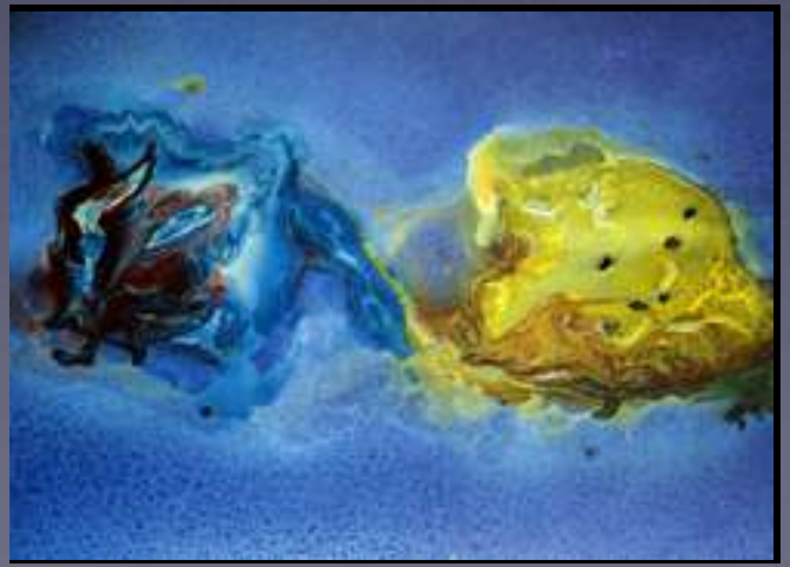
Non-relativistic Multi-scale Systems with Effective Quantum Field Theories

NORA BRAMBILLA



PHYSIK DEPARTMENT TUM
T30F

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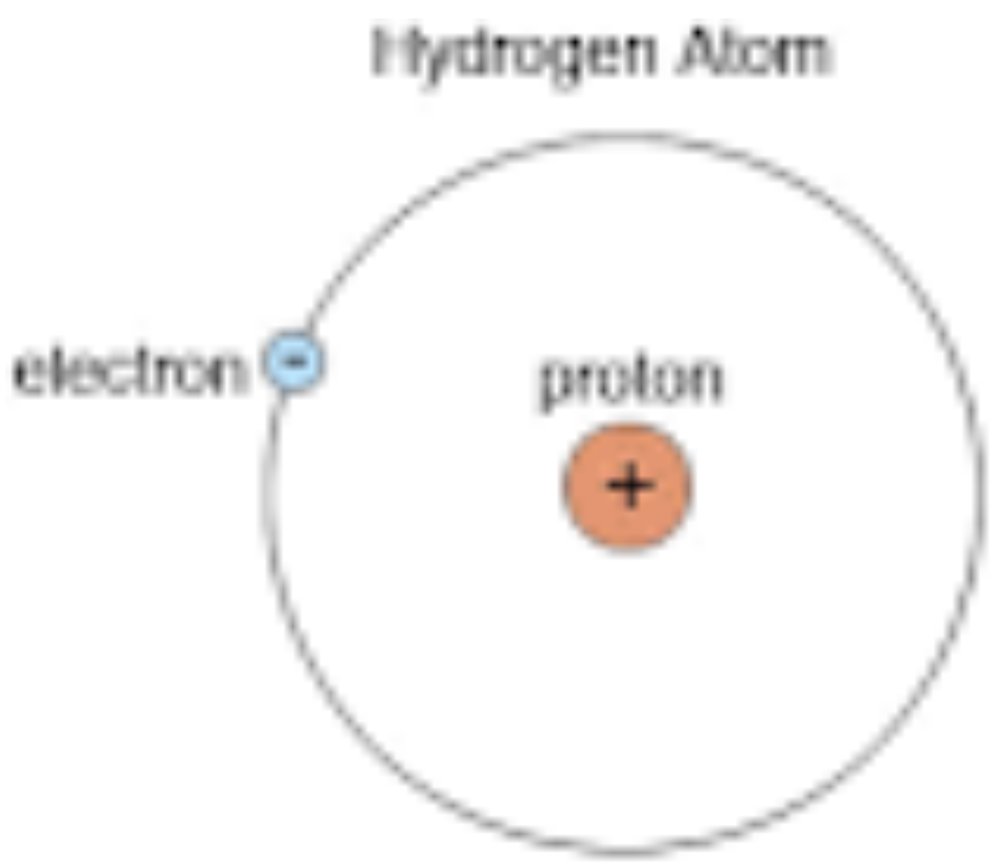
Exotics X Y Z

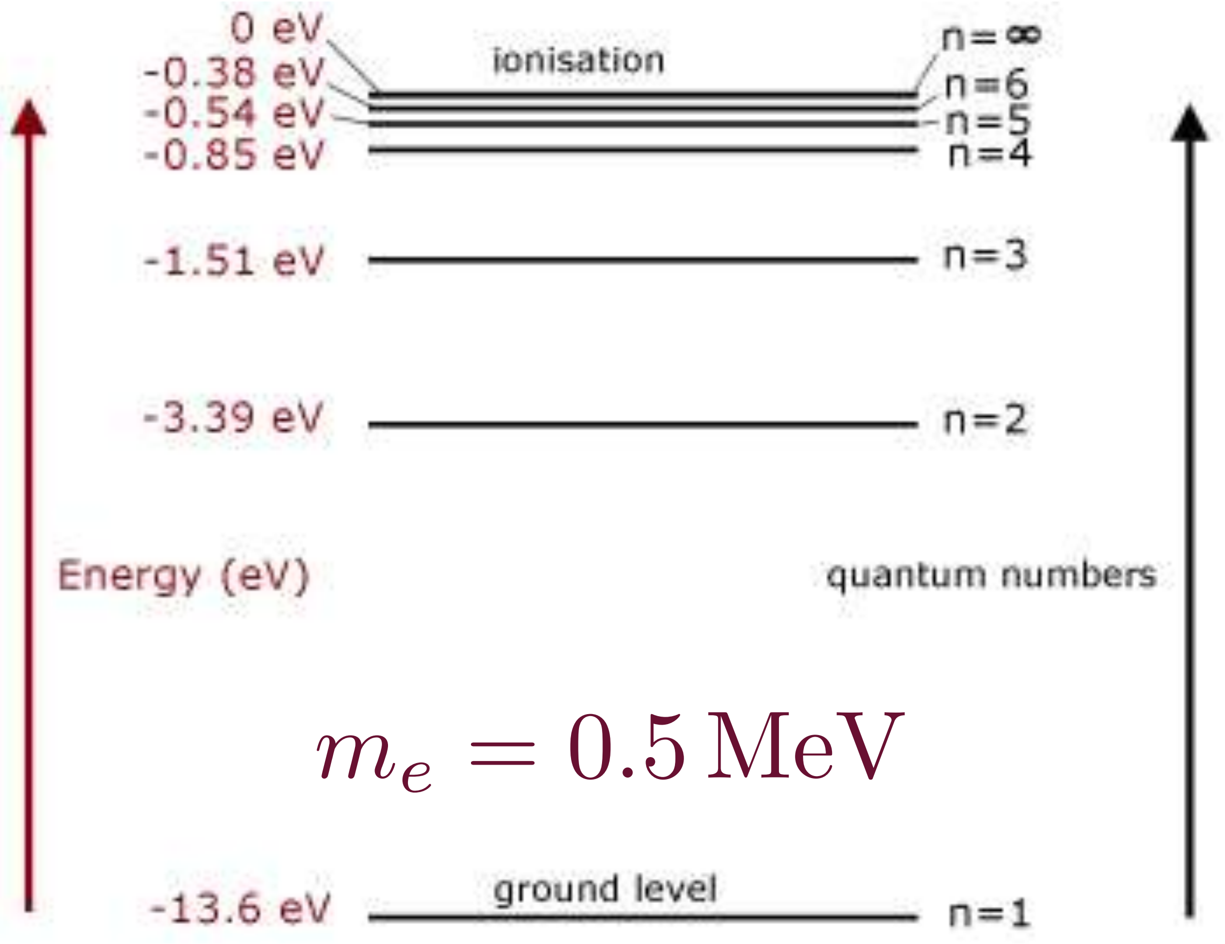
Quarkonium production

Nonequilibrium evolution of NR systems in a medium

(Quarkonium in QGP, Dark Matter in Early Universe)

Prototype of NR states: Hydrogen Atom





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for the hydrogen atom

$$v = \alpha = \frac{e^2}{4\pi}$$

fine structure constant

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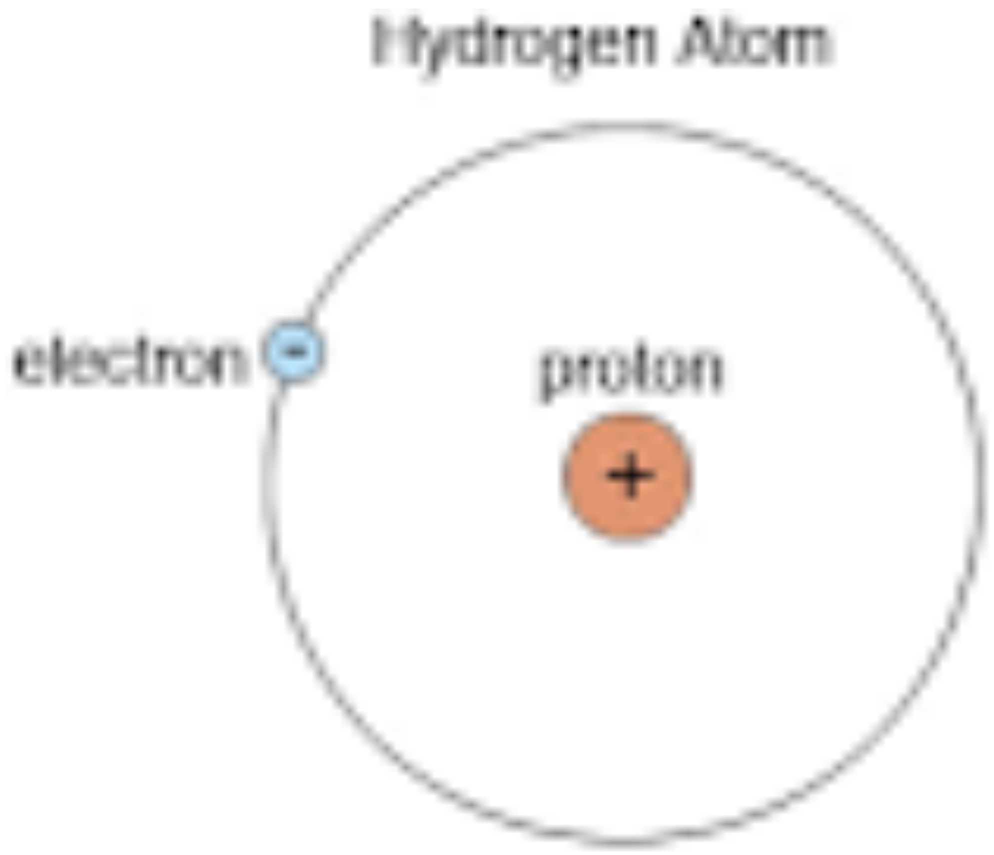

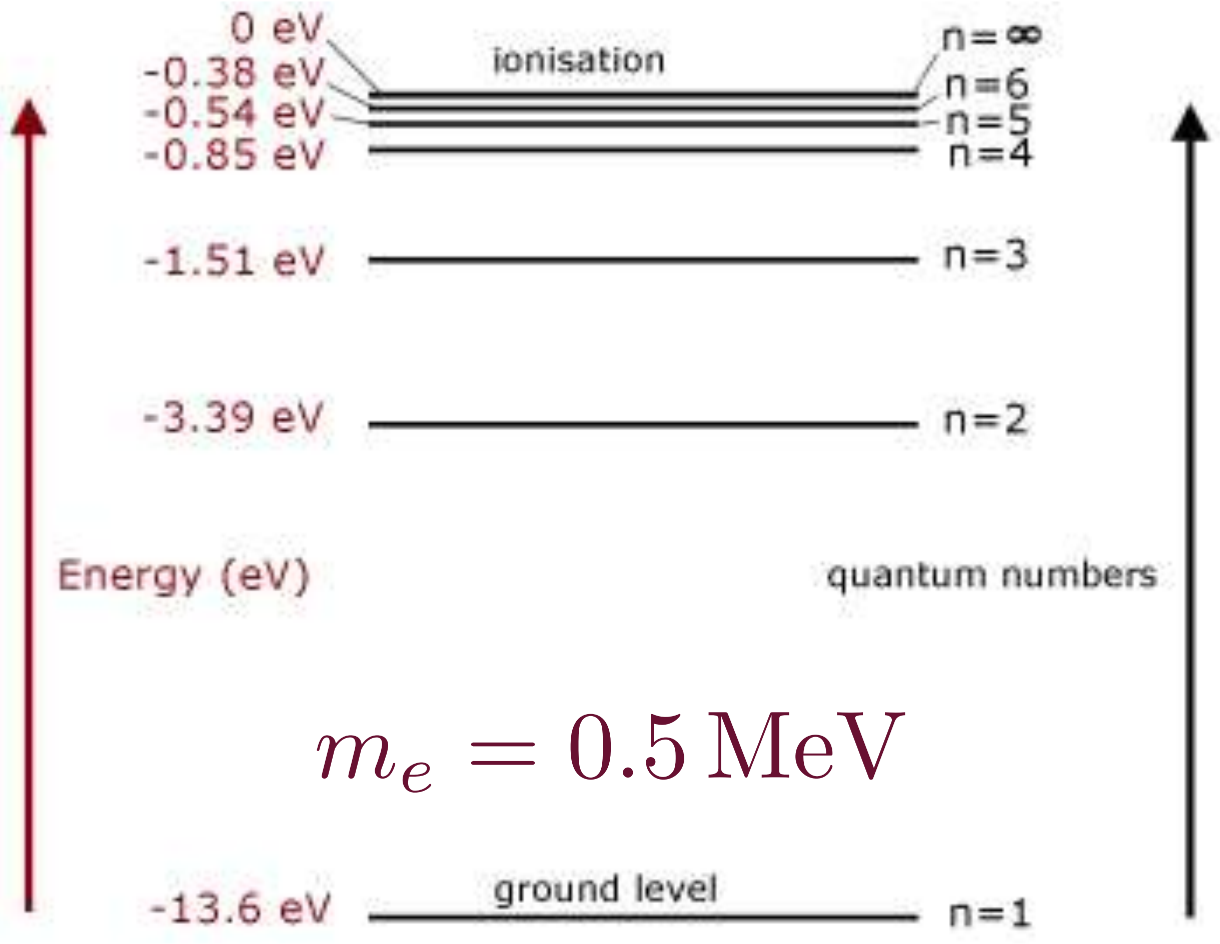

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The relevant scales of the non-relativistic bound state dynamics are

- $E \sim \frac{\mathbf{p}^2}{2m} \sim V \sim mv^2$,
- $p \sim 1/r \sim mv$;

a crucial observation: if v(elocity)$\ll 1$, then $m \gg mv \gg mv^2$.

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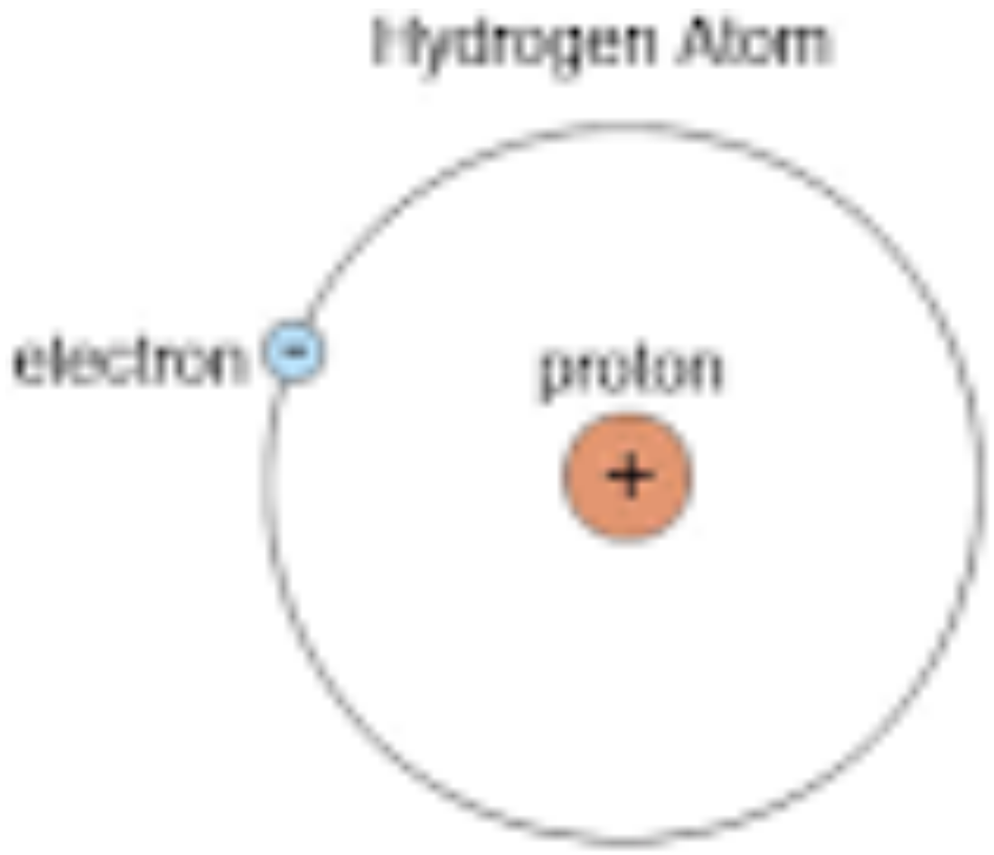

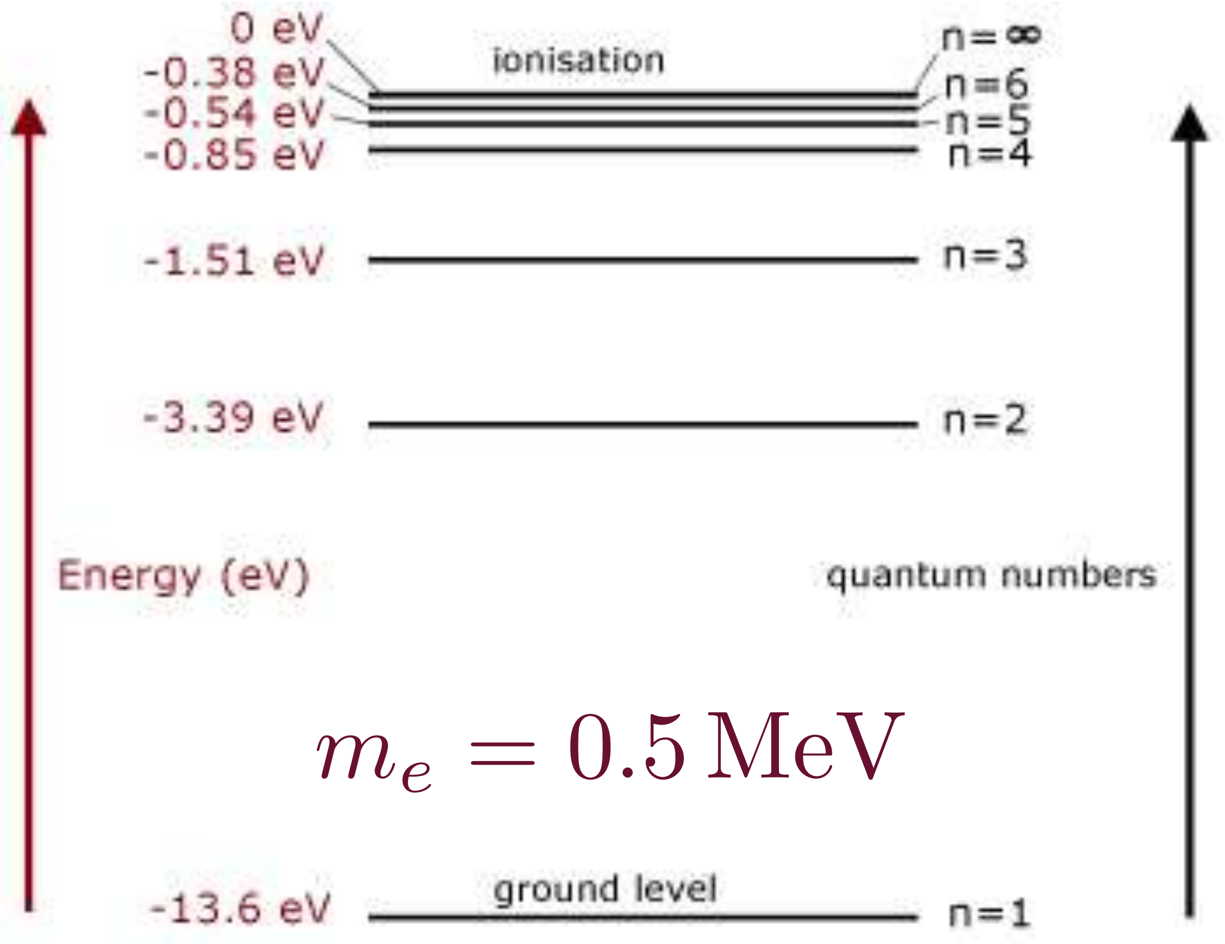

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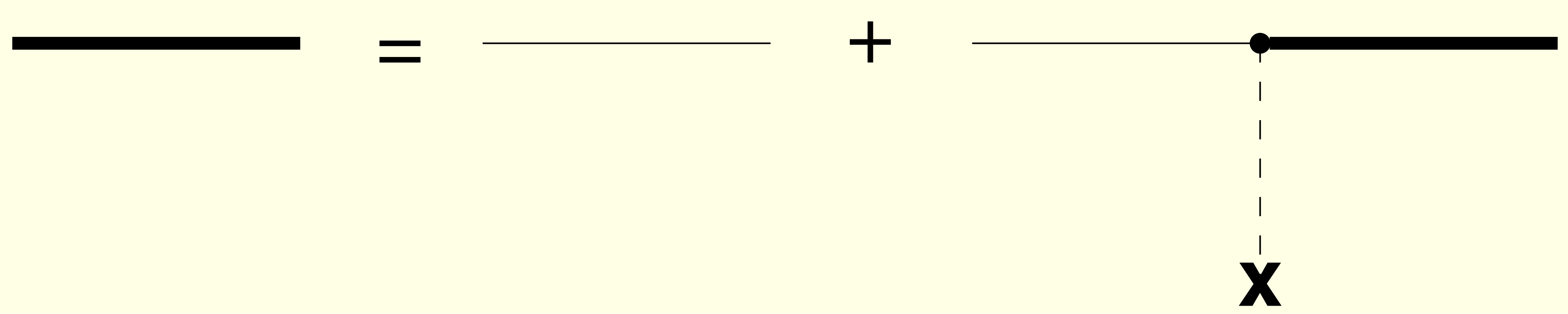
Non-relativistic (NR) bound states accompanied the history of the quantum theory from its inception to the establishing of the quantum theory of fields

Nonrelativistic Quantum Theory of bound states

- 1926 Schrödinger equation: $\left(\frac{\mathbf{p}^2}{2m} + V\right)\phi = E\phi$

$$\begin{cases} g = g_0 + g_0(-iV)g \\ g_0 = \frac{i}{E-\mathbf{p}^2/(2m)} \end{cases}$$



- 1927 Pauli equation: $\left(\frac{(\mathbf{p} - e\mathbf{A})^2}{2m} + V - \frac{\boldsymbol{\sigma}\cdot e\mathbf{B}}{2m}\right)\phi = E\phi$

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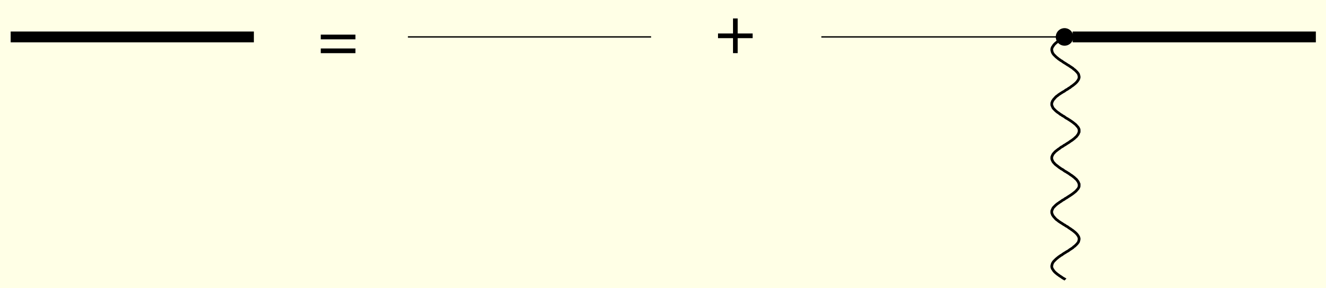

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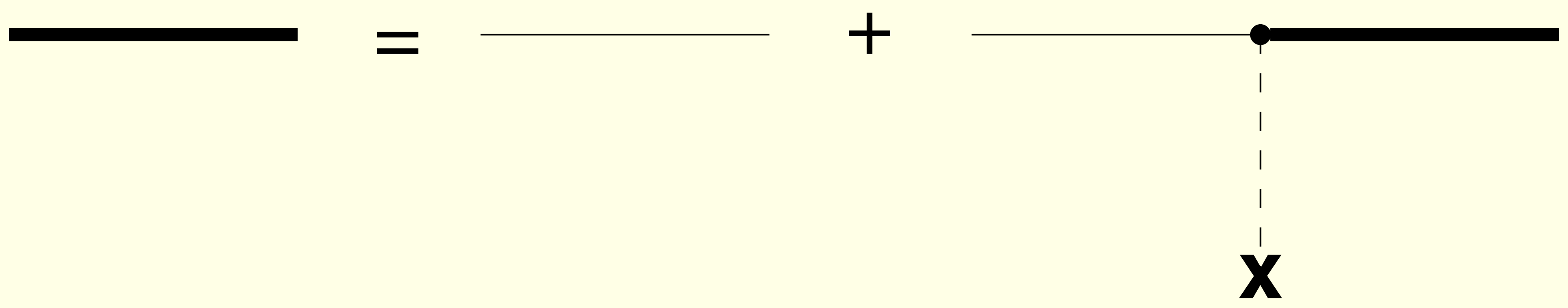

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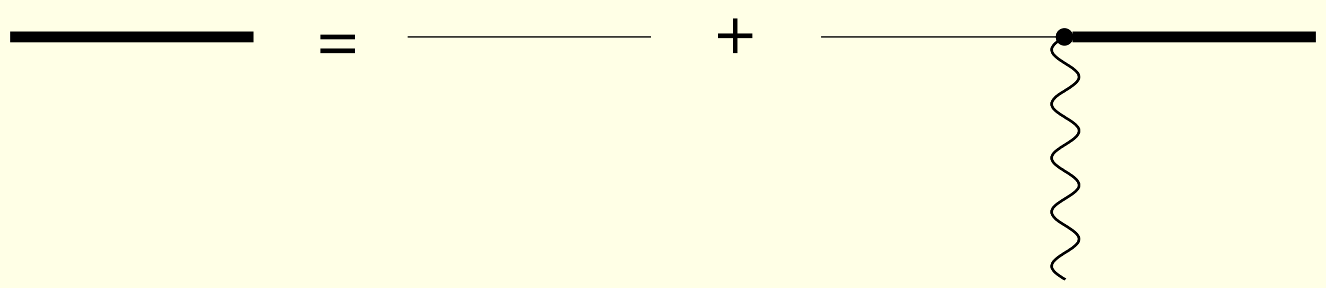

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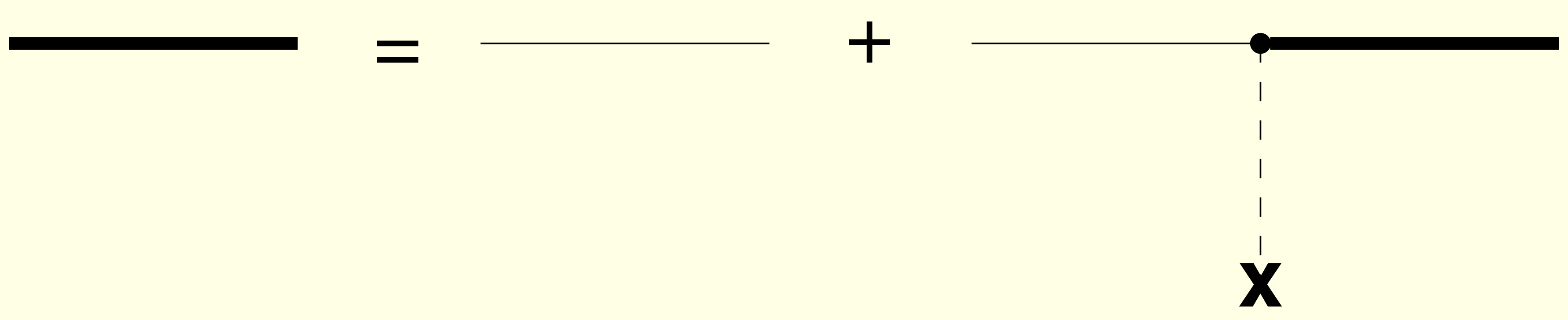

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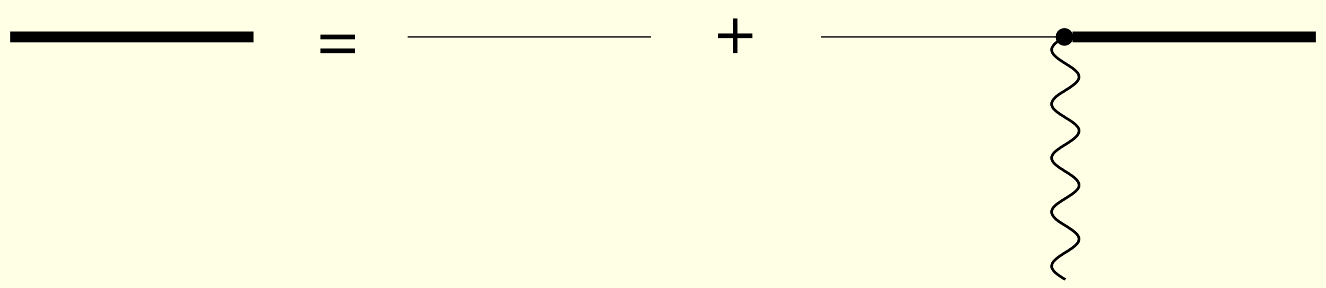

to address relativistic corrections in 1/m

it produces spin-corrections (spin-orbit), it does not give the Lamb shift (radiative corrections)

Relativistic Quantum Field Theory of bound states

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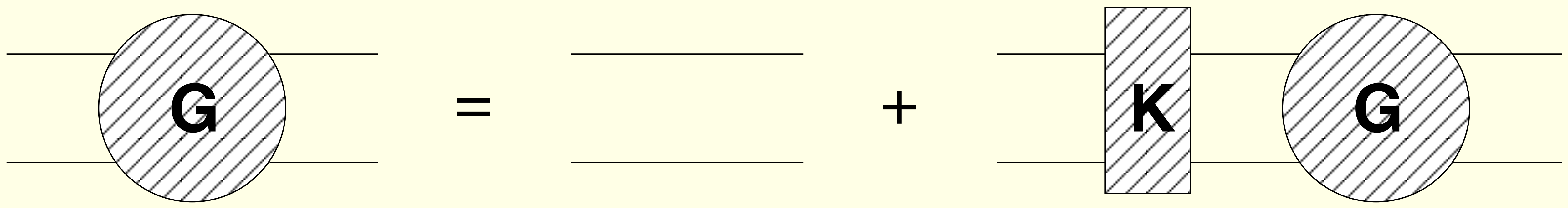

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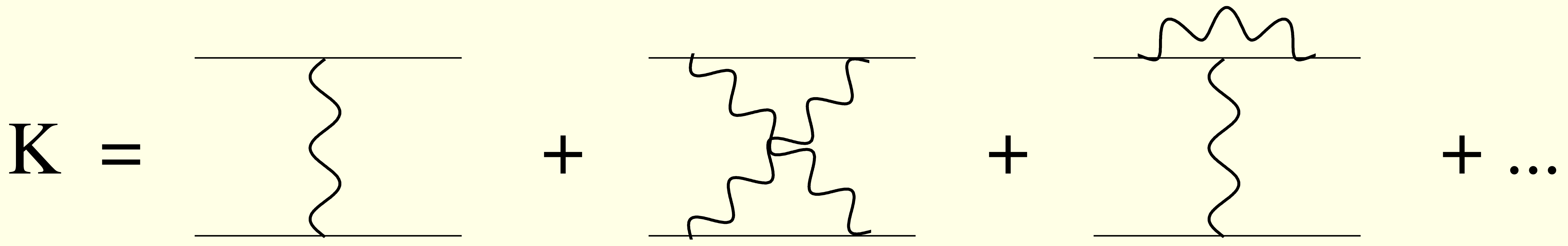

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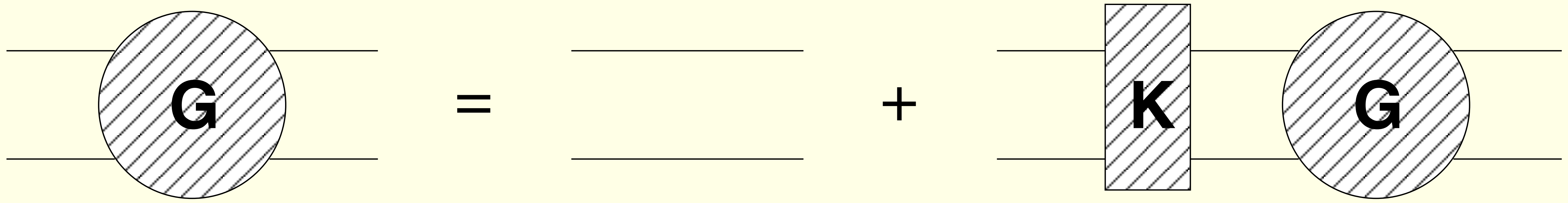

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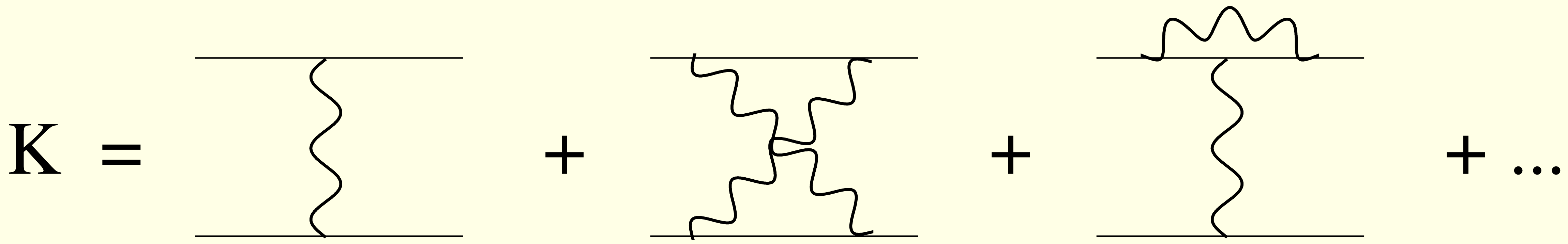

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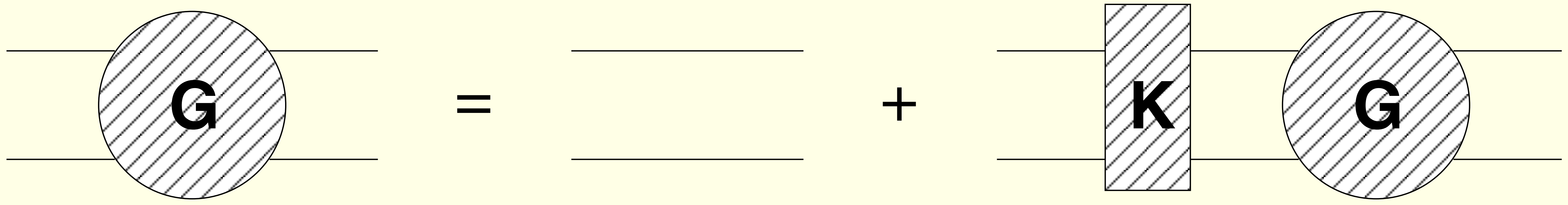

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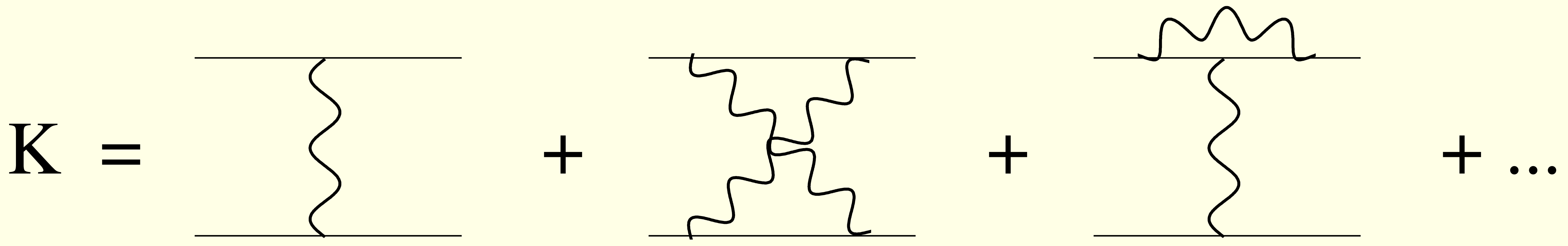

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Ex.

- It shows the difficulty of the approach the fact that going from the calculation of the $m\alpha^5$ correction in the hyperfine splitting of the positronium ground state to the $m\alpha^6 \ln \alpha$ term took twenty-five years!
  - Karplus Klein PR 87(52)848, Caswell Lepage PRA (20)(79)36 Bodwin Yennie PR 43(78)267

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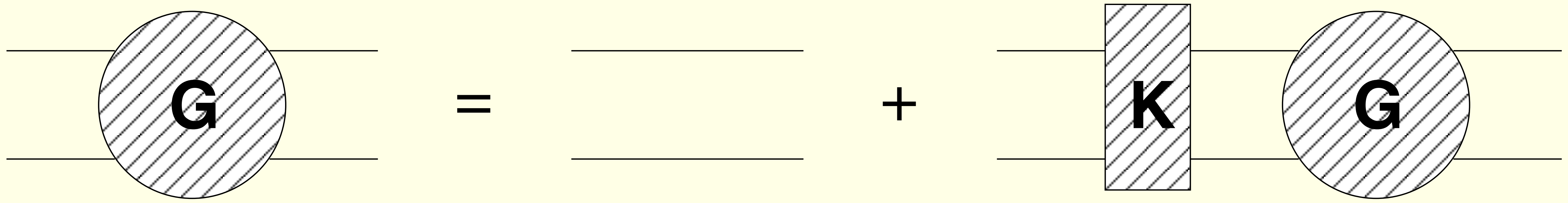

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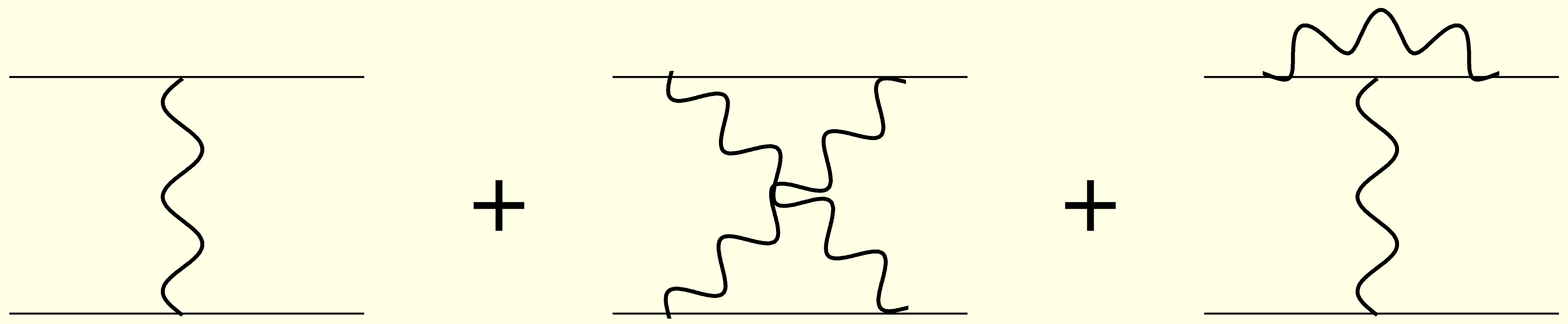

+ ...

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B. It is very poorly suited to achieve factorization (especially important in QCD)

THE EFT APPROACH: PNREFT

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Relevant for: atomic physics: Hydrogen atom (e.g. proton radius), positronium (e.g. width, hfs), muonium ttbar threshold production; Dark matter annihilation and production close to threshold; SUSY particles annihilation and production; QQbar, QQq and QQQ with small radius; extraction of SM parameters

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II. In QCD (or in a strongly coupled theory) it factorizes automatically high energy contributions (perturbative) from low-energy (nonperturbative, thermal) ones

Relevant for: pionium and precision chiral dynamics; nucleon-nucleon systems; Quarkonium, Exotic X, Y, Z states, Quarkonium in hot QCD medium in heavy ion collisions; NR states in early universe; confinement and nonperturbative effects

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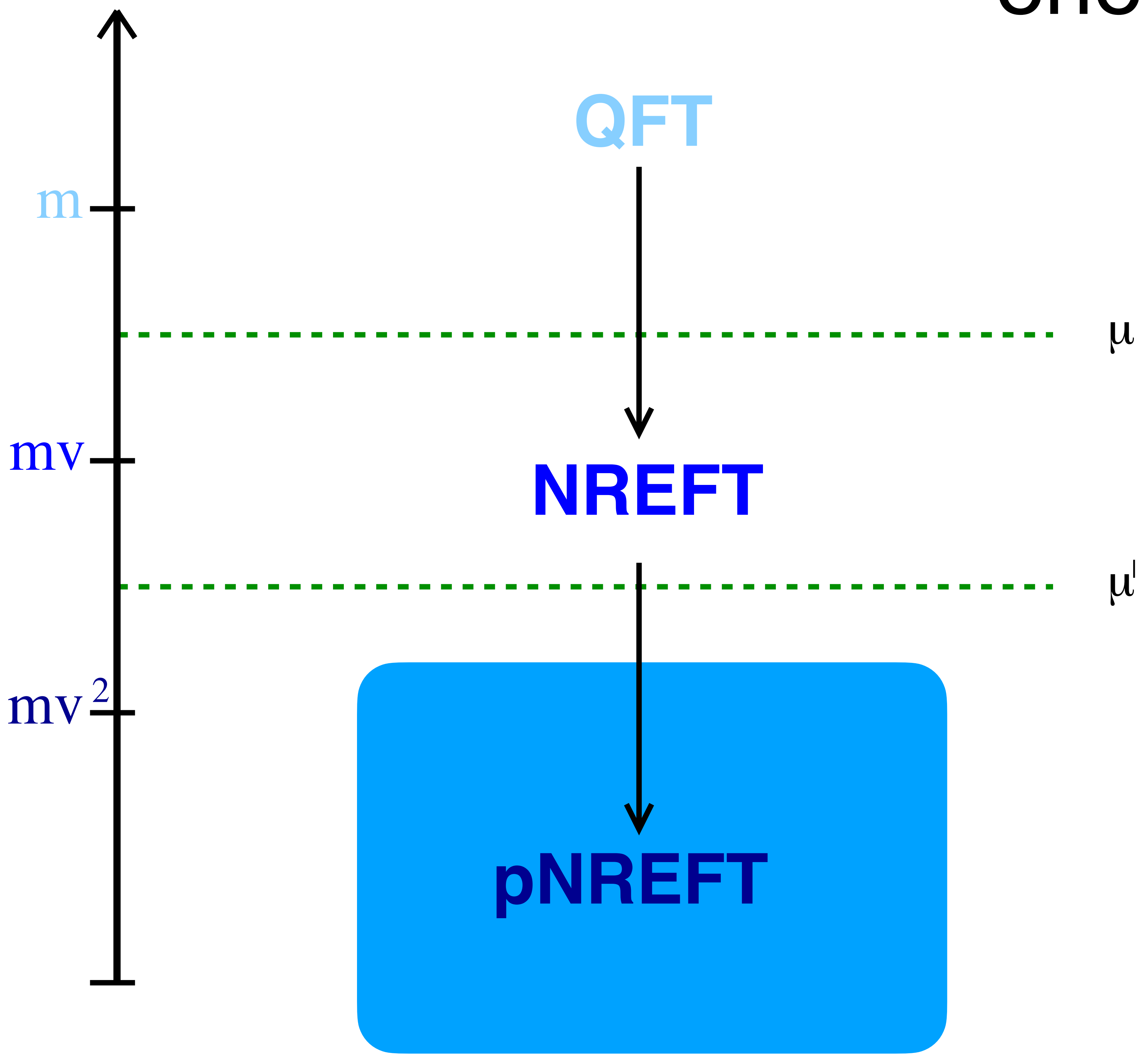
IV. It allows to define in QFT objects of great importance like potentials

More conceptually V. It provides a field theoretical foundation of the Schroedinger eq.:

the Lagrangian $$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

separates the Schroedinger dynamics of the two particle field ϕ from the low energy dynamics encoded in $\Delta\mathcal{L}$



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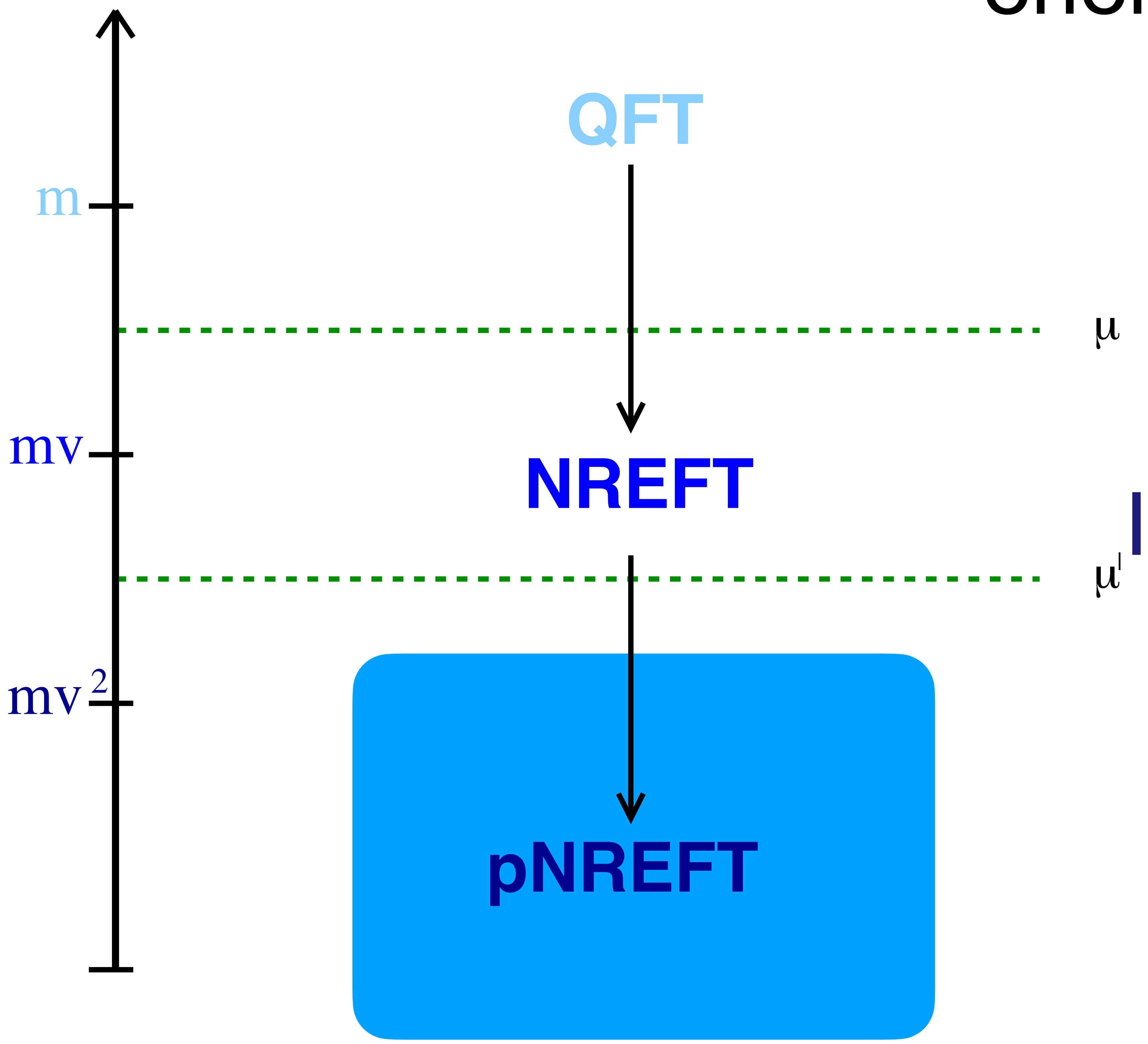

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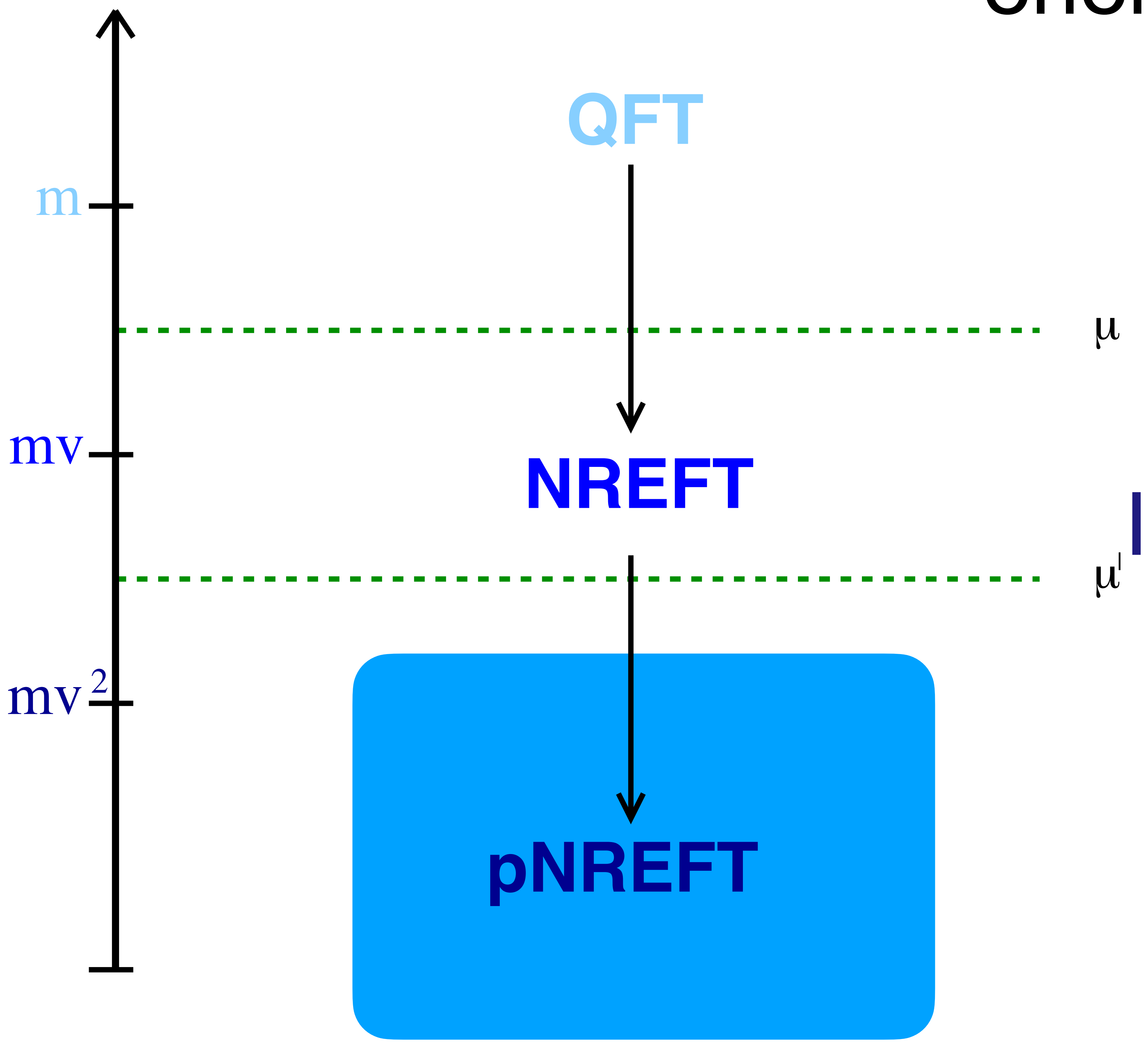

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if QFT = QED, pNRQED (Pineda,Soto 1998) gives a proper version of Quantum Mechanics

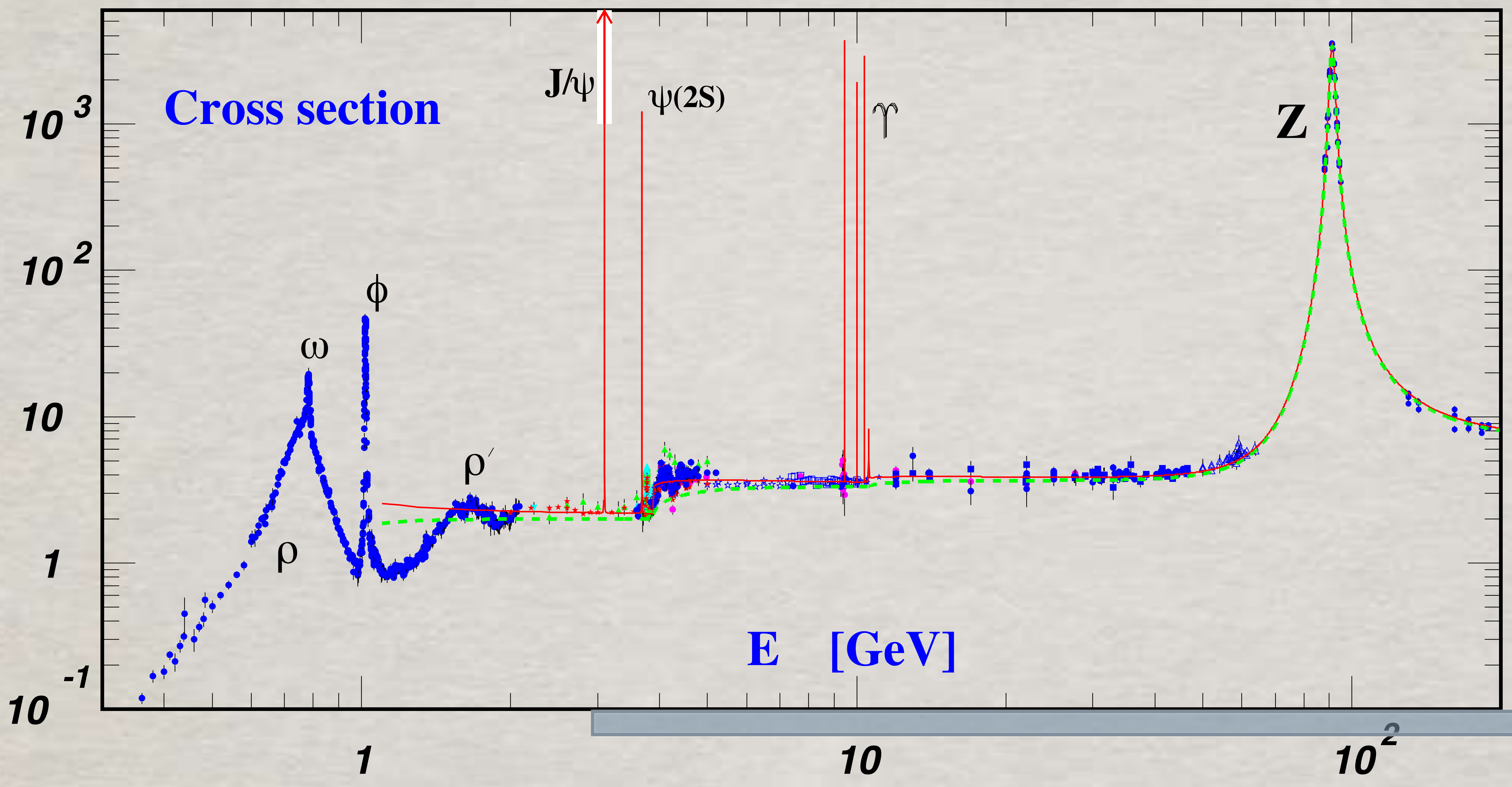
A rich physical example of NR state: quarkonium

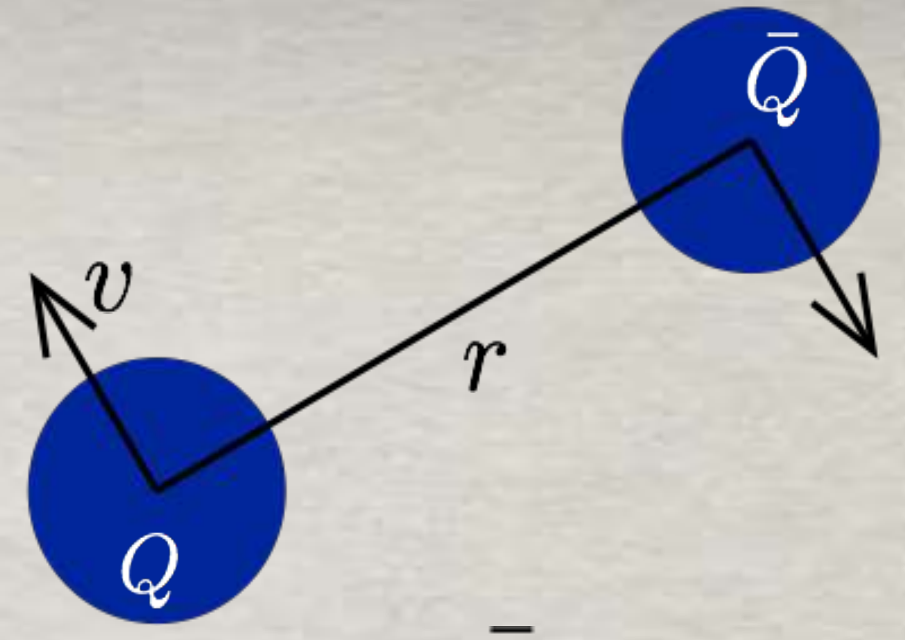
in 1974 the J/psi discovery triggered the
November revolution:
charm discovery and confirmation of asymptotic freedom

Today it is a golden probe of strong interactions

From the physical point of view it is a pretty interesting system, that add $\Lambda_{\rm QCD}$
to the other scales and requires two different versions of pNREFT,
weakly coupled and strongly coupled, for $mv > \Lambda_{\rm QCD}$ and $mv \sim \Lambda_{\rm QCD}$ respectively

Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

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$m_b \sim 5$GeV

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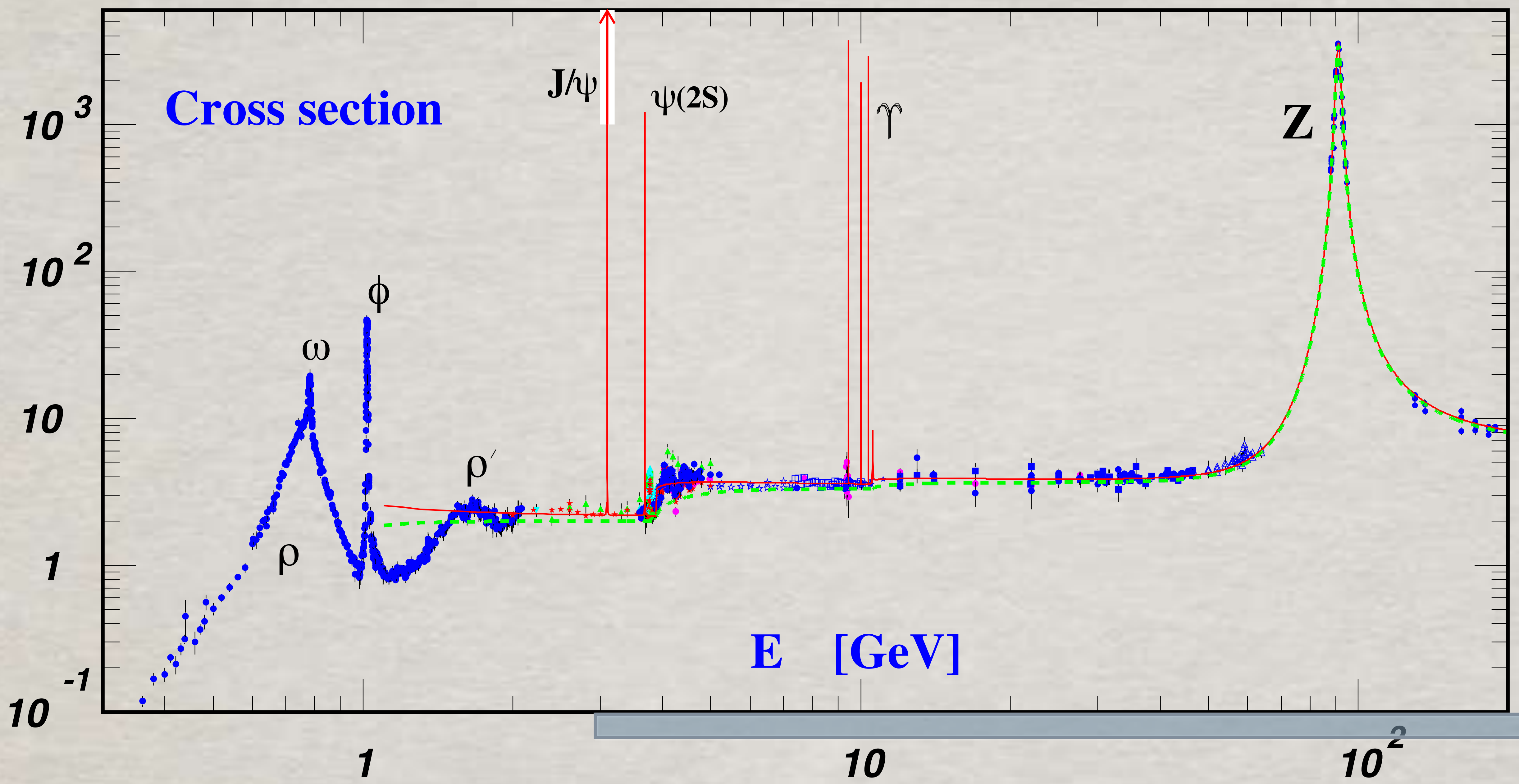

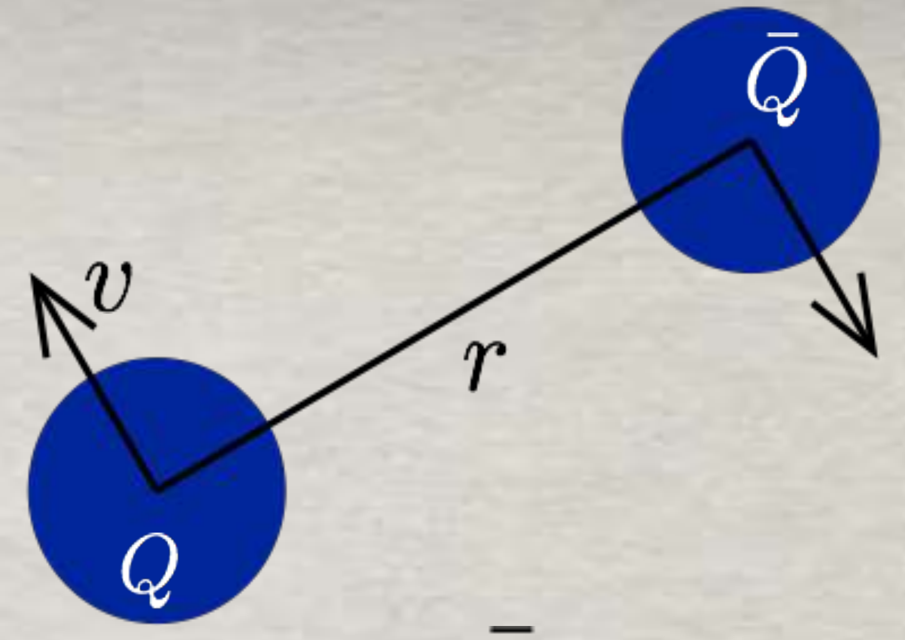

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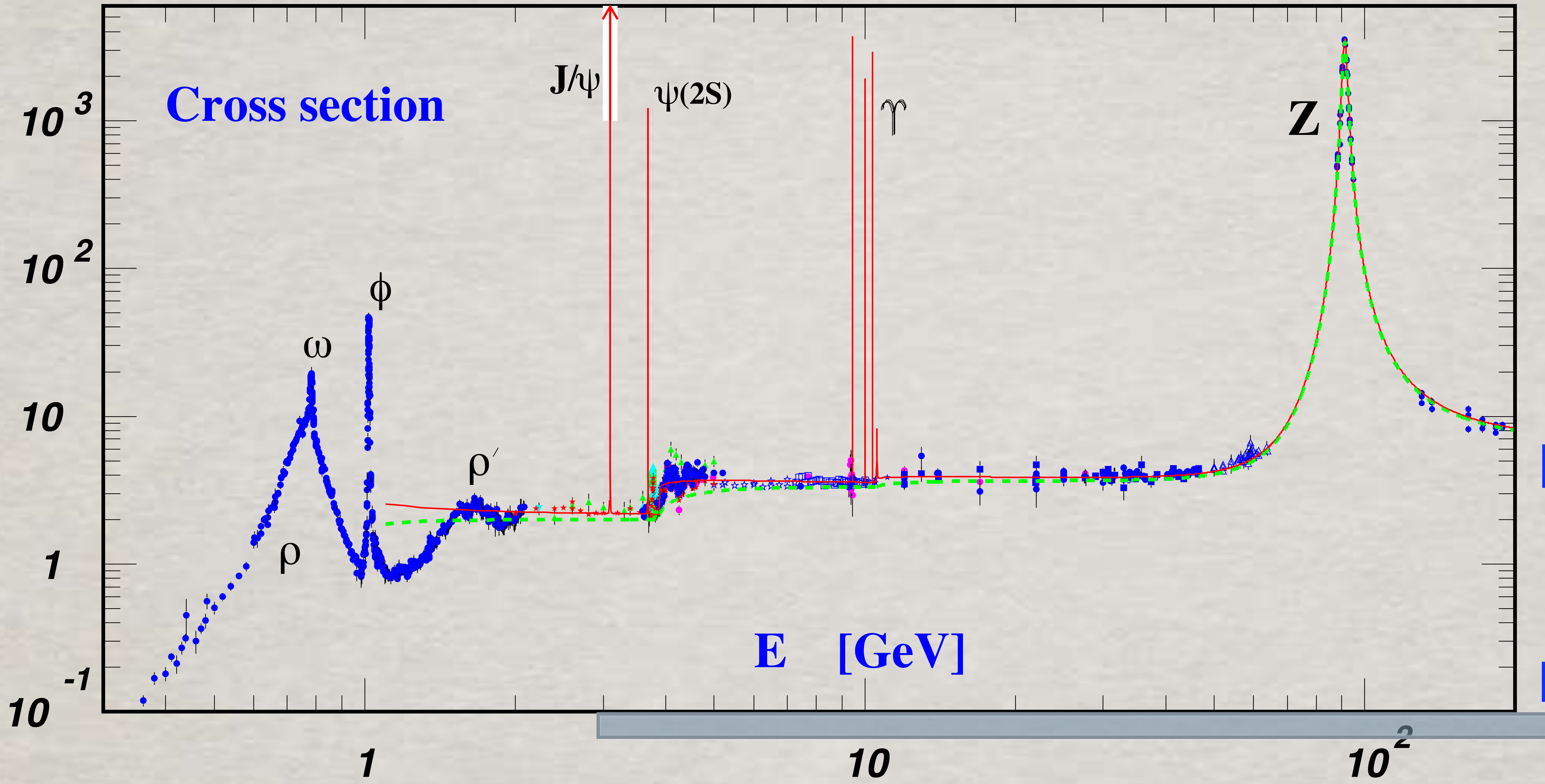

