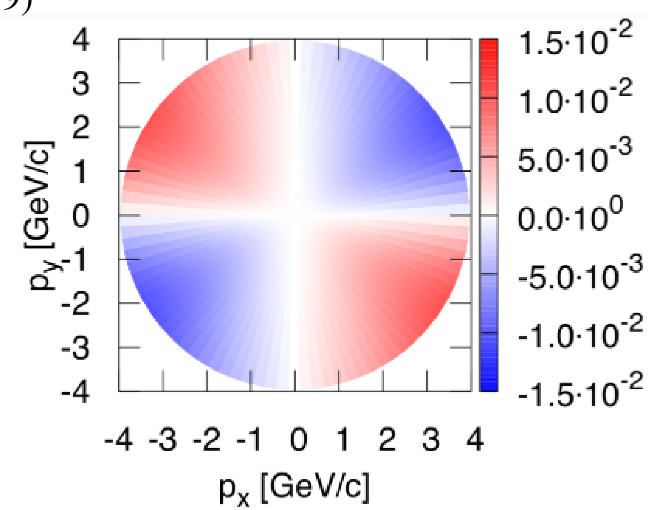


Niida T. Nucl. Phys. A982:511 (2019)

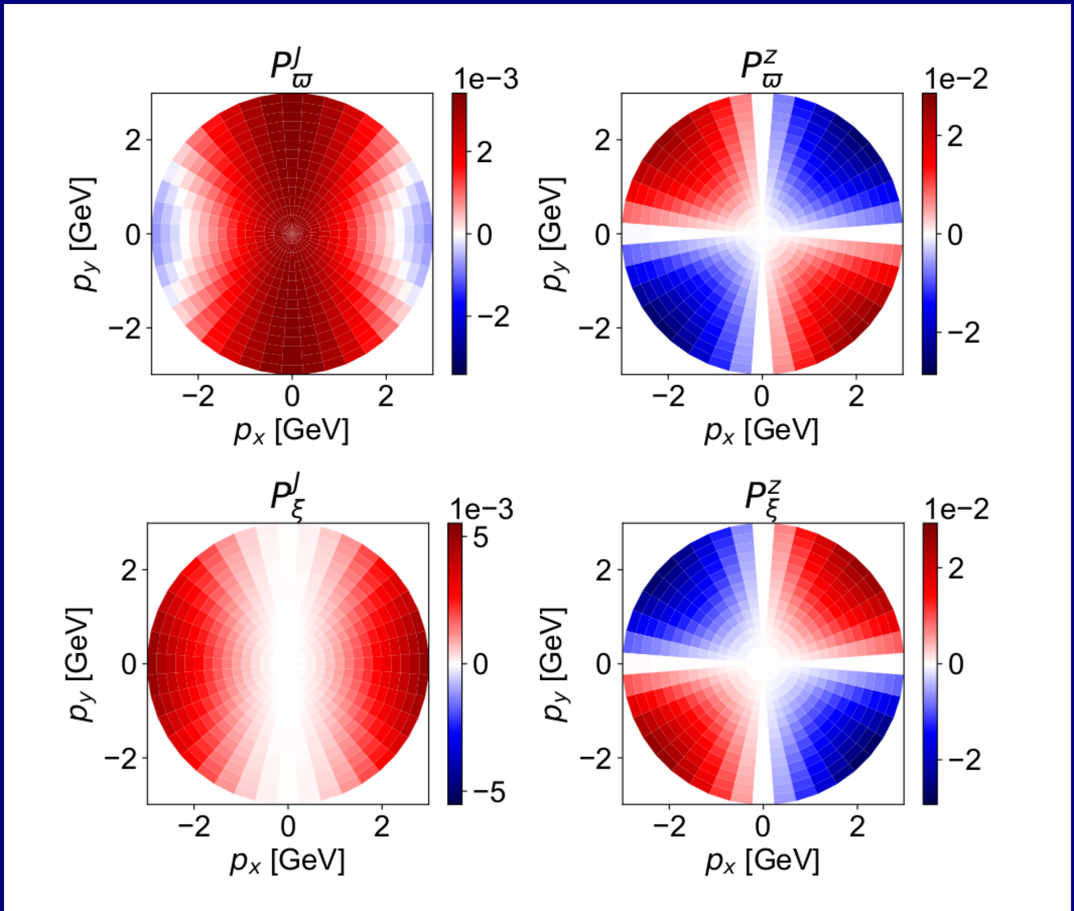


Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)



