Topological metals

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CATS
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Where do gapless excitations come from?

Where do gapless excitations come from?

- Liquid of interacting bosons, e.g. liquid helium.

$$\epsilon(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu$$

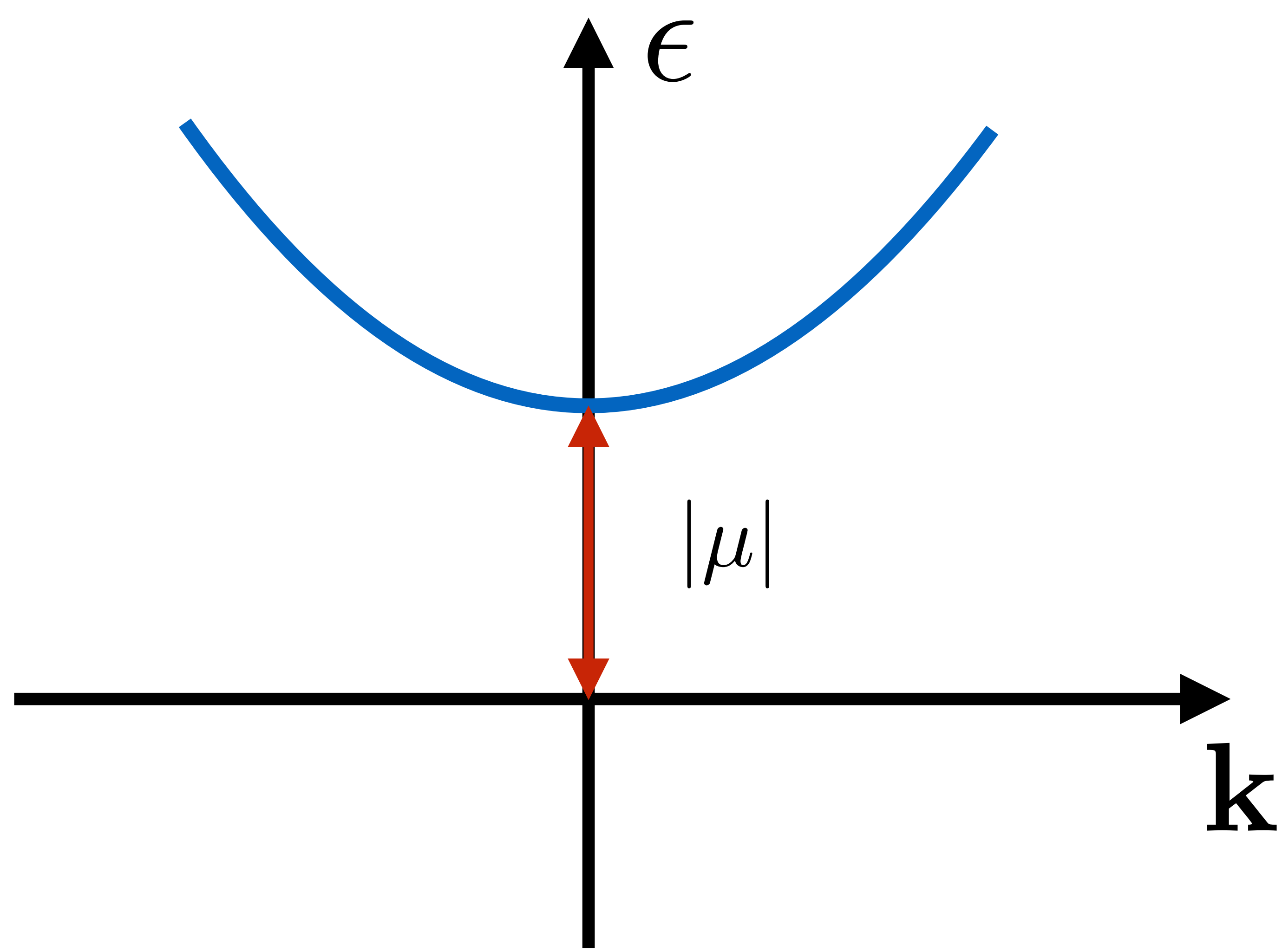
Where do gapless excitations come from?

- Chemical potential is large and negative at high T, thus all bosons have high energy relative to it, no low energy excitations.

$$\epsilon(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu$$

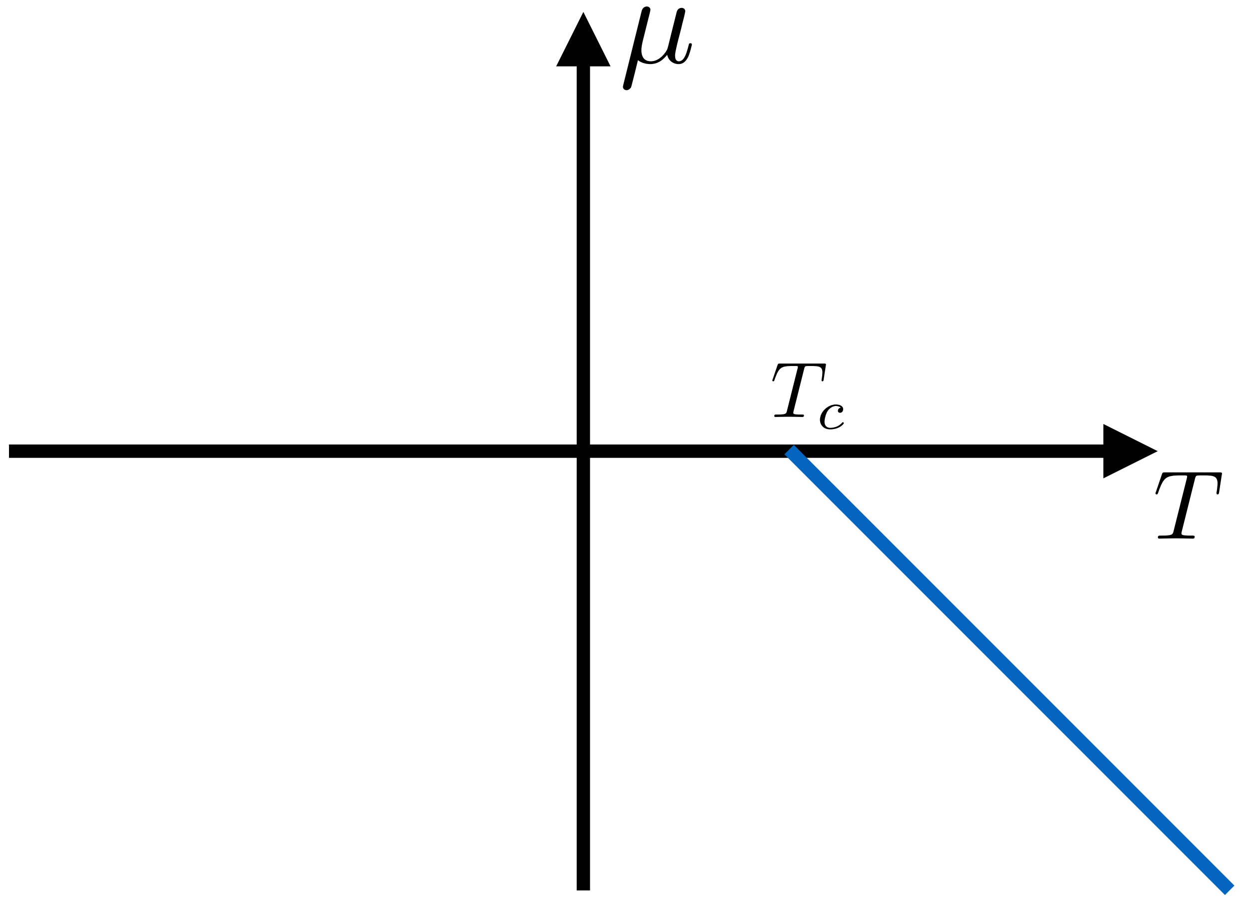
$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$

$$F = E - TS$$



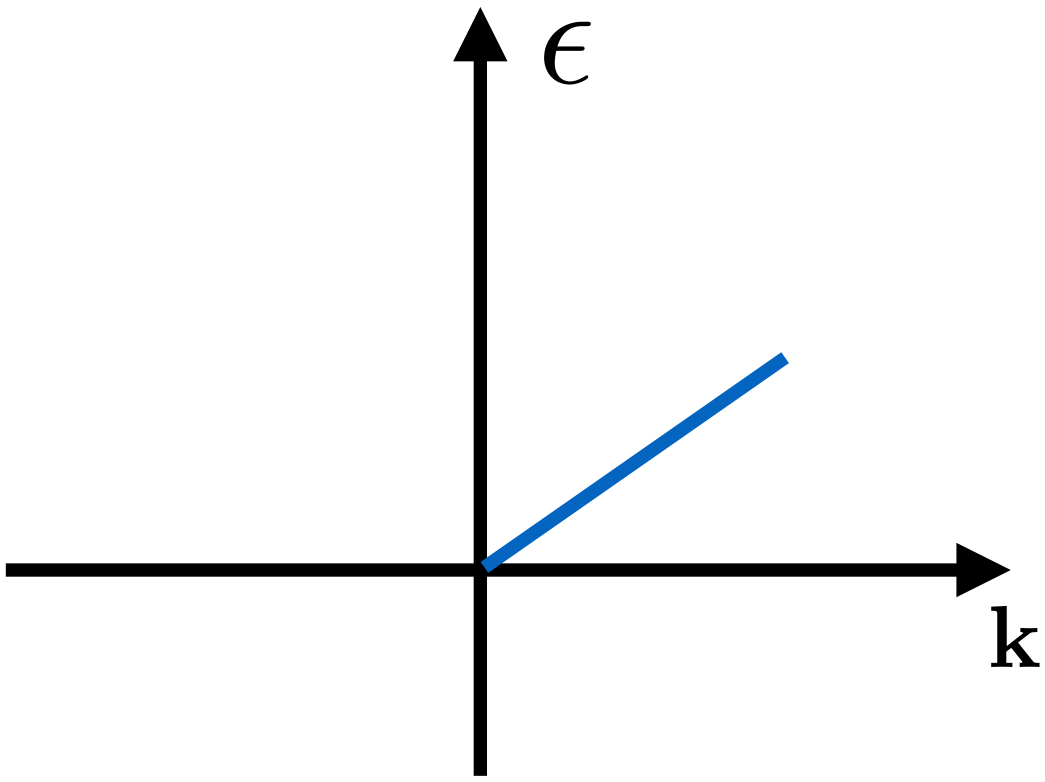
Where do gapless excitations come from?

- Transition to superfluid at low T.



Where do gapless excitations come from?

- Excitations are phonons with a gapless linear dispersion.

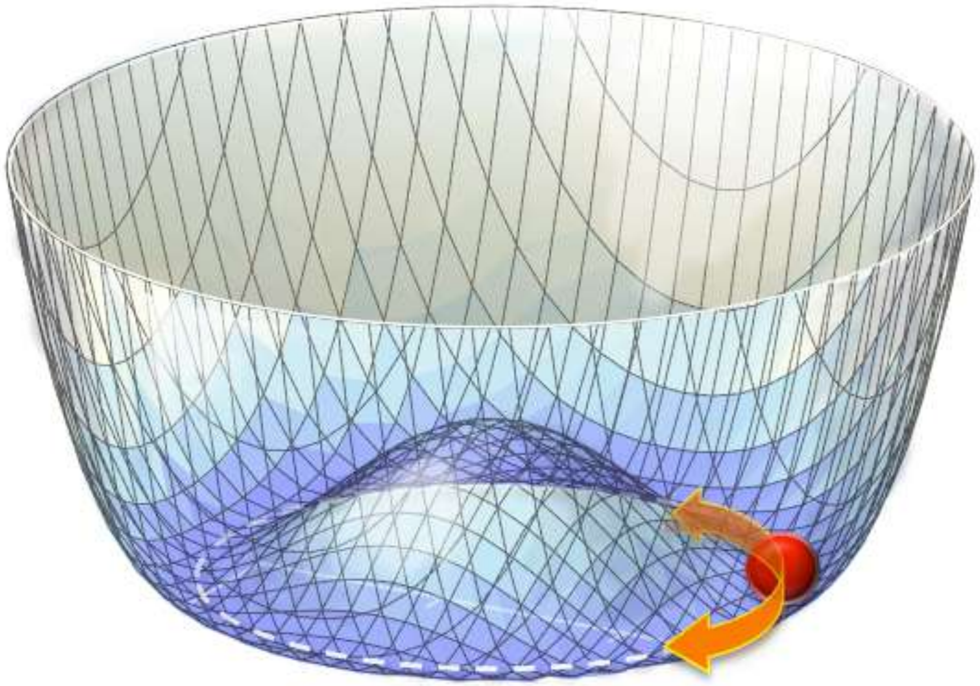


$$\epsilon(\mathbf{k}) = vk$$

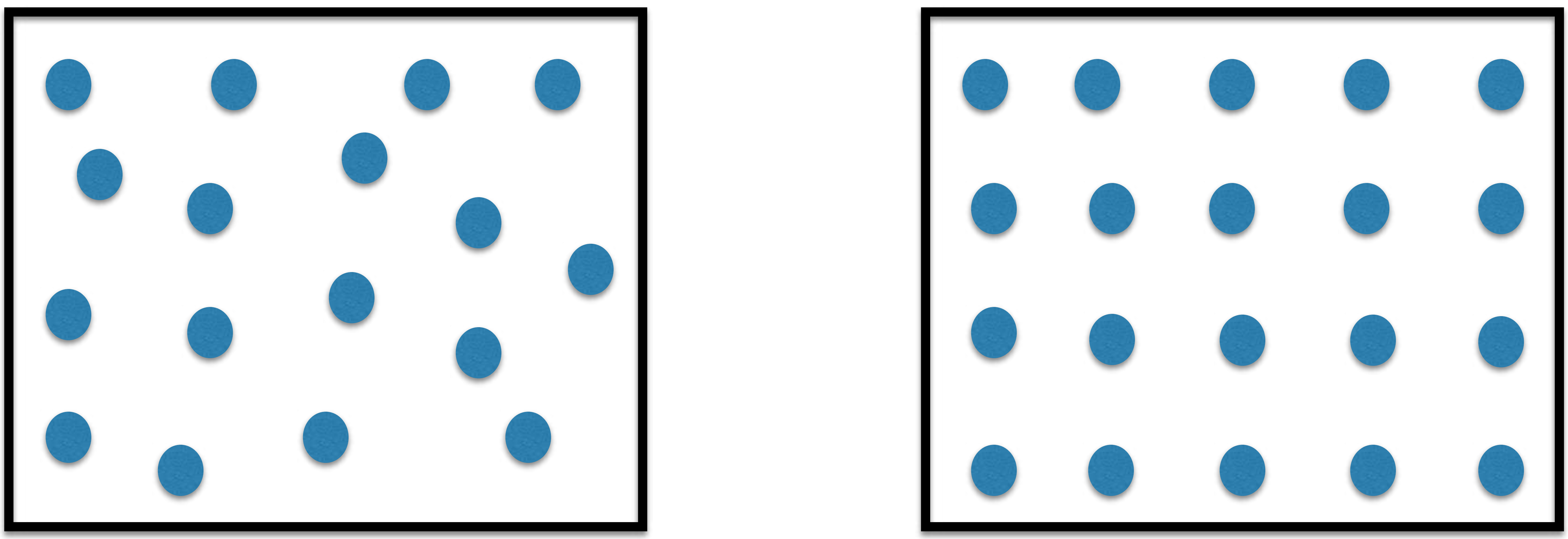
Where do gapless excitations come from?

