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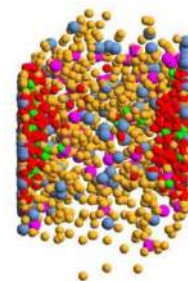
GOETHE  
UNIVERSITÄT  
FRANKFURT AM MAIN

# Exploring the QGP at finite baryon chemical potential in and out of equilibrium

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*Theoretical Physics Colloquium (on-line),  
hosted by Prof. Igor Shovkovy at the Arizona State University.*

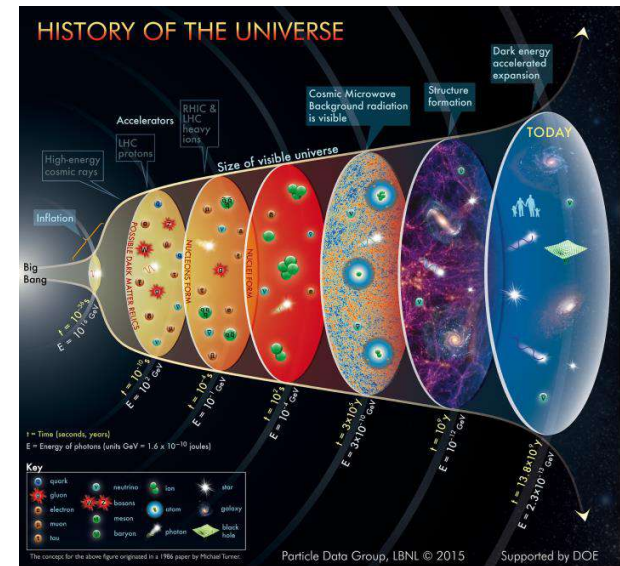
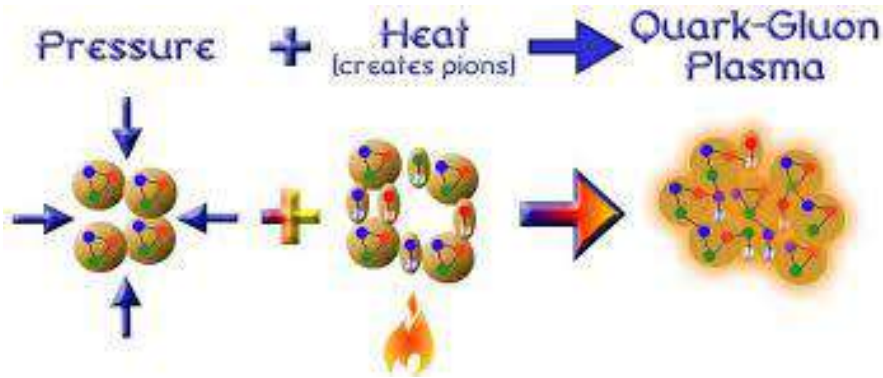
*20 January, 2021*



# Experiment: Heavy-ion collisions

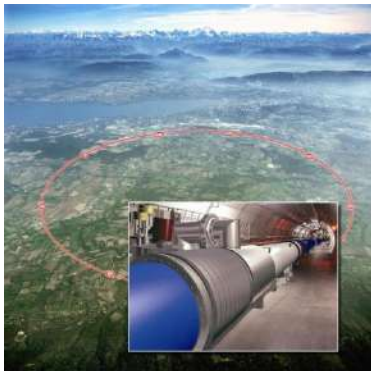
## Heavy-ion collision experiments

→ ,re-creation‘ of the Big Bang conditions in laboratory:  
matter at high **pressure** and **temperature**

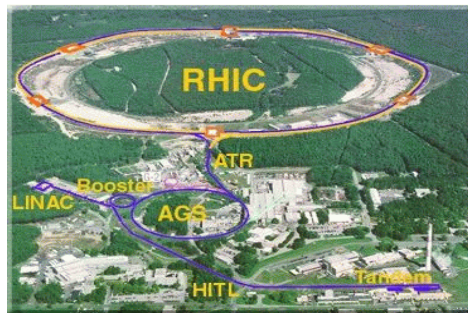


## Heavy-ion accelerators:

Large Hadron Collider -  
**LHC** (CERN):  
Pb+Pb up to 574 A TeV



Relativistic-Heavy-Ion-Collider -  
**RHIC** (Brookhaven):  
Au+Au up to 21.3 A TeV



Facility for Antiproton and Ion  
Research – **FAIR** (Darmstadt)  
(Under construction)  
Au+Au up to 10 (30) A GeV

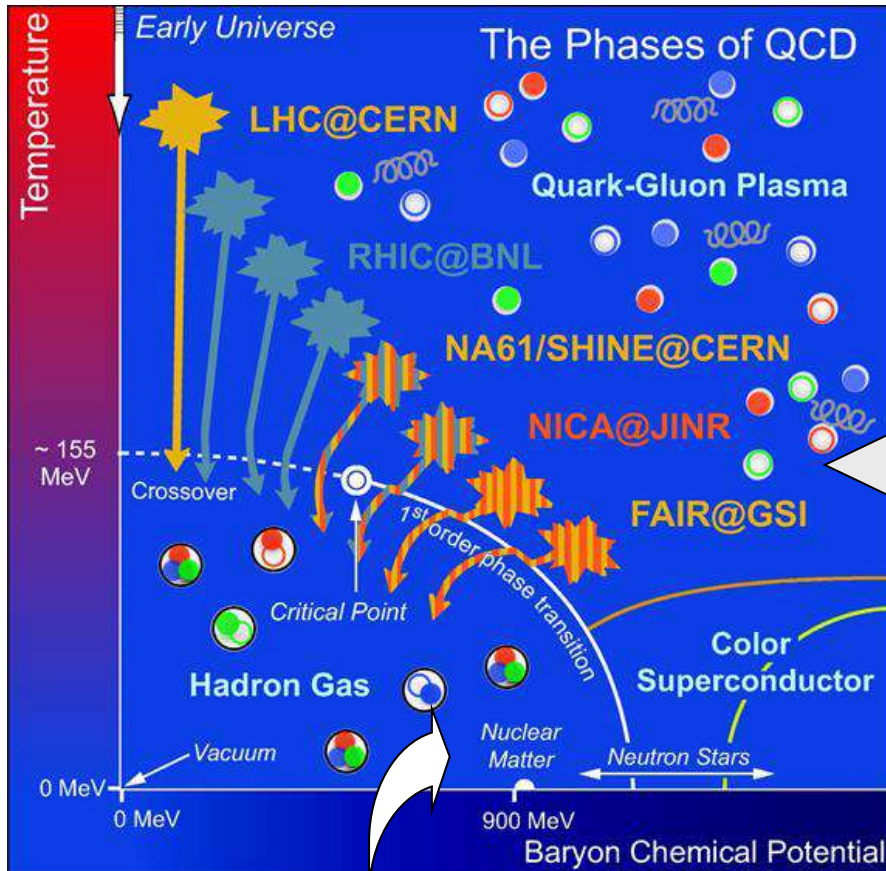


Nuclotron-based Ion Collider  
fAcility – **NICA** (Dubna)  
(Under construction)  
Au+Au up to 60 A GeV

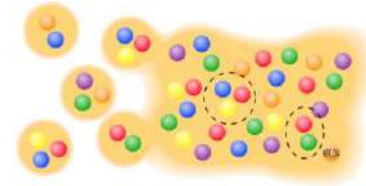


# The ,holy grail‘ of heavy-ion physics:

The phase diagram of QCD → thermal properties of QCD in the  $(T, \mu_B)$  plain



- **Equation-of-State** of hot and dense matter?
- Study of the **phase transition** from hadronic to partonic matter – **Quark-Gluon-Plasma**



- Search for a **critical point**
- Search for signatures of **chiral symmetry restoration**
- Study of the **in-medium properties of hadrons** at high baryon density and temperature



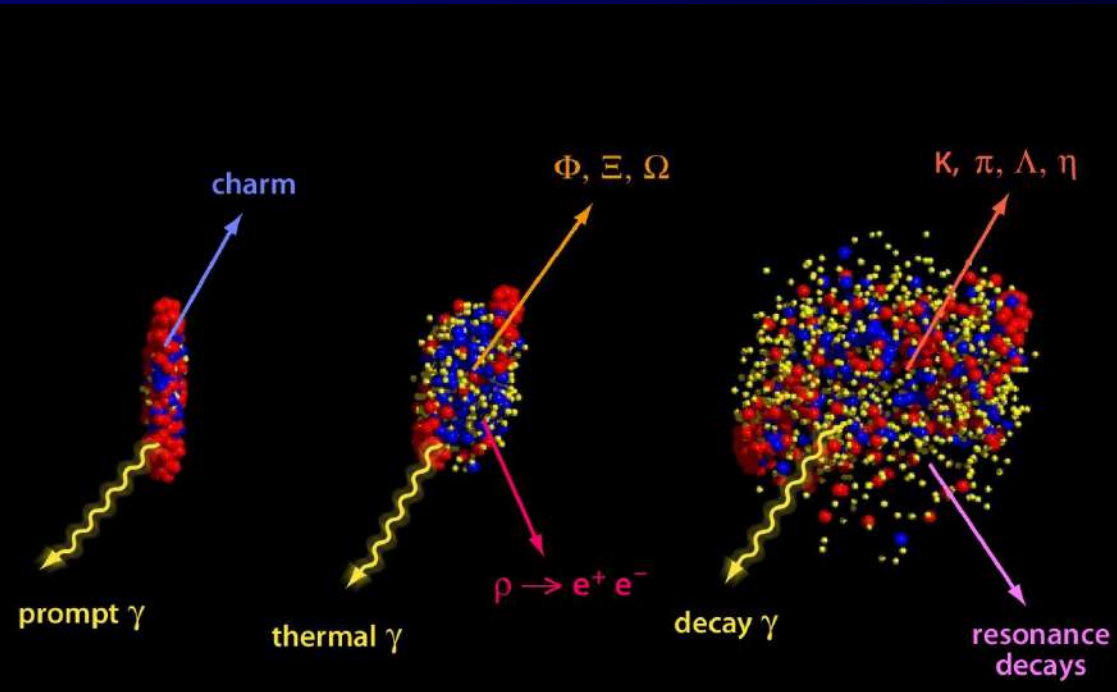
## Signals of the phase transition:

- Multi-strange particle enhancement in A+A
- Charm suppression
- Collective flow ( $v_1, v_2$ )
- Thermal dileptons
- Jet quenching and angular correlations
- High  $p_T$  suppression of hadrons
- Nonstatistical event-by-event fluctuations and correlations
- ...

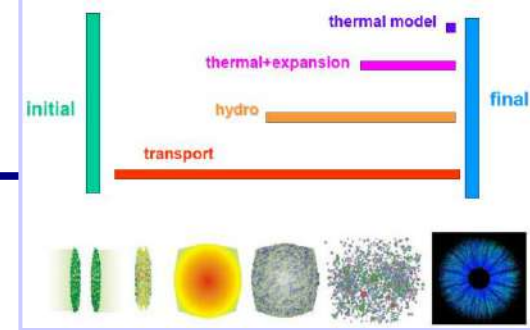
**Experiment:** measures final hadrons and leptons

How to learn about physics from data?

Compare with theory!



# Basic models for heavy-ion collisions



- Statistical models:

**basic assumption:** system is described by a (grand) canonical ensemble of non-interacting fermions and bosons in **thermal and chemical equilibrium**  
= **thermal hadron gas at freeze-out** with common  $T$  and  $\mu_B$

[ - : no dynamical information]

- Hydrodynamical models:

**basic assumption:** conservation laws + equation of state (EoS);  
assumption of **local thermal and chemical equilibrium**

- Interactions are ,hidden' in properties of the **fluid** described by **transport coefficients** (shear and bulk viscosity  $\eta$ ,  $\zeta$ , ..), which is **'input'** for the hydro models

[ - : simplified dynamics]

- Microscopic transport models:

**based on transport theory of relativistic quantum many-body systems**

- **Explicitly account for the interactions of all degrees of freedom** (hadrons and partons)  
in terms of cross sections and potentials

- Provide a unique dynamical description of **strongly interaction matter**

**in- and out-of equilibrium:**

- **In-equilibrium:** transport coefficients are calculated in a box – controlled by IQCD

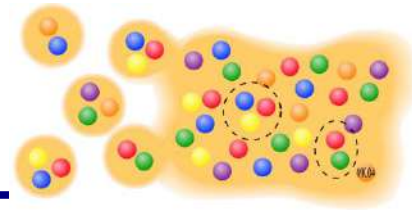
- **Nonequilibrium dynamics** – controlled by HIC

Actual solutions: Monte Carlo simulations

[+ : full dynamics | - : very complicated]

# Microscopic modeling of HICs

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**Goal:** microscopic transport description of the **partonic** and **hadronic phases** of heavy-ion collisions

**Problems:**

- ❑ What are the **properties** of the **QGP** degrees of freedom?
- ❑ How to solve the **hadronization** problem?
- ❑ What is an appropriate **transport theory** ?

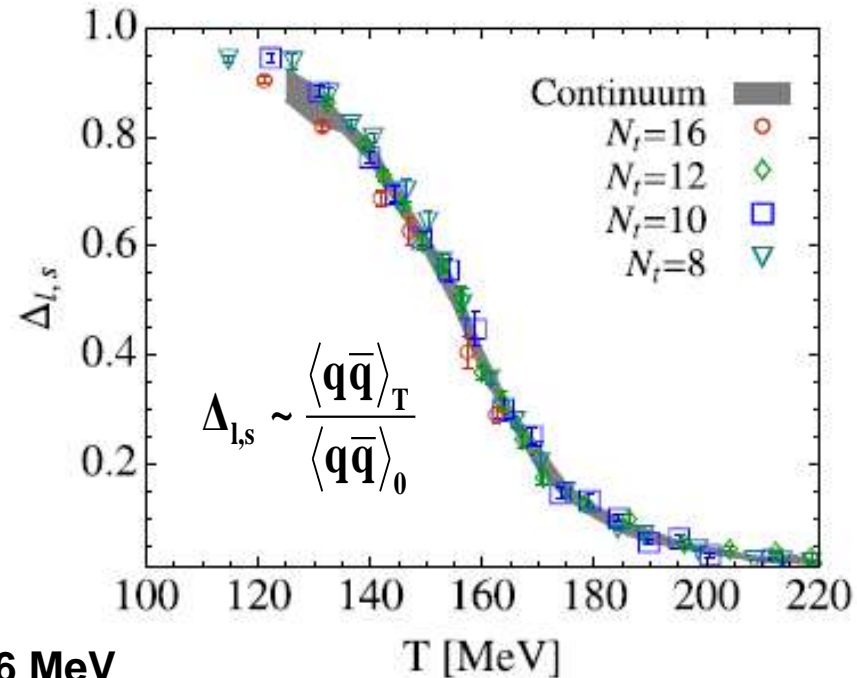
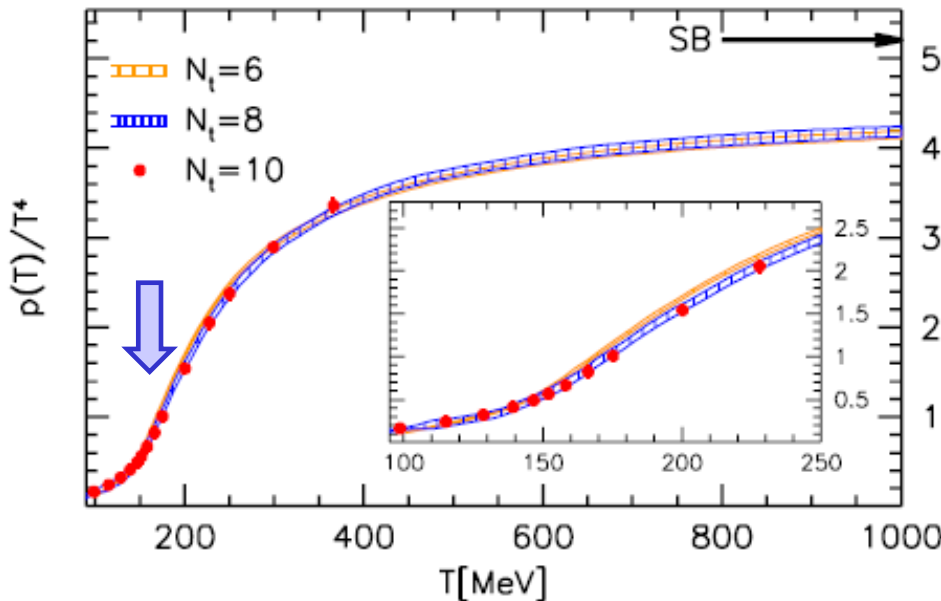
# Information from lattice QCD at $\mu_B=0$

I. deconfinement phase transition with increasing temperature

+

II. chiral symmetry restoration with increasing temperature

IQCD BMW collaboration:  $\mu_B=0$



□ **EoS: Crossover:** hadron gas  $\rightarrow$  QGP,  $T_c = 156$  MeV

□ **Scalar quark condensate  $\langle \bar{q}q \rangle$**  is viewed as an **order parameter** for the restoration of chiral symmetry:

$$\langle \bar{q}q \rangle = \begin{cases} \neq 0 & \text{chiral non-symmetric phase;} \\ = 0 & \text{chiral symmetric phase.} \end{cases}$$

$\rightarrow$  both transitions occur at about the same temperature  $T_c$  for low chemical potentials

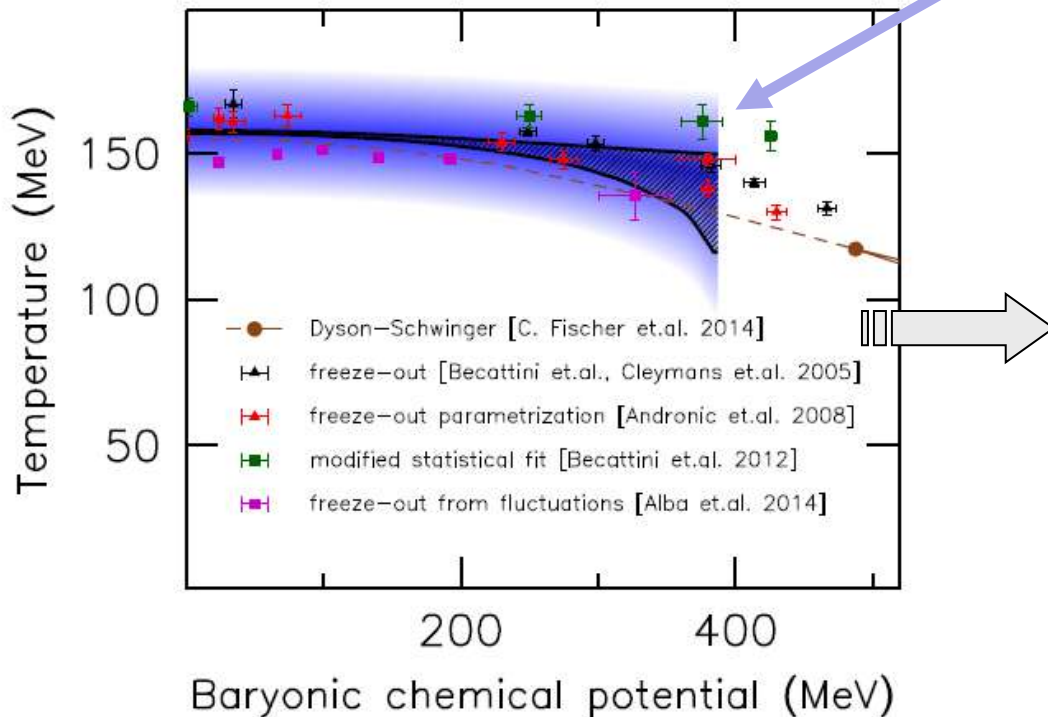
# Thermodynamics of QCD at finite T and $\mu_B$

Theory:  $\rightarrow$  thermal properties of QCD in the  $(T, \mu_B)$  plain

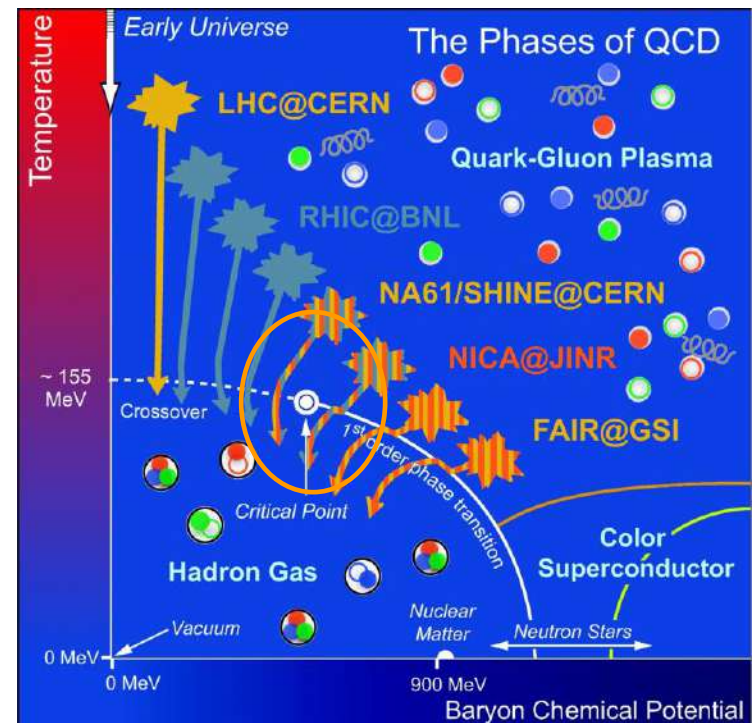
$\rightarrow$  **lattice QCD** – limited to low  $\mu_B < 400$  MeV - ‘sign problem’ of IQCD at finite  $\mu_B$

**Taylor expansion** allows for IQCD calculations for  $\mu_q/T \ll 1$  ( $\mu_B = 3\mu_q$ )

IQCD: J. Guenther et al., Nucl. Phys. A 967 (2017) 720

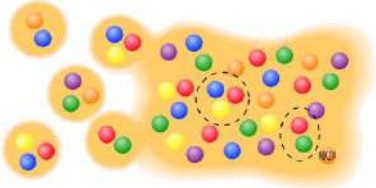


Possible phase diagram of QCD



$\rightarrow$  Lattice QCD results: up to  $\mu_B < 400$  MeV:  
**Crossover**: hadron gas  $\rightarrow$  QGP





# Degrees-of-freedom of the QGP

For the microscopic transport description of the system one **needs to know all degrees of freedom** as well as their properties and interactions!