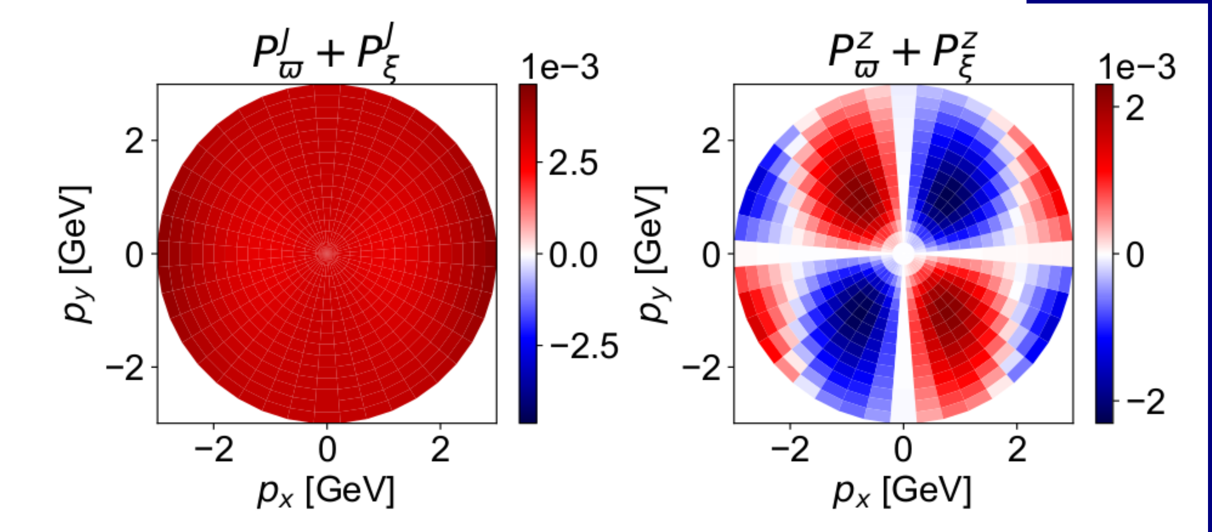


New calculations



Based on the hydrodynamic code VHLLE (author I. Karpenko) tuned to reproduce Au-Au momentum spectra at RHIC top energy.
Similar output with ECHO-QGP (main author G. Inghirami).

Right pattern!



Not sufficient to restore the agreement between data and model

Calculations fully consistent with:
B. Fu, S. Liu, L. Pang, H. Song and Y. Yin, arXiv:2103.10403.

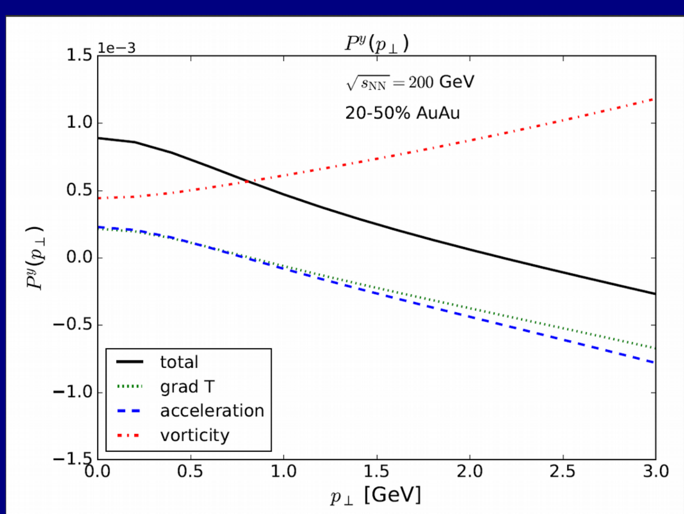
Isothermal local equilibrium

The most appropriate setting for relativistic heavy ion collisions at very high energy!

Both thermal shear and thermal vorticity include temperature gradients

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \qquad \xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu) \qquad \beta^\mu = (1/T)u^\mu$$

Thermal gradients do contribute to the polarization



$$\mathbf{S}^* \propto \frac{\hbar}{KT^2}\mathbf{u} \times \nabla T + \frac{\hbar}{KT}(\boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{v}\mathbf{u}/c^2) + \frac{\hbar}{KT}\mathbf{A} \times \mathbf{u}/c^2$$

I. Karpenko, Nucl. Phys. A 982 (2019), 519-522, talk given at QM 2018

Is it the best thing to do?

The formulae we have derived are based on a Taylor expansion of the density operator