- The origin of phonons is spontaneous symmetry breaking.

$$\langle \phi(\mathbf{r},t) \rangle = \langle \sqrt{\rho} e^{i\theta(\mathbf{r},t)} \rangle \neq 0$$



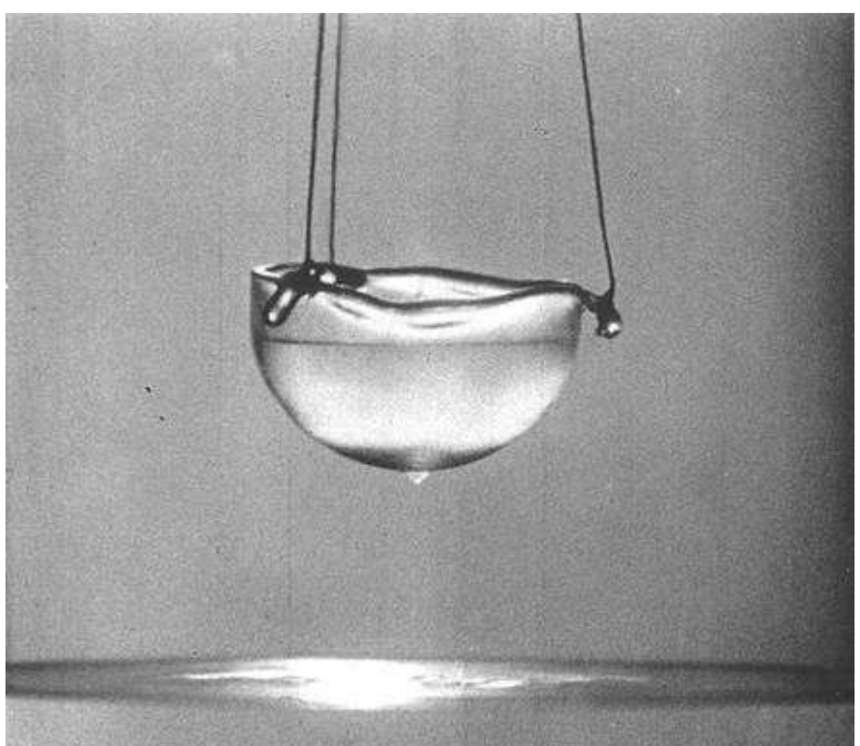
Gapless excitations in crystals



- Crystal breaks translational symmetry of space, has gapless acoustic phonons.

Where do gapless excitations come from?

- Lesson 1: gapless excitations do not arise accidentally, there must be a reason.
- Lesson 2: their existence is associated with interesting physics, such as superfluidity, crystallinity, etc.

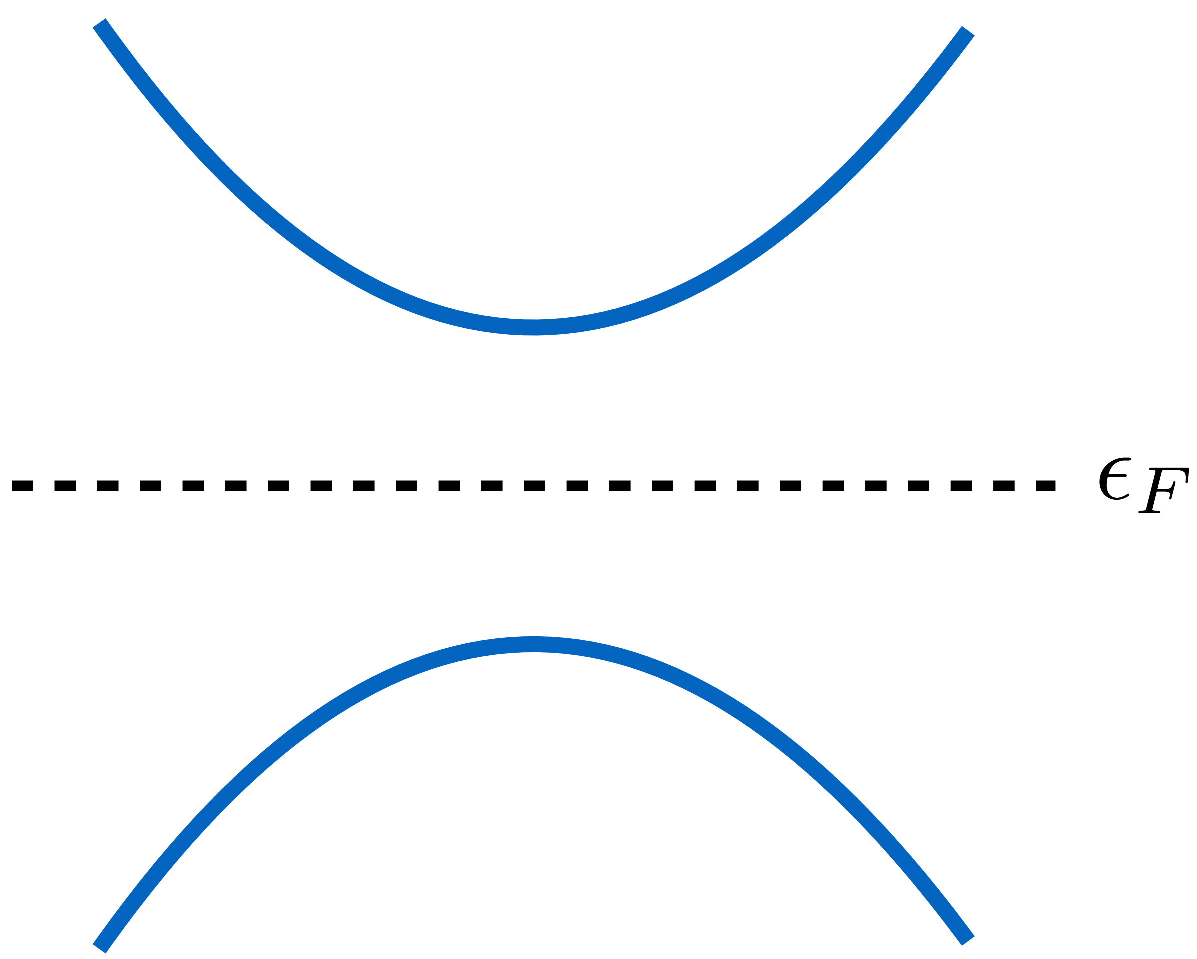


Gapless fermions

- Energy spectrum of electrons in solids has form of bands separated by bandgaps.
- Electrons obey Pauli principle.
- Number of states in a band is always 2 times the number of unit cells.

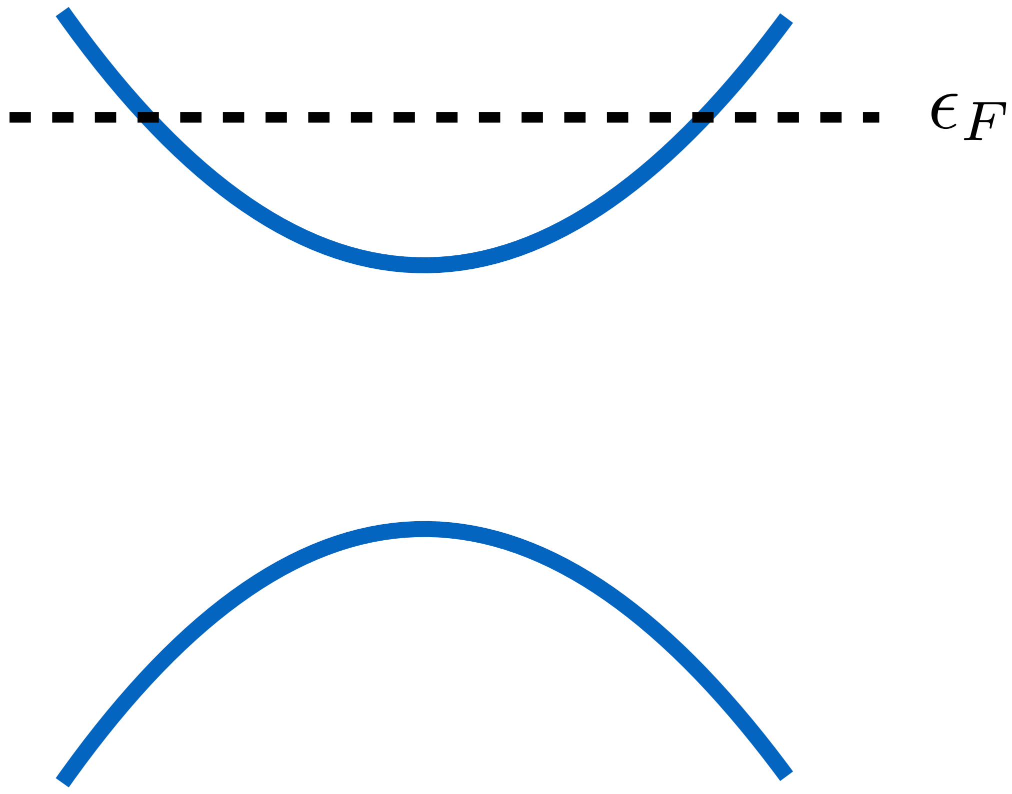
Gapless fermions

- Even integer number of electrons per unit cell: insulator, no gapless excitations.



Gapless fermions

- Fractional (not even integer) number of electrons per unit cell: metal, Fermi surface of gapless excitations.



$$\frac{2V_F}{(2\pi)^d} = \frac{\nu - 2n}{v_c}$$

Luttinger

Metals

- Metals are fun and useful.

Insulators

- Insulators are boring.

Where do gapless excitations come from?

- Recent understanding: gapless excitations may also appear for topological reasons.

- Insulators are not as boring as one might have thought: there are two distinct classes of insulators, not adiabatically connected to each other.

The Nobel Prize in Physics 2016

© Trinity Hall, Cambridge University. Photo: Kiloran Howard
David J. Thouless
Prize share: 1/2

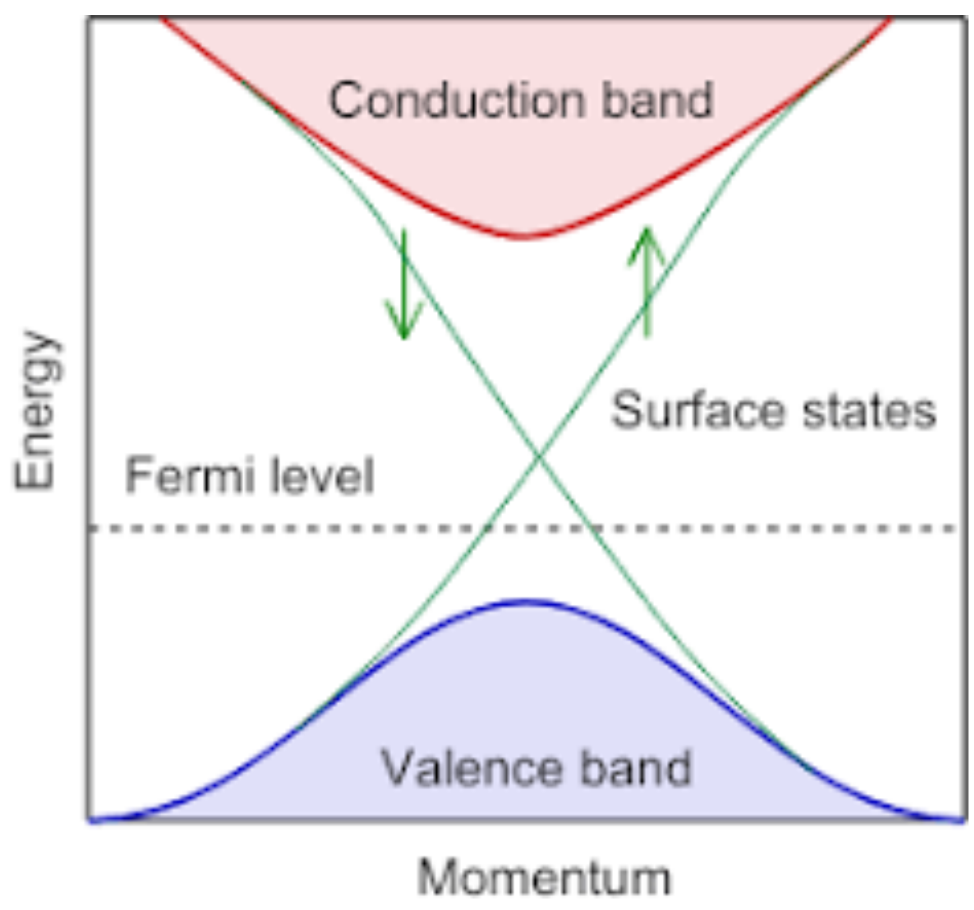
Photo: Princeton University, Comms. Office, D. Applewhite
F. Duncan M. Haldane
Prize share: 1/4

Ill: N. Elmehed. © Nobel Media 2016
J. Michael Kosterlitz
Prize share: 1/4

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz *"for theoretical discoveries of topological phase transitions and topological phases of matter"*.

Topological insulators

- Transition between a topological and an ordinary insulator is only possible through closing the gap, which leads to gapless surface states on topological insulators.



Bulk topological metals

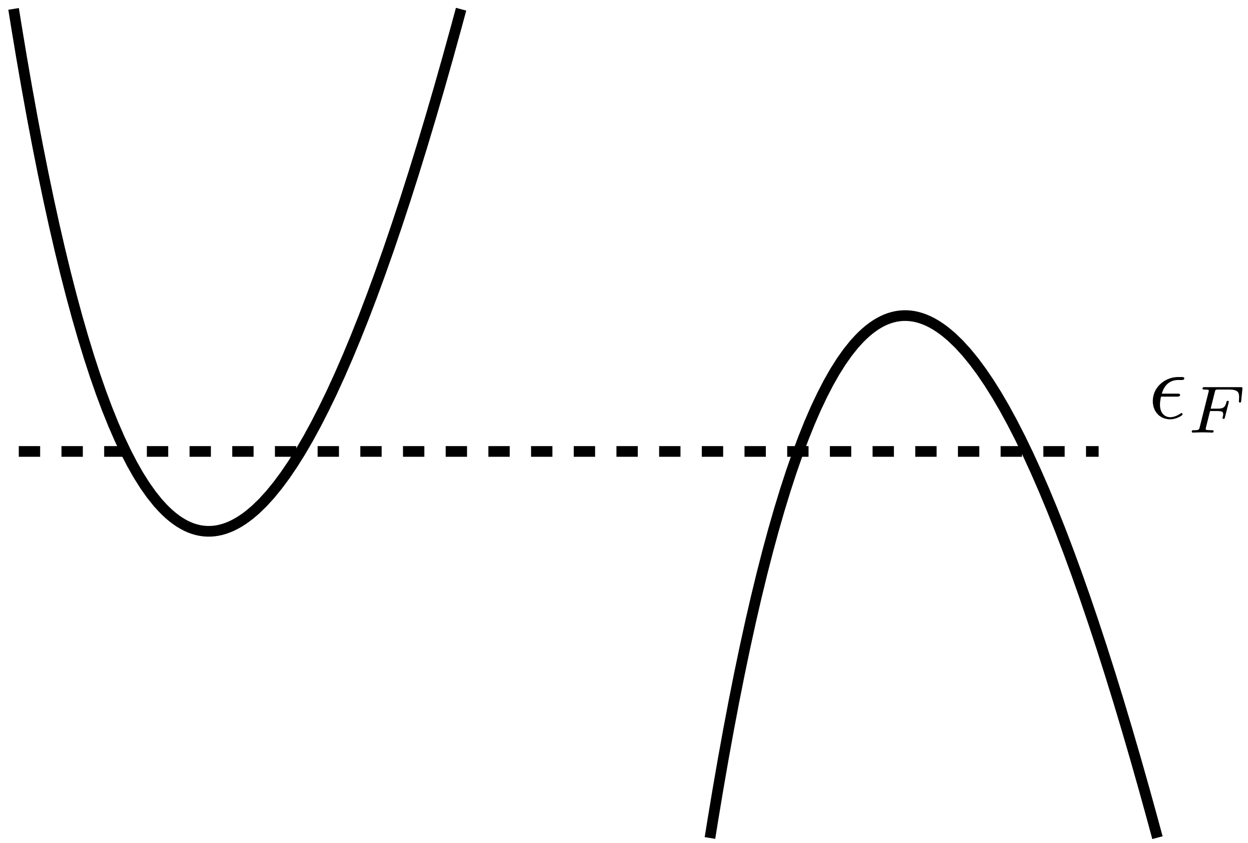
- Can bulk 3D metals be topologically nontrivial in the same sense?

Bulk topological metals

- Can bulk 3D metals be topologically nontrivial in the same sense?

- Can we have a bulk 3D topologically-protected metal when the material should be an insulator by band filling?