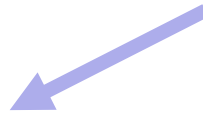
❖ **IQCD** gives **QGP EoS** at finite  $(T, \mu_B)$



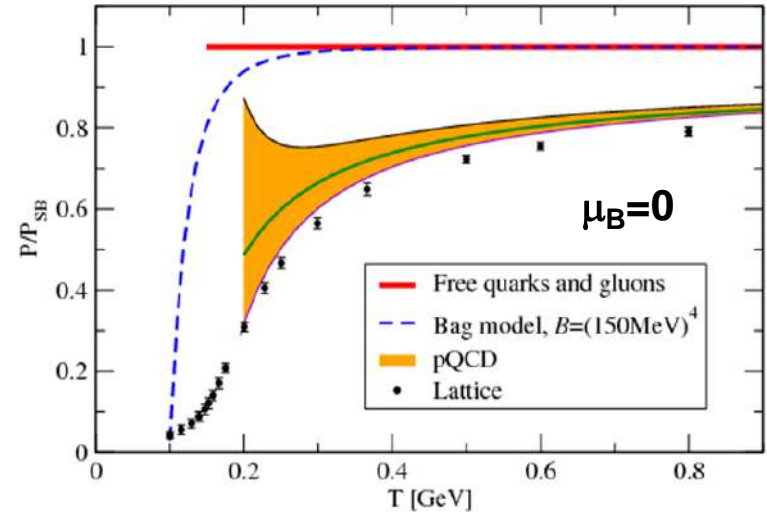
**! needs to be interpreted in terms of degrees-of-freedom**

**pQCD:**

- weakly interacting system
- massless quarks and gluons



Present solution - **effective models**



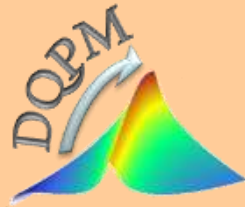
Non-perturbative QCD ← pQCD



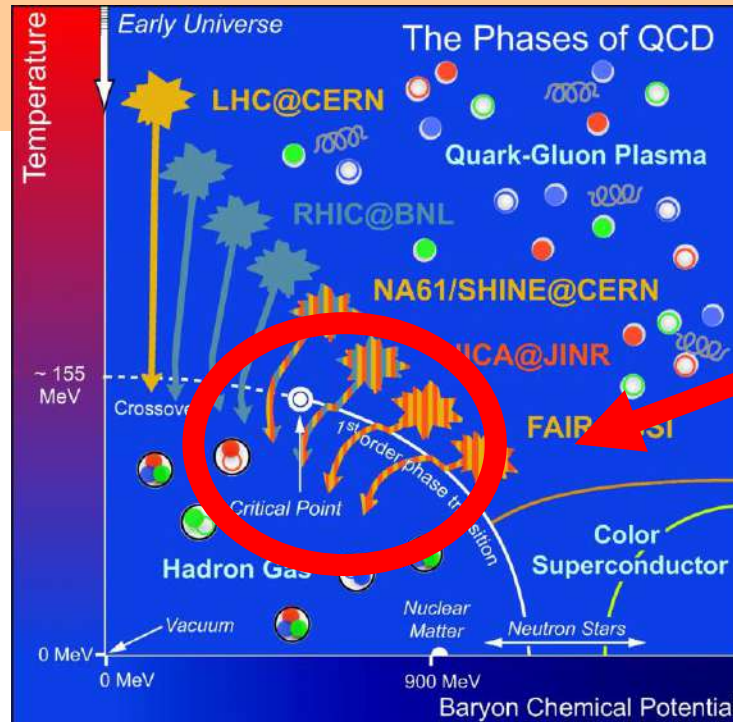
**Thermal QCD**

= QCD at high parton densities:

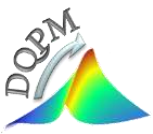
- strongly** interacting system
- massive quarks and gluons
- ➔ **quasiparticles**
- = **effective degrees-of-freedom**



# DQPM ( $T, \mu_q$ )



**finite  $\mu_q$**



# Dynamical QuasiParticle Model (DQPM)

**DQPM** – effective model for the description of **non-perturbative** (strongly interacting) QCD based on **IQCD EoS**

**Degrees-of-freedom:** strongly interacting **dynamical quasiparticles** - quarks and gluons

**Theoretical basis :**

□ ,resummed‘ single-particle Green’s functions → quark (gluon) propagator (2PI) :

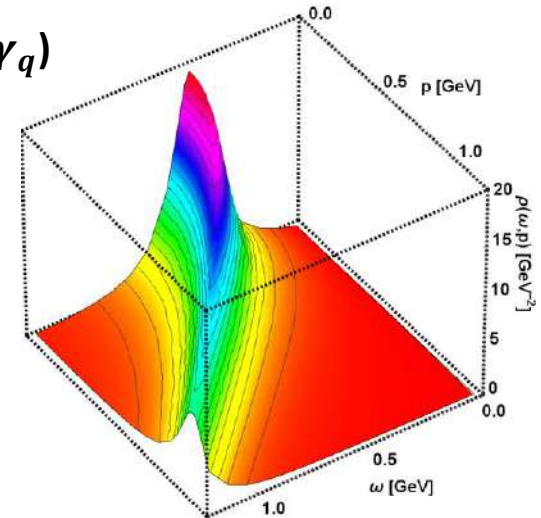
$$\begin{aligned} \text{gluon propagator: } \Delta^{-1} = P^2 - \Pi \quad & \& \quad \text{quark propagator } S_q^{-1} = P^2 - \Sigma_q \\ \text{gluon self-energy: } \Pi = M_g^2 - i2\gamma_g\omega \quad & \& \quad \text{quark self-energy: } \Sigma_q = M_q^2 - i2\gamma_q\omega \end{aligned}$$

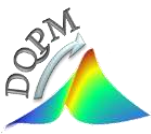
**Properties of the quasiparticles** are specified by scalar **complex self-energies:**

$Re\Sigma_q$  : **thermal masses** ( $M_g, M_q$ );  $Im\Sigma_q$  : **interaction widths** ( $\gamma_g, \gamma_q$ )

→ spectral functions  $\rho_q = -2ImS_q$  → Lorentzian form:

$$\begin{aligned} \rho_j(\omega, \mathbf{p}) &= \frac{\gamma_j}{\tilde{E}_j} \left( \frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2} \right) \\ &\equiv \frac{4\omega\gamma_j}{(\omega^2 - \mathbf{p}^2 - M_j^2)^2 + 4\gamma_j^2\omega^2} \quad \tilde{E}_j^2(\mathbf{p}) = \mathbf{p}^2 + M_j^2 - \gamma_j^2 \end{aligned}$$



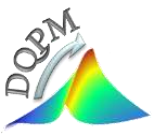


# Dynamical QuasiParticle Model (DQPM)

## Theoretical basis → realization :

- ❑ introduce an **ansatz** (HTL; with few parameters) for the  $(T, \mu_B)$  dependence of **masses/widths**
  - ❑ ansatz for coupling constant  $g$  based on the IQCD entropy density as a function of  $T$  at  $\mu_B=0$
  - ❑ scaling hypothesis for **critical temperature**  $T_c(\mu_q)$  :
  - ❑ evaluate the **QGP thermodynamics** in equilibrium using the Kadanoff-Baym theory
  - ❑ fix DQPM parameters by comparison to the entropy density  $s$ , pressure  $P$ , energy density  $\varepsilon$  from DQPM to **IQCD results** at  $\mu_B=0$
- obtain the **properties of the QGP** at  $(T, \mu_B)$





# Parton properties

- Modeling of the quark/gluon **masses** and **widths** (inspired by HTL calculations)

## Masses:

$$M_{q(\bar{q})}^2(T, \mu_B) = \frac{N_c^2 - 1}{8N_c} g^2(T, \mu_B) \left( T^2 + \frac{\mu_q^2}{\pi^2} \right)$$

$$M_g^2(T, \mu_B) = \frac{g^2(T, \mu_B)}{6} \left( \left( N_c + \frac{1}{2} N_f \right) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)$$

## Widths:

$$\gamma_{q(\bar{q})}(T, \mu_B) = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} \frac{g^2(T, \mu_B) T}{8\pi} \ln \left( \frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

$$\gamma_g(T, \mu_B) = \frac{1}{3} N_c \frac{g^2(T, \mu_B) T}{8\pi} \ln \left( \frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

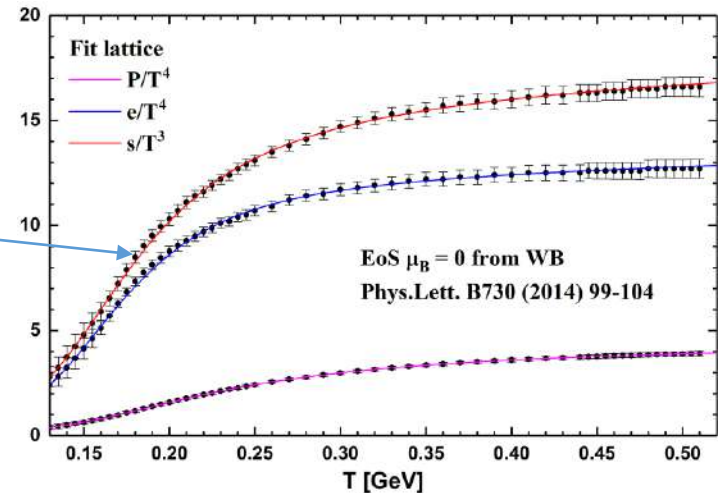
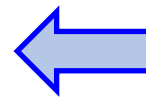
➔ **DQPM :**

only **one parameter** ( $c = 14.4$ )  
+  $(T, \mu_B)$ - dependent **coupling constant** has to be determined from lattice results

- **Coupling g:** input - IQCD **entropy density**  $s$  function of  $T$  at  $\mu_B=0$

$$g^2(s/s_{SB}) = d \left( (s/s_{SB})^e - 1 \right)^f$$

$$s_{SB}^{QCD} = 19/9 \pi^2 T^3$$



# DQPM at finite $(T, \mu_q)$ : scaling hypothesis

- Scaling hypothesis for the effective temperature  $T^*$  for  $N_f = N_c = 3$

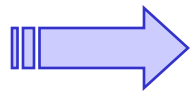
$$\mu_u = \mu_d = \mu_s = \mu_q$$

$$T^{*2} = T^2 + \frac{\mu_q^2}{\pi^2}$$

- Coupling:

$$g(T/T_c(\mu=0)) \longrightarrow g(T^*/T_c(\mu))$$

- Critical temperature  $T_c(\mu_q)$ : obtained by assuming a constant energy density  $\varepsilon$  for the system at  $T=T_c(\mu_q)$ , where  $\varepsilon$  at  $T_c(\mu_q=0)=156$  GeV is fixed by IQCD at  $\mu_q=0$



$$\frac{T_c(\mu_q)}{T_c(\mu_q=0)} = \sqrt{1 - \alpha \mu_q^2} \approx 1 - \alpha/2 \mu_q^2 + \dots$$

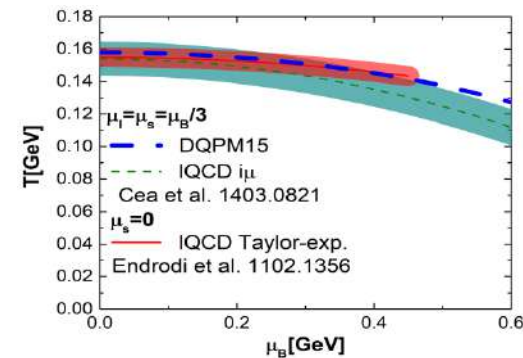
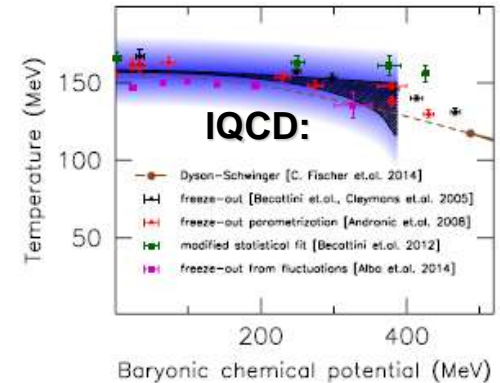
$$\alpha \approx 8.79 \text{ GeV}^{-2}$$

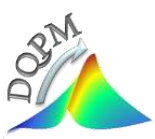
! Consistent with lattice QCD:

IQCD: C. Bonati et al., PRC90 (2014) 114025

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left( \frac{\mu_B}{T_c} \right)^2 + \dots$$

$$\text{IQCD } \kappa = 0.013(2) \longleftrightarrow \kappa_{DQPM} \approx 0.0122$$





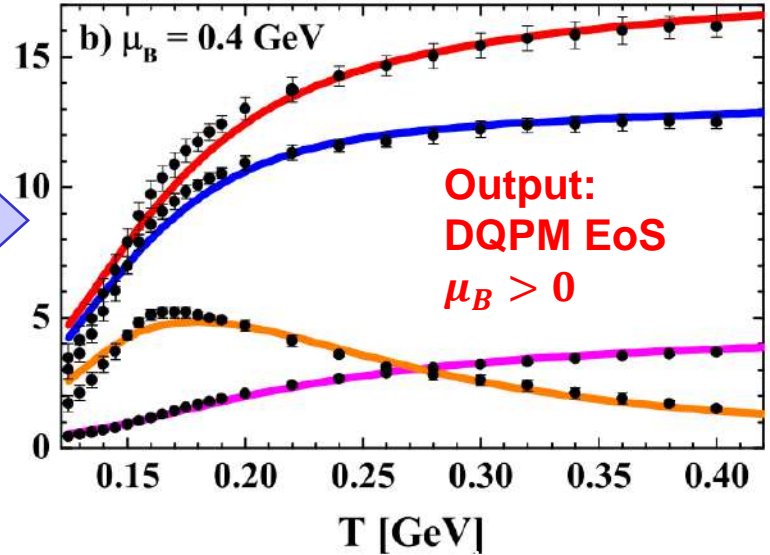
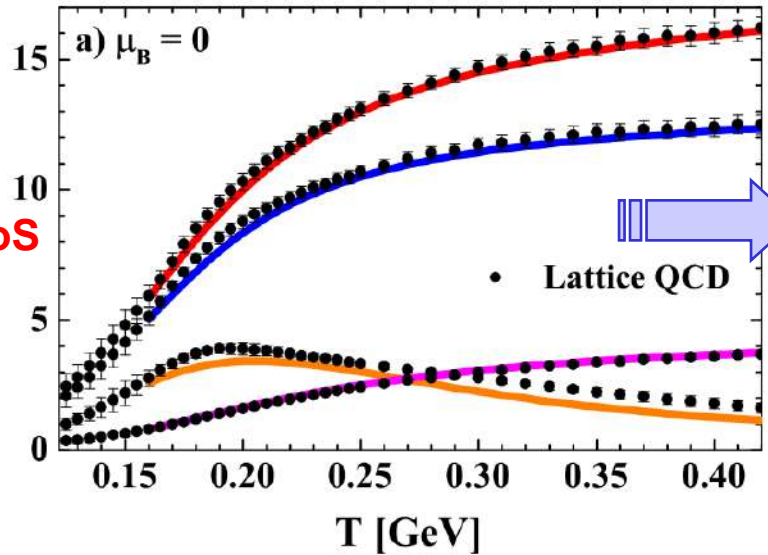
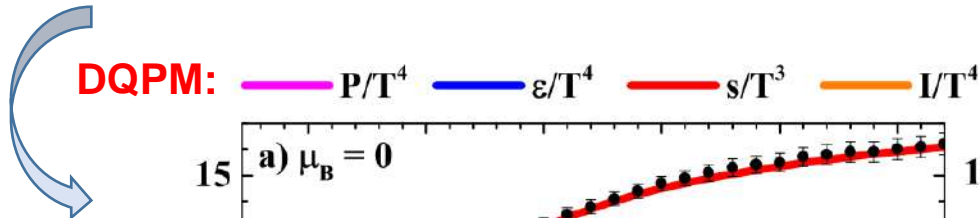
# DQPM thermodynamics at finite $(T, \mu_q)$

- Entropy and baryon density in the quasiparticle limit (G. Baym 1998):

$$s^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ d_q \frac{\partial n_B}{\partial T} (\text{Im}(\ln -\Delta^{-1}) + \text{Im} \Pi \text{Re} \Delta) \right. \\ \left. + \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial T} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial T} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

$$n^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

Blaziot, Iancu, Rebhan, Phys. Rev. D 63 (2001) 065003

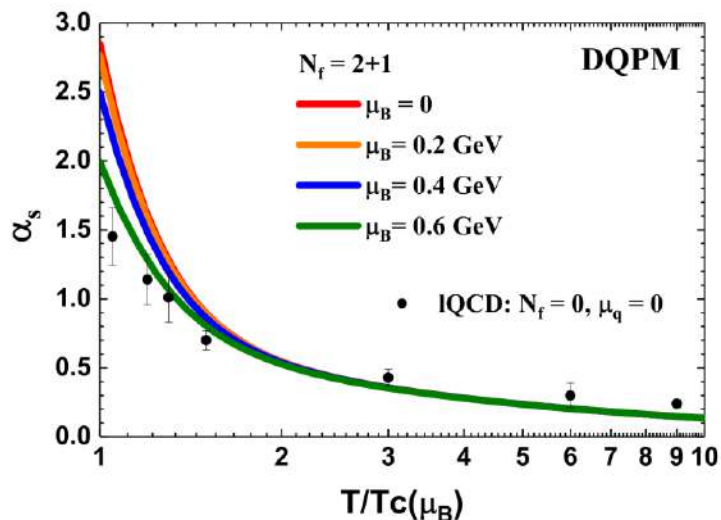


Input:  
lattice EoS  
 $\mu_B = 0$

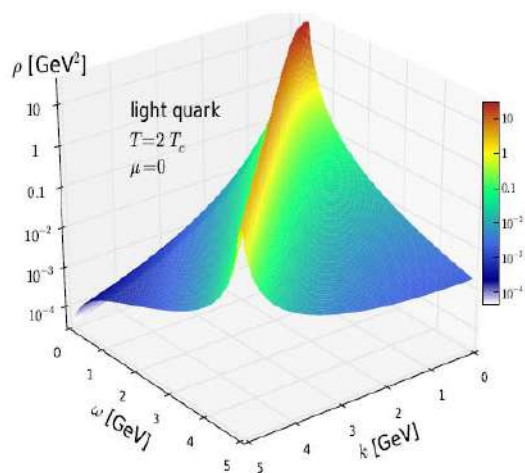
Output:  
DQPM EoS  
 $\mu_B > 0$

# DQPM: parton properties

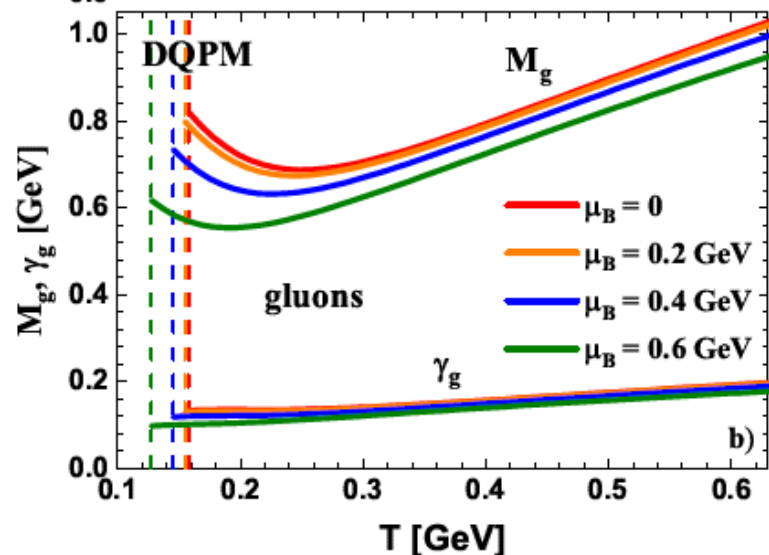
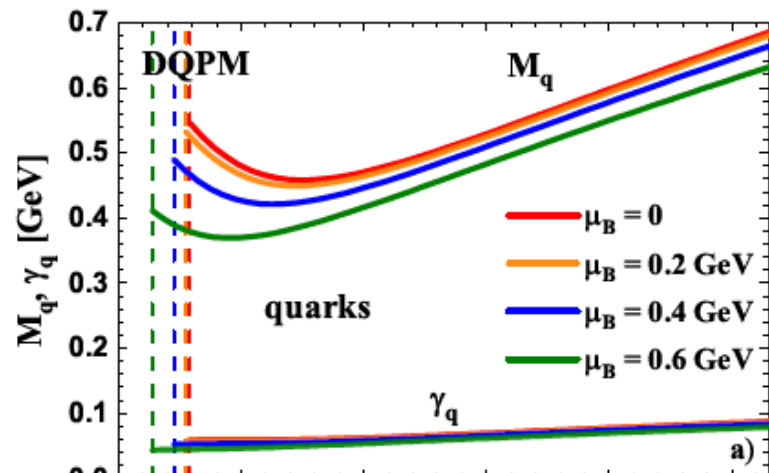
## □ Coupling as a function of $(T, \mu_B)$