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

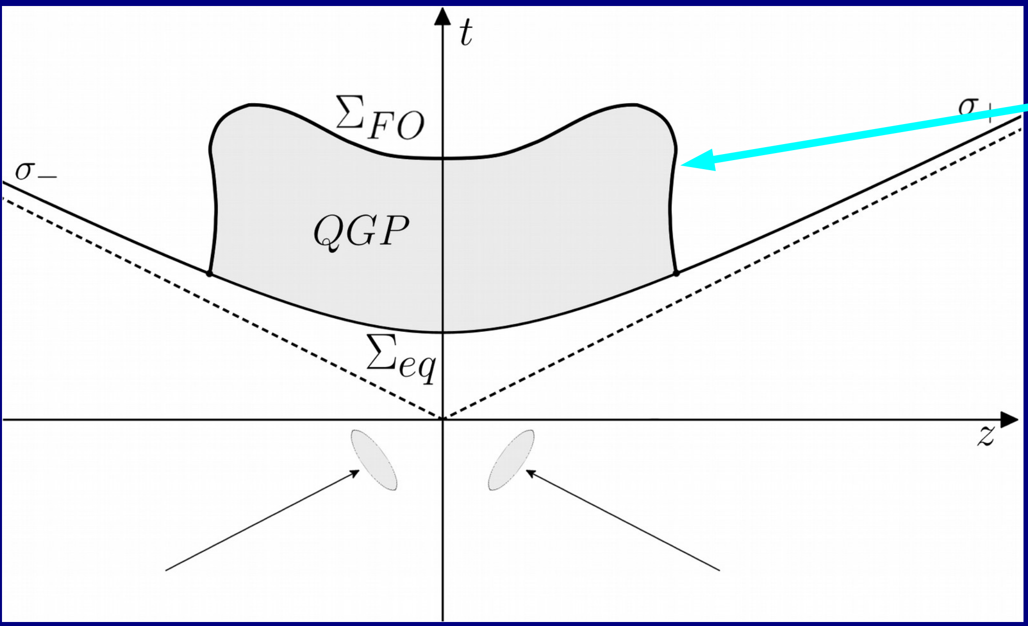
$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \ldots]$$

This is generally correct, but it is an approximation after all.

Can we find a better approximation for our special case?

Isothermal hadronization



At high energy, Σ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T) u^\mu$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \, \widehat{T}^{\mu\nu} \beta_\nu \right] = \frac{1}{Z} \exp\left[- \frac{1}{T} \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \, \widehat{T}^{\mu\nu} u_\nu \right]$$

NOW u (and just u) can be expanded!

$$u_\nu(y) = u_\nu(x) + \partial_\lambda u_\nu(x) (y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2T}(\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x)) \widehat{J}^{\mu\nu}_x - \frac{1}{2T}(\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x)) \widehat{Q}^{\mu\nu}_x + \ldots]$$

Spin mean vector at leading order with isothermal local equilibrium (ILE)

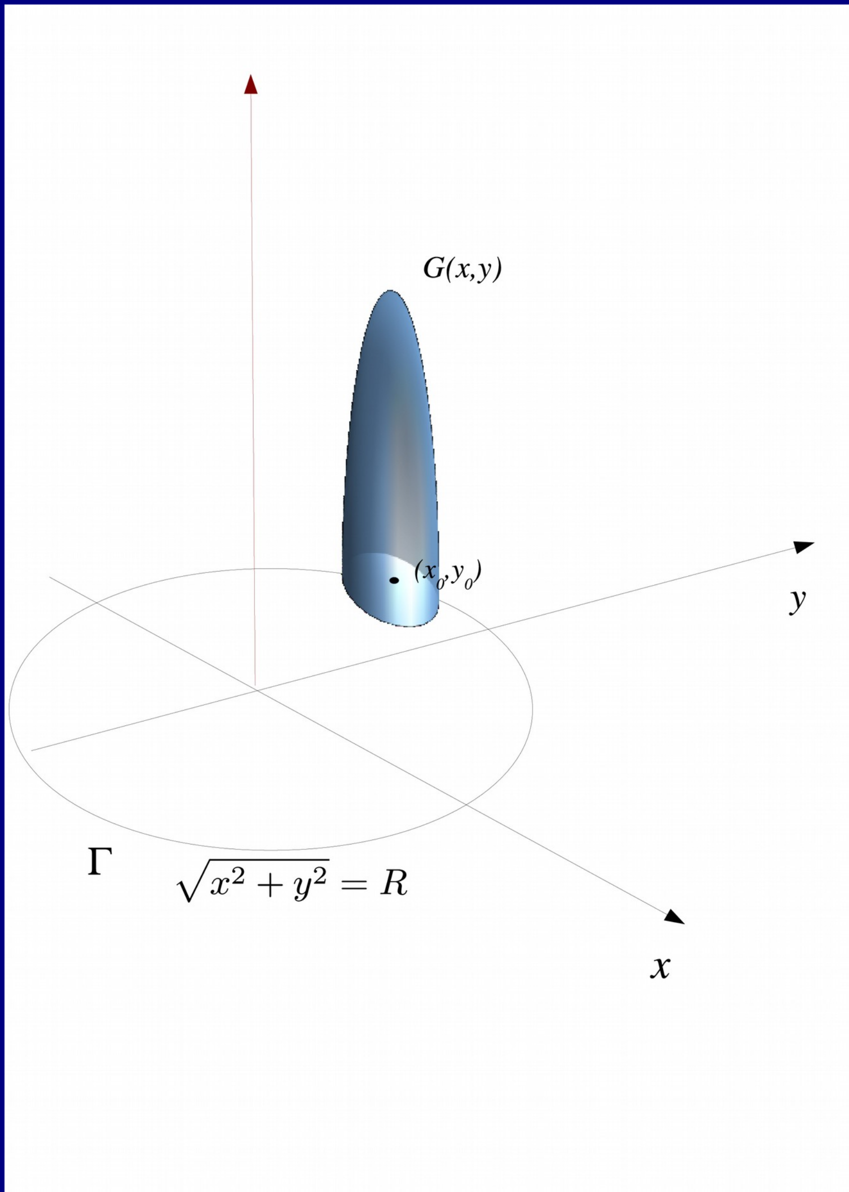
Readily found by replacing the gradients of β with those of *u*