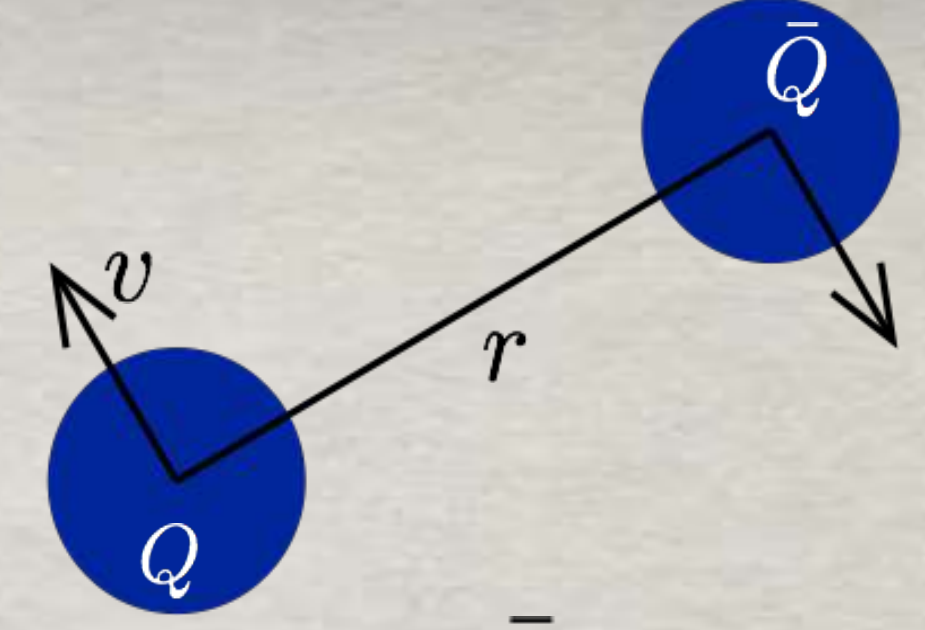

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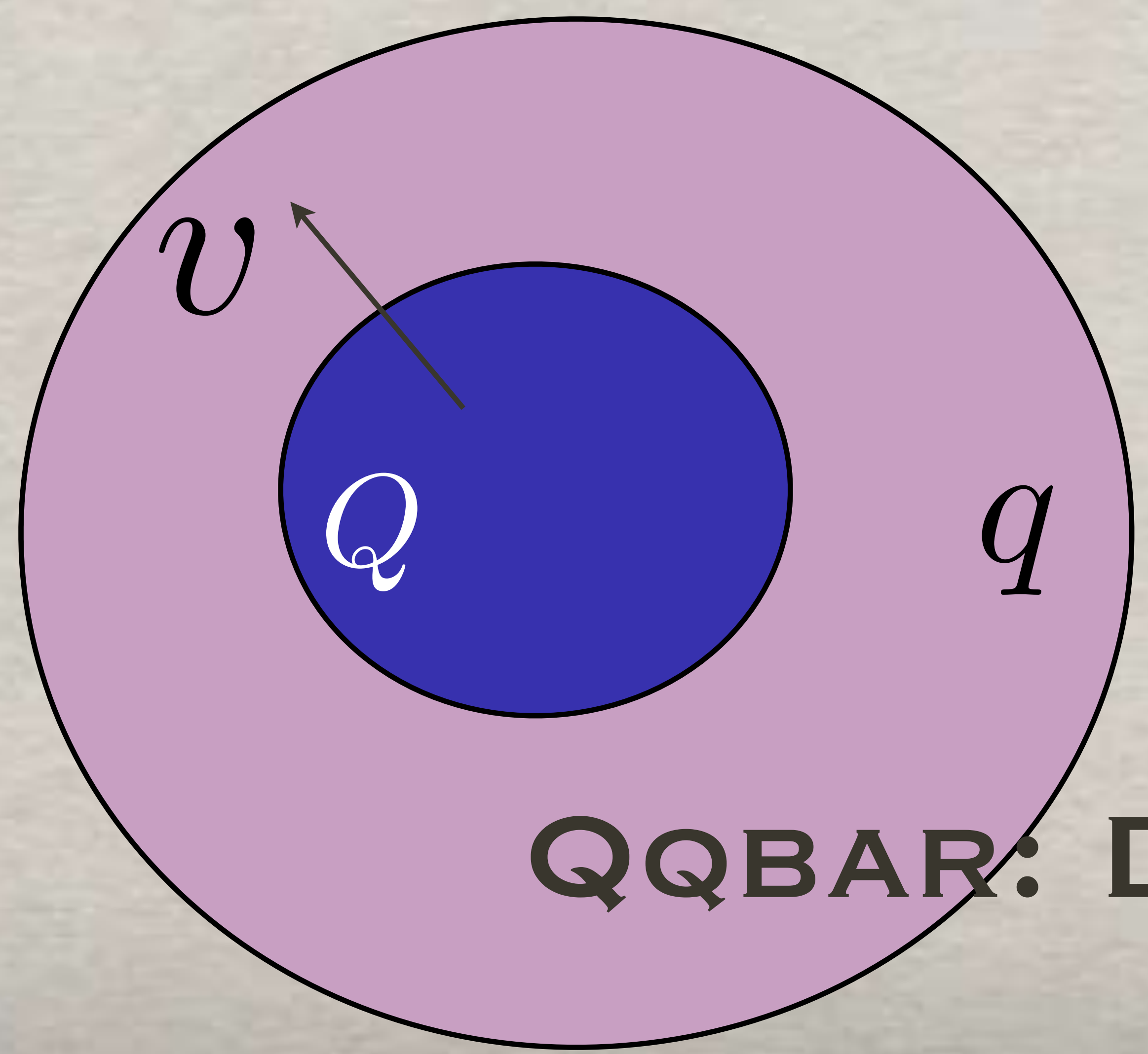
Heavy quarkonium is very different from heavy-light hadrons



different physics from the heavy light meson where only two scales exist m and Λ_{QCD}

Qqbar: D, B mesons

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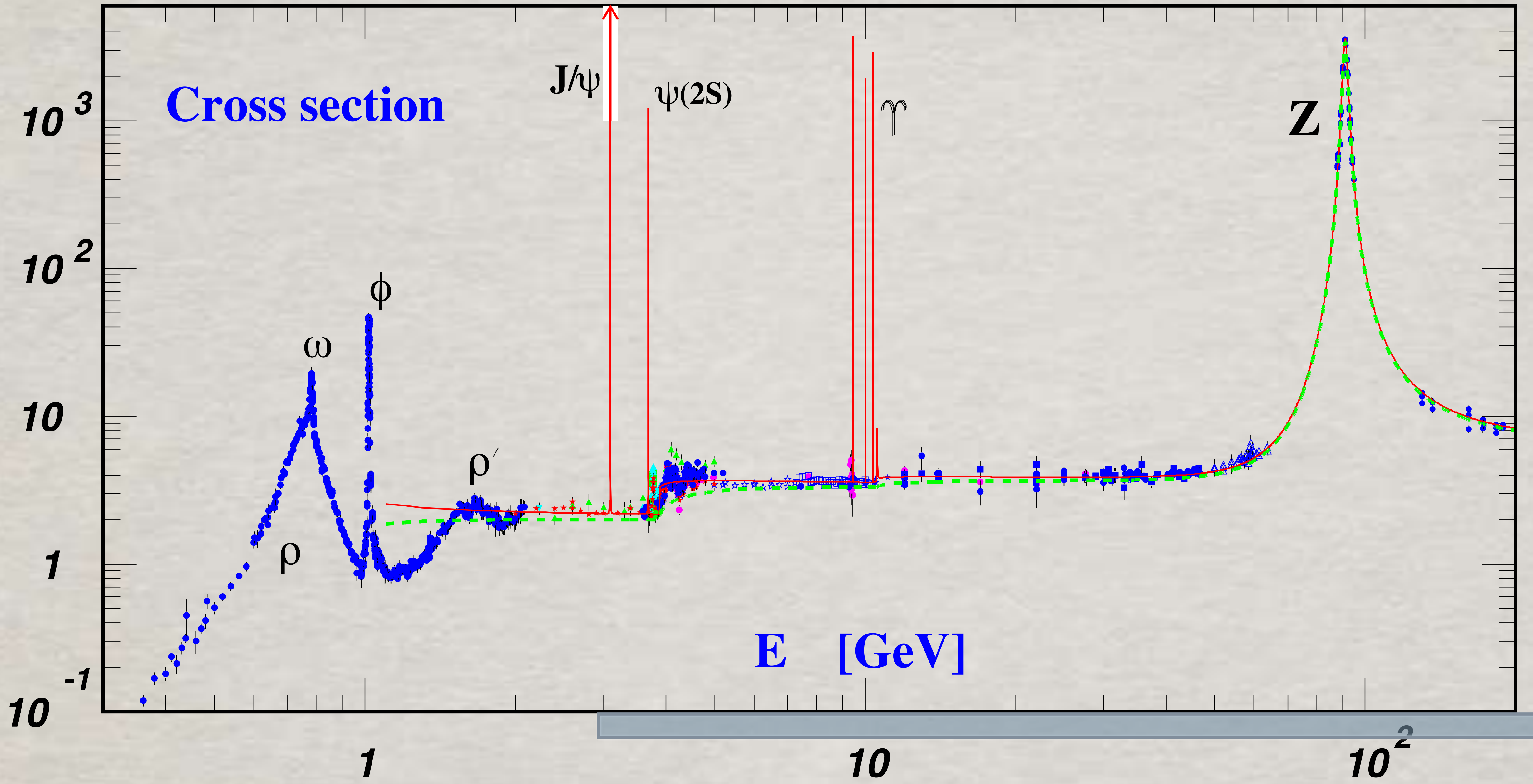

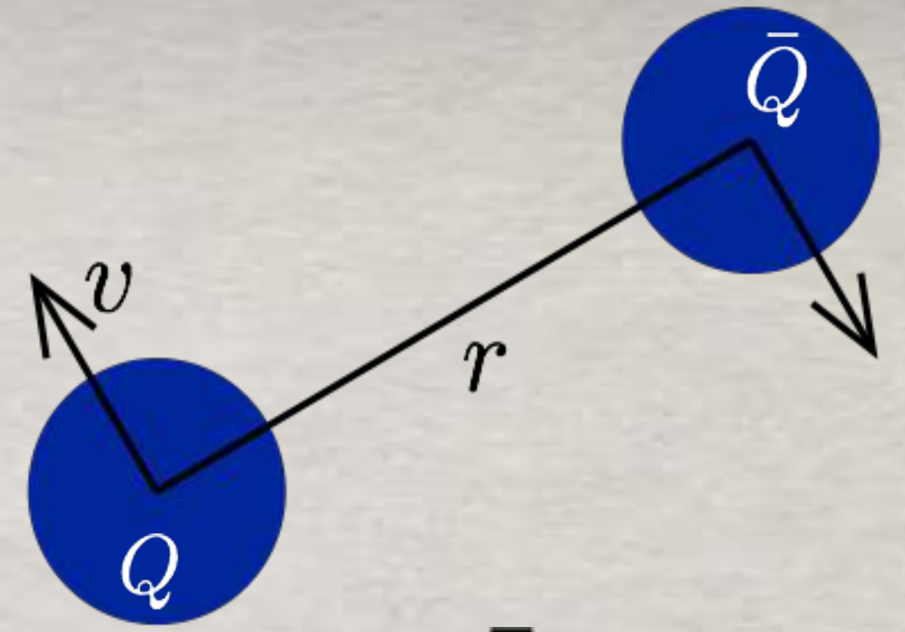

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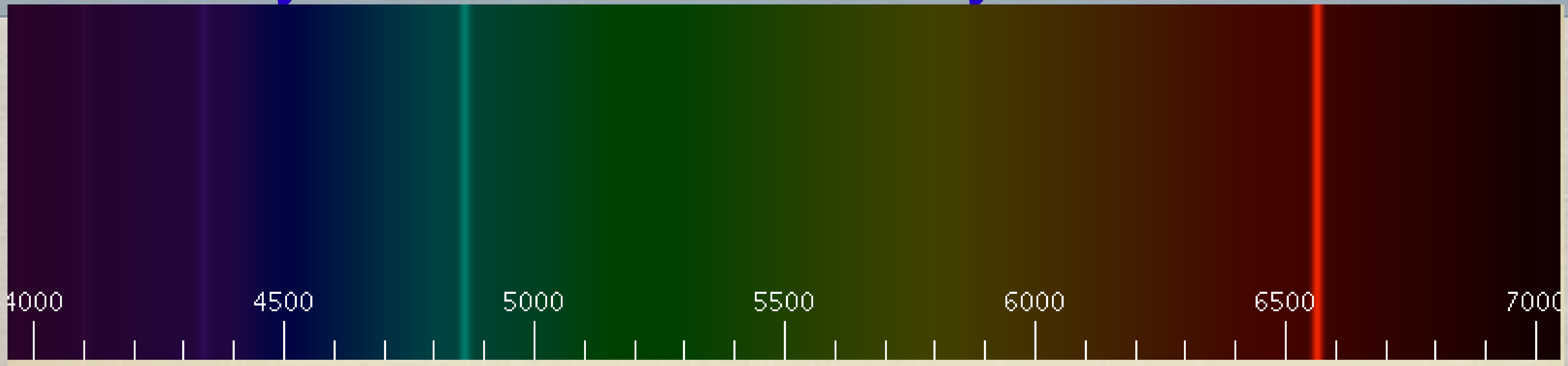
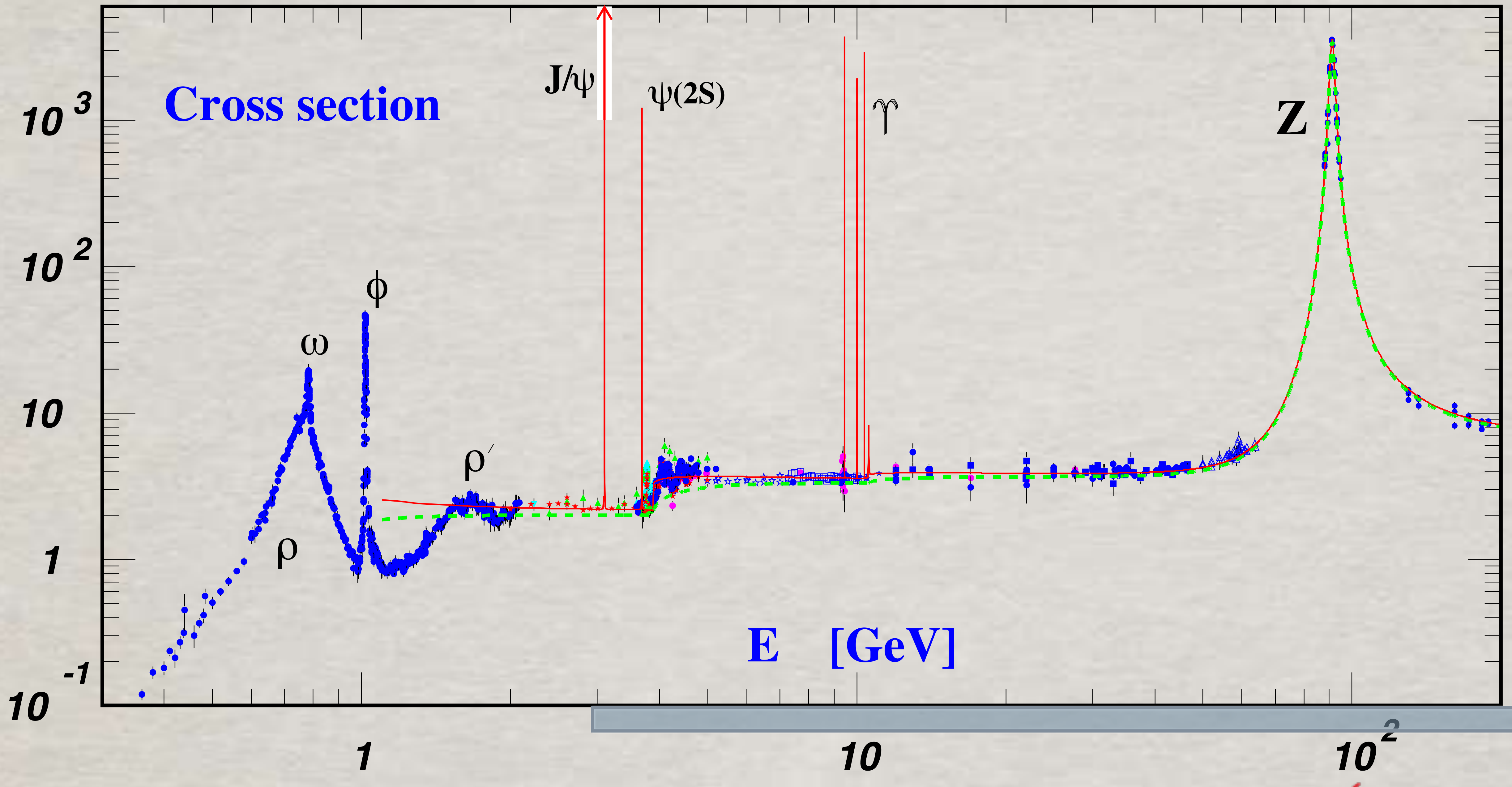
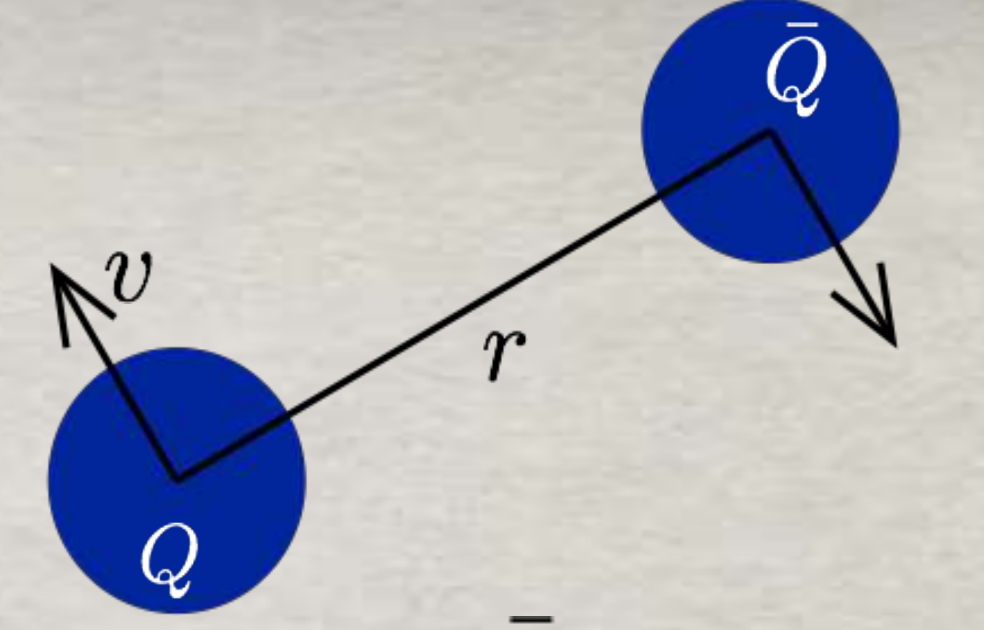


ELECTROMAGNETIC BOUND STATES: ATOMS, MOLECULES,

Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

$m_c \sim 1.5$GeV

$m_b \sim 5$GeV

$m_t \sim 170$GeV

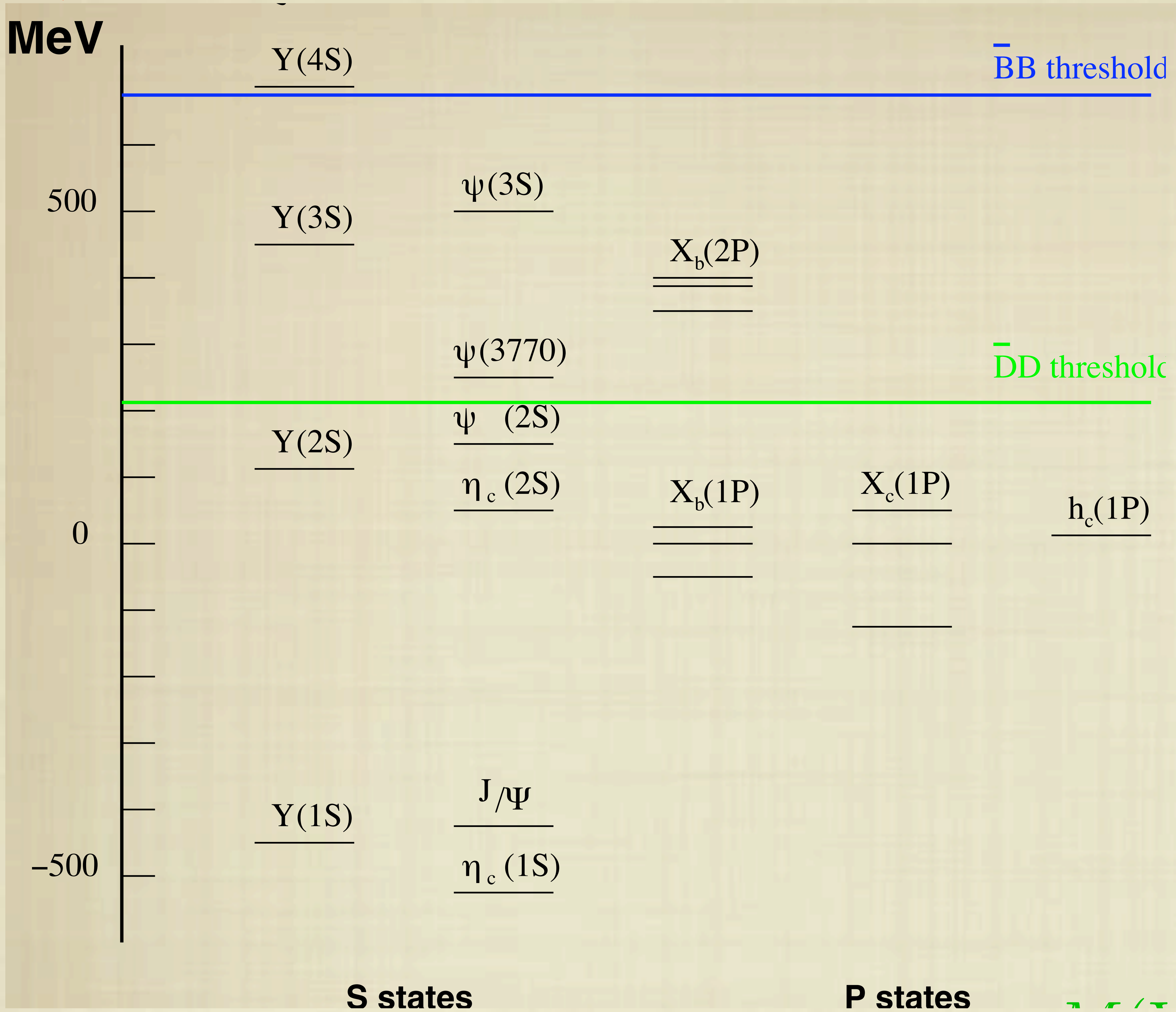
A large scale $m_Q \gg \Lambda_{QCD}$ $\alpha_s(m_Q) \ll 1$

Heavy quarkonia are nonrelativistic bound systems: multiscale systems

many scales: a challenge and an opportunity

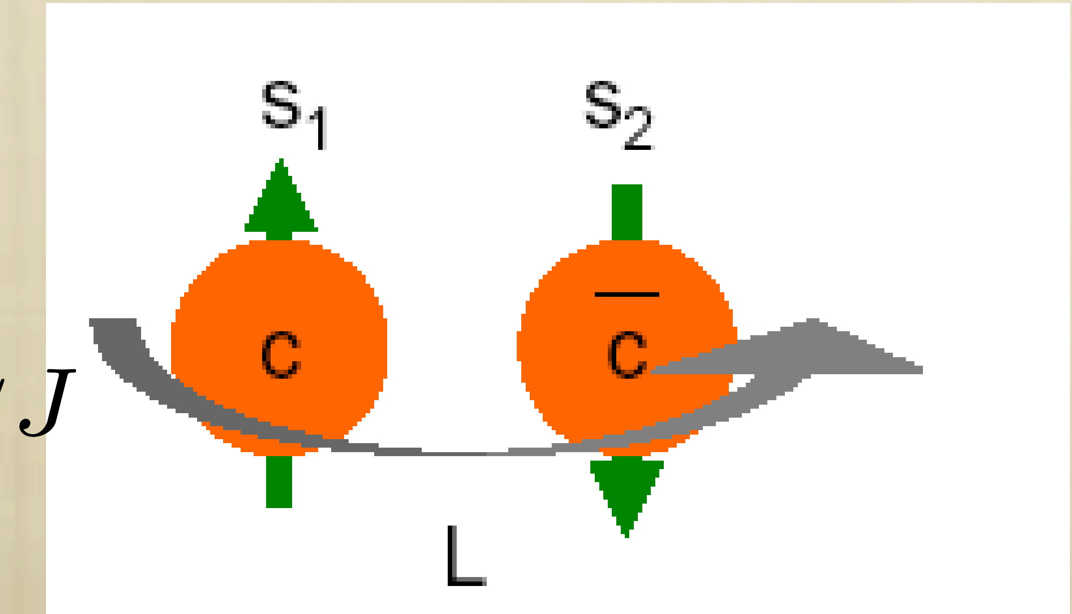
Electromagnetic bound states: atoms, molecules,

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

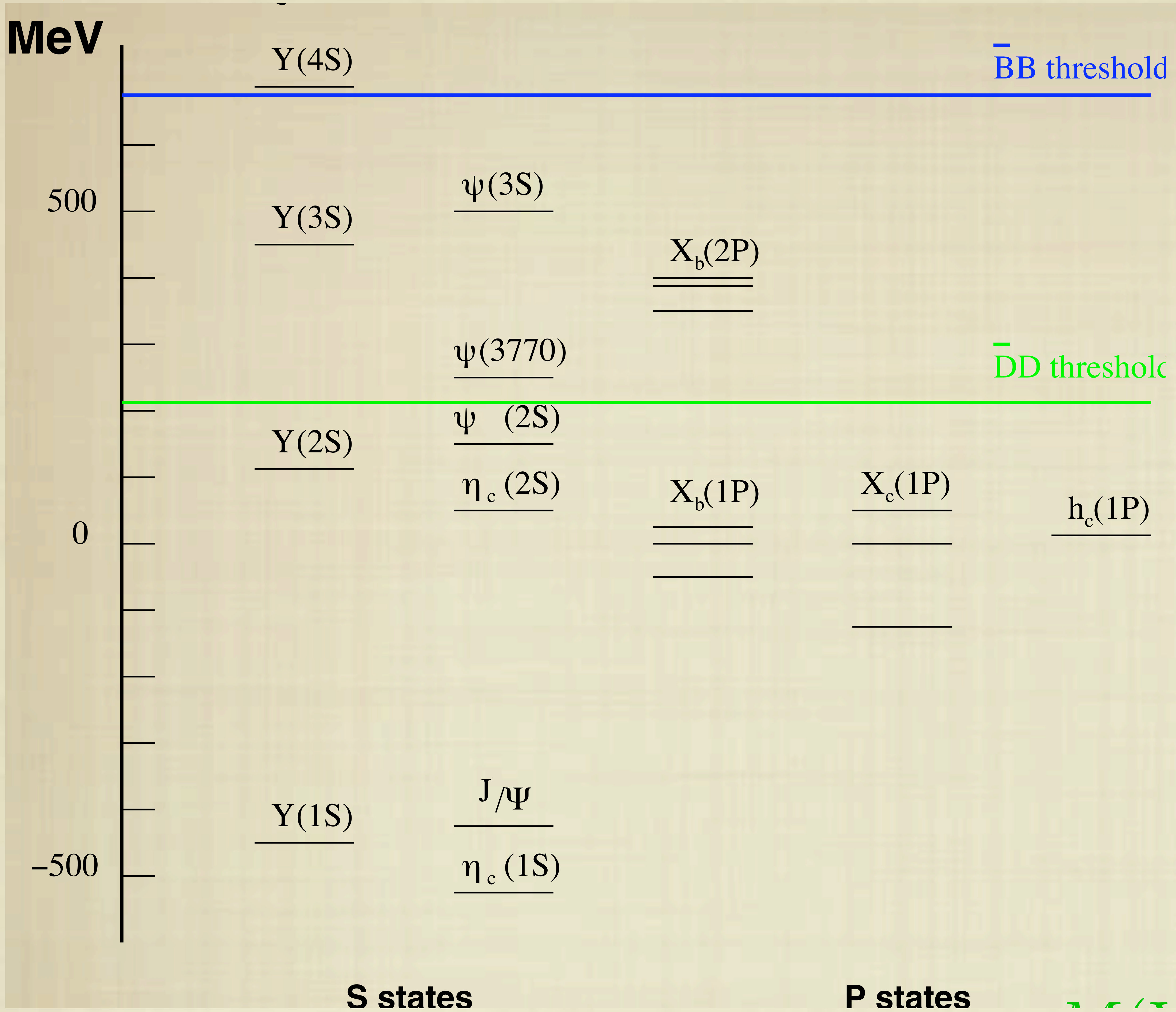
$$^{2S+1}L_J$$



$$M(Y(1S)) = 9460\,\mathrm{GeV}$$

$$M(J/\Psi) = 3097\,\mathrm{GeV}$$

Quarkonium scales



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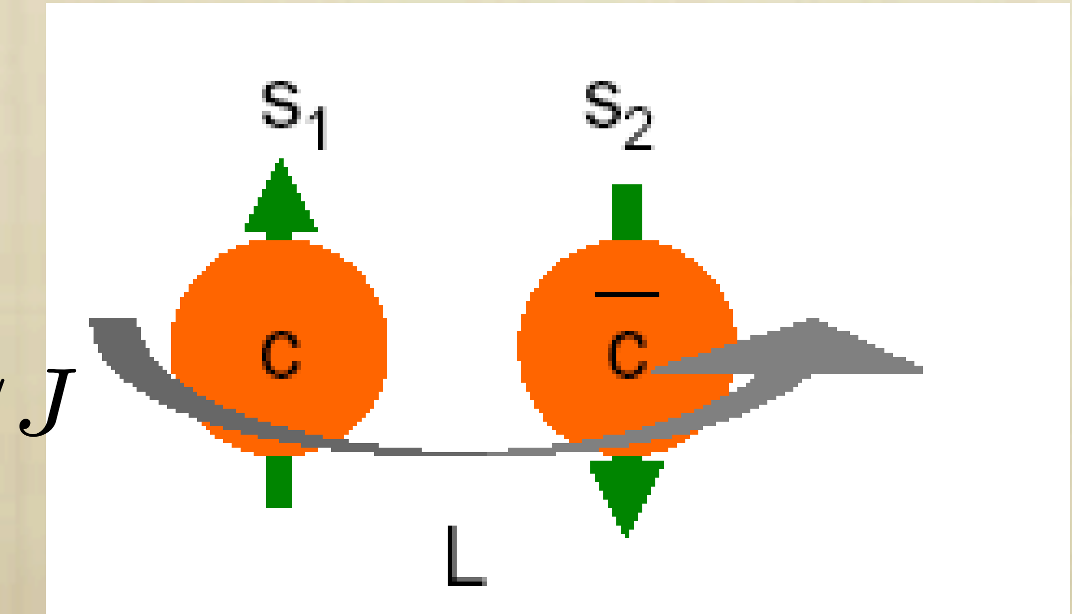

The mass scale is perturbative

$$m_Q \gg \Lambda_{\rm QCD}$$

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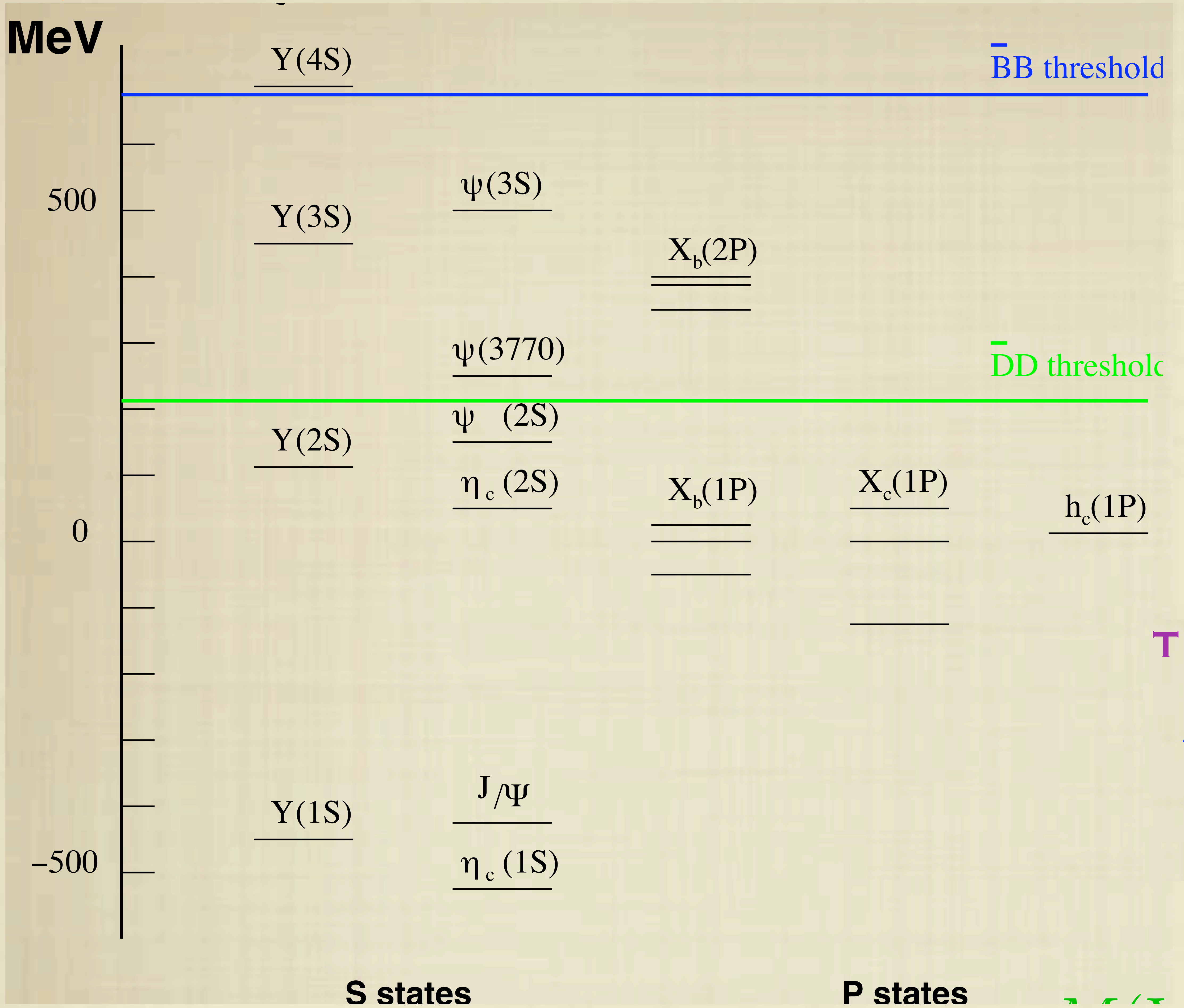

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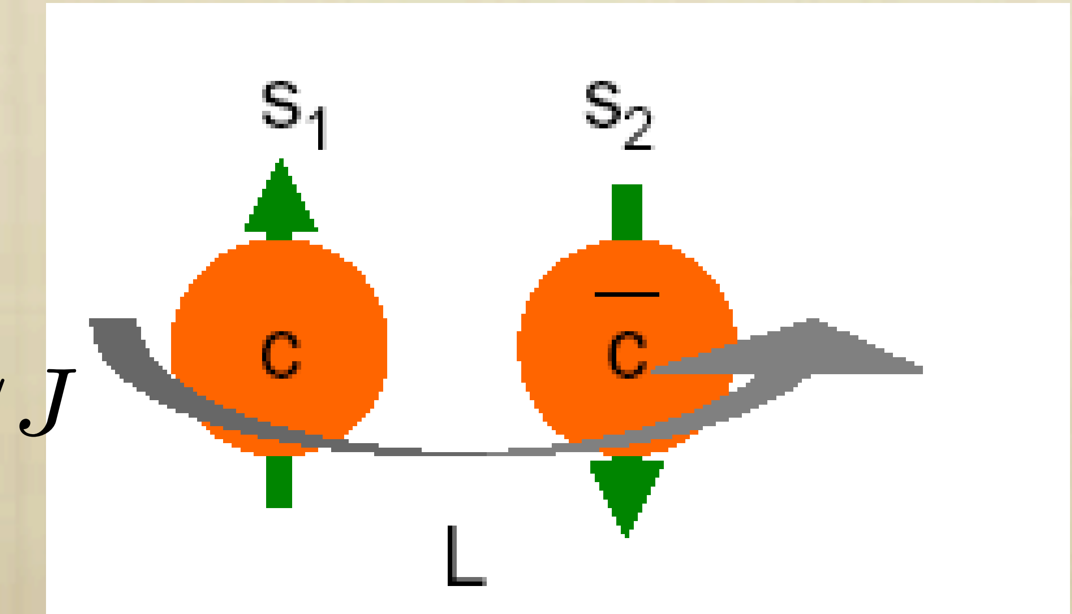

THE SYSTEM IS NONRELATIVISTIC(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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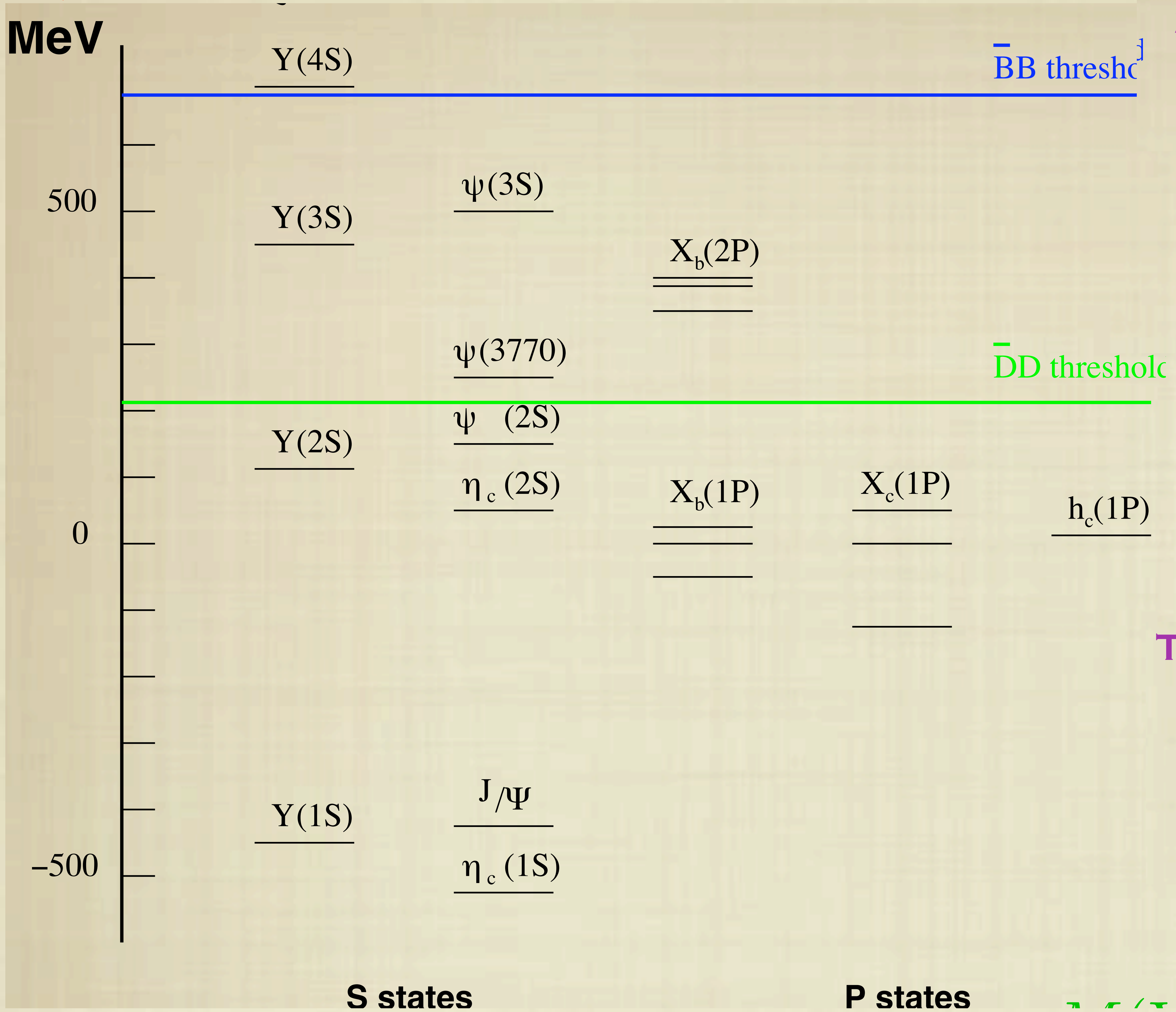

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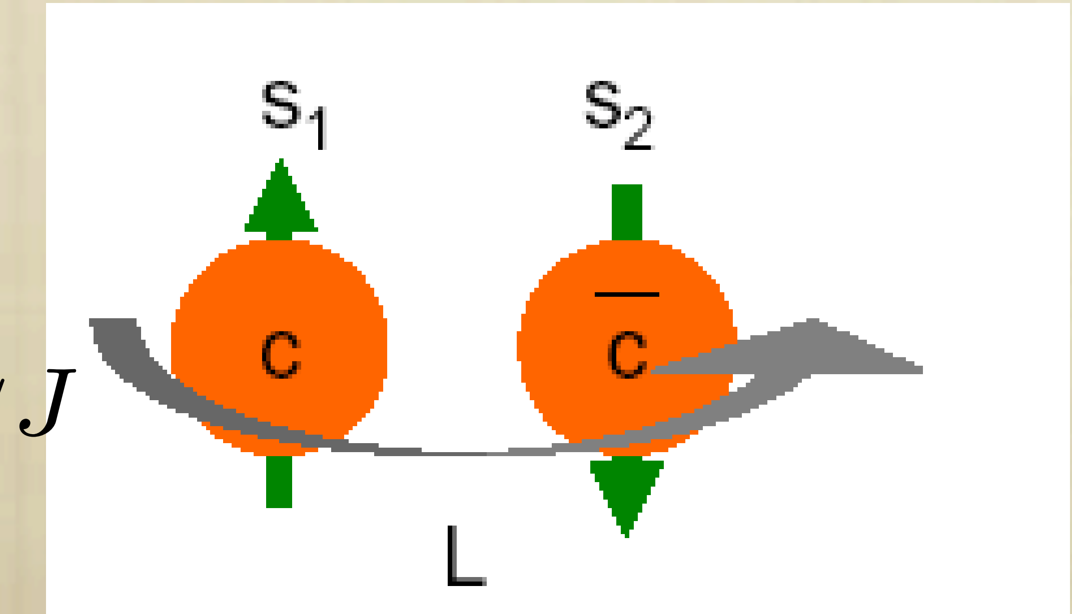

NR BOUND STATES HAVE AT LEAST 3 SCALES

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$$mv \sim r^{-1}$$

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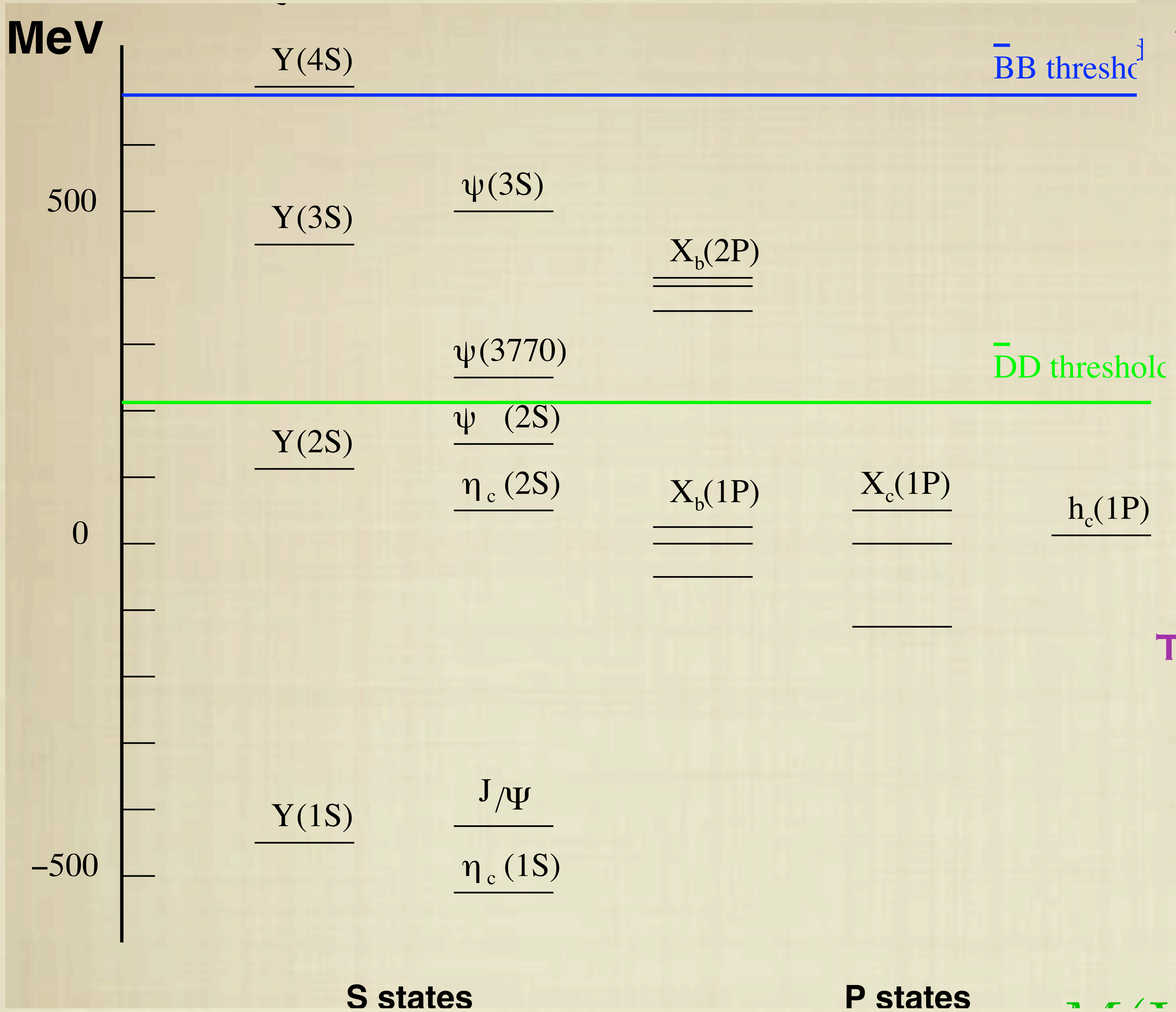

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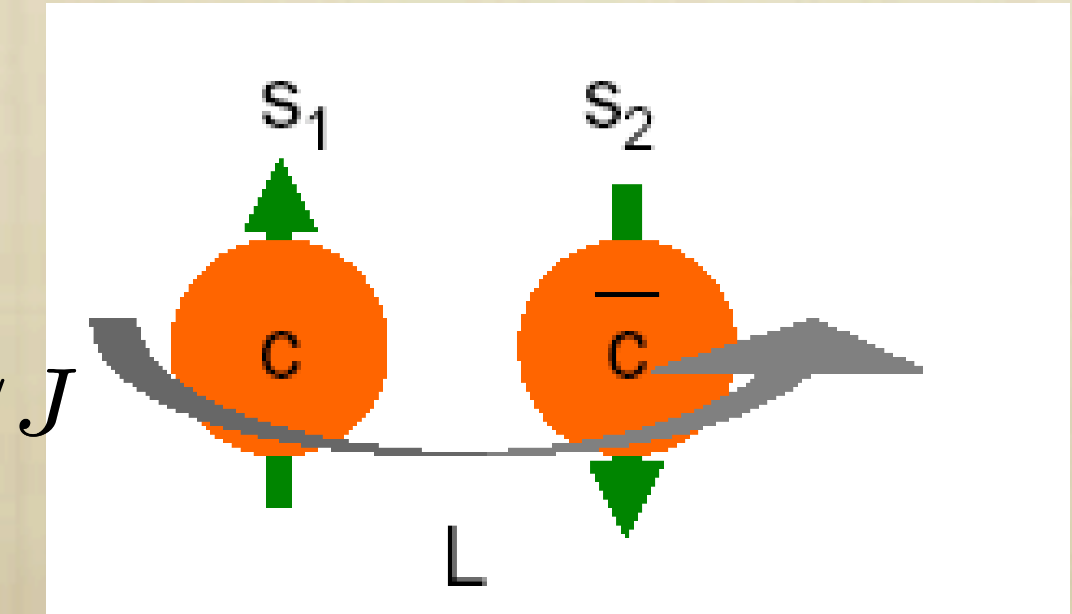

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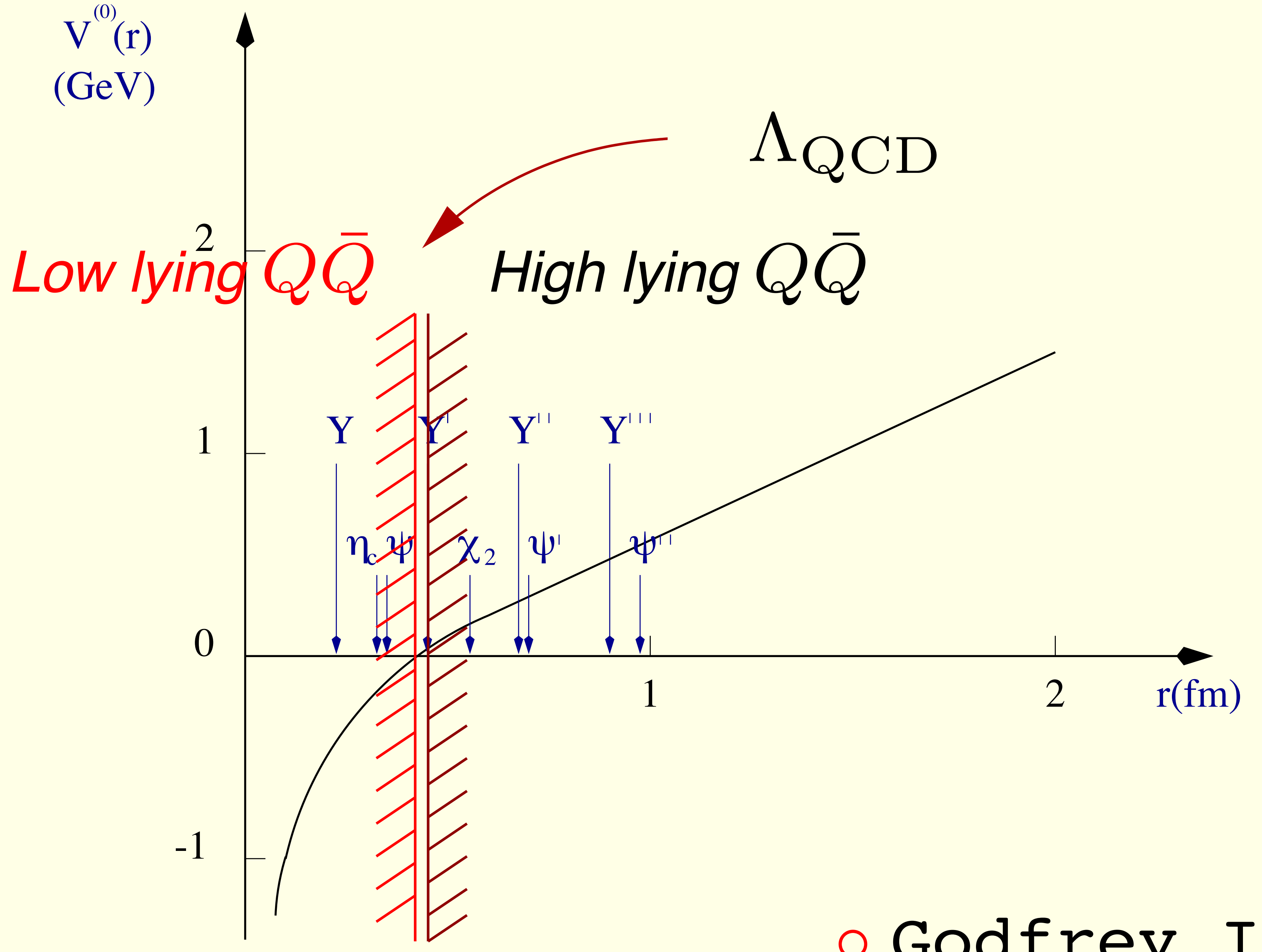
Quarkonium as a confinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



  - Godfrey Isgur PRD 32(85)189

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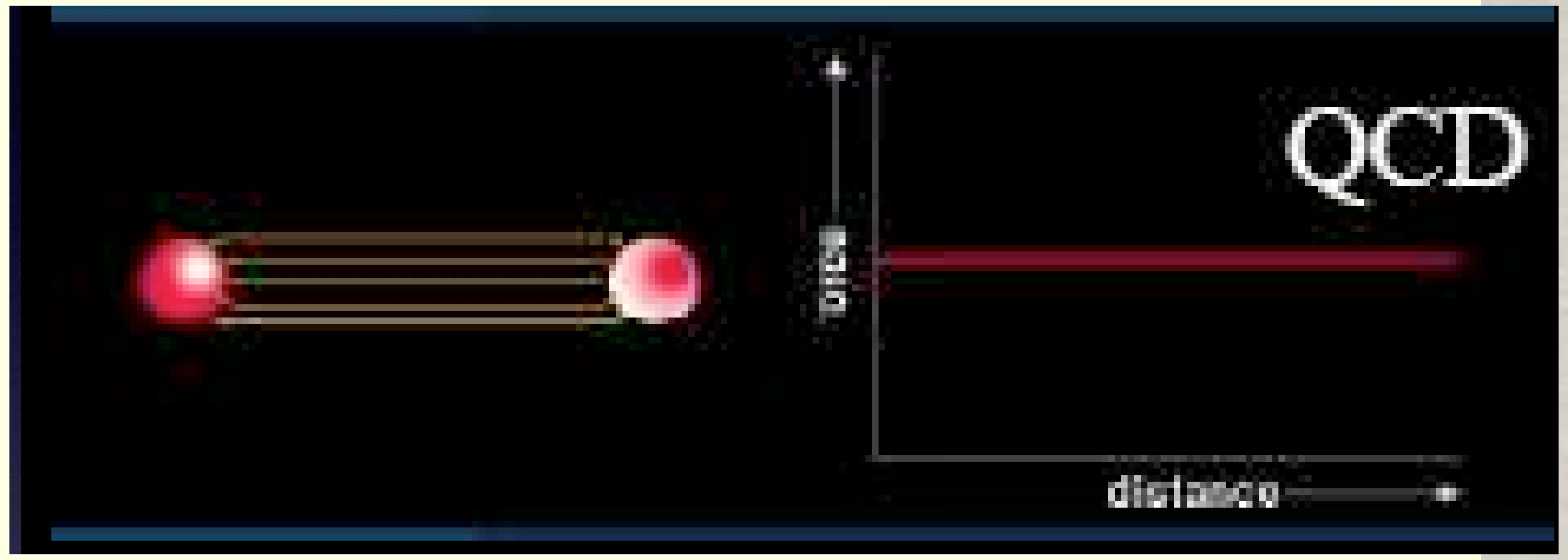

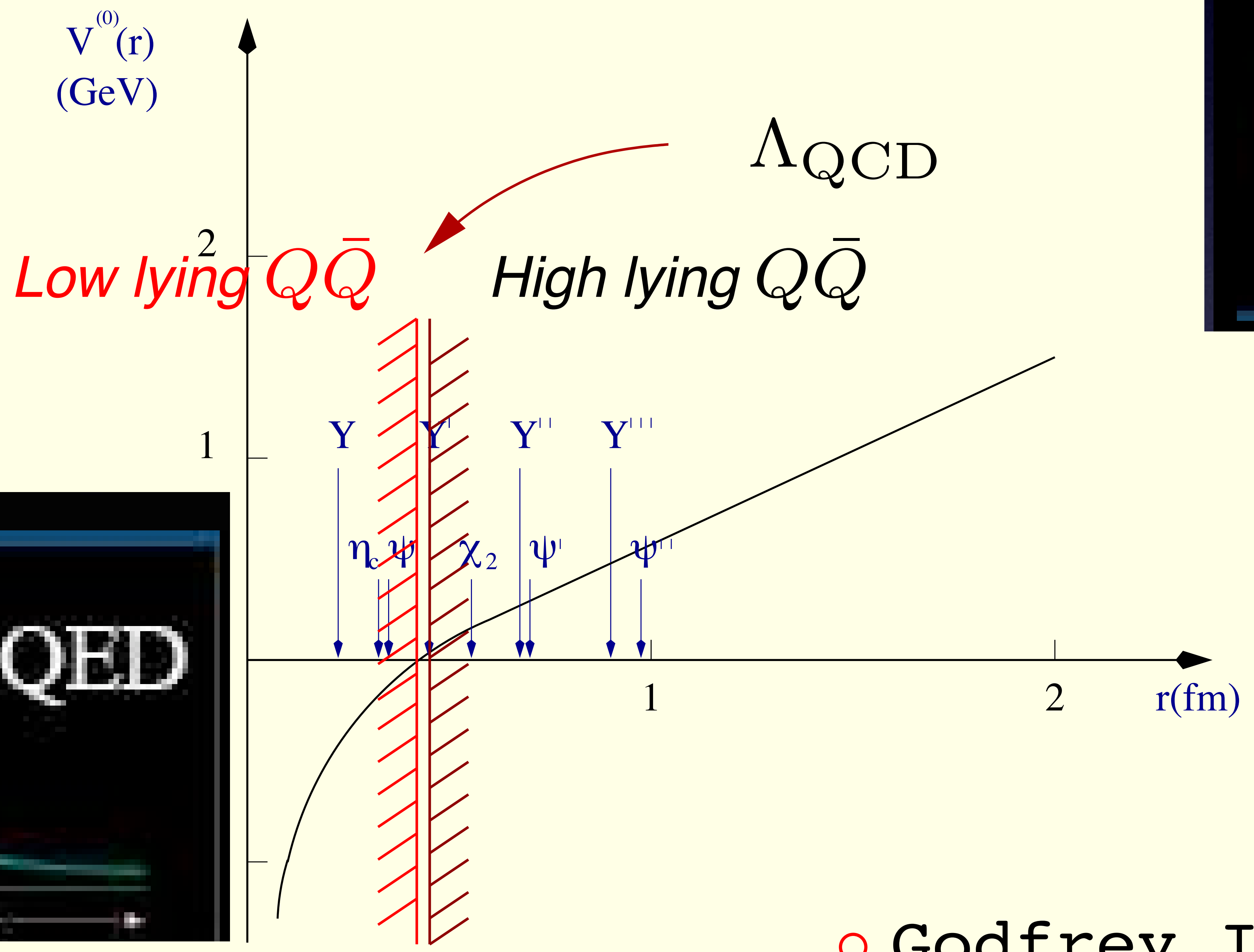

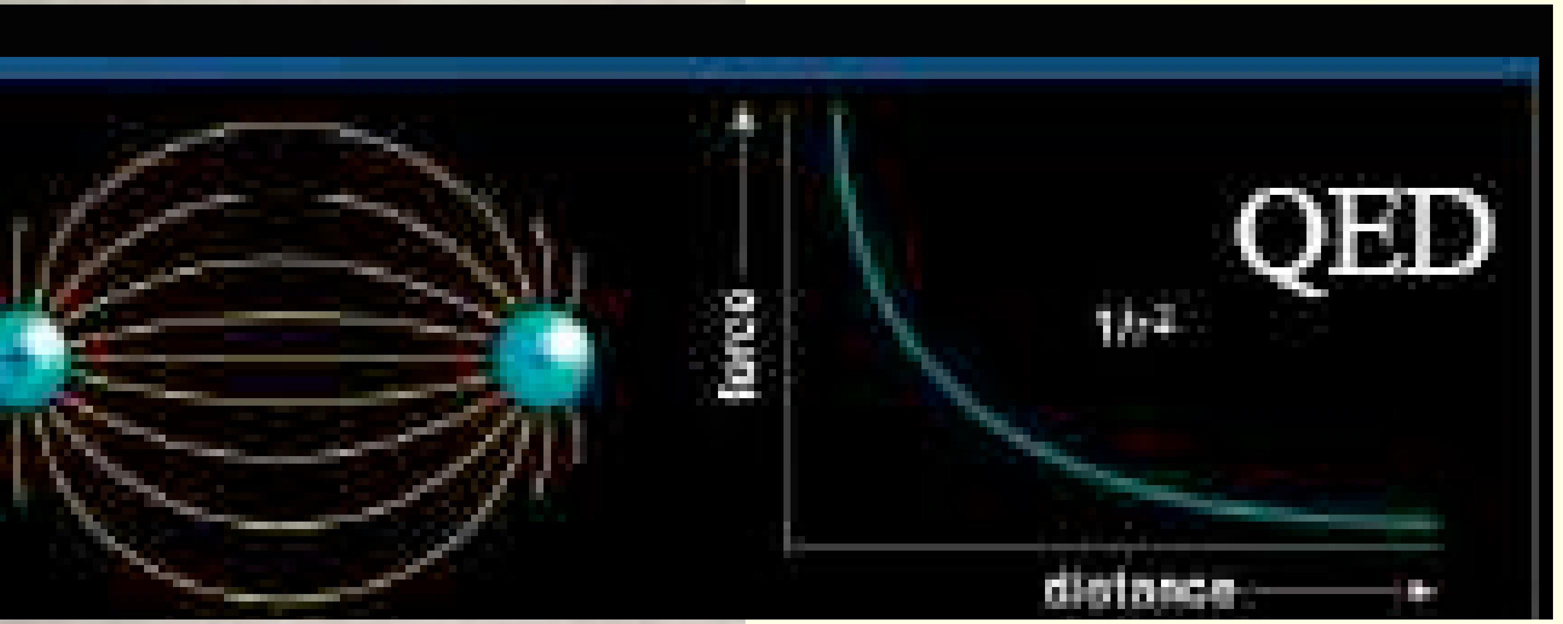

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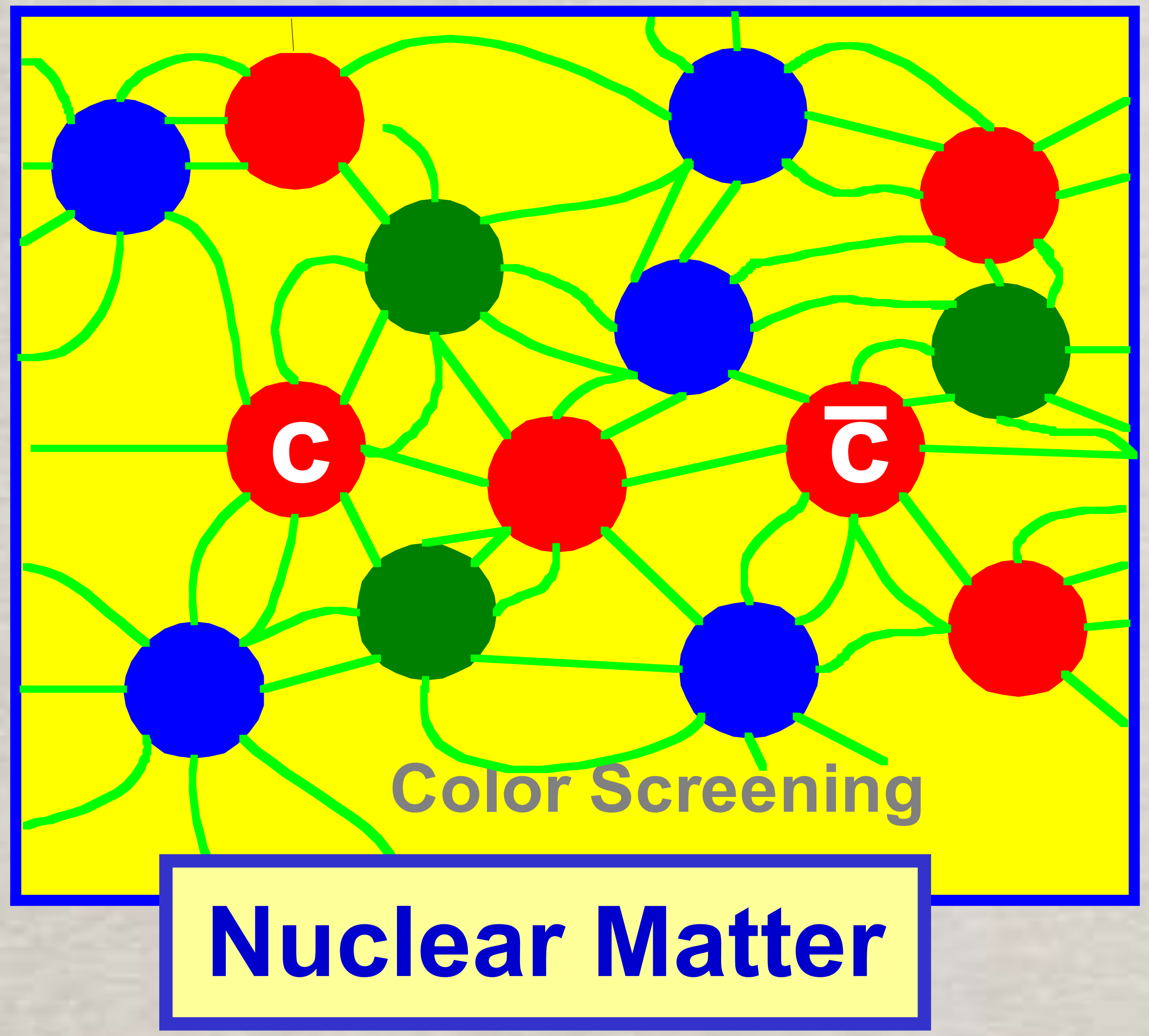
Quarkonium as a deconfinement probe

At finite temperature T they are sensitive to the formation of a quark gluon plasma via color screening



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$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

Matsui Satz 1986

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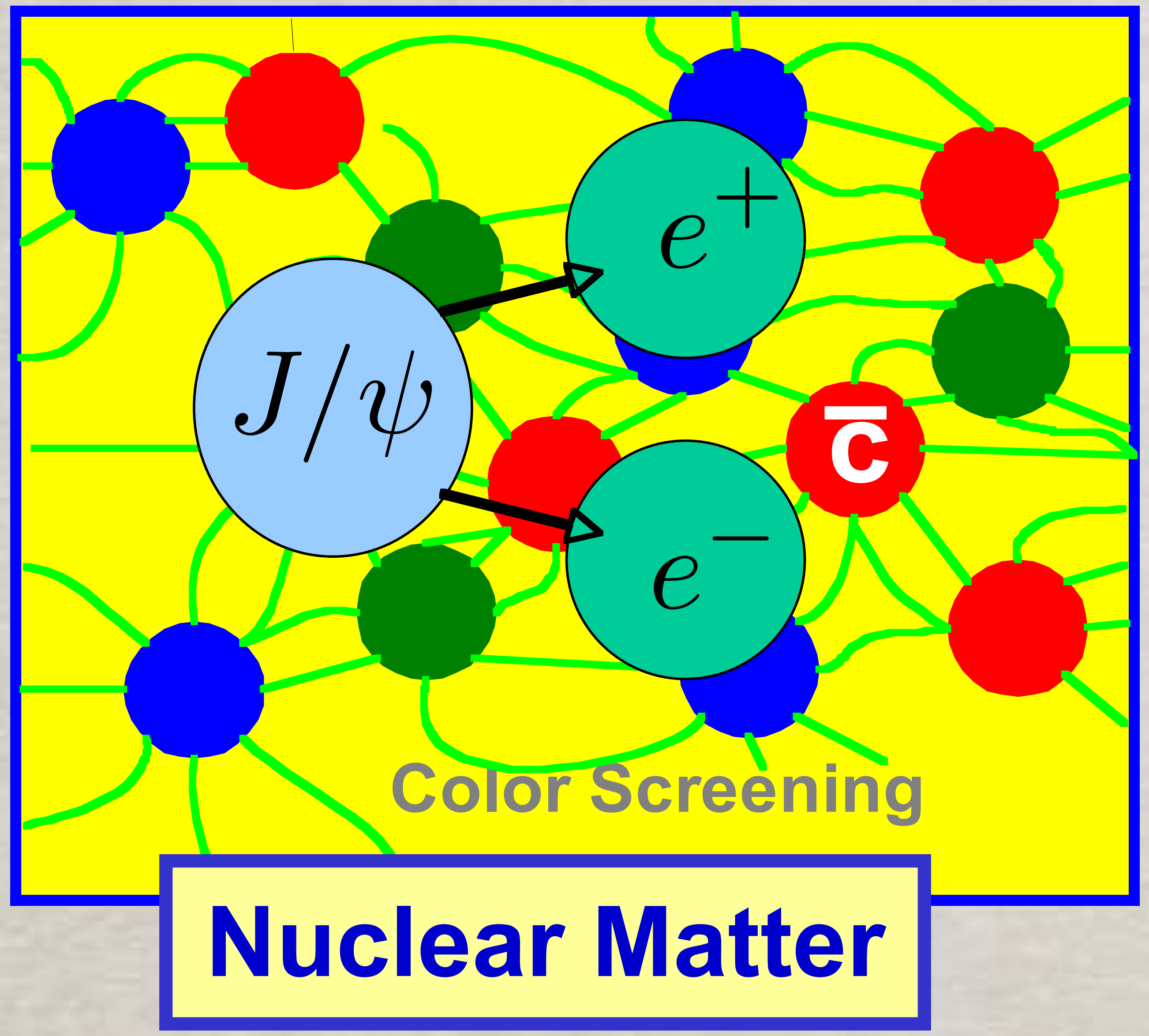