→ Lorentzian spectral function:



## □ Masses and widths as a function of $(T, \mu_B)$





# Partonic interactions: matrix elements

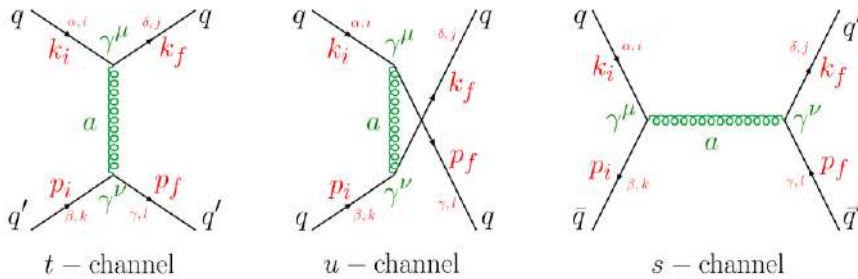
DQPM partonic cross sections  $\rightarrow$  **leading order diagrams**

**Propagators** for massive bosons and fermions:

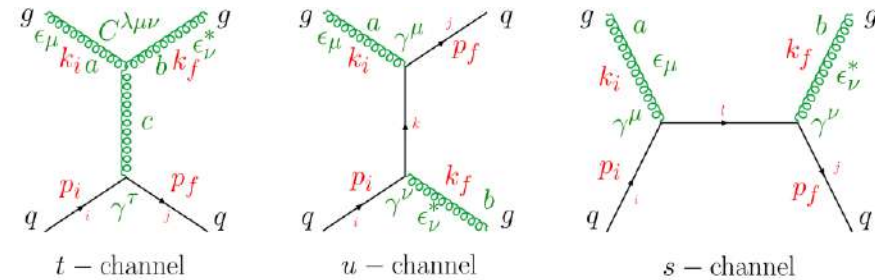
$$\frac{\mu, a \quad \nu, b}{q} = -i\delta_{ab} \frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0}$$

$$\begin{array}{c} i \\ \longrightarrow \\ q \end{array} \begin{array}{c} j \\ \longrightarrow \\ q \end{array} = i\delta_{ij} \frac{q + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}$$

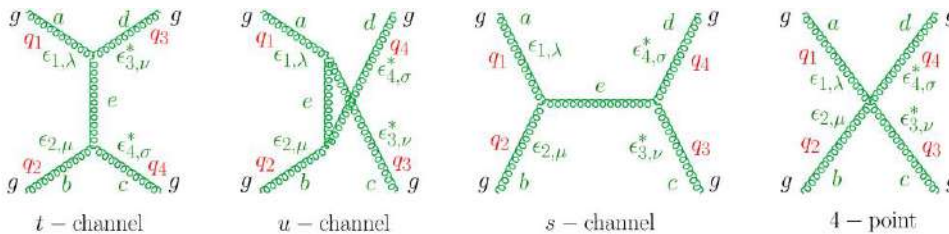
**qq'  $\rightarrow$  qq' scattering**



**gq  $\rightarrow$  gq scattering**



**gg  $\rightarrow$  gg scattering**

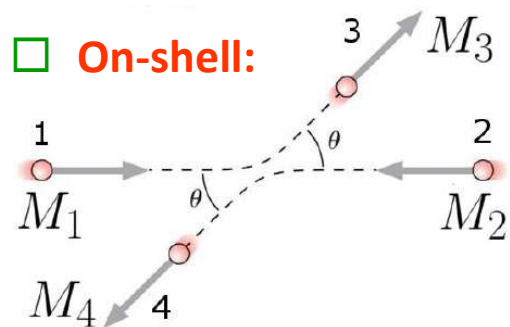


H. Berrehrah et al, PRC 93 (2016) 044914,  
Int.J.Mod.Phys. E25 (2016) 1642003,



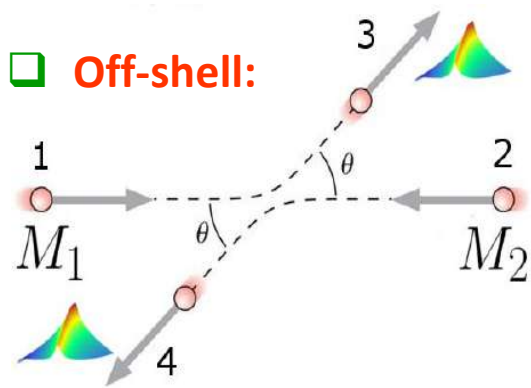
P. Moreau et al., PRC100 (2019) 014911

# Differential cross sections



**Initial masses: pole masses**

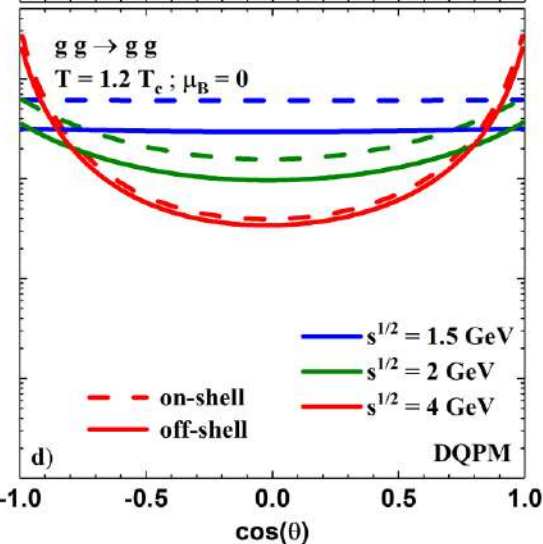
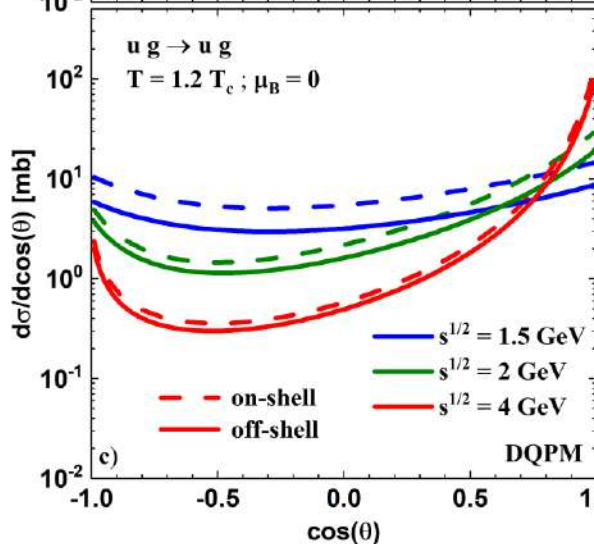
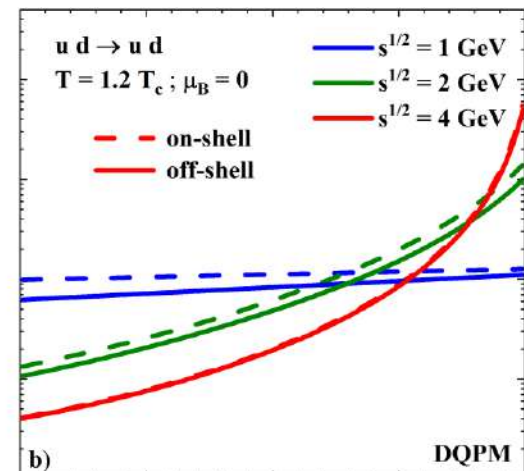
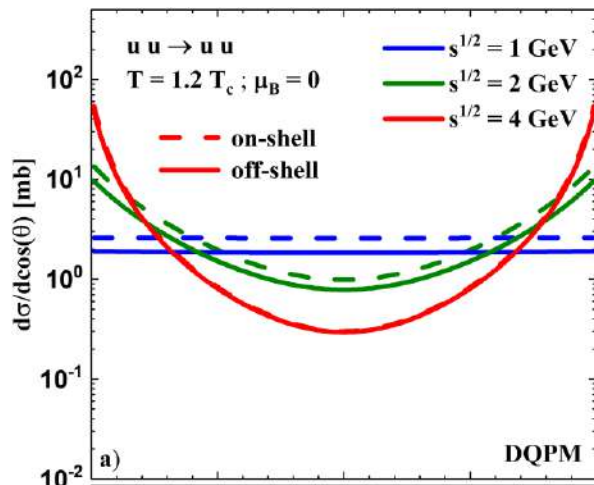
**Final masses: pole masses**



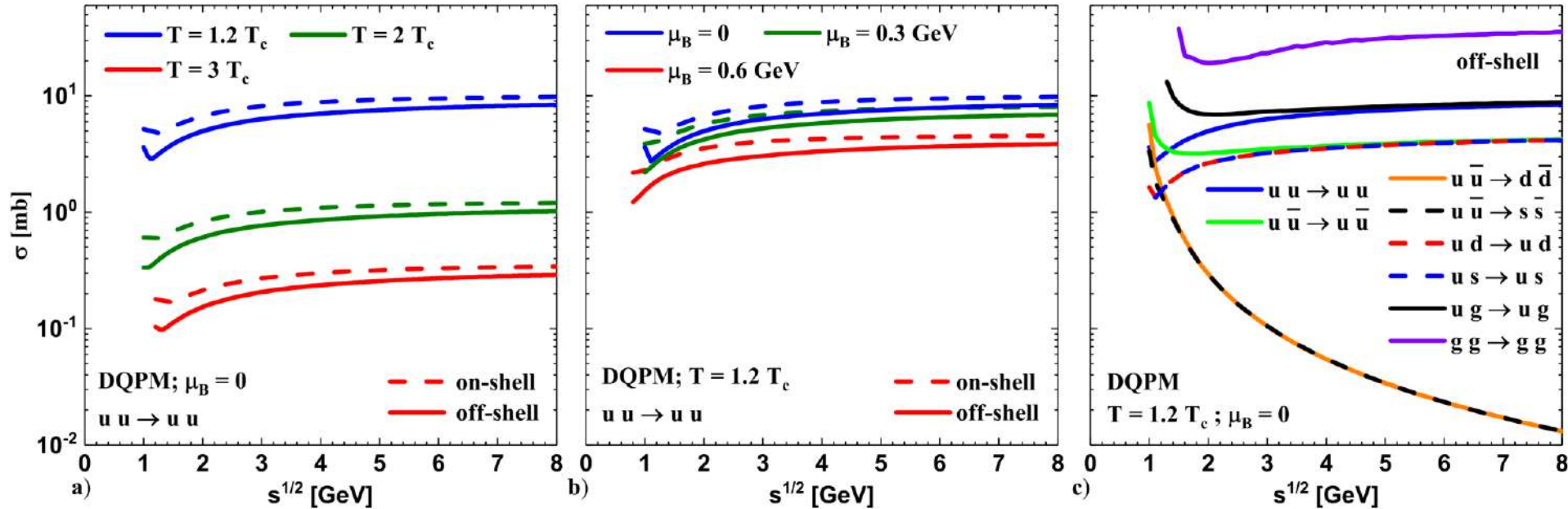
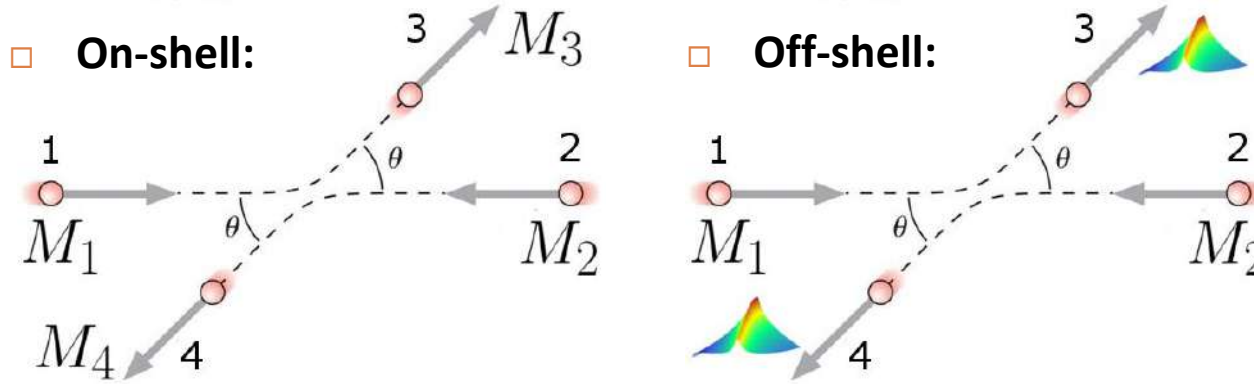
**Initial masses: pole masses**

**Final masses: integrated over spectral functions**

- At lower  $s$ : off-shell  $\sigma <$  on-shell  $\sigma$  since  $\omega_3 + \omega_4 < \sqrt{s}$



# Total cross section



# DQPM: Mean-field potential for quasiparticles

Space-like part of energy-momentum tensor  $T_{\mu\nu}$  defines the **potential energy density**:

$$V_p(T, \mu_q) = T_{g-}^{00}(T, \mu_q) + T_{q-}^{00}(T, \mu_q) + T_{\bar{q}-}^{00}(T, \mu_q)$$

**space-like gluons** + **space-like quarks+antiquarks**

$$\hat{T}_{R_g^\pm} \dots = d_g \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} 2\omega \rho_g(\omega) \Theta(\omega) n_B(\omega/T) \Theta(\pm P^2) \dots$$

$$\hat{T}_{R_q^\pm} \dots = d_q \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} 2\omega \rho_q(\omega) \Theta(\omega) n_F((\omega - \mu_q)/T) \Theta(\pm P^2) \dots$$

$$\hat{T}_{R_{\bar{q}}^\pm} \dots = d_{\bar{q}} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} 2\omega \rho_{\bar{q}}(\omega) \Theta(\omega) n_F((\omega + \mu_q)/T) \Theta(\pm P^2) \dots$$

→ **mean-field scalar potential (1PI)** for quarks and gluons ( $U_q, U_g$ ) vs **parton scalar density**  $\rho_s$ :

$$U_s(\rho_s) = \frac{dV_p(\rho_s)}{d\rho_s} \quad \rho_s = N_g^+ + N_q^+ + N_{\bar{q}}^+$$

$$U_q = U_s, \quad U_g \sim 2U_s$$

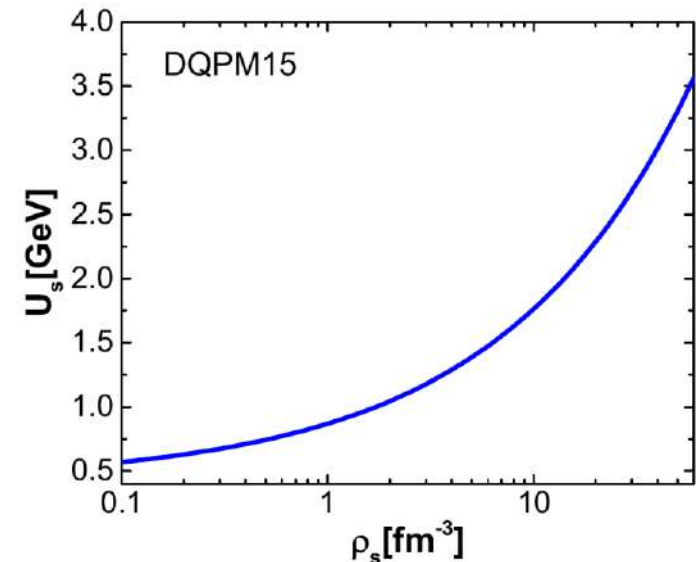
**Quasiparticle potentials ( $U_q, U_g$ ) are repulsive !**

→ **the force acting on a quasiparticle j:**

$$F \sim M_j/E_j \nabla U_s(x) = M_j/E_j dU_s/d\rho_s \nabla \rho_s(x)$$

$$j = g, q, \bar{q}$$

→ **accelerates particles**

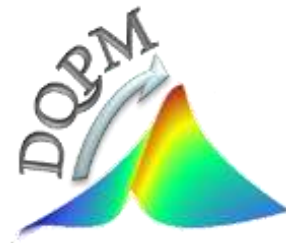




# QGP near equilibrium

DQPM ( $T, \mu_q$ ):

transport properties at finite ( $T, \mu_q$ )



# The properties of QGP in HICs → transport coefficients

Properties of the QGP near equilibrium are characterized by **transport coefficients**

Shear  $\eta$ , bulk viscosity  $\zeta$ , ... are 'input' for the **viscous hydrodynamic models!**

## Hydrodynamics

$$T^{\mu\nu} = -P g^{\mu\nu} + w u^\mu u^\nu + \Delta T^{\mu\nu}$$

$$J_B^\mu = n_B u^\mu + \Delta J_B^\mu$$

$$\begin{cases} \partial_\mu J_B^\mu = 0 \\ \partial_\mu T^{\mu\nu} = 0 \end{cases}$$

input for hydro simulations

$$\Delta T^{\mu\nu} = \eta \left( D^\mu u^\nu + D^\nu u^\mu + \frac{2}{3} \Delta^{\mu\nu} \partial_\rho u^\rho \right) - \zeta \Delta^{\mu\nu} \partial_\rho u^\rho$$

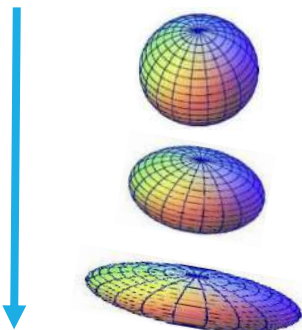
$$\Delta J_B^\mu = \kappa_B D^\mu \left( \frac{\mu_B}{T} \right)$$

$$D^\mu = \Delta^{\alpha\nu} \partial_\nu \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

### Shear viscosity

Resistance to 'deformation'

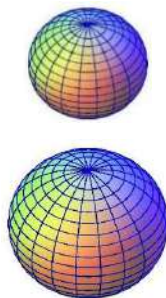
$$\eta \nabla^{\langle\mu} u^{\nu\rangle}$$



### Bulk viscosity

Resistance to expansion

$$-\zeta \nabla u$$

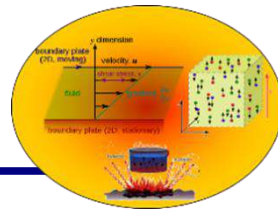


Baryon/electric charge  
diffusion coefficients

$$\kappa_B \nabla^\mu \frac{\mu_B}{T}$$



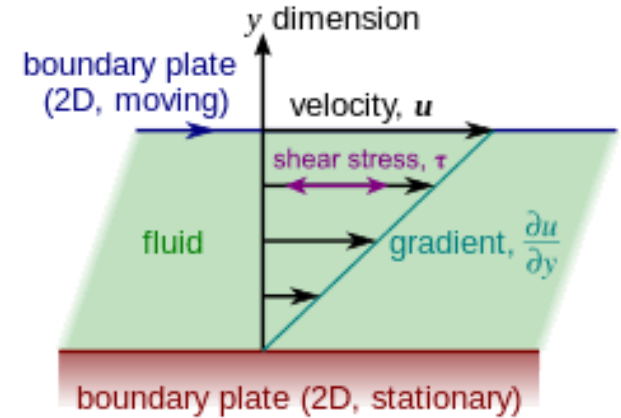
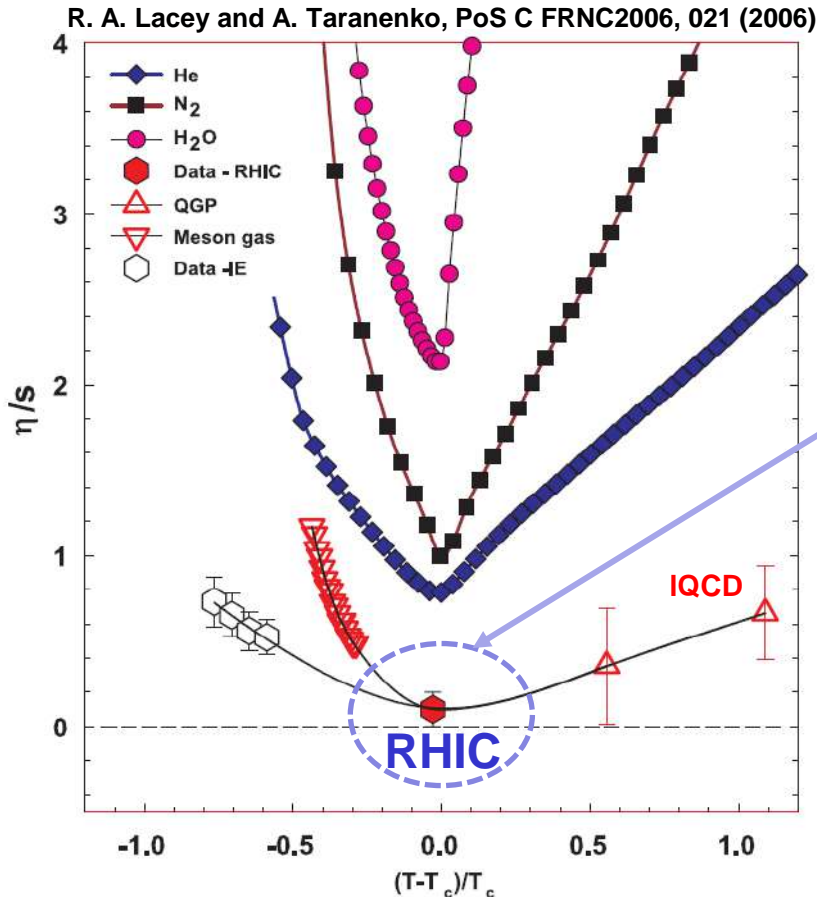
# The properties of QGP from HIC - shear viscosity



The **shear viscosity** of a system measures its **resistance** to flow.