$$S^\mu_{\rm ILE}(p) = \qquad (
- \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8 m T_{\rm dec} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$

$$\omega_{\rho\sigma} = \frac{1}{2} \left(\partial_\sigma u_\rho - \partial_\rho u_\sigma \right)$$

$$\Xi_{\rho\sigma} = \frac{1}{2} \left(\partial_\sigma u_\rho + \partial_\rho u_\sigma \right)$$

Understand the point: a simple example



Task: approximate the integral

$$W = \int_\Gamma \mathrm{e}^{\sqrt{x^2+y^2}} G(x,y)ds$$

where *G(x,y)* is a peaked function around the point (x_0,y_0) on the circle.

Since G is peaked, one can Taylor expand the exponent about (x_0,y_0)

$$\begin{aligned} W &\simeq \mathrm{e}^{\sqrt{x_0^2+y_0^2}} \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{\nabla r|_{(x_0,y_0)}\cdot(\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s \end{aligned}$$

But it is just pointless if we integrate over the circle!

$$W = \mathrm{e}^R \int_\Gamma G(x,y)\mathrm{d}s$$

In the previous example, the Taylor expansion at first order introduces an undesired term:

$$\mathrm{W} = \mathrm{e}^{R} \int_\Gamma G(x,y)\mathrm{d}s \qquad\qquad W \simeq \mathrm{e}^{R} \int_\Gamma \mathrm{e}^{\nabla r|_{(x_0,y_0)}\cdot(\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s$$

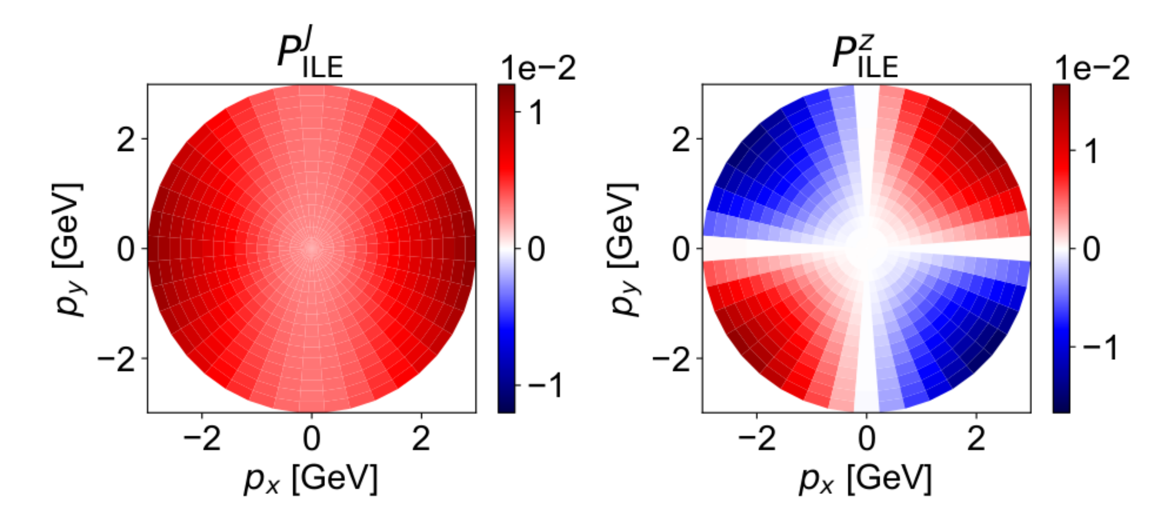
exact — With gradient of r expansion

which is proportional to the gradient of the constant quantity on the circle, perpendicular to the integration line. This term does not vanish in the integration!

Similarly, for an isothermal hadronization, the inclusion of temperature gradients results in an additional, undesirable contribution proportional to the gradient of *T*, perpendicular to Σ_{FO} :