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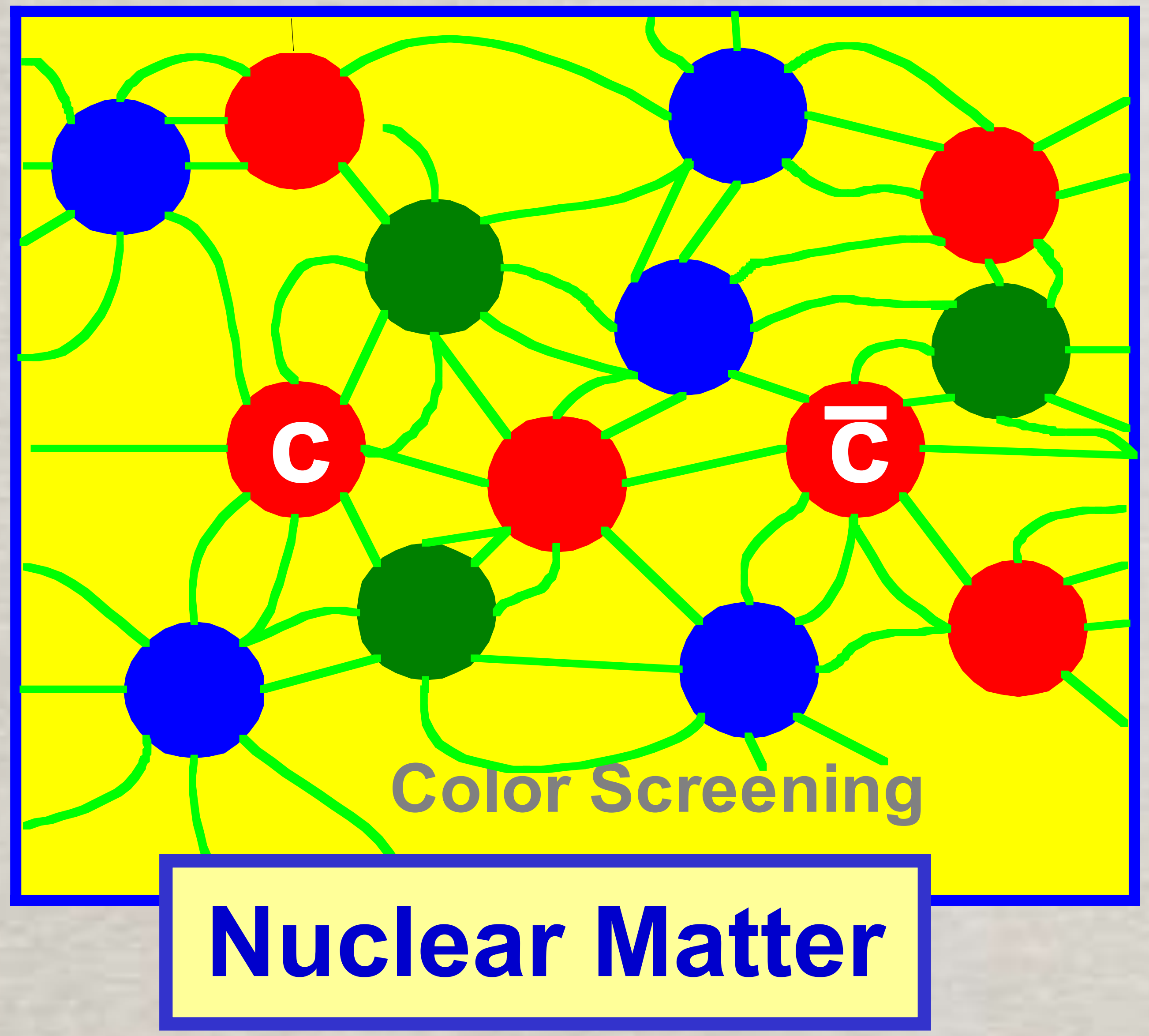

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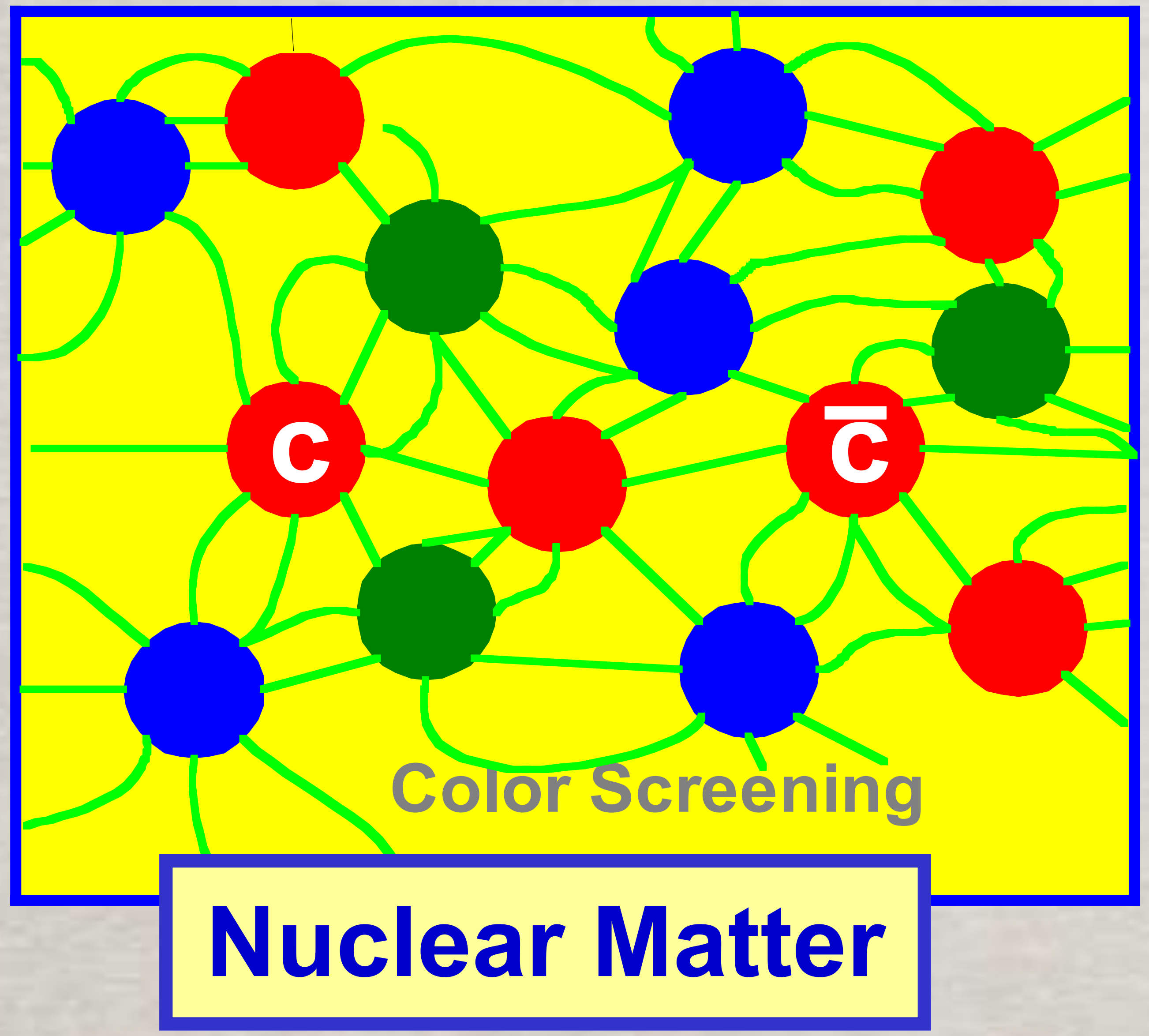

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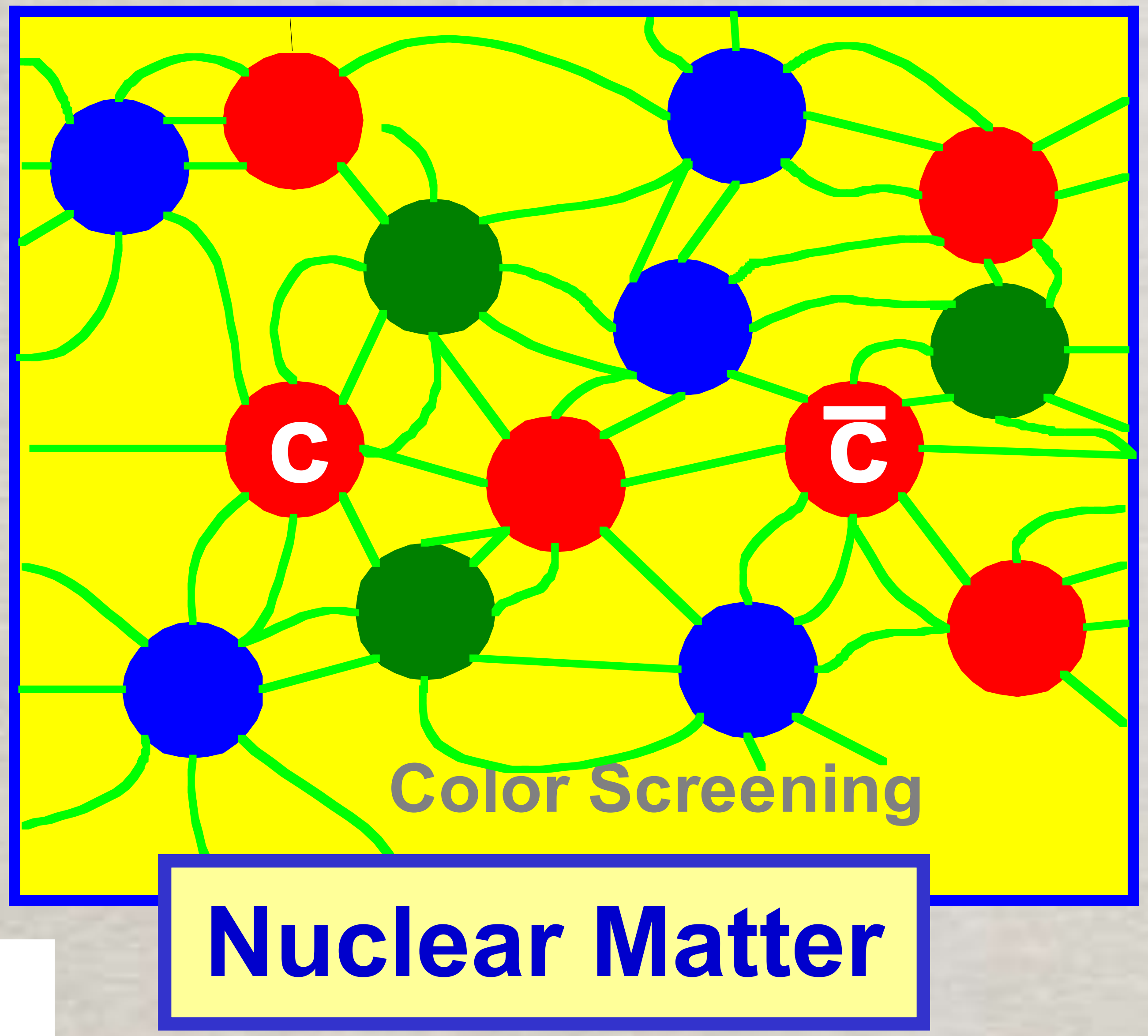

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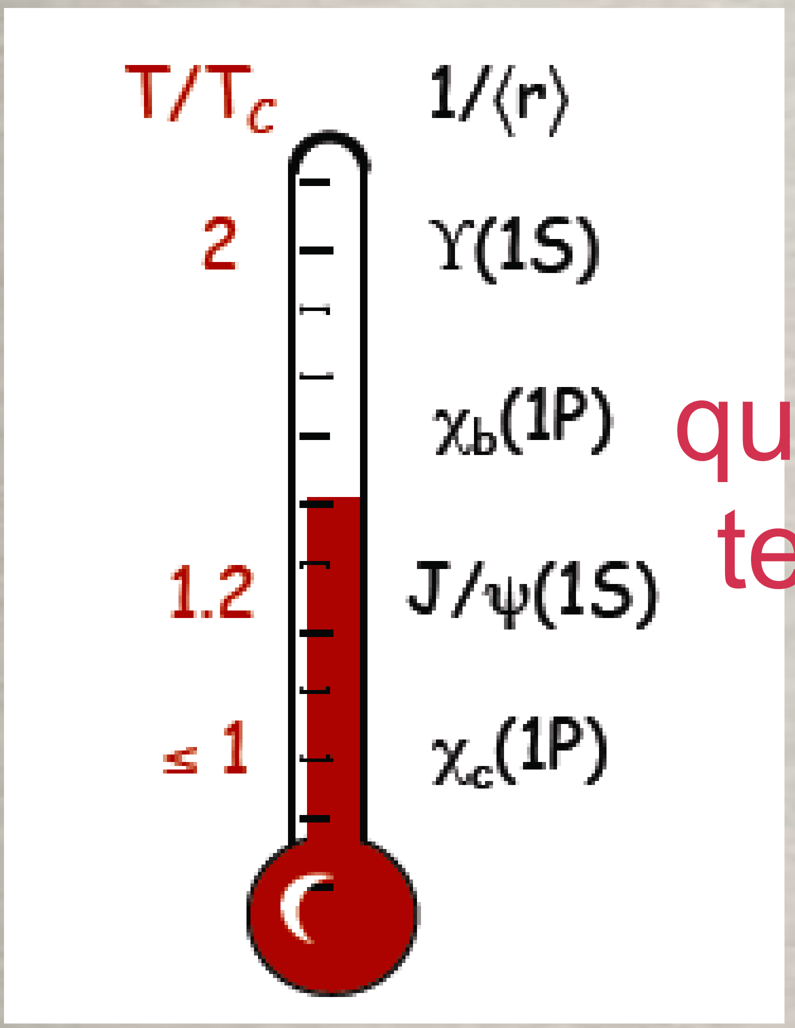

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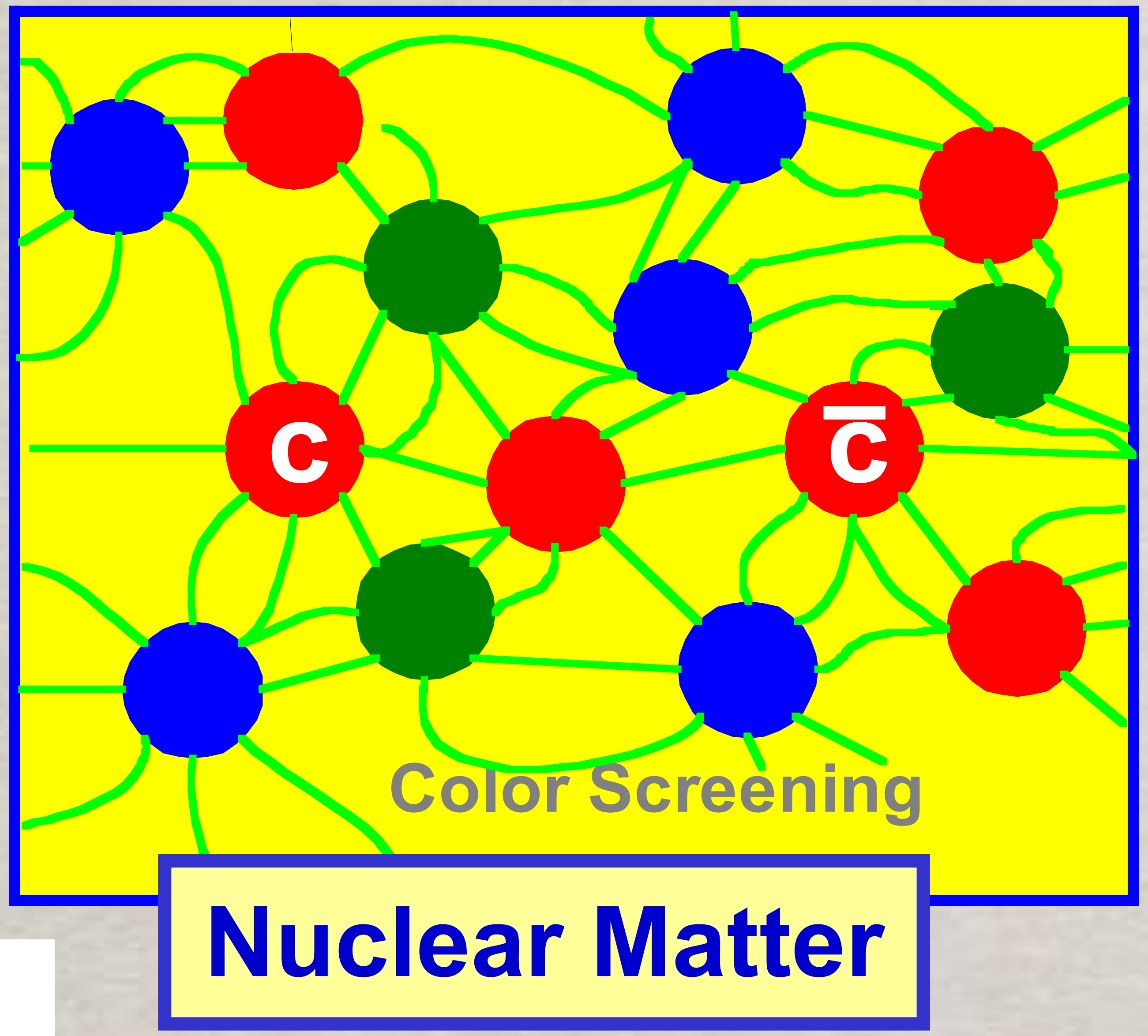

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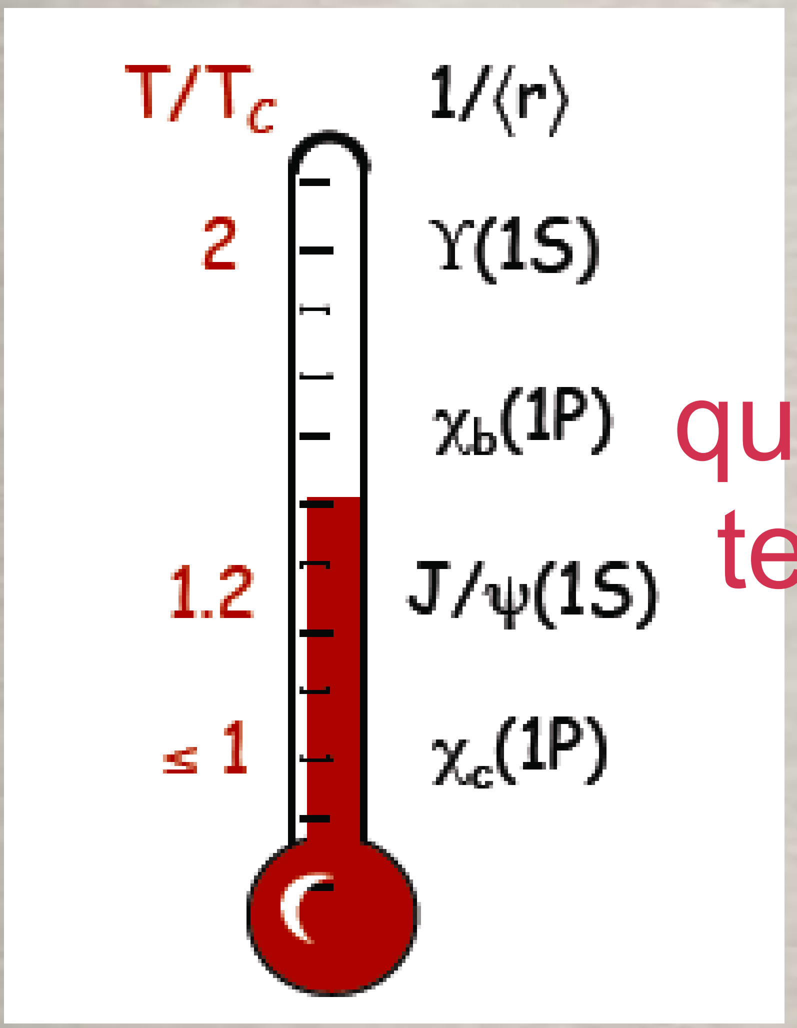
nuclear modificatio

$$R_{AA} = \frac{Yield^{q\bar{q}}_{AA}}{\langle N_{coll} \rangle \times Yield^{q\bar{q}}_{pp}}$$



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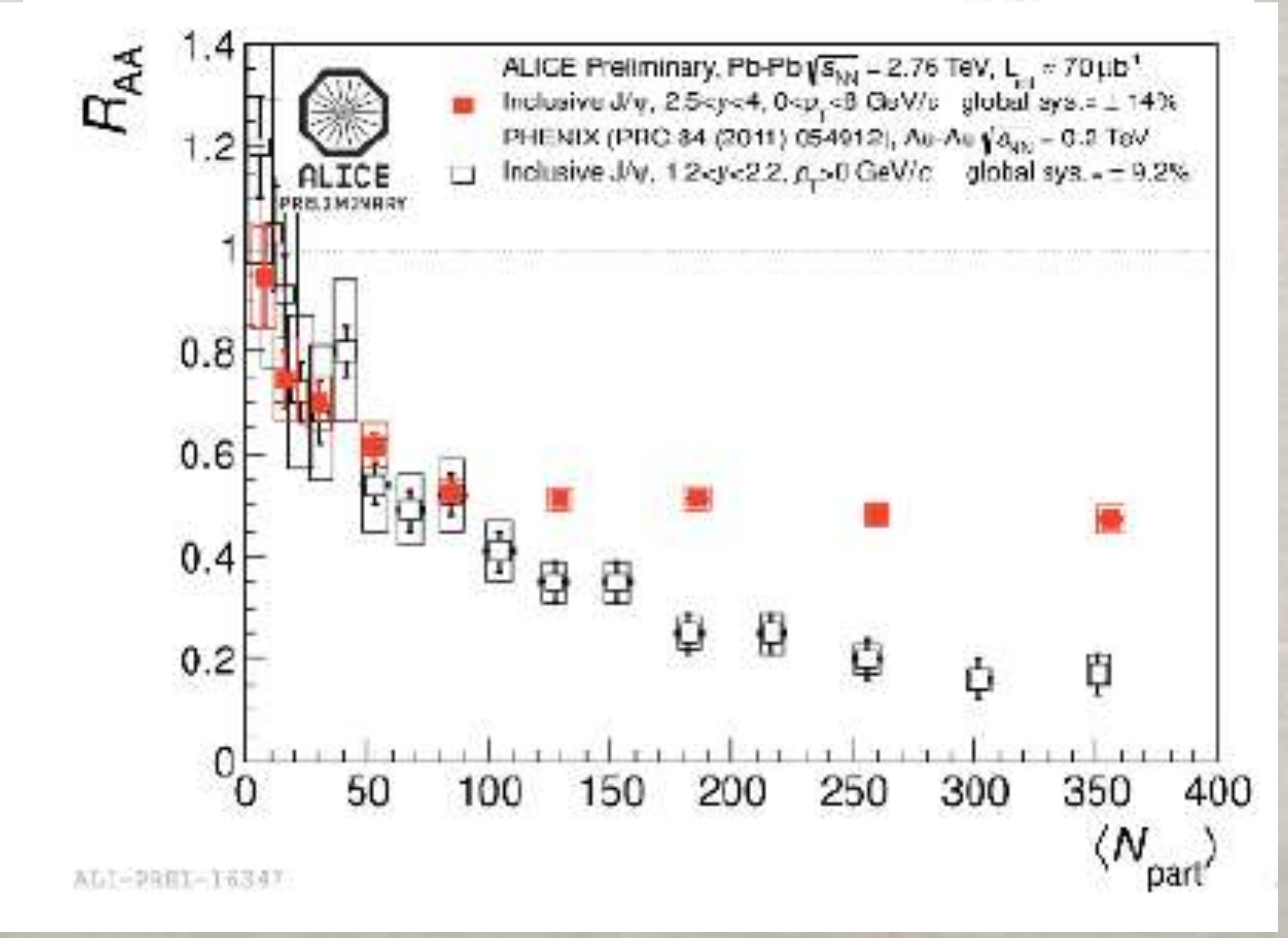

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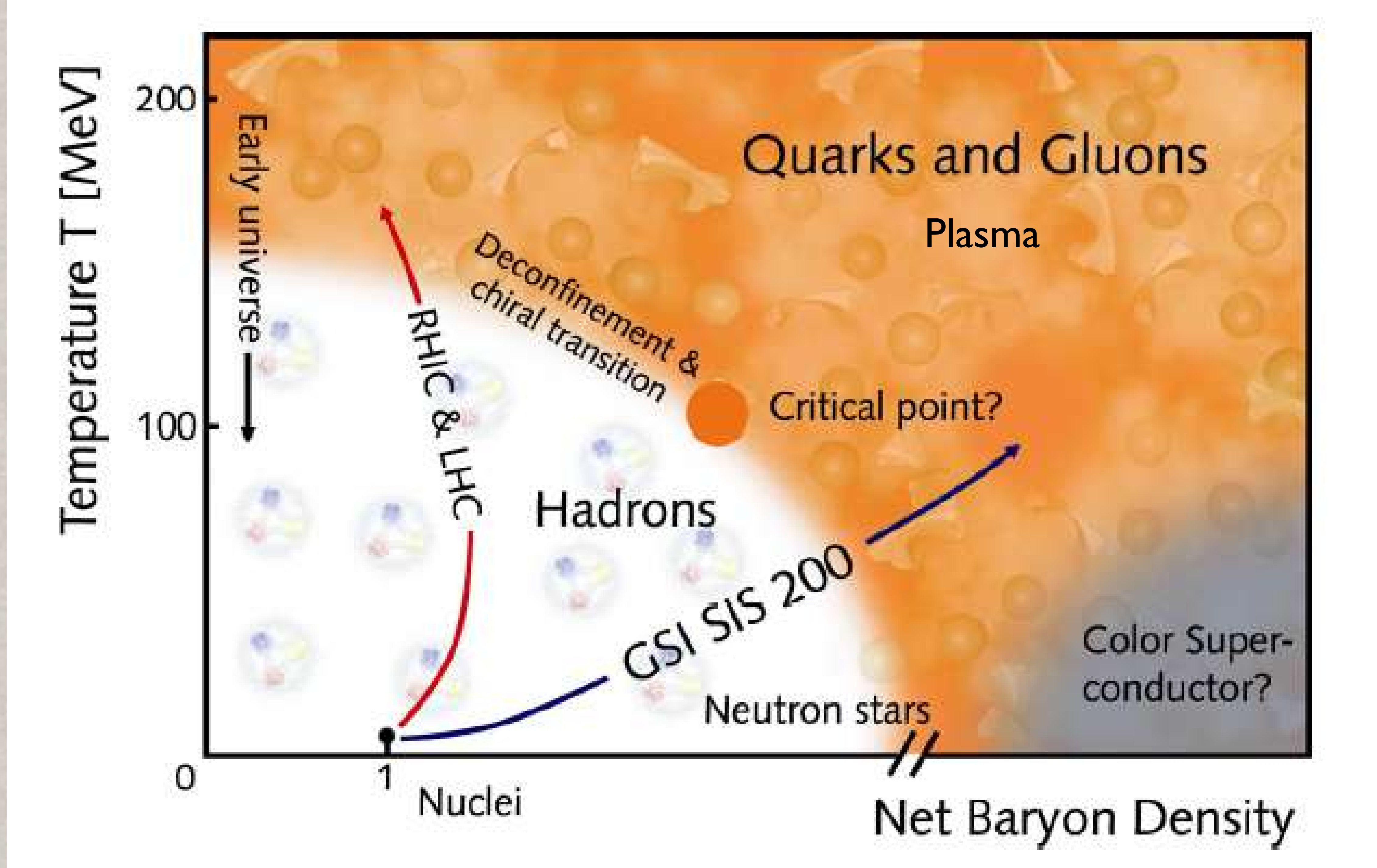
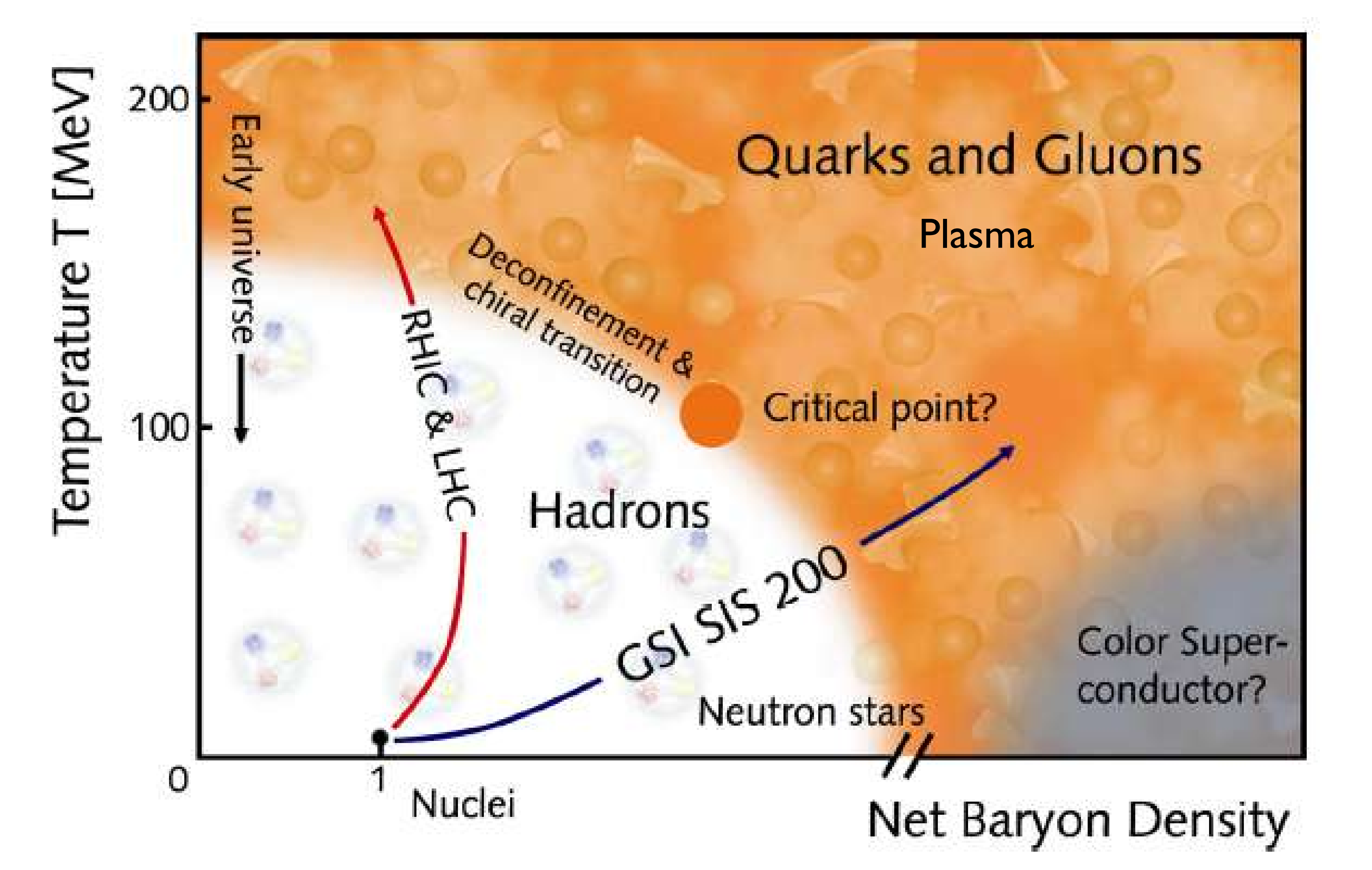


Nuclear Matter

Quarkonium as a deconfinement probe



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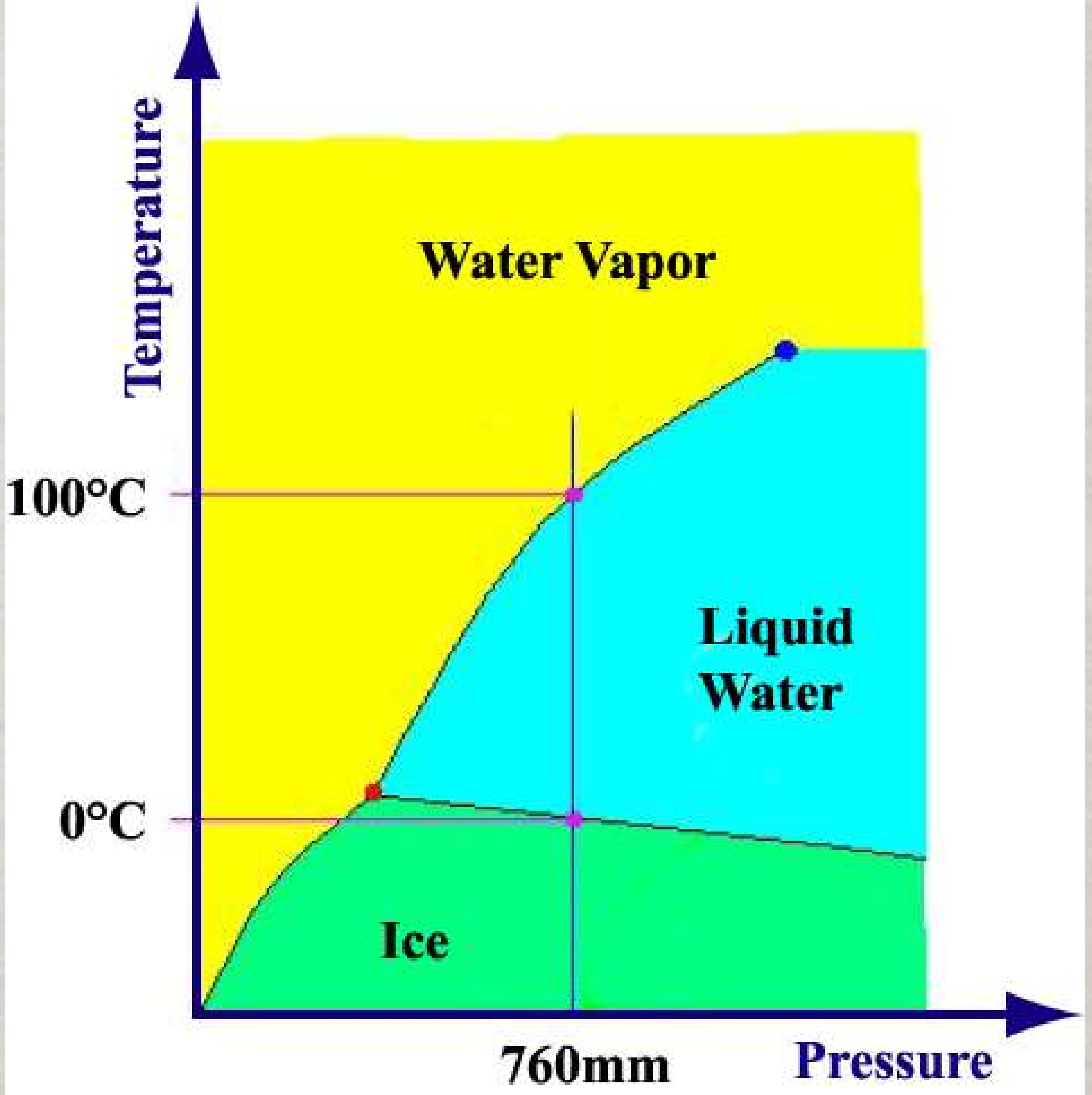
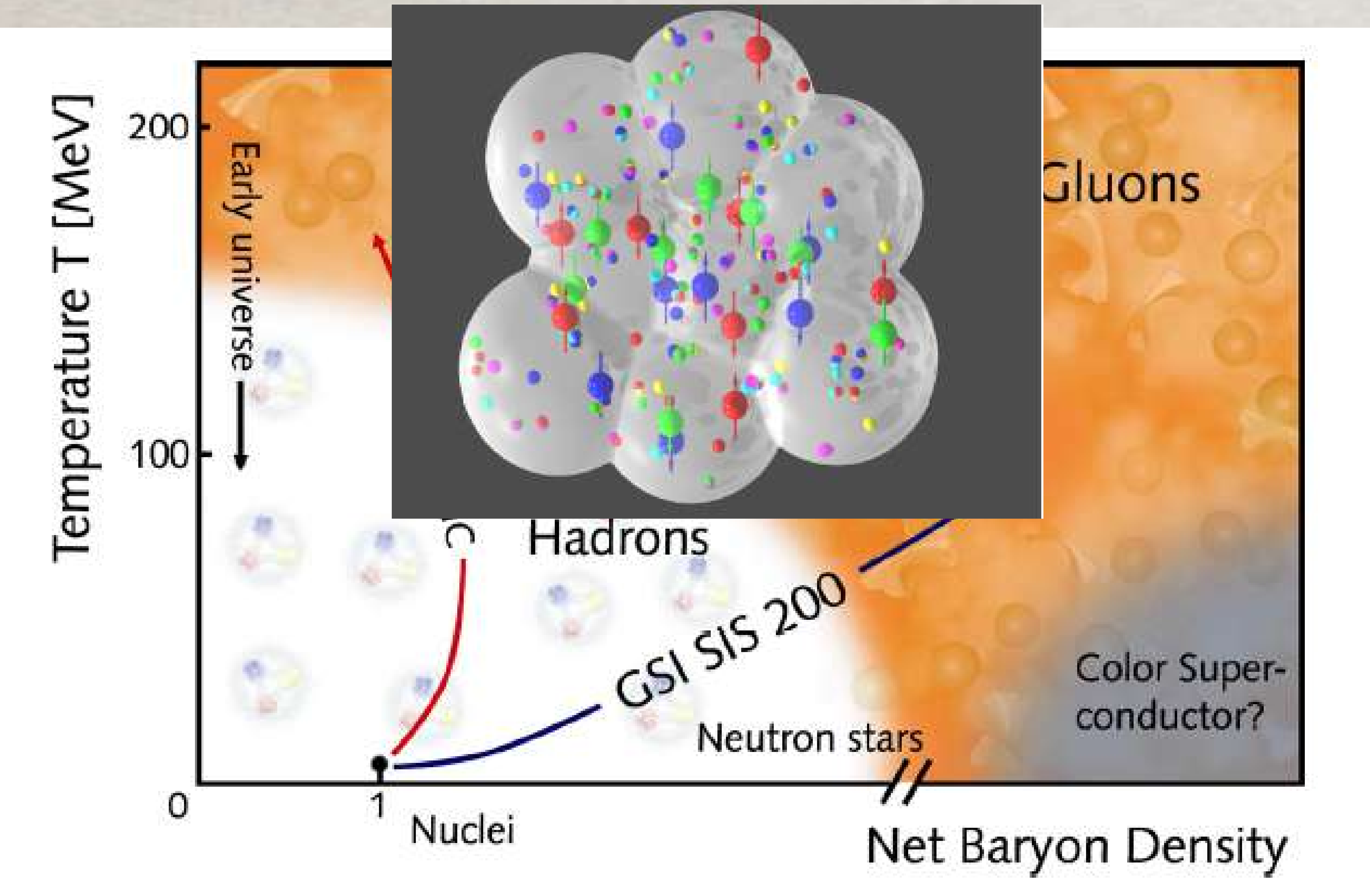


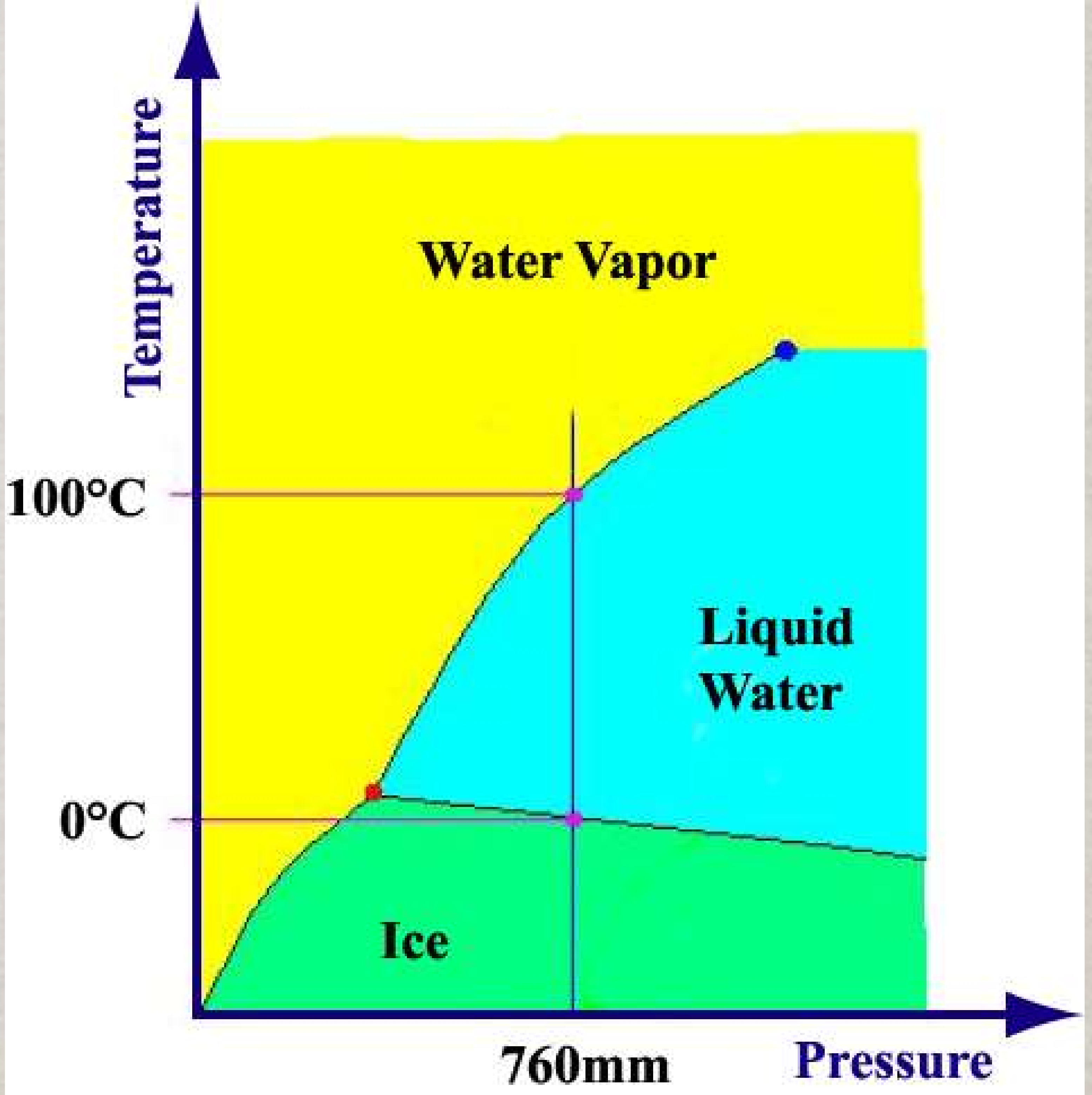
Water

Quarkonium as a deconfinement probe



Nuclear Matter

nuclear matter compressed/heated
T ~170 Mev $\sim 10^{12}\mathrm{K}$
Energy density ~1Gev/fm^3



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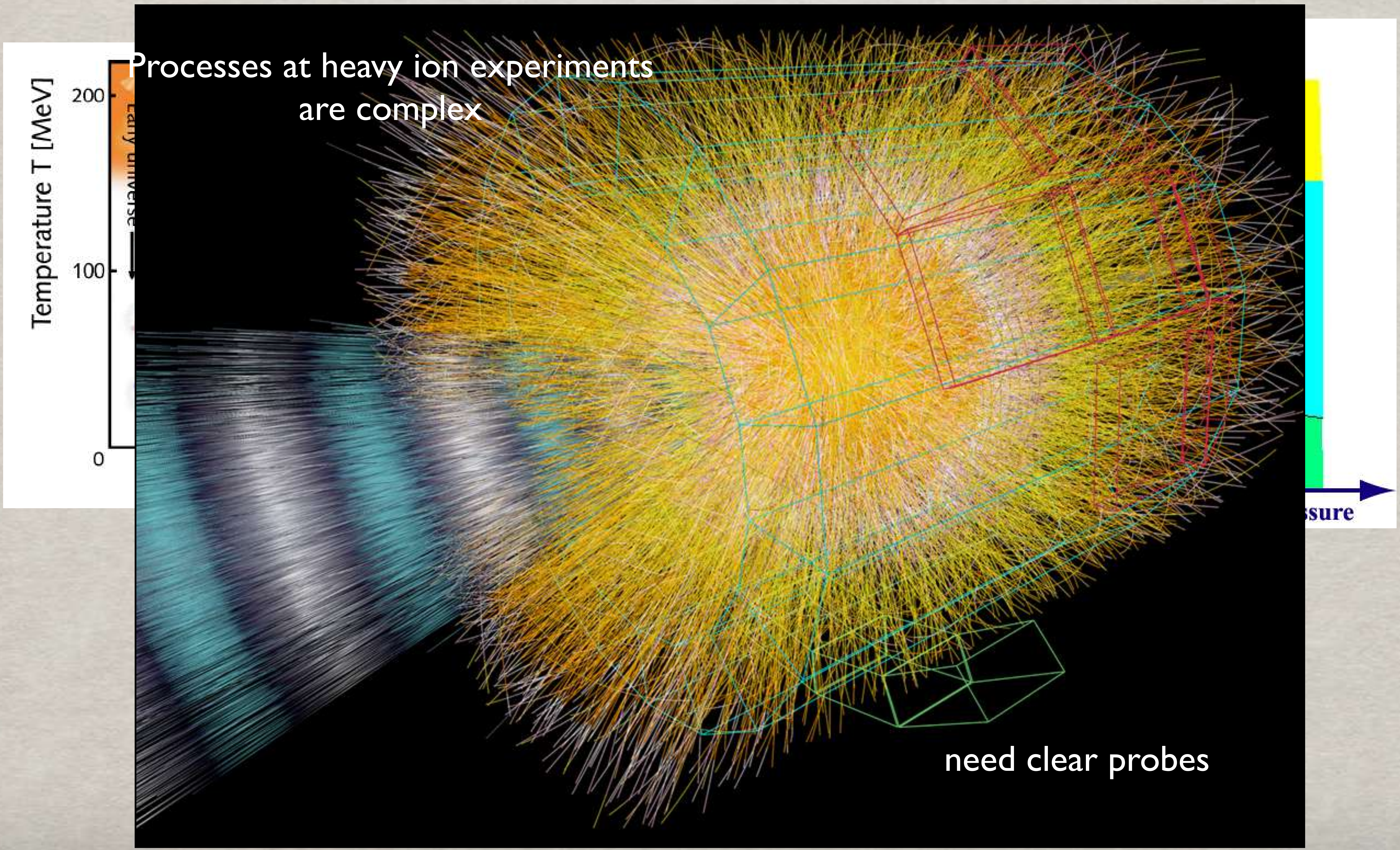

Processes at heavy ion experiments are complex

need clear probes

Quarkonium as an exploration tool of physics of Standard Model and beyond

- Quarkonium can serve for the precise extraction of Standard Model parameters: heavy quark masses and strong coupling constant α_s

- Quarkonium in its exotic manifestations probes the nonstandard characteristics of a nonabelian gauge theory: hybrids, multi quark configurations

- The large m makes Quarkonium an ideal probe of new light particles

BaBar light-Higgs & dark-photon searches

Mode	Mass range (GeV)	BF upper limit (90% CL)
$\Upsilon(2S,3S) \to \gamma A^0, A^0 \to \mu^+\mu^-$	$0.21 < m_A < 9.3$	$(0.3 - 8.3) \times 10^{-6}$
$\Upsilon(3S) \to \gamma A^0, A^0 \to \tau^+\tau^-$	$4.0 < m_A < 10.1$	$(1.5 - 16) \times 10^{-5}$
$\Upsilon(2S,3S) \to \gamma A^0, A^0 \to$ hadrons	$0.3 < m_A < 7.0$	$(0.1 - 8) \times 10^{-5}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to \chi\bar{\chi}$	$m_\chi < 4.5\,\mathrm{GeV}$	$(0.5 - 24) \times 10^{-5}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to$ invisible	$m_A < 9.2\,\mathrm{GeV}$	$(1.9 - 37) \times 10^{-6}$
$\Upsilon(3S) \to \gamma A^0, A^0 \to$ invisible	$m_A < 9.2\,\mathrm{GeV}$	$(0.7 - 31) \times 10^{-6}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to g\bar{g}$	$m_A < 9.0\,\mathrm{GeV}$	$10^{-6} - 10^{-2}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to s\bar{s}$	$m_A < 9.0\,\mathrm{GeV}$	$10^{-5} - 10^{-3}$

invisible decays of Y(1S) at Belle

Experimental data on quarkonium are abundant!

B-FACTORIES (Belle, BABAR): Heavy Mesons Factories

Fermilab CDF, D0, E835   Hera

RHIC (Star, Phenix), NA60

CLEO-c BES tau charm factories

CLEO-III bottomonium factory

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CMS ATLAS LHCb

heavy ions experiments ALICE at CERN, RHIC

BELLEII at SuperKEKB

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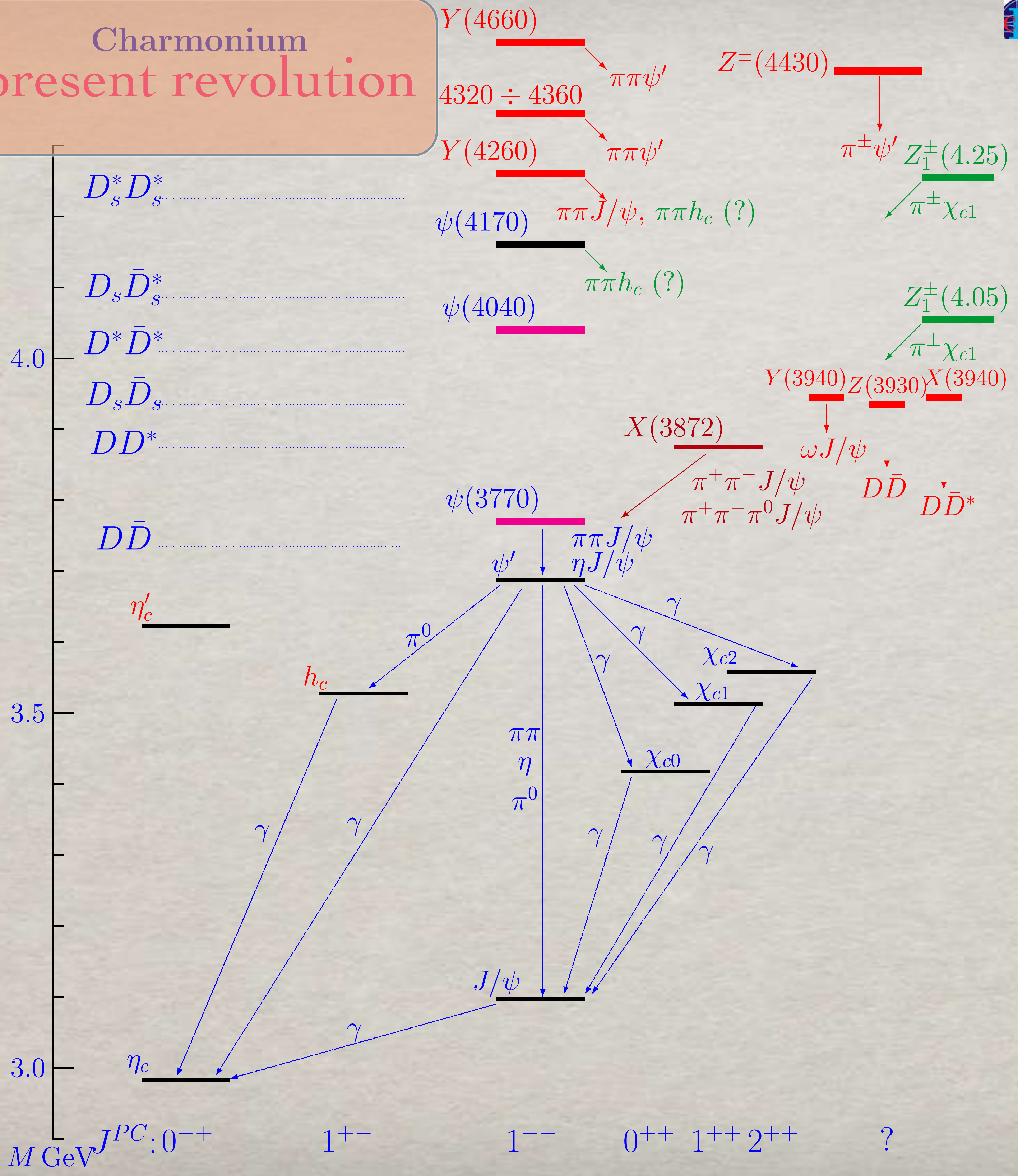
and in the future PANDA at FAIR, Electron Ion Collider, target experiments

Charmonium
the present revolution



THE COLLIDER DETECTOR AT FERMILAB

DØ

BELLE

BABAR Collaboration Home Page

BES

CLEO

ATLAS

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LHCb

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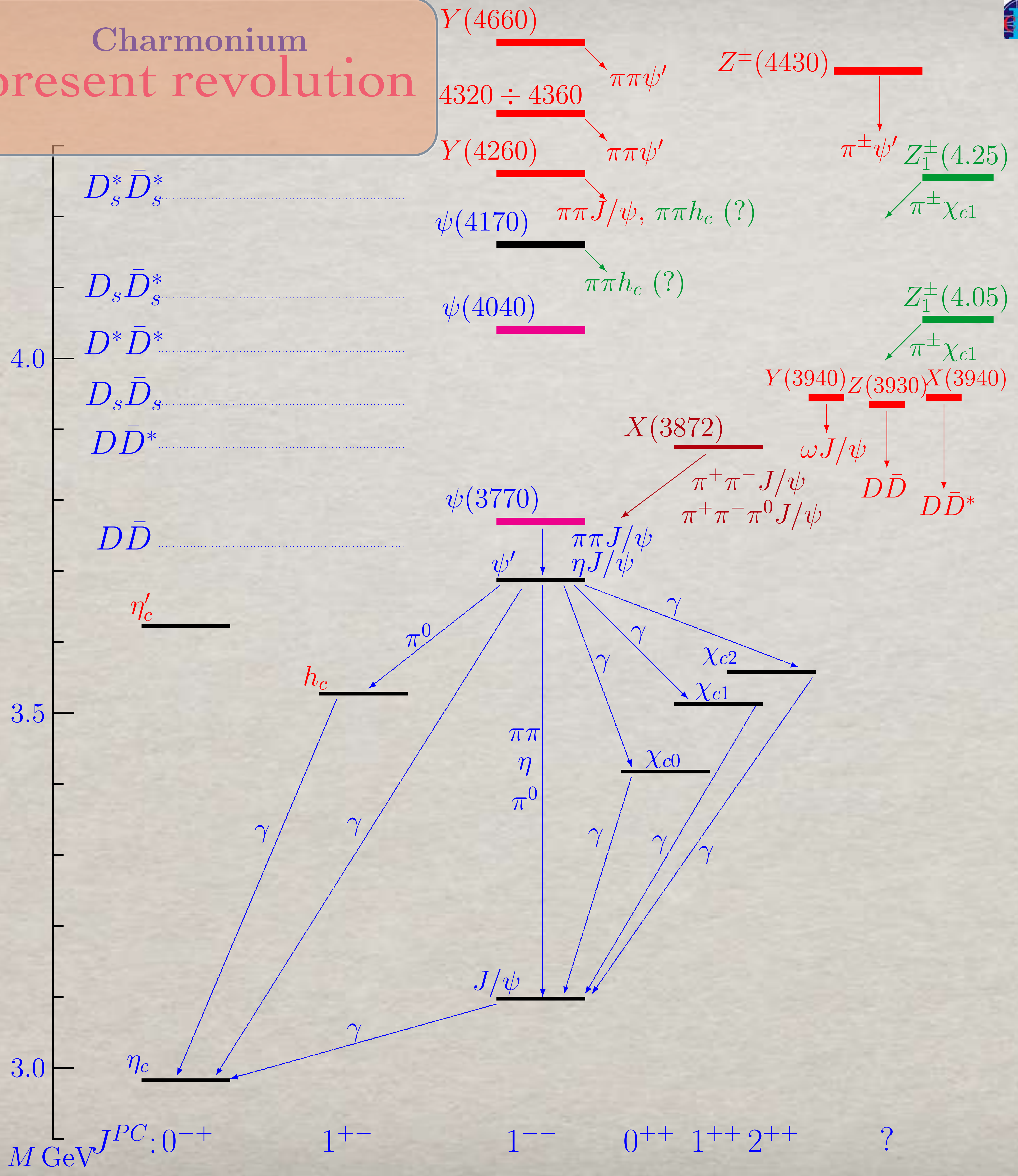

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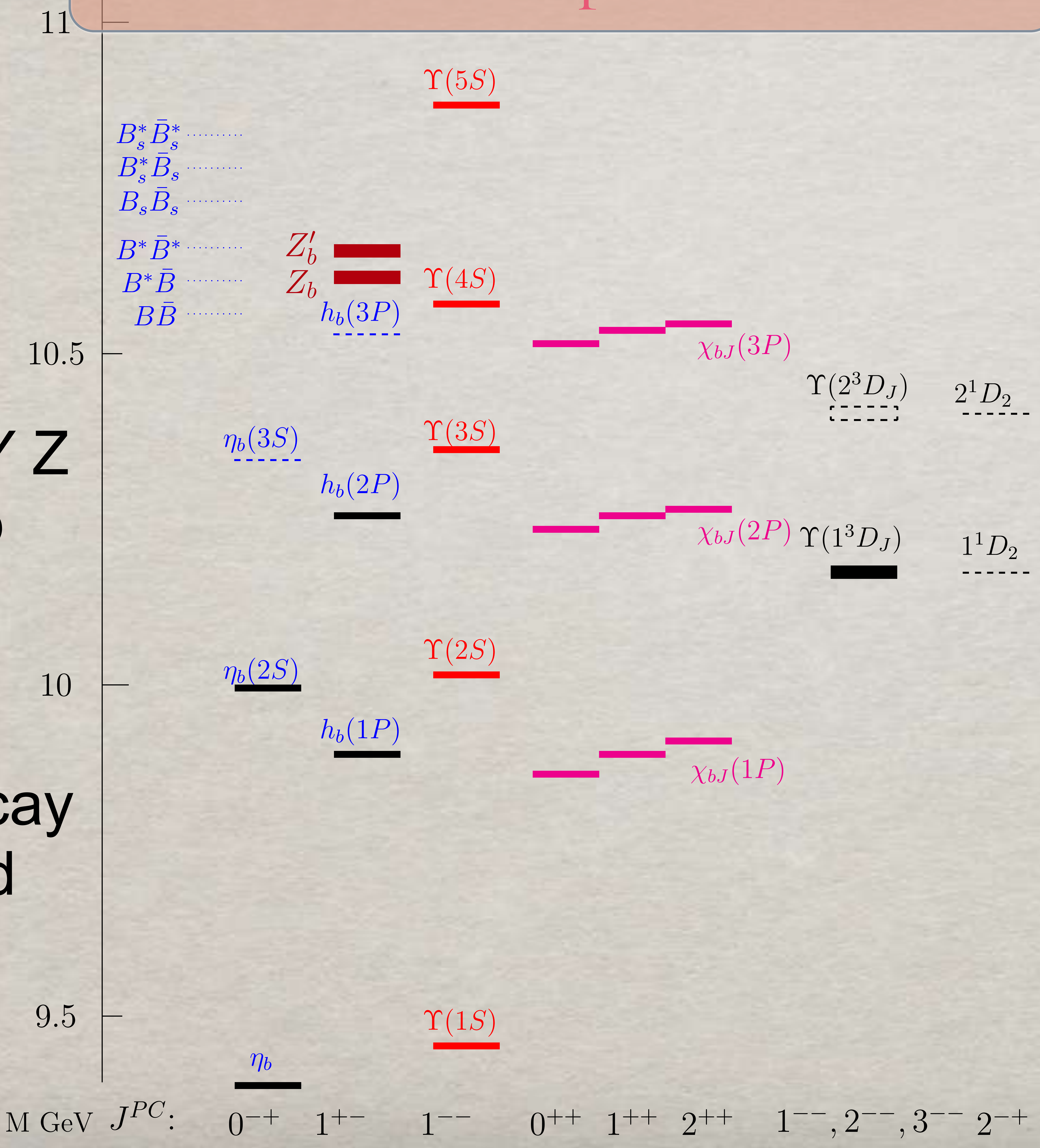
bottomonium: the present revolution



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CLEO

QCD theory of quarkonium: a very challenging problem

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Close to the bound state $\alpha_s \sim v$

QCD THEORY OF QUARKONIUM: A VERY CHALLENGING PROBLEM

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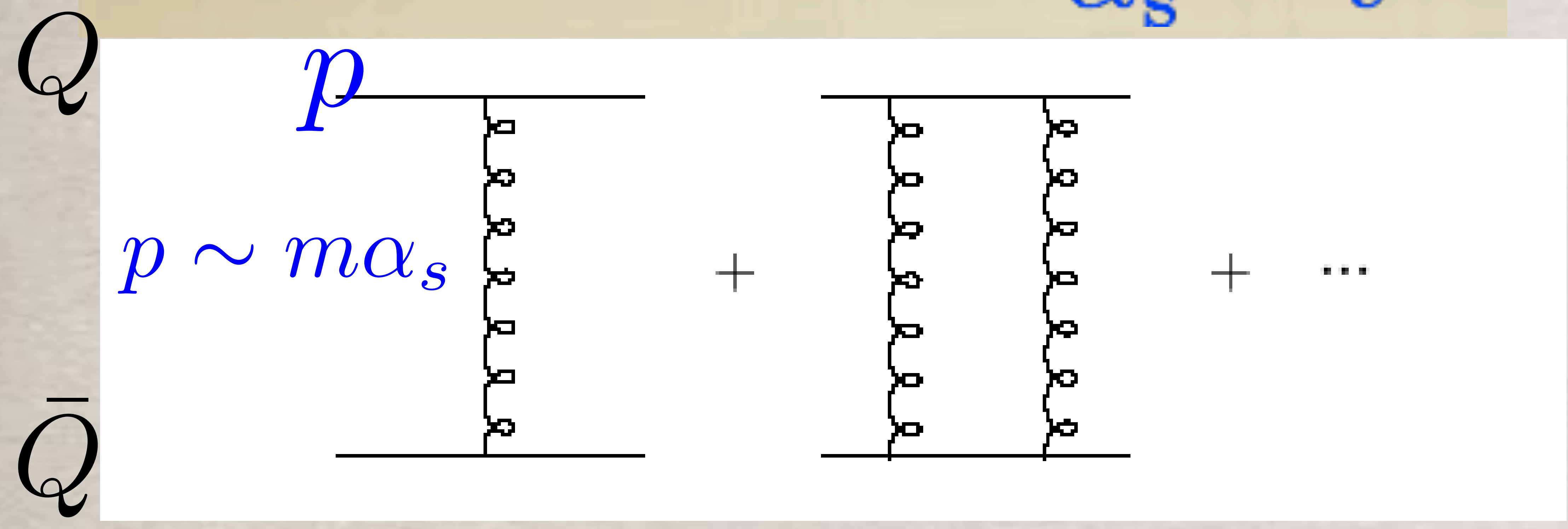

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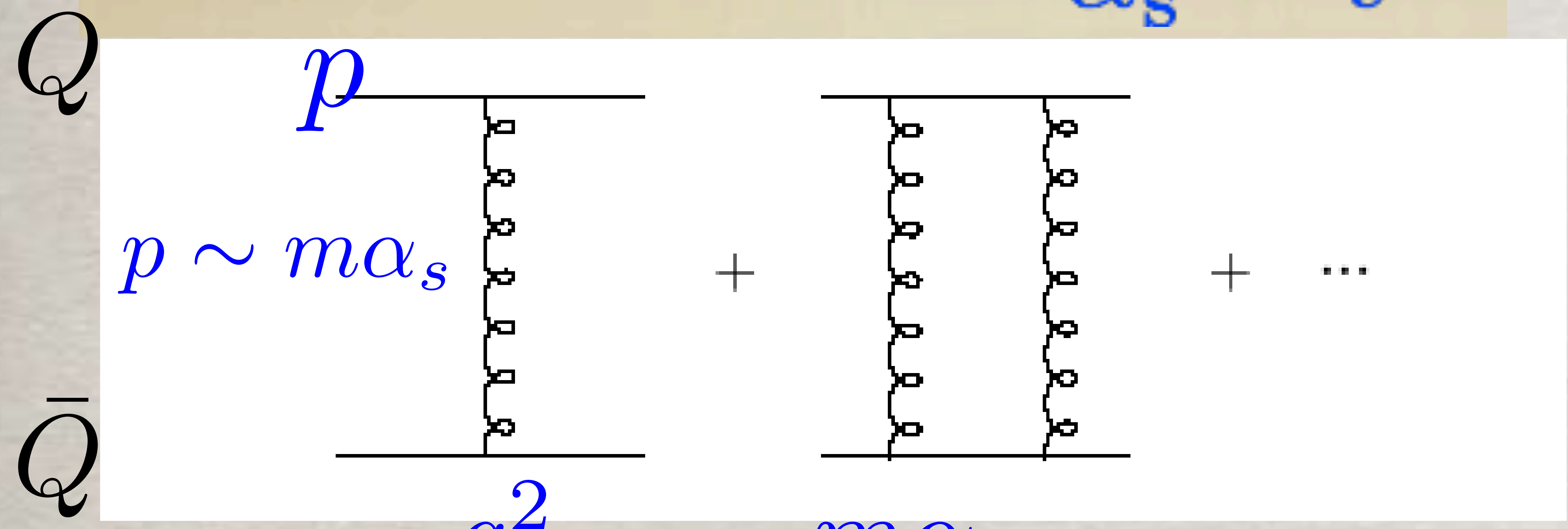

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

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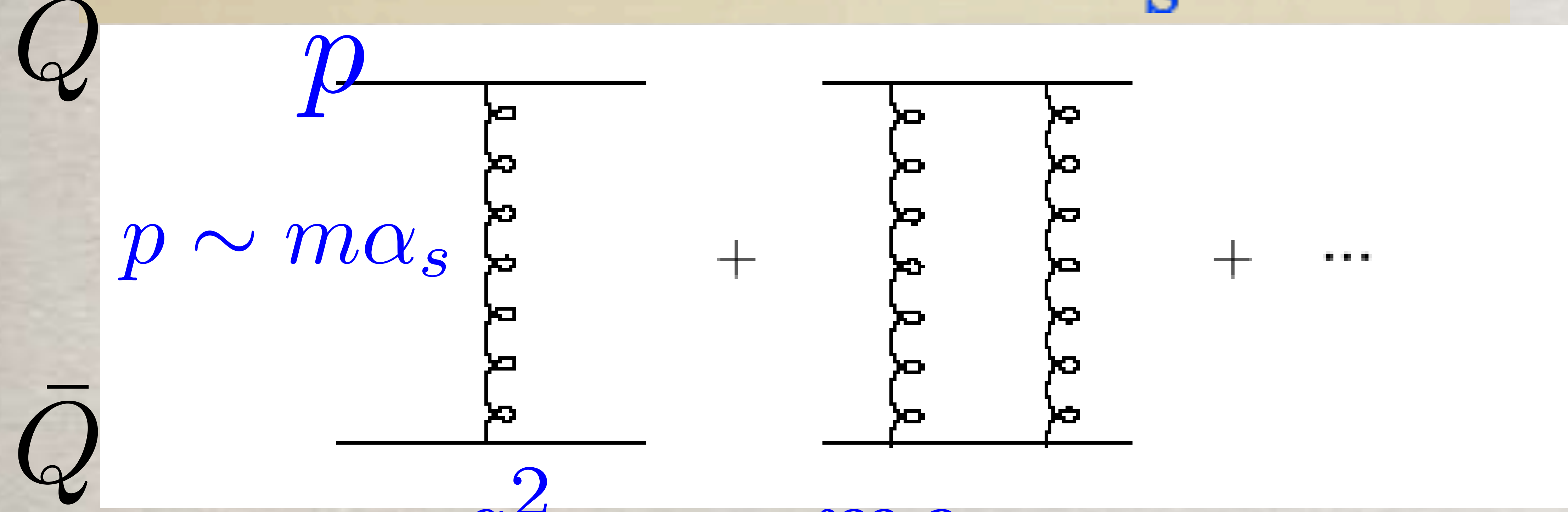