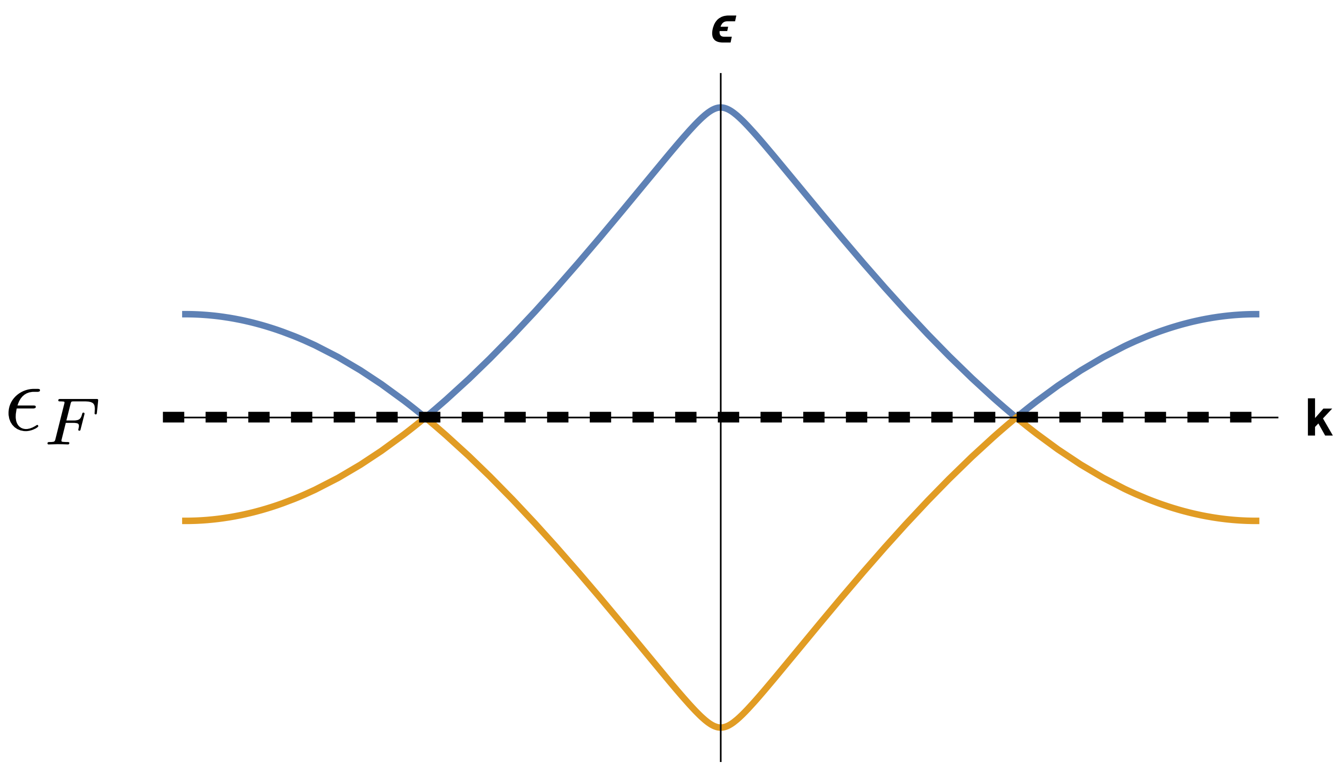
Accidental semimetal



- Bands can overlap: materials with even number of electrons per unit cell often fail to be insulators.

Weyl semimetal

- Weyl semimetal: gapless topological phase which arises in 3D materials lacking time-reversal or inversion symmetries.



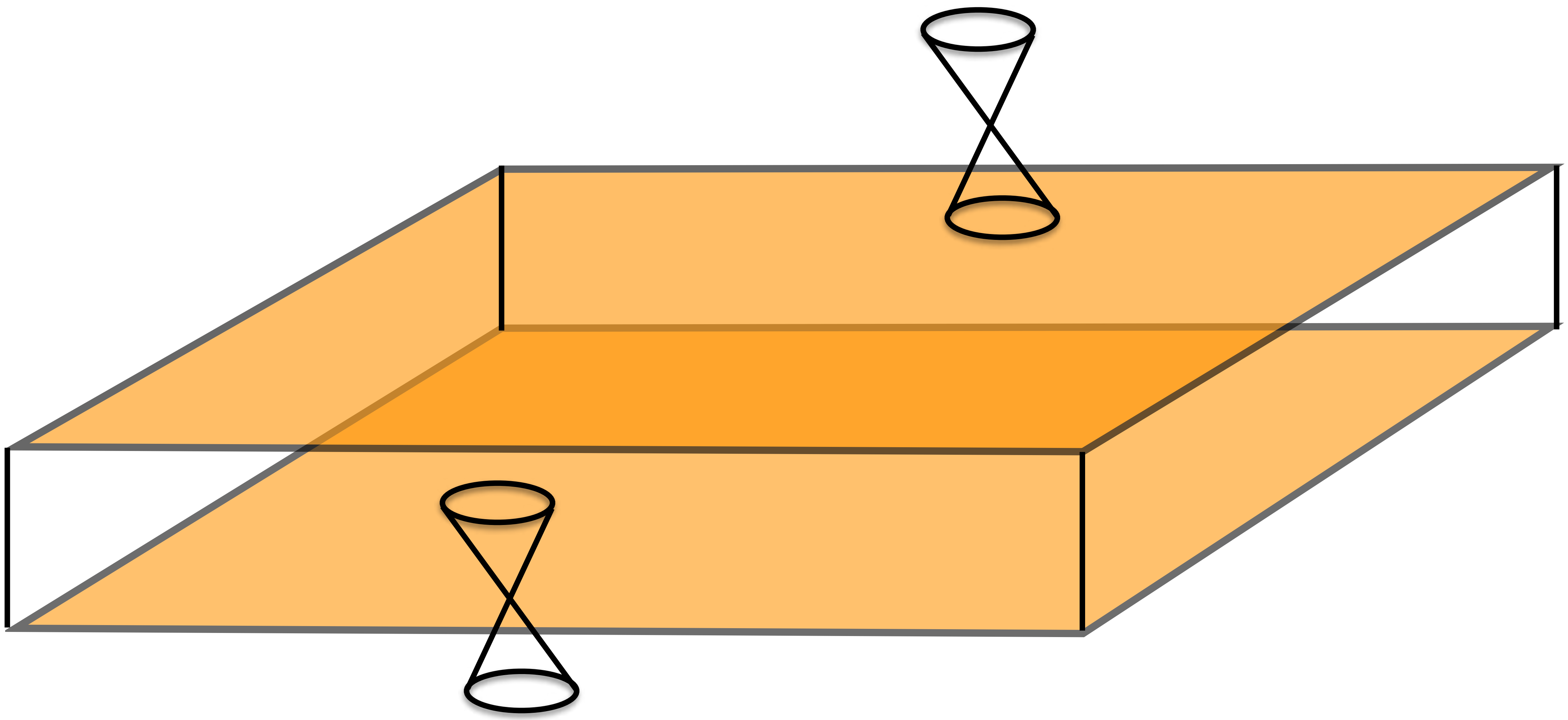
Murakami, 2007

Wan et al., 2011

AAB & Balents, 2011

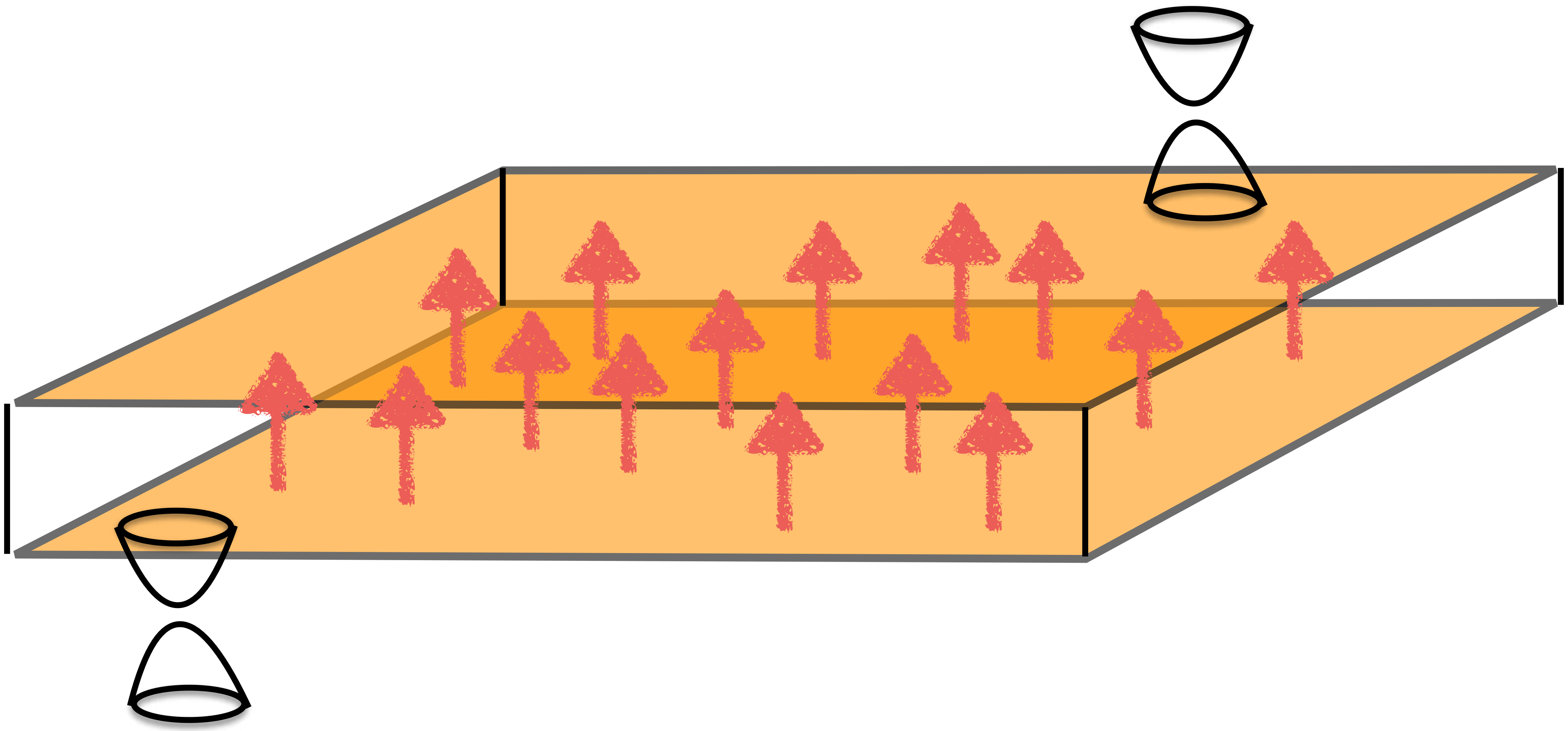
- Exists unaviodably as an intermediate phase between a topological and ordinary insulator in 3D.

Quantum Anomalous Hall Insulator



Thin film of 3D TI: gapless Dirac surface states protected by TRS

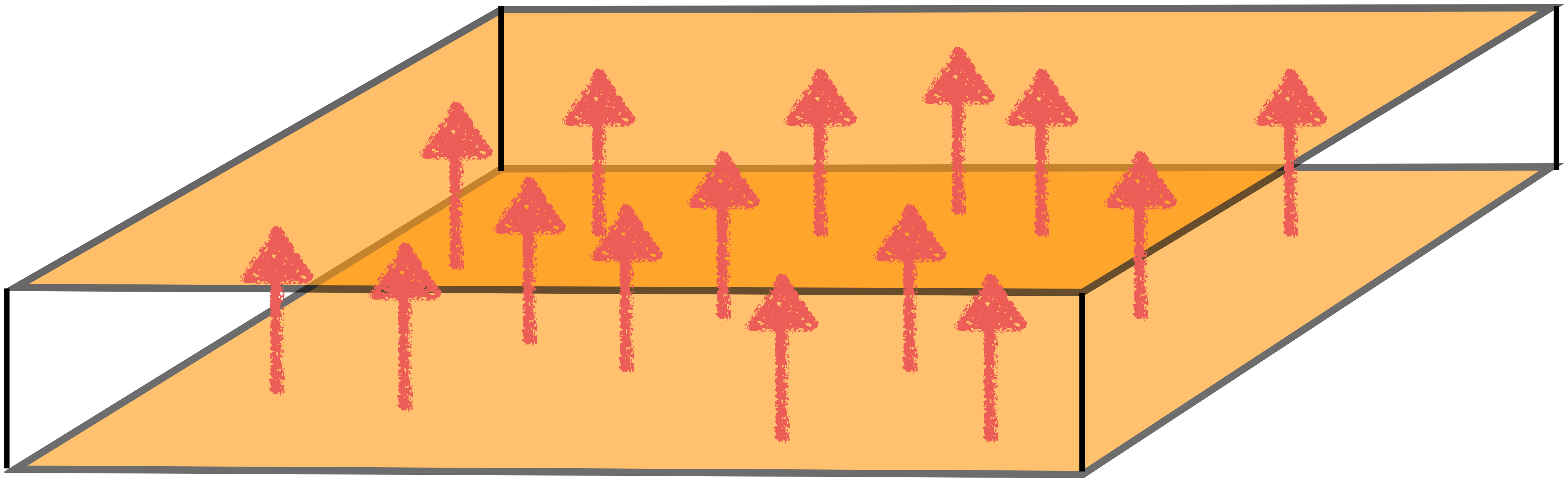
Quantum Anomalous Hall Insulator



Break TRS by doping with magnetic impurities, or can use a magnetic TI.