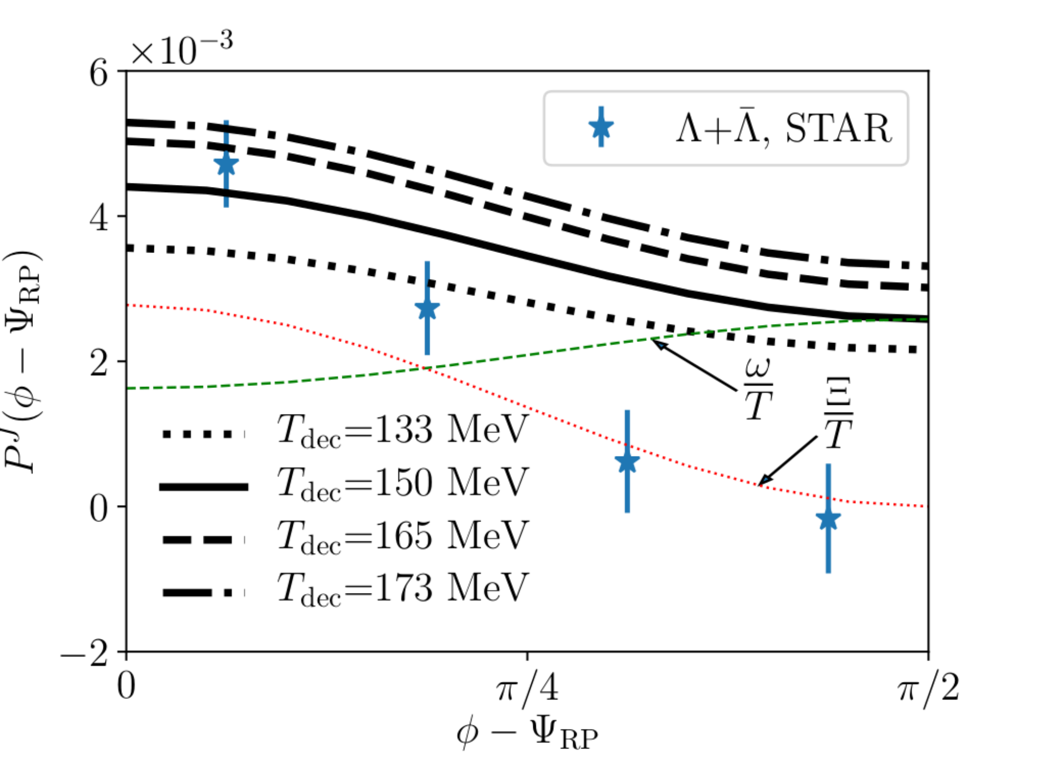
$$\frac{1}{2}[(\partial_\mu T)u_\nu(x) - (\partial_\nu T)u_\mu(x)]\widehat{J}_x^{\mu\nu} + \frac{1}{2}[(\partial_\mu T)u_\nu(x) + (\partial_\nu T)u_\mu(x)]\widehat{Q}_x^{\mu\nu}$$

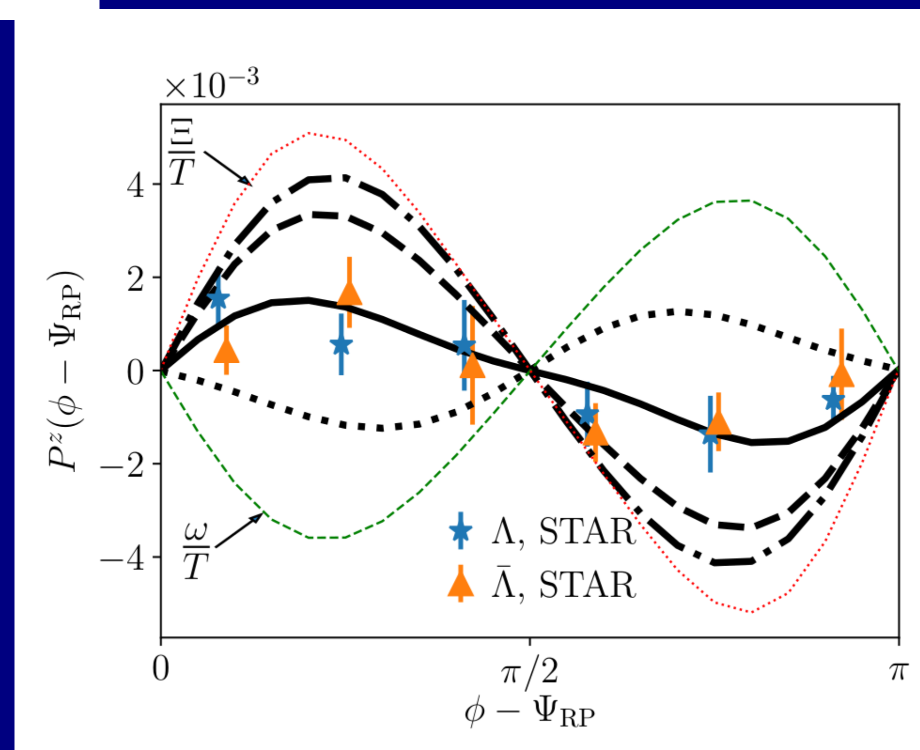
Isothermal local equilibrium: results



Apply the new formula (for primary hadrons)