**Wikipedia:** Viscosity can be conceptualized as quantifying the **frictional force** that arises between adjacent layers of fluid that are in relative motion.

Compilation of the ratio of shear viscosity to entropy density for various substances:



Exp. data + IQCD:

$\eta/s$  near  $T_c$  is very small !

→ **QGP** : close to an **ideal liquid**, not a gas of weakly interacting quarks and gluons

→ **QGP**: **strongly-interacting matter**

# pQCD: shear viscosity $\eta$

## QCD: Pure Yang-Mills (only gluons)

LO (Leading order) perturbative QCD calculations:

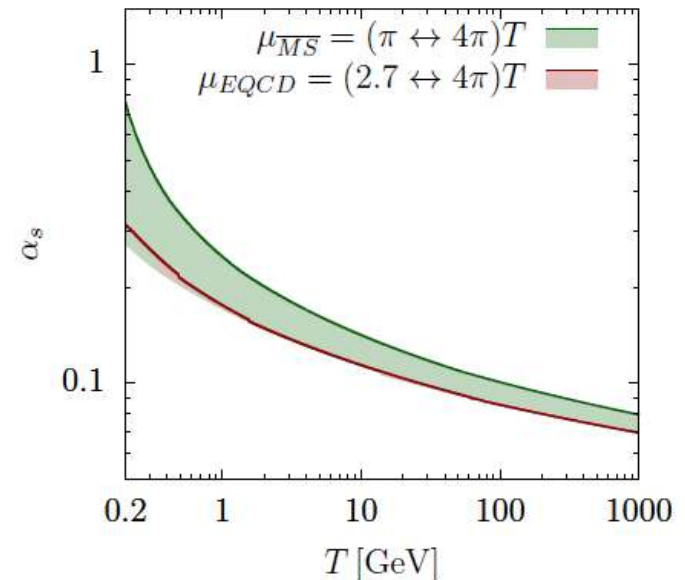
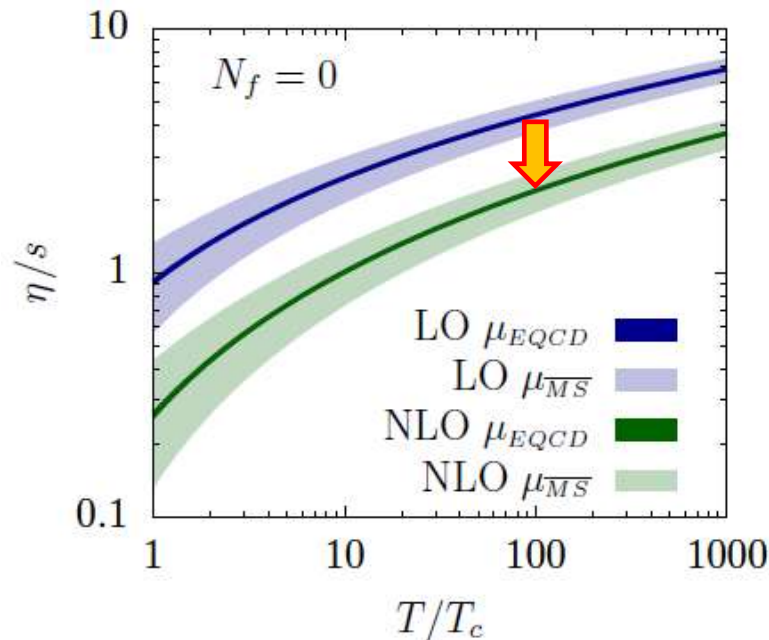
$\eta/s > 0.5$  at T near  $T_c$  'AMY': P.B. Arnold, G.D. Moore and L.G. Yaffe, JHEP 11 (2000) 001

NLO (Next-to-leading order):

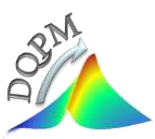
(J. Ghiglieri, G.D. Moore, D. Teaney, JHEP 1803 (2018) 179):

“The next-to-leading order corrections are large and bring  $\eta/s$  down by more than a factor of 3 at physically relevant couplings.

The perturbative expansion is problematic even at  $T \sim 100$  GeV”



→ from pQCD to effective models of QCD!



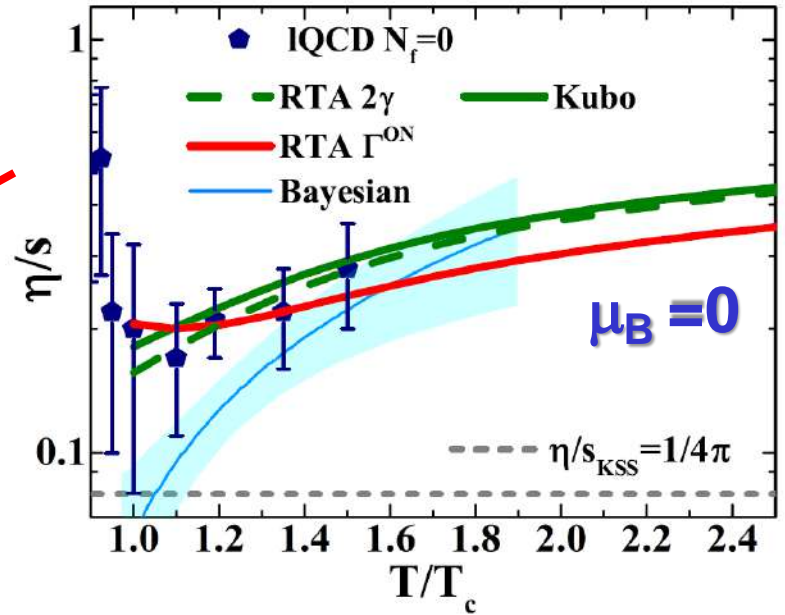
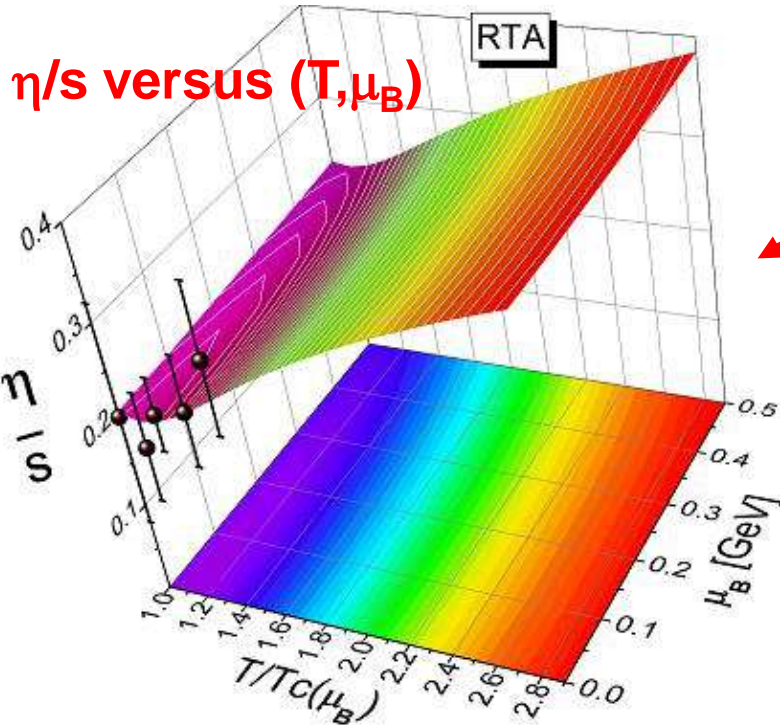
# Transport coefficients: shear viscosity $\eta$

## ➤ Relaxation Time Approximation

$$\eta^{\text{RTA}}(T, \mu_B) = \frac{1}{15T} \sum_{i=q, \bar{q}, g} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^4}{E_i^2} \tau_i(\mathbf{p}, T, \mu_B) d_i (1 \pm f_i) f_i$$

$$1) \tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

$$2) \tau_i(T, \mu_B) = \frac{1}{2\gamma_i(T, \mu_B)}$$



Hydro: Bayesian analysis, S. Bass et al., NPA967 (2017) 67  
Kovtun-Son-Starinets bound :  $(\eta/s)_{\text{KSS}} = 1/(4\pi)$

## ➤ Weak dependence of shear viscosity on $\mu_B$



# Transport coefficients: bulk viscosity $\zeta$

## ➤ Relaxation Time Approximation

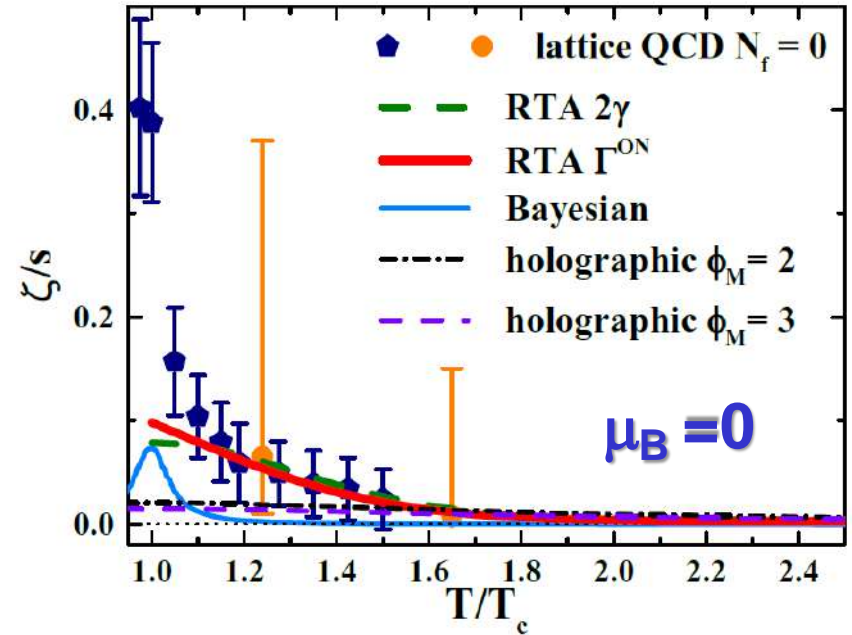
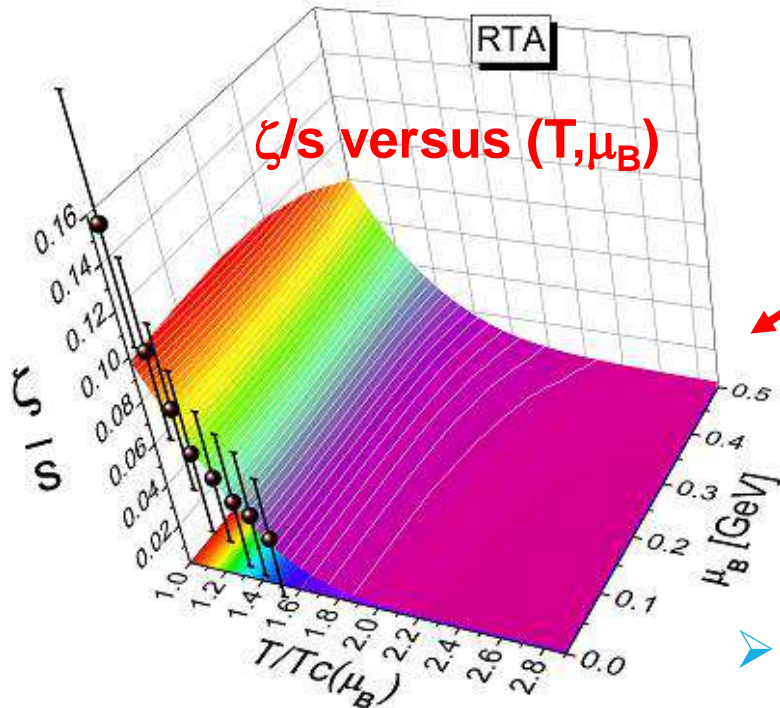
$$\zeta^{\text{RTA}}(T, \mu_B) = \frac{1}{9T} \sum_{i=q, \bar{q}, g} \int \frac{d^3p}{(2\pi)^3} \tau_i(\mathbf{p}, T, \mu_B)$$

Relaxation times

From DQPM parametrization

$$\times \frac{d_i(1 \pm f_i)f_i}{E_i^2} \left( \mathbf{p}^2 - 3c_s^2 \left( E_i^2 - T^2 \frac{dm_i^2}{dT^2} \right) \right)^2$$

Speed of sound



Lattice: Astrakhantsev et al, Phys.Rev. D98 (2018) 054515

## ➤ Weak dependence of bulk viscosity on $\mu_B$

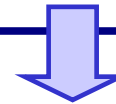
# Transport coefficients: electric conductivity $\sigma_e/T$

$\sigma_0 \rightarrow$  Probe of **electric properties of the QGP**

➤ **Relaxation Time Approximation**

$$\sigma_0^{\text{RTA}}(T, \mu_B) = \frac{e^2}{3T} \sum_{i=q, \bar{q}} q_i^2 \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^2}{E_i^2} \times \tau_i(\mathbf{p}, T, \mu_B) d_i(1 - f_i) f_i$$

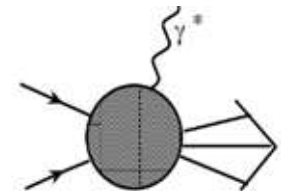
- the **QCD matter** even at  $T \sim T_c$  is a **much better electric conductor than Cu or Ag** (at room temperature) by a factor of **500** !



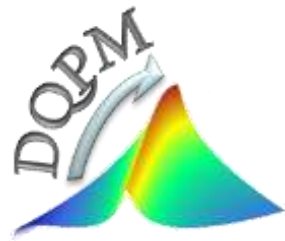
**Exp. observables – photon and dilepton spectra**

☐ **Photon emission:** rates for  $q_0 \rightarrow 0$  are related to **electric conductivity  $\sigma_0$**

$$q_0 \left. \frac{dR}{d^4x d^3q} \right|_{q_0 \rightarrow 0} = \frac{T}{4\pi^3} \sigma_0$$



**QGP:**  
**in-equilibrium → off-equilibrium**  
**microscopic transport theory!**



# From weakly to strongly interacting systems

Properties of matter (on hadronic and partonic levels) in heavy-ion collisions:

**QGP** – strongly interacting system! Degrees of freedom – dressed partons!

**Hadronic matter** – in-medium effects – modification of hadron properties at finite  $T, \mu_B$  (vector mesons, strange mesons)

Many-body theory:

**Strong interaction** → large width = short life-time

→ broad spectral function → quantum object

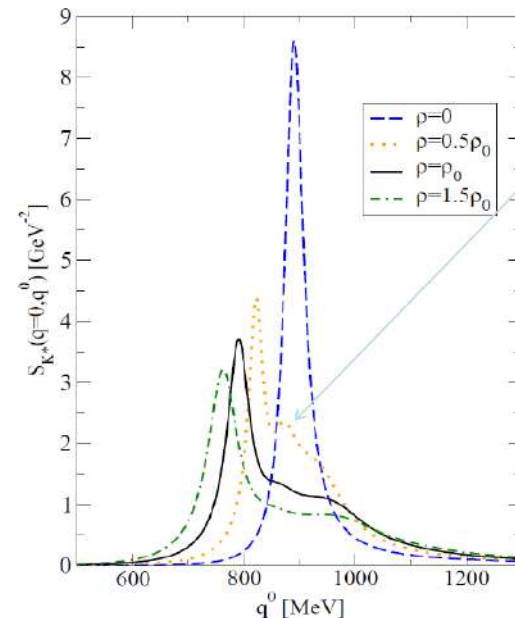
▪ How to describe the dynamics of broad strongly interacting quantum states in transport theory?

□ semi-classical BUU

first order gradient expansion of quantum Kadanoff-Baym equations

□ generalized transport equations based on Kadanoff-Baym dynamics

Kbar\* spectral function



$\Lambda(1783)N^{-1}$   
and  
 $\Sigma(1830)N^{-1}$   
excitations

# Dynamical description of strongly interacting systems

Quantum field theory →

**Kadanoff-Baym dynamics** for resummed single-particle Green functions  $S^<$

$$\hat{S}_{0x}^{-1} S_{xy}^< = \sum_{xz}^{ret} \odot S_{zy}^< + \sum_{xz}^< \odot S_{zy}^{adv} \quad (1962)$$

Green functions  $S^</math> / self-energies  $\Sigma$ :$

$$iS_{xy}^< = \eta \langle \{ \Phi^+(y) \Phi(x) \} \rangle$$

$$iS_{xy}^> = \langle \{ \Phi(y) \Phi^+(x) \} \rangle$$

$$iS_{xy}^c = \langle T^c \{ \Phi(x) \Phi^+(y) \} \rangle \text{ - causal}$$

$$iS_{xy}^a = \langle T^a \{ \Phi(x) \Phi^+(y) \} \rangle \text{ - anticausal}$$

Integration over the intermediate spacetime

$$S_{xy}^{ret} = S_{xy}^c - S_{xy}^< = S_{xy}^> - S_{xy}^a \text{ - retarded}$$

$$S_{xy}^{adv} = S_{xy}^c - S_{xy}^> = S_{xy}^< - S_{xy}^a \text{ - advanced}$$

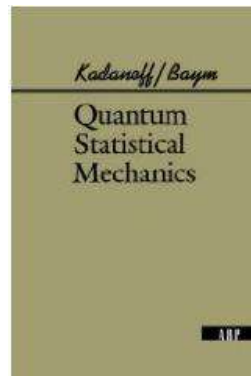
$$\eta = \pm 1 \text{ (bosons / fermions)}$$

$$T^a (T^c) \text{ - (anti-)time - ordering operator}$$

$$\hat{S}_{0x}^{-1} \equiv -(\partial_x^\mu \partial_\mu^x + M_0^2)$$

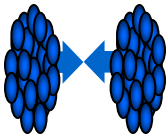


Leo Kadanoff



Gordon Baym





# From Kadanoff-Baym equations to generalized transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

## Generalized transport equations (GTE):

$$\underbrace{\diamond \{ P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{\text{ret}} \}}_{\text{drift term}} \underbrace{\{ S_{XP}^< \}}_{\text{Vlasov term}} - \underbrace{\diamond \{ \Sigma_{XP}^< \} \{ \text{Re} S_{XP}^{\text{ret}} \}}_{\text{backflow term}} = \frac{i}{2} [ \Sigma_{XP}^> S_{XP}^< - \Sigma_{XP}^< S_{XP}^> ] \quad \text{collision term} = \text{,gain' - ,loss' term}$$

**Backflow term** incorporates the **off-shell** behavior in the particle propagation  
**! vanishes in the quasiparticle limit**  $A_{XP} \rightarrow \delta(p^2 - M^2)$

□ GTE: Propagation of the Green's function  $iS_{XP}^< = A_{XP} N_{XP}$ , which carries information not only on the **number of particles** ( $N_{XP}$ ), but also on their **properties**, interactions and correlations (via  $A_{XP}$ )

□ **Spectral function:**

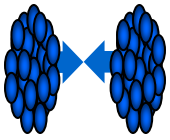
$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{\text{ret}})^2 + \Gamma_{XP}^2/4}$$

$\Gamma_{XP} = -\text{Im} \Sigma_{XP}^{\text{ret}} = 2 p_0 \Gamma$  - **,width' of spectral function**  
 = **reaction rate** of particle (at space-time position X)

4-dimensional generalization of the Poisson-bracket:

$$\diamond \{ F_1 \} \{ F_2 \} := \frac{1}{2} \left( \frac{\partial F_1}{\partial X_\mu} \frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right)$$

□ **Life time**  $\tau = \frac{\hbar c}{\Gamma}$



# General testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

□ Employ **testparticle Ansatz** for the real valued quantity  $i S_{XP}^<$

$$F_{XP} = A_{XP} N_{XP} = i S_{XP}^< \sim \sum_{i=1}^N \delta^{(3)}(\vec{X} - \vec{X}_i(t)) \delta^{(3)}(\vec{P} - \vec{P}_i(t)) \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine **equations of motion** !