Quantum Anomalous Hall Insulator

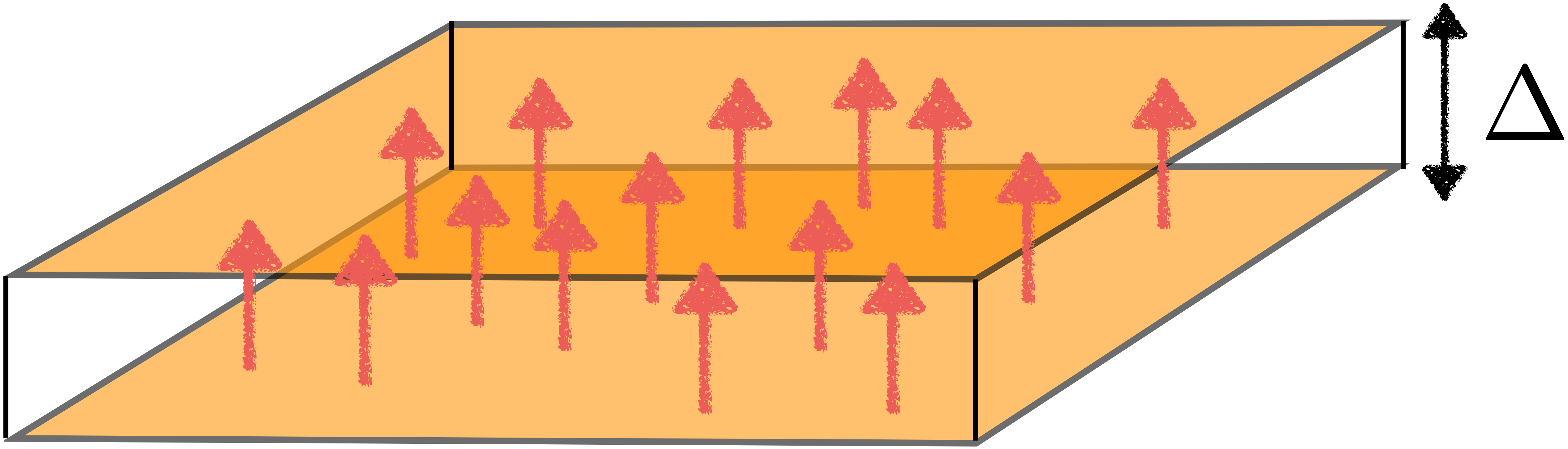
$$\sigma_{xy}^{T} = \frac{e^2}{2h}$$



$$\sigma_{xy}^{B} = \frac{e^2}{2h}$$

$$\sigma_{xy} = \sigma_{xy}^{T} + \sigma_{xy}^{B} = \frac{e^2}{h}$$

Quantum Anomalous Hall Insulator

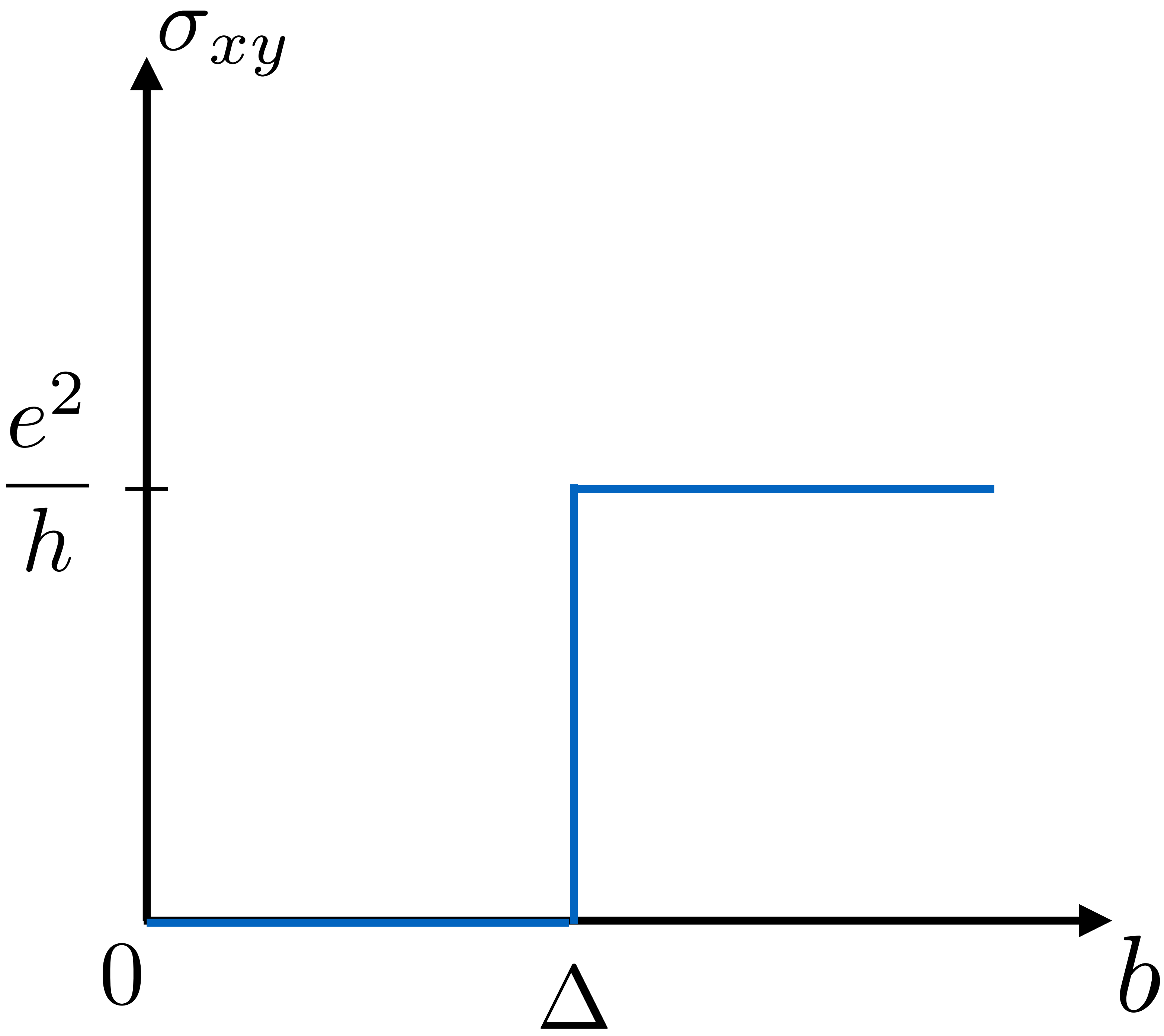


$$b < \Delta \qquad \sigma_{xy} = 0$$

$$b > \Delta \qquad \sigma_{xy} = \frac{e^2}{h}$$

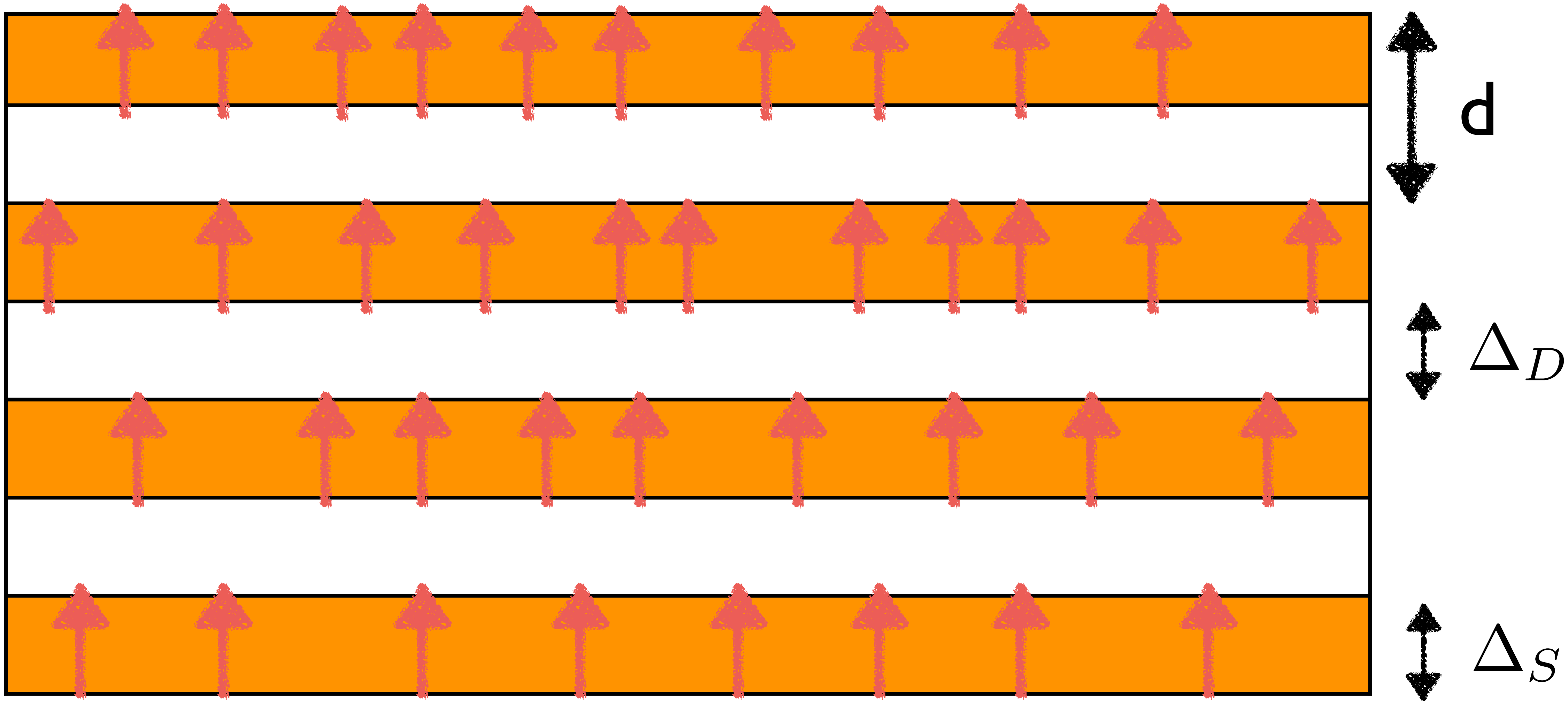
Haldane, 1988 Yu et al., 2010 Chang et al., 2013

Plateau transition in 2D

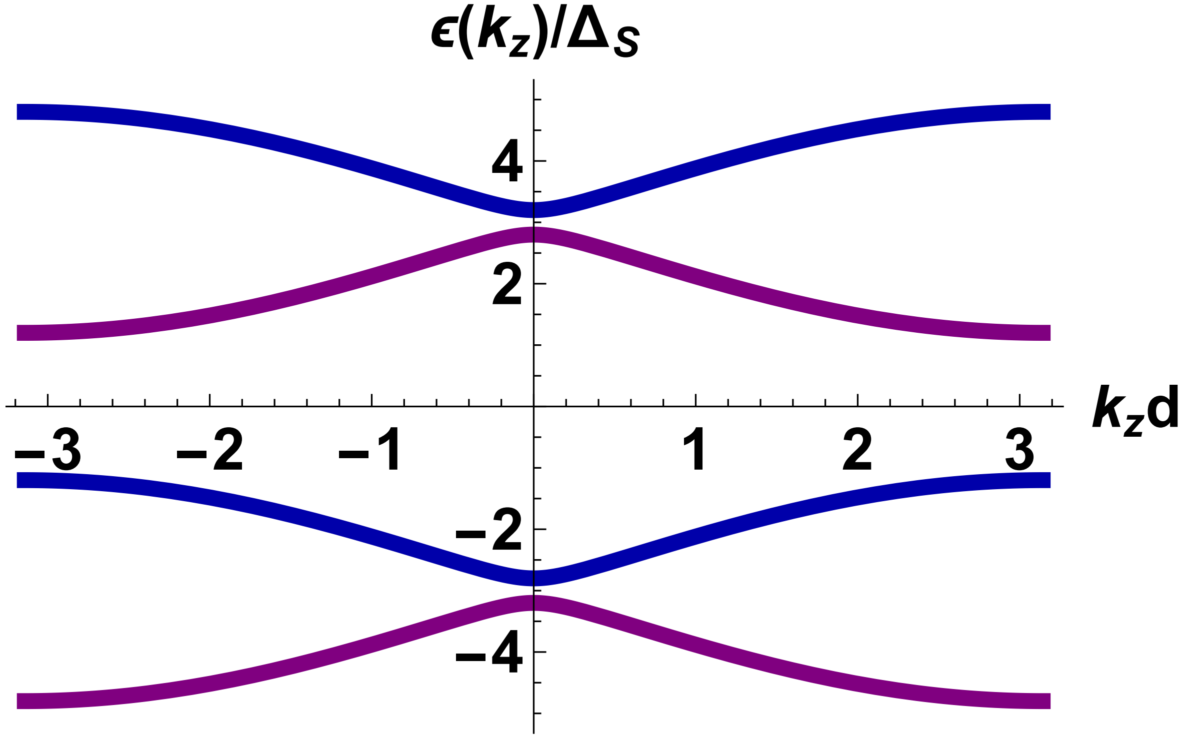


From 2D QAH insulator to 3D Weyl semimetal

- Stack of 2D QAH insulators, separated by ordinary insulator spacers.



3D Integer Quantum Hall Effect



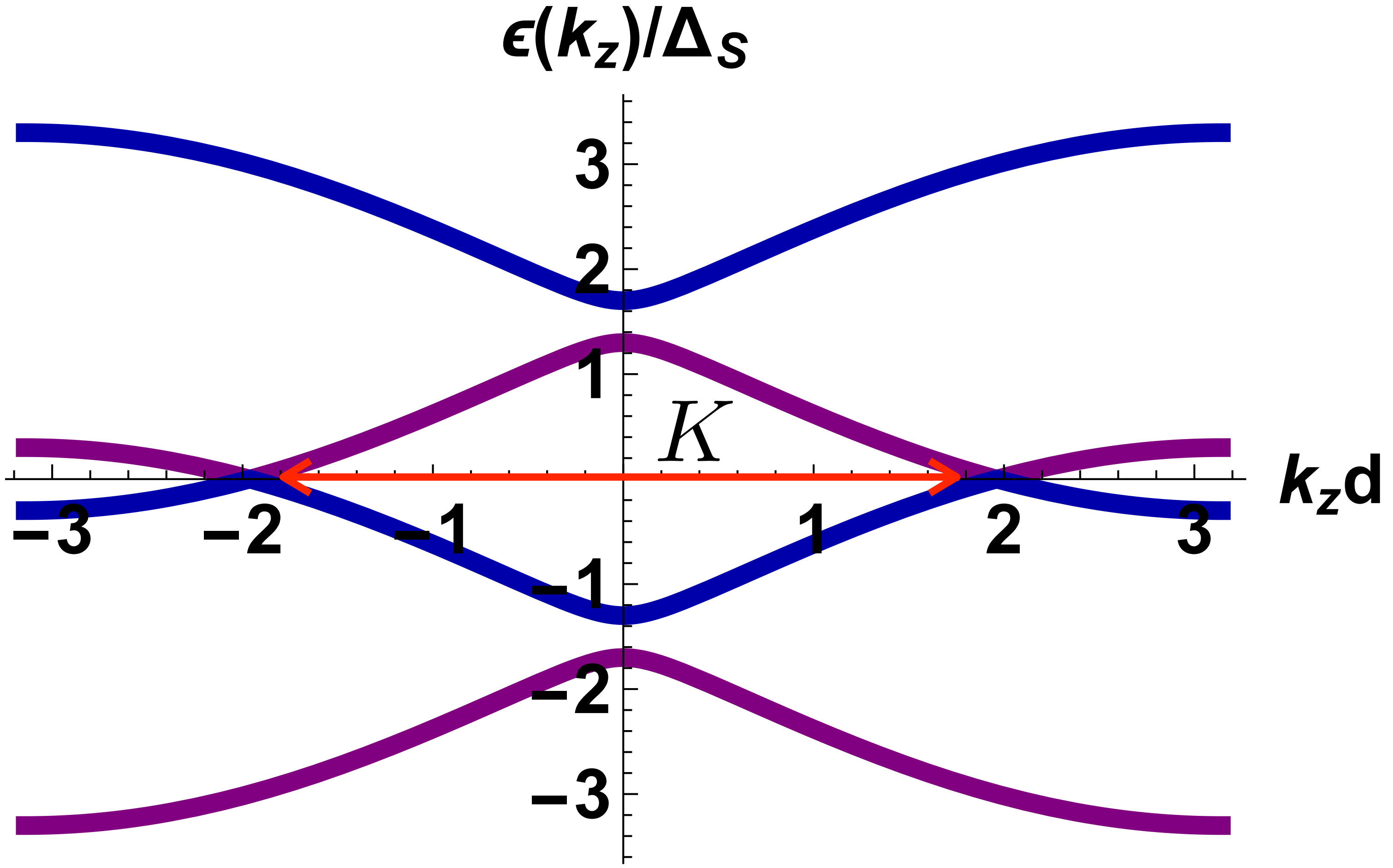
$$\sigma_{xy} = \frac{e^2}{h}\frac{G}{2\pi}$$

$$G = \frac{2\pi}{d}$$

Kohmoto, Halperin, Wu

- Hall conductivity involves a wavevector, transition to zero must happen smoothly.

3D Integer Quantum Hall Effect

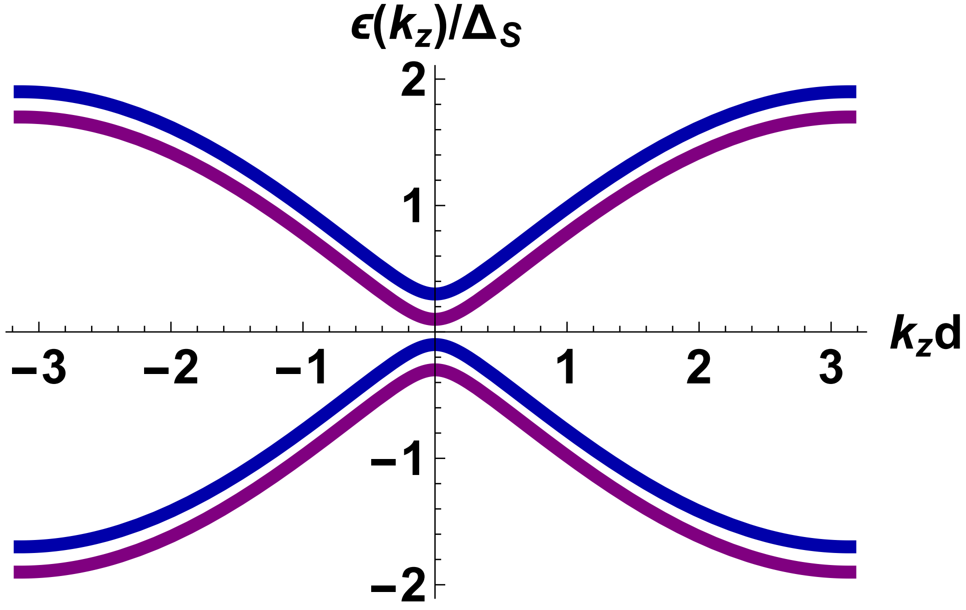


$$\sigma_{xy} = \frac{e^2}{h}\frac{K}{2\pi}$$

AAB & Balents

- Hall conductivity is proportional to the distance between the nodes and varies smoothly.