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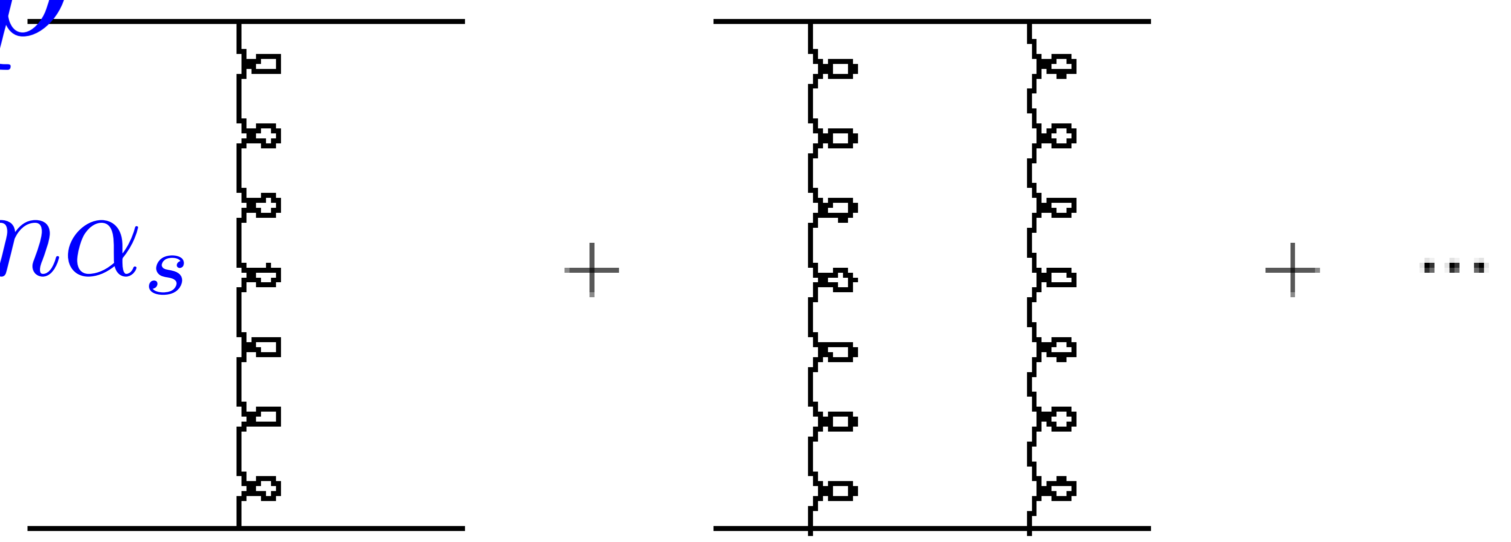
$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

QCD THEORY OF QUARKONIUM: A VERY CHALLENGING PROBLEM

Close to the bound state $\alpha_s \sim v$

Q p

$p \sim m\alpha_s$



$\bar{Q}$

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- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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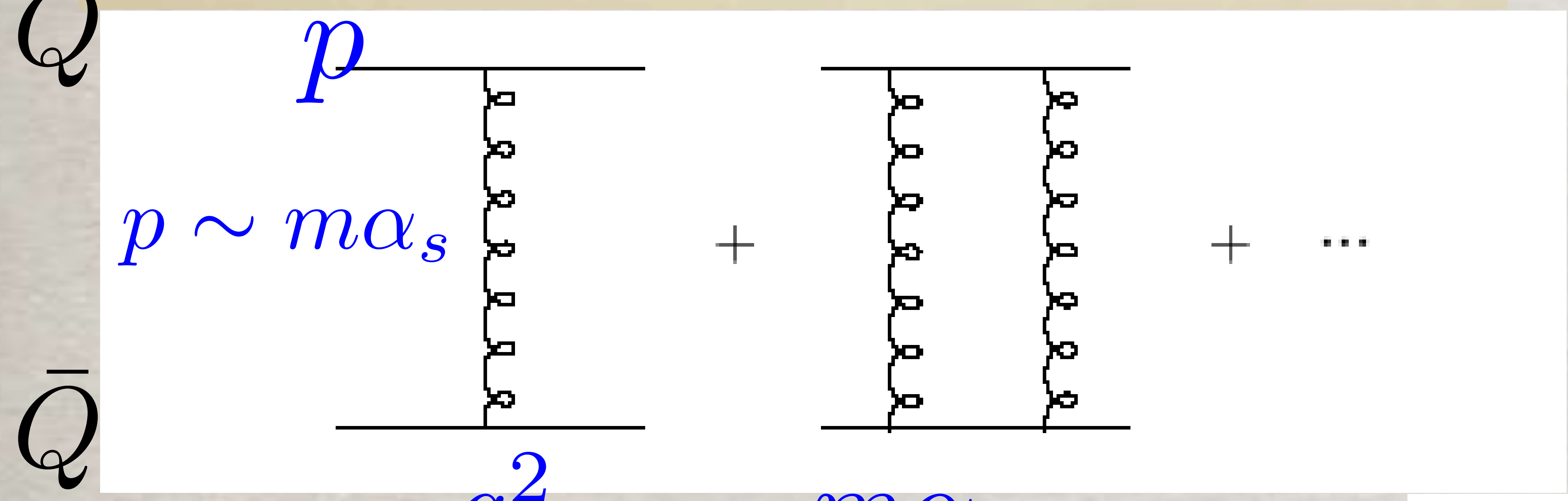

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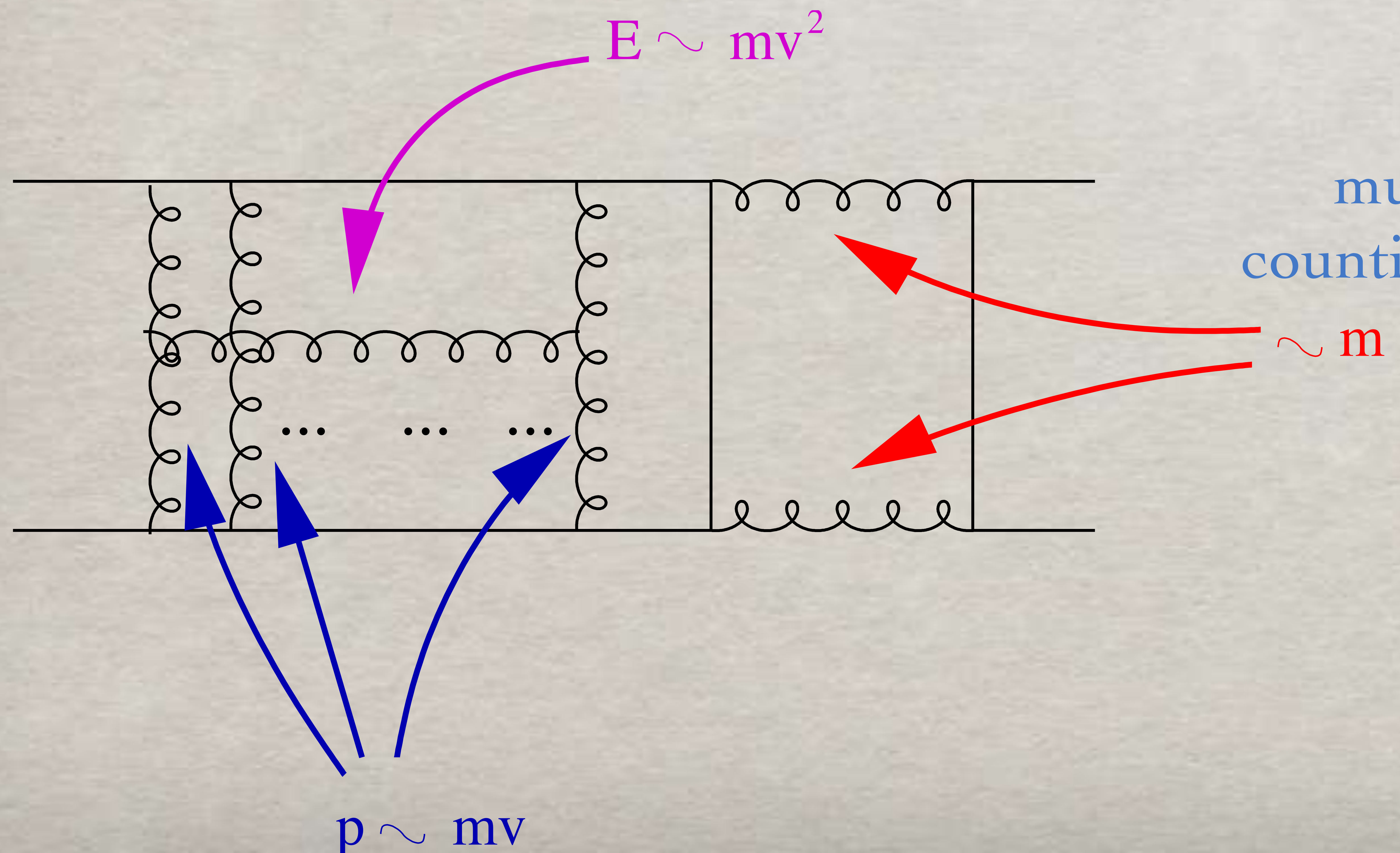


multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

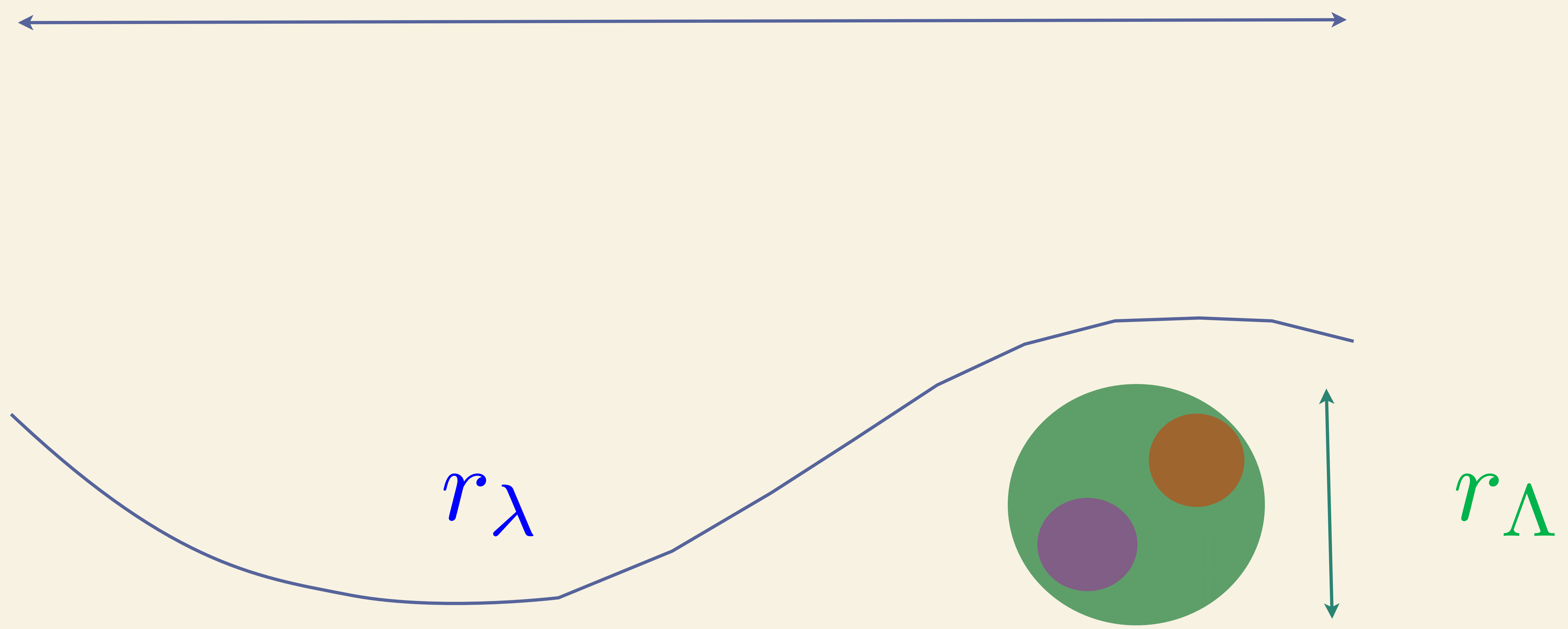
Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

SOLUTION:

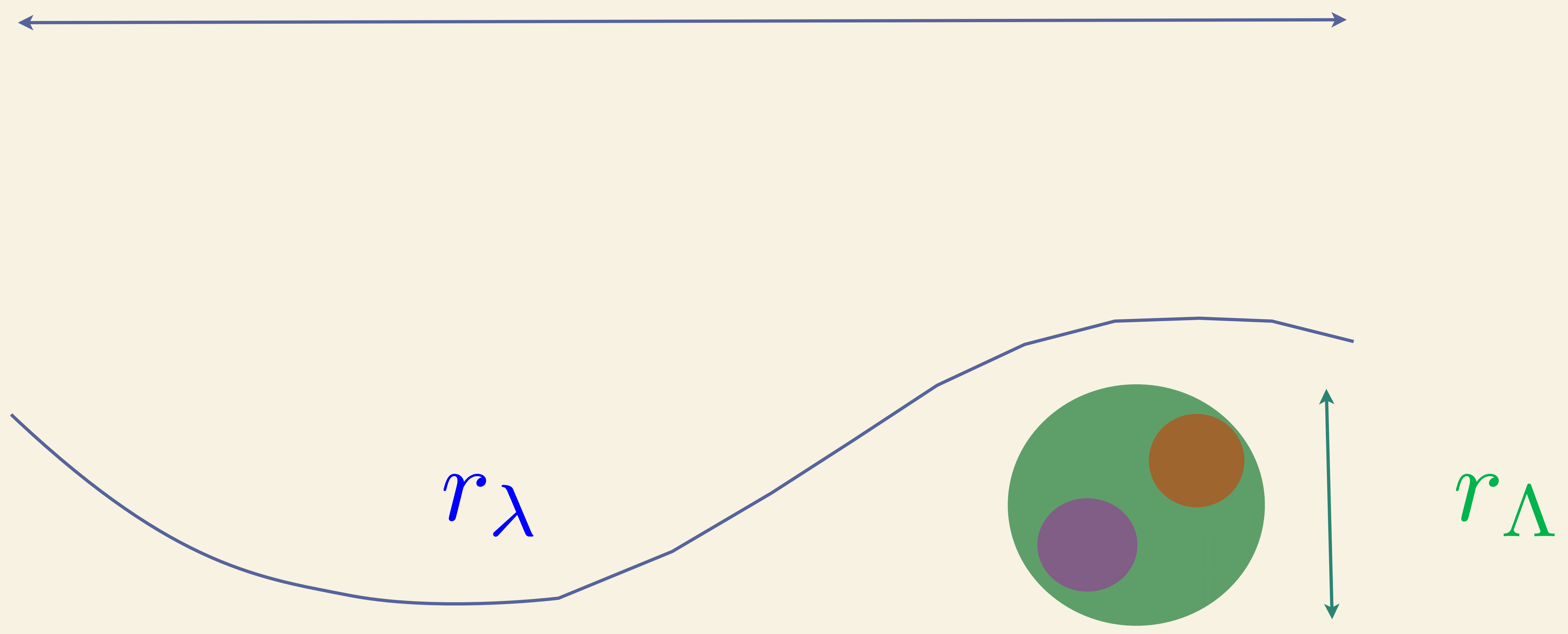
FORMULATE A
HIERARCHY OF EFFECTIVE FIELD THEORIES
IN CORRESPONDENCE OF
THE HIERARCHY OF SCALES

EFT for a system with two scales



An effective field theory makes the expansion in λ/Λ explicit at the Lagrangian level.

EFT for a system with two scales



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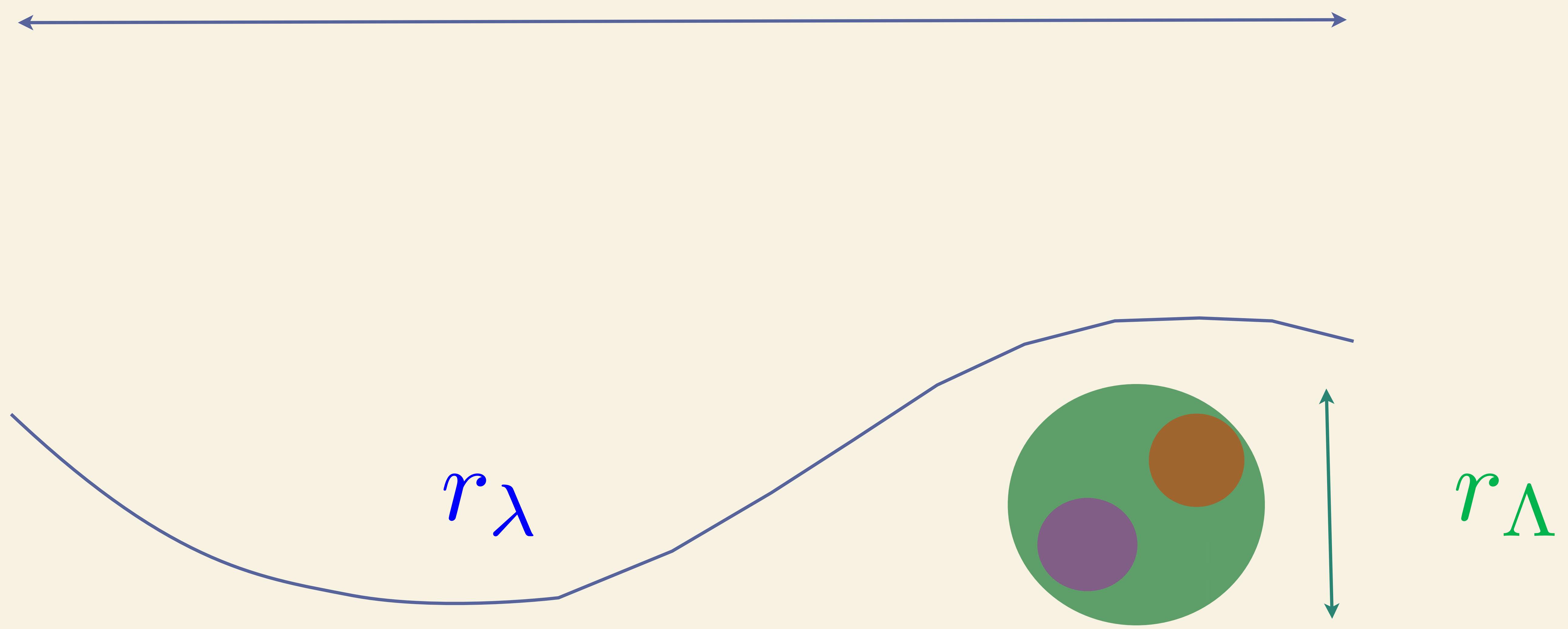

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RANGE OF VALIDITY OF THE EFT: ENERGY $< \mu$

$\Rightarrow$ $\mathcal{L}_{\rm EFT}$ is made of all operators O_n that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}$.

Effective Field Theories

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(\Lambda, \mu) \frac{O_n(\mu, \lambda)}{\Lambda^n}$$

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low energy operator

Wilson coefficient

large scale

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- The matching coefficients $c_n(\Lambda/\mu)$ encode the non-analytic behaviour in Λ. They are calculated by imposing that $\mathcal{L}_{\rm EFT}$ and $\mathcal{L}$ describe the same physics at any finite order in the expansion: matching procedure.

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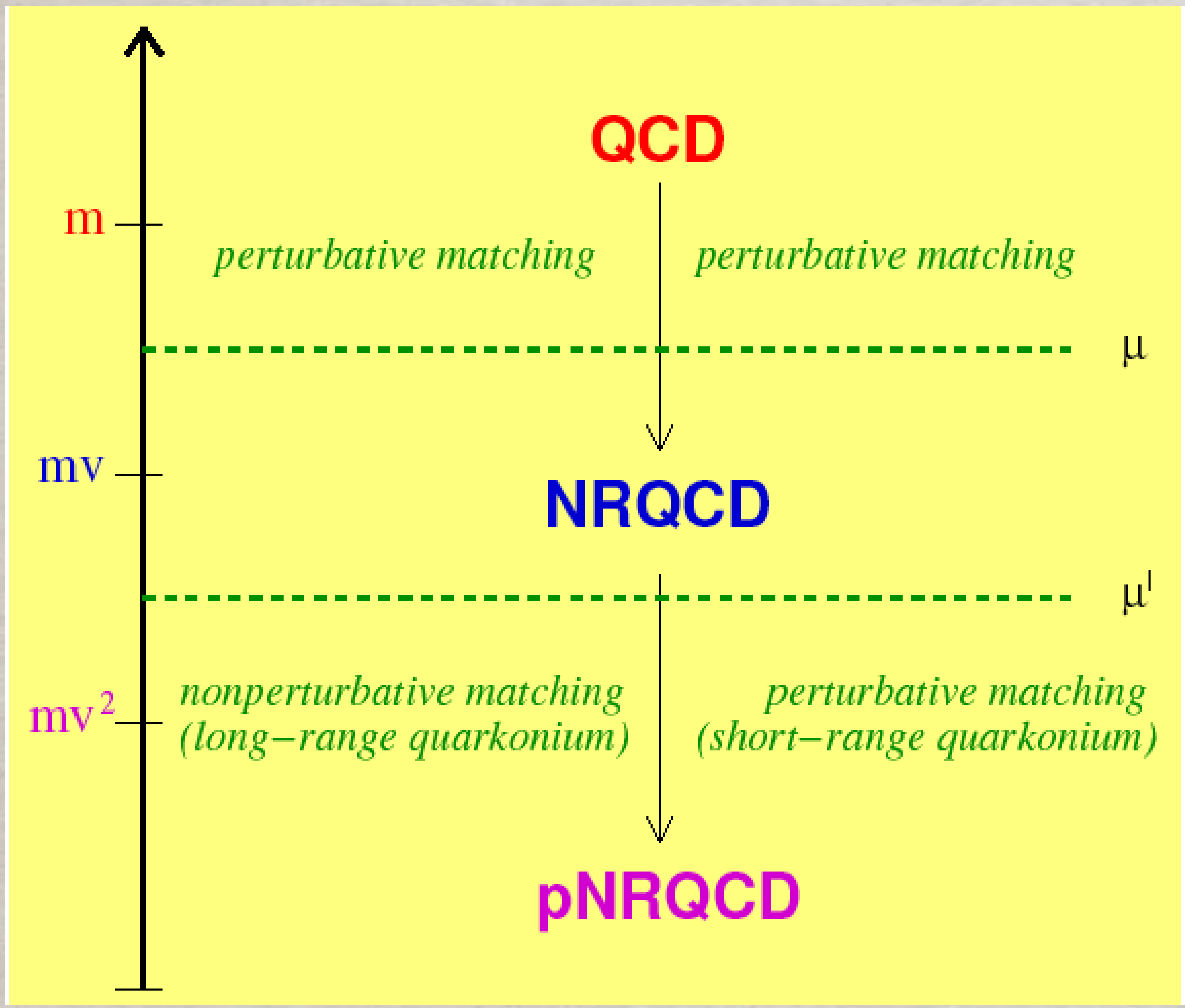
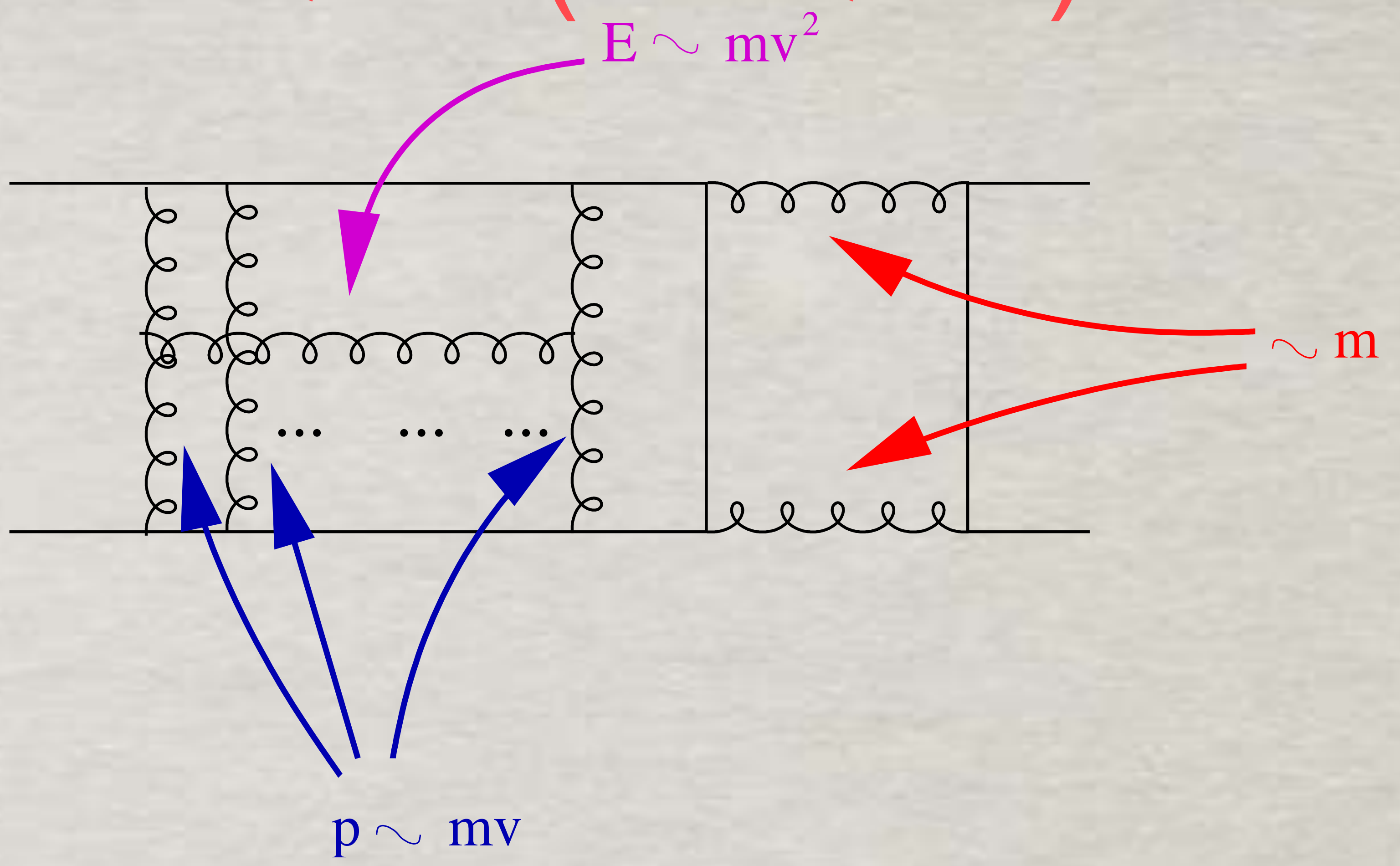
Matching

- If $\Lambda \gg \Lambda_{\text{QCD}}$ then $c_n(\Lambda/\mu)$ may be calculated in perturbation theory.

- Symmetries of the system become manifest;
- Large $\log(\Lambda/\lambda)$ can be resummed via RG. (Renormalization group)

Quarkonium with NREFTs: Non Relativistic QCD (NRQCD)





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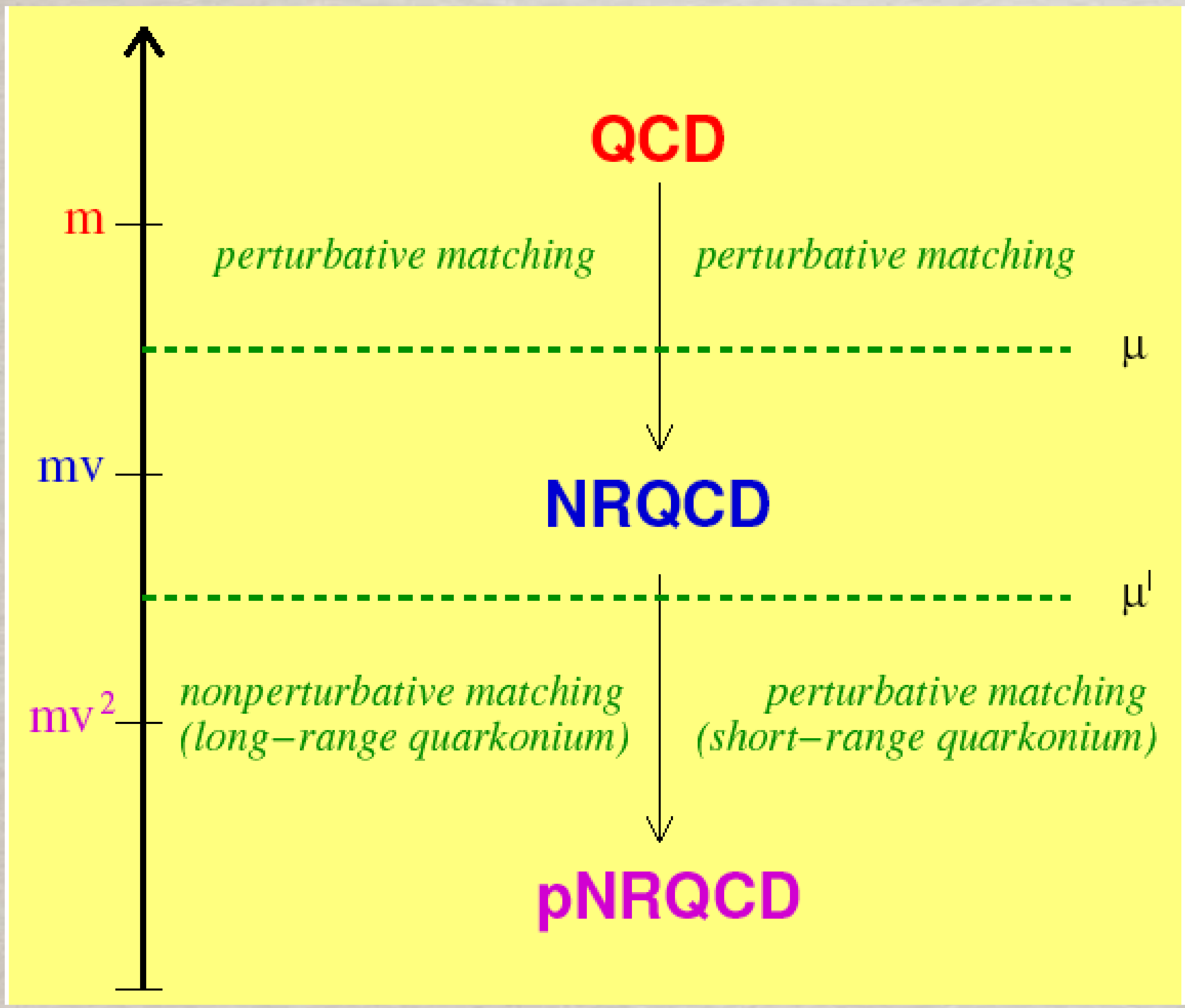

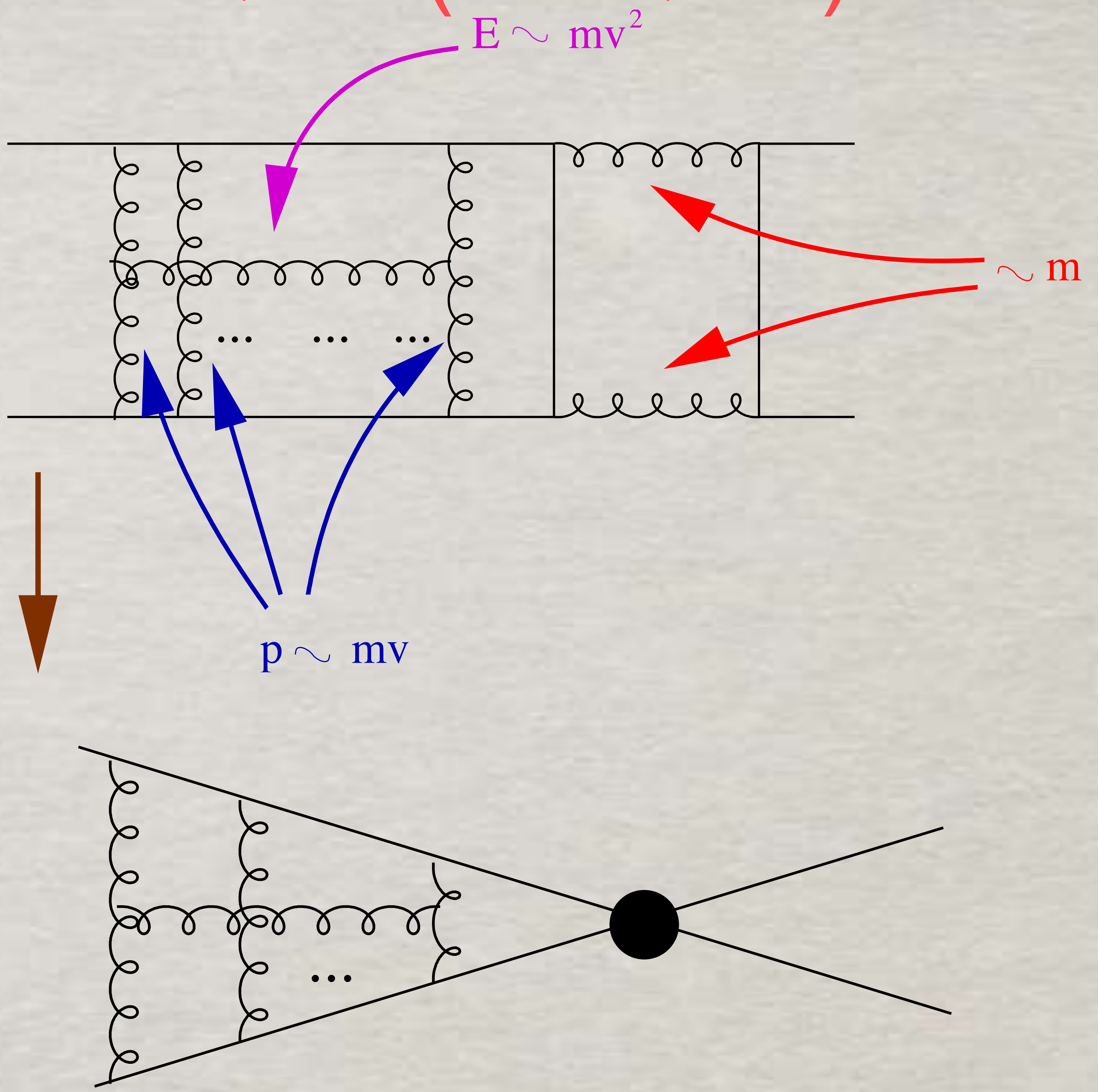

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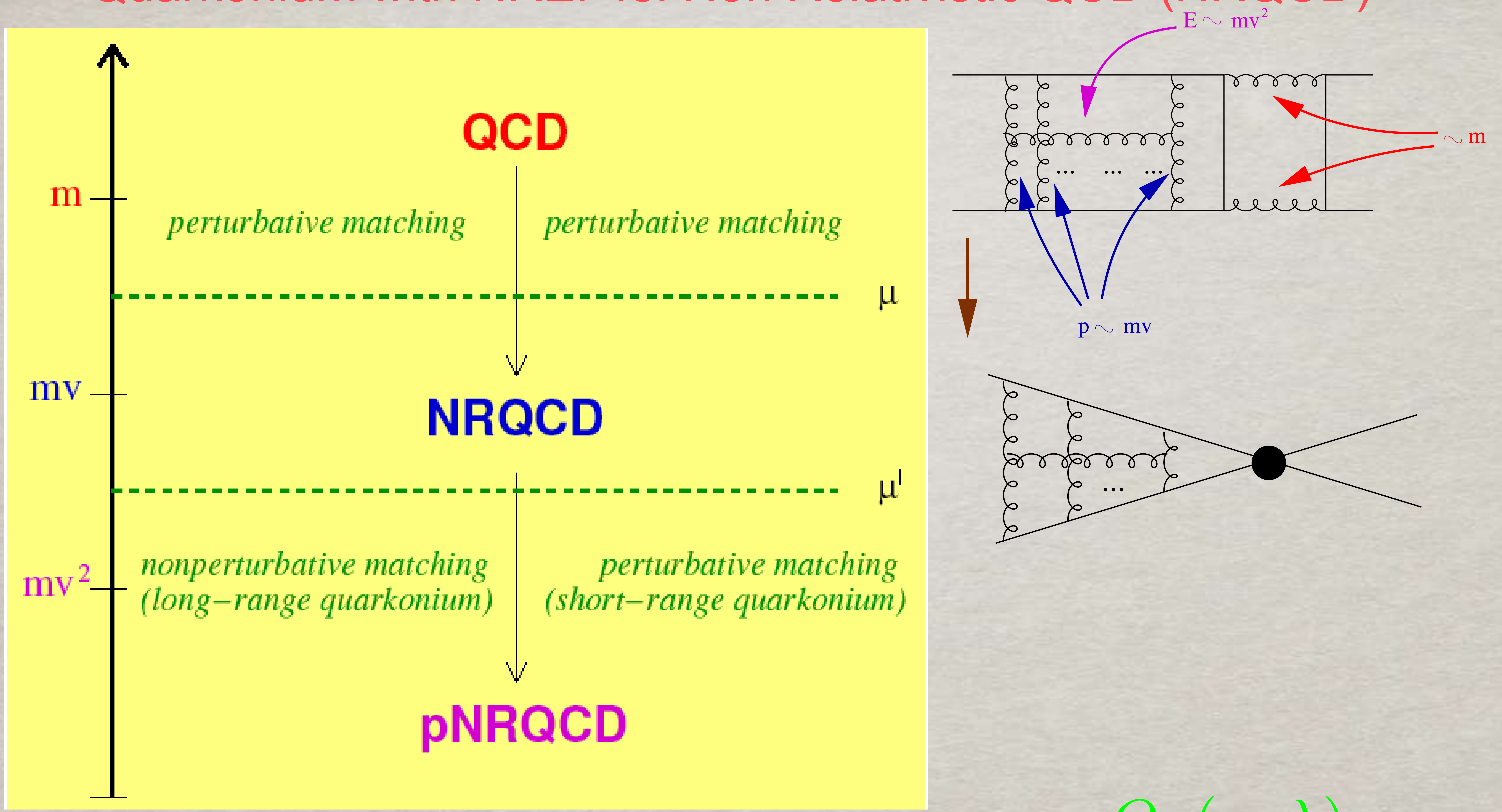
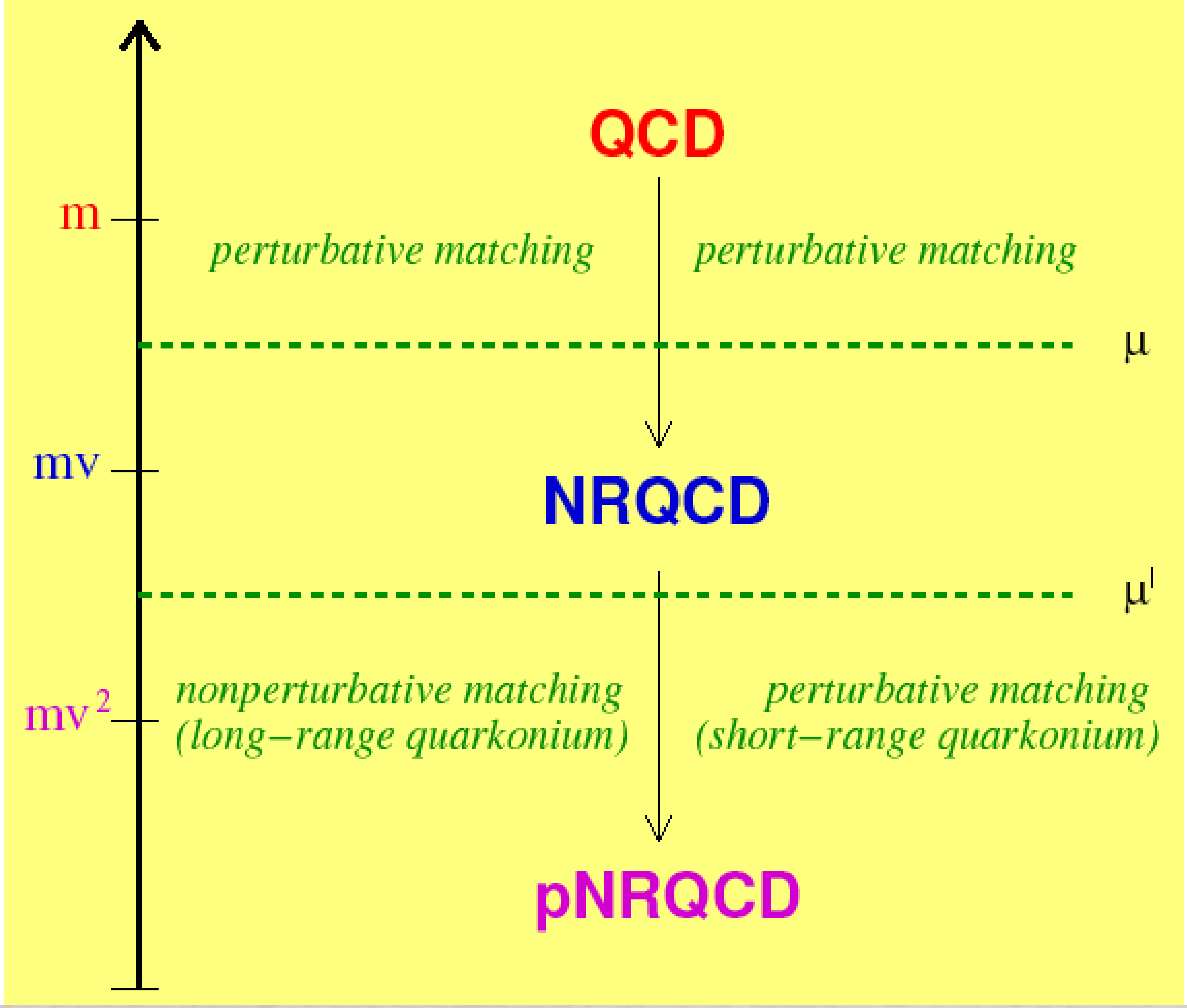
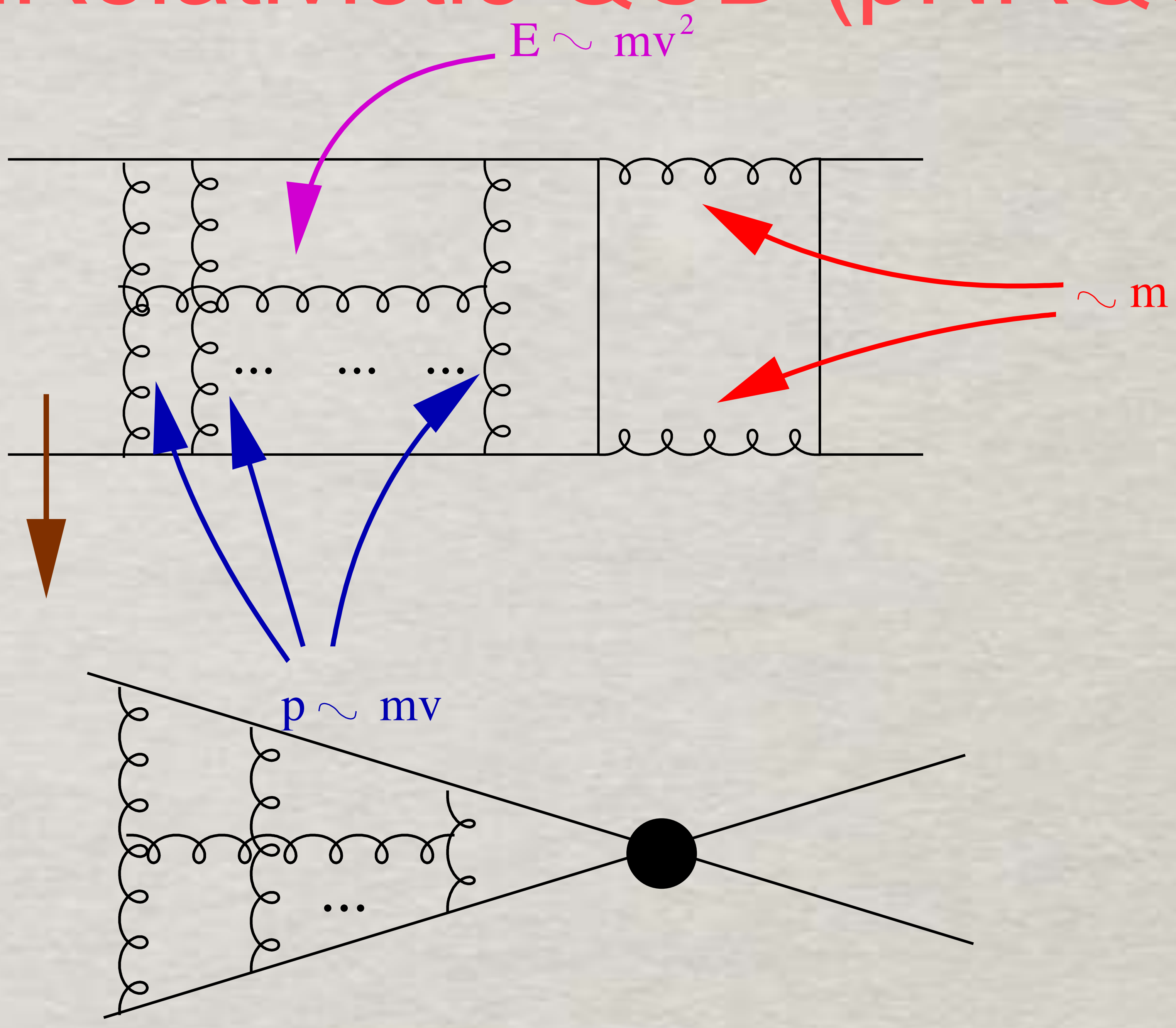


$$\mathcal{L}_{\mathrm{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu, \lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





(with perturbative matching)

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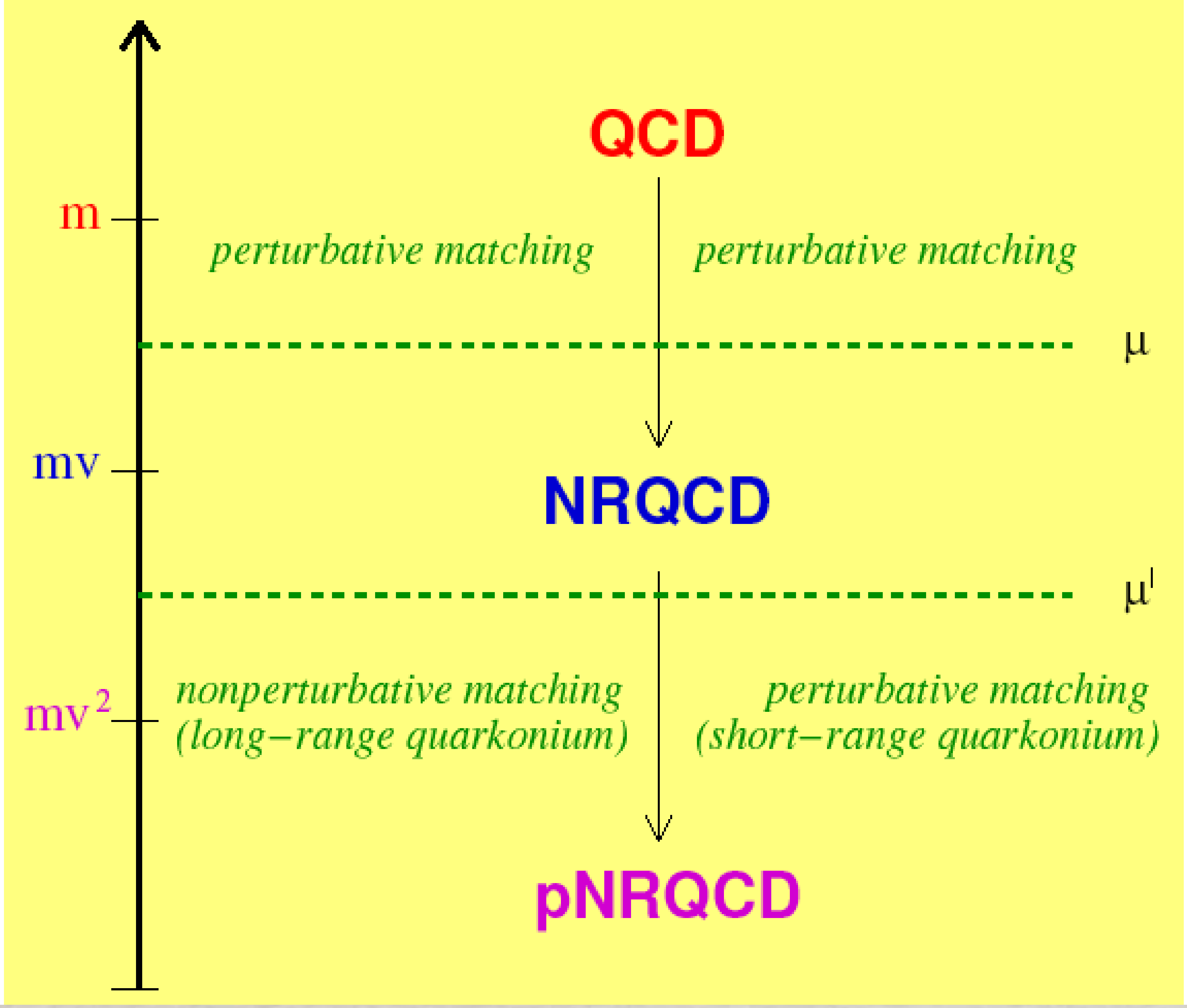

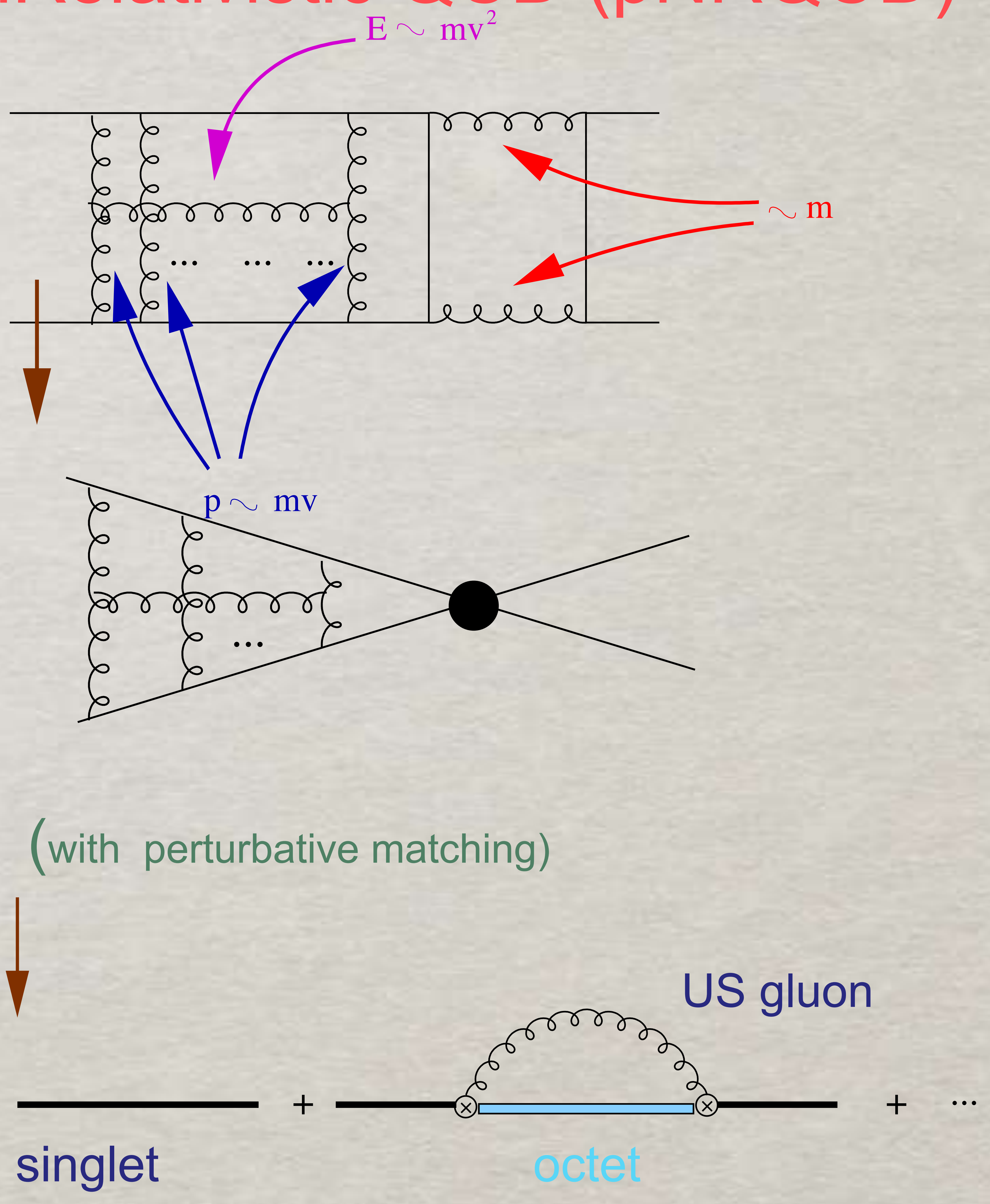

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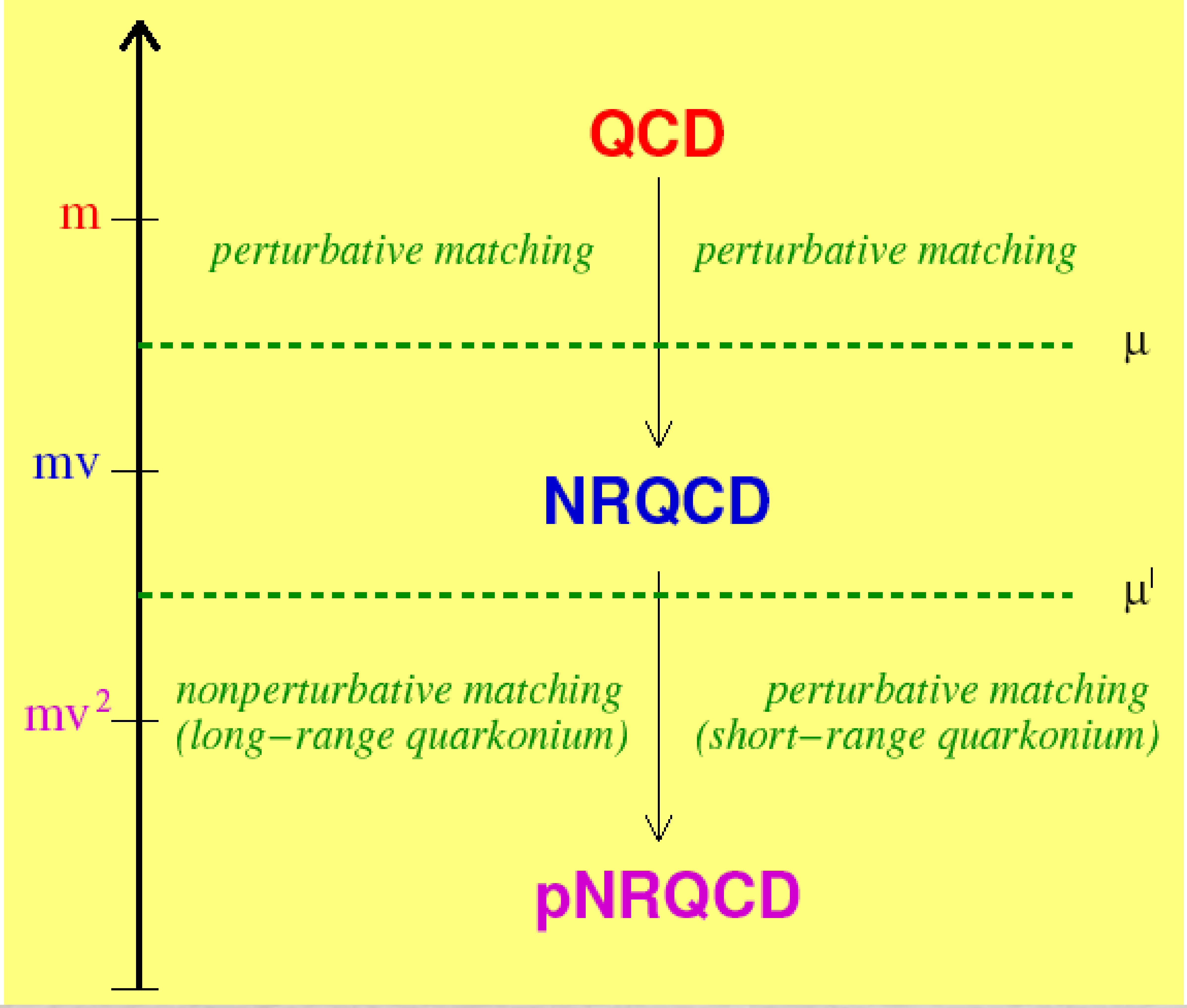

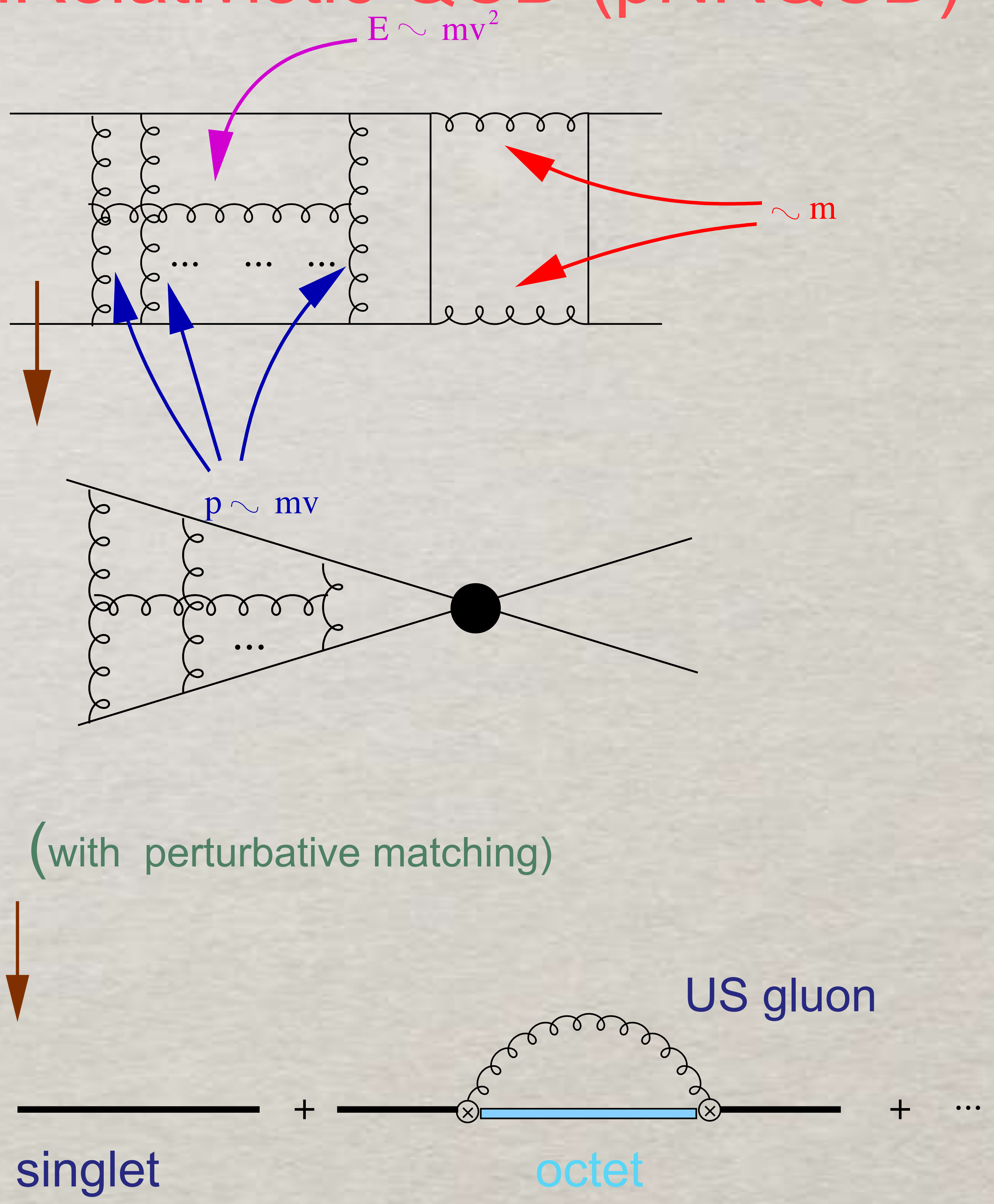

$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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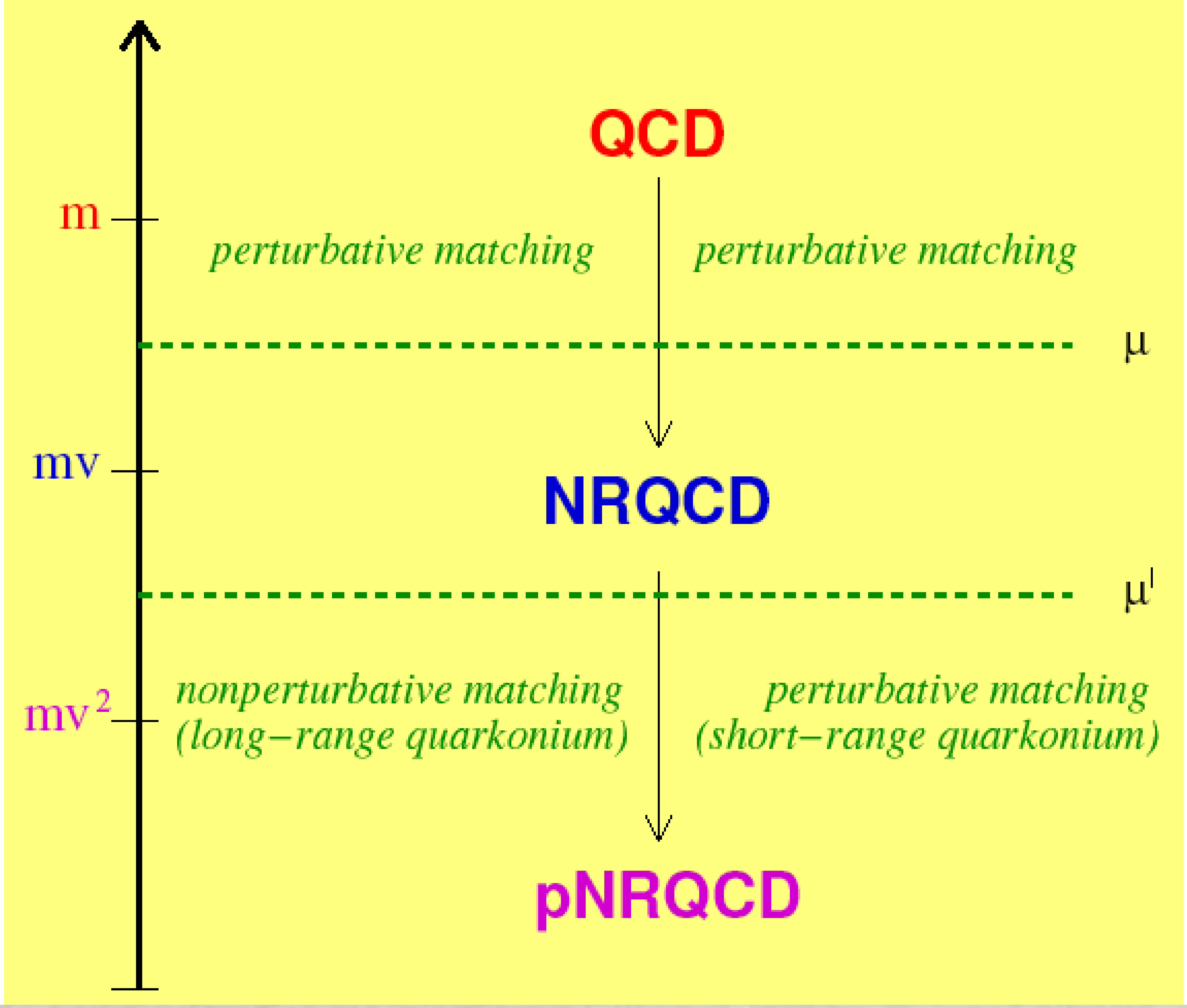

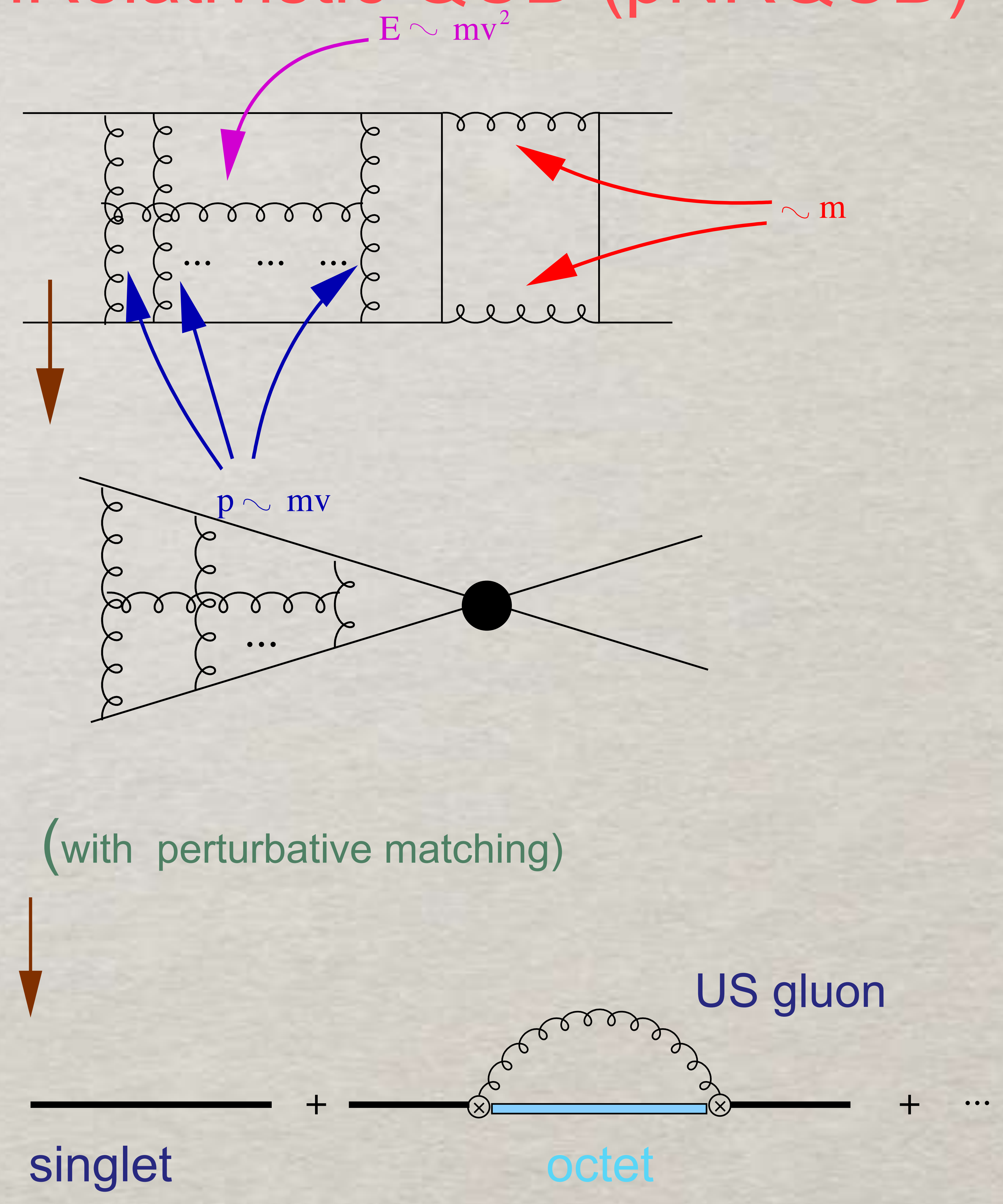

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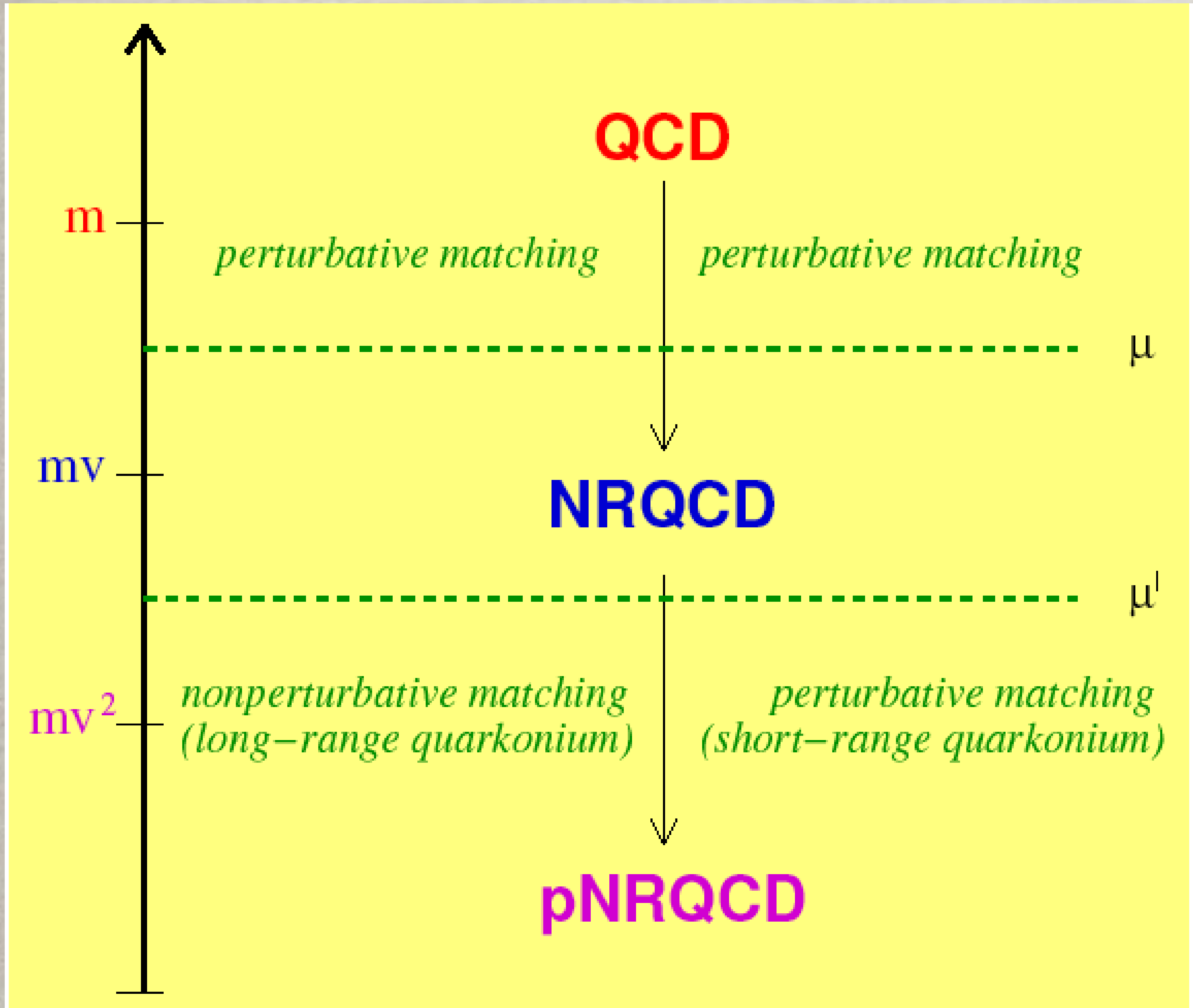