→ **General testparticle Cassing's off-shell equations of motion for the time-like particles:**

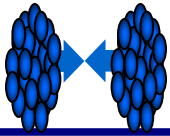
$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ 2\vec{P}_i + \vec{\nabla}_{P_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \vec{\nabla}_{X_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \frac{\partial \text{Re}\Sigma_{(i)}^{\text{ret}}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with  $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[ \frac{\partial}{\partial \epsilon_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$



# Collision term in off-shell transport models

**Collision term** for reaction 1+2->3+4:

$$I_{coll}(X, \vec{P}, M^2) = Tr_2 Tr_3 Tr_4 \underbrace{A(X, \vec{P}, M^2) A(X, \vec{P}_2, M_2^2) A(X, \vec{P}_3, M_3^2) A(X, \vec{P}_4, M_4^2)}_{|G((\vec{P}, M^2) + (\vec{P}_2, M_2^2) \rightarrow (\vec{P}_3, M_3^2) + (\vec{P}_4, M_4^2))|_{\mathcal{A}, \mathcal{S}}^2} \delta^{(4)}(P + P_2 - P_3 - P_4)$$

$$[ \underbrace{N_{X\vec{P}_3 M_3^2} N_{X\vec{P}_4 M_4^2} \bar{f}_{X\vec{P} M^2} \bar{f}_{X\vec{P}_2 M_2^2}}_{\text{,gain' term}} - \underbrace{N_{X\vec{P} M^2} N_{X\vec{P}_2 M_2^2} \bar{f}_{X\vec{P}_3 M_3^2} \bar{f}_{X\vec{P}_4 M_4^2}}_{\text{,loss' term}} ]$$

**with**  $\bar{f}_{X\vec{P} M^2} = 1 + \eta N_{X\vec{P} M^2}$  and  $\eta = \pm 1$  for bosons/fermions, respectively.

**The trace over particles 2,3,4 reads explicitly**

**for fermions**

$$Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3 P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$$

**for bosons**

$$Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3 P_2 \frac{dP_{0,2}^2}{2}$$

additional integration

The transport approach and the particle spectral functions are fully determined once the **in-medium transition amplitudes G** are known in their **off-shell dependence!**



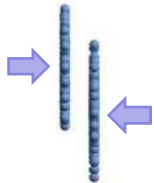
# Parton-Hadron-String-Dynamics (PHSD)



**PHSD** is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

**Dynamics:** based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

Initial A+A collision



Initial A+A collisions :

$N+N \rightarrow$  **string formation**  $\rightarrow$  decay to pre-hadrons + leading hadrons

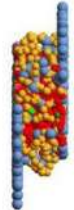
Formation of QGP stage if local  $\varepsilon > \varepsilon_{\text{critical}}$  :

dissolution of **pre-hadrons**  $\rightarrow$  partons

Partonic phase - QGP:

QGP is described by the **Dynamical QuasiParticle Model (DQPM)** matched to reproduce **lattice QCD EoS** for finite  $T$  and  $\mu_B$  (crossover)

Partonic phase



- **Degrees-of-freedom:** strongly interacting quasiparticles: **massive quarks and gluons ( $g, q, q_{\text{bar}}$ )** with sizeable collisional widths in a self-generated mean-field potential

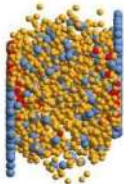
- **Interactions:** (quasi-)elastic and inelastic collisions of partons

Hadronization to colorless **off-shell mesons and baryons:**

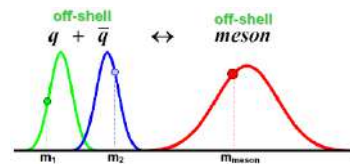
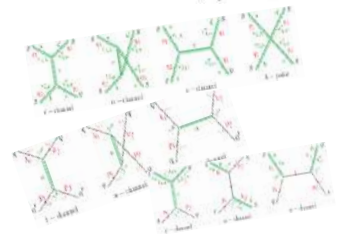
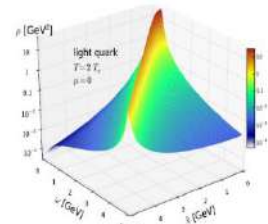
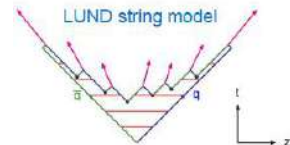
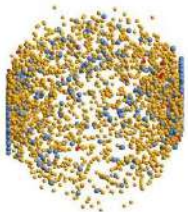
Strict 4-momentum and quantum number conservation

Hadronic phase: **hadron-hadron interactions – off-shell HSD**

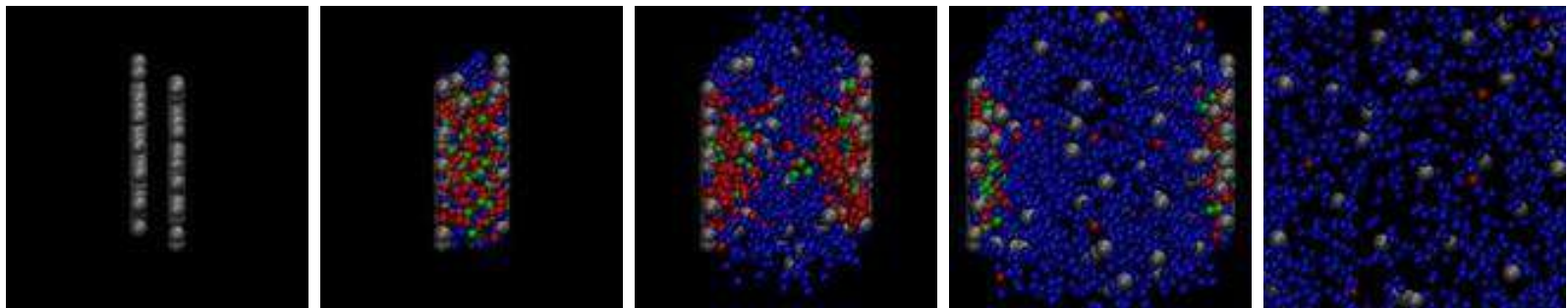
Hadronization



Hadronic phase



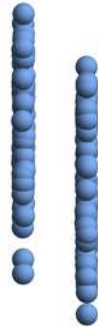
# Traces of the QGP in observables in high energy heavy-ion collisions









# Stages of a collision in PHSD

$t = 0.05 \text{ fm}/c$



$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$  – Section view

-  Baryons (394)
-  Antibaryons ( 0)
-  Mesons ( 0)
-  Quarks ( 0)
-  Gluons ( 0)

# Stages of a collision in PHSD





---

$t = 1.6512 \text{ fm}/c$



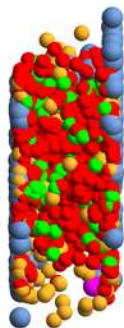
$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$  – Section view

-  Baryons (394)
-  Antibaryons ( 0)
-  Mesons (1523)
-  Quarks (4553)
-  Gluons (368)

# Stages of a collision in PHSD

$t = 3.91921 \text{ fm}/c$



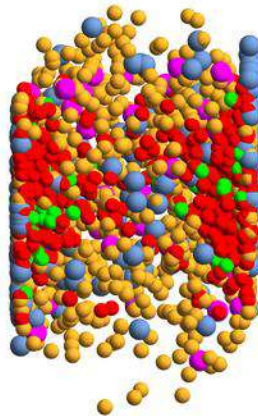
$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$  - Section view

-  Baryons (426)
-  Antibaryons ( 29)
-  Mesons (1189)
-  Quarks (4459)
-  Gluons (783)

# Stages of a collision in PHSD

$t = 7.31921 \text{ fm}/c$



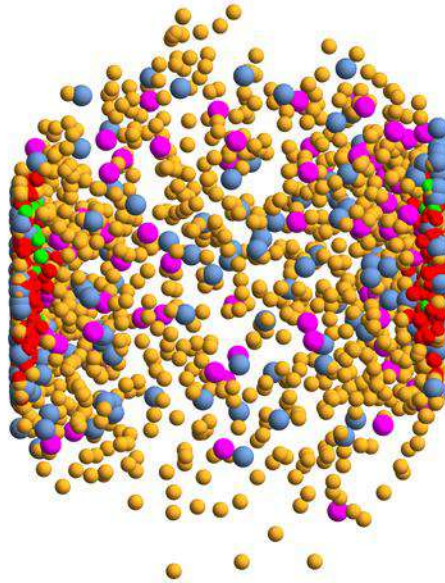
$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$  – Section view

-  Baryons (540)
-  Antibaryons (120)
-  Mesons (2481)
-  Quarks (2901)
-  Gluons (492)

# Stages of a collision in PHSD

$t = 12.0192 \text{ fm}/c$



$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$  – Section view

 Baryons (626)

 Antibaryons (202)

 Mesons (3357)

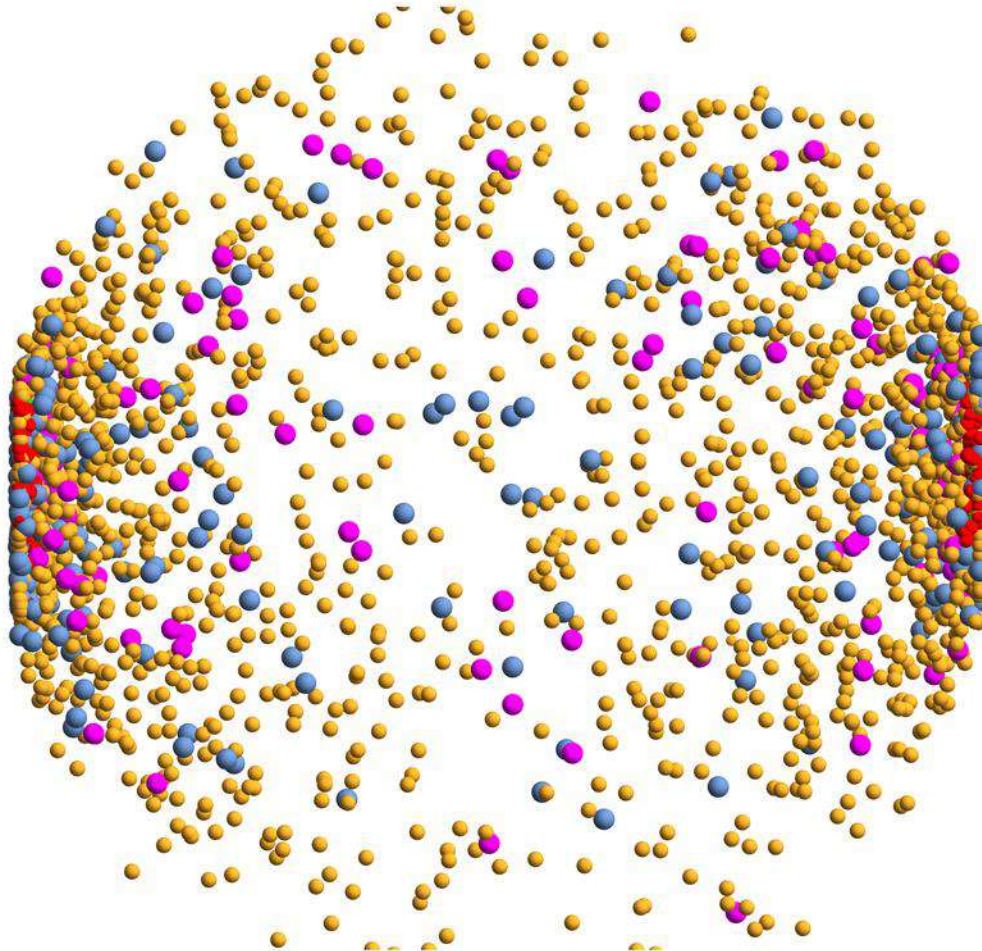
 Quarks (1835)

 Gluons (269)



# Stages of a collision in PHSD

$t = 25.5191 \text{ fm}/c$



$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$

$b = 2.2 \text{ fm}$  - Section view

 Baryons (710)

 Antibaryons (272)

 Mesons (4343)

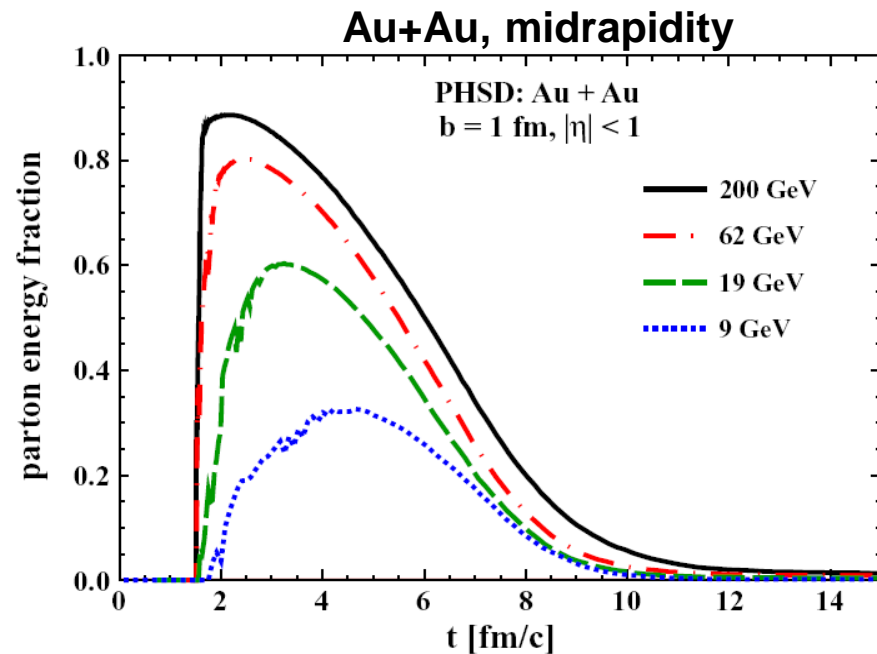
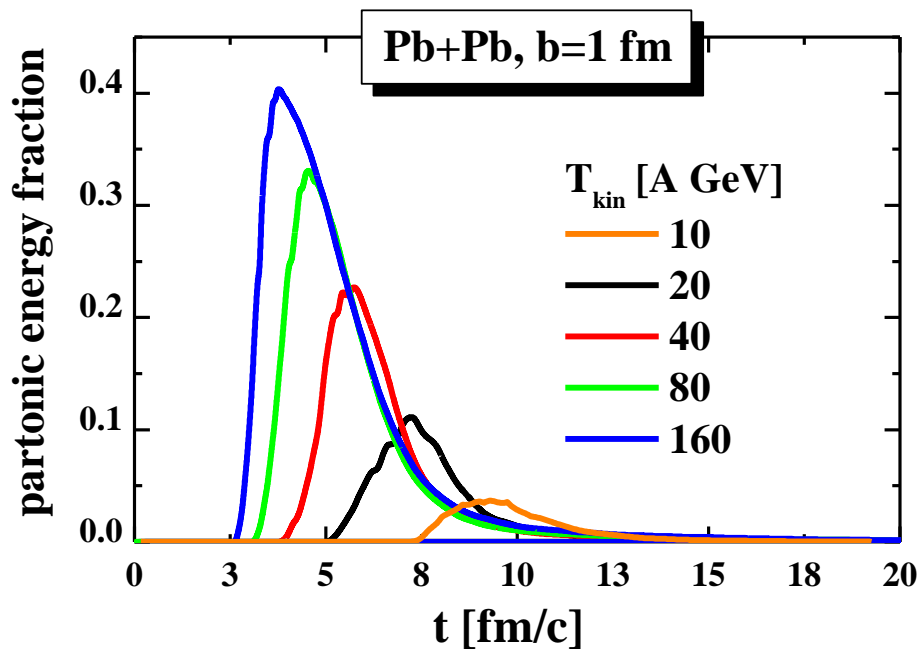
 Quarks ( 899)

 Gluons ( 46)



# Partonic energy fraction in central A+A

Time evolution of the partonic energy fraction vs energy

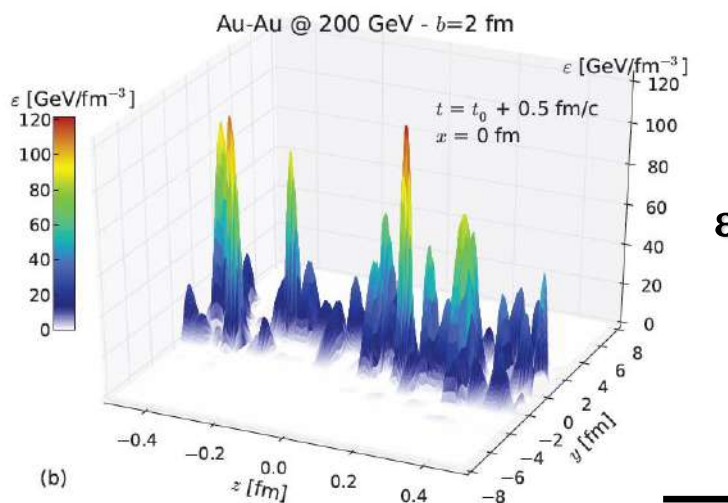


- Strong increase of partonic phase with energy from AGS to RHIC
- SPS: Pb+Pb, 160 A GeV: only about 40% of the converted energy goes to partons; the rest is contained in the large hadronic corona and leading partons
- RHIC: Au+Au, 21.3 A TeV: up to 90% - QGP



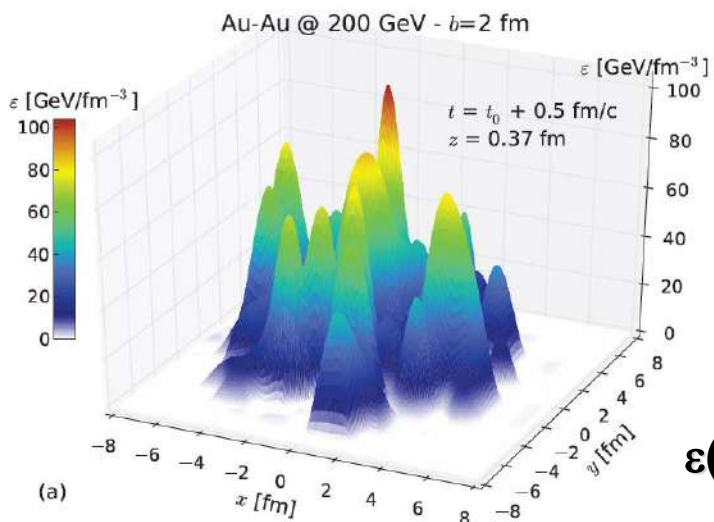
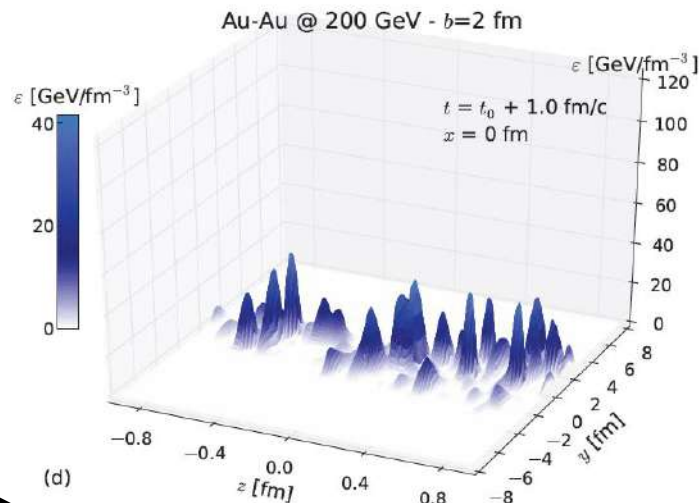
# Time evolution of energy density

PHSD: 1 event Au+Au, 200 GeV, b = 2 fm

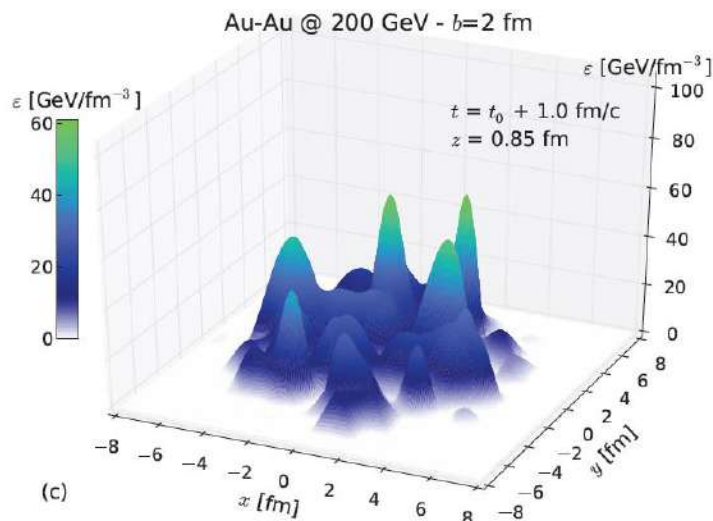


$\epsilon(x=0, y, z)$

time



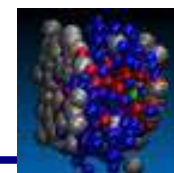
$\epsilon(x, y, z=0)$



$\Delta V: \Delta x = \Delta y = 1 \text{ fm}, \Delta z = 1/\gamma \text{ fm}$

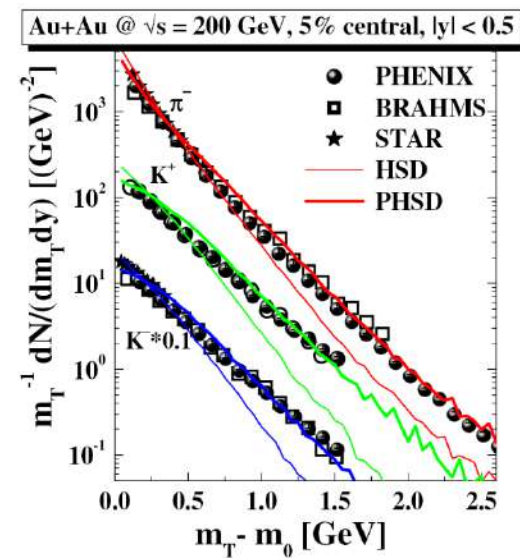
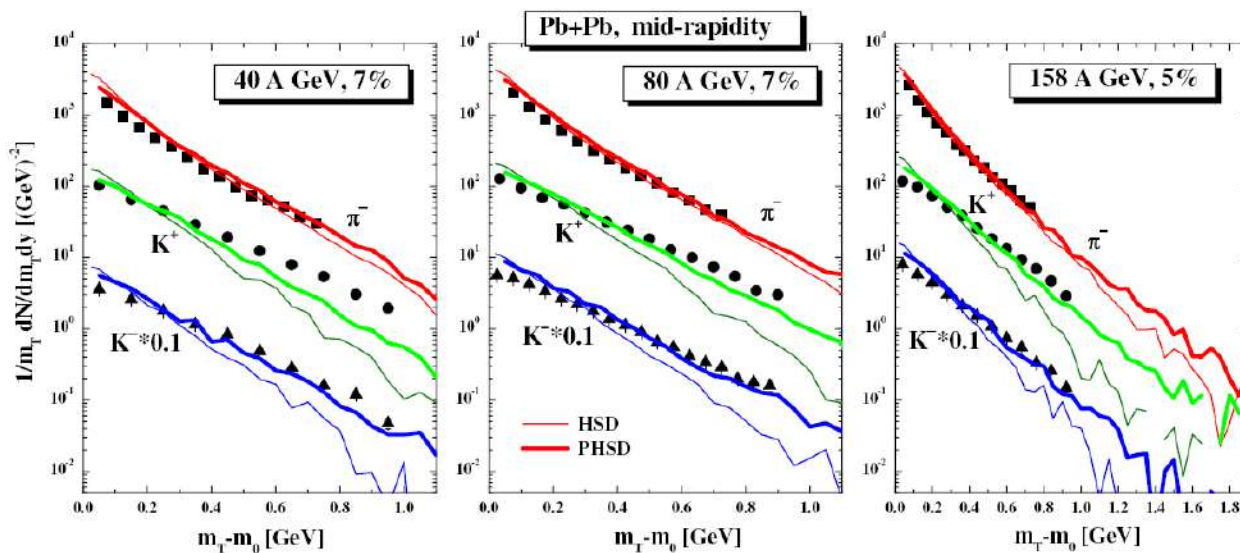
R. Marty et al., PRC 92 (2015) 015201