3D Integer Quantum Hall Effect

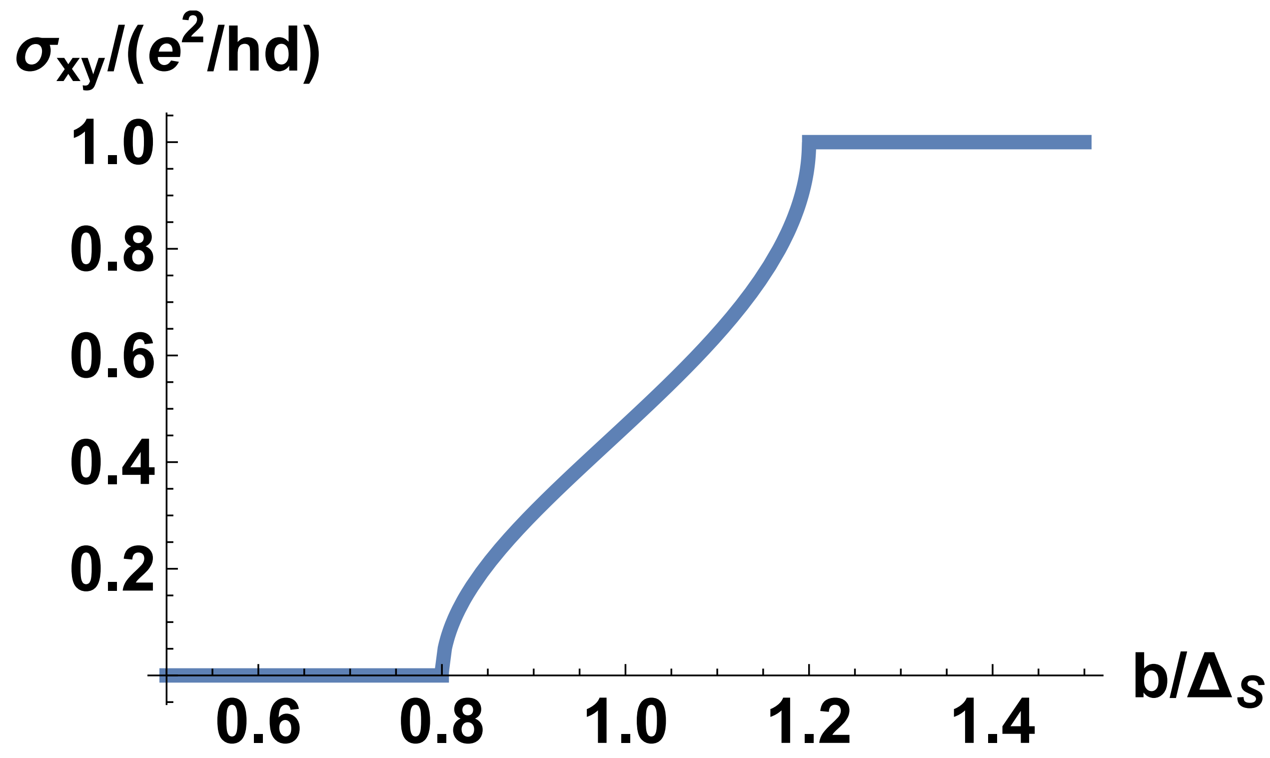


$$\sigma_{xy} = 0$$

- Hall conductivity is proportional to the distance between the nodes and varies smoothly.

"Plateau transition" in 3D

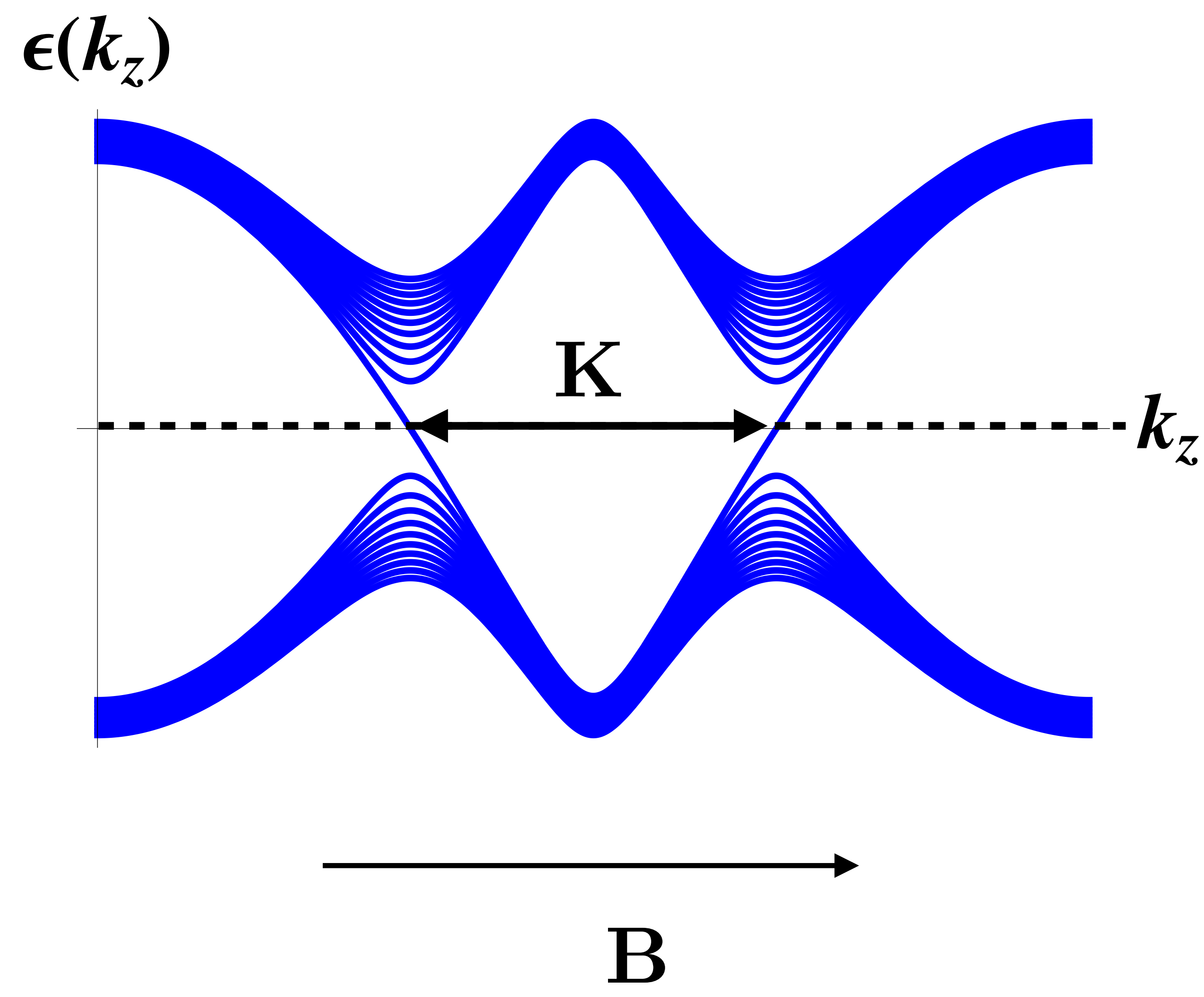
$$\sigma_{xy} = \frac{e^2}{h}\frac{K}{2\pi}$$



- Plateau transition is sharp in 2D, but broadens into Weyl semimetal phase in 3D.

Chiral anomaly

- Extra Landau level below the Fermi energy in between the Weyl nodes.



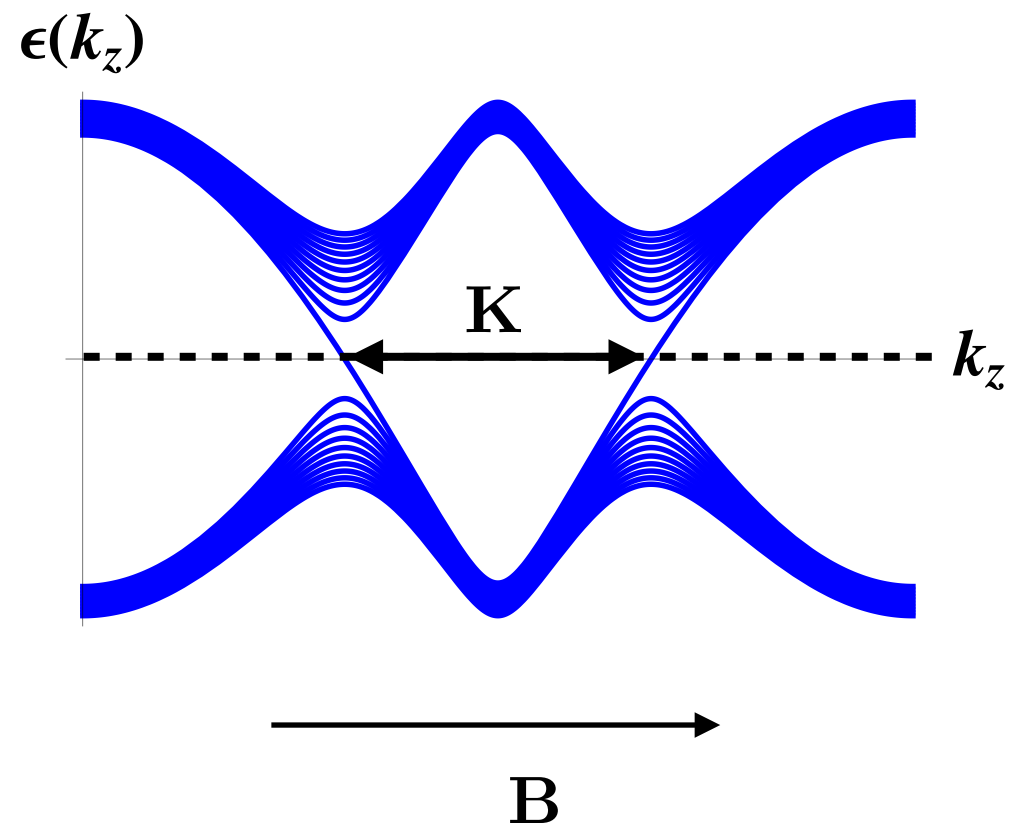
$$\sigma_{xy} = e\frac{\partial n}{\partial B} = e\frac{K}{2\pi\hbar}\frac{\partial}{\partial B}\frac{1}{2\pi\ell_B^2} = \frac{e^2}{h}\frac{K}{2\pi}$$

- Hall conductivity is a derivative of the Luttinger volume with respect to the magnetic field.

Chiral anomaly

- "Fractional" Hall conductivity in the absence of a Fermi surface inevitably implies Weyl nodes.

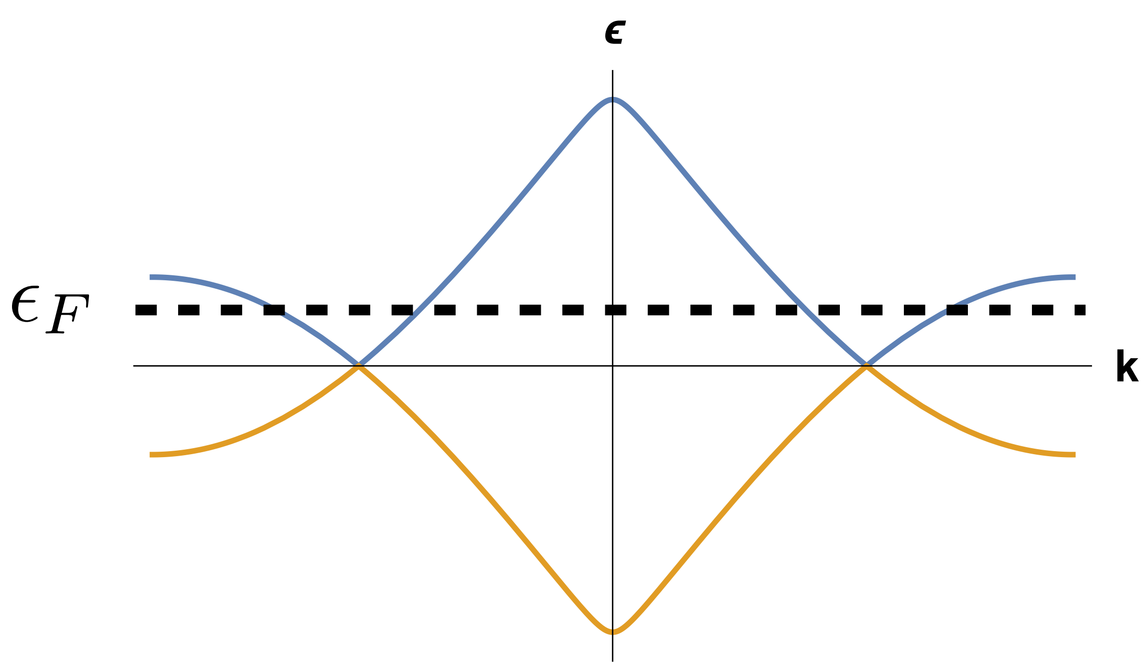
- Extra Landau level below the Fermi energy in between the Weyl nodes.



$$\sigma_{xy} = e\frac{\partial n}{\partial B} = e\frac{K}{2\pi\hbar}\frac{\partial}{\partial B}\frac{1}{2\pi\ell_B^2} = \frac{e^2}{h}\frac{K}{2\pi}$$

Chiral anomaly

- Left-handed and right-handed charges should be separately conserved.