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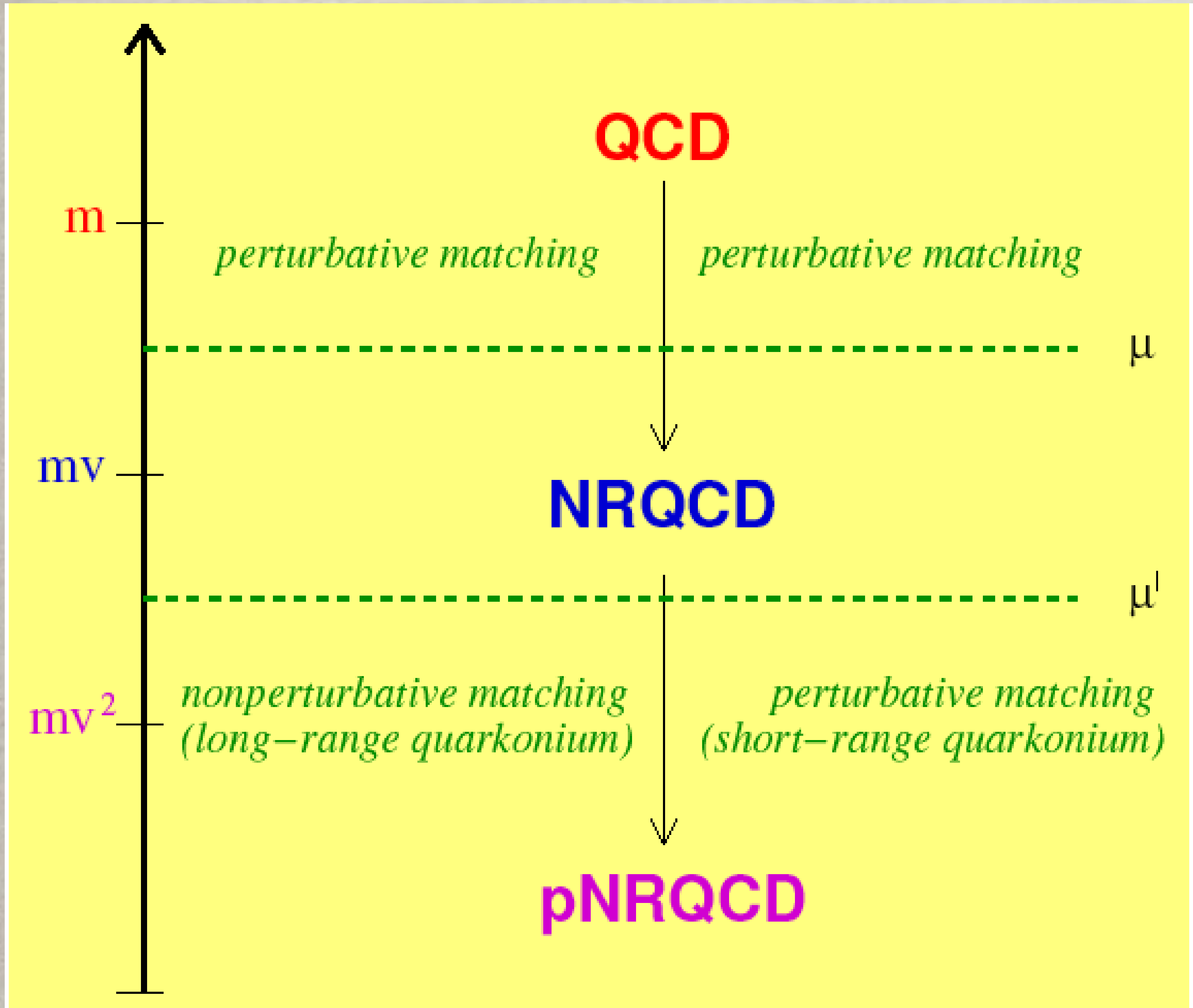

In QCD another scale is relevant Λ_{QCD}

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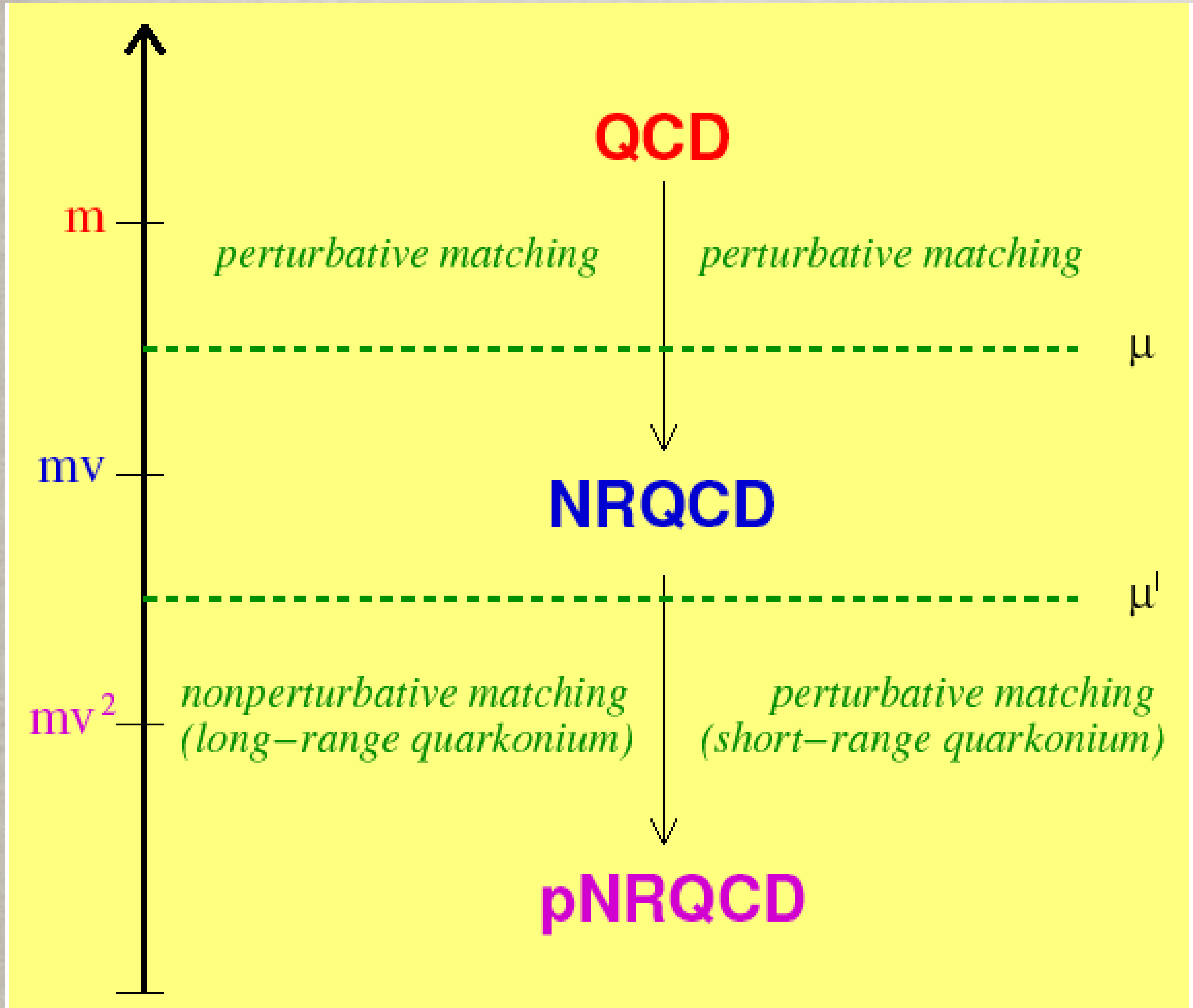

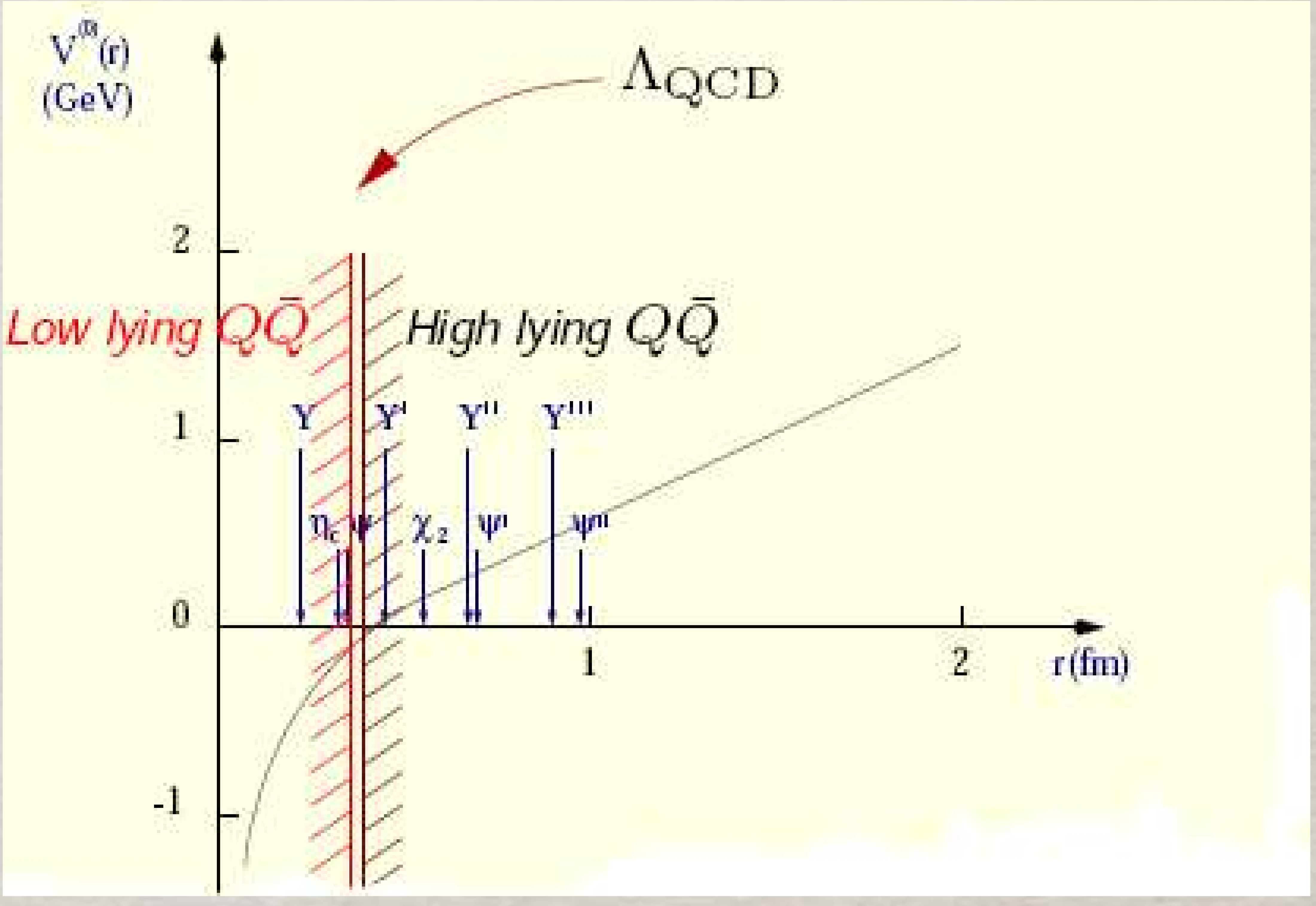

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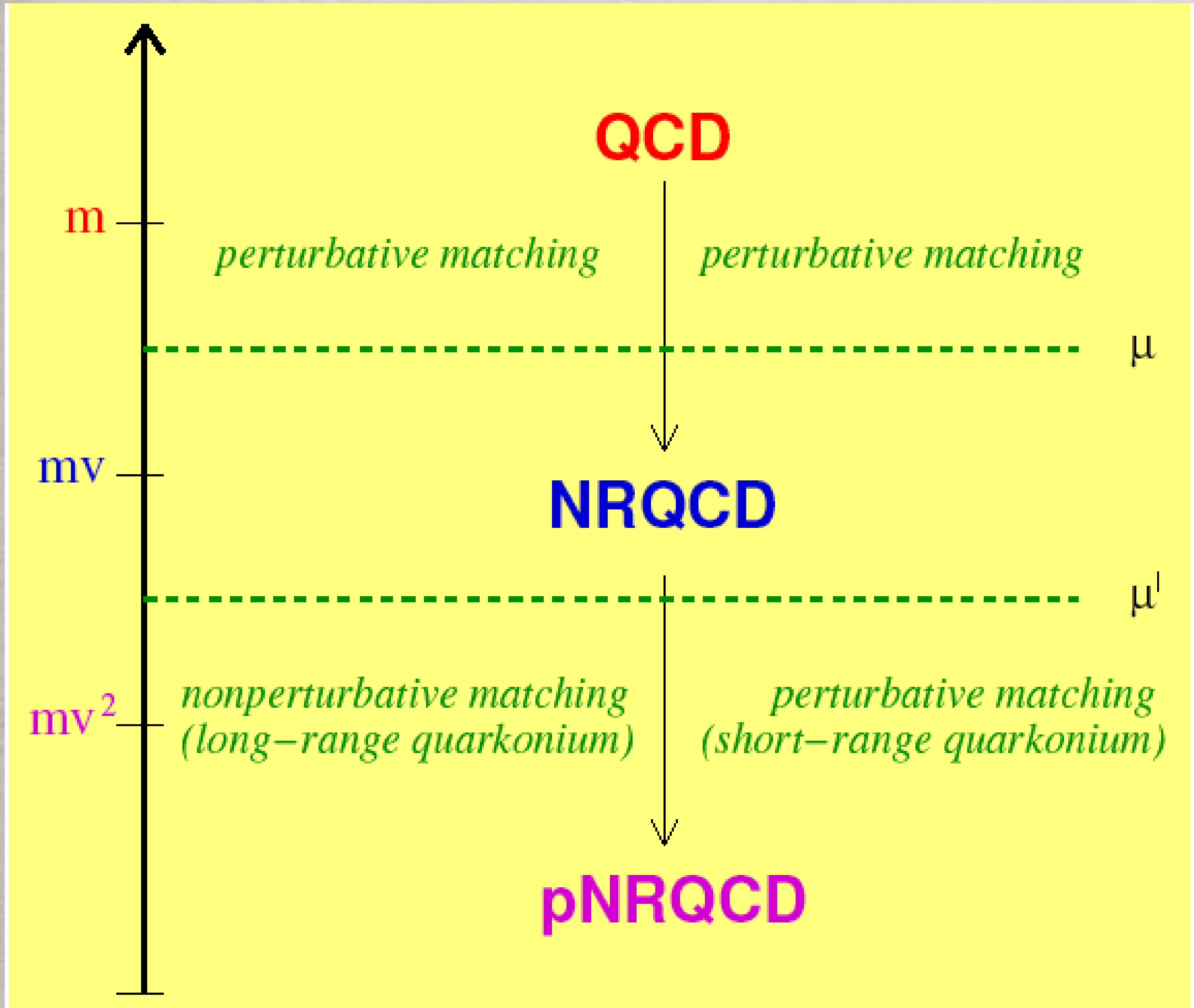

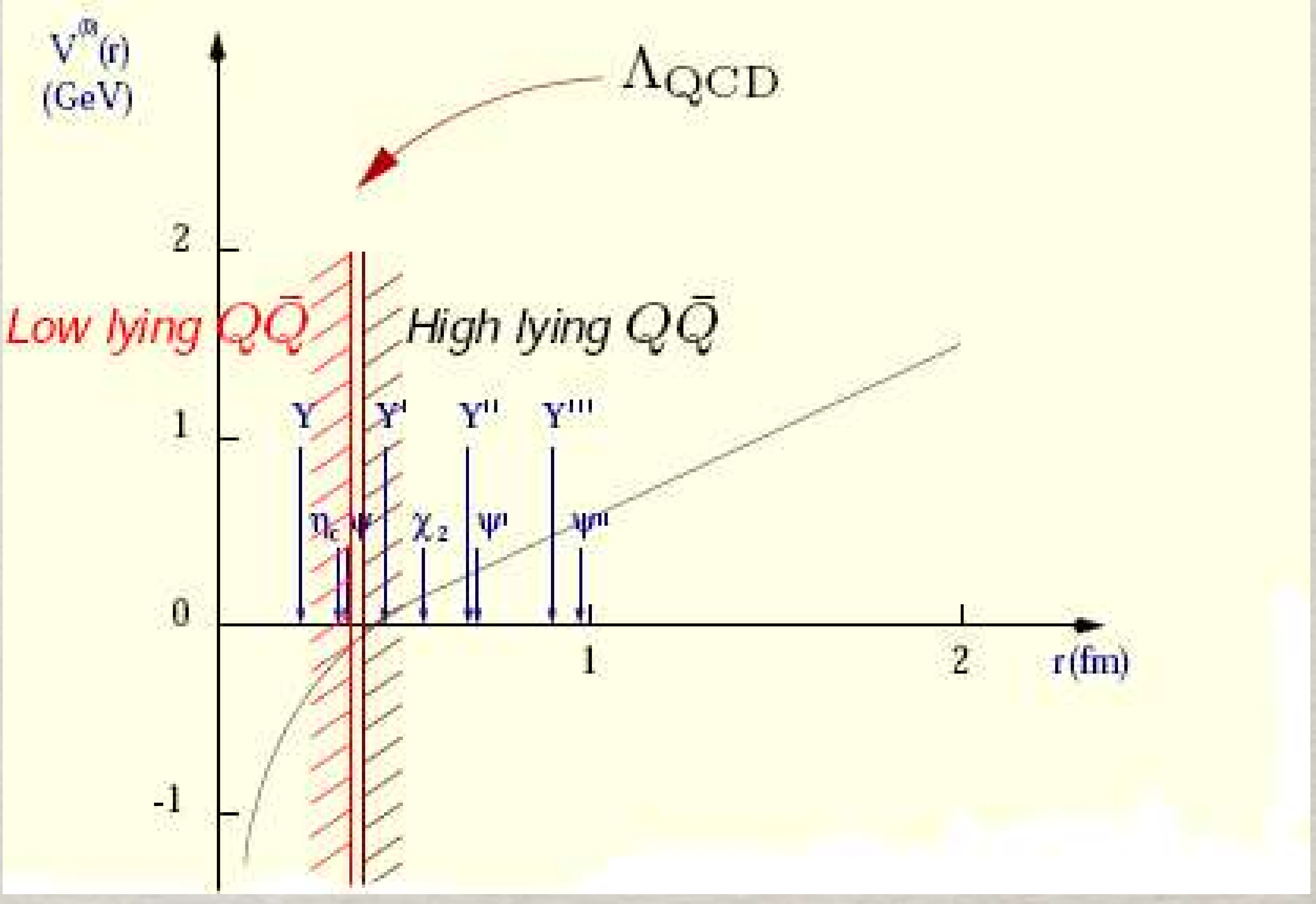

A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
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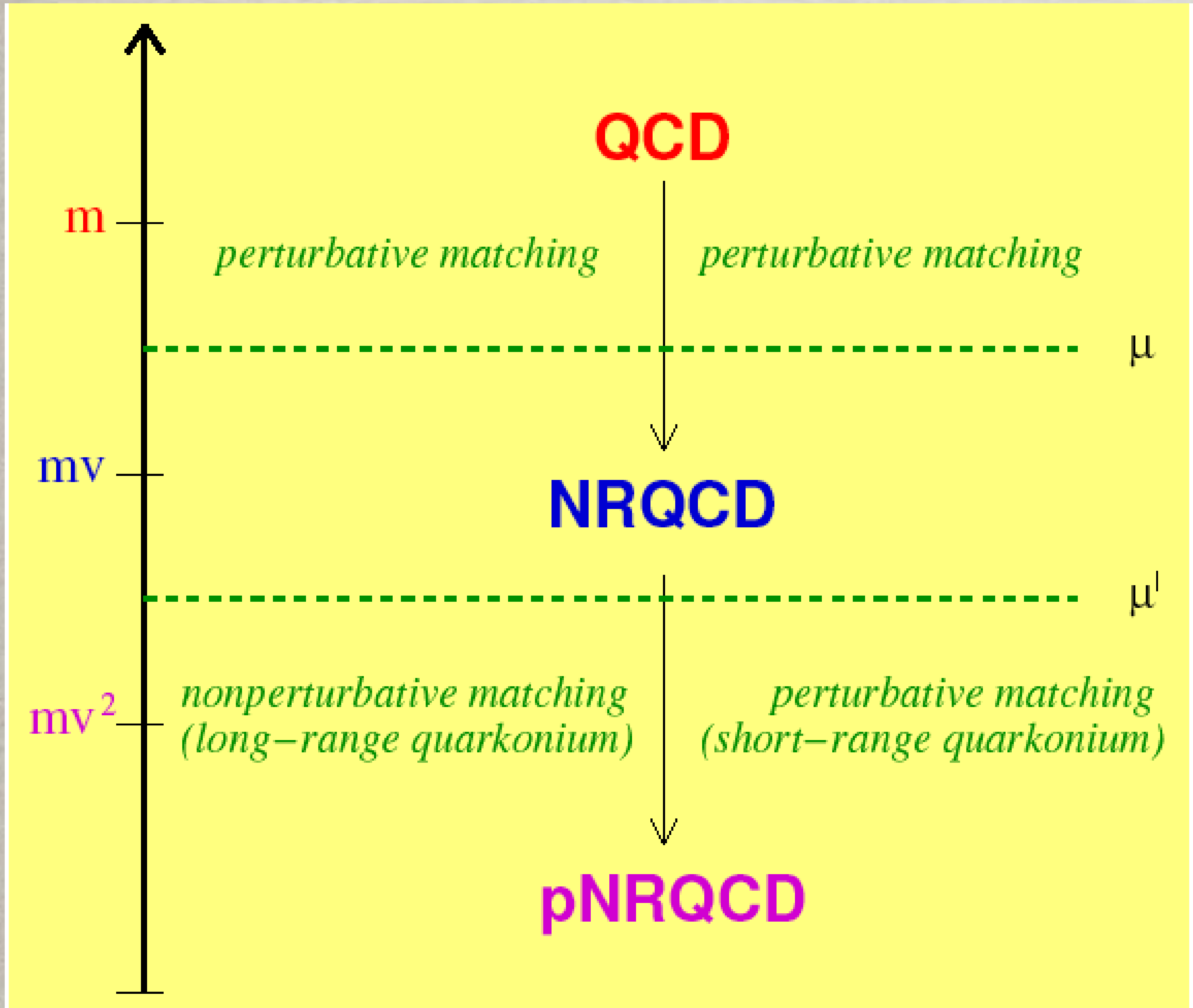

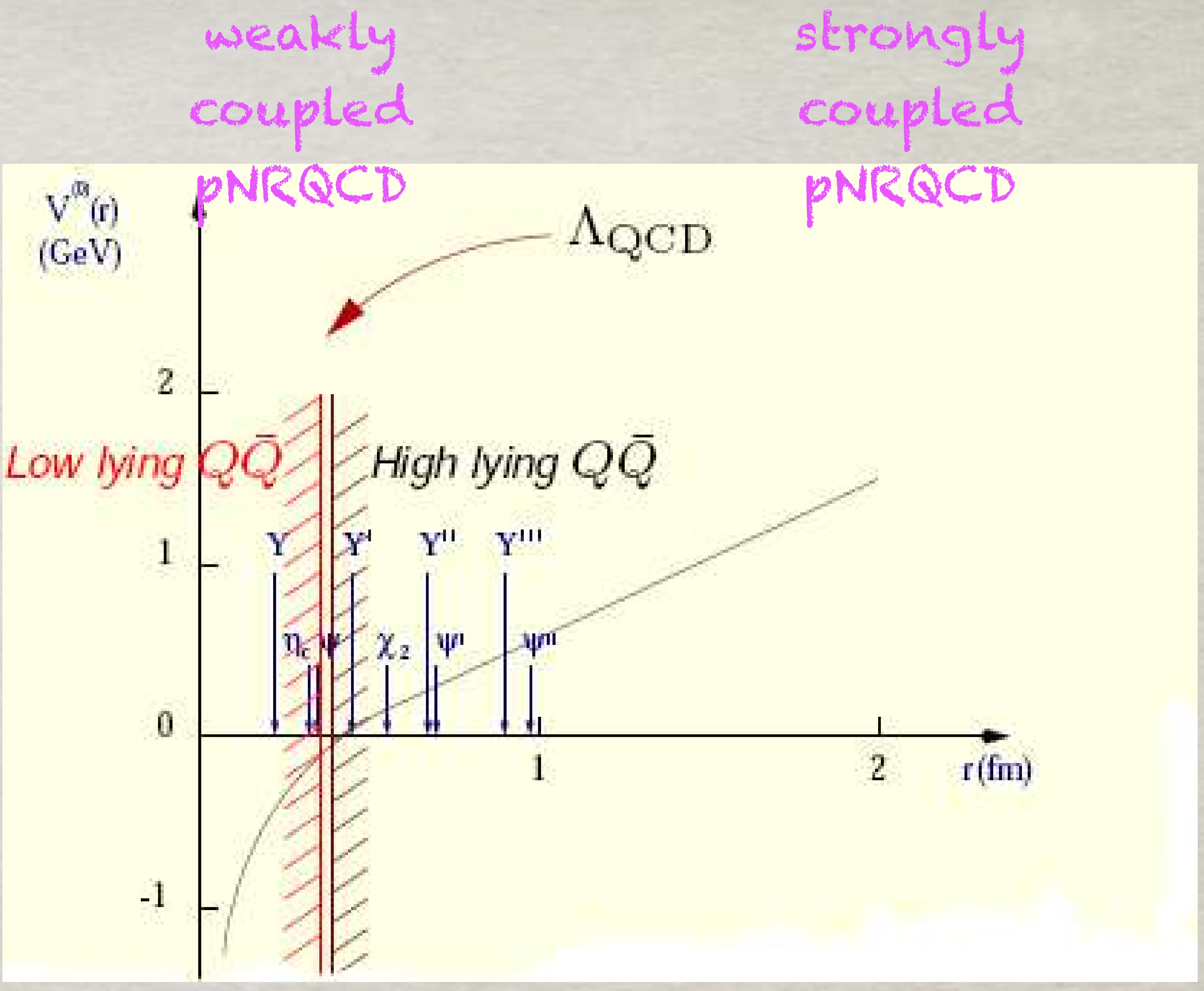

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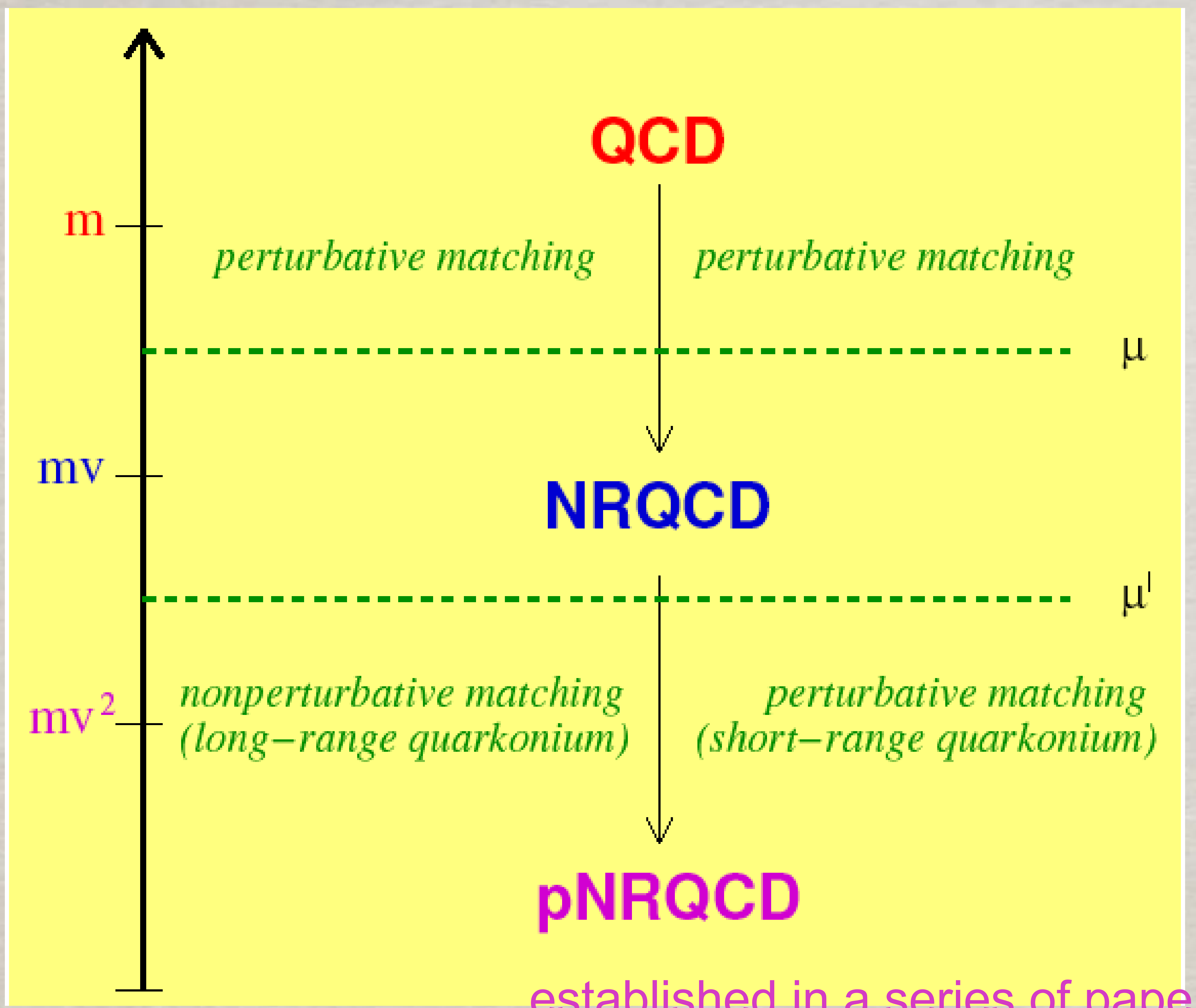

Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95......

Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

established in a series of papers:
Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, et al. 00–020
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

NRQCD

(Caswell Lepage 86, Thacker, Lepage 88, 91, Bodwin Braaten Lepage 95)

$$\mathcal{L}^{\rm NRQCD} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + ic_s \frac{\mathbf{S}\cdot[\mathbf{D}\times, g\mathbf{E}]}{4m^2} + \cdots \right)\psi + \chi^\dagger(\quad \ldots \quad)\chi$$

$$+O(1/m^3)$$

$$+\sum_K \frac{f}{m^2}\psi^\dagger\, K\, \chi\chi^\dagger\, K\, \psi + \cdots \qquad -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i\not{D}\, q + \ldots$$

ψ (χ) is the field that annihilates (creates) the (anti)fermion

expansion in v and $\alpha_s(m)$

the relevant dynamical scales are : mv, mv^2

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$$1 + (\cdots)\,\alpha_s + \ldots$$

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$$f = \mathrm{Re} f + i \mathrm{Im} f$$

Imaginary parts give the decay

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$$|H\rangle = (|(Q\bar{Q})_1\rangle + |(Q\bar{Q})_8 g\rangle + \cdots) \otimes |nljs\rangle$$

quarkonium state H

$$\psi^\dagger K^{(n)} \chi\chi^\dagger K'^{(n)}\psi = \begin{cases} O_1(^{2S+1}L_J) \\ O_8(^{2S+1}L_J) \end{cases}$$

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NRQCD had a tremendous impact on spectrum lattice calculations, has given a theoretical framework for quarkonium production at colliders and for decays

pNRQCD

pNRQCD is the EFT for nonrelativistic quark-antiquark pairs ($Q\bar{Q}$) near threshold.

with respect to
$$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

- QFT = QCD
- It is obtained by integrating out hard and soft gluons with p or E scaling like m, mv.
- The d.o.f. are $Q\bar{Q}$ pairs (sometimes cast in color singlet S and color octet O) and ultrasoft modes (e.g. light quarks, low-energy gluons):
  $\phi = S$
- The Lagrangian is organized as an expansion in $1/m$ and r.
- The form of $\Delta\mathcal{L}$ and of the ultrasoft modes depends on the low energy dynamics.
- The power counting is
  $\rightarrow p \sim 1/r \sim mv$ (soft scale),
  $\rightarrow E \sim \mathbf{p}^2/2m \sim V^{(0)} \sim \mathbf{P}_{\rm cm} \sim 1/\mathbf{R}_{\rm cm} \sim mv^2$ (ultrasoft scale),
  $\rightarrow$ operators in $\Delta\mathcal{L}$ scale like $(mv^2)^{\rm dimension}$.

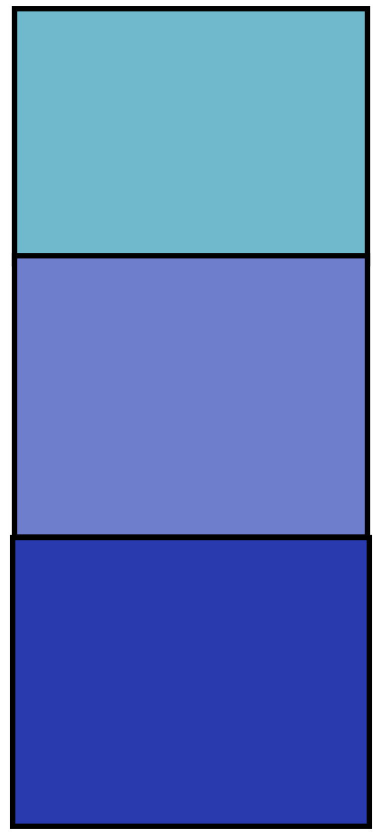
◦ Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275
◦ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

Low energy (nonperturbative) factorized effects depend on the size of the physical system

$$r \ll \frac{1}{\Lambda_{QCD}}$$

lowest quarkonia states

The EFT factorizes the low energy nonperturbative part.
Depending on the physical system:



	$\langle F^2(0) \rangle$	$\Upsilon(1S)$, ...	local condensates
$\Lambda_{\rm QCD} \sim t$	$\langle F(t)F(0) \rangle$	annihilations, short range $c\bar{c}$, $b\bar{b}$, gluelumps, ...	non local condensates
$\Lambda_{\rm QCD} \sim loop$	$\langle e^{ig\oint dx A(x)} \rangle$	long range $c\bar{c}$, $b\bar{b}$, hybrids, glueballs, ...	Wilson loops

The more extended the physical object, the more we probe the non-perturbative vacuum.

excited quarkonia states

$$r \sim \frac{1}{\Lambda_{QCD}}$$

$$r \ll \frac{1}{\Lambda_{QCD}} \qquad T$$

quarkonia in a hot medium

$$r \sim \frac{1}{\Lambda_{QCD}}$$

quarkonia and exotics close and above threshold

Low energy (nonperturbative) facto... effects depend on the size of the... system

THESE OBJECTS SHOULD BE CALCULATED ON THE LATTICE which is why we founded the TUMQCD lattice collaboration

The EFT factorizes the low...

Depending on the phy...

...condensates

...ions, short range $c\bar{c}$, $b\bar{b}$, gluelumps, ...

non local condensates

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Wilson loops

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lowest quarkonia states

excited quarkonia states

$$r \sim \frac{1}{\Lambda_{QCD}}$$

quarkonia and exotics close and above threshold

$$r \sim \frac{1}{\Lambda_{QCD}}$$

$$r \ll \frac{1}{\Lambda_{QCD}} \quad T$$

quarkonia in a hot medium

Weakly coupled pNRQCD: $mv \gg \Lambda_{\rm QCD}$

Quarkonia or QQq, QQQ systems with a small radius

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—precision calculations of observables : spectra, decays, transitions

—precise extraction of standard model parameters: alphas, m_c, m_b,m_t

WEAKLY COUPLED PNRQCD: $mv \gg \Lambda_{\rm QCD}$

QUARKONIA OR QQY, QQQ SYSTEMS WITH A SMALL RADIUS

—PRECISION CALCULATIONS OF OBSERVABLES : SPECTRA, DECAYS, TRANSITIONS

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WEAKLY COUPLED PNREFT CAN BE APPLIED TO ANY NR SYSTEM OF ANY NATURE: SUSY PARTICLES, DM PARTICLES...

Weakly coupled pNRQCD

- Pineda Soto NP PS 64 (1998) 428
  Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:
$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla}A(R,t) + \dots$

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r\, {\rm Tr}\left\{S^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots\right)S + O^\dagger\left(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots\right)O + \right.$$

LO in r

$$\left. + V_A(S^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger \mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\right\} + \dots$$

NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

Weakly coupled pNRQCD

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NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

The matching coefficients are the Coulomb potential

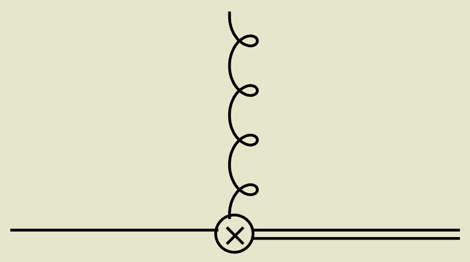
$$V_S(r) = -C_F\frac{\alpha_s}{r} + \ldots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_s}{r} + \ldots,$$

$$V_A = 1 + \mathcal{O}(\alpha_s^2),\ V_B = 1 + \mathcal{O}(\alpha_s^2).$$

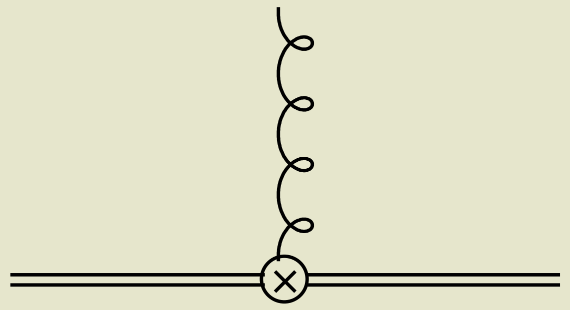
Feynman rules

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V)}$$

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$$



$= {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S}$



$= {\rm O}^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, {\rm O}\}$

pNRQCD

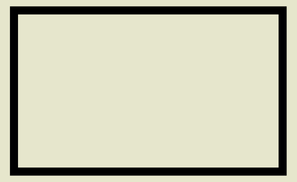
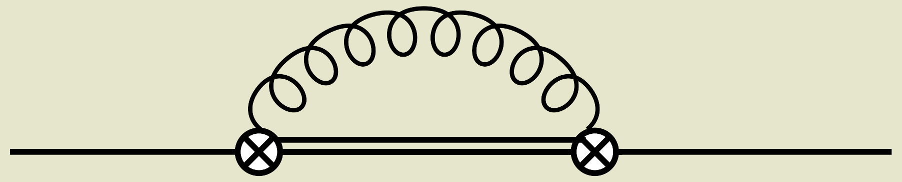
- pNRQCD provides a QM description from field theory: the Schroedinger equation and the potentials appear once all scales above the binding energy have been integrated out: this provides a clear definition of the potential

- The EFT accounts for non-potential terms as well. They provide loop corrections to the leading potential picture. Retardation effects are typically related to the nonperturbative physics

- The Quantum Mechanical divergences are cancelled by the NRQCD matching coefficients.

- Poincare’ invariance is intact and is realized via exact relations among the matching coefficients (potentials)

QCD singlet static potential and singlet static energy

$$V^{(0)}(r,\mu') = \lim_{T\to\infty}\frac{i}{T}\ln\Big\langle$$  $$\Big\rangle -$$  $$+\cdots$$

$$= E_0(r) + \frac{i}{N}\int_0^\infty dt\, e^{-it(V_o-V)}\langle \mathrm{Tr}\,\mathbf{r}\cdot g\mathbf{E}(t)\,\mathbf{r}\cdot g\mathbf{E}(0)\rangle(\mu') + \cdots$$

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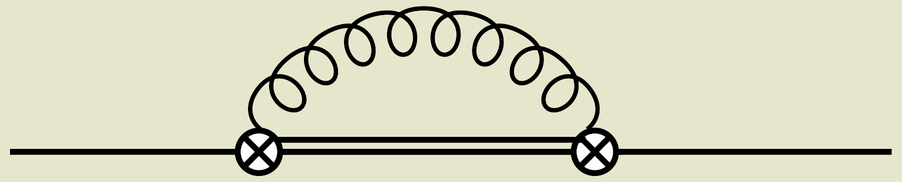

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The potential is a Wilson coefficient of the EFT.
In general, it undergoes renormalization, develops scale dependence and satisfies renormalization group equations, which allow to resum large logarithms.

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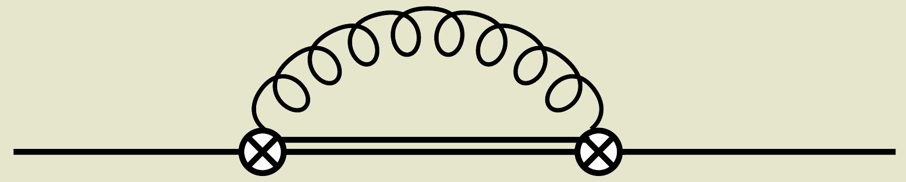
$$V^{(0)}(r,\mu') = \lim_{T\to\infty} \frac{i}{T} \ln \langle \Box \rangle -$$  $$+ \cdots$$

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The static energy $E_0(r)$ is known at three loops:

- Anzai Kiyo Sumino PRL 104 (2010) 112003
  A.Smirnov V.Smirnov Steinhauser PRL 104 (2010) 112002

$$E_0(r) = \Lambda_s - \frac{C_F\alpha_s}{r}(1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3 \ln\alpha_s + \#\alpha_s^4 \ln^2\alpha_s + \#\alpha_s^4 \ln\alpha_s + \dots)$$

$\ln\alpha_s$ in E_0 signals the cancellation of contributions coming from soft and ultrasoft gluons:

$$\ln\alpha_s = \ln\frac{\mu'}{1/r} + \ln\frac{\alpha_s/r}{\mu'}$$

Infrared logarithms in the potential may be computed in the EFT solving the ADM problem.

- Appelquist Dine Muzinich PRD 17 (1978) 2074

Quarkonium singlet static potential at N^4LO

$$
\begin{aligned}
V_s(r,\mu) \;=\; & -C_F \frac{\alpha_s(1/r)}{r}\left[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2\right. \\
& + \left(\frac{16\,\pi^2}{3} C_A^3 \,\ln r\mu + a_3\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 \\
& \left. + \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\, C_A^3 \beta_0(-5 + 6\ln 2)\right)\ln r\mu + a_4\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\right]
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a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $ln r\mu$ N.B. Pineda, Soto, Vairo 99

a_4^{L2}, a_4^L N.B., Garcia, Soto, Vairo 06

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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3LOOPS REDUCES TO 1 LOOP IN THE EFT

a_4^{L2}, a_4^L N.B., Garcia, Sot

4LOOPS REDUCES TO 2LOOPS IN THE EFT

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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Two problems:

1)Bad convergence of the series due to large beta_0 terms

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The eft cures both:

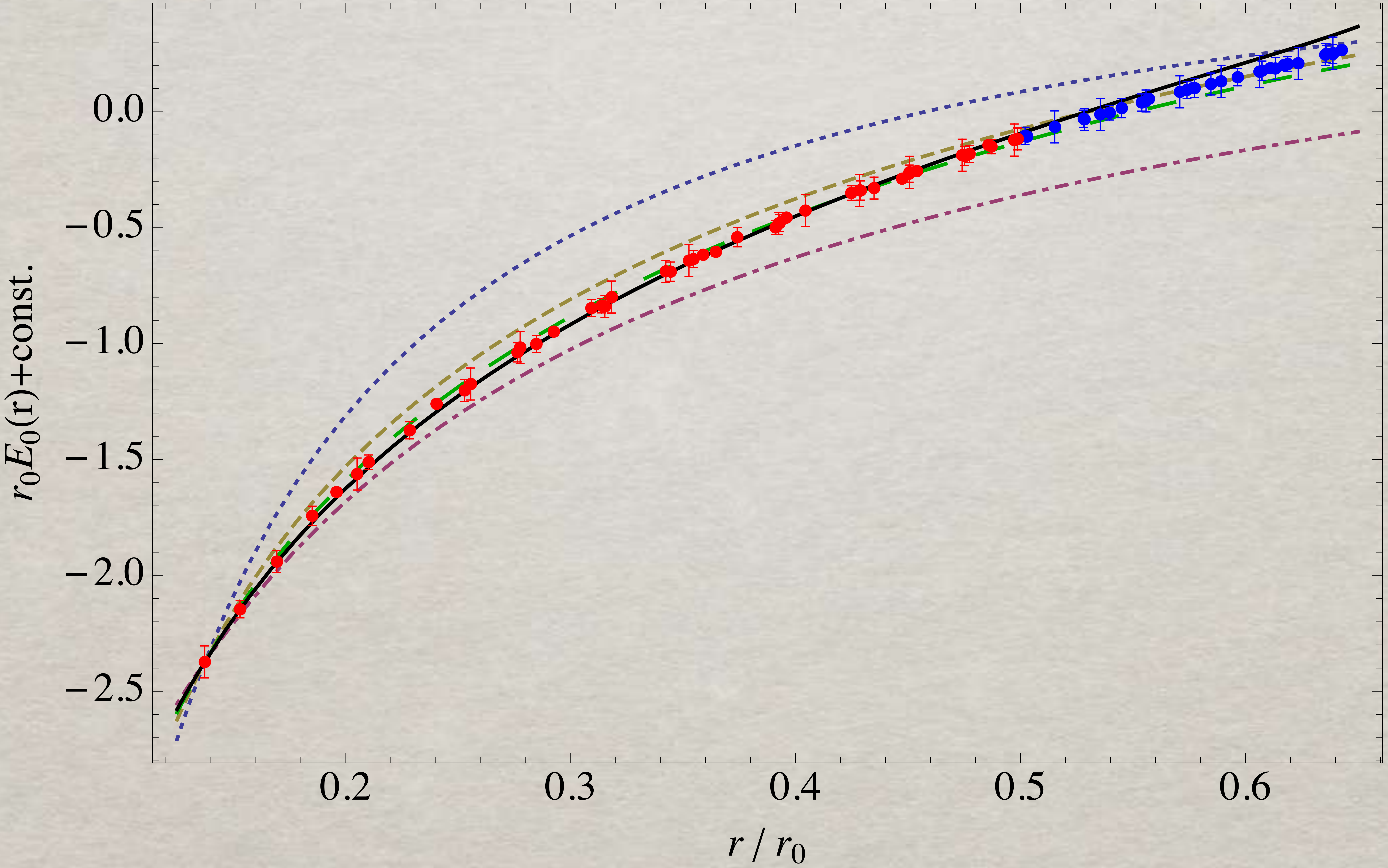
1) Renormalon subtracted scheme

Beneke 98, Hoang, Lee 99, Pineda 01, N.B. Pineda Soto, Vairo 09

2) Renormalization group summation of the logs up to N^3LL $(\alpha_s^{4+n} \ln^n \alpha_s)$ N. B Garcia, Soto Vairo 2007, 2009, Pineda, Soto

QQbar singlet static energy at N^3LI in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014, with Weber 2019



QQbar singlet static energy at N^3LL in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

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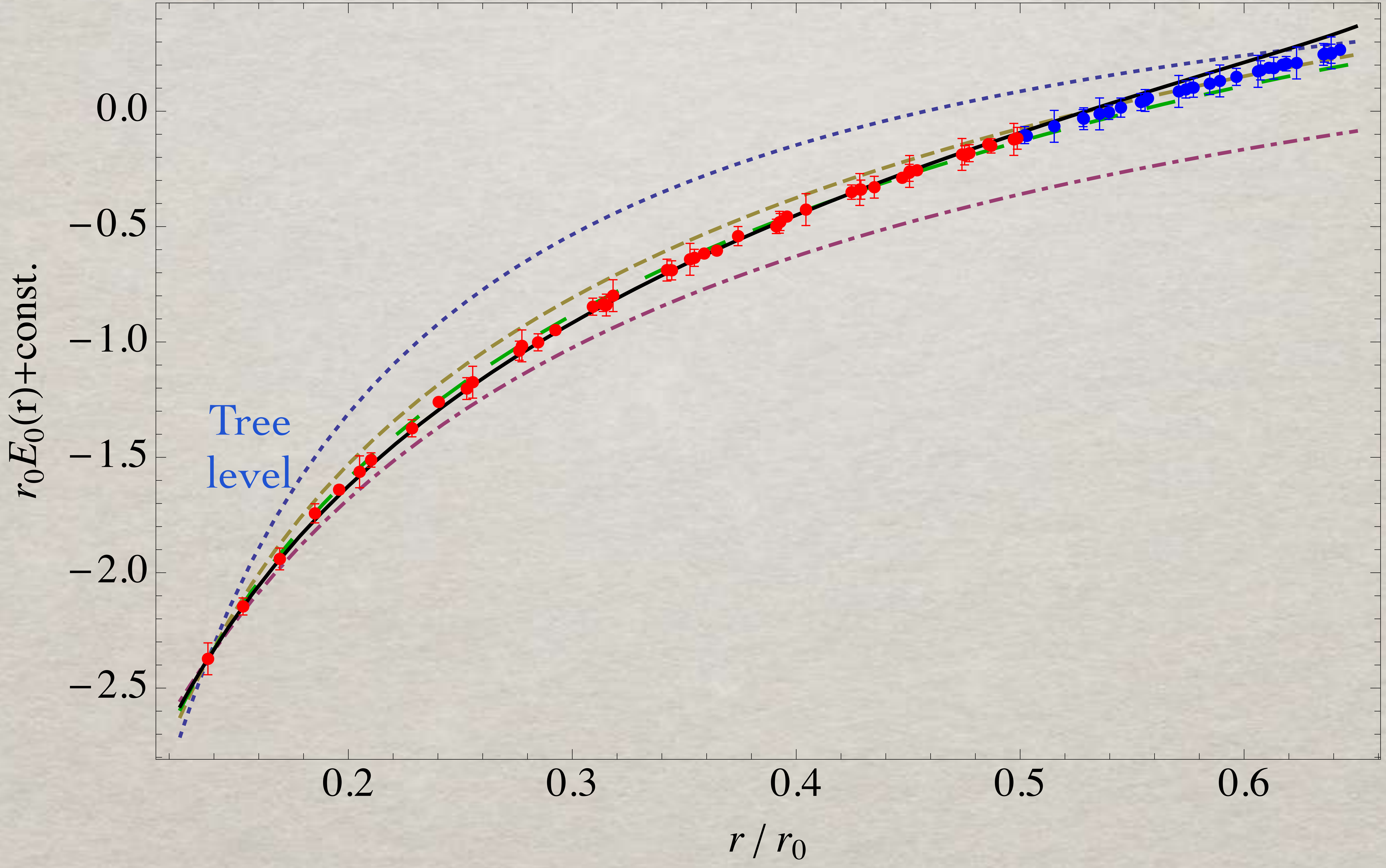

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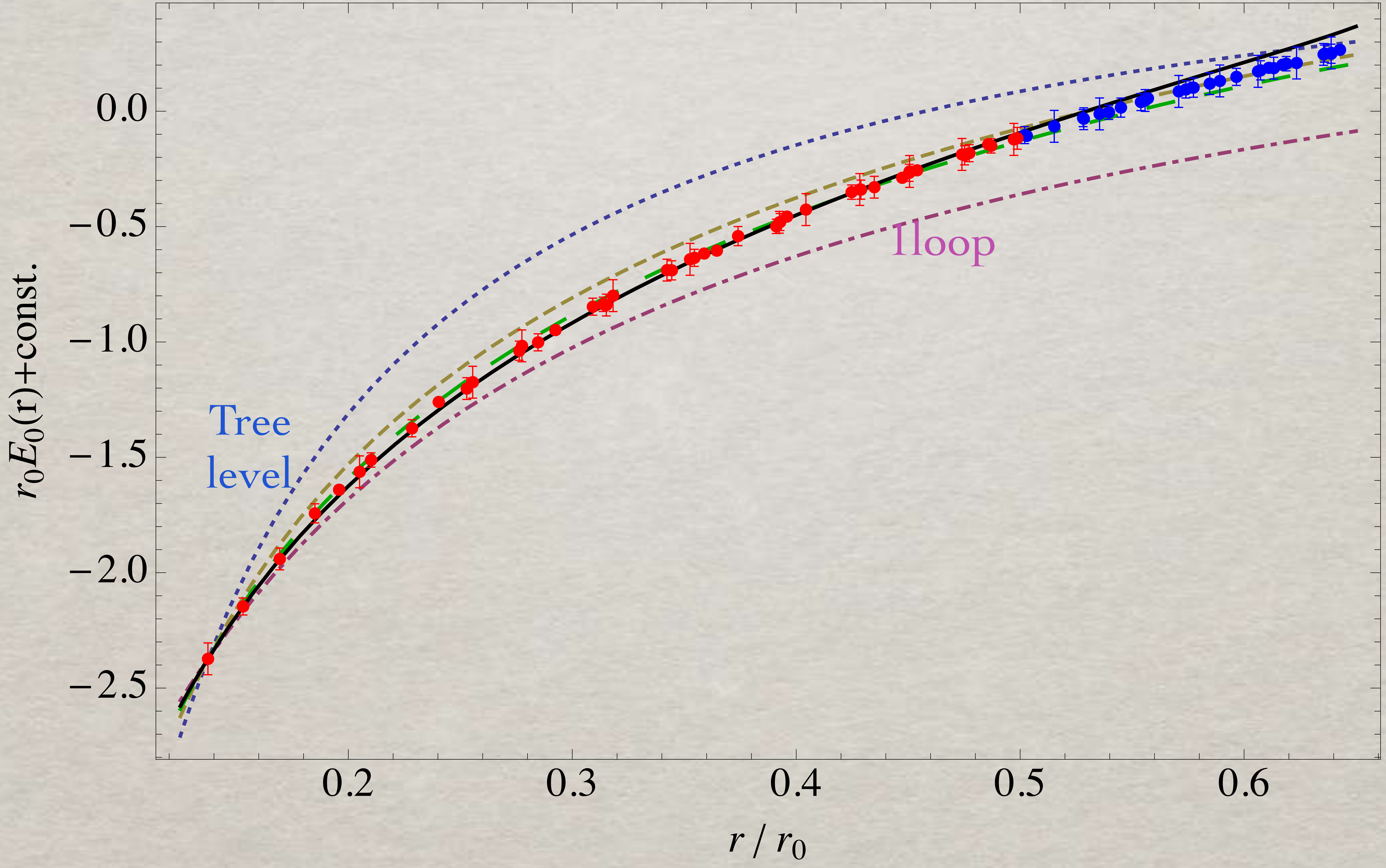

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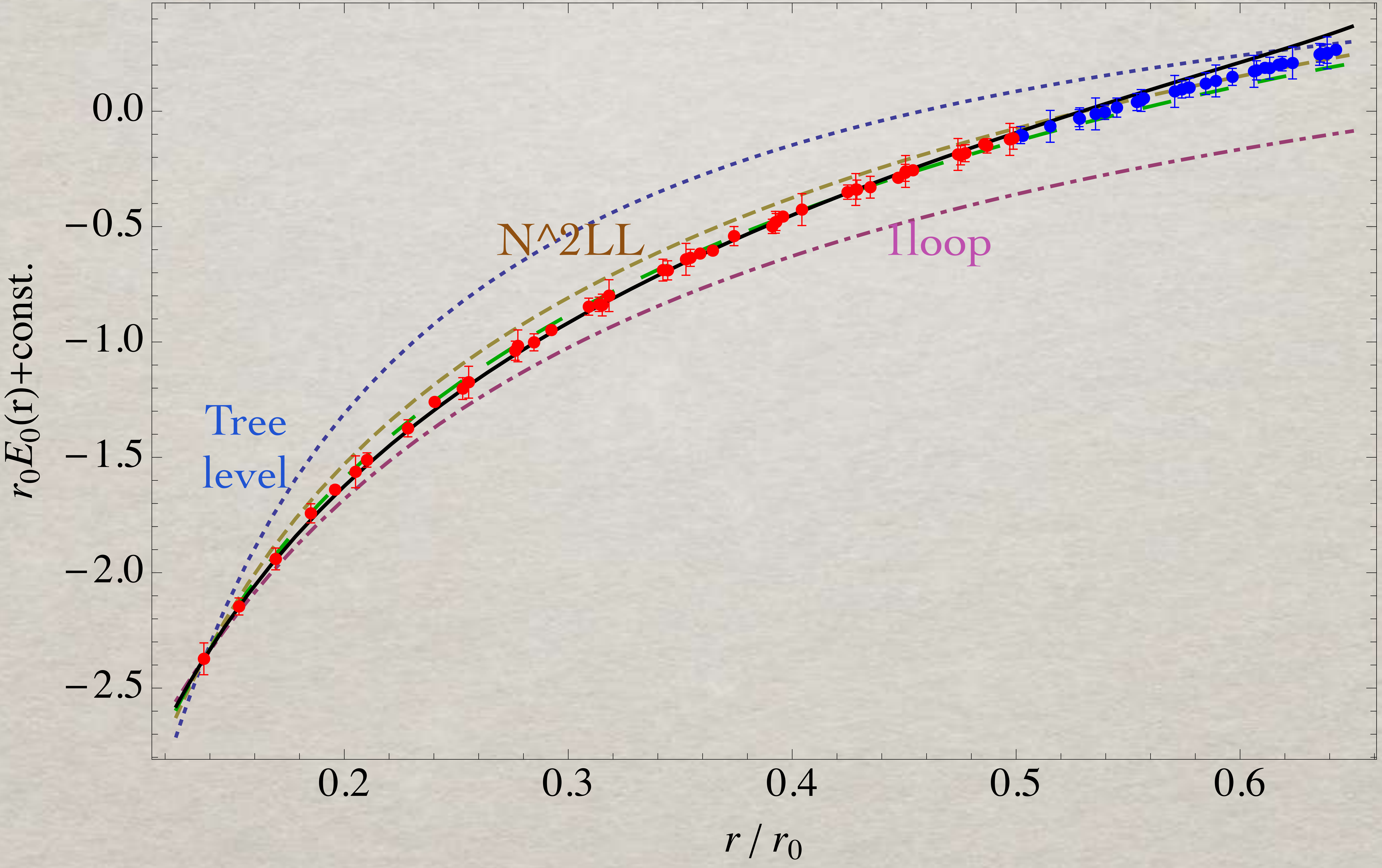

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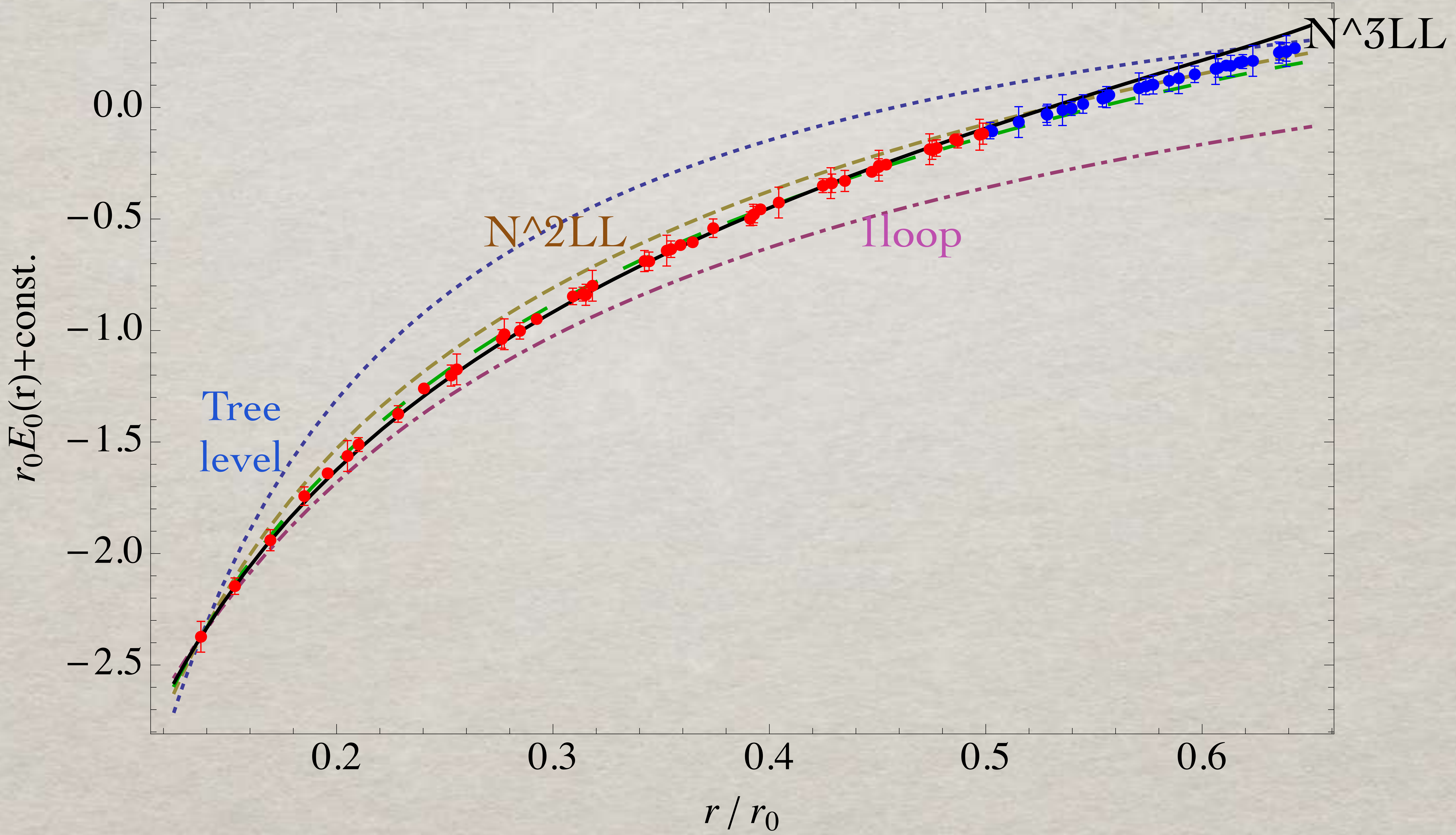

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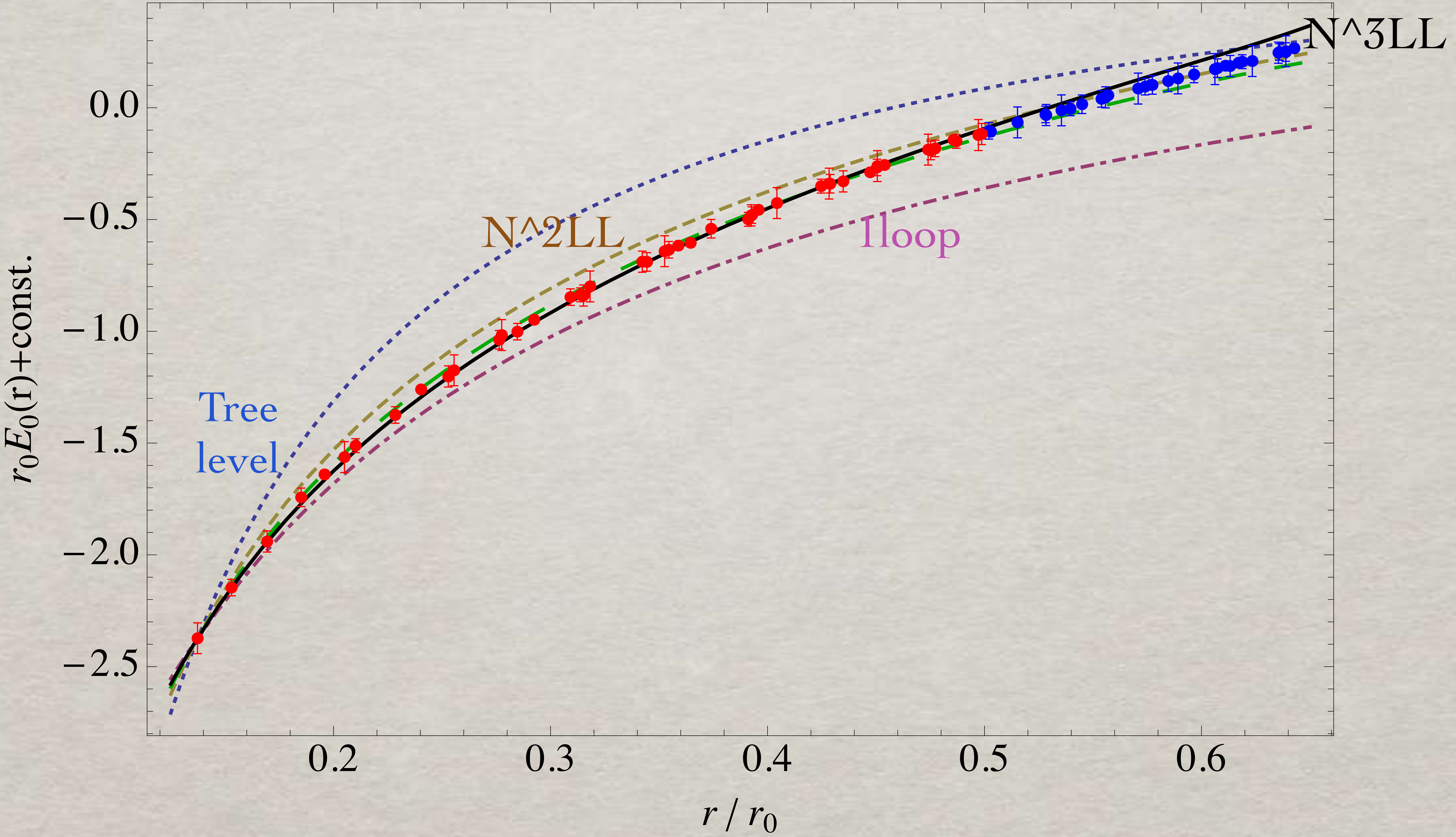

Good convergence to the lattice data

Lattice data less accurate in the unquenched case

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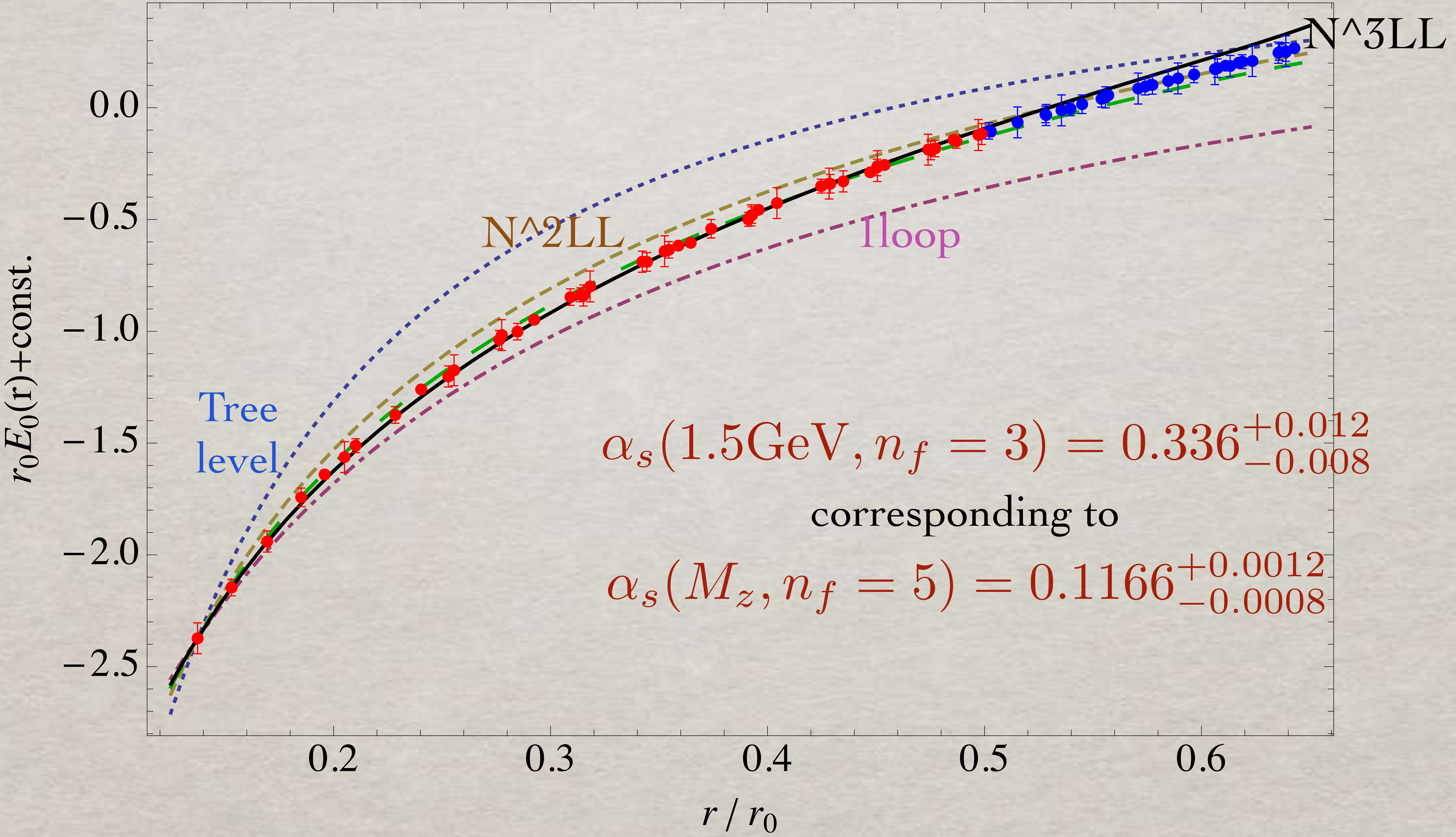

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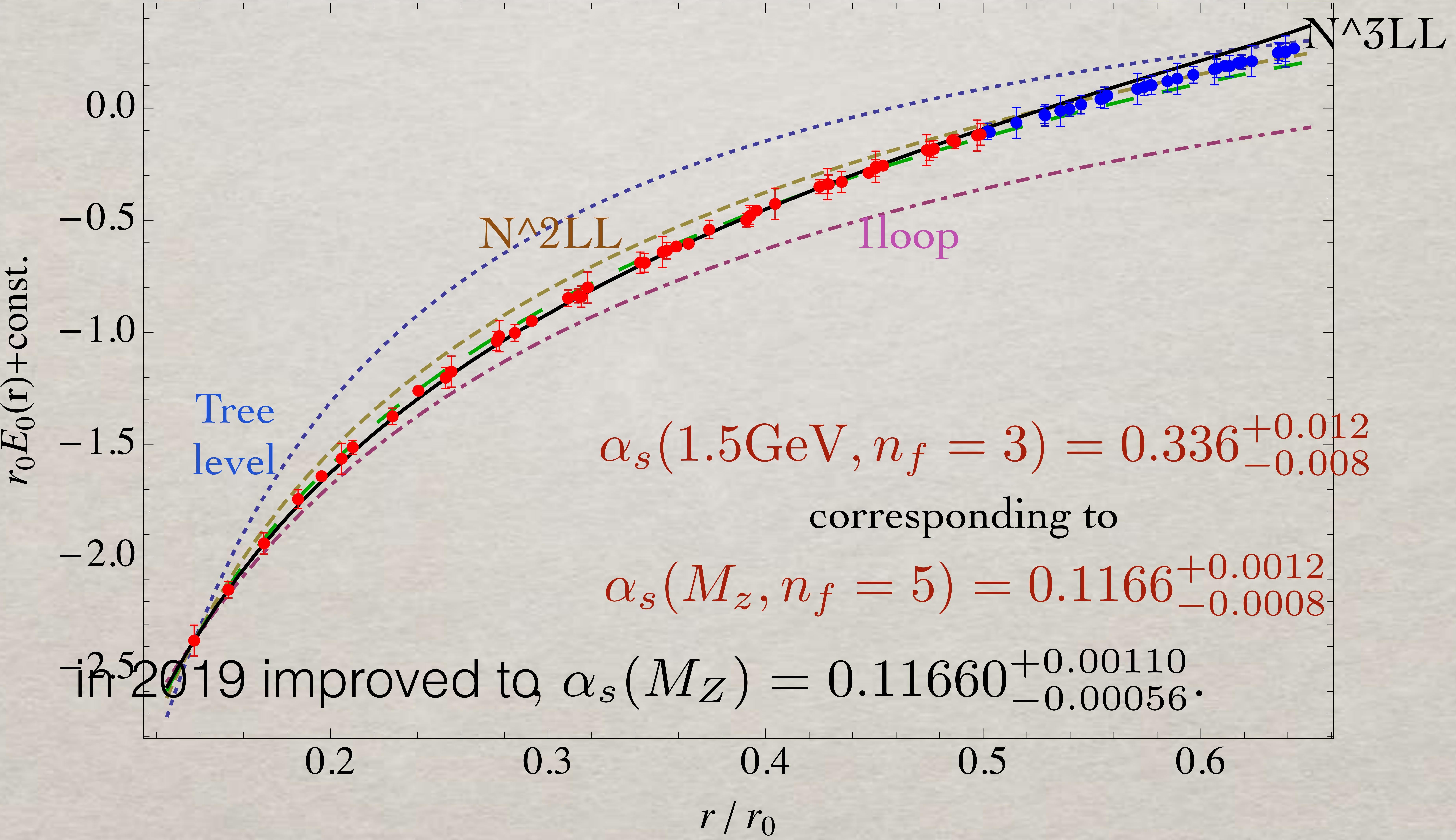

$\alpha_s(1.5\text{GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$

corresponding to

$\alpha_s(M_z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$

in 2019 improved to $\alpha_s(M_Z) = 0.11660^{+0.00110}_{-0.00056}$.