## Central Pb + Pb at SPS energies

## Central Au+Au at RHIC



- PHSD gives **harder  $m_T$  spectra** and works better than HSD (without QGP) at high energies – RHIC, SPS (and top FAIR, NICA)
- however, at **low SPS** (and low FAIR, NICA) energies the **effect of the partonic phase decreases** due to the decrease of the partonic fraction

W. Cassing & E. Bratkovskaya, NPA 831 (2009) 215

E. Bratkovskaya, W. Cassing, V. Konchakovski, O. Linnyk, NPA856 (2011) 162

# Anisotropy coefficients $v_n$

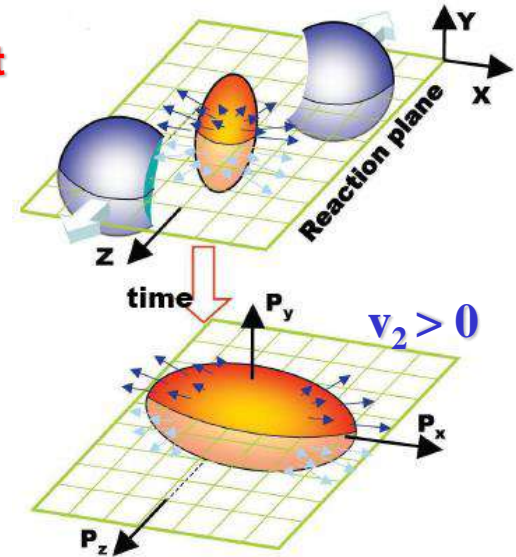
Non central Au+Au collisions :

□ interaction between constituents leads to a **pressure gradient**  
 → spatial asymmetry is converted to an asymmetry in momentum space → **collective flow**

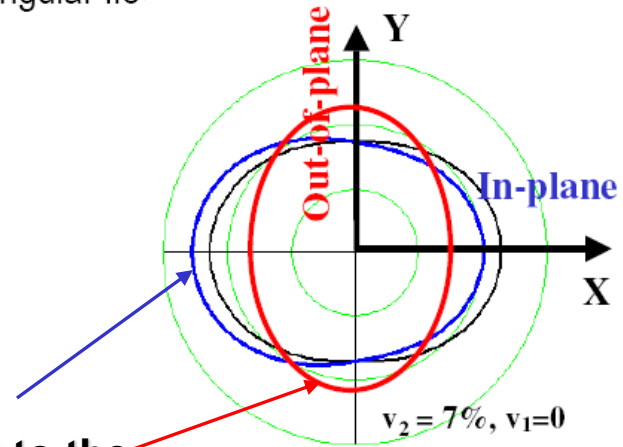
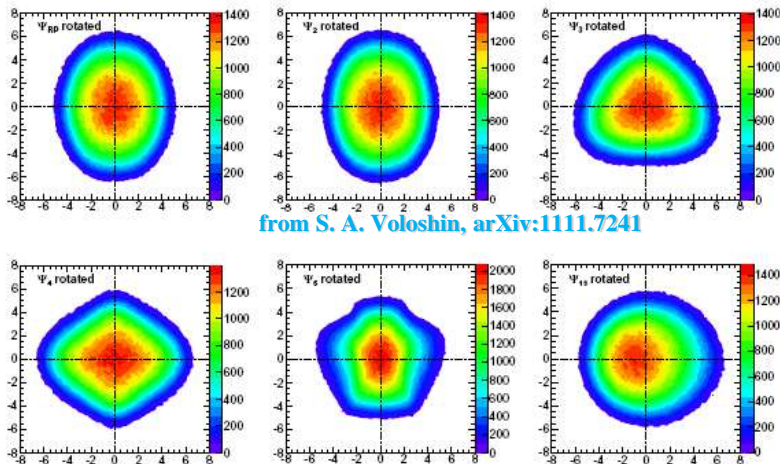
$$\frac{dN}{d\varphi} \propto \left( 1 + 2 \sum_{n=1}^{+\infty} v_n \cos[n(\varphi - \psi_n)] \right)$$

$$v_n = \left\langle \cos n(\varphi - \psi_n) \right\rangle, \quad n = 1, 2, 3, \dots$$

$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle, \quad v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



$v_1$ : directed flow  
 $v_2$ : elliptic flow  
 $v_3$ : triangular flow



$v_2 > 0$  indicates **in-plane** emission of particles

$v_2 < 0$  corresponds to a **squeeze-out** perpendicular to the reaction plane (**out-of-plane** emission)

$v_2 = 7\%, v_1 = -7\%$   
 $v_2 = -7\%, v_1 = 0$

# Hydrodynamic models: elliptic flow $v_2$

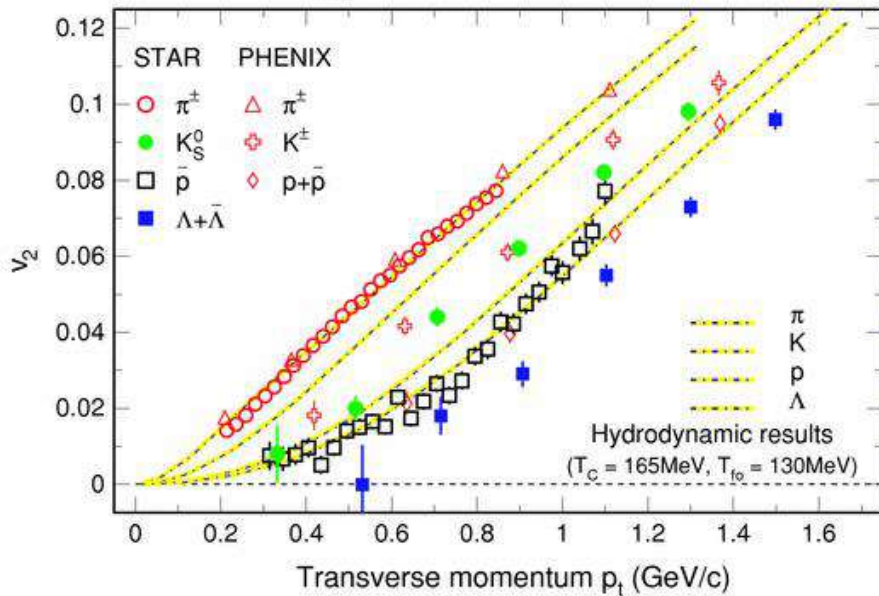
Comparison between hydro simulations and experimental data for the elliptic flow

Ideal hydrodynamic

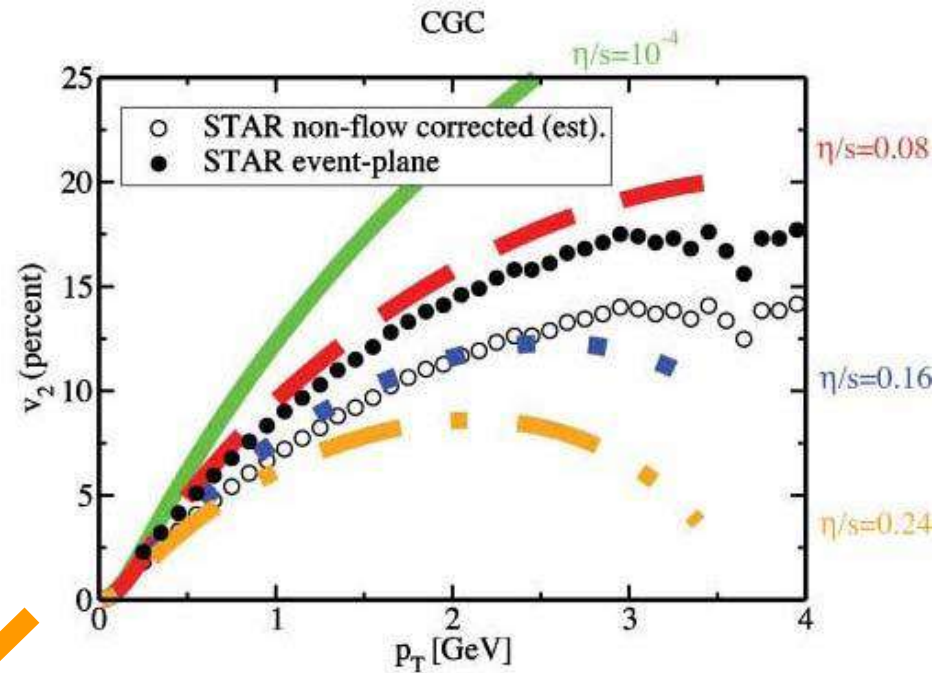


Viscous hydrodynamics

$\sqrt{s}_{NN} = 200\text{GeV } ^{197}\text{Au} + ^{197}\text{Au}$  at RHIC



Heinz:2001



Luzum,Romatschke:2008

**Ideal hydro:** reproduces exp. data at low  $p_T$ , overestimates  $v_2$  at  $p_T > 1.2$  GeV/c

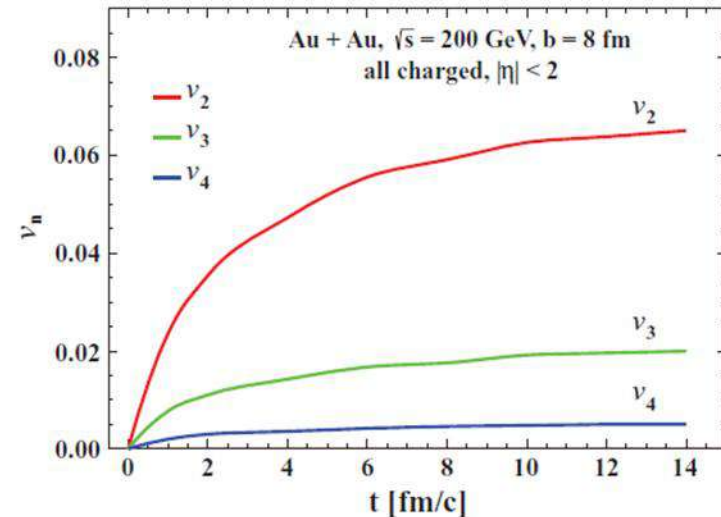
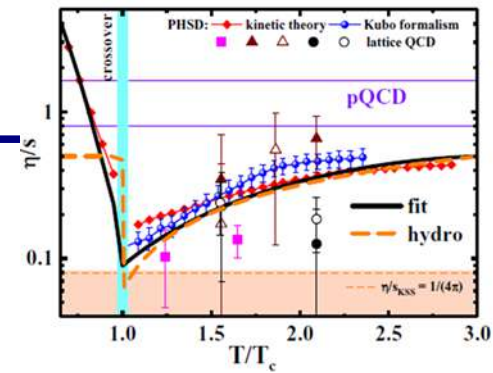
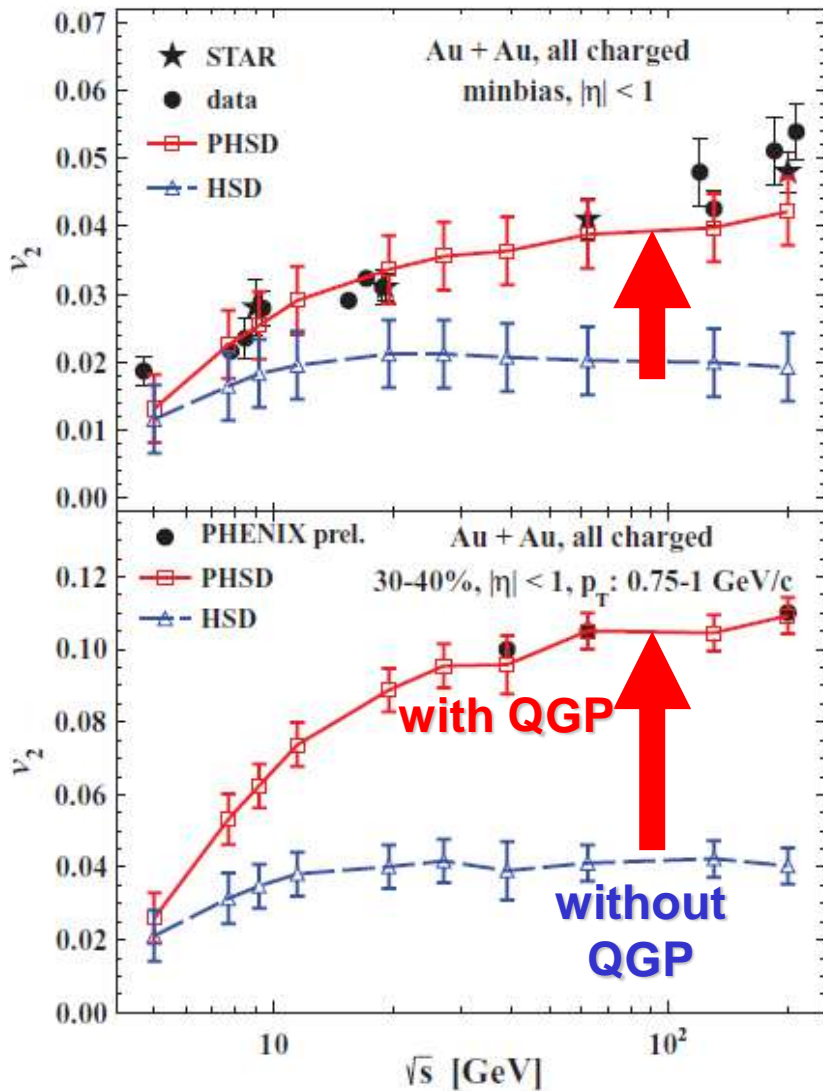
→ **Viscosity of QGP** has to be accounted for → **viscous hydro**

**Elliptic flow  $v_2$  is sensitive to  $\eta/s$**





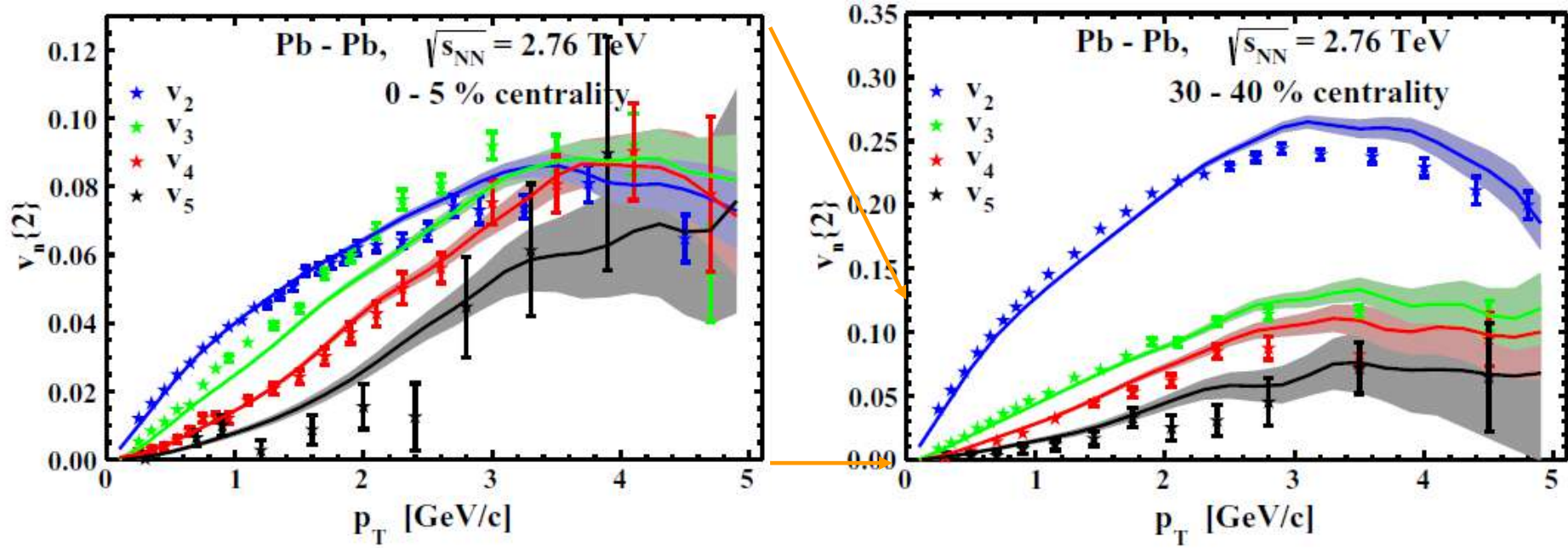
# Transport model PHSD: elliptic flow $v_2$



- $v_2$  in PHSD is larger than in HSD due to the repulsive scalar mean-field potential  $U_s(\rho)$  for partons
- $v_2$  grows with bombarding energy due to the increase of the parton fraction



# $V_n$ ( $n=2,3,4,5$ ) of charged particles from PHSD at LHC

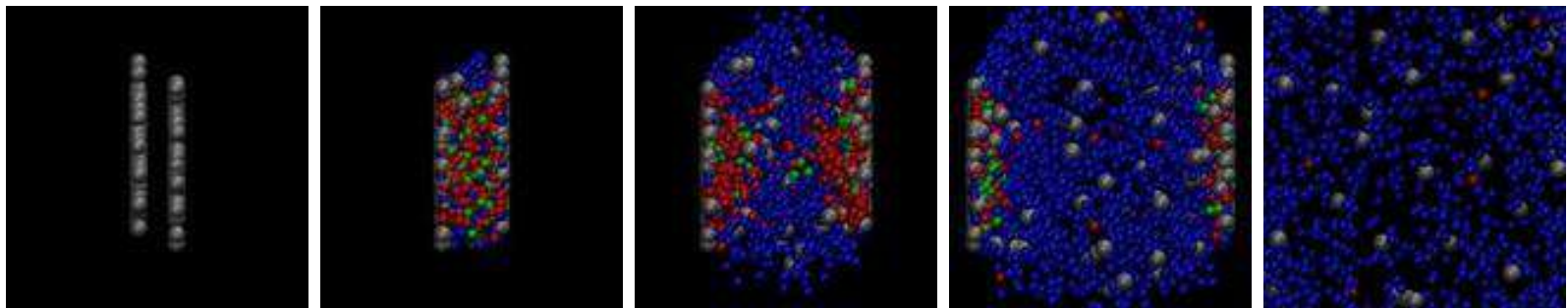


- PHSD: increase of  $v_n$  ( $n=2,3,4,5$ ) with  $p_T$
- $v_2$  increases with decreasing centrality
- $v_n$  ( $n=3,4,5$ ) show weak centrality dependence

symbols – ALICE  
PRL 107 (2011) 032301  
lines – PHSD (e-by-e)

$v_n$  ( $n=3,4,5$ ) develops by interactions in the QGP and in the final hadronic phase

# Traces of the QGP at finite $\mu_q$ in observables in high energy heavy-ion collisions



# Extraction of $(T, \mu_B)$ in PHSD

□ For each cell in PHSD :

In order to extract  $(T, \mu_B)$  use **IQCD** relations (up to 4<sup>th</sup> order) - **Taylor series** :

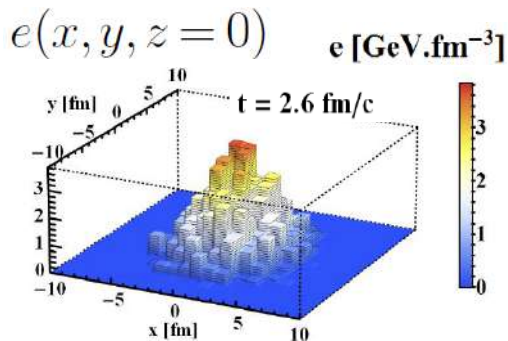
(1)  
**IQCD**

$$\left\{ \begin{array}{l} \frac{n_B}{T^3} \approx \chi_2^B(T) \left( \frac{\mu_B}{T} \right) + \dots \\ \Delta\epsilon/T^4 \approx \frac{1}{2} \left( T \frac{\partial \chi_2^B(T)}{\partial T} + 3\chi_2^B(T) \right) \left( \frac{\mu_B}{T} \right)^2 + \dots \end{array} \right.$$