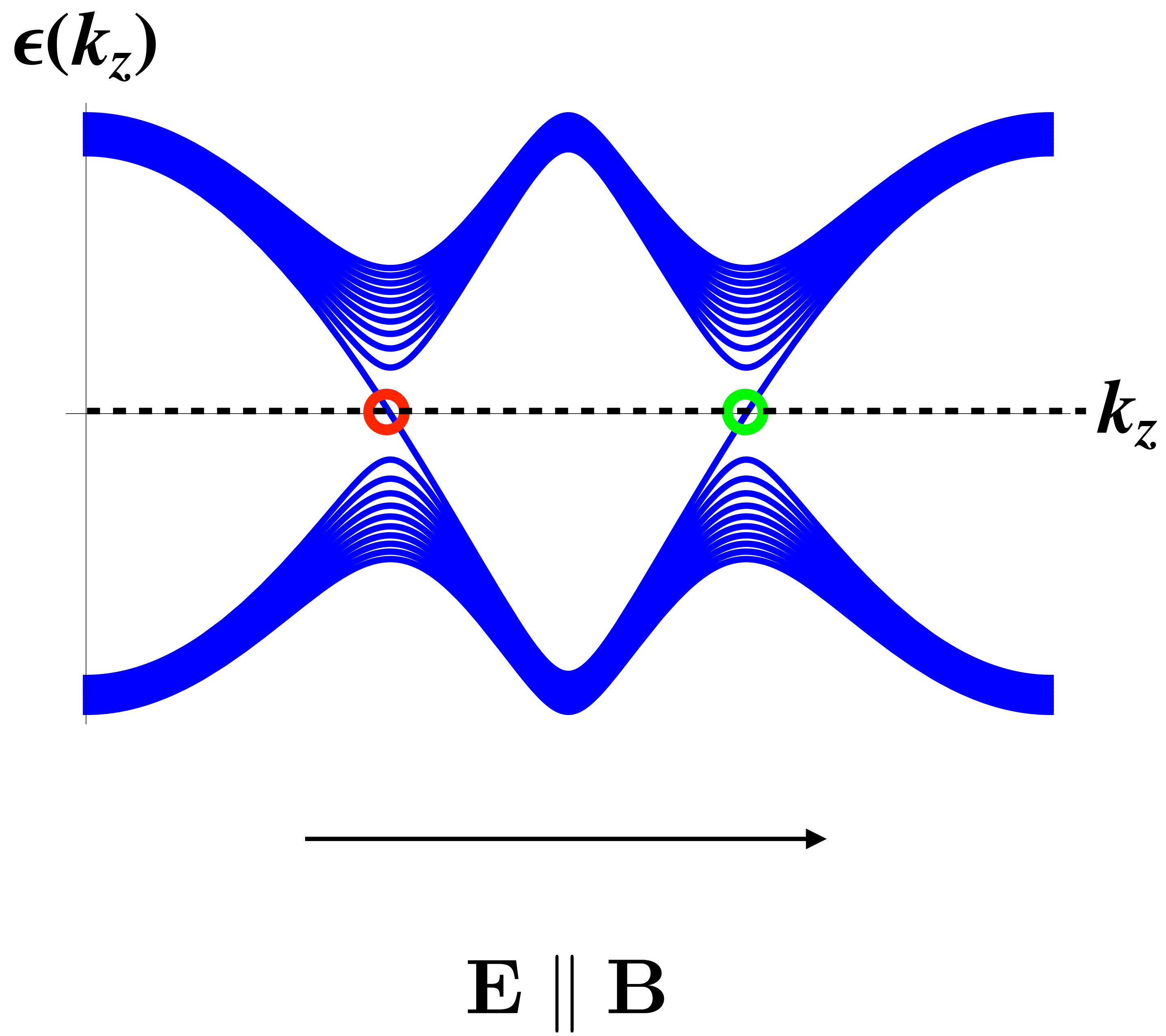


Chiral anomaly

- Conservation is violated in the presence of collinear electric and magnetic fields.

$$\frac{\partial n_R}{\partial t} = \frac{1}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$$

$$\frac{\partial n_L}{\partial t} = -\frac{1}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$$



Density response in an ordinary metal

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2(n + gV)$$

- Diffusion and drift current: $$\mathbf{j} = eD\boldsymbol{\nabla}n + egD\boldsymbol{\nabla}V = eD\boldsymbol{\nabla}n + \sigma\mathbf{E}$$

- Einstein relation: $$\sigma = e^2 gD$$

Density response in a Weyl metal

Total charge: $n = n_R + n_L$

Chiral charge: $n_c = n_R - n_L$

- If both were conserved, both would obey independent continuity (diffusion) equations:

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2 n \qquad \frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2 n_c - \frac{n_c}{\tau_c}$$

Density response in topological metal

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2(n + gV) + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n_c + gV_c)$$

$$\frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2(n_c + gV_c) - \frac{n_c + gV_c}{\tau_c} + \boldsymbol{\Gamma} \cdot \boldsymbol{\nabla}(n + gV)$$

- New transport coefficients:

$$n = n_R + n_L$$

$$n_c = n_R - n_L$$

$$\boldsymbol{\Gamma} = \frac{e\mathbf{B}}{2\pi^2 g}$$

Chiral charge relaxation time: $\tau_c \gg \tau$

Density response in topological metal

$$\frac{\partial n}{\partial t} = D\boldsymbol{\nabla}^2(n + gV) + \boldsymbol{\Gamma}\cdot\boldsymbol{\nabla}(n_c + gV_c)$$

$$\frac{\partial n_c}{\partial t} = D\boldsymbol{\nabla}^2(n_c + gV_c) - \frac{n_c + gV_c}{\tau_c} + \boldsymbol{\Gamma}\cdot\boldsymbol{\nabla}(n + gV)$$

- First derivatives will dominate at long length scales. This leads to propagating density modes and quasiballistic conductance.

Diffusion propagator

$$\mathcal{D}^{-1}(q,\omega) = \begin{pmatrix} -i\omega\tau + Dq^2\tau & -i\Gamma q\tau \\ -i\Gamma q\tau & -i\omega\tau + \tau/\tau_c + Dq^2\tau \end{pmatrix}$$

- Poles of the diffusion propagator determine the long-distance, long-time dynamics of the system.

Diffusion eigenmodes

$$i\omega_{\pm} = Dq^2 + \frac{1}{2\tau_c} \pm \sqrt{\frac{1}{4\tau_c^2} - \Gamma^2 q^2}$$

$$q < \frac{1}{2\Gamma\tau_c}$$

- Ordinary diffusion of conserved electric and almost conserved chiral charges:

$$i\omega_{+} = Dq^2 + \frac{1}{\tau_c} \qquad\qquad i\omega_{-} = Dq^2$$

Diffusion eigenmodes

$$i\omega_{\pm} = Dq^2 + \frac{1}{2\tau_c} \pm \sqrt{\frac{1}{4\tau_c^2} - \Gamma^2 q^2}$$

$$q > \frac{1}{2\Gamma\tau_c}$$

- Get a propagating mode:

$$\omega \approx \Gamma q - iDq^2$$

Propagating mode

- Get a propagating mode:

$$\omega \approx \Gamma q - iDq^2$$

- This mode is weakly damped as long as:

$$q < \frac{\Gamma}{D} = \frac{1}{L_a}$$

$$L_a = \frac{D}{\Gamma} \sim \ell (k_F \ell_B)^2$$

$$\ell_B = \sqrt{\hbar c / eB}$$

Propagating mode

- Linearly-dispersing propagating mode:

$$\omega = \Gamma q$$

- Exists as long as: $\frac{1}{L_*} < q < \frac{1}{L_a}$

$$L_* = \frac{L_c^2}{L_a} \qquad L_c = \sqrt{D\tau_c}$$ chiral charge diffusion length

- The existence of such a propagating mode in the diffusive transport regime in weak magnetic fields is a qualitatively new feature of topological metals.

Propagating modes

- First time and first space derivative: wave equation rather than diffusion.

$$\frac{\partial n}{\partial t} = \Gamma \frac{\partial n_c}{\partial z} \qquad \frac{\partial n_c}{\partial t} = \Gamma \frac{\partial n}{\partial z}$$

$$n_R = \frac{1}{2}(n + n_c) \qquad n_L = \frac{1}{2}(n - n_c)$$

$$\frac{\partial n_R}{\partial t} = \Gamma \frac{\partial n_R}{\partial z} \qquad \frac{\partial n_L}{\partial t} = -\Gamma \frac{\partial n_L}{\partial z}$$

- A pair of propagating chiral density modes.

Chiral Magnetic Effect

$$\mathbf{j} = \frac{\sigma}{e}\boldsymbol{\nabla}\mu + eg\mu_c\boldsymbol{\Gamma}$$

Kharzeev et al.

$$\sigma = \frac{ne^2\tau}{m}$$

involves irreversible randomization of momentum, dissipative.

- Second term is nondissipative.

Chiral Magnetic Effect

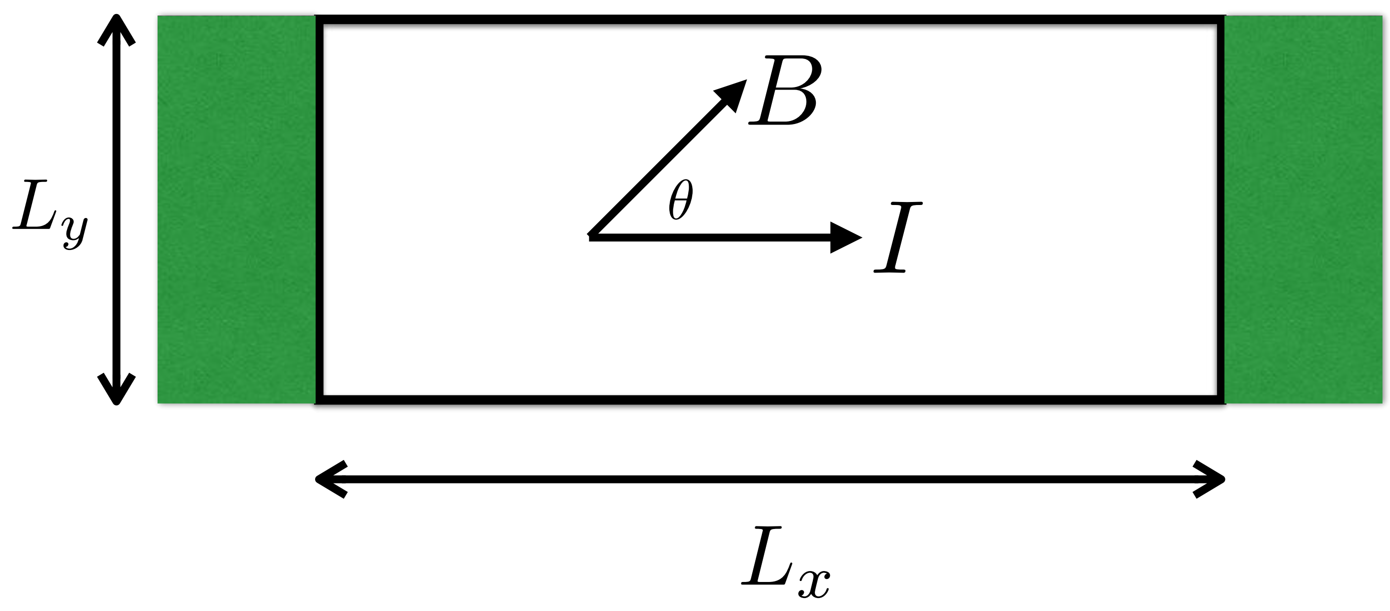
$$\mathbf{j} = \frac{\sigma}{e}\boldsymbol{\nabla}\mu + eg\mu_c\boldsymbol{\Gamma}$$

Kharzeev et al.

- Second term is nondissipative.

$$\mathbf{j} = -\frac{c}{4\pi\lambda^2}\mathbf{A}$$

Anisotropic MR



Kharzeev et al.

- Chiral magnetic effect: $$\mathbf{j} = \frac{\sigma}{e}\boldsymbol{\nabla}\mu + eg\mu_c\boldsymbol{\Gamma}$$

$$\rho_{xx} = \rho_\perp - \Delta\rho\cos^2\theta,$$
$$\rho_{yx} = -\Delta\rho\sin\theta\cos\theta,$$

$$\Delta\rho = \rho_\perp - \rho_\|$$

$$\rho_\perp = 1/\sigma$$

Anisotropic MR

Negative LMR: $\rho_{xx} = \rho_\perp - \Delta\rho\cos^2\theta$

Son & Spivak AAB

Planar Hall Effect: $\rho_{yx} = -\Delta\rho\sin\theta\cos\theta$

AAB $\Delta\rho = \rho_\perp - \rho_\parallel = \frac{1}{\sigma}\frac{(L_c/L_a)^2}{1+(L_c/L_a)^2}$

$$L_a = \frac{D}{\Gamma} \sim \ell(k_F\ell_B)^2$$

$$\ell_B = \sqrt{\hbar c/eB}$$

purely quantum phenomena!

Anisotropic MR

$$\Delta\rho = \rho_\perp - \rho_\parallel = \frac{1}{\sigma}\frac{(L_c/L_a)^2}{1+(L_c/L_a)^2}$$

- AMR has opposite sign to what is typically seen in ferromagnets and much larger magnitude.

$$\frac{\Delta\rho}{\rho_\perp} \approx 50\%$$

largest AMR in a FM metal in U_3As_4

- Several hundred percent in Na_3Bi, N.P. Ong et al.

$$\frac{\Delta\rho}{\rho_\parallel} = \frac{(L_c/L_a)^4}{1+(L_c/L_a)^2}$$

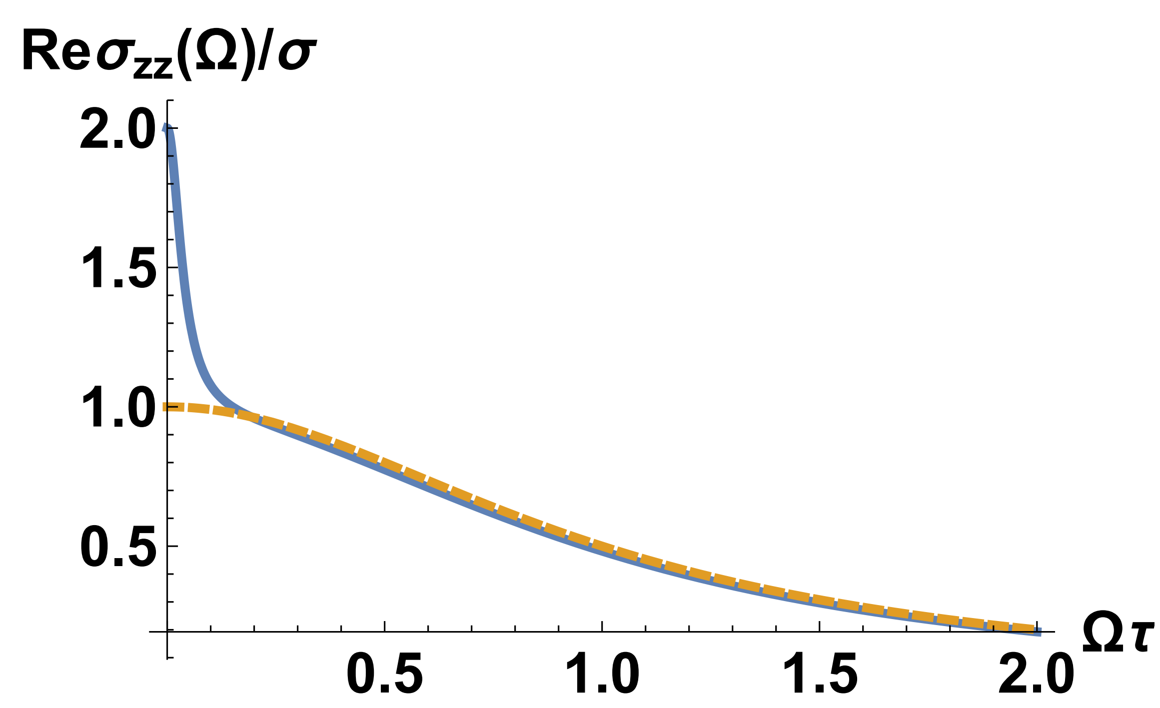
Optical conductivity

- From charge continuity equation:

$$\sigma_{zz}(\omega) = \lim_{q \to 0} \frac{ie^2\omega}{q^2} \chi_{00}(q, \omega)$$

$$\mathrm{Re}\sigma_{zz}(\omega) = \frac{\sigma}{1+\omega^2\tau^2}\left[1 + \left(\frac{L_c}{L_a}\right)^2 \frac{1-\omega^2\tau\tau_c}{1+\omega^2\tau_c^2}\right]$$

AAB



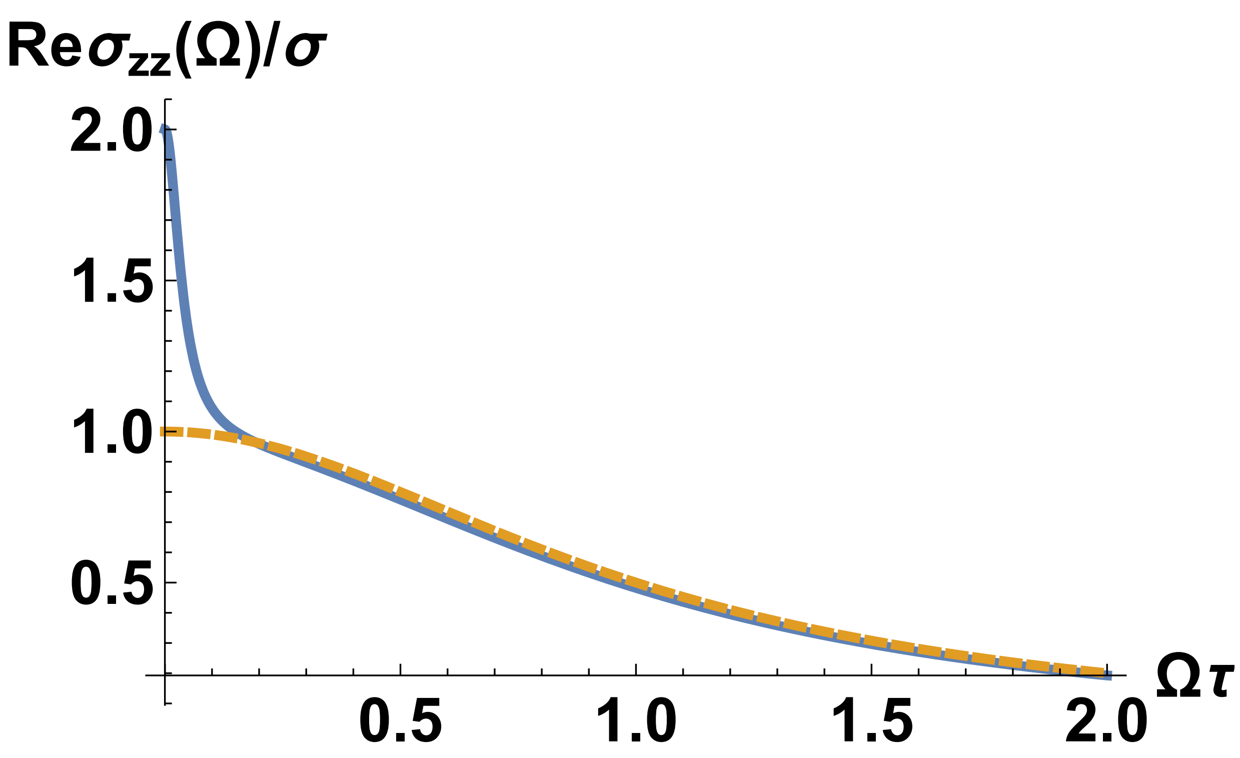
Optical conductivity

- Transfer of spectral weight to a narrow low-frequency peak.