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All the potentials can be calculated in the matching

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

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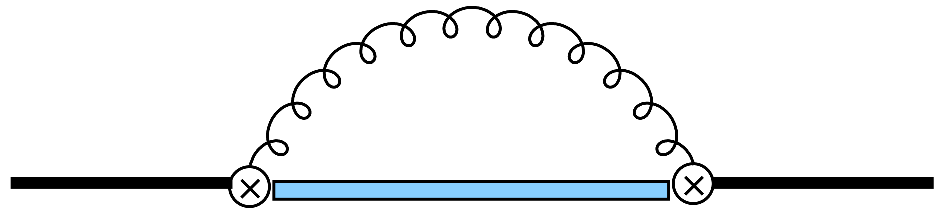
Energies at order m alpha^5 (NNNLO)

$$E_n = 2m + \langle n| \frac{p^2}{m} + V_s |n\rangle + \langle n|$$  $$|n\rangle$$

$$\sim e^{i\Lambda_{\rm QCD} t}$$

$$E_n = \langle n|H_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r} e^{it(E_n^{(0)} - H_o)}\mathbf{r}|n\rangle \, \langle \mathbf{E}(t)\, \mathbf{E}(0)\rangle(\mu)$$

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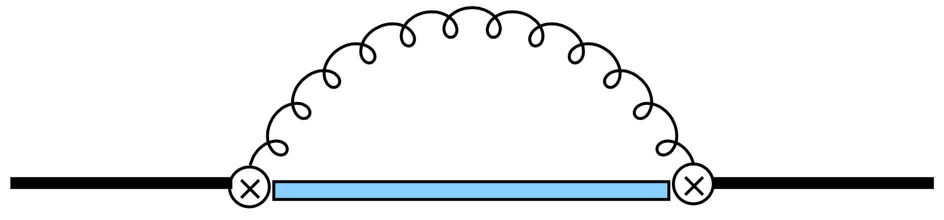
$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

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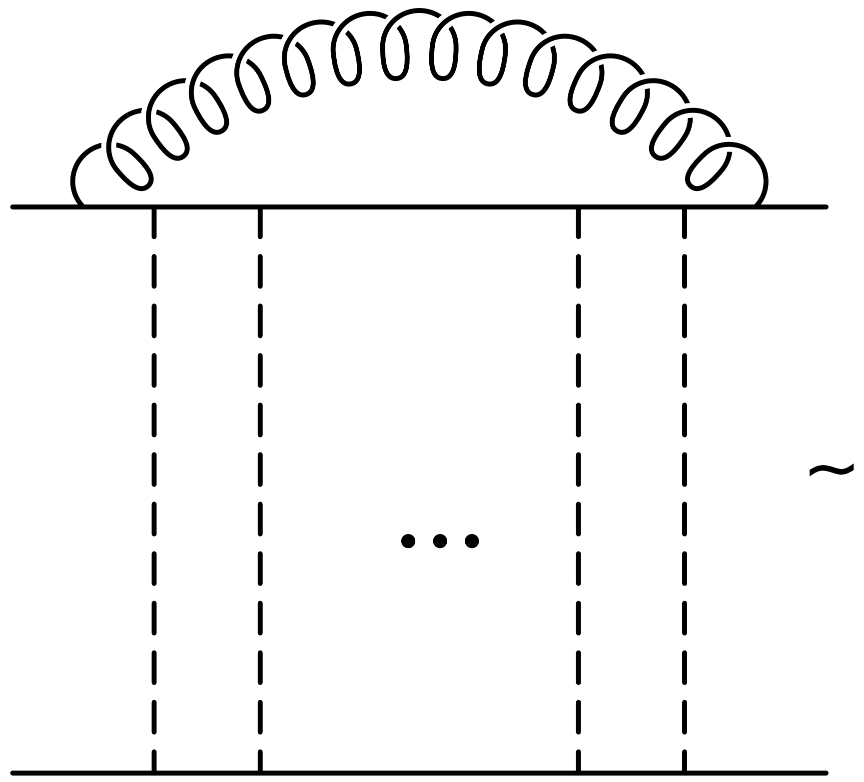
$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

local condensates

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

Σ  $\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

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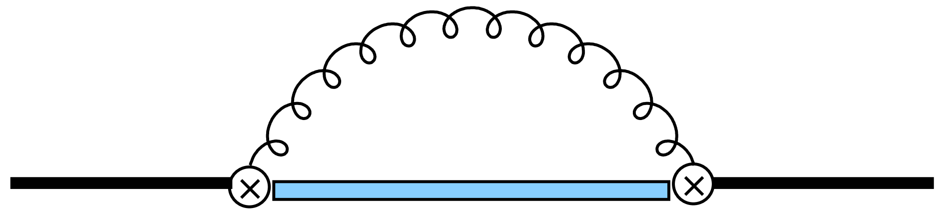
$$E_n = 2m + \langle n | \frac{p^2}{m} + V_s | n \rangle + \langle n |$$  $$| n \rangle$$

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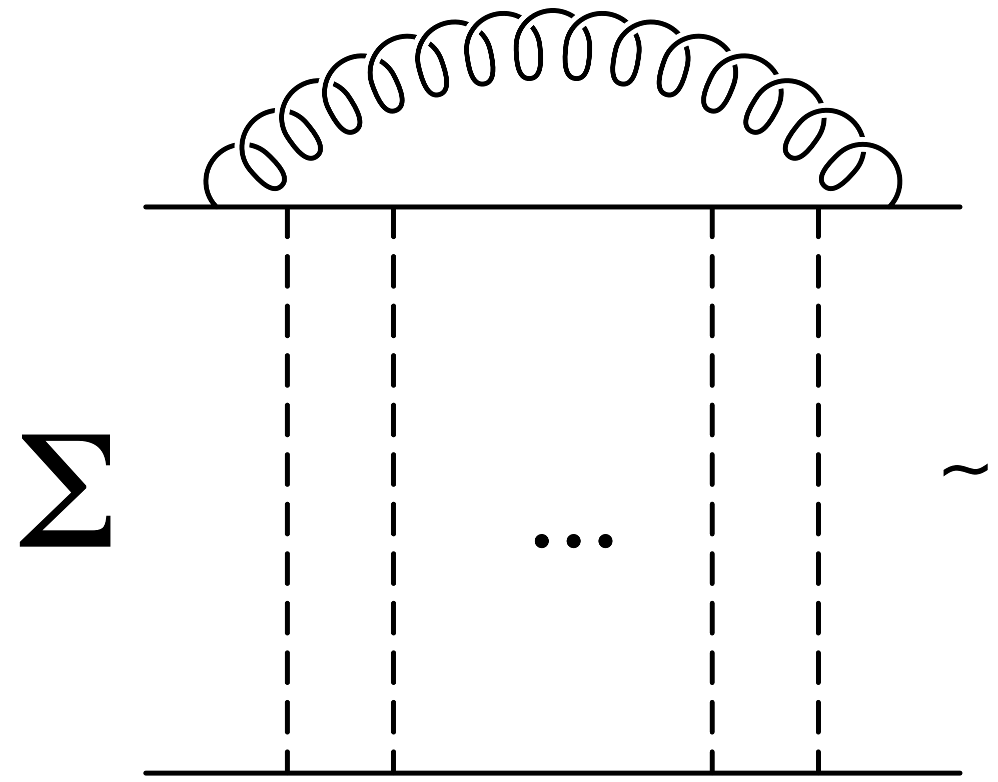

$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

—>used to extract precise (NNNLO) determination of m_c and m_b

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Quarkonia or QQq, QQQ systems with a larger radius below the strong decay threshold

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Quarkonia or QQq, QQQ systems with a larger radius below the strong decay threshold

—Factorization of high energy and low energy contributions, only glue dependent : to be calculated on the lattice
—Study of confinement effects

—Exploit the bound state dynamics to reduce the nonperturbative unknowns: application to quarkonium production

Strongly coupled pNRQCD: $mv \sim \Lambda_{QCD}$

Quarkonia or QQq, QQQ systems with a larger radius
below the strong decay threshold

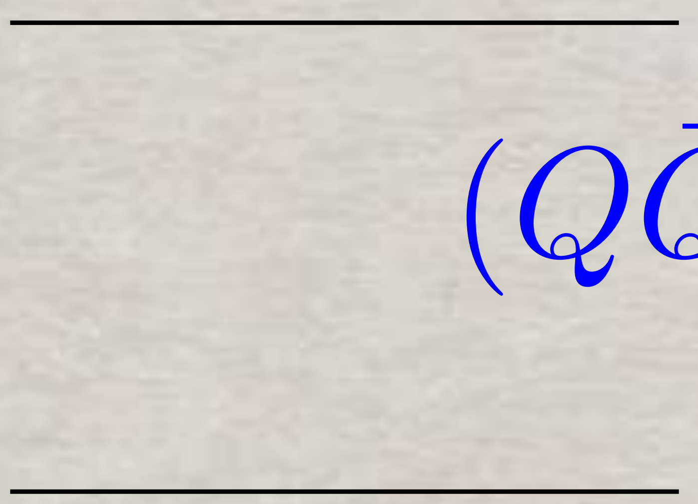
—Factorization of high energy and low energy
contributions, only glue dependent :
to be calculated on the lattice
—study of confinement effects

—exploit the bound state dynamics to reduce the
nonperturbative unknowns: application to
quarkonium production

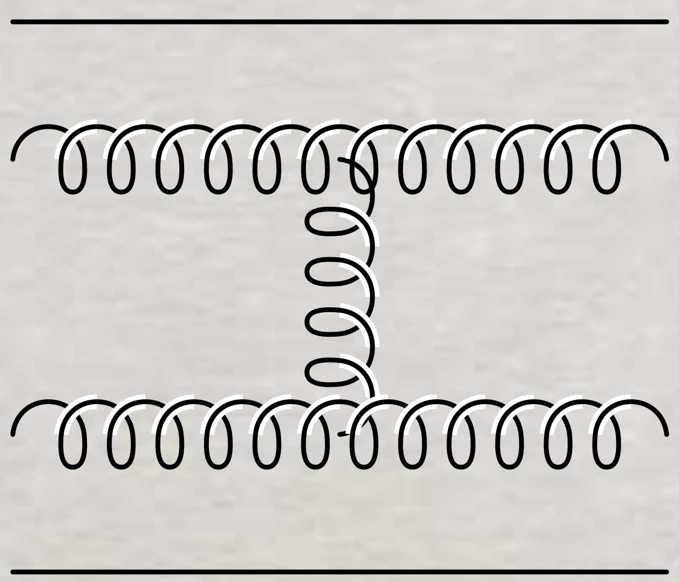
Strongly coupled pNREFT can be applied to any strongly
coupled NR system: applications to BSM physics and strings

Strongly coupled pNRQCD: Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$

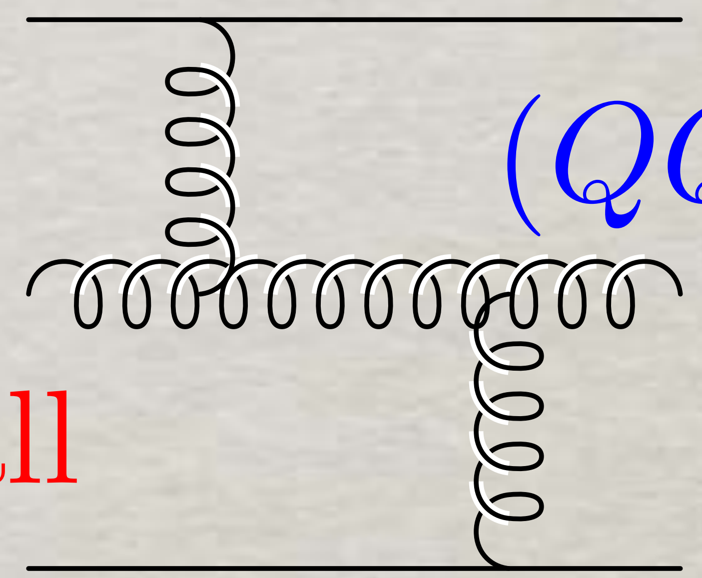
The degrees of freedoms now are



$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \text{Glueball}$



$(Q\bar{Q})_8 G$

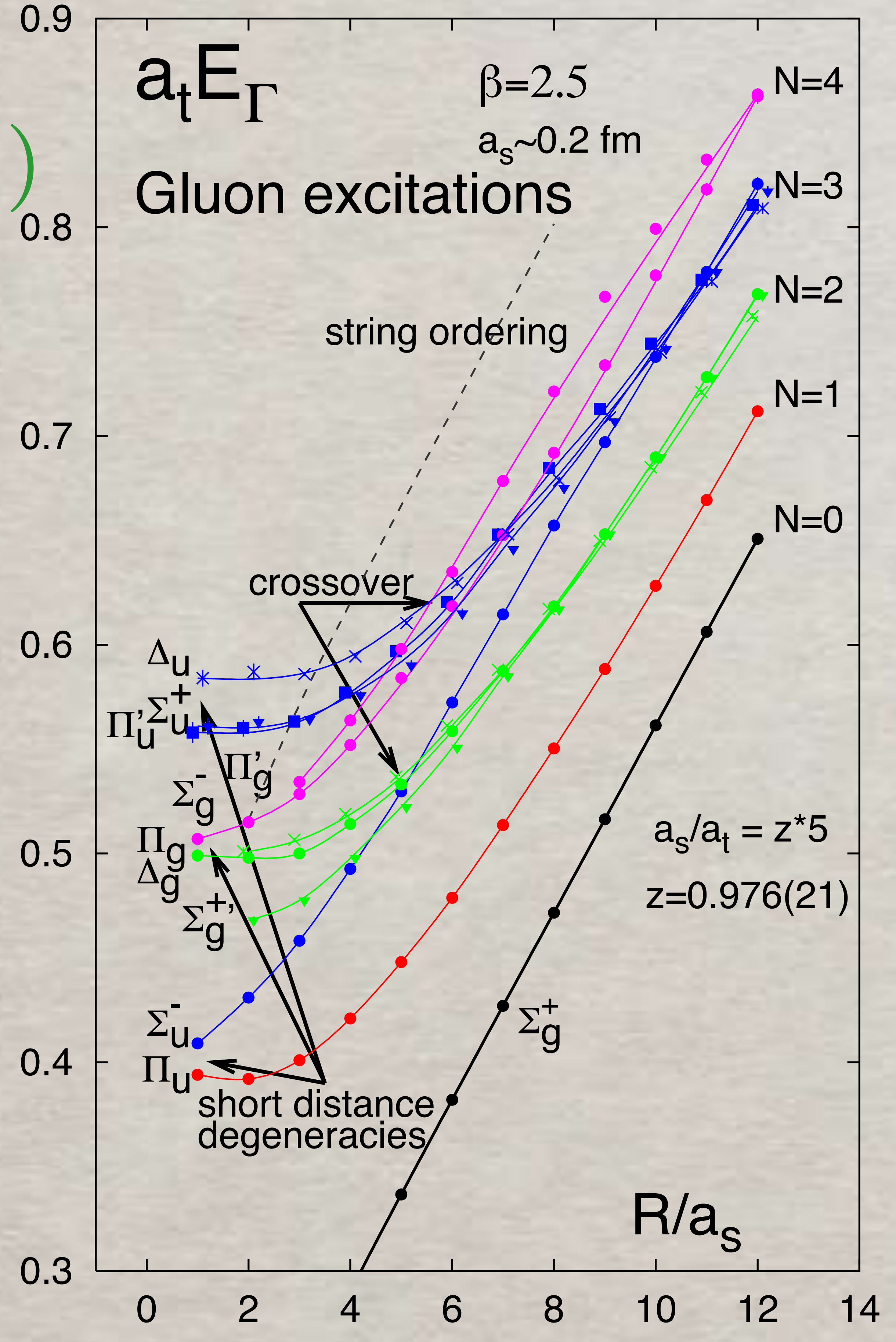
hybrid

with gluons at the scale $\Lambda_{\rm QCD}$

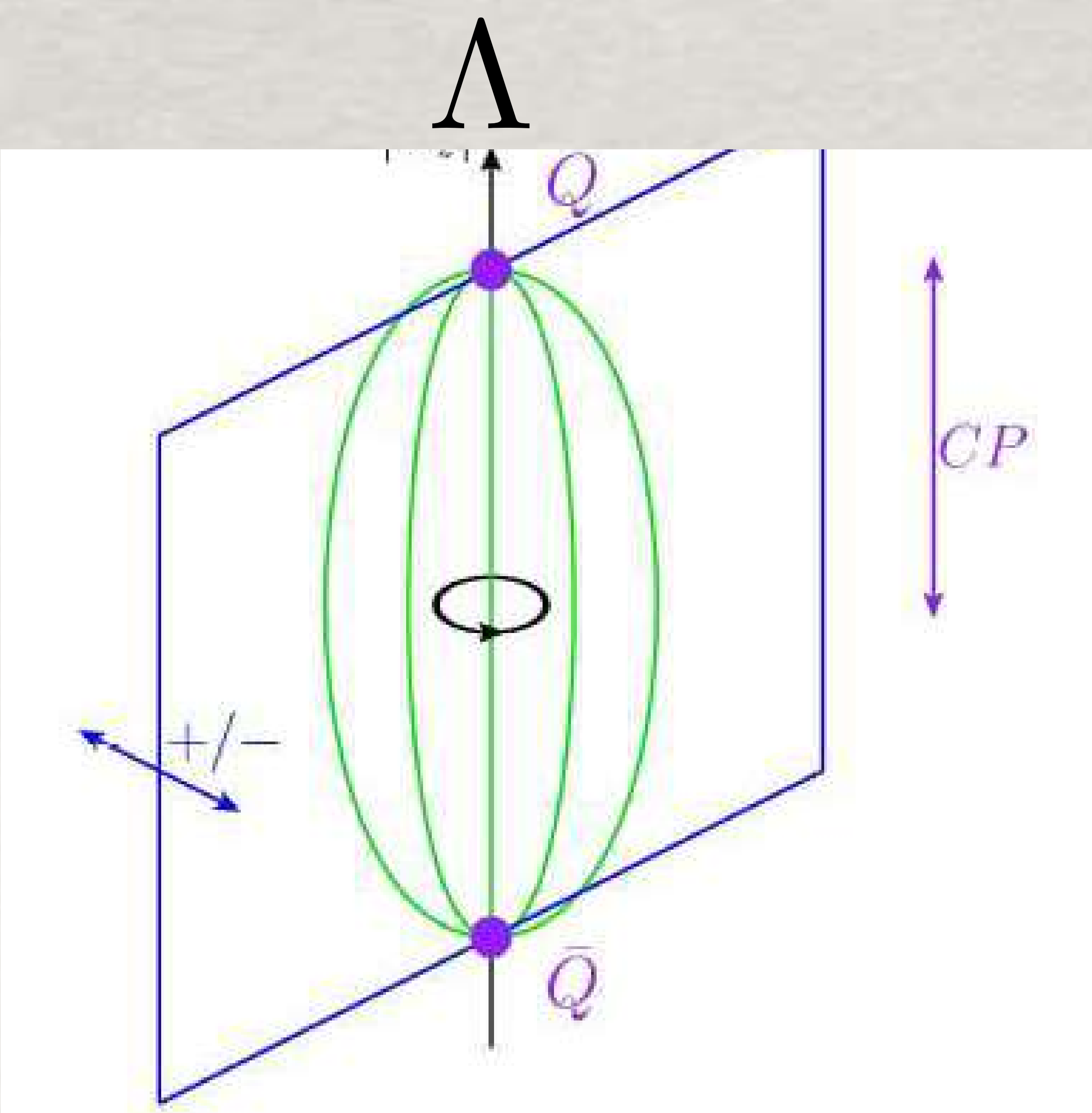
Strongly coupled pNRQCD: Hitting the scale Λ_{QCD} $r \sim \Lambda_{QCD}^{-1}$

Static NRQCD spectrum from lattice QCD

$E_n^{(0)}$



$r = |\mathbf{x}_1 - \mathbf{x}_2|$



Static states classified by symmetry group $D_{\infty h}$
Representations labeled Λ_η^σ $-- > n$

- Λ rotational quantum number $|\lambda| = |\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \dots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta \dots$
- η eigenvalue of CP: $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)

K is the angular momentum of the light degrees of freedom;same symmetry as the diatomic molecule

NRQCD

$$\mathcal{H}^{(0)} |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2) |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2) |n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

NRQCD states

$|\underline{0}; \mathbf{x}_1 \mathbf{x}_2\rangle - >$ $|(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

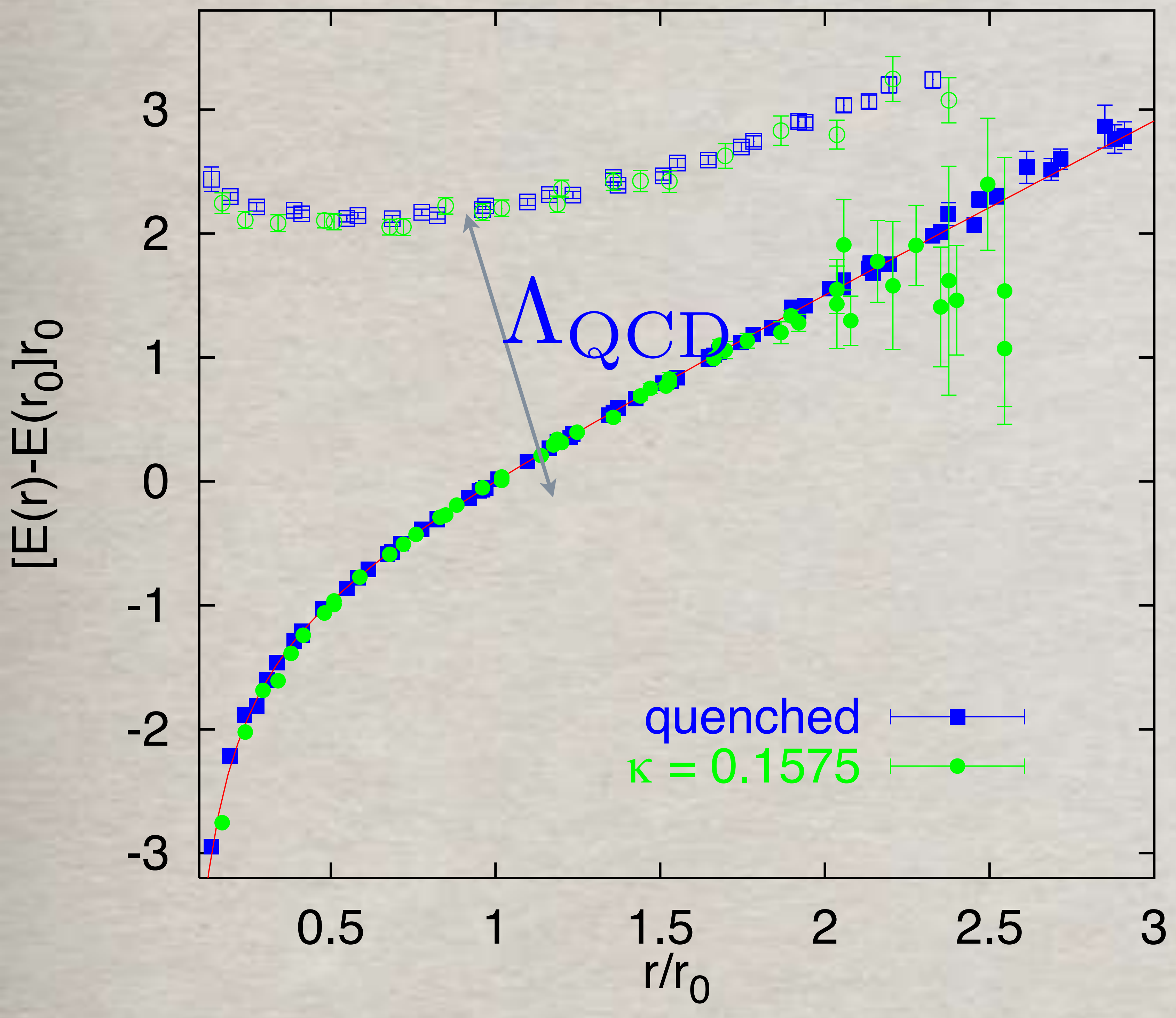
$|\underline{n} > 0; \mathbf{x}_1 \mathbf{x}_2\rangle - >$ $|(Q\bar{Q}) g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

Juge Kuti Mornigstar 98-06

$mv \sim \Lambda_{QCD}$

- pNRQCD and the potentials come from integrating out all scales up to mv^2

Bali et al. 98



- gluonic excitations develop a gap Λ_{QCD} and are integrated out

Brambilla Pineda Soto Vairo 00

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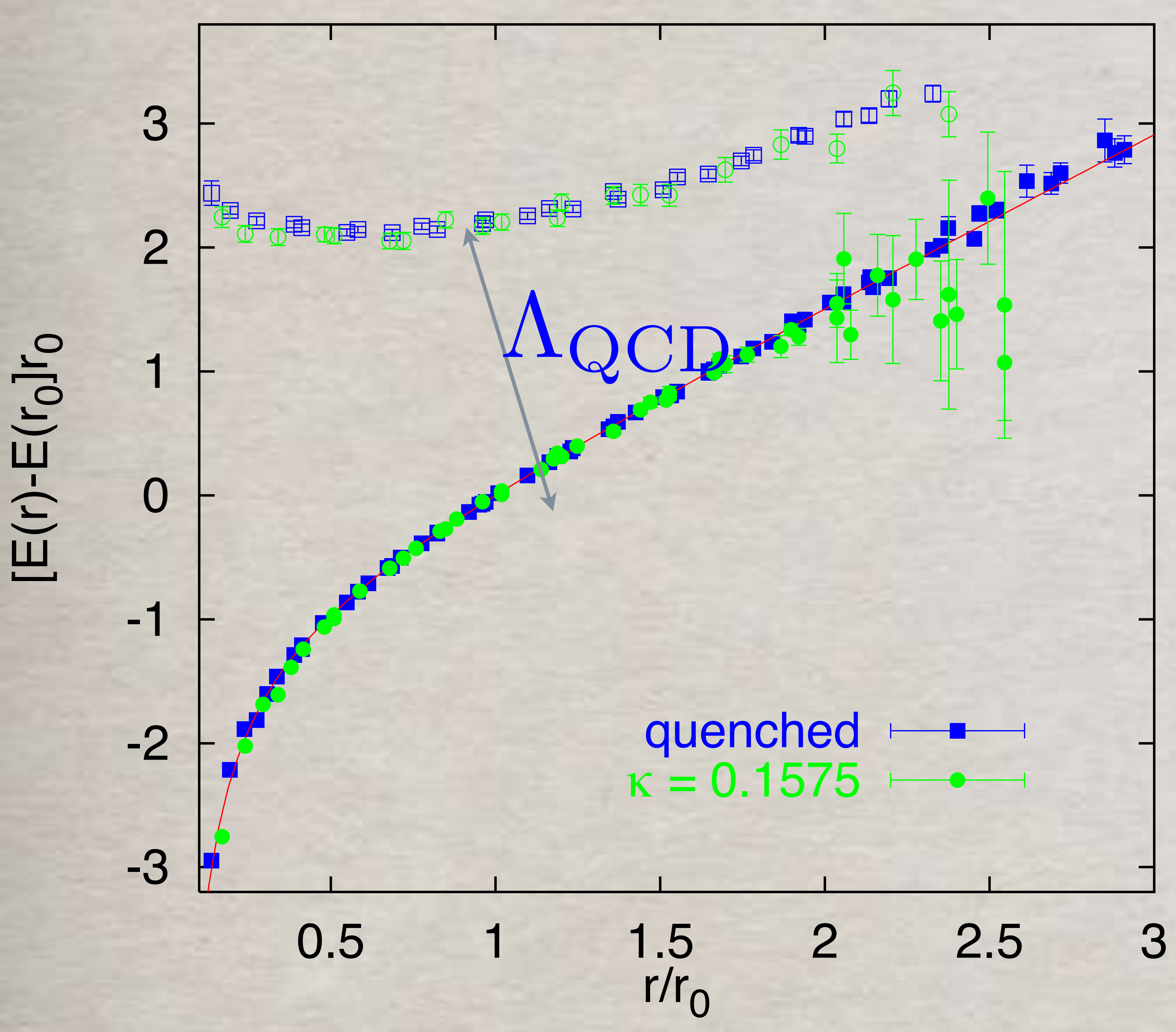

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⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

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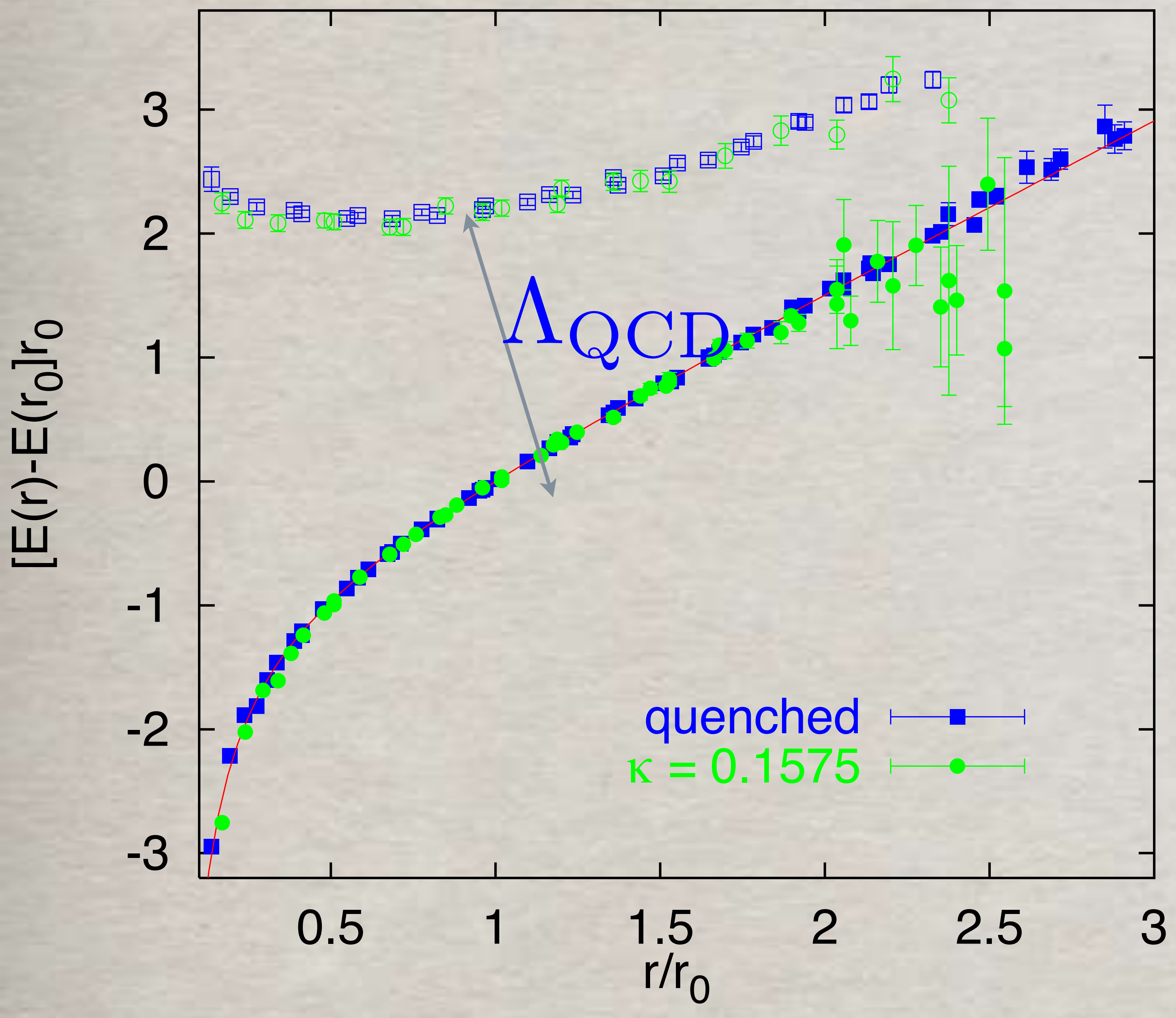

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$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

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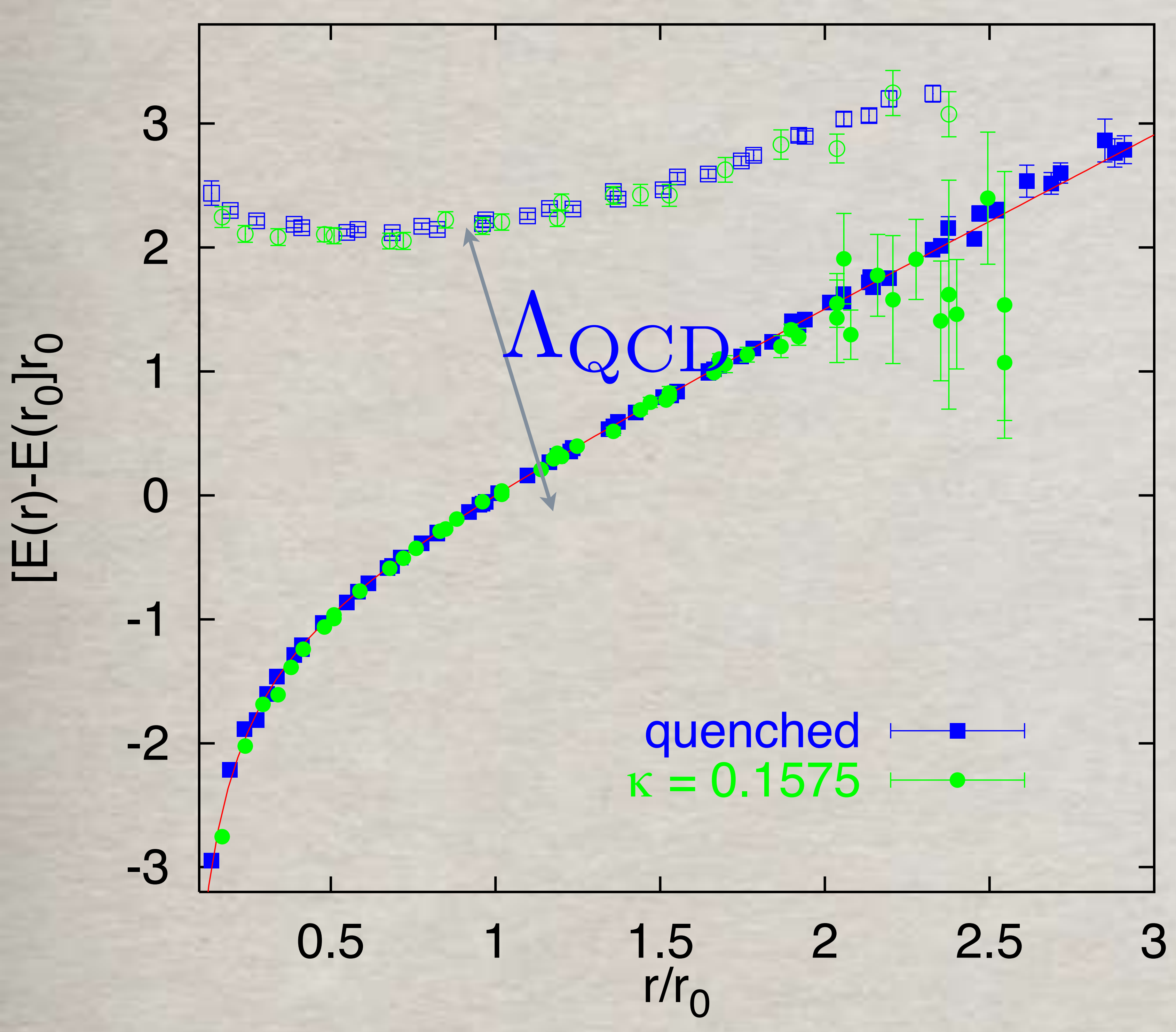

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- A pure potential description emerges from the EFT
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops, that are low energy pure gluonic correlators: all the flavour dependence is pulled out

Brambilla Pineda Soto Vairo 00

The matching condition is: $\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$

and from this we obtain the

Quarkonium singlet potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

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Quarkonium singlet potential

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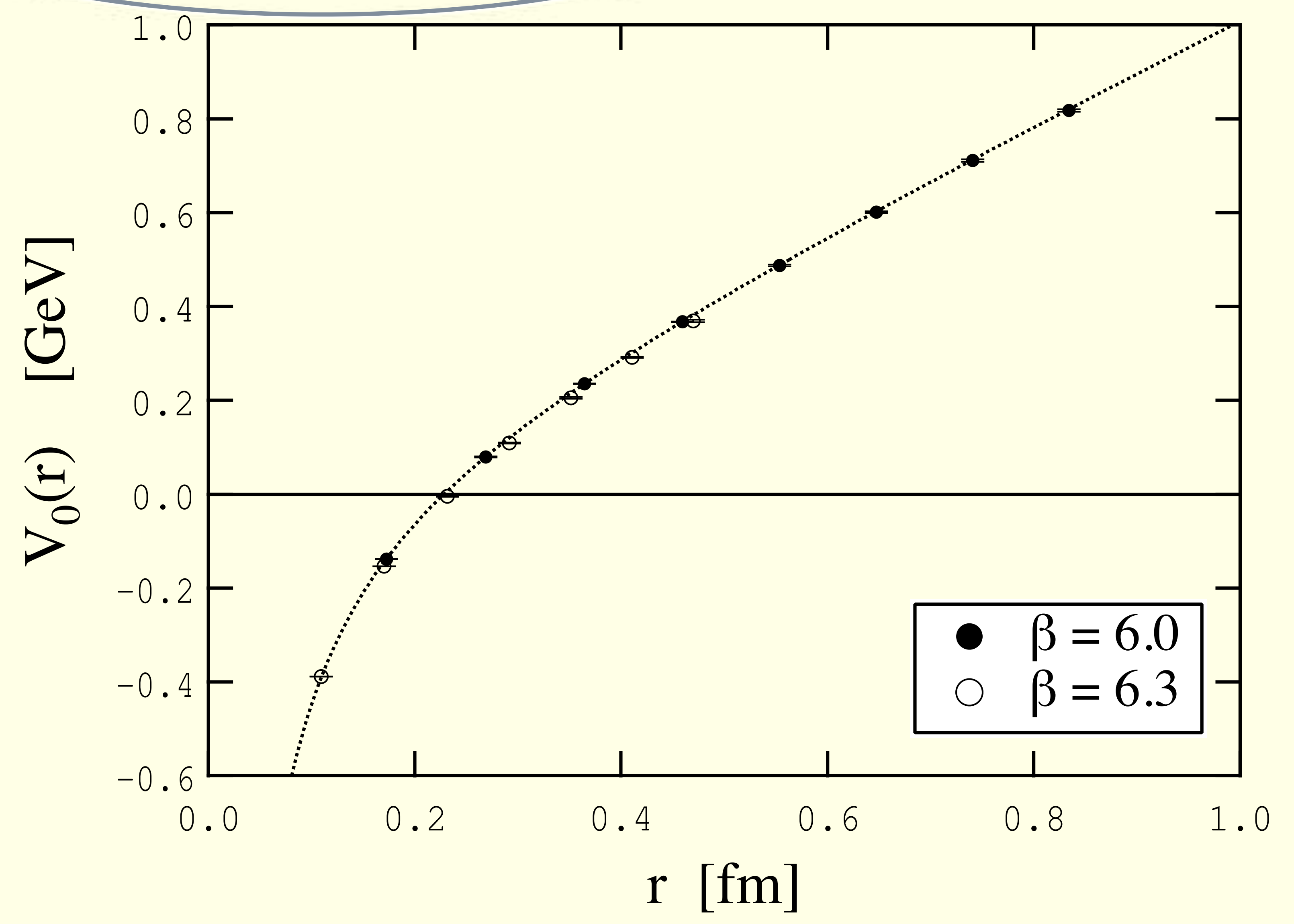
$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r\times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle$$

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$



○ Koma Koma NPB 769(07)79

QCD Spin dependent potentials

$$V_{\rm SD}^{(2)} = \frac{1}{r}\left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt\, t \,\langle \square^{E\,B}_{i\;j} \rangle - \frac{1}{2} V_s^{(0)\prime} \right) (\mathbf{S}_1 + \mathbf{S}_2)\cdot \mathbf{L}$$

$$-{c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \square_{i\;j} \rangle - \frac{\delta_{ij}}{3} \langle \square \rangle \right) \times \left(\mathbf{S}_1\cdot\mathbf{S}_2 - 3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) \right)$$

$$+\left(\frac{2}{3} {c_F}^2 i \int_0^\infty dt \langle \square \rangle - 4(d_2 + C_F d_4)\delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1\cdot\mathbf{S}_2$$

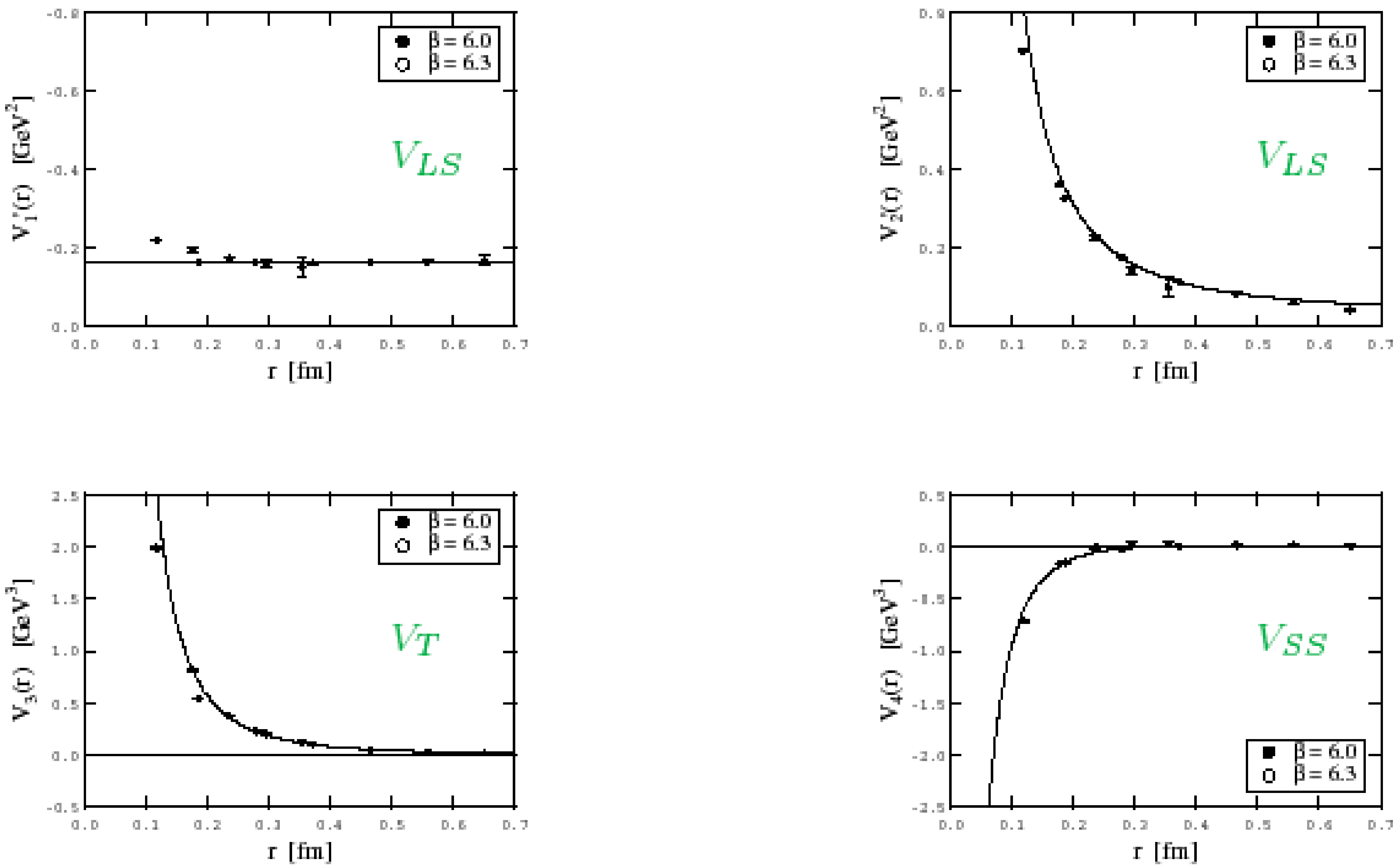
Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

-factorization: the NRQCD matching coefficients encode the physics at the large scale m, the potentials are given in terms of low energy nonperturbative Wilson loops. They depend only on the glue, only one lattice calculation to get the spectrum of charmonium bottomonium and Bc

-the spin dependent potential has the usual structure with spin-orbit, tensor and spin-spin terms. The spin-orbit term has a confining contribution: they appear at order 1/m^2

-the spin dependent potentials in the Schroedinger eq. give the multiplet spin structure

Spin dependent potentials

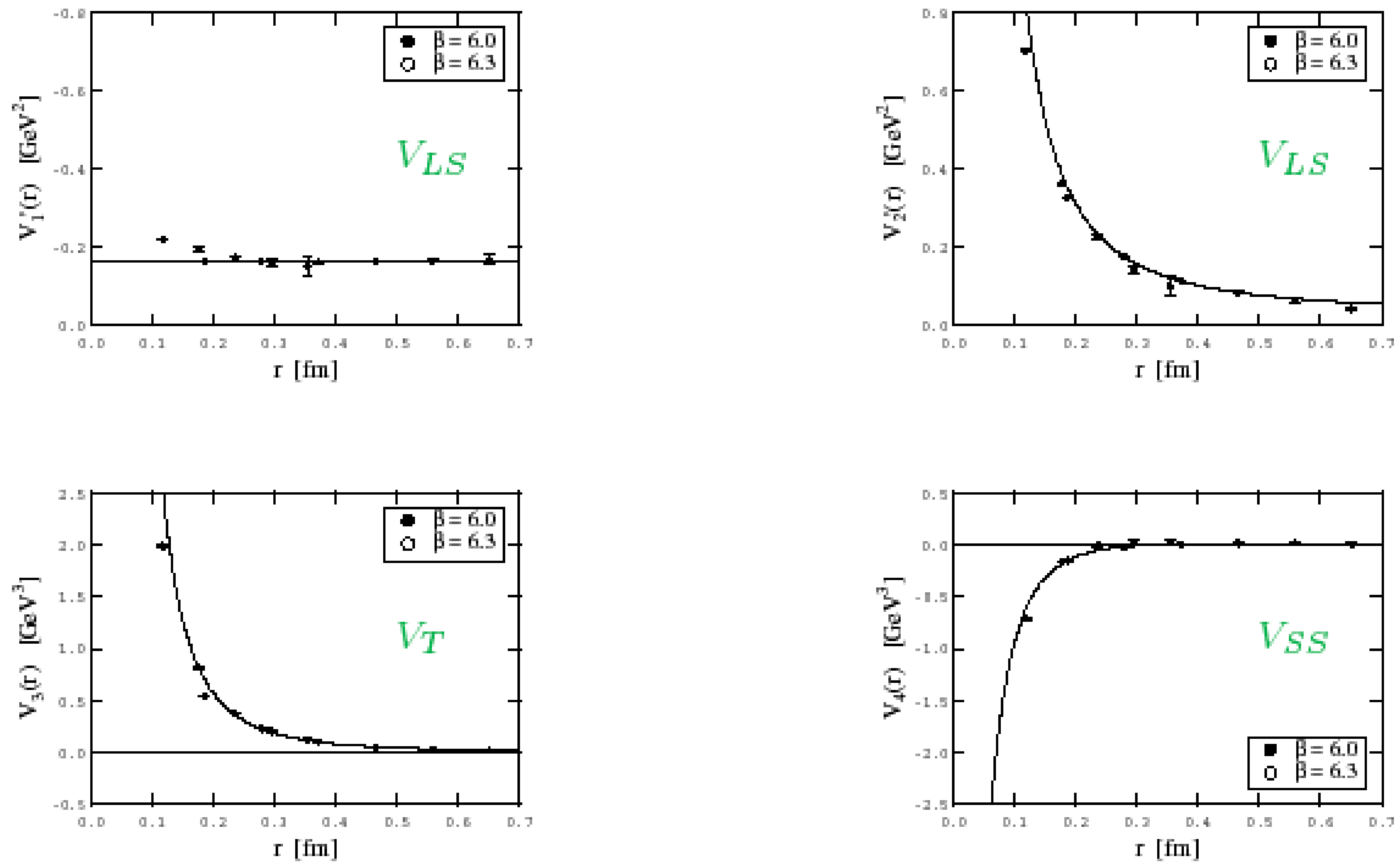


Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

N. B., Martinez, vairo 2014

Spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

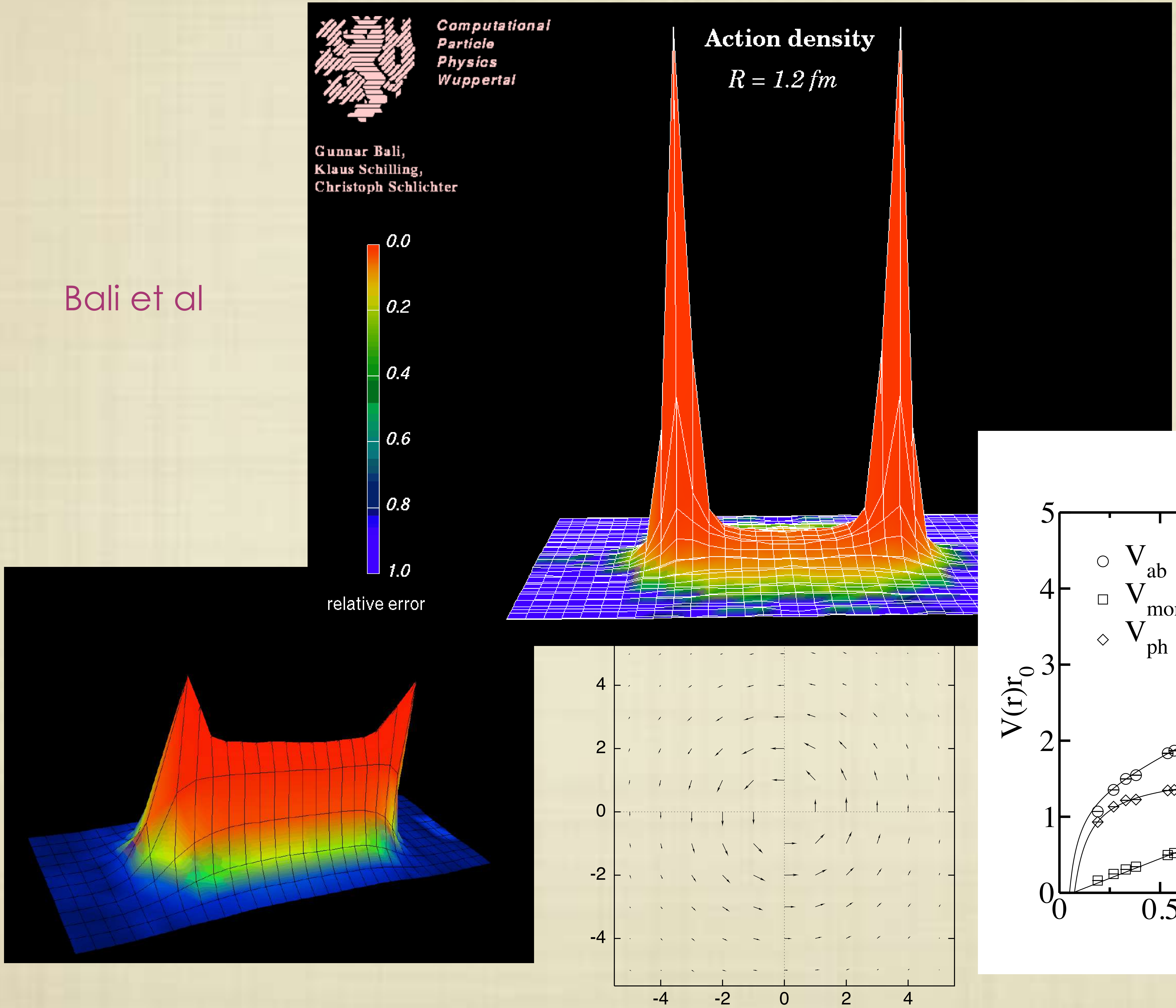
Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

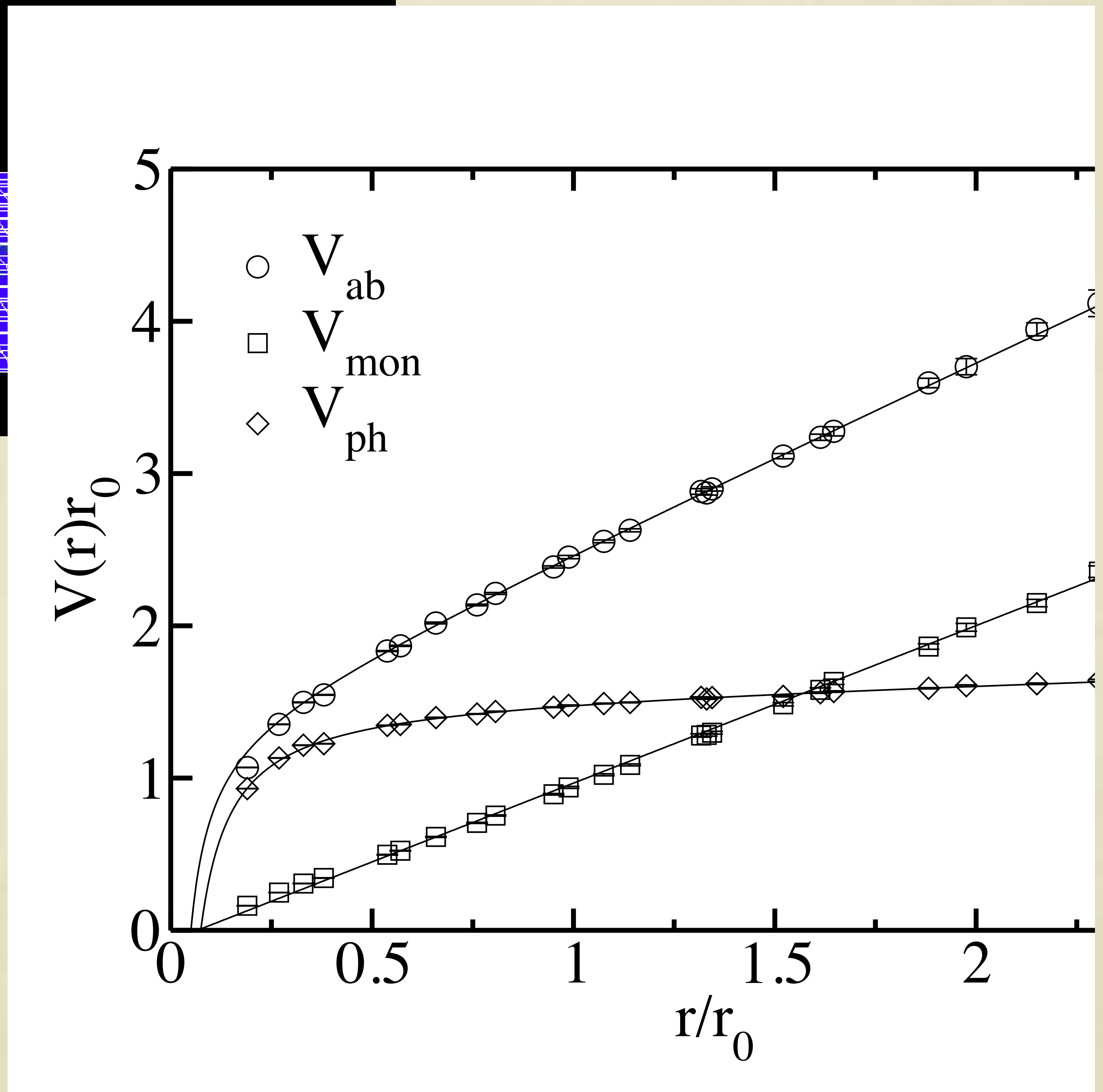
N. B., Martinez, vairo 2014

Low energy physics factorized in Wilson loops: can be used to probe the confinement mechanism

Bali et al



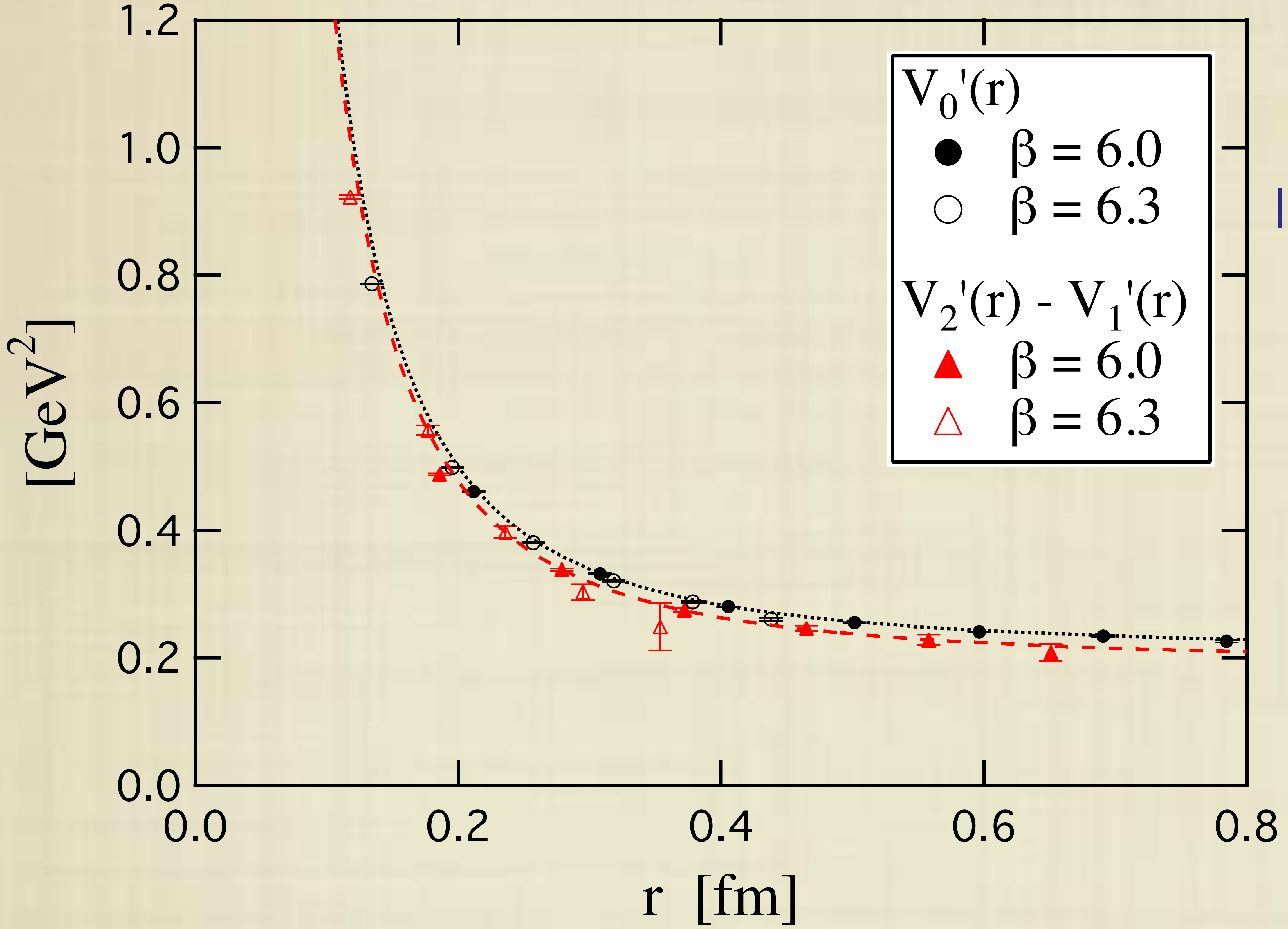
Boryakov et al. 04



Exact relations from Poincare' invariance

The EFT is still Poincare' invariant-> this induces relations among the potentials