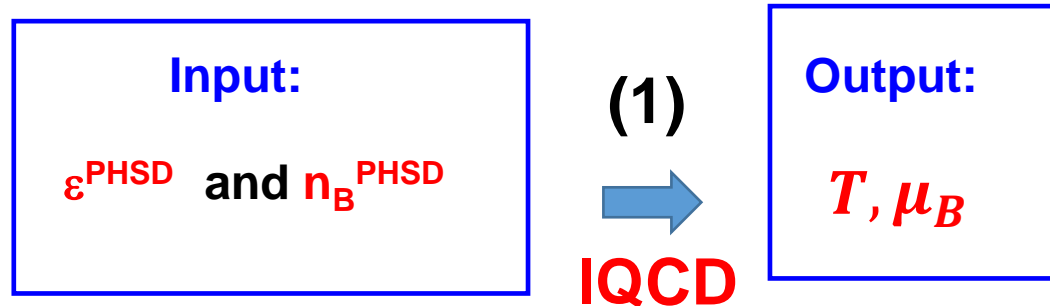
\* Use baryon number susceptibilities  $\chi_n$  from IQCD

➔ obtain  $(T, \mu_B)$  by solving the system of coupled equations using  $\epsilon^{\text{PHSD}}$  and  $n_B^{\text{PHSD}}$

\* Done by the Newton-Raphson method



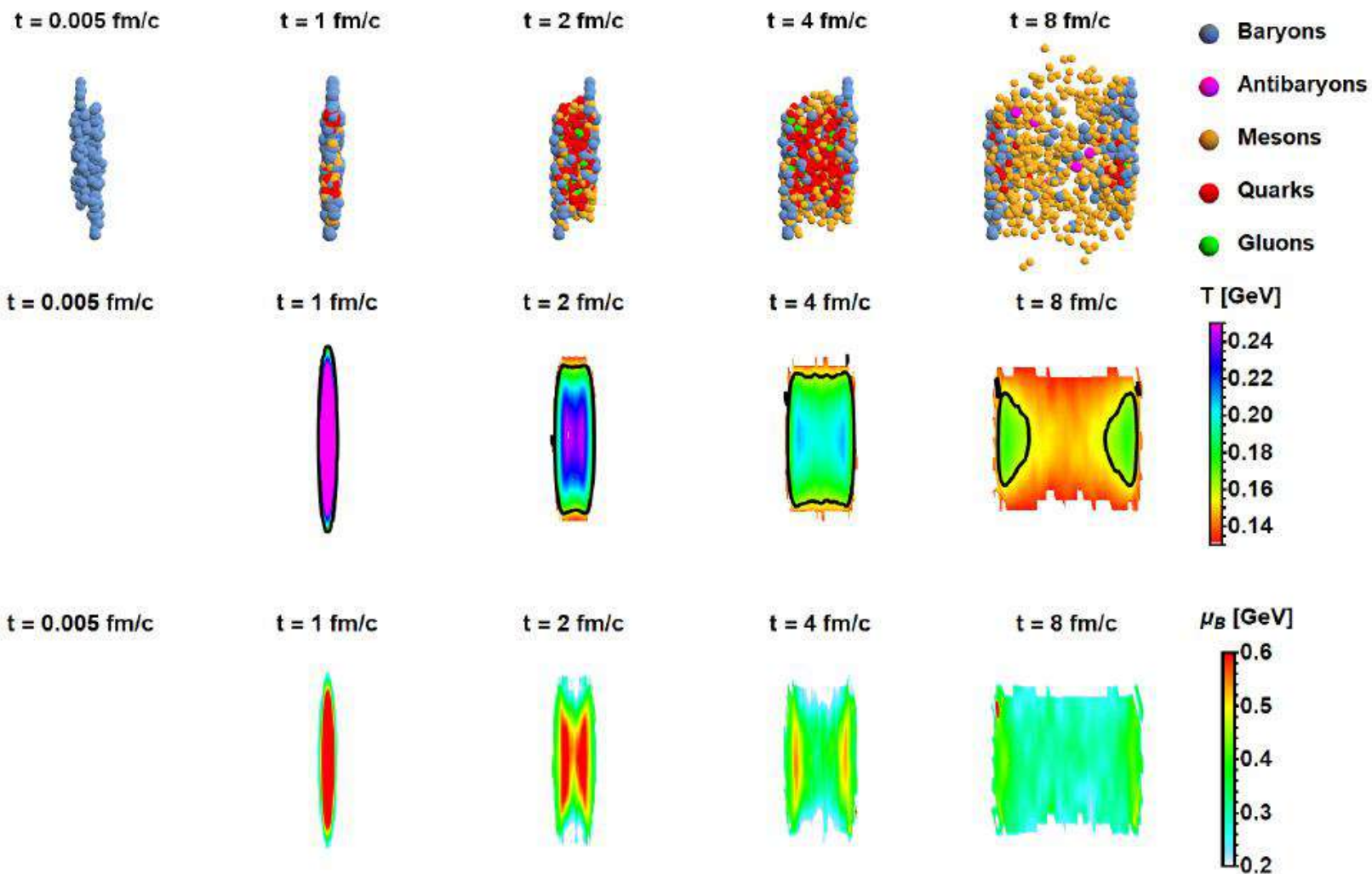
Au+Au, 200 GeV, b=6 fm



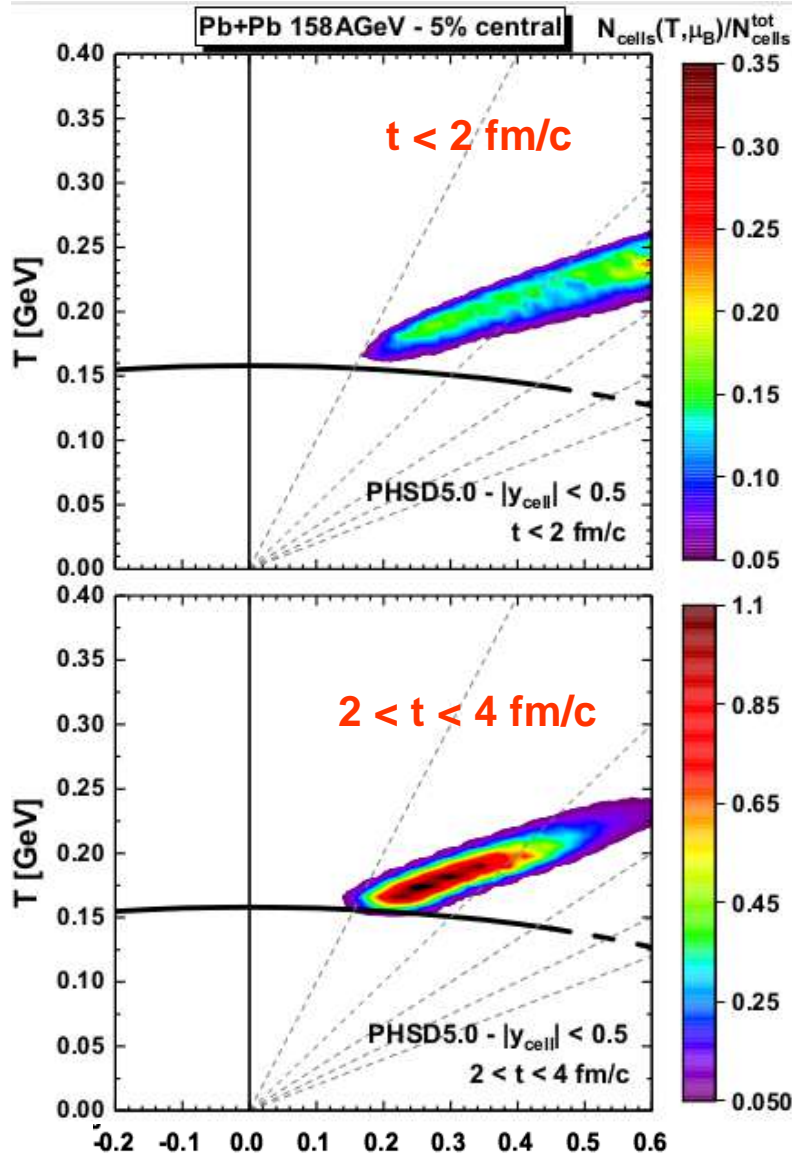


# Illustration for a HIC ( $\sqrt{s_{NN}} = 19.6 \text{ GeV}$ )

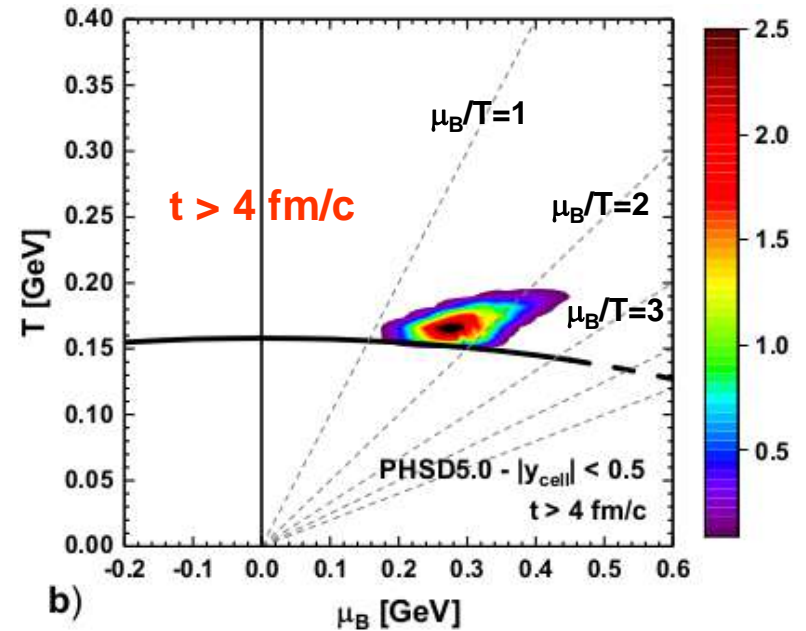
Au + Au  $\sqrt{s_{NN}} = 19.6 \text{ GeV}$  –  $b = 2 \text{ fm}$  – Section view



# Illustration for a HIC ( $\sqrt{s_{NN}} = 17$ GeV)



Distribution of cells ( $|y_{\text{cell}}| < 0.5$ ) with  $T > T_C$

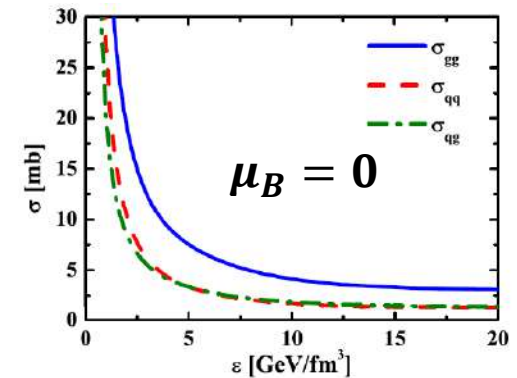




➤ Comparison between three different results:

1) PHSD 4.0 : only  $\sigma(T)$  and  $\rho(T)$

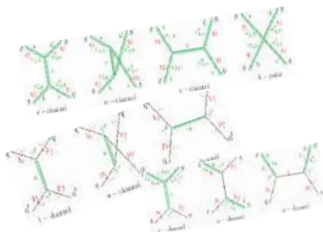
$\sigma(T)$  – parton interaction cross sections  
 $\rho(T)$  – spectral function of partons  
 → (masses and widths)



2) PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \underline{\mu_B = 0})$  and  $\rho(T, \mu_B = 0)$

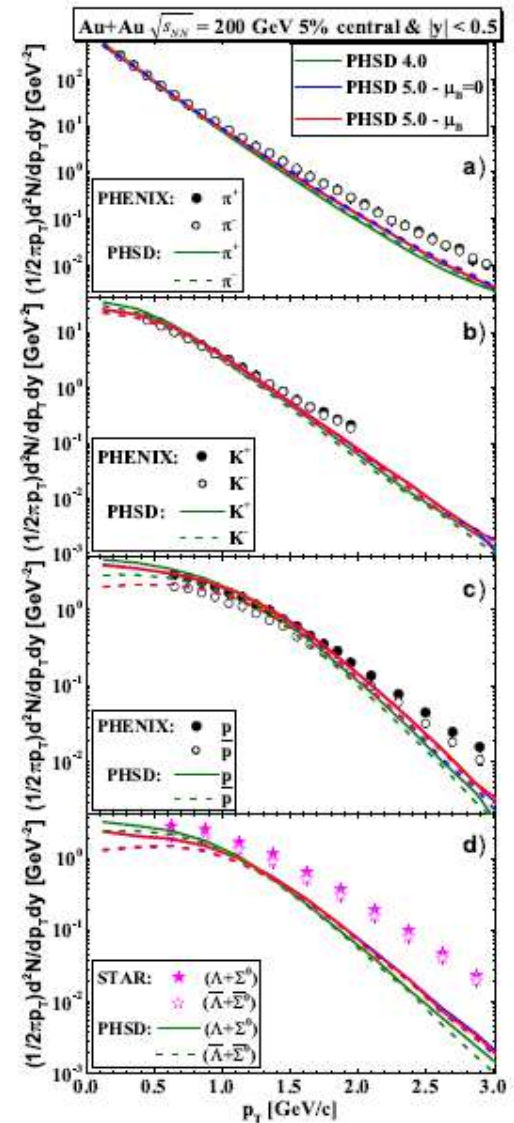
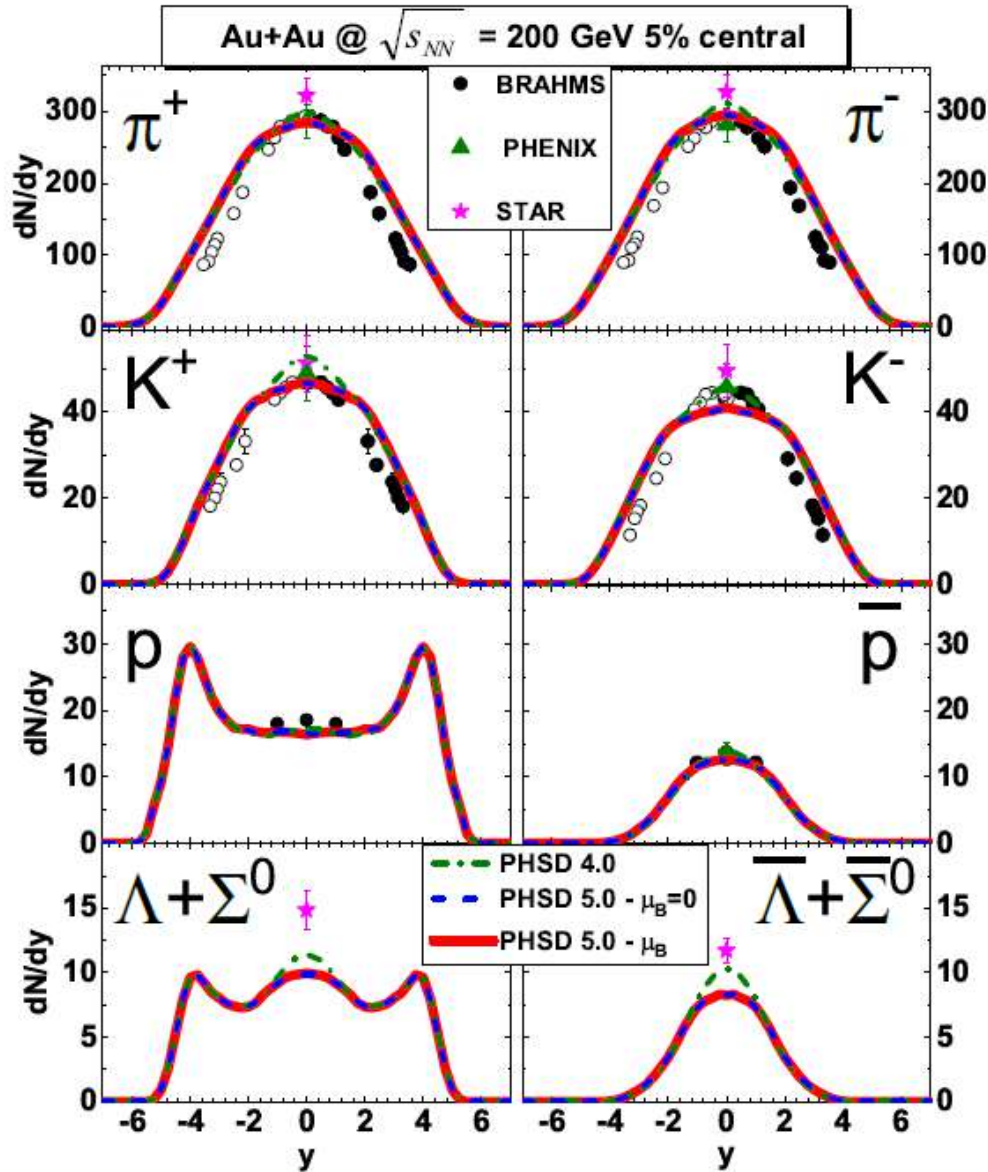
In v.5.0: + angular dependence of diff. partonic cross sections

3) PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B)$  and  $\rho(T, \mu_B)$





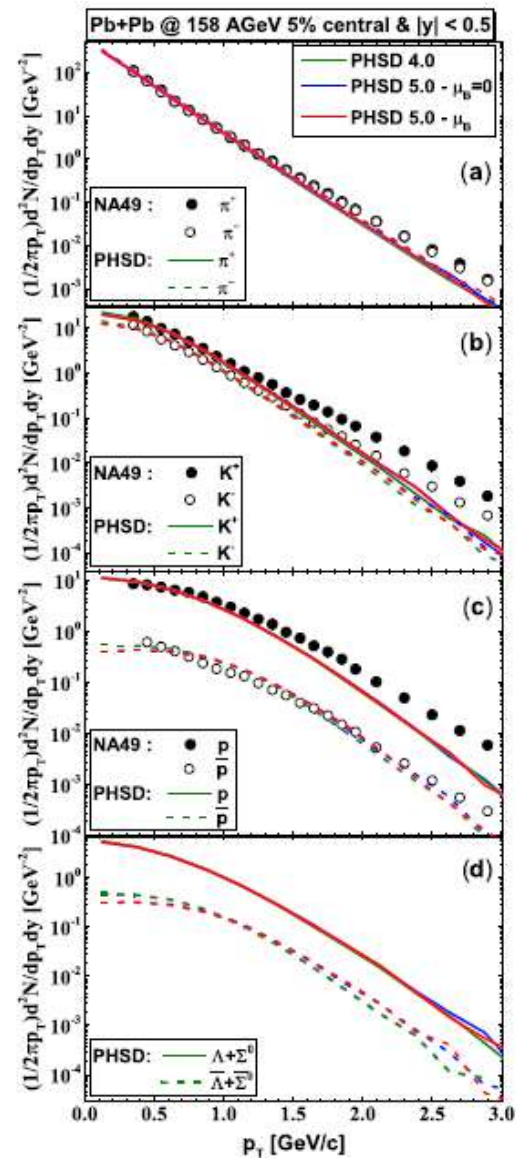
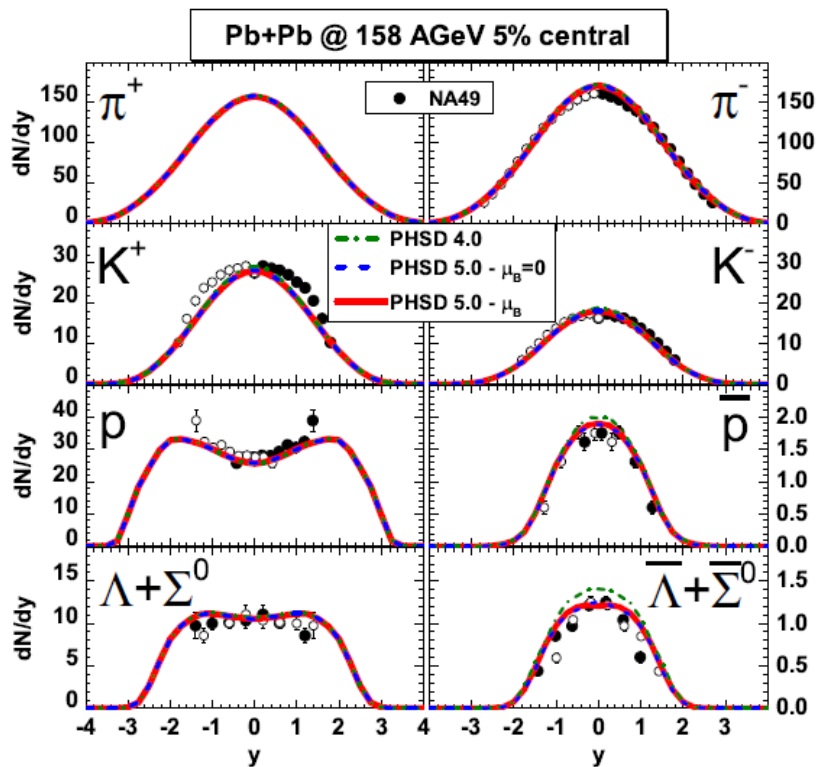
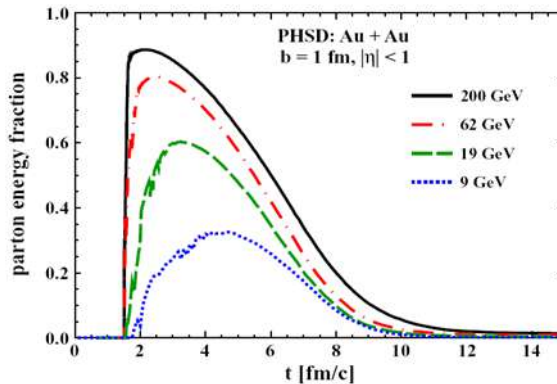
# Results for HICs ( $\sqrt{s_{NN}} = 200$ GeV)