$$\mathrm{Re}\sigma_{zz}(\omega) = \frac{\sigma}{1+\omega^2\tau^2}\left[1+\left(\frac{L_c}{L_a}\right)^2 \frac{1-\omega^2\tau\tau_c}{1+\omega^2\tau_c^2}\right]$$

- Drude weight is preserved.

$$\int_0^\infty d\omega\,\mathrm{Re}\sigma(\omega) = \frac{\pi\sigma}{2\tau}$$



Optical conductivity

Probing charge pumping and relaxation of the chiral anomaly in a Dirac semimetal

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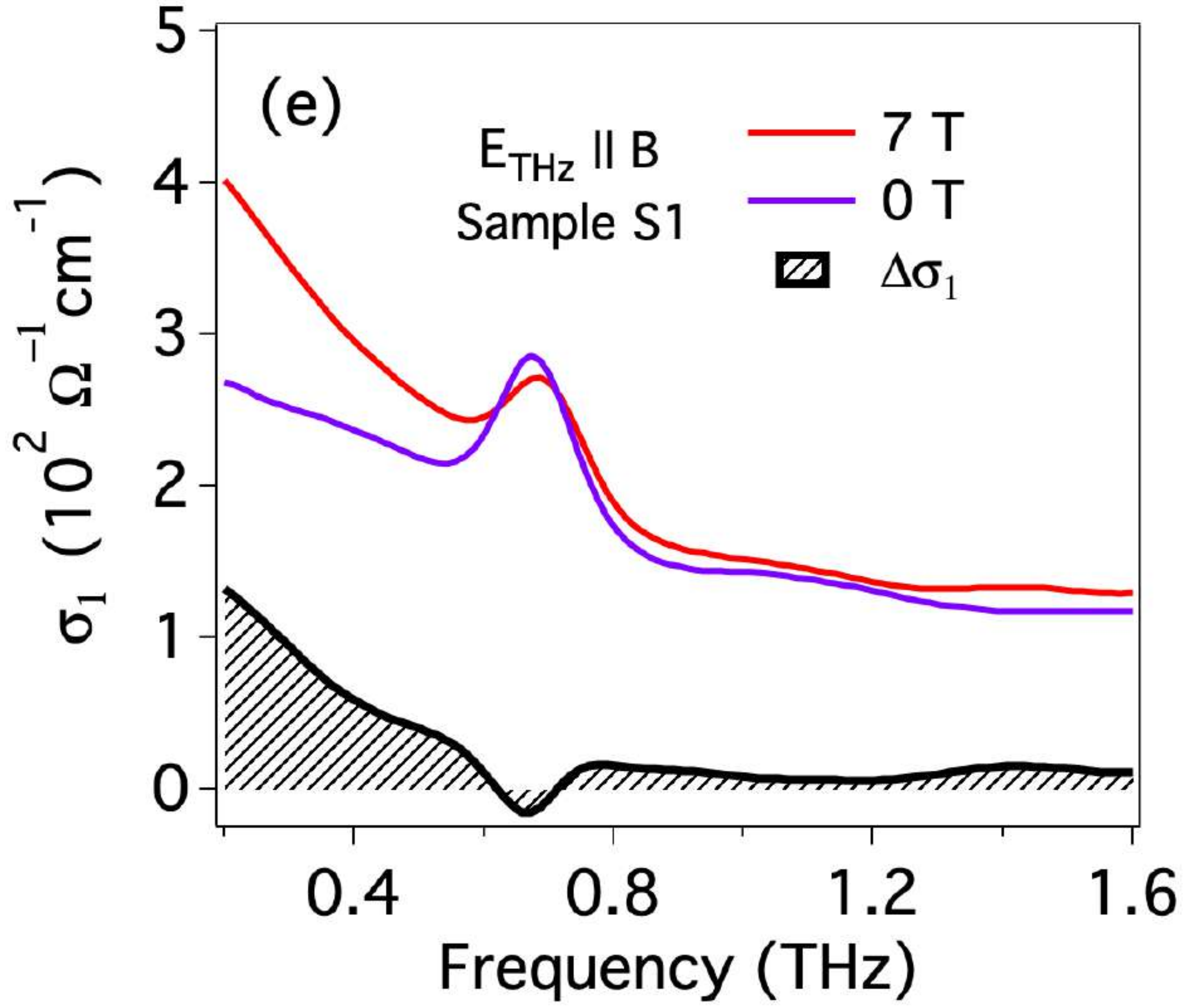
(Dated: October 31, 2019)



Chiral anomaly and interactions

- Chiral anomaly inevitably implies Weyl nodes in case of weak interactions.
- Does this remain true when the interactions are not weak?
- In other words, can we gap out the Weyl nodes while preserving the chiral anomaly and while not breaking any symmetries?

3D Fractional Quantum Hall Effect

PHYSICAL REVIEW LETTERS **124,** 096603 (2020)

Fractional Quantum Hall Effect in Weyl Semimetals

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(Received 8 July 2019; accepted 11 February 2020; published 6 March 2020)

- We can "defeat" the anomaly, but at the cost of fractionalizing electrons.

3D Fractional Quantum Hall Effect

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- We can "defeat" the anomaly, but at the cost of fractionalizing electrons.
- This is analogous to asking if we can have a gapped Mott insulator not breaking any symmetries at odd integer electron filling per unit cell.

Vortex condensation

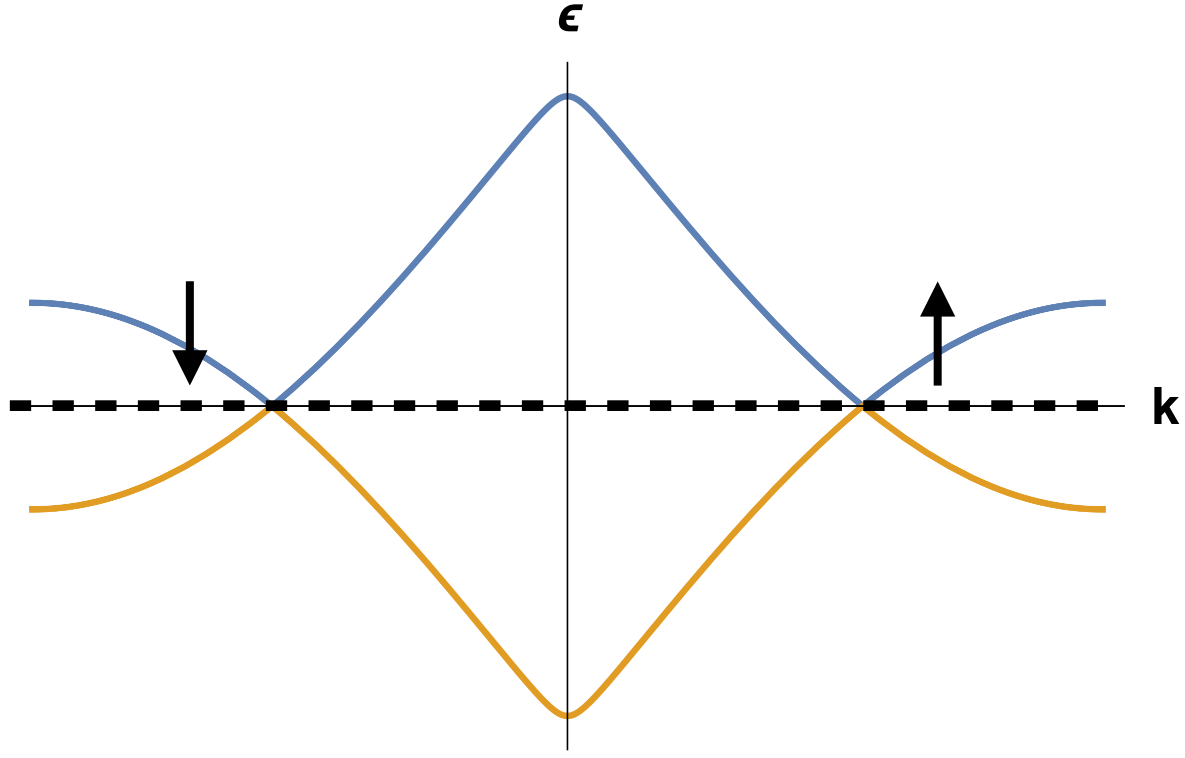
- Induce fully gapped superconductivity in Weyl semimetal.

- Destroy SC coherence by condensing vortices while keeping the pairing gap: this produces an insulator.

- Chiral anomaly places strong restrictions on the procedure and prohibits a simple insulator, has to have topological order.

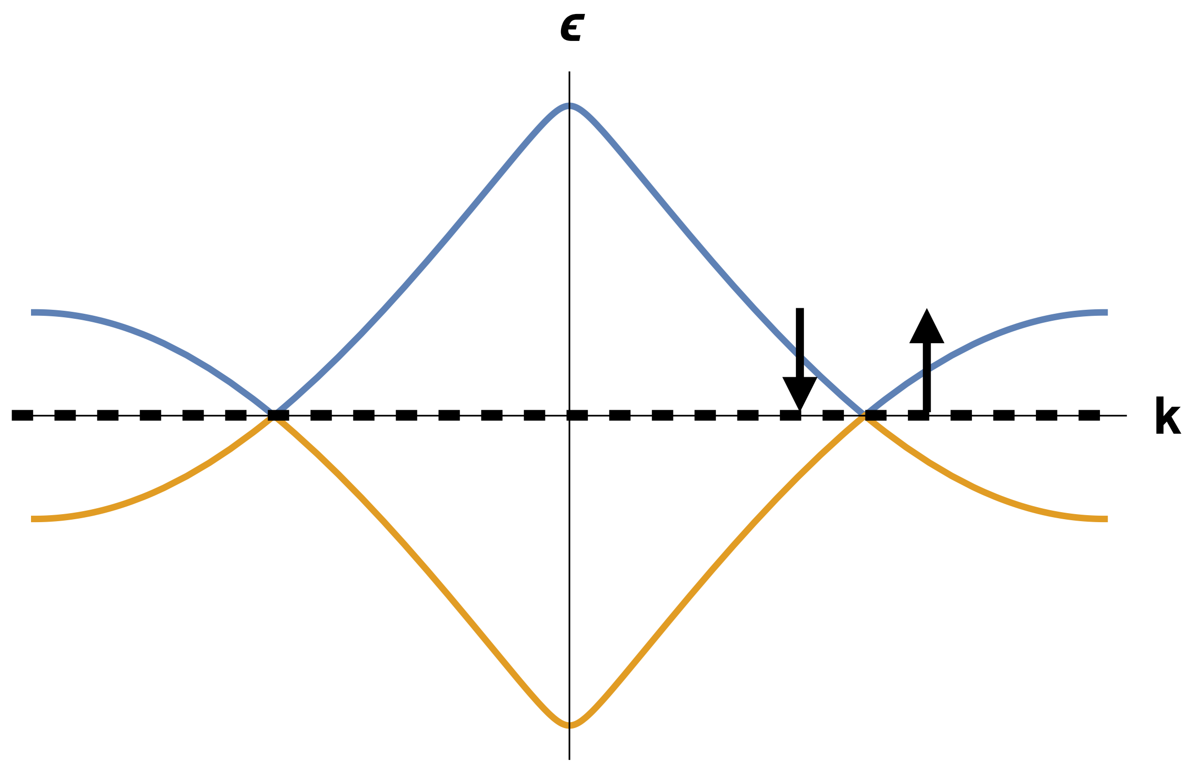
Weyl superconductor

- BCS: pairing k and -k states, i.e. internodal pairing.



Weyl superconductor

- FFLO: pairing states on the opposite side of each Weyl point, i.e. intranodal pairing.



BCS pairing

- BCS pairing can not open a gap, since the two chiralities are not mixed by the pairing term:

$$H = v_F \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \tau^z \boldsymbol{\sigma} \cdot \mathbf{k} c_{\mathbf{k}} + \Delta \sum_{\mathbf{k}} (c_{\mathbf{k}R}^{\dagger} i\sigma^y c_{-\mathbf{k}L}^{\dagger} + h.c.)$$

$$\psi_{\mathbf{k}} = (c_{\mathbf{k}R\uparrow}, c_{\mathbf{k}R\downarrow}, c_{-\mathbf{k}L\downarrow}^{\dagger}, -c_{-\mathbf{k}L\uparrow}^{\dagger})$$

$$H = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^{\dagger} (v_F \boldsymbol{\sigma} \cdot \mathbf{k} + \Delta \tau^x) \psi_{\mathbf{k}}$$

Meng & Balents

FFLO pairing

- FFLO does open a gap, but breaks translational symmetry:

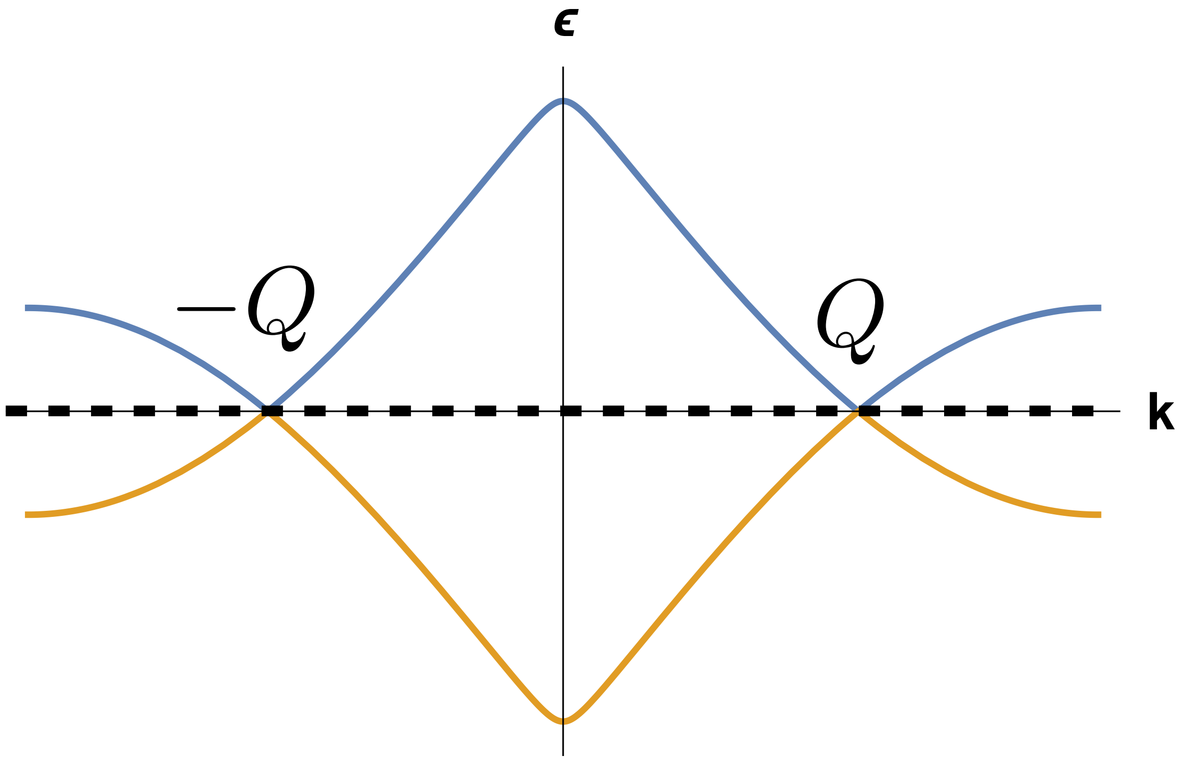
$$H = v_F \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \boldsymbol{\sigma} \cdot \mathbf{k}\, c_{\mathbf{k}} + \Delta \sum_{\mathbf{k}} (c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow})$$

$$\psi_{\mathbf{k}} = (c_{\mathbf{k}\uparrow}, c_{\mathbf{k}\downarrow}, c^\dagger_{-\mathbf{k}\downarrow}, -c^\dagger_{-\mathbf{k}\uparrow})$$

$$H = \frac{1}{2} \sum_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} (v_F \tau^z \boldsymbol{\sigma} \cdot \mathbf{k} + \Delta \tau^x) \psi_{\mathbf{k}}$$