Koma and Koma 2006



e. g. $V_0'(r) = V_2'(r) - V_1'(r)$

Gromes relation

It is a check of the lattice calculation

many other relations among potentials in the EFT

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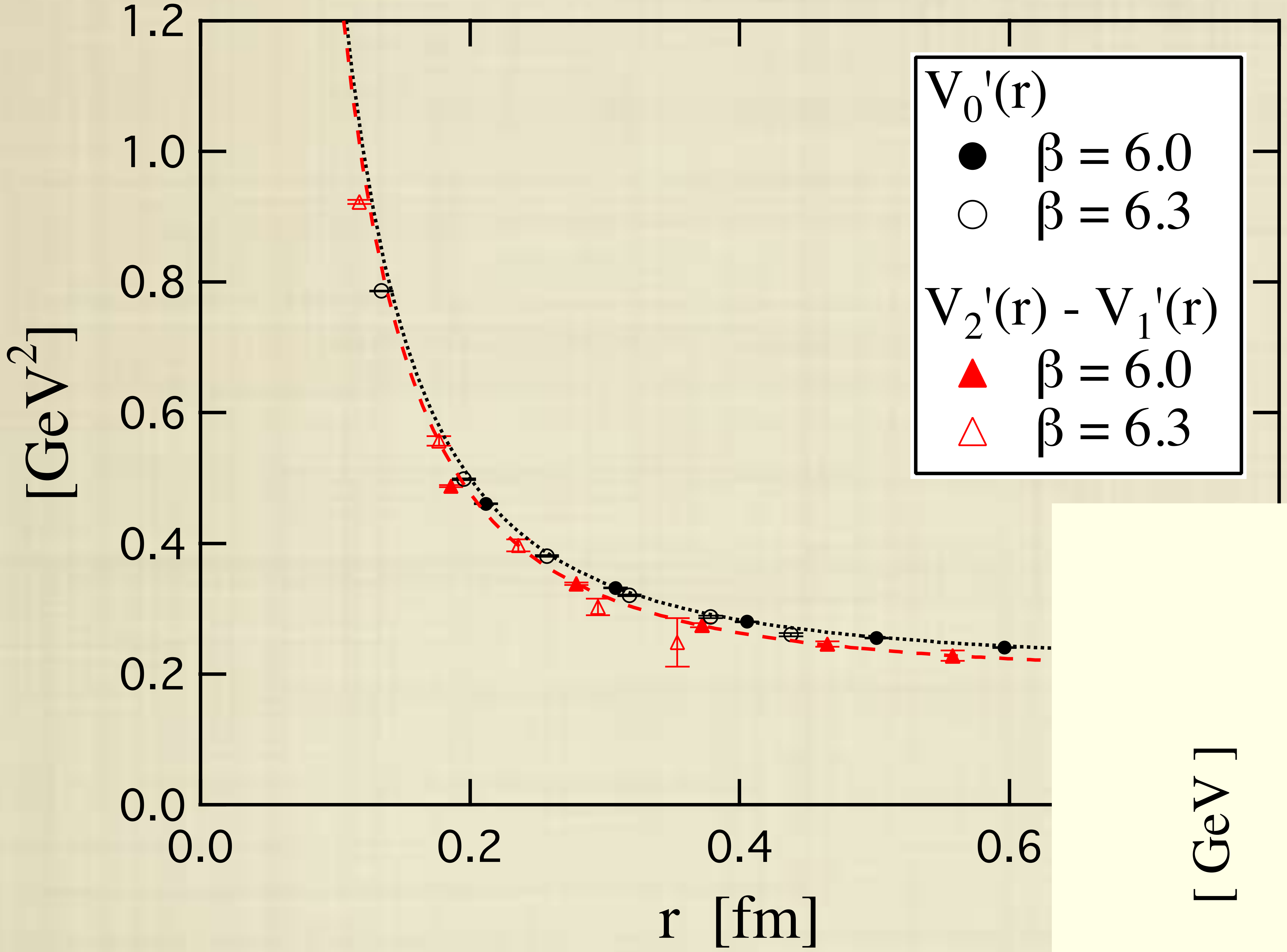

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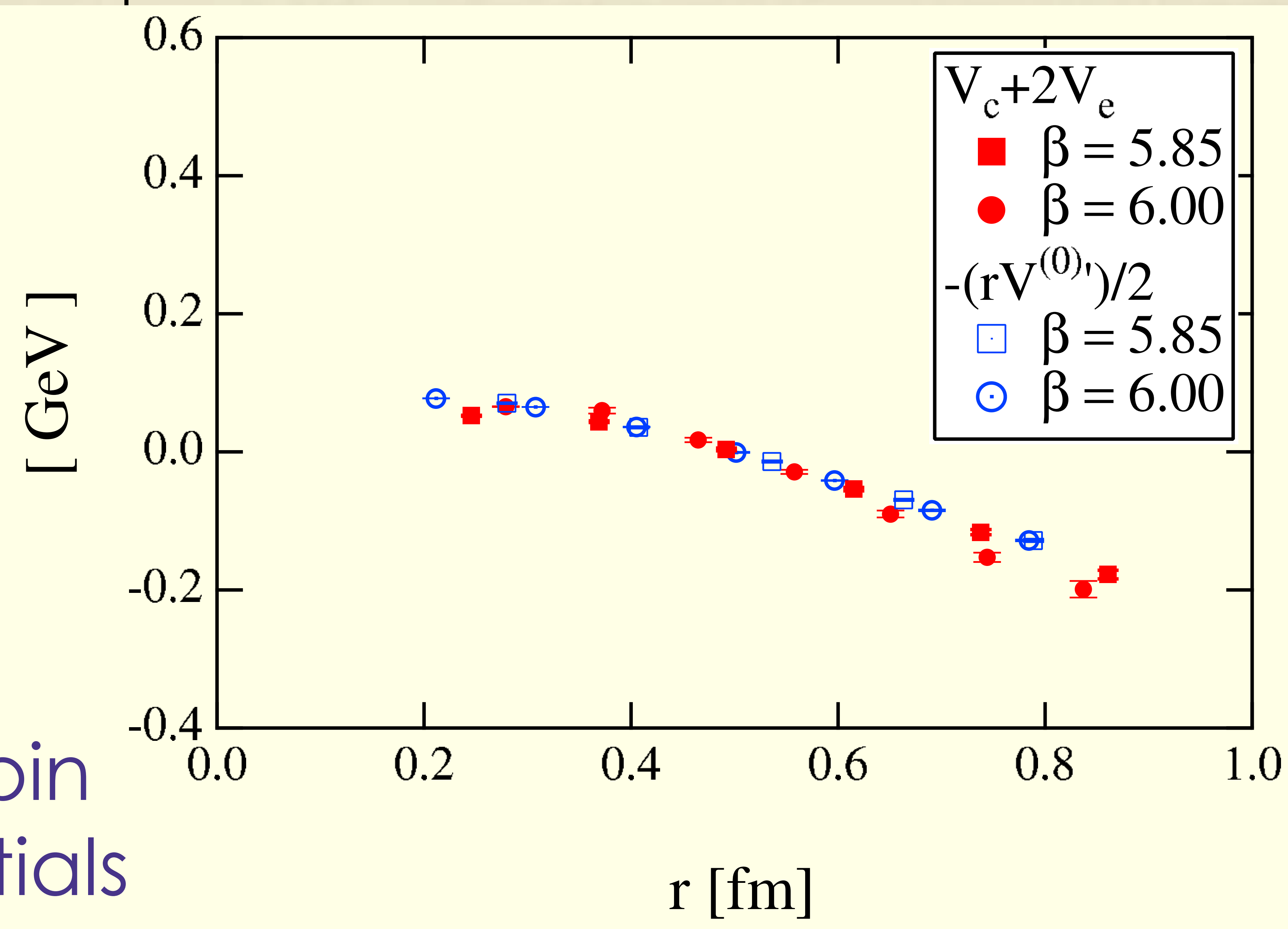

Koma Koma Wittig PoS LAT2007 (2007) 111

relations involving spin independent potentials

Applications of strongly coupled pNRQCD include: Quarkonium Production at LHC

Intense work in the theory community:

Qiu, Nayak, Sterman, Butenschon Kniehl , Bodwin , Hee Soh, Chung, J. Lee, Kuang Ta Chao, Y. Q. Ma, Gong Wang, Fleming, Mehen, Yu Jia, Braaten, Lansberg, Leibovich, Rothstein…

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NRQCD factorization formula for quarkonium production
valid for large p_T   Bodwin Braaten Lepage 1995

cross section
$$\sigma(H) = \sum_n F_n \langle 0|\mathcal{O}_n^H|0\rangle.$$

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short distance coefficients
partonic hard scattering cross section
convoluted with parton distribution

long distance matrix elements
(LDME)
give the probability of a qqbar pair with certain quantum number to evolve into a final quarkonium H

they are vacuum expectation values of four fermion operators with color singlet and color octet contributions and a projection over quarkonium plus X in the middle

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One problem is the proliferation of LDMEs:
nonperturbative objects
that cannot evaluated on the lattice
and should be extracted from the data,
they depend on the considered quarkonium state

Intense work in the theory community:

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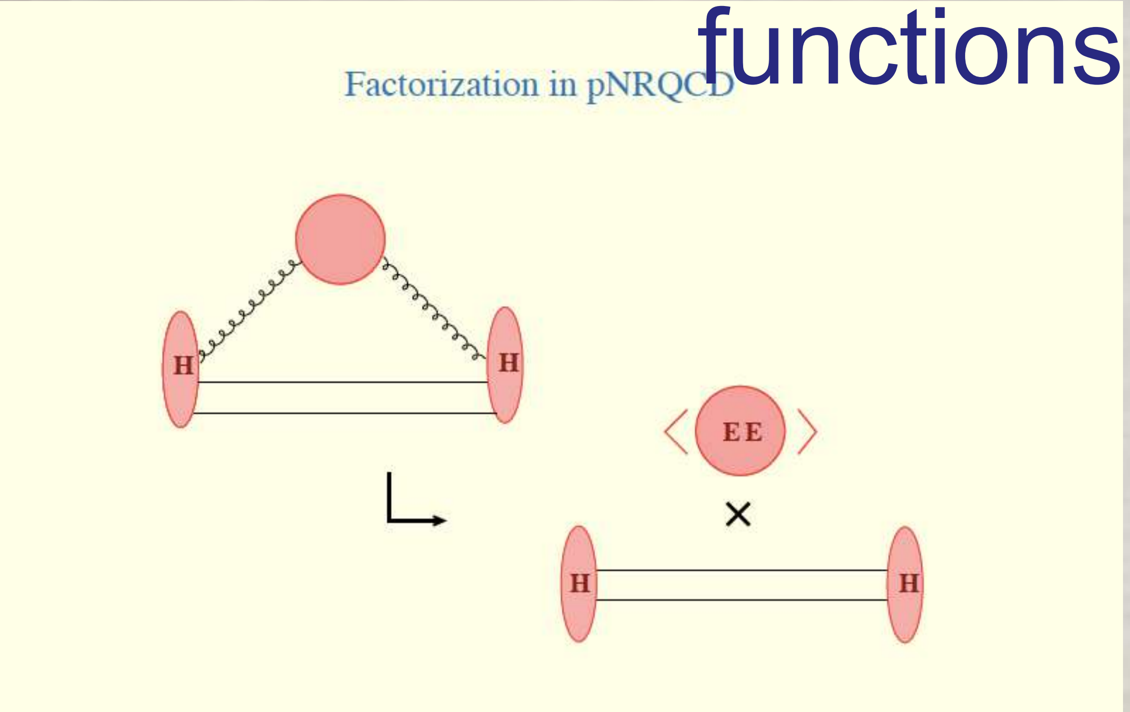
Factorization of LDMEs in pNRQCD : the NRQCD LDMEs are factorized in terms of wave functions and universal nonperturbative correlators depending only on the glue



- The number of nonperturbative unknowns is reduced by half
- The nonperturbative unknowns are correlators of gluonic fields that can be calculated on the lattice

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

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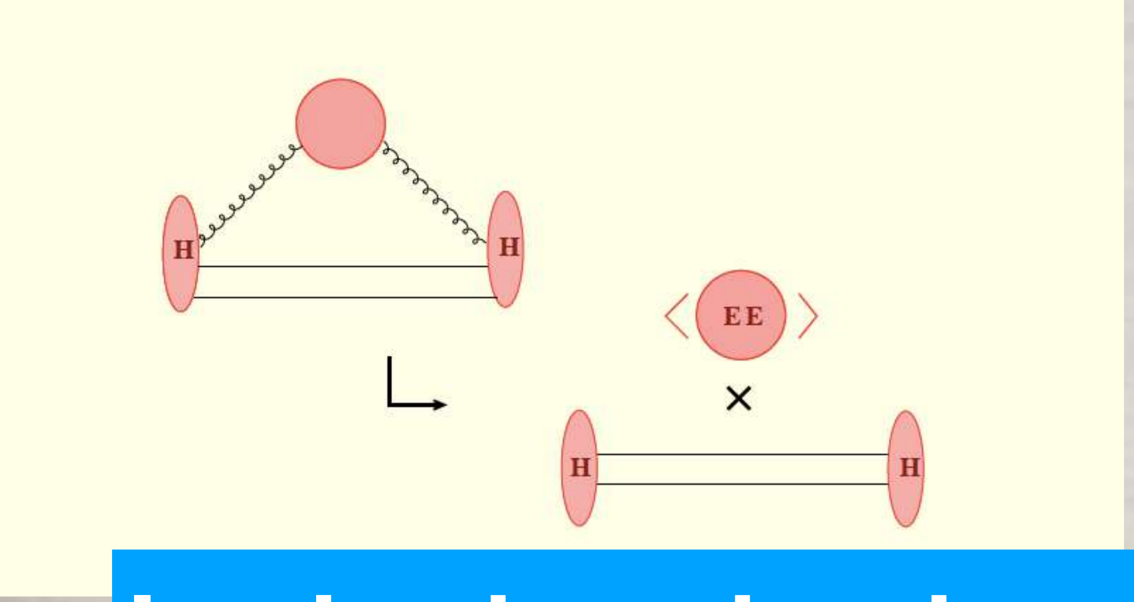

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Inclusive hadroproduction of p wave quarkonia

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle.$$

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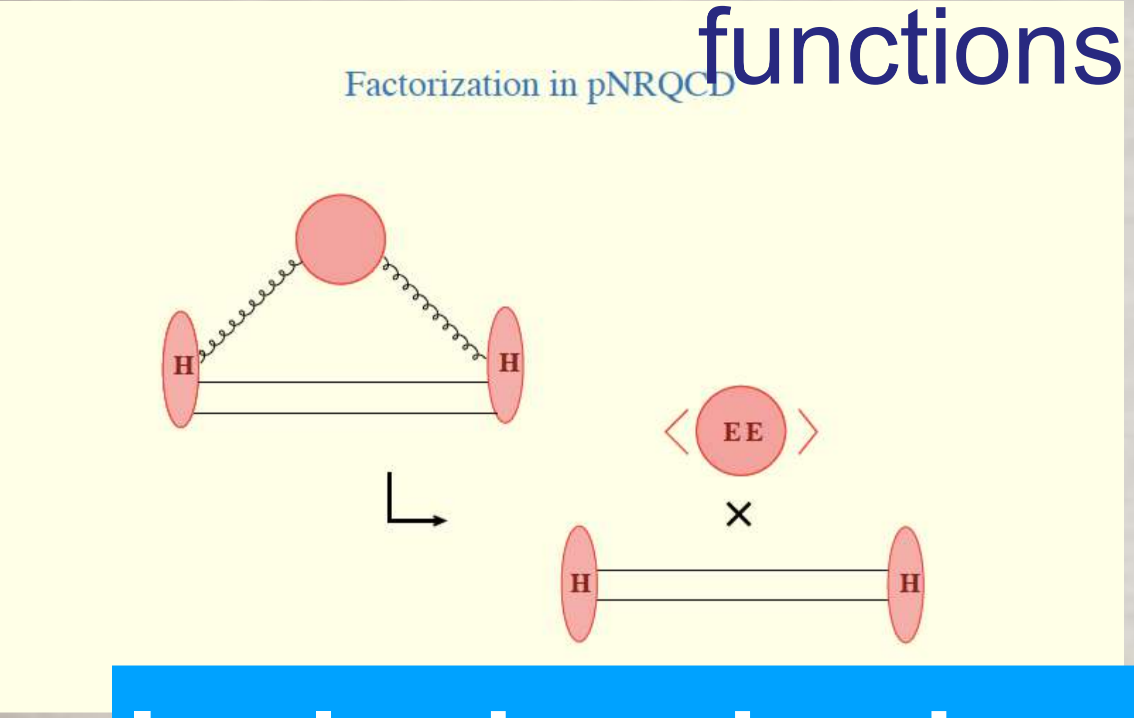

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$$+ (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})} \quad \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \frac{\mathcal{E}}{9N_c m^2}$$

- The dimensionless correlator $\mathcal{E}$ is defined in terms of chromoelectric fields gE with Wilson lines Φ extending to infinity in the ℓ direction.

$$\mathcal{E} = \frac{3}{N_c}\int_0^\infty t\,dt\int_0^\infty t'\,dt'\langle\Omega|\Phi_\ell^{\dagger ab}\Phi_0^{\dagger da}(0,t)gE^{d,i}(t)gE^{e,i}(t')\Phi_0^{ec}(t',0)\Phi_\ell^{bc}|\Omega\rangle$$

- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

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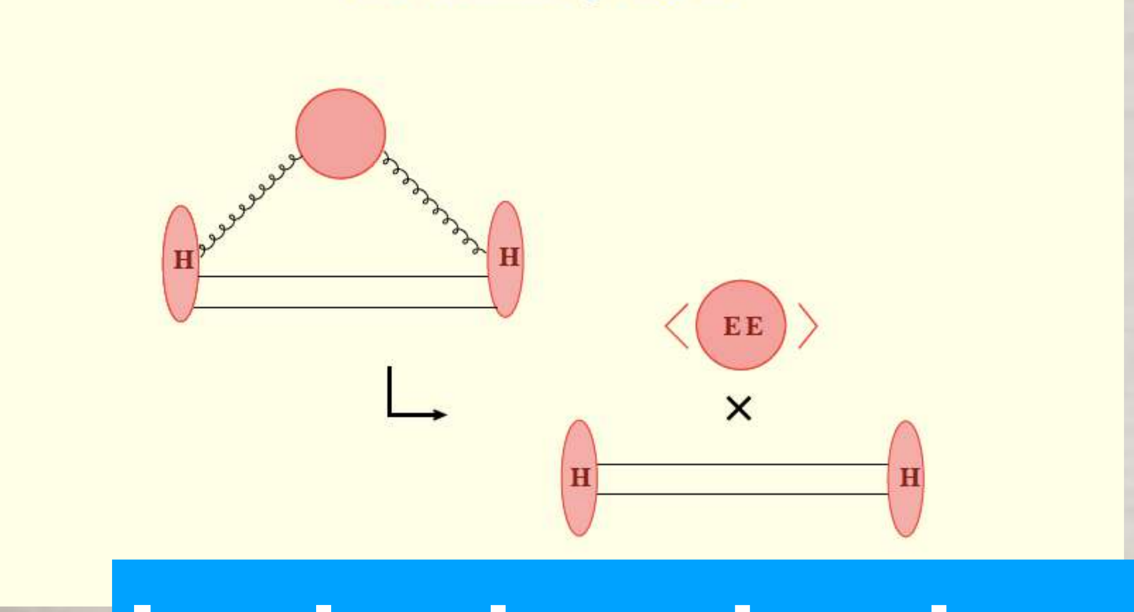

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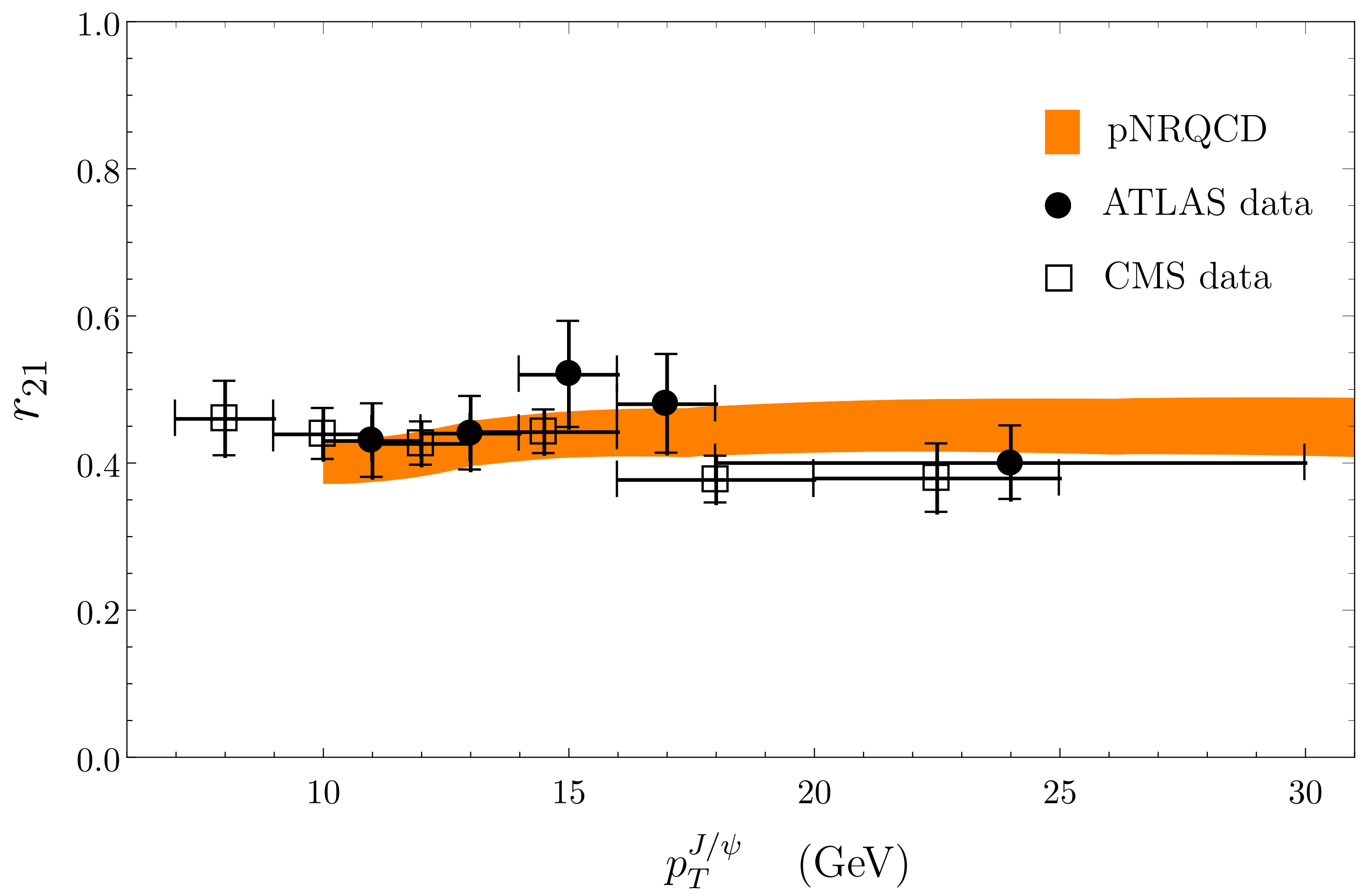
- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

$\mathcal{E}$ is a universal quantity that ***does not depend on quark flavor or radial excitation***. Determination of $\mathcal{E}$ directly leads to ***determination of all*** χ_{cJ} ***and*** $\chi_{bJ}(nP)$ ***cross sections, as well as*** h_c ***and*** h_b ***production rates***.

->good description of data at ATLAS and CMS

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

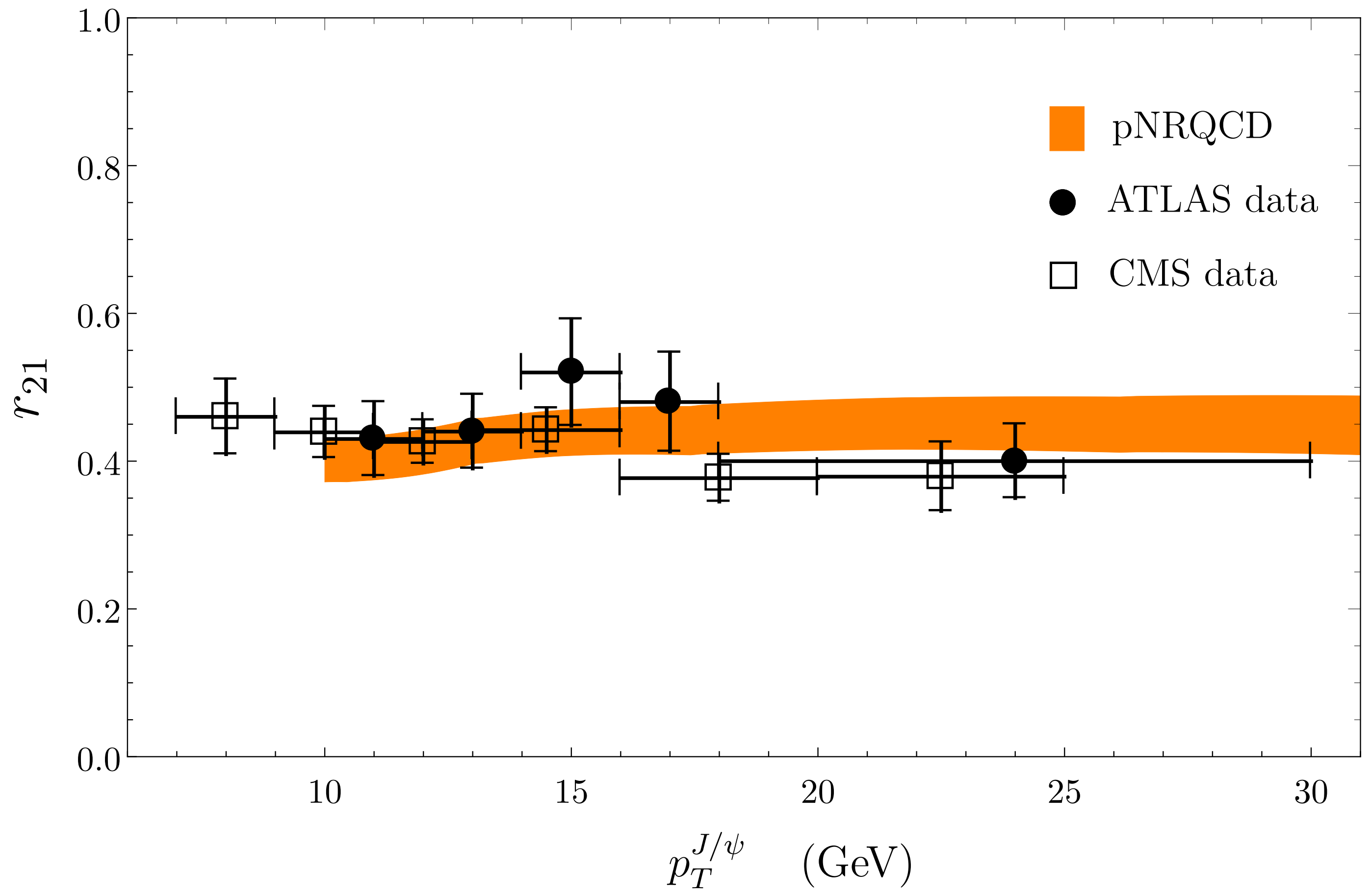
Ratio r_{21} of χ_{c2} and χ_{c1} cross sections at the LHC (**CMS, ATLAS)**



fix here $\mathcal{E}$ and predict

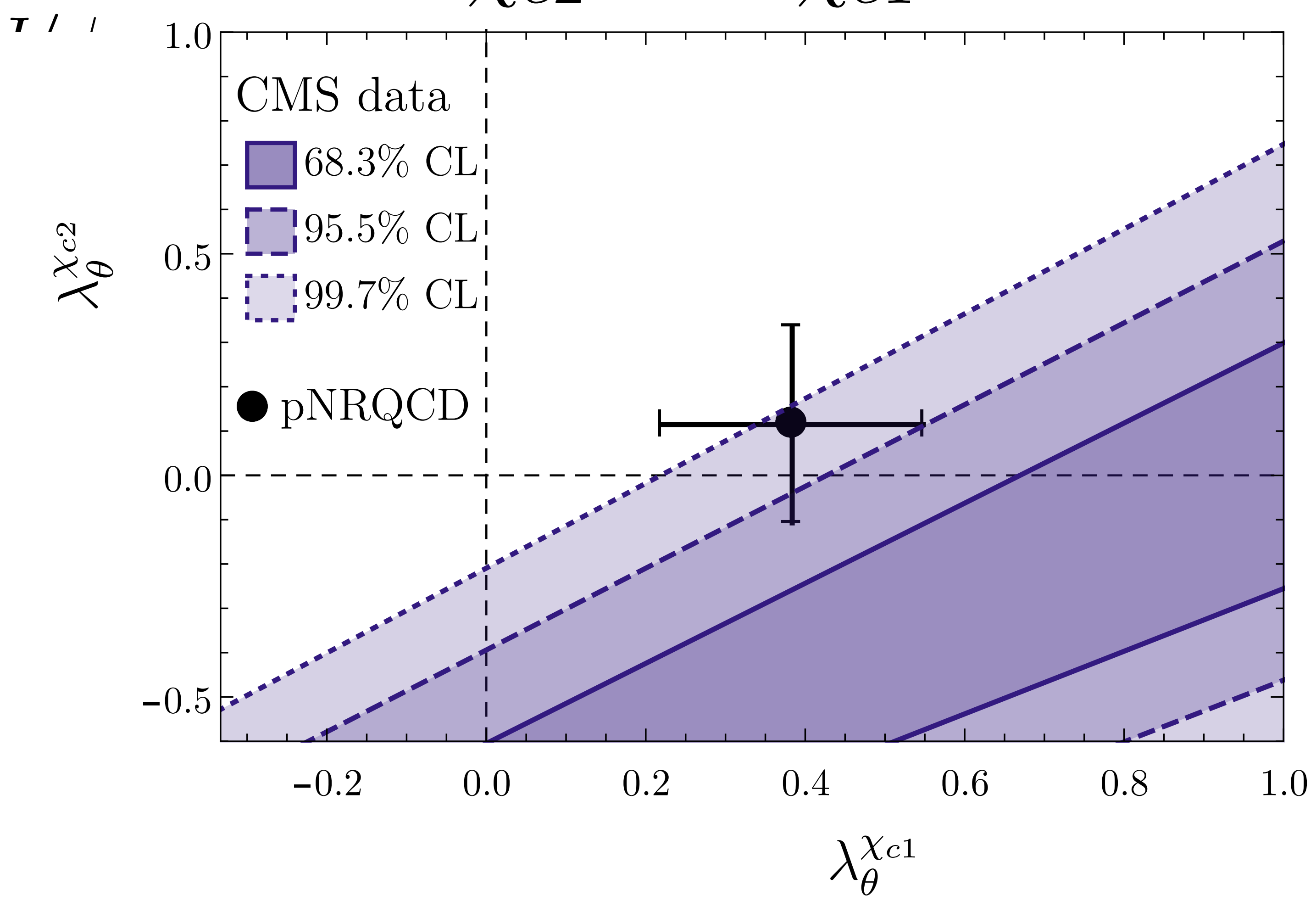
we are currently investigating the J/psi case

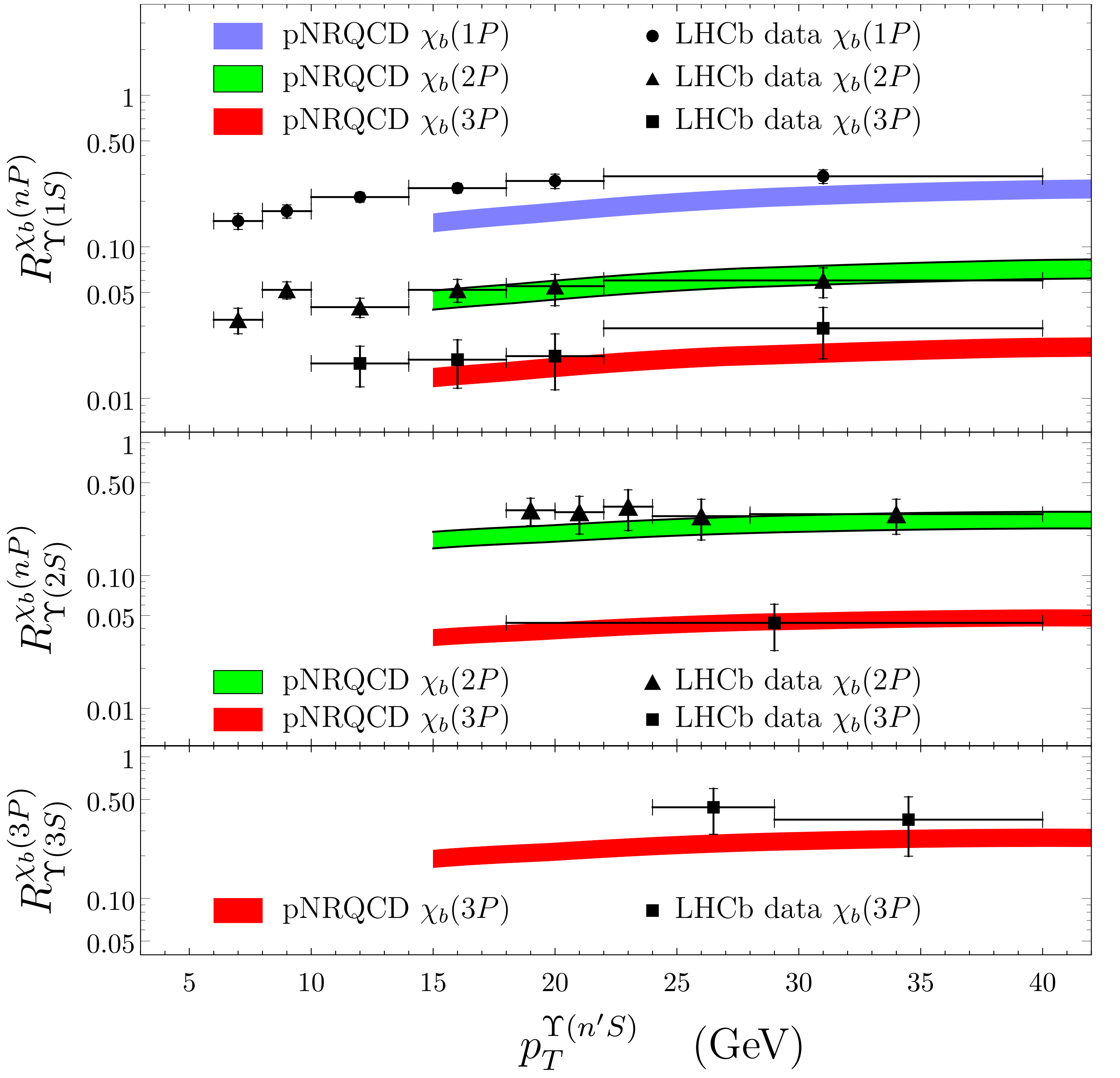
Ratio r_{21} of χ_{c2} and χ_{c1} cross sections at the LHC (CMS, ATLAS)



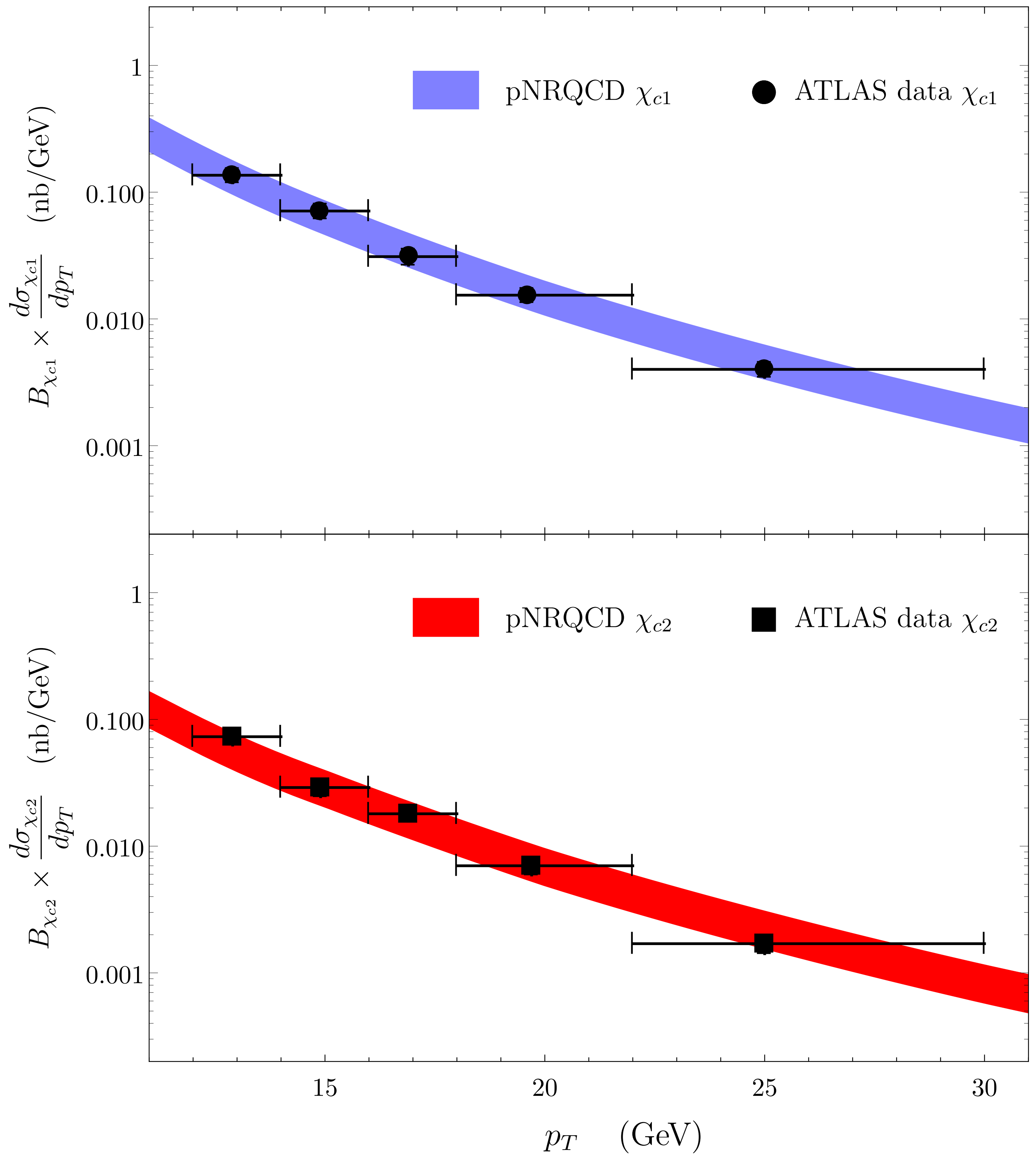
fix here $\mathcal{E}$ and predict

Polarization of χ_{c2} and χ_{c1} at the LHC

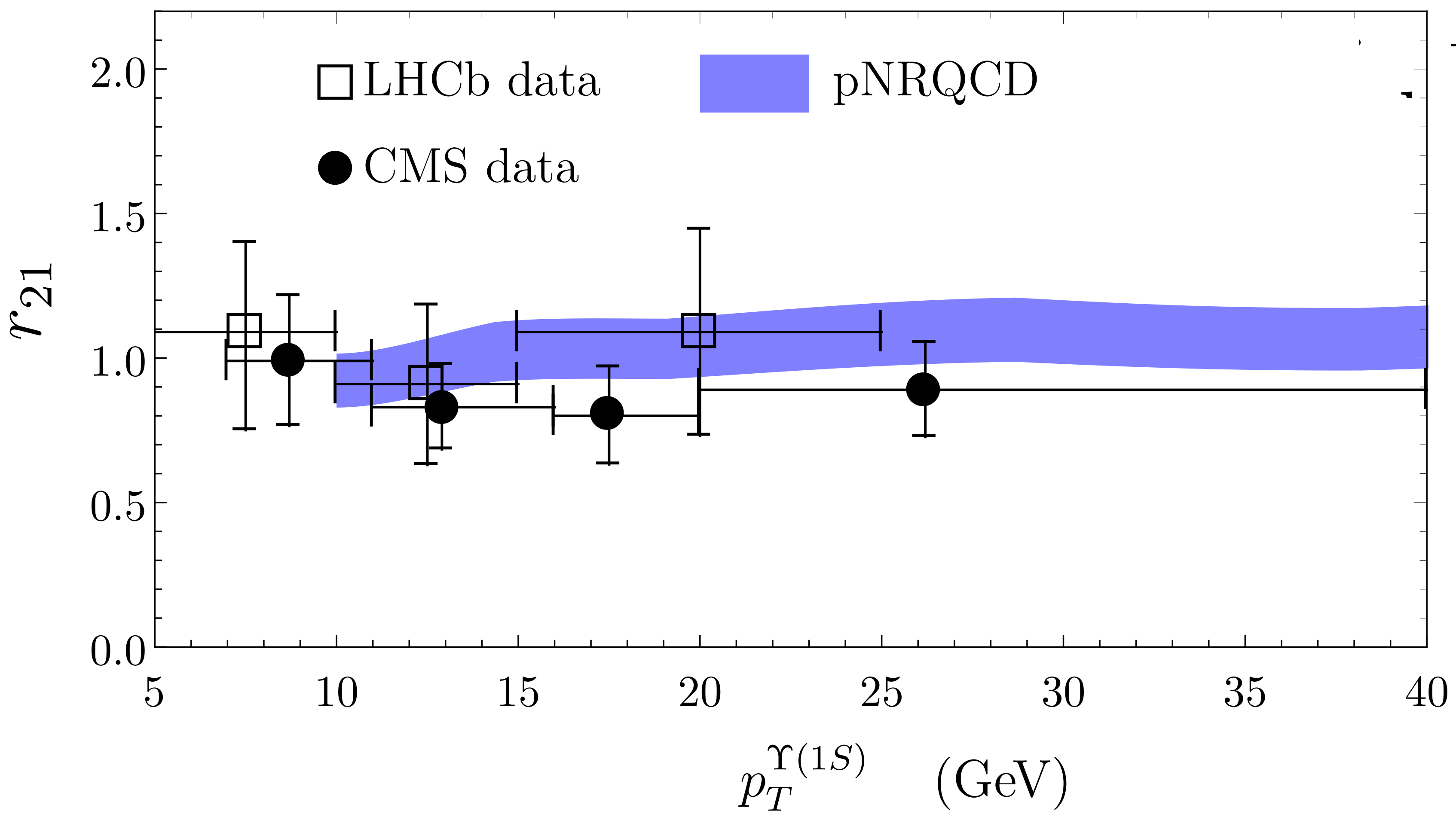




Feeddown fractions $R^{\chi_b(nP)}_{\Upsilon(n'S)}$ at the LHC



Production cross sections of χ_{c2} and χ_{c1} at the LHC



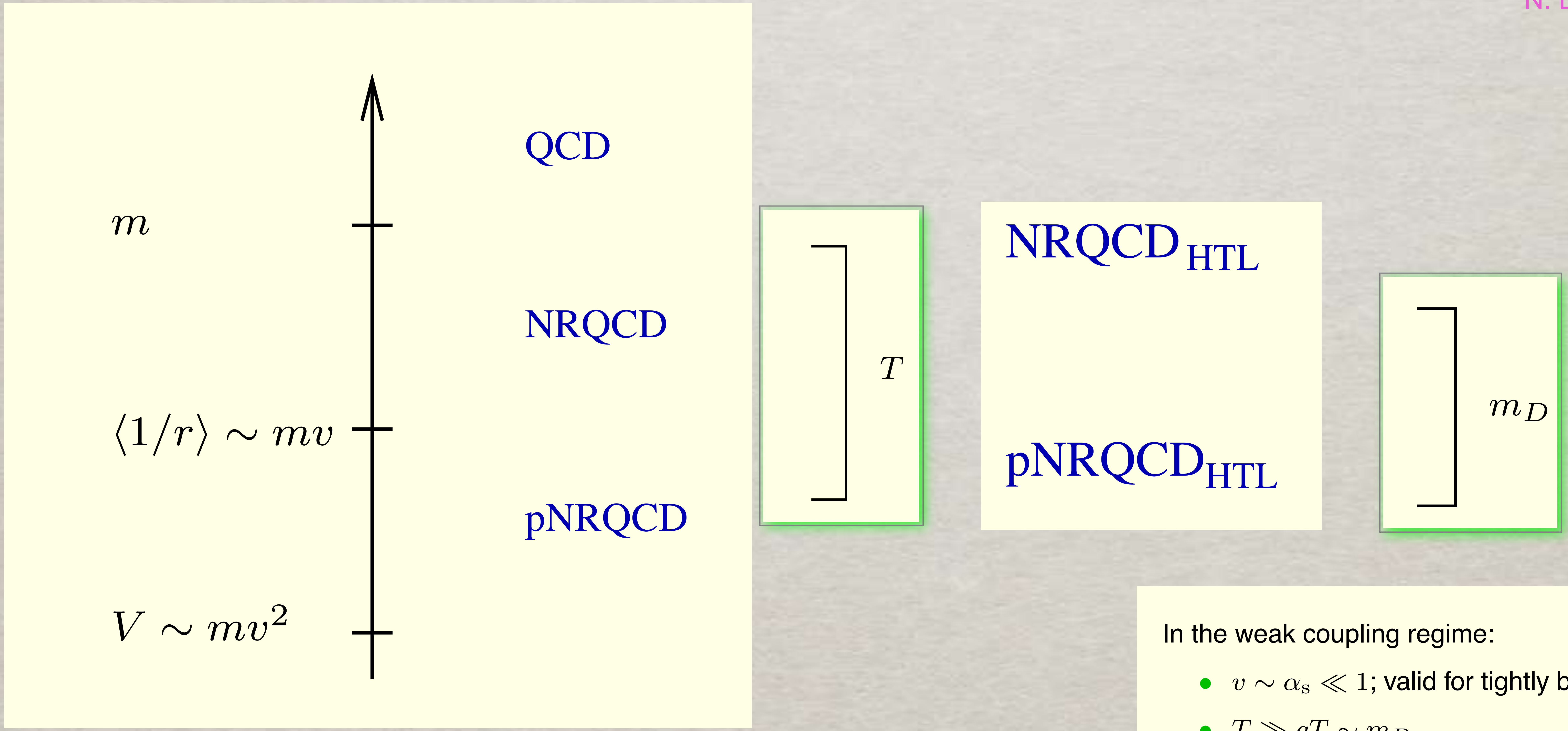
Ratio of χ_{b2} and χ_{b1} cross sections at the LHC (

we are currently investigating the J/psi case

Notice: additional scales smaller than m can be integrated out combining with other EFTs

Example: quarkonium in thermal medium, T<m, the thermal medium has scales T and m_d=gT=> integrate out T produces Hard Thermal EFT (HTL)

N. B., Ghiglieri, Petreczky, Vairo 08



if T > E the potential has thermal effects

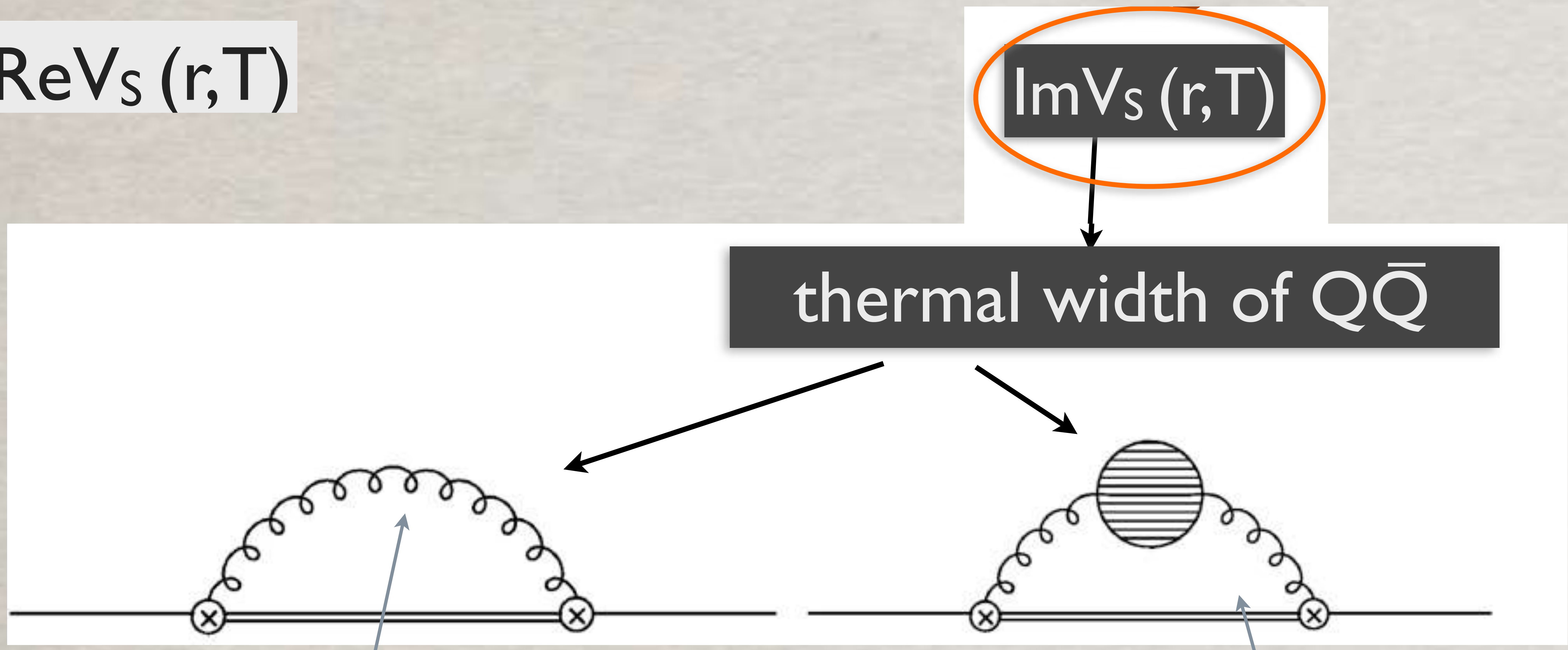
In the weak coupling regime:

- $v \sim \alpha_s \ll 1$; valid for tightly bound states: $\Upsilon(1S)$, J/ψ, ...
- $T \gg gT \sim m_D$.

Effects due to the scale Λ_{QCD} will not be considered.

The potential V(r,T) dictates throught the Schroedinger equation the real time evolution of the QQbar pair in the medium-> use pNRQCD to define and calculate it

$ReV_S (r,T)$



Singlet-to-octet

N.B Ghiglieri, Petreczky, Vairo 2008

(gluo dissociation)

Landau damping

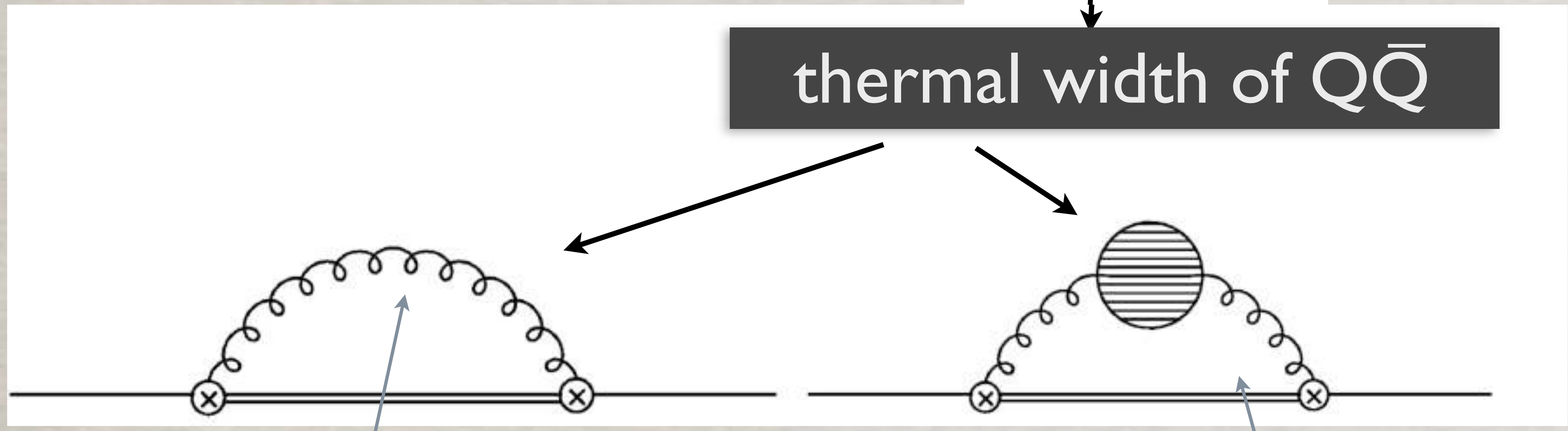
Laine et al 07, Escobedo Soto 07

(inelastic parton scattering)

The potential V(r,T) dictates throught the Schroedinger equation the real time evolution of the QQbar pair in the medium-> use pNRQCD to define and calculate it

ReVs (r,T)

ImVs (r,T)



Singlet-to-octet

N.B Ghiglieri, Petreczky, Vairo 2008

(gluo dissociation)

Landau damping

Laine et al 07, Escobedo Soto 07

(inelastic parton scattering)

The EFT produces:

=> T effects can be different from screening

=> dissociation of quarkonium is driven by the imaginary parts of the potentials instead than by Debye screening

=> a technology to calculate systemically thermal energies and widths: spectrum of quarkonium at finite T at alpha_s^5

Notice: pNREFT is the lowest energy EFT for a single NR system but in the interaction between two NR systems more energy scales can be integrated out giving interaction potentials:

WEFT the EFT for bound-state—bound state-interaction

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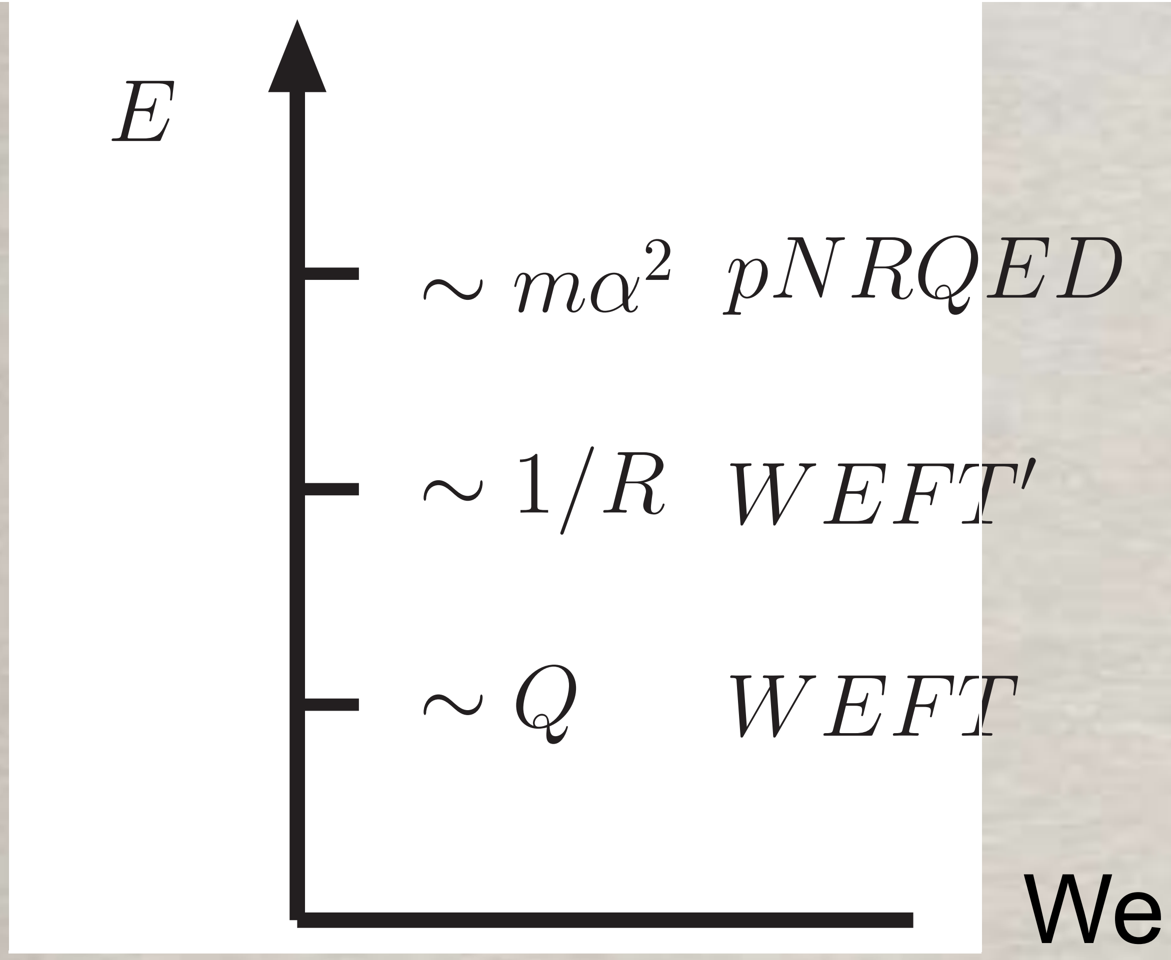
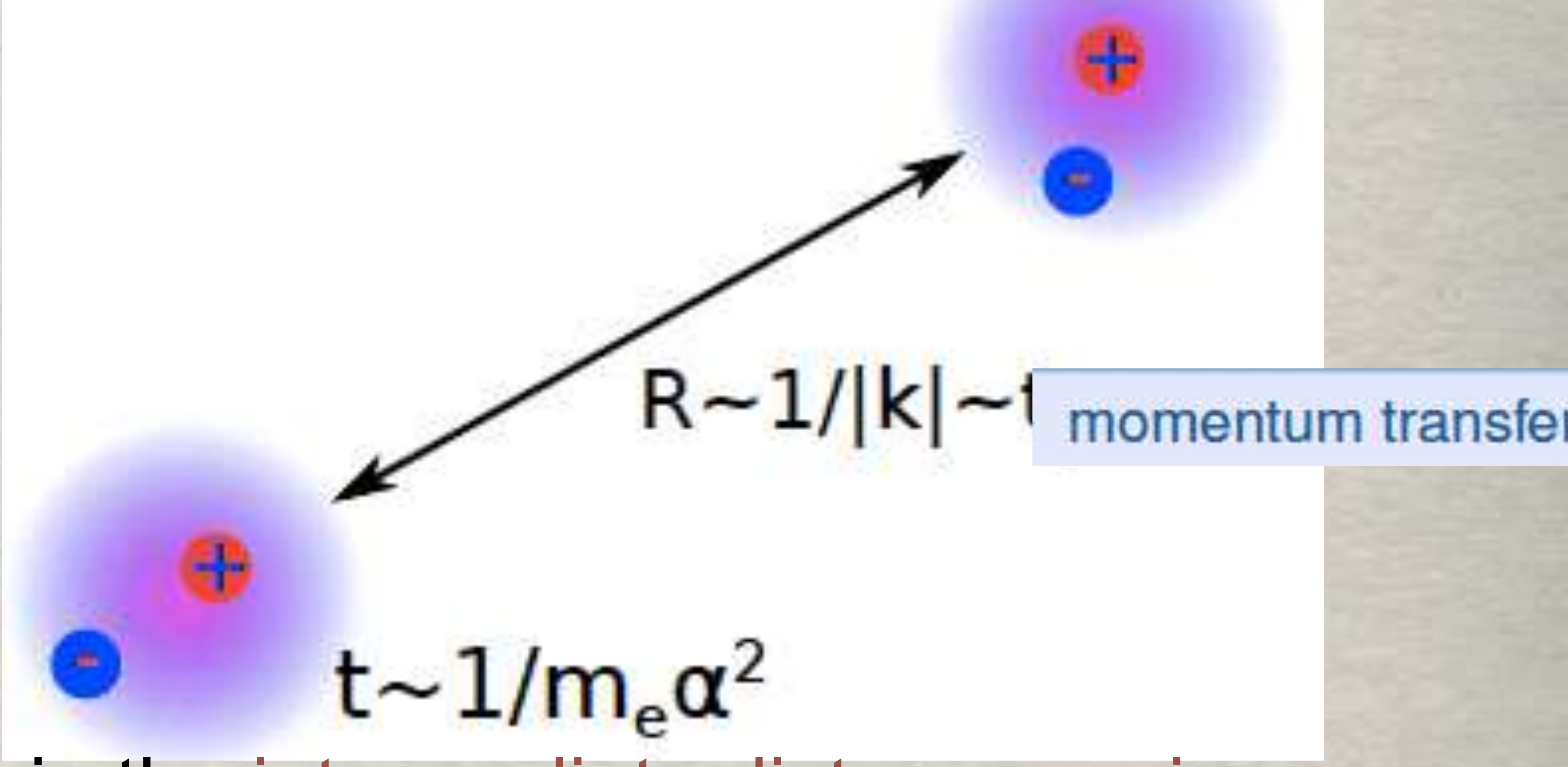


WEFT is matched to pNREFT

Van der Walls EFT

N.B., V. Shtabovenko, J. Tarrus, A. Vairo 1704.03476



We have obtained the van der Waals potential also in the intermediate distance region (limits for short and large distance reproduce London and Casimir Polder)

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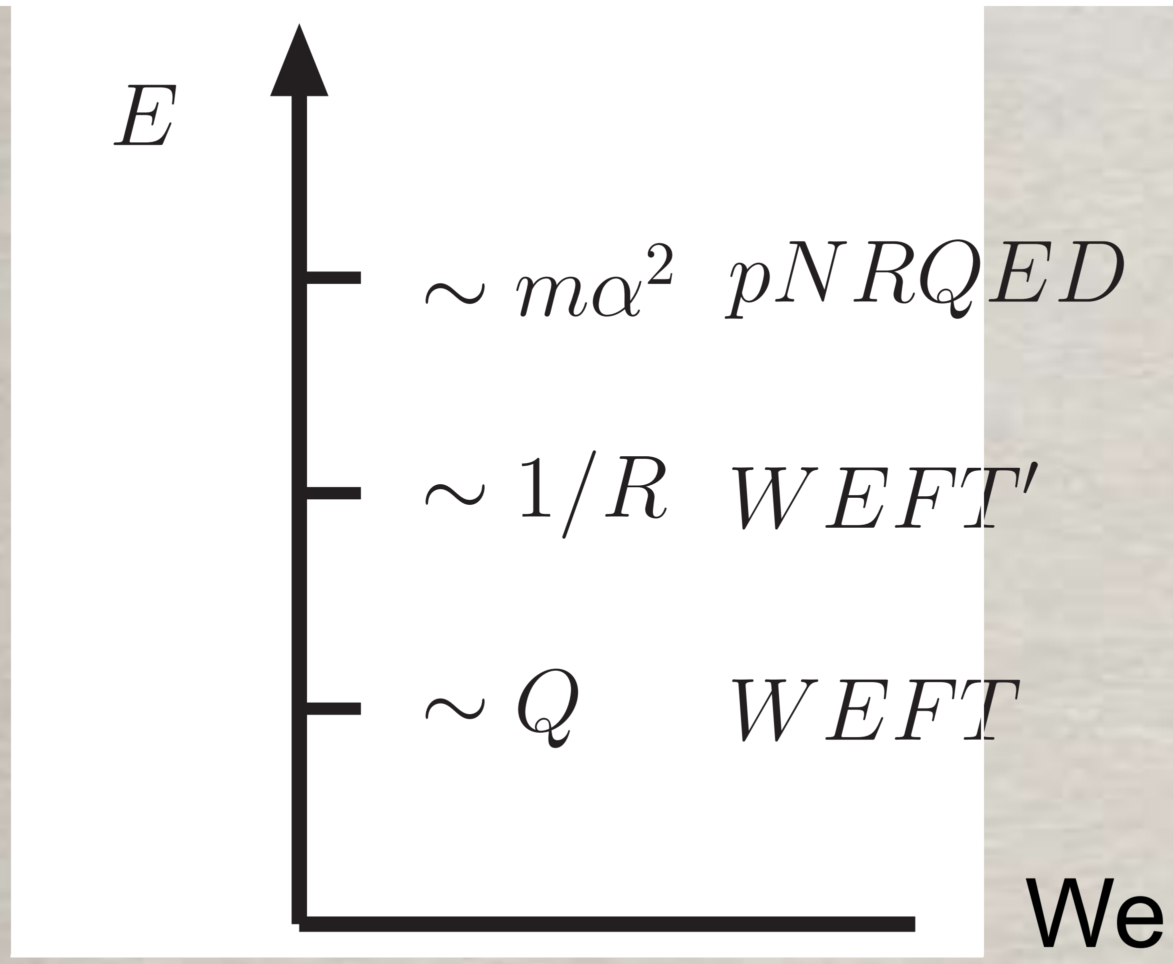

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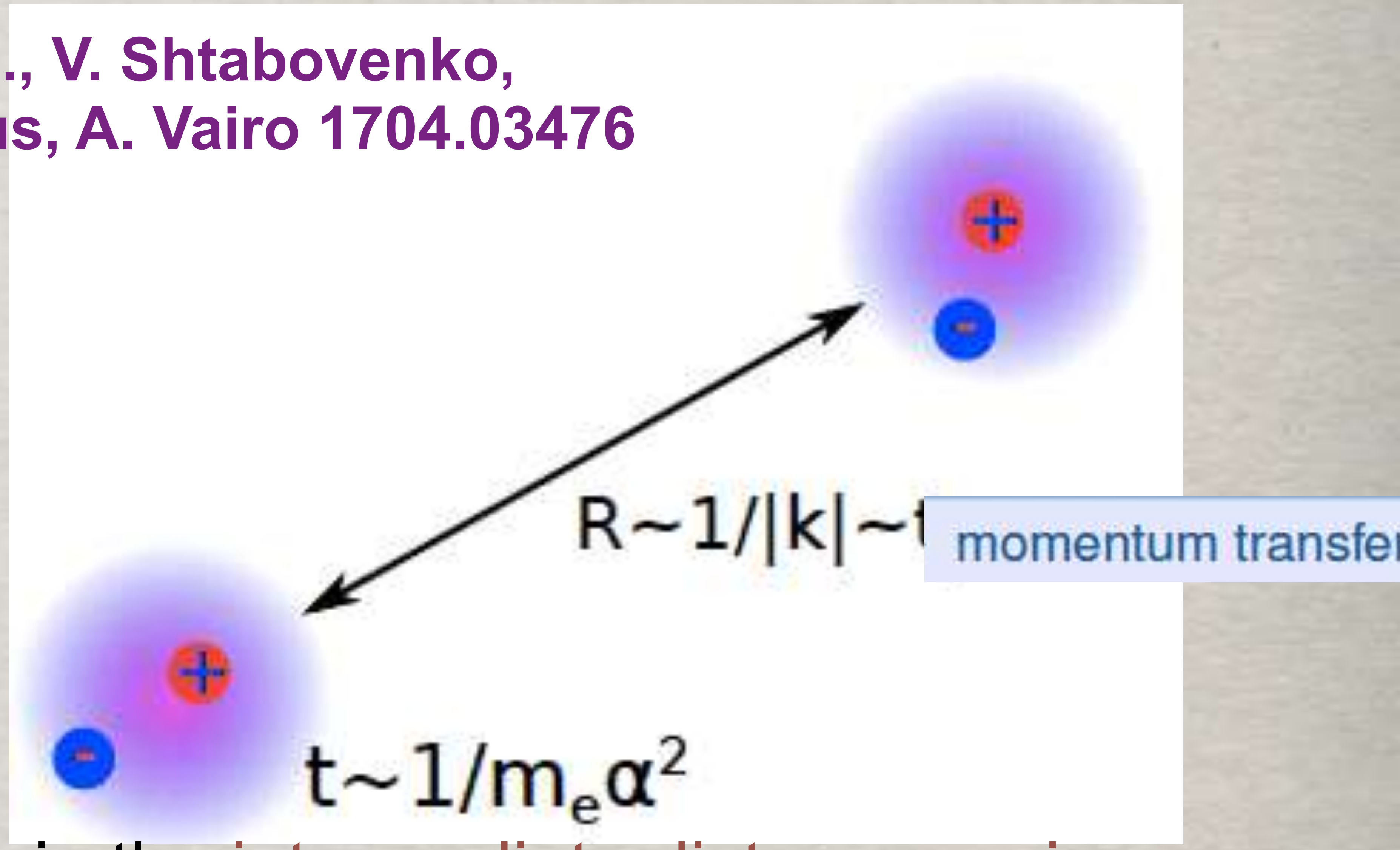

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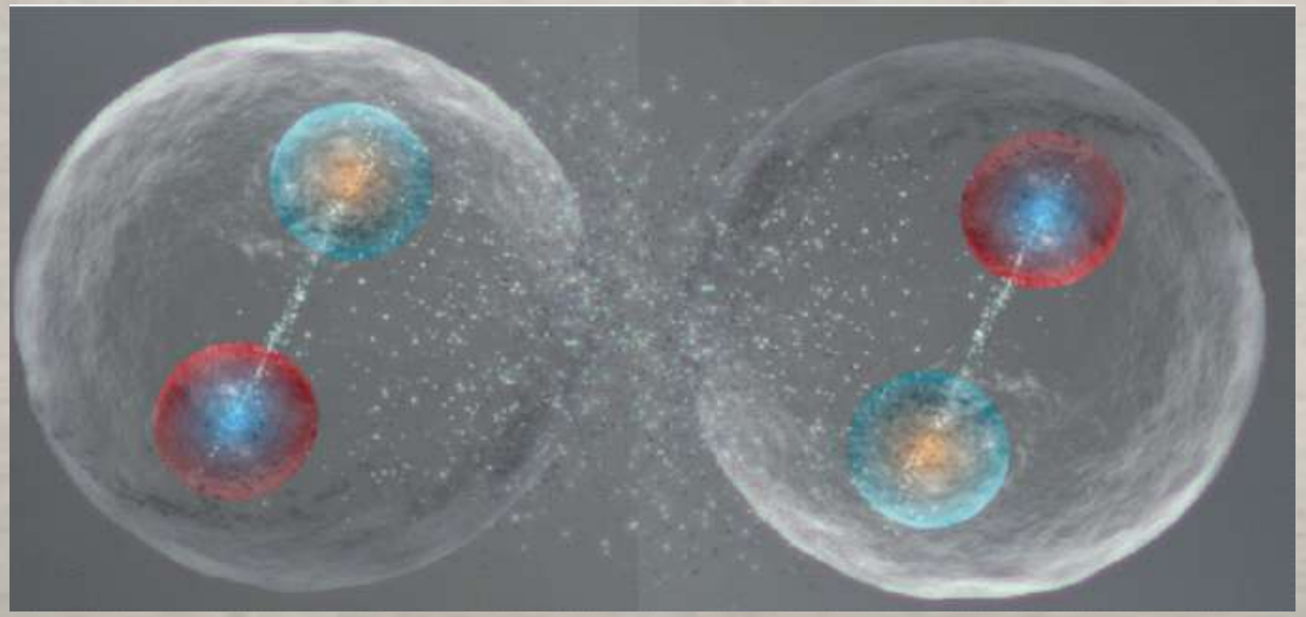
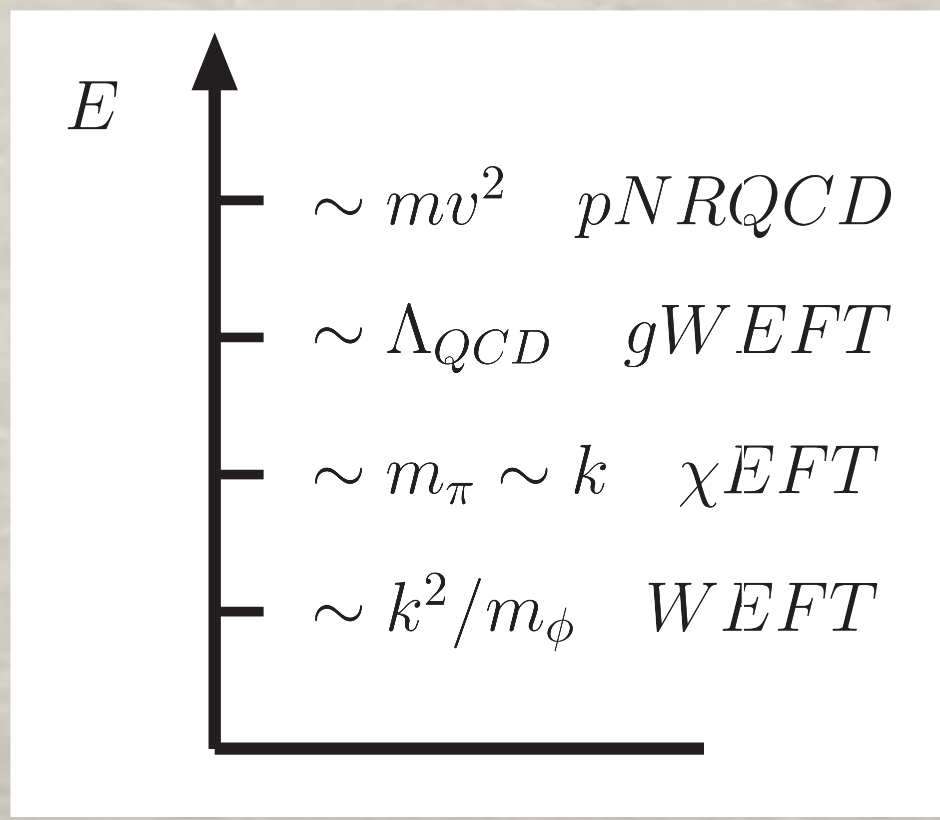
we obtained WEFT in QCD for $\eta_b - \eta_b$



Chromopolarizability & color van der Waals forces

N.B., G. Krein, J. Tarrus, A. Vairo 2015



more scales to integrate out in QCD

THE FRONTIER OF THE NR BOUND STATE:

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NR PAIRS AND LIGHT DEGREES FREEDOM:
X, Y, Z EXOTICS OBSERVED AT COLLIDERS

BO (BORN-OPPENHEIMER) EFT

N ,B., Berwein, Tarrus, Vairo 1510.04299, Oncala, Soto 1702.03900, N. B., Krein, Tarrus, Vairo, 1707.09647, Soto, Tarrus, 2005.00552, N.B., W.K. Lai, Segovia, Tarrus, Vairo 1805.07713, 1908.11699

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NR PAIRS IN NONEQUILIBRIUM EVOLUTION IN A MEDIUM (QGP, EARLY UNIVERSE) THAT TRIGGER DECAYS AND RECOMBINATIONS

PNREFT PLUS OPEN QUANTUM SYSTEM: LINBLAD EQUATION

N.B; Escobedo, Soto, Vairo. 1711.04515, 1612.07248; N.B. Escobedo, Vairo, Vander Griend 1903.08063; N.B. Escobedo , Strickland, Vairo, Vander Griend, Weber, 2012.01240; Yao, Mehen 2009.02408, 1811.07027; Sharma 2020…

nonequilibrium evolution of quarkonium in a strongly coupled QGP

nonequilibrium evolution of quarkonium in a strongly coupled QGP

work in the hierarchy and in real time formalism

$$M \gg \frac{1}{r} \sim M\alpha_{\rm s} \gg T \sim gT \gg \text{any other scale}, \qquad v \sim \alpha_{\rm s}$$

use a Coulombic quarkonium to test the strongly coupled plasma

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- Subsystem: heavy quarks/quarkonium
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N.B., J. Soto, M. Escobedo, A. Vairo 2016, 2018 (1612.07248, 1711.04515)

We may define a density matrix in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

$$\langle \mathbf{r}', \mathbf{R}'|\rho_s(t';t)|\mathbf{r}, \mathbf{R}\rangle \equiv \mathrm{Tr}\{\rho_{\text{full}}(t_0)\, S^\dagger(t, \mathbf{r}, \mathbf{R}) S(t', \mathbf{r}', \mathbf{R}')\}$$

$$\langle \mathbf{r}', \mathbf{R}'|\rho_o(t';t)|\mathbf{r}, \mathbf{R}\rangle \frac{\delta^{ab}}{8} \equiv \mathrm{Tr}\{\rho_{\text{full}}(t_0)\, O^{a\dagger}(t, \mathbf{r}, \mathbf{R}) O^b(t', \mathbf{r}', \mathbf{R}')\}$$

$t_0 \approx 0.6$ fm is the time formation of the plasma.