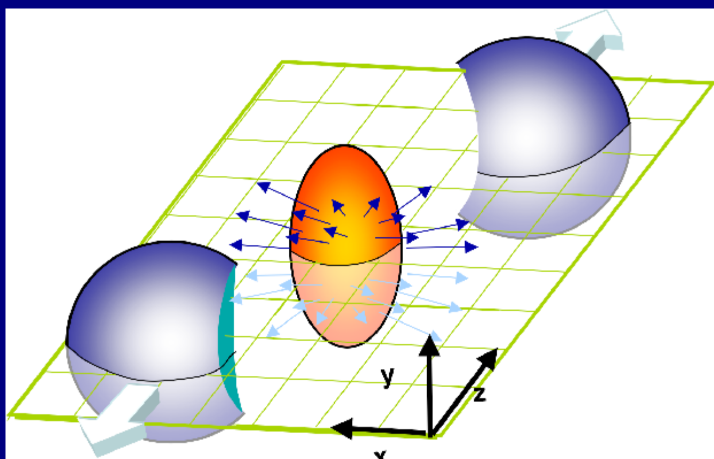
$$S^\mu_{\rm ILE}(p) = - \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F(1-n_F) \left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}\right]}{8 m T_{\rm dec} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$



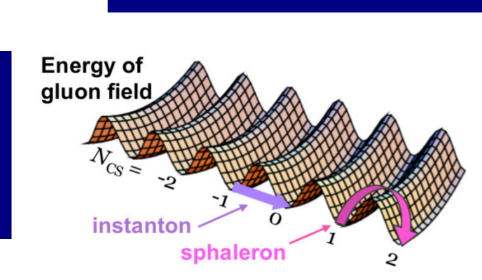


Polarization and local parity violation

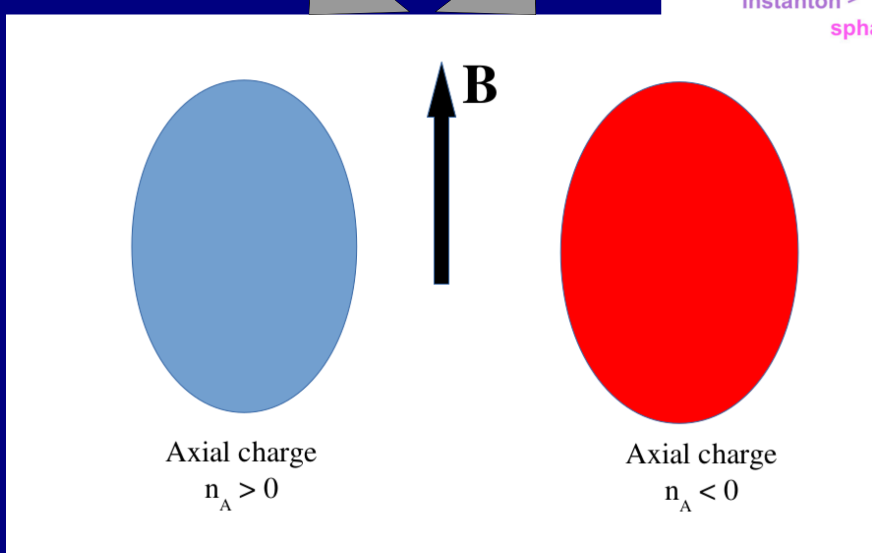
F.B., M. Buzzegoli, A. Palermo, G. Prokhorov 2009.13449



A peripheral collision is a system invariant by reflection. However, there might be event-by-event ("local") parity breaking due to non-perturbative QCD topological transition induced by high T



Axial imbalance = parity violation long sought in relativistic heavy ion collisions through the Chiral Magnetic Effect



$$\mathbf{j} = \frac{\mu_A}{2\pi}\mathbf{B}$$

Local parity violation has become a synonimous of CME

Investigating local parity violation with spin without the mediation of the EM field

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{eq}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A \widehat{j}^\mu_A \right) \right]$$

Leading order with linear response theory

$$S^\mu(p) = S^\mu_\chi(p) + S^\mu_\varpi(p)$$

$$S^\mu_\chi(p) \simeq \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p \, \zeta_\mathrm{A} n_\mathrm{F} (1 - n_\mathrm{F})}{\int_\Sigma \mathrm{d}\Sigma \cdot p \, n_\mathrm{F}} \frac{\varepsilon p^\mu - m^2 \hat{t}^\mu}{m\varepsilon} \leftarrow \text{Axial imbalance}$$

$$S^\mu_\varpi(p) = \frac{1}{8m} \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F (1 - n_F) \partial_\rho \beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F} \leftarrow \text{Thermal vorticity}$$

$$\zeta_A = \frac{\mu_A}{T}$$

g_h

axial charge of hadron h

ζ_A changes sign event by event

Average over multiple events

$$\langle\langle S^\mu(p) \rangle\rangle = \cancel{\langle\langle S^\mu_\chi(p) \rangle\rangle} + \langle\langle S^\mu_\varpi(p) \rangle\rangle$$

$$\langle\langle \zeta_\mathrm{A} \rangle\rangle = 0 \qquad \langle\langle \zeta_\mathrm{A}^2 \rangle\rangle \neq 0$$