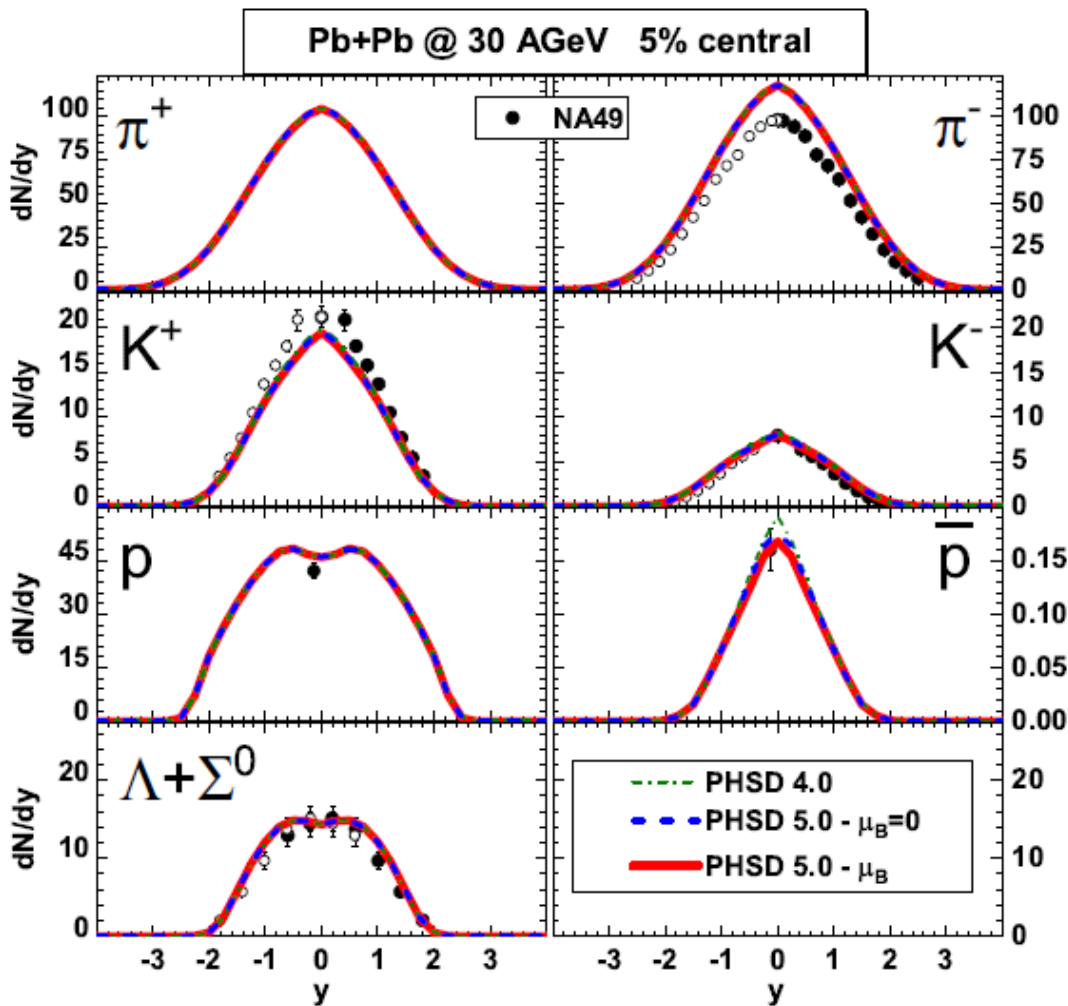
# Results for HICs ( $\sqrt{s_{NN}} = 17$ GeV)

High- $\mu_B$  regions are probed at **low  $\sqrt{s_{NN}}$  or high rapidity regions**  
 But, **QGP fraction is small at low  $\sqrt{s_{NN}}$**

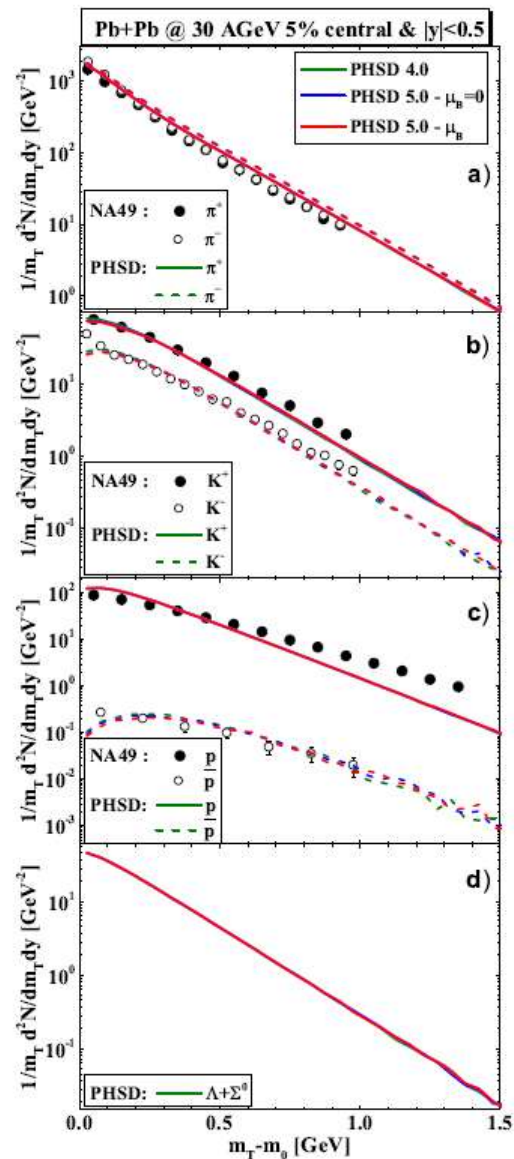




# Results for HICs ( $\sqrt{s_{NN}} = 7.6$ GeV)



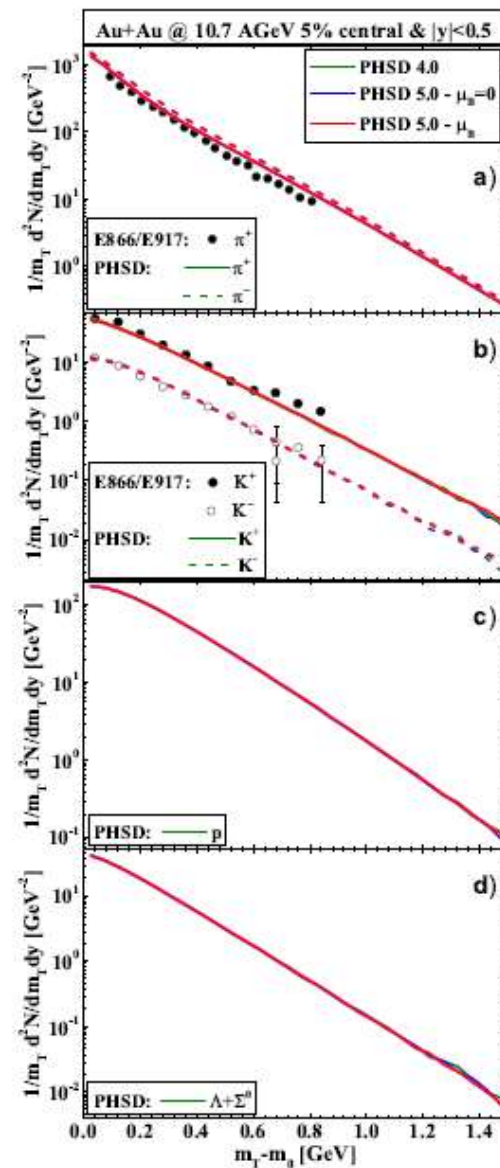
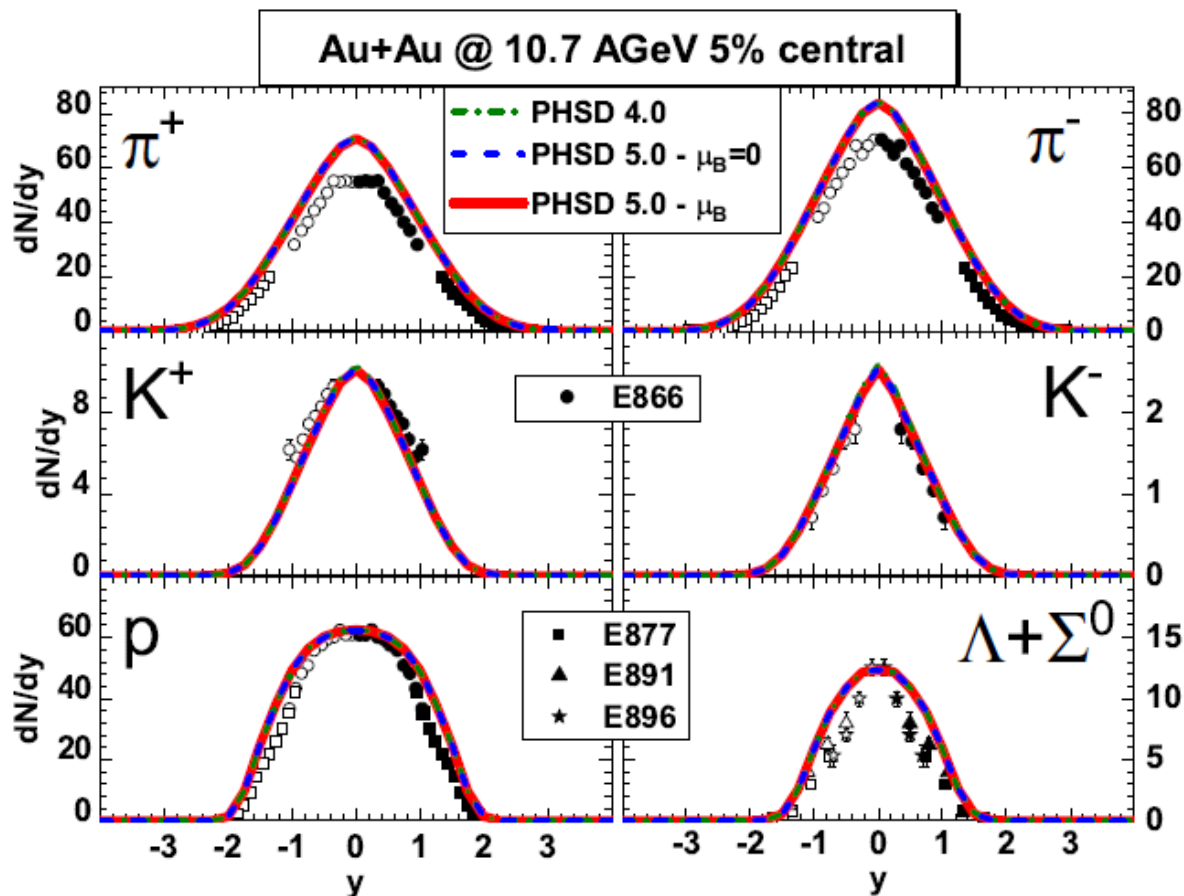
➤ very weak dependence of 'bulk' observables on  $\mu_B$





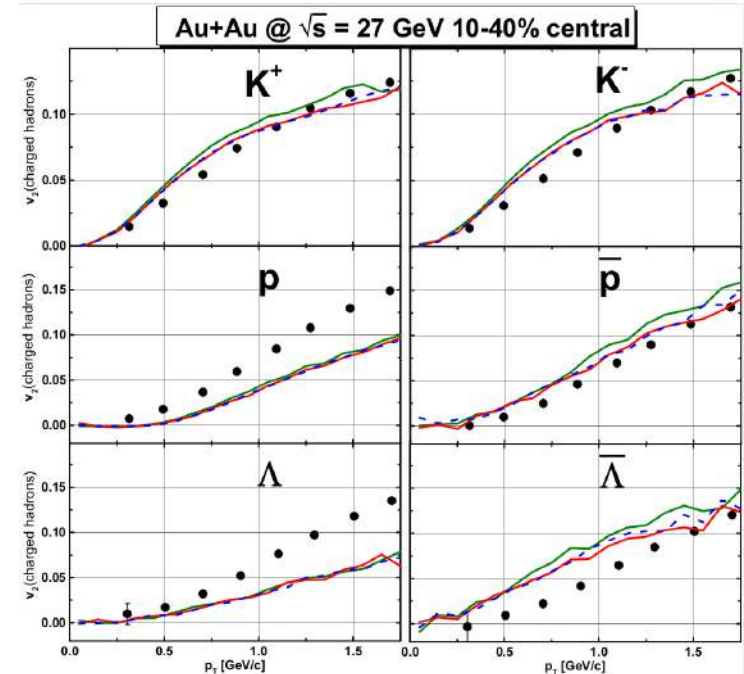
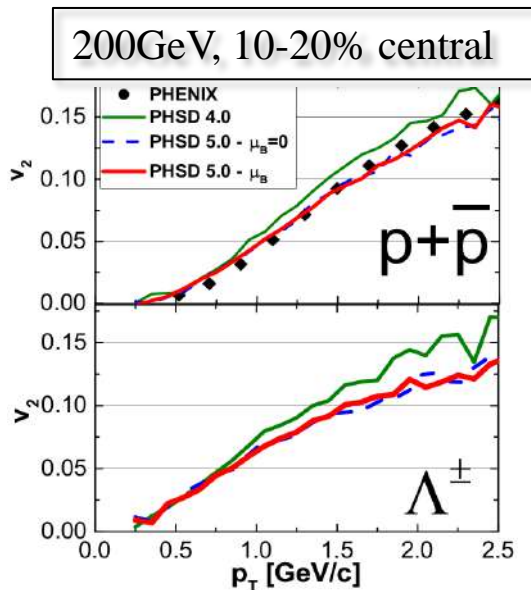
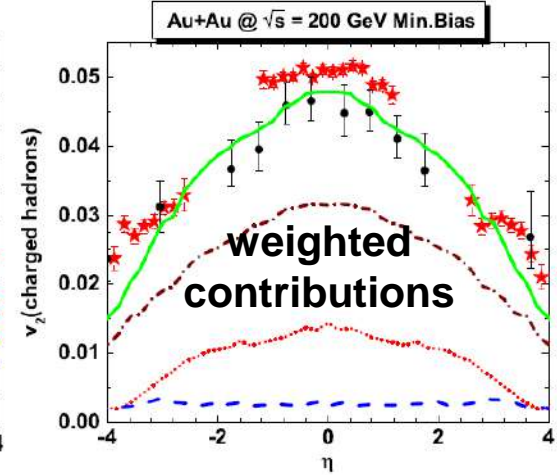
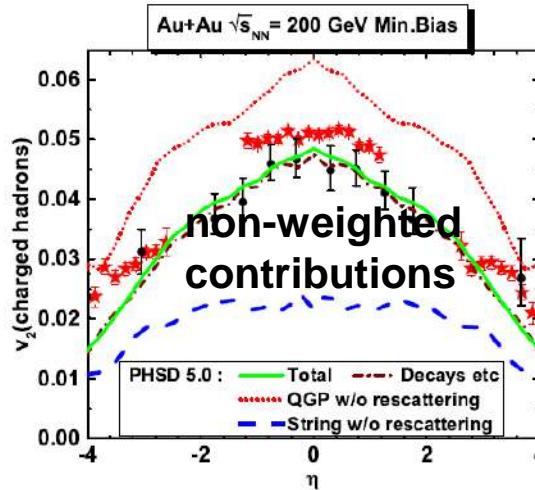
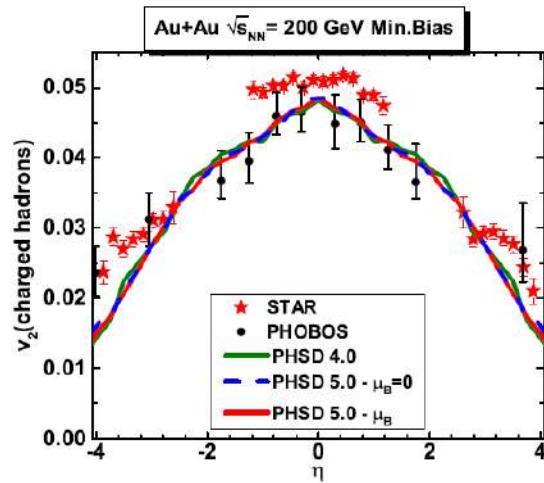


# Results for HICs ( $\sqrt{s_{NN}} = 4.86$ GeV)



➤ very **weak dependence** of 'bulk' observables on  $\mu_B$

# Elliptic flow $v_2$ ( $\sqrt{s_{NN}} = 200 \text{ GeV vs } 27 \text{ GeV}$ )



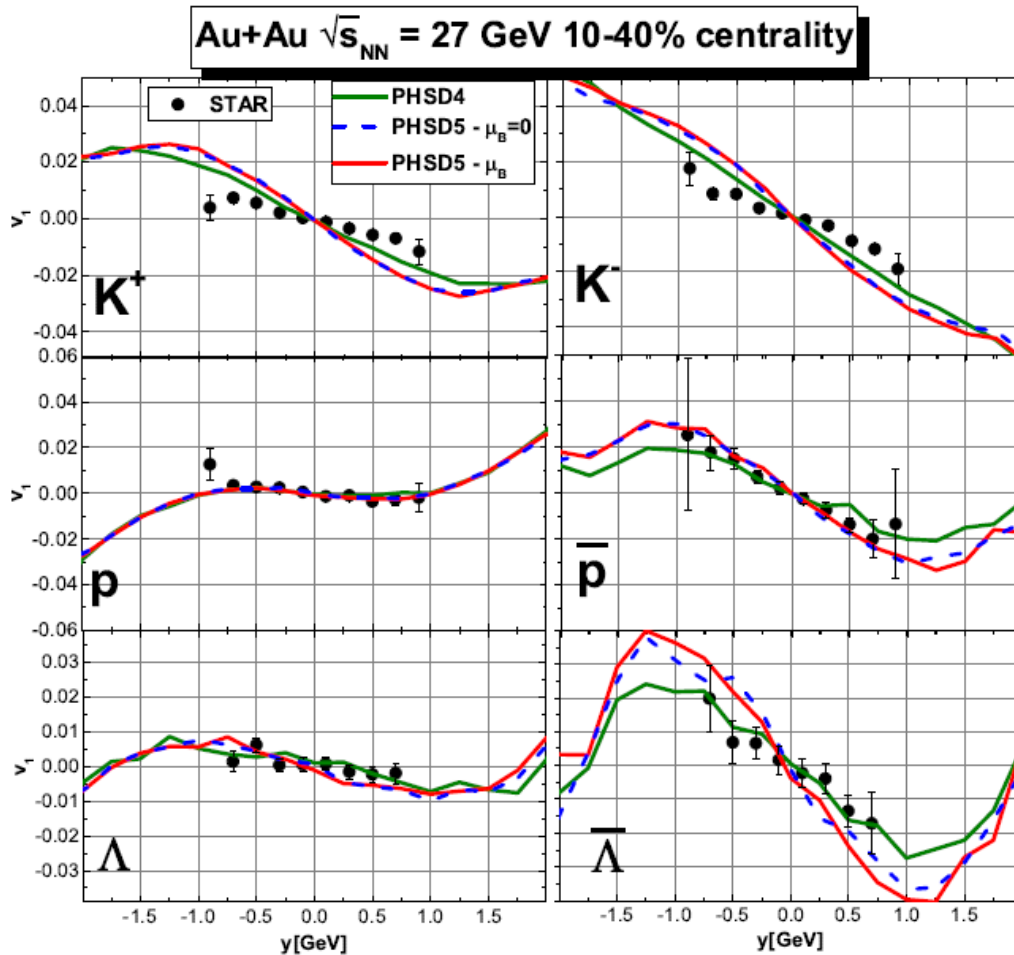
➔ Small effect of  $\mu_B$  dependence on  $v_2$





# Results for $v_1$ for HICs ( $\sqrt{s_{NN}} = 27$ GeV)

$v_1$



Messages from  $v_1, v_2$  analysis:

- weak dependence of  $v_1, v_2$  on  $\mu_B$
- small influence on  $v_1, v_2$  of explicit  $\sqrt{s}$ -dependence of total partonic cross sections  $\sigma$  + angular dependence of  $d\sigma/d\cos\theta$  due to the relatively small QGP volume
- strong flavor dependence of  $v_1, v_2$

O. Soloveva et al., arXiv:2001.07951, MDPI Particles 2020, 3, 178



# Messages from studies of QGP at $T, \mu_B$

- ❑  $(T, \mu_B)$ -dependent **partonic** cross sections and masses/widths of quarks and gluons have been implemented in PHSD
- ❑ **High- $\mu_B$**  region is probed at **low bombarding energies** or high rapidity regions
- ❑ But, **QGP fraction is small at low bombarding energies**:
  - ➔ no effects of  $(T, \mu_B)$ -dependent partonic cross sections and masses/widths seen in '**bulk**' **observables** –  $dN/dy$ ,  $p_T$ -spectra
- ❑ Flow harmonics  $v_1, v_2$  show :  
visible sensitivity to the explicit  **$\sqrt{s}$ -dependence** of total partonic cross sections  $\sigma$   
+ **angular dependence** of  $d\sigma/d\cos\theta$ , however, **weak** dependence on  $\mu_B$
- ❑ **Outlook**:
  - More precise EoS at large  $\mu_B$
  - Possible 1<sup>st</sup> order phase transition at even larger  $\mu_B$ ?!

**High- $\mu_B$**  region of QCD phase diagram ➔ **challenge for FAIR, NICA, BES RHIC**

# Thanks to:

# PHSD group - 2021



DAAD



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**Marlene Nahgang, Klaus Werner**

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**Barcelona University:**  
**Laura Tolos**

**Duke University:**  
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**Catania University:**  
**Vincenzo Greco**



**PHSD home page:**  
<http://theory.gsi.de/~ebratkov/phsd-project/PHSD/index1.html>