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The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

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the descriptions is fully quantum and nonabelian

nonequilibrium evolution of quarkonium: density matrix evolution

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

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evolution with t is evolution with T or any other parameter characterising the QGP out of equilibrium

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- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$-i\Sigma_{s,o}(t) + i\Sigma^\dagger_{s,o}(t) = 2\,\mathrm{Re}\,(-i\Sigma_{s,o}(t)) = 2\delta m_{s,o}(t)$$

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- Ξ_{so} accounts for the production of singlets through the decay of octets, and Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.

nonequilibrium evolution of quarkonium: Linblad equations

If $E \ll T \sim m_D$ the Lindblad equation for a strongly coupled plasma reads

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i \left(C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\}\right)$$

C collapse operators

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$

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the sQGP is characterised by two nonperturbative parameters (transport coefficients) kappa and gamma that must be calculated on the lattice

κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{18}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\, \langle \mathrm{T}\, E^{a,i}(s,\mathbf{0})\, \phi^{ab}(s,0)\, E^{b,i}(0,\mathbf{0})\rangle$$

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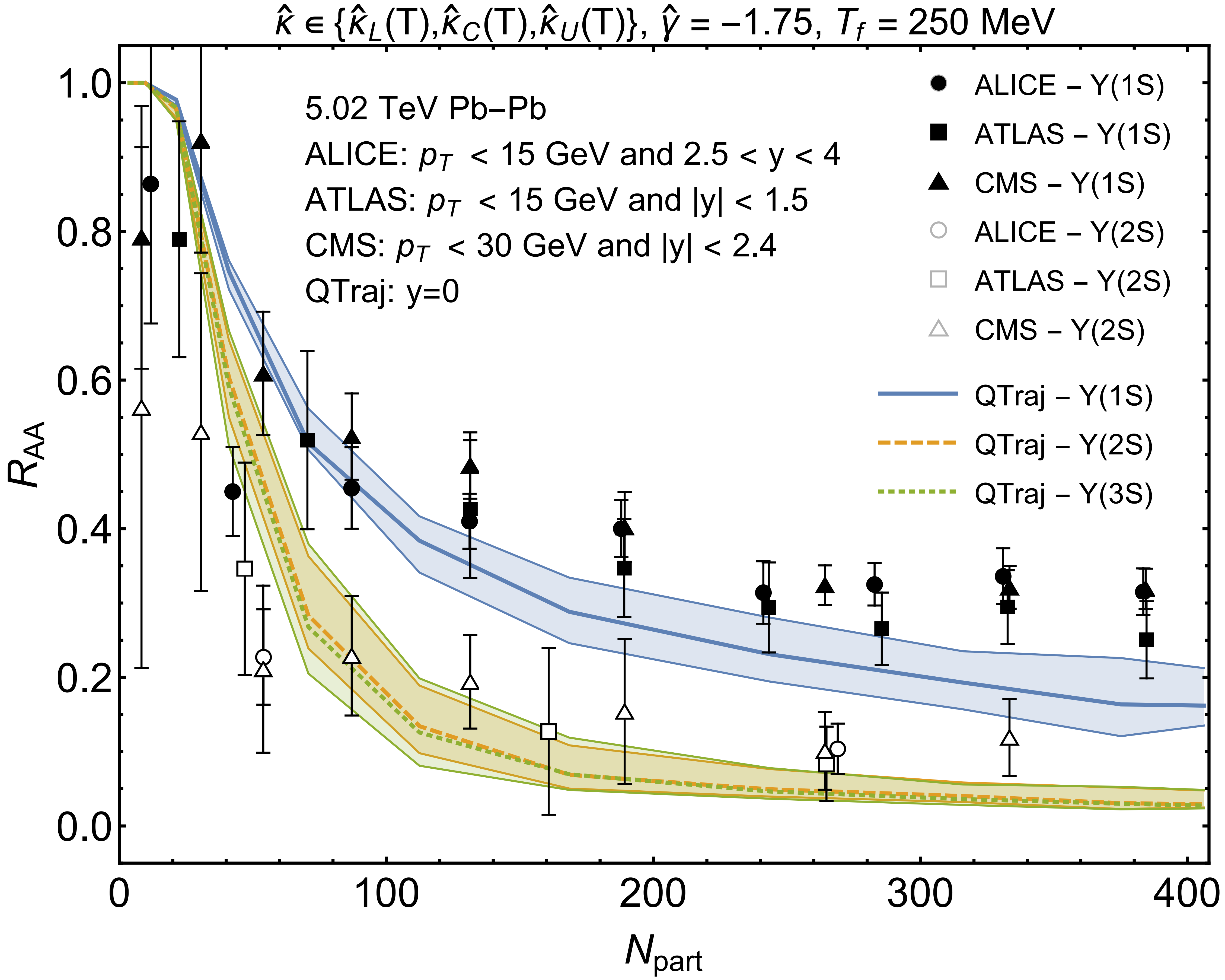
the EFT allows to use lattice QCD equilibrium calculation to study the non equilibrium evolution! EFT is intermediate layer to non equilibrium

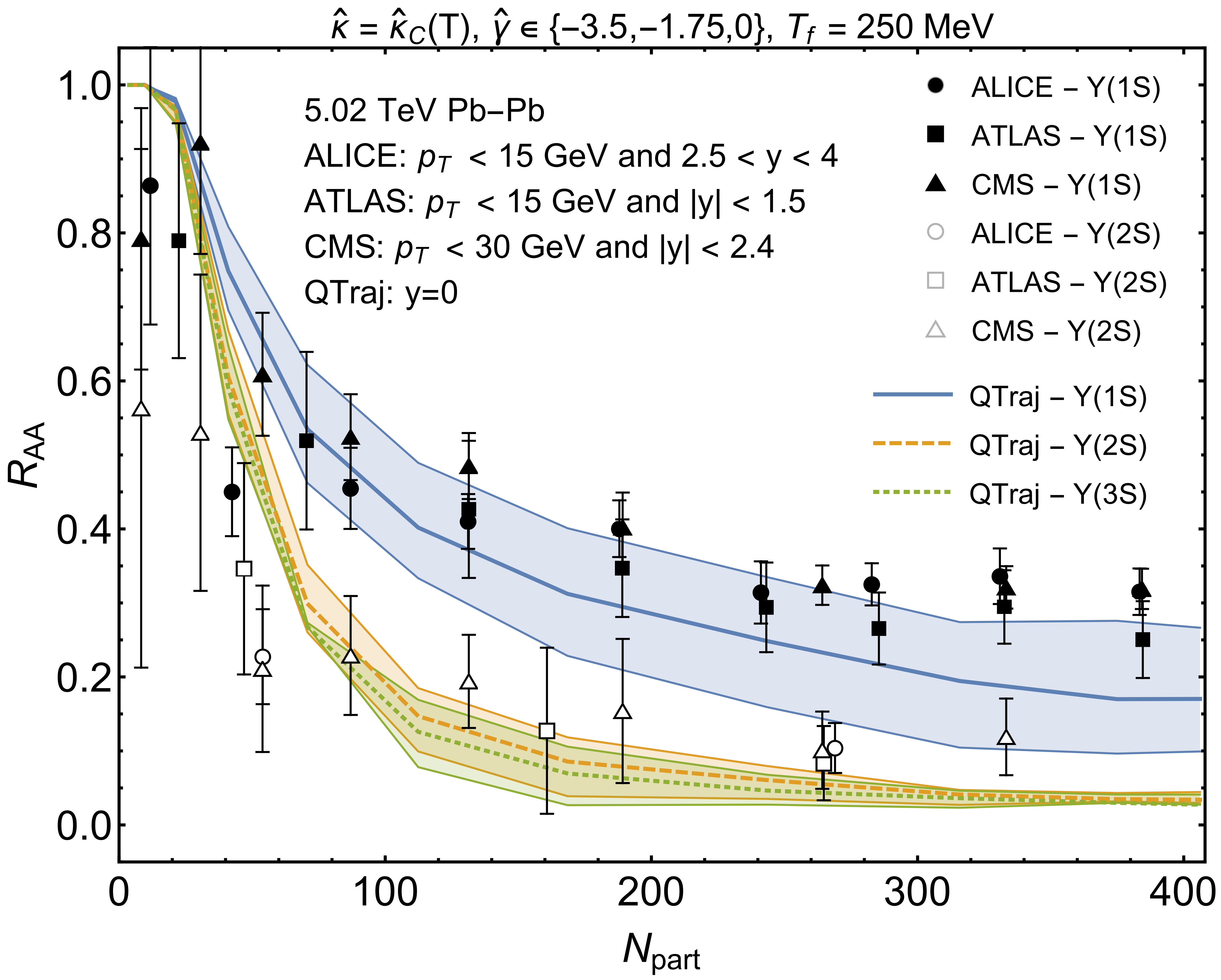
nonequilibrium evolution of quarkonium in medium: nuclear modification factor R_AA

We compute the nuclear modification factor R_{AA}:

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$

calculation with no free parameters, results depends on kappa function of T (calculated on the lattice) and gamma (extracted from the lattice)

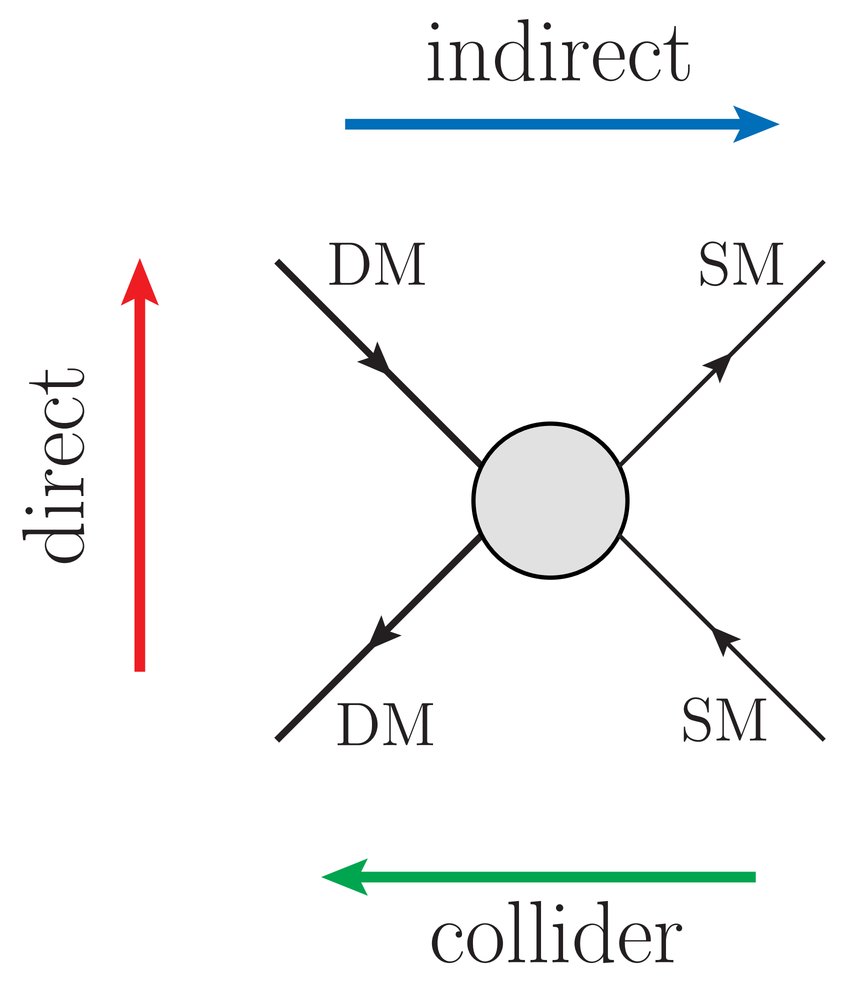




R_AA of singlet Bottomonium in comparison to ALICE, ATLAS and CMS data, left plot bands from variation in kappa, right plot variation in gamma —> we can use R_AA to learn about the QGP!

N.B. Escobedo , Strickland, Vairo, Vander Griend, Weber, 2012.01240

These results and approach could be applied to the study of the non equilibrium evolution of dark matter annihilation and formation in the early universe and after



- DM as a particle: many candidates
- Any model has to comply with

$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

Thermal freeze-out

- Boltzmann equation for DM (χ)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

- relevant processes $\chi\chi \leftrightarrow \mathrm{SM\,SM}$
- $\langle \sigma v \rangle$: input from particle physics with $v \sim \sqrt{T/M} < 1$

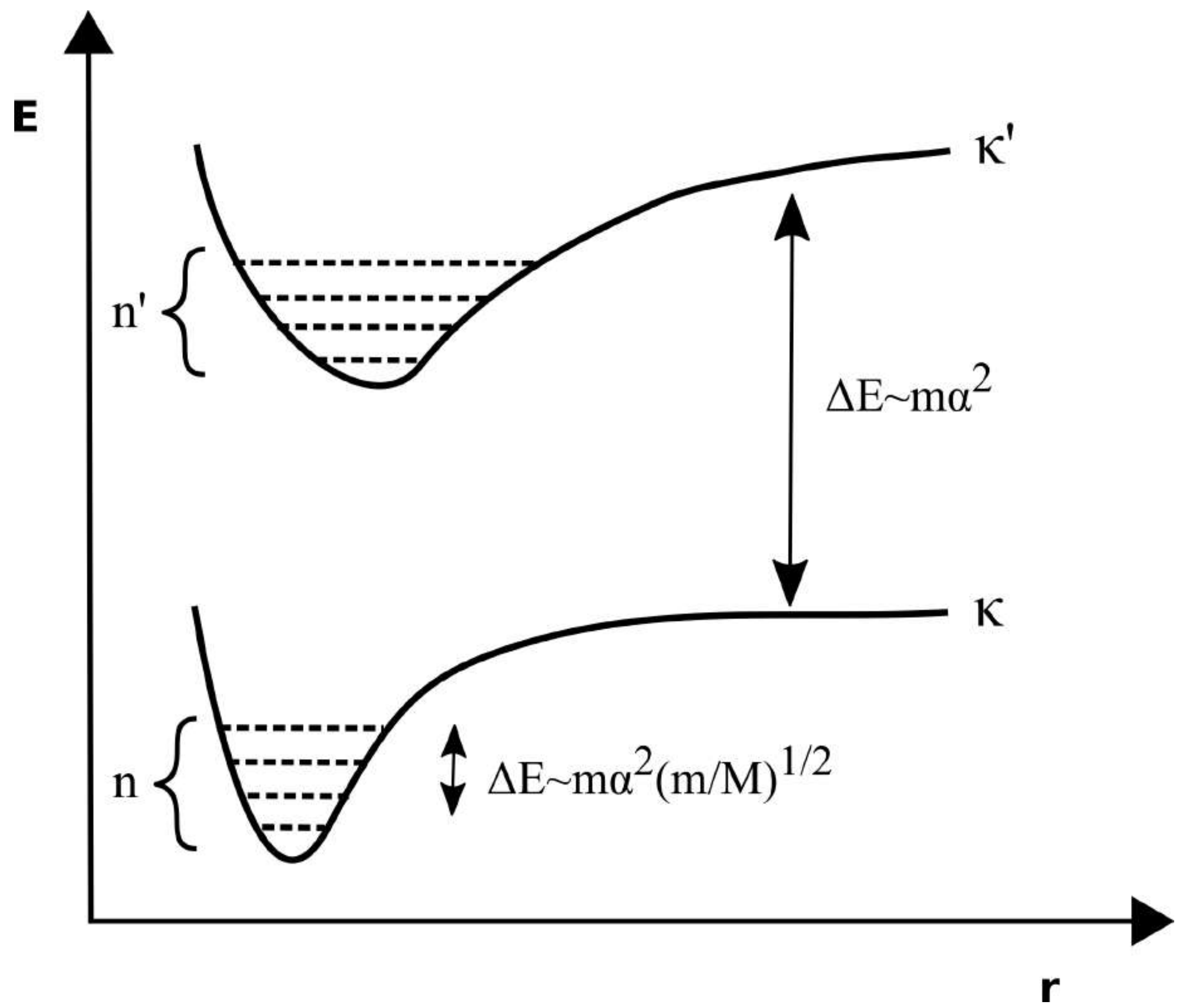
$$\langle \sigma v \rangle \approx \langle a + bv^2 + \dots \rangle \Rightarrow \boxed{\langle \sigma v \rangle^{(0)} \approx \frac{\alpha^2}{M^2}}$$

BOEFT: EFT for nonrelativistic pairs and light d.o.f.

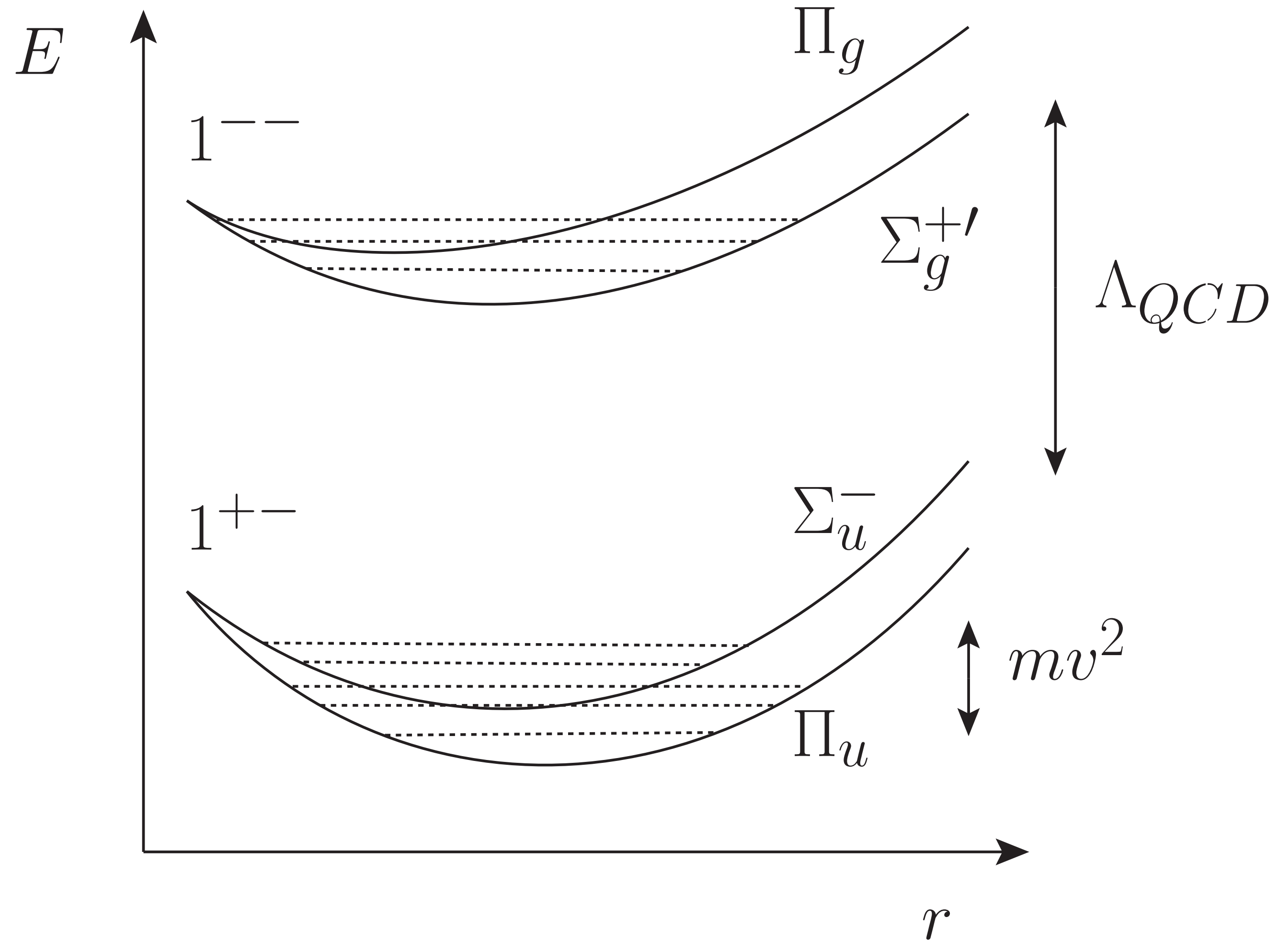
Consider bound states of two nonrelativistic particles and some light d.o.f., e.g., molecules/quarkonium hybrids ($Q\bar{Q}g$ states)or tetraquarks ($Q\bar{Q}q\bar{q}$ states):

- electron/gluon fields change adiabatically in the presence of heavy quarks/nuclei. The heavy quarks/nuclei interaction may be described at leading order in the non-relativistic expansion by an effective potential V_κ between static sources, where κ labels different excitations of the light d.o.f.
- a plethora of states can be built on each of the potentials V_κ by solving the corresponding Schrödinger equation.

This picture goes also under the name of Born-Oppenheimer approximation. Starting from pNRQED/pNRQCD the Born-Oppenheimer approximation can be made rigorous and cast into a suitable nonrelativistic EFT called Born–Oppenheimer EFT (BOEFT).



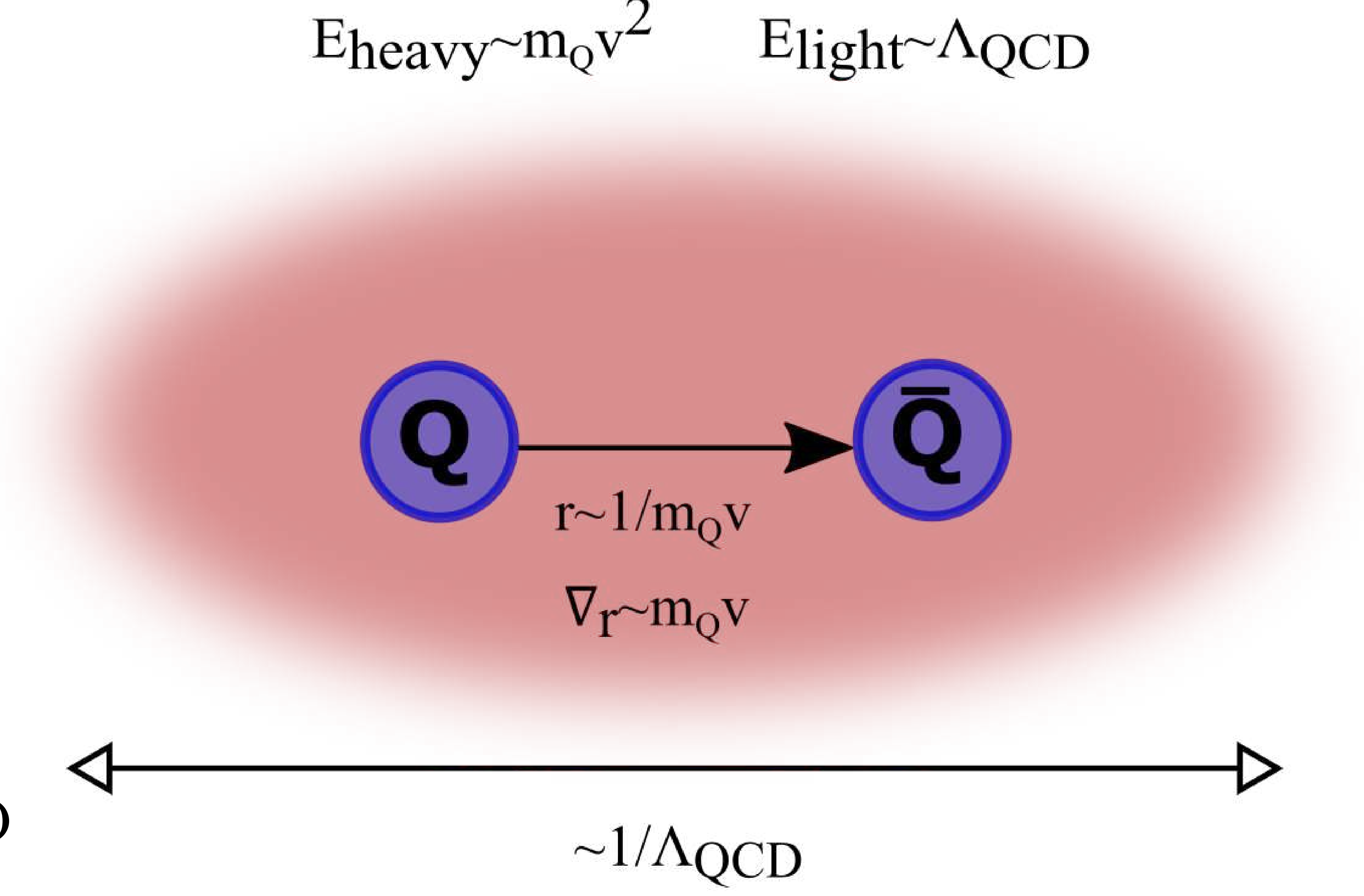
Michael et al. 1983,
Juge, Kuti, Mornigstar 1997, 1998,
Braaten,Langsmack, Smith 2014



hybrids

two different scales

$\Lambda_{QCD} \gg mv^2$



analogous to

$E_{electrons} \gg E_{nuclei}$

in QED

we proceed to integrate out 1/r and then Λ_{QCD}

(the two scales can also be integrated out simultaneously see Soto, Tarrus 2020)

Λ_{QCD} is nonperturbative but we can use the lattice static energies

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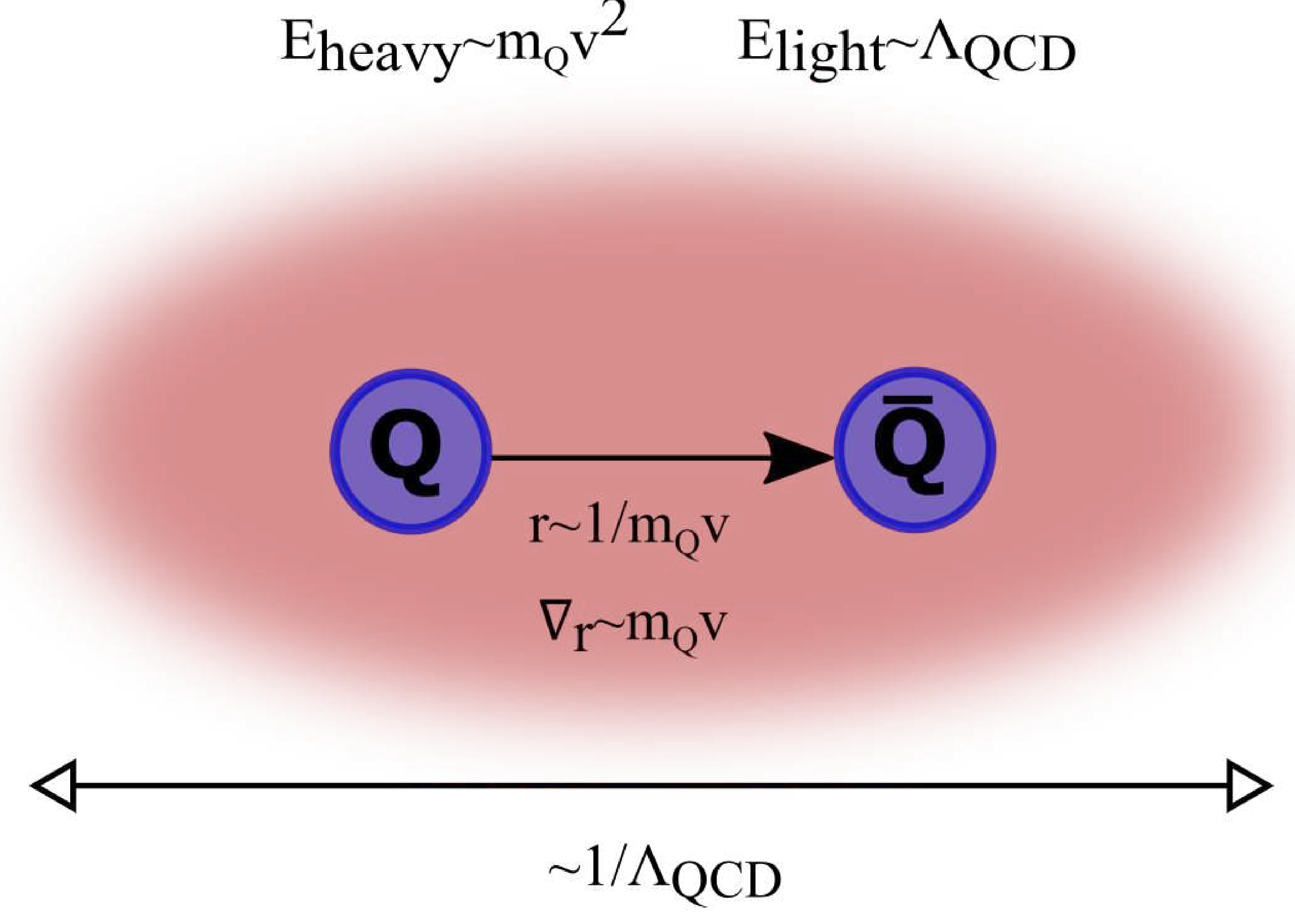

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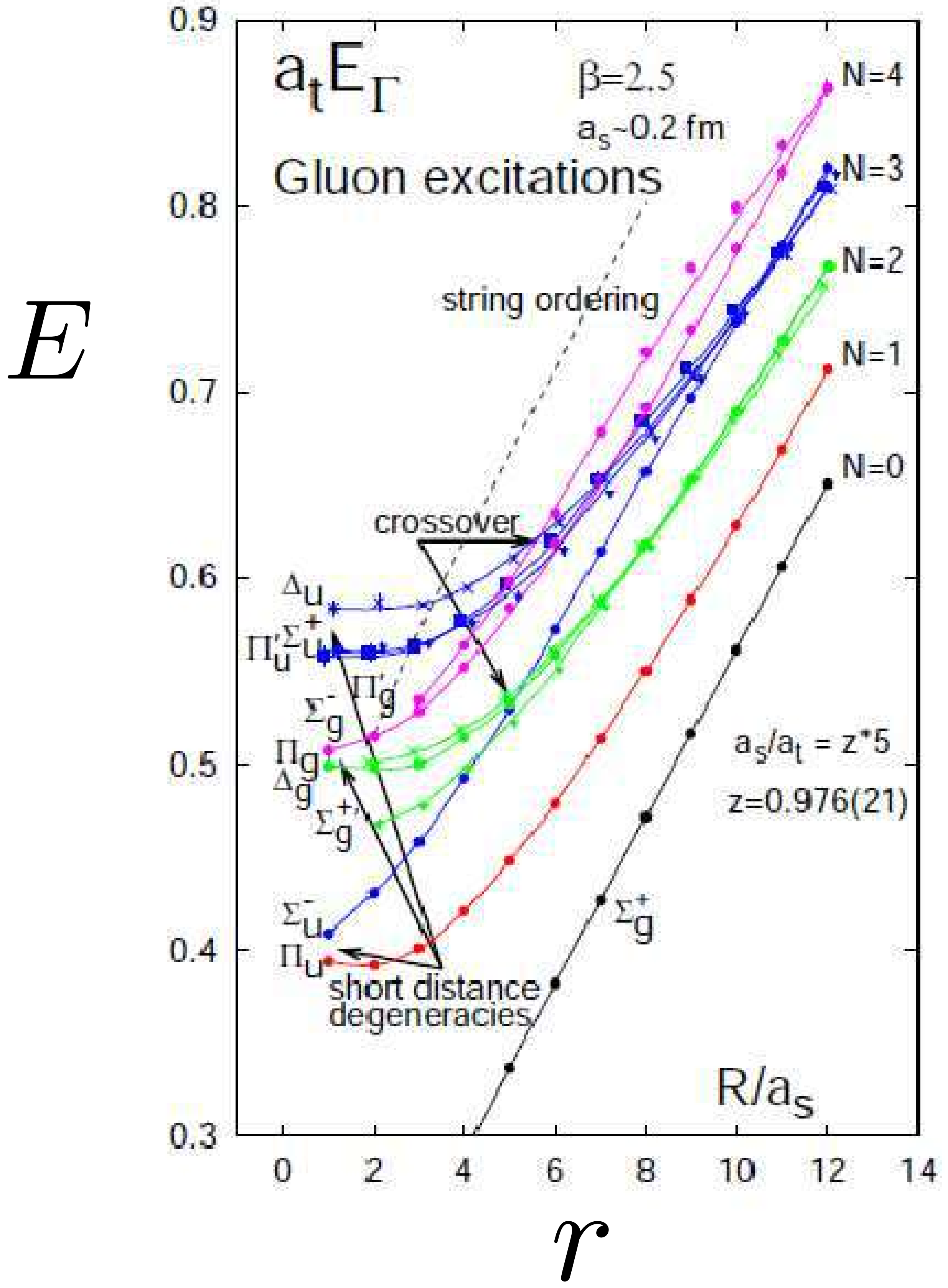

d out

20)

Λ_{QCD} is nonperturbative but we can use the lattice static energies

- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.

- Juge Kuti Morningstar PRL 90 (2003) 161601

  Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 03450

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_{\lambda}\frac{\boldsymbol{\nabla}_r^2}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \frac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \frac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'}V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$.

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- Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \dfrac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \dfrac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$.

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0 \Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}^2_r}{m} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda \left(\frac{\boldsymbol{\nabla}^2_r}{m}\right)\hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}^2_r}{m} + C^{\text{nad}}_{1+-\lambda\lambda'}$$

with $C^{\text{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda\left[\dfrac{\boldsymbol{\nabla}^2_r}{m}, \hat{r}^i_{\lambda'}\right]$ called the nonadiabatic coupling.

Spectrum: general consideration

- The Schrödinger equation mixes states with the same parity.
  A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, however, there Λ-doubling is a subleading effect, while it is a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.

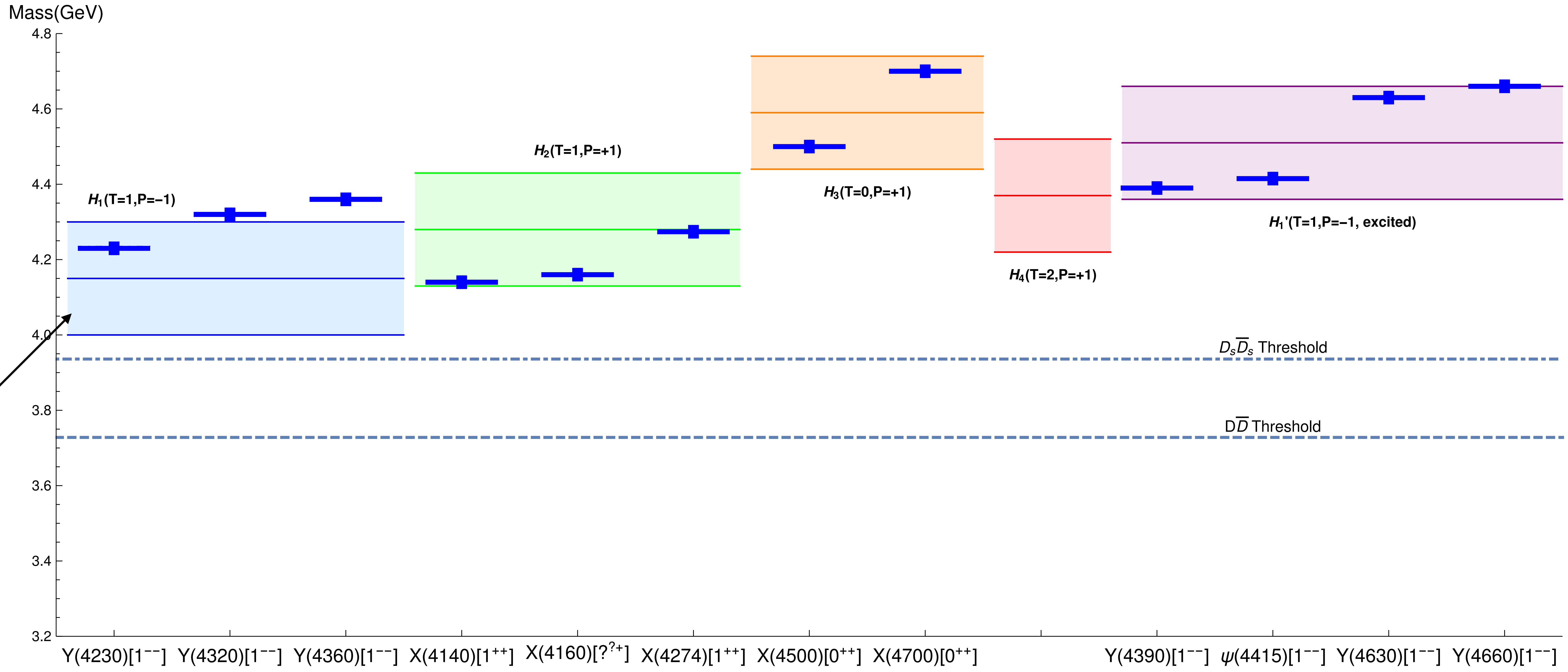
Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}$, E_{Π_u}
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
H_3	0	0^{++}	1^{+-}	$E_{\Sigma_u^-}$
H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}$, E_{Π_u}

T is the sum of the orbital angular momentum of the quark-antiquark pair and the gluonic angular momentum; $T=0$ state turns out not to be the lowest mass state.

we can calculate the structure of the hybrids multiplets

- Braaten PRL 111 (2013) 162003
  Braaten Langmack Smith PRD 90 (2014) 014044

Quarkonium hybrid states vs experiments I



the bands come from the uncertainty in the lattice determination of the gluelump masses

neutral exotics charmonium states

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019 updated in Brambilla Eidelman Hanhart Nefediev Shen Thomas Vairo Yuan arXiv:1907.11747

Hybrid spin-dependent potentials at order $1/m$ and $1/m^2$

$$V^{(1)}_{1+-\,\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S}$$
$$+ V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$
$$V^{(2)}_{1+-\,\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'}$$
$$+ V^{(2)}_{LS\,c}(r)\left[\hat{r}_{\lambda}\cdot\boldsymbol{r}\,(\boldsymbol{p}\times\boldsymbol{S})\cdot\hat{r}_{\lambda'} + \hat{r}_{\lambda}\cdot(\boldsymbol{p}\times\boldsymbol{S})\,\hat{r}_{\lambda'}\cdot\boldsymbol{r}\right]$$
$$+ V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons
and $\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

Hybrid spin-dependent potentials at order $1/m$ and $1/m^2$

- The non perturbative part of the potentials depends on six nonperturbative correlators that could be calculated on the lattice directly

-The only flavour dependence is carried by the NRQCD matching coefficients

$$V^{(1)}_{1+-\,\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S}$$

$$+\,V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S}+\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

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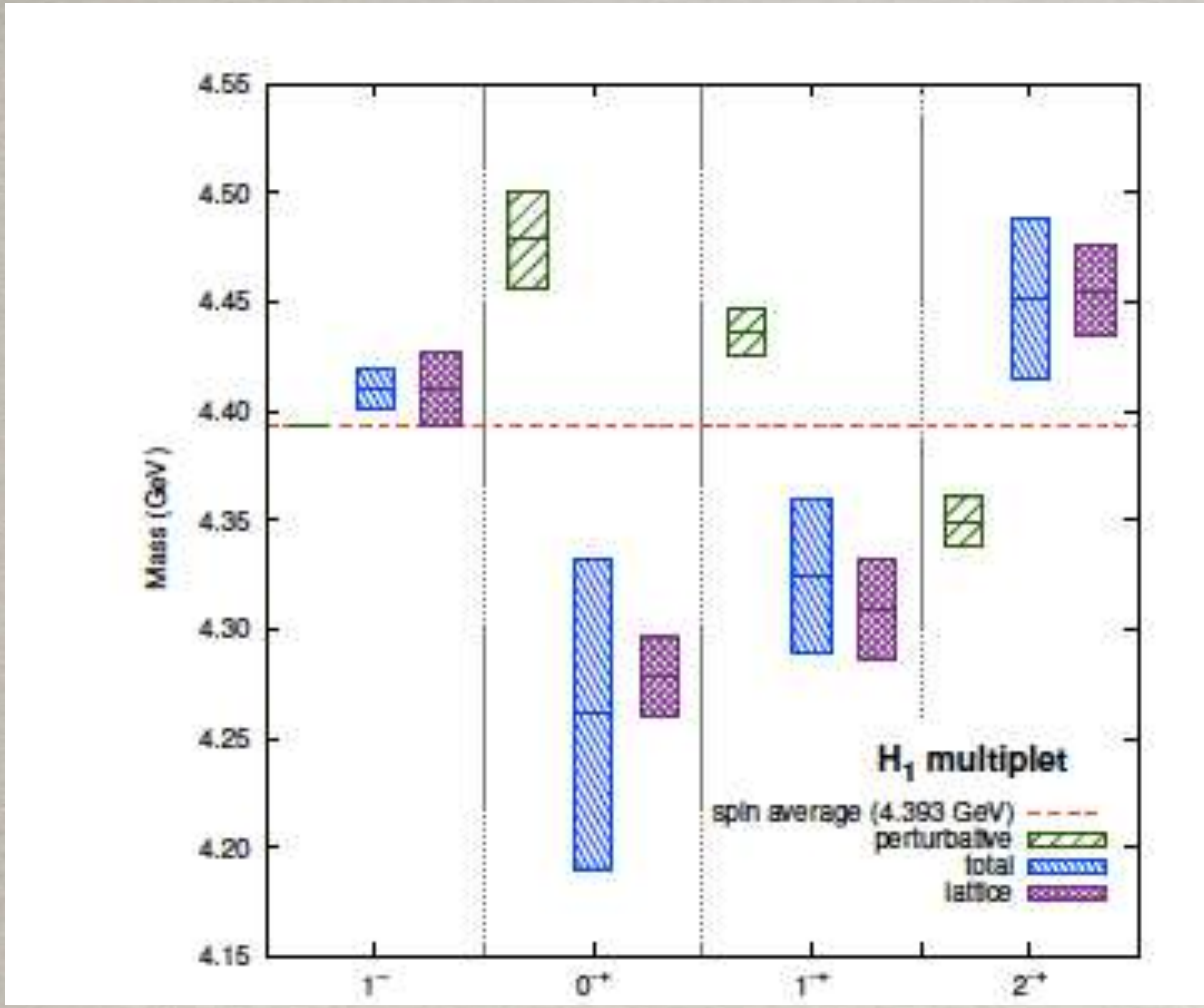
We can use lattice data on charmonium hybrids spin multiplets to fix the nonperturbative and predict bottomonium hybrids spin multiplets



Charmonium Hybrids Multiplets H_1

lattice data from

G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].

with a pion of about 240 MeV

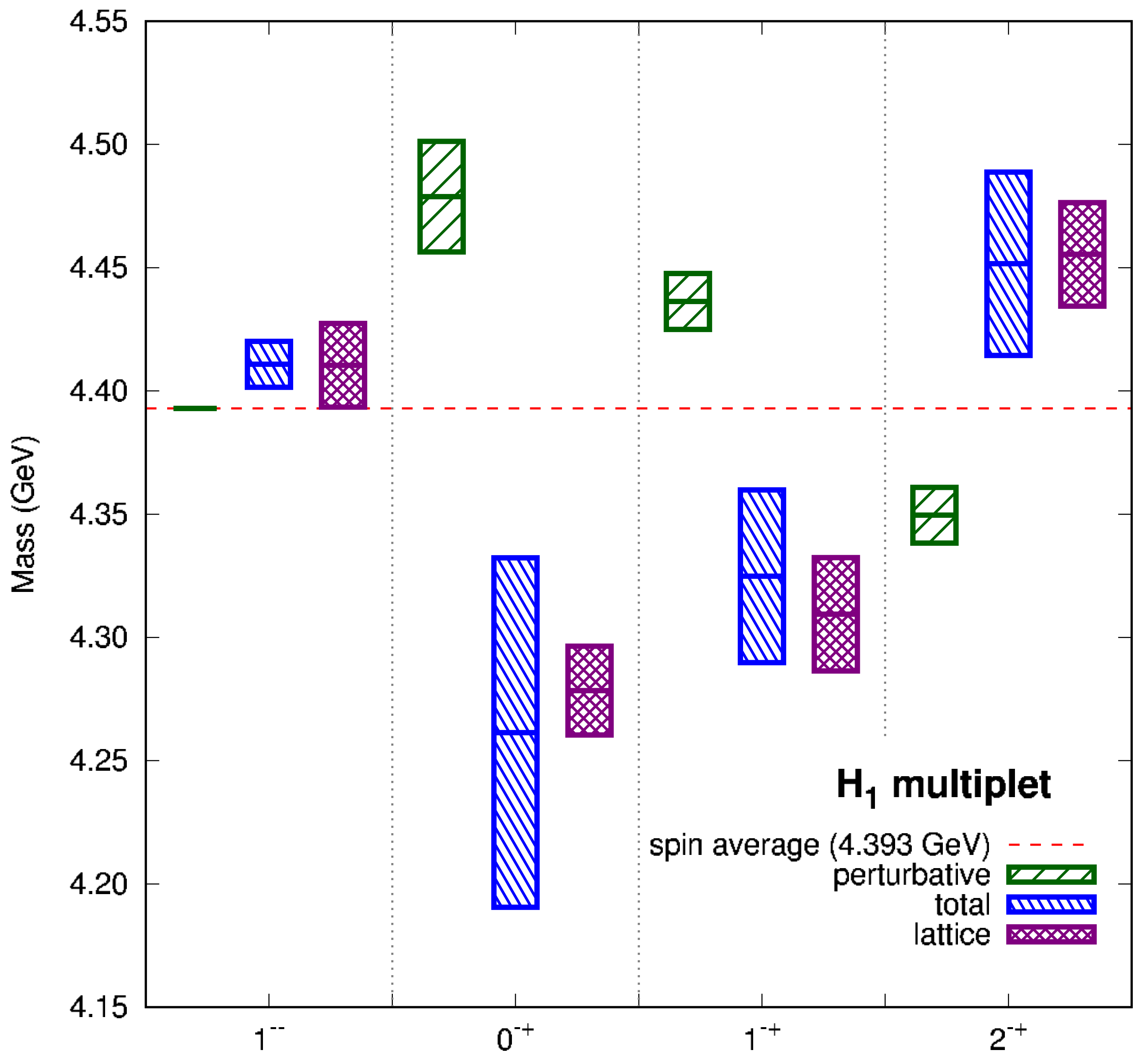
We fit the nonperturbative correlators on the lattice data: violet boxes are the nonperturbative contributions

the perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m (*note that all models are inspired to the perturbative part..)*

Power counting: we include terms up to order Lambda^3/m^2 and m v^4 to the spin splittings

height of the boxes is an estimate of the uncertainty:
estimated by the parametric size of higher order corrections, m alpha_s^5
for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

Charmonium H_1 hybrid spin splittings



fix the nonperturbative unknowns from a charmonium hybrid calculation

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from Liu et al JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]

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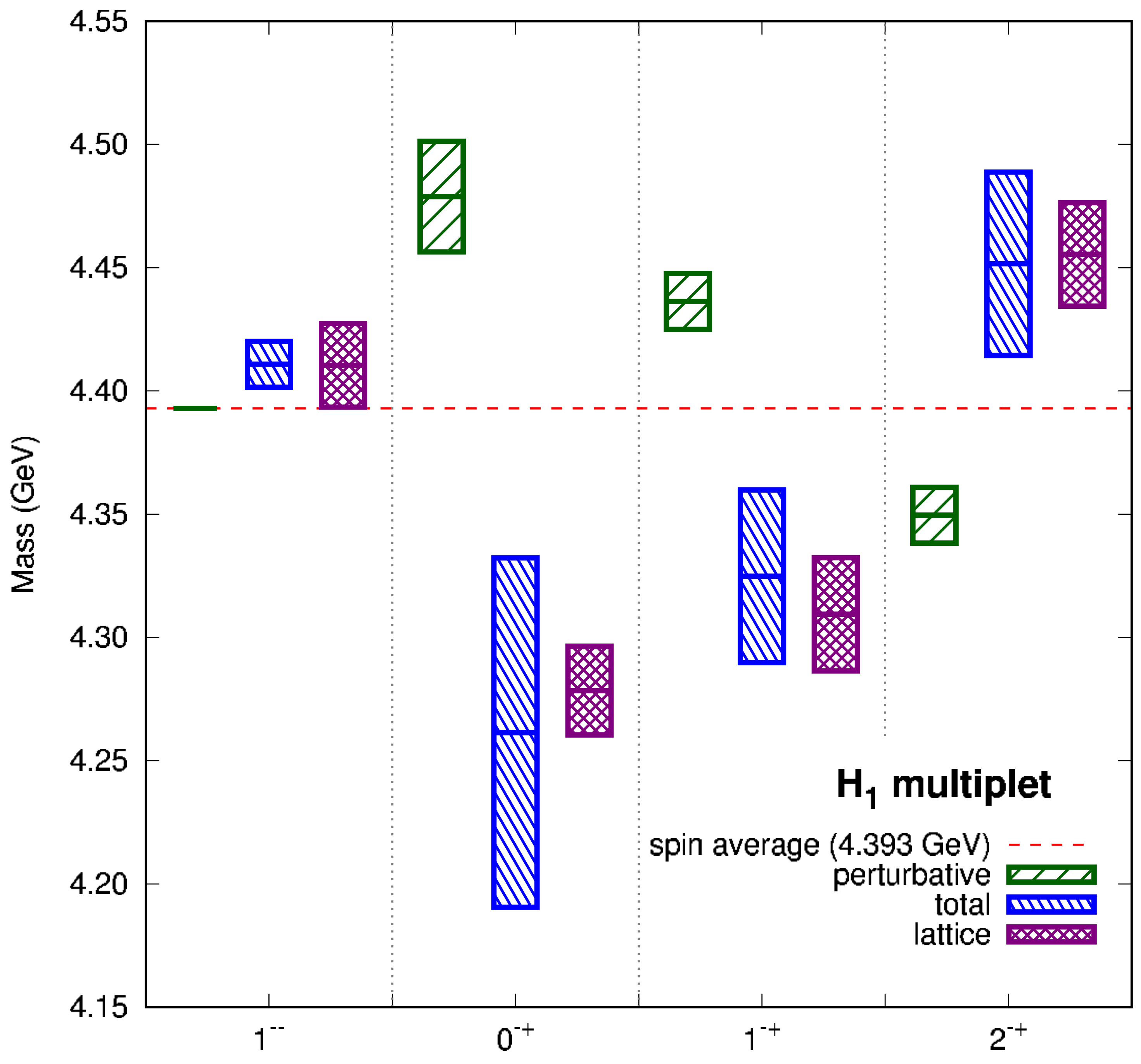

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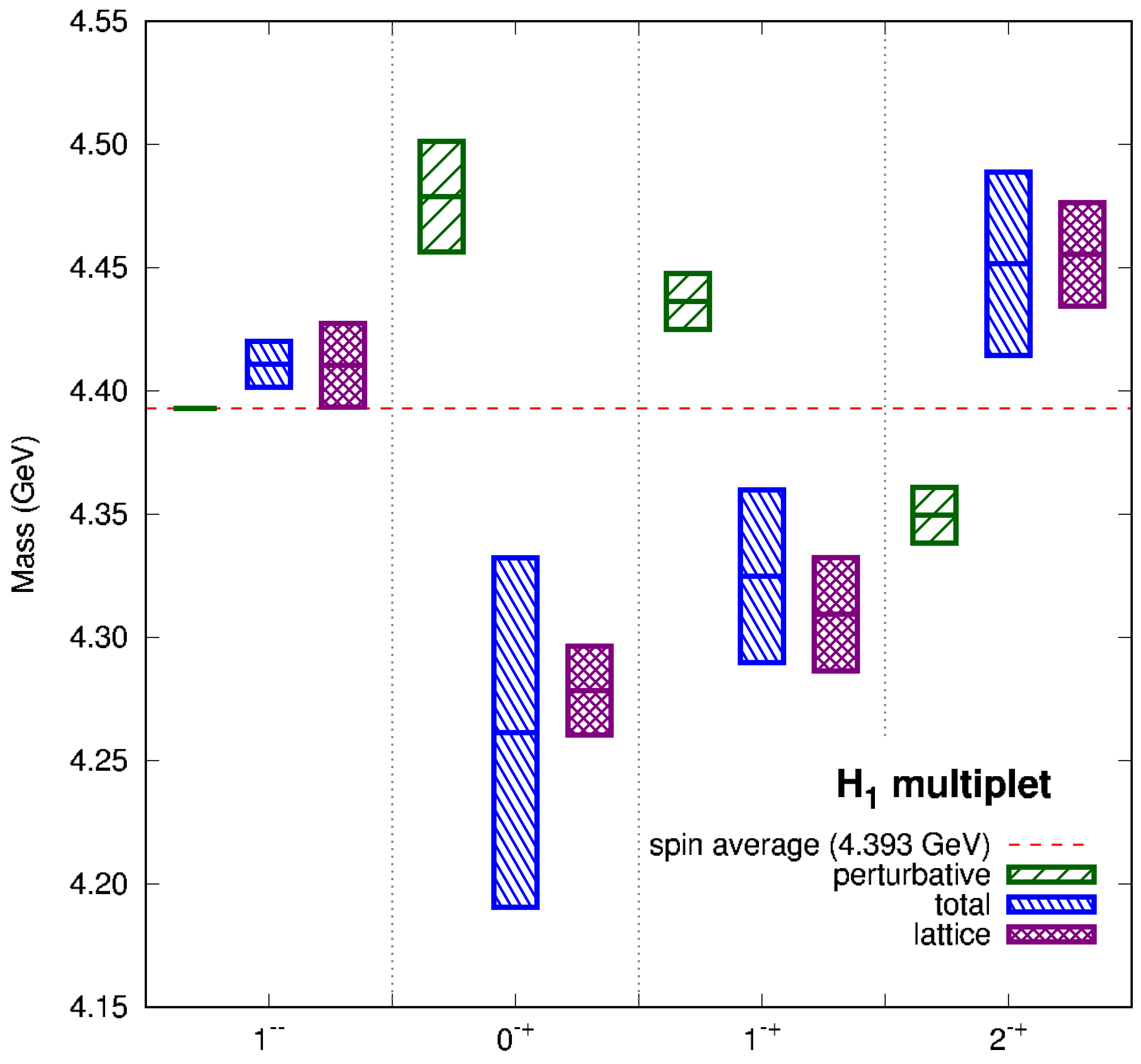

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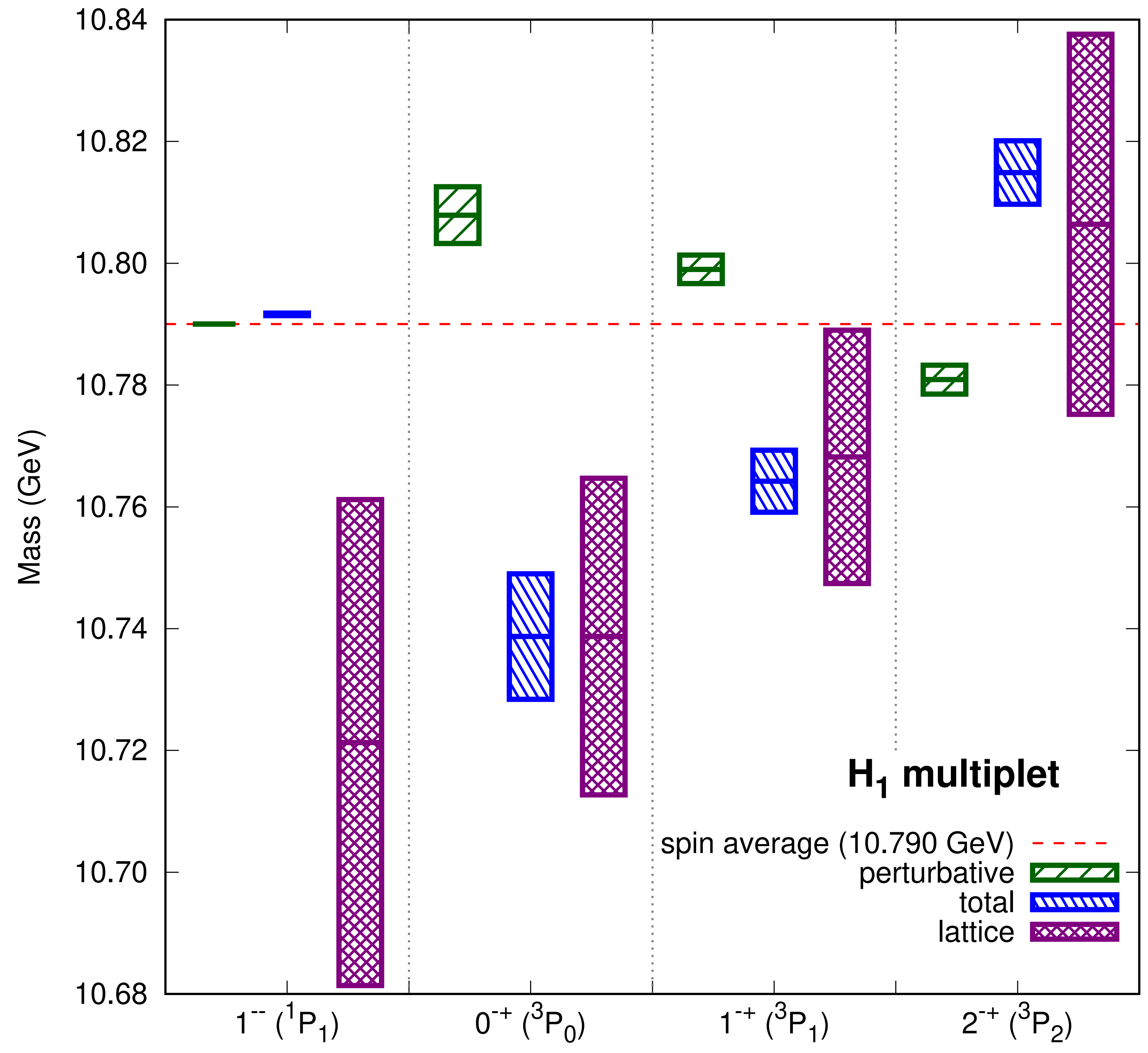

fix the nonperturbative unknowns from
a charmonium hybrid calculation

blue EFT predictions,
violet actual lattice calculation

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from Liu et al JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]

- Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV
  unpublished plot by J. Segovia

Outlook

pNREFT gives us a powerful tool to address NR bound and threshold states in QFT

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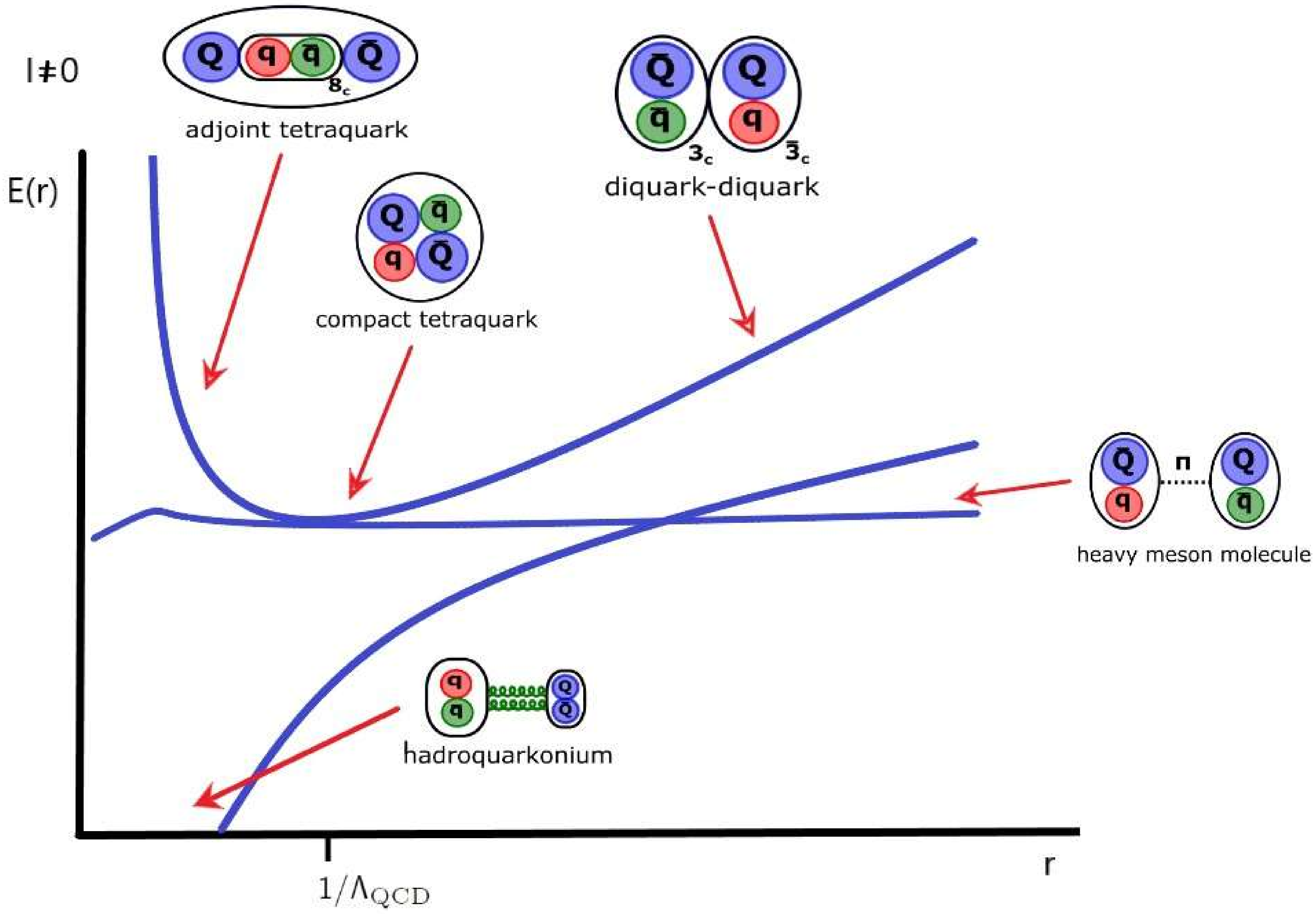
pNREFT is a flexible and versatile tool that could be applied to the realm of atomic, condensed matter, BSM…. any physics wherever NR states play a role

one can imagine a situation of this type where the BOEFT comprehends all the different phenomenological models in different dynamical regions

Static energies for tetraquarks (schematic):



Courtesy J. Tarrús Castellà

Lattice calculations of these objects are needed: we started such calculations in our TUMQCD collaboration