Axial contribution modifies helicity pattern

In the rest frame of the hadron

$$\mathbf{S}_{0,\chi} = \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p \, \zeta_A n_\mathrm{F} (1 - n_\mathrm{F})}{\int_\Sigma \mathrm{d}\Sigma \cdot p \, n_\mathrm{F}} \hat{\mathbf{p}} \equiv F_\chi(\mathbf{p}) \hat{\mathbf{p}}$$

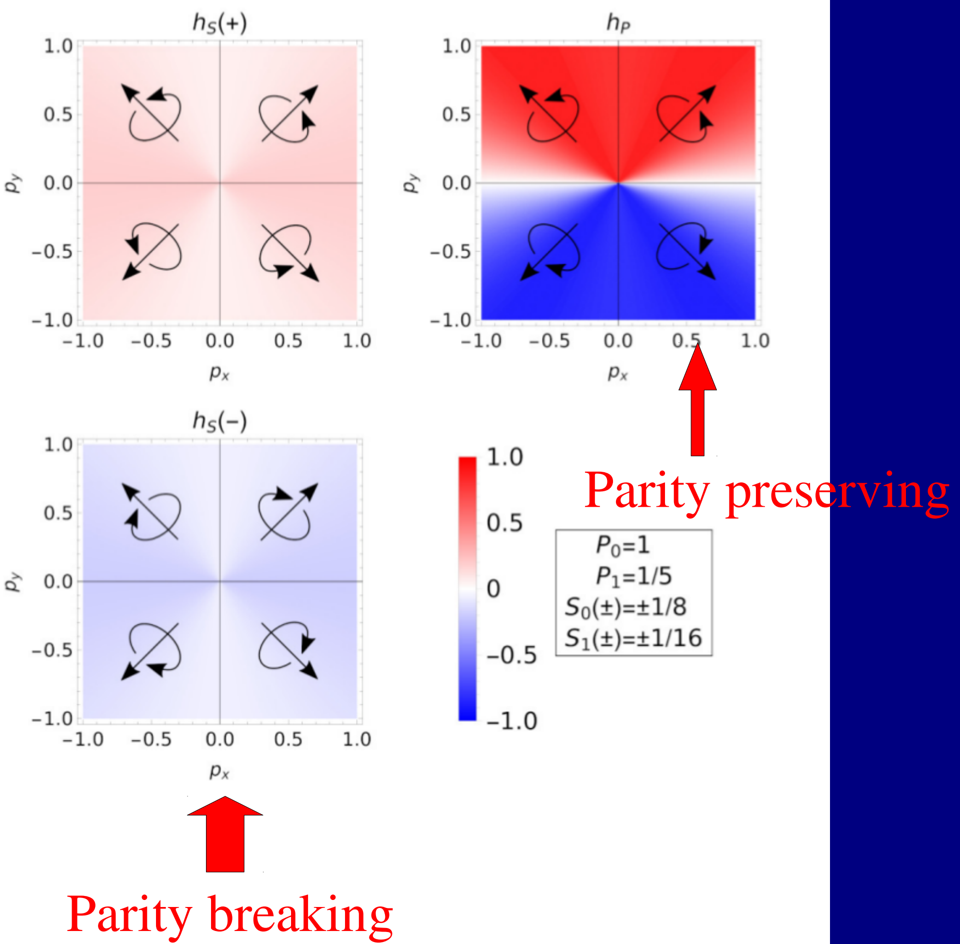
MODEL-INDEPENDENT ANALYSIS

$h = h_\mathrm{P} + h_\mathrm{S}$

From rotational symmetry $\phi \to \pi - \phi$ and reflection properties $\phi \to \pi + \phi$:

$$h_P(p_T, \phi) = \sum_k P_k(p_T) \sin[(2k+1)\phi]$$

$$h_S(p_T, \phi) = \sum_k S_k(p_T) \cos[2k\phi]$$

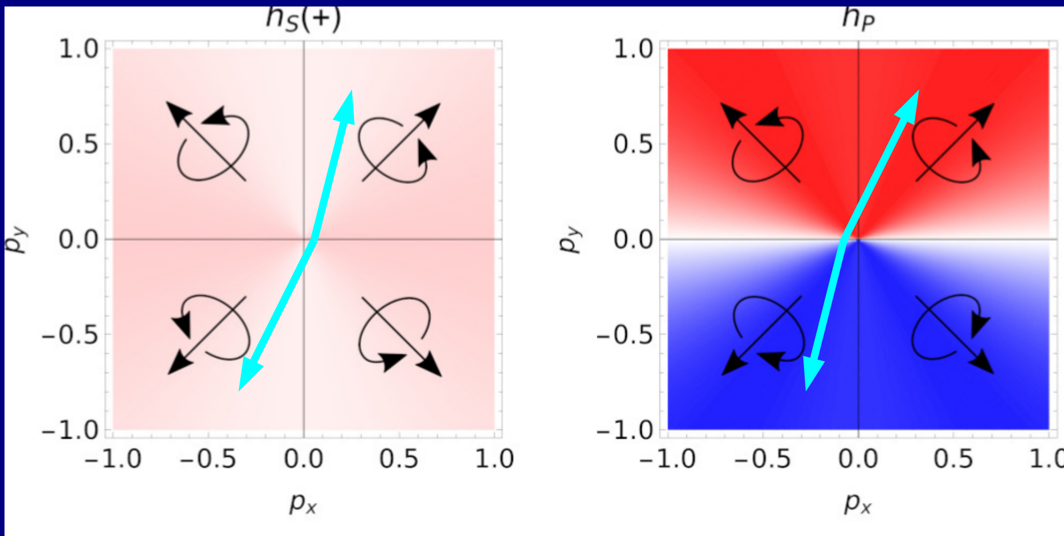


Local parity violation $S_k(p_T) \neq 0$

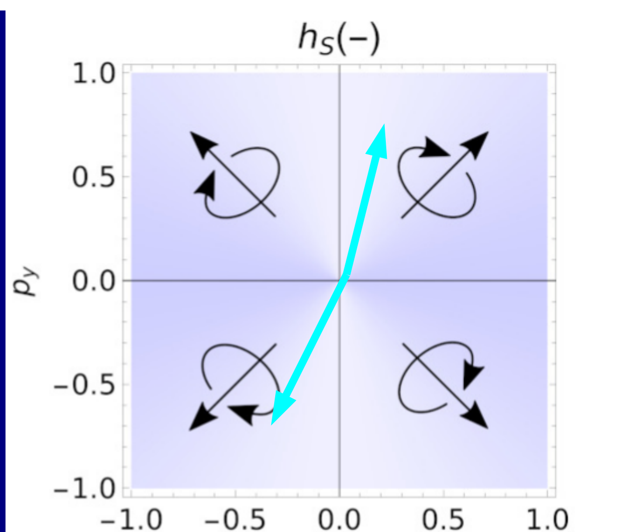
Global parity conservation $\langle\langle S_k(p_T) \rangle\rangle = 0$

Azimuthal analysis of helicity and helicity correlations

Helicity can be measured by projecting the p momentum in the Λ rest frame onto the momentum of the Λ in the QGP frame

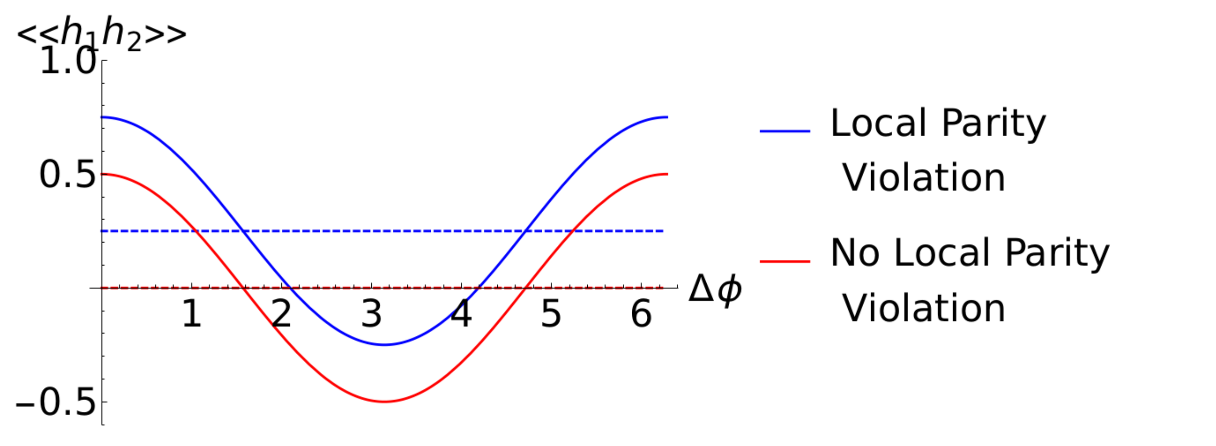


Studying the helicity of hyperons with large angle difference in the same event can reveal the local parity violation



Helicity-helicity azimuthal 2P correlation

$$\langle h_1 h_2(\Delta\phi)\rangle \simeq \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \left(\bar{S}_0^2 + \bar{P}_0^2 \sin^2\phi \cos\Delta\phi\right) = \bar{S}_0^2 + \frac{1}{2}\bar{P}_0^2 \cos\Delta\phi$$



Summary and outlook

- Quantum statistical mechanics is an essential tool to properly deal with spin physics in relativistic fluids
- Local thermodynamic equilibrium: new unexpected term relating spin polarization with the symmetric gradient of four-temperature, independently found in a very similar study. Under further investigation.
- Local polarization puzzles can be solved by including this term in the numerical analysis along with the isothermal hadronization setting: the “best vorticity” or “best gradient” depends on the hadronization conditions.
- Important phenomenological consequences: local equilibrium (“ideal fluid” picture) seems to hold in the spin sector too
- Sensitivity to hadronization temperature: spin as a probe of the plasma formation and decay
- Spin as an independent probe of local parity violation in HIC