FFLO pairing

- FFLO does open a gap, but breaks translational symmetry:



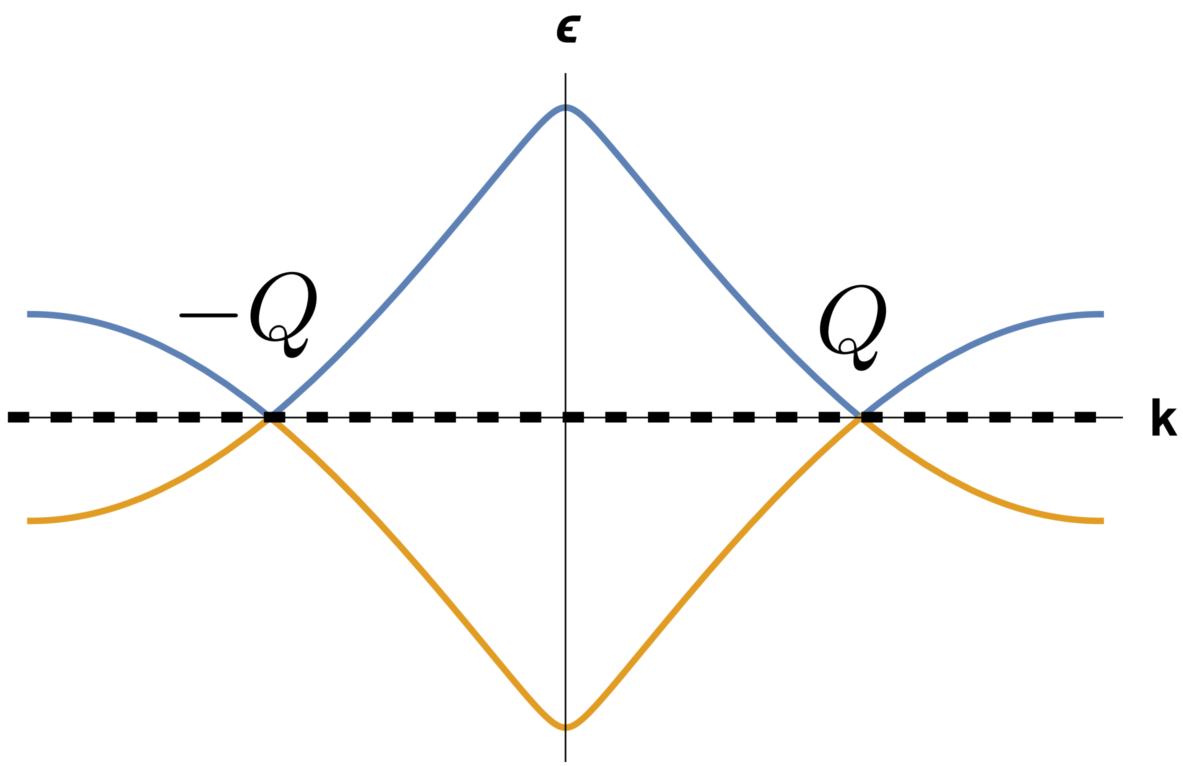
$$\Delta(\mathbf{Q}) \sim \sum_{\mathbf{k}} \langle c^\dagger_{\mathbf{Q}+\mathbf{k}} c^\dagger_{\mathbf{Q}-\mathbf{k}} \rangle$$

carries momentum 2Q.

$$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$$

carries momentum 4Q.

FFLO pairing



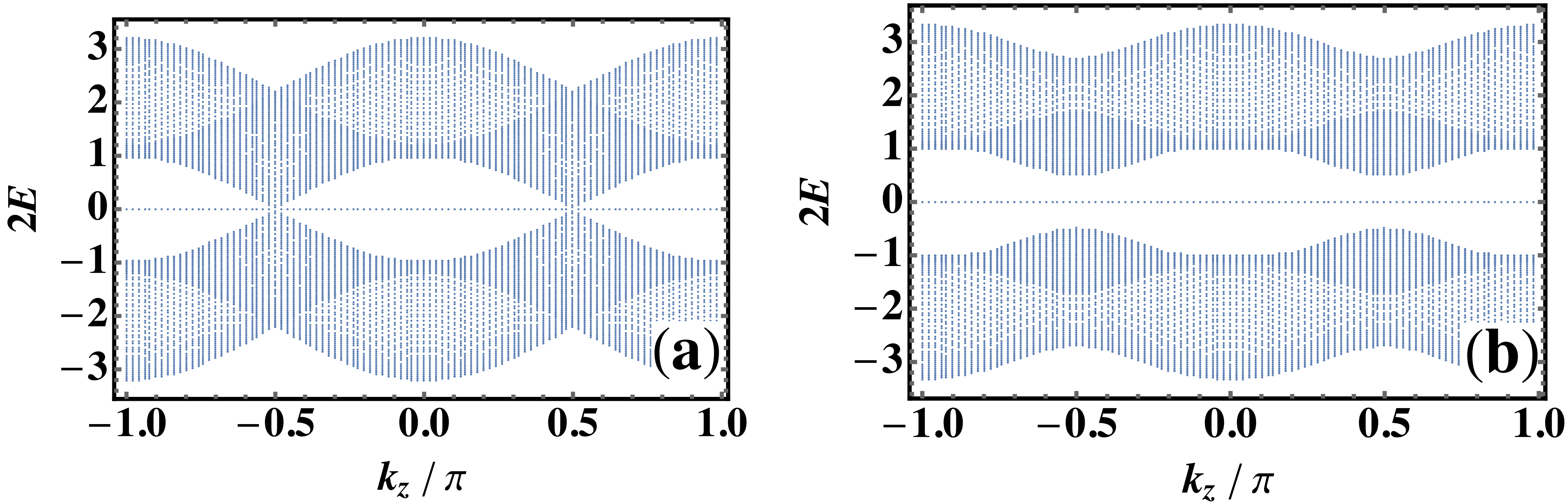
$$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$$

carries momentum 4Q.

- This breaks translational symmetry, unless $\mathbf{Q} = \mathbf{G}/4$
- In other words, FFLO does not break translational symmetry when Weyl node separation is exactly half the BZ size.

Majorana arc

- Fermi arc becomes Majorana arc, which occupies twice the momentum interval of the Fermi arc, i.e. 4Q.

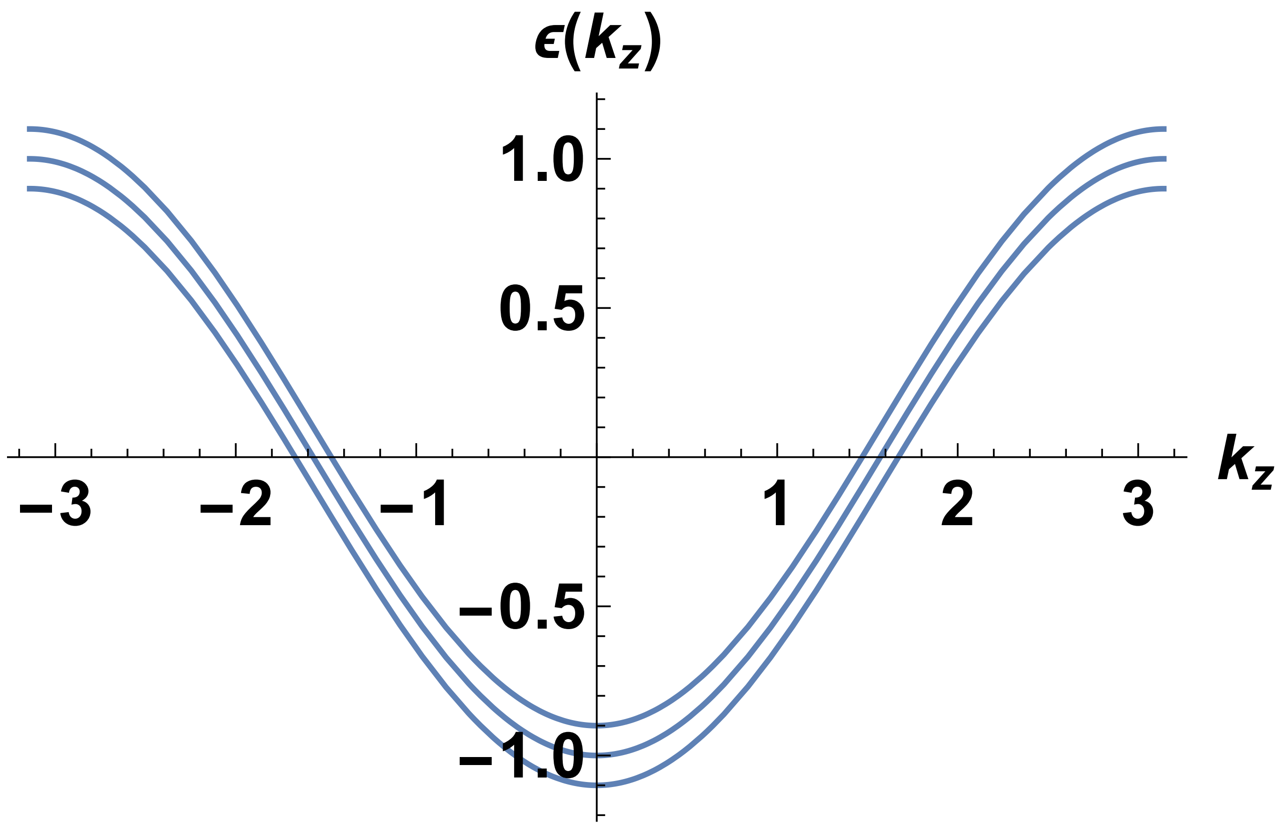


$$\kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Vortex condensation in FFLO state

- n-fold vortex (Φ=nhc/2e) in FFLO paired state: get n chiral Majorana modes in the vortex core.

$$\epsilon_p(k_z) = \epsilon_F \left(1 - \frac{2p}{n+1}\right) + v_F k_z. \qquad p = 1, \ldots, n.$$

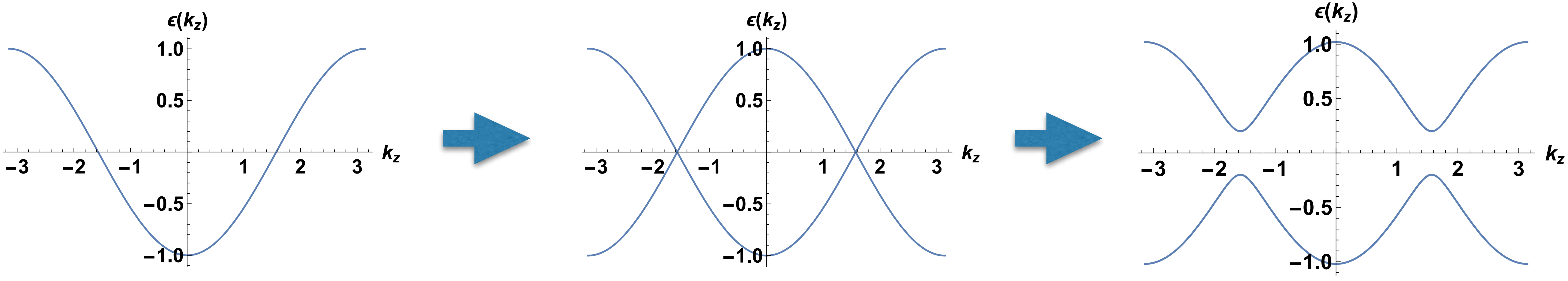


Callan & Harvey

Jackiw & Rossi

Vortex condensation in FFLO state

- Any even number 2n of Majorana vortex modes may be combined into n 1D Weyl fermion modes, which are gapped out by pairing:



Vortex condensation in FFLO state

- Any even number 2n of Majorana vortex modes may be combined into n 1D Weyl fermion modes, which are gapped out by pairing:

$$H = v_F \sum_{k_z} [k_z c^\dagger_{k_z} \tau^z c_{k_z} + \Delta (c^\dagger_{k_z} i\tau^y c^\dagger_{-k_z} + \mathrm{h.c.})/2]$$

- An odd number of Majorana modes can not be eliminated without breaking translational symmetry, thus a fundamental SC vortex may not be condensed.

$$\Phi = \frac{hc}{2e} = \pi \qquad \hbar = c = e = 1$$

Vortex condensation in FFLO state

- A double vortex does not have Majorana modes, but may still not be condensed.

- This follows from the fact that the insulating state we want to obtain must preserve the chiral anomaly, i.e. must have a Hall conductivity of half conductivity quantum per atomic plane:

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi}$$

Vortex condensation in FFLO state

- A vortex will induce a charge when intersecting an atomic plane:

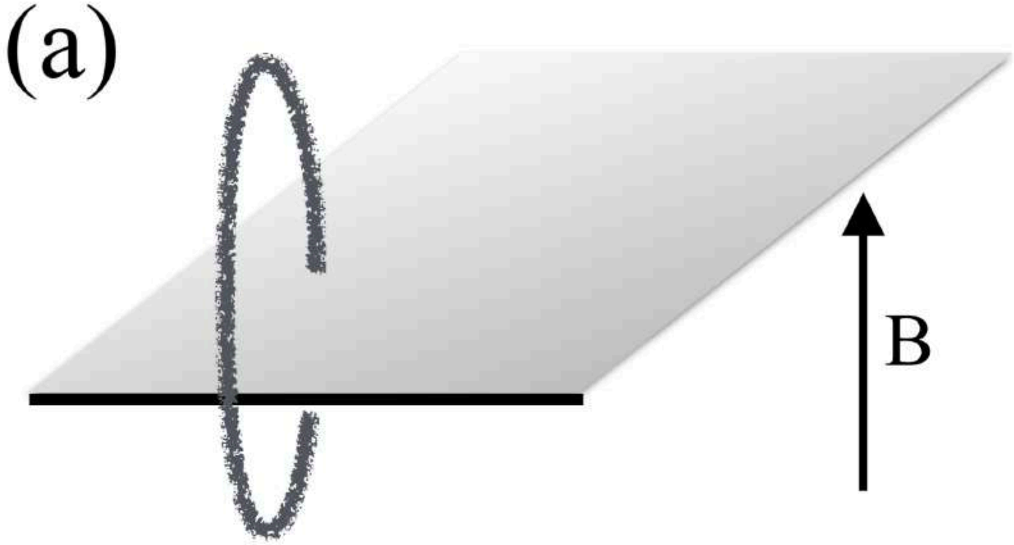
$$\mathcal{L} = \frac{\sigma_{xy}}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

$$n = \frac{\Phi}{4\pi} = \frac{1}{2}$$

- Since vortex is a loop, it will always intersect any atomic plane twice, inducing a pair of opposite charges, whose effect will thus cancel.

Vortex condensation in FFLO state

- But consider a crystal with a dislocation.



- In this case vortex loop may intersect the extra half-plane only once, inducing uncompensated 1/2 charge.

Vortex condensation in FFLO state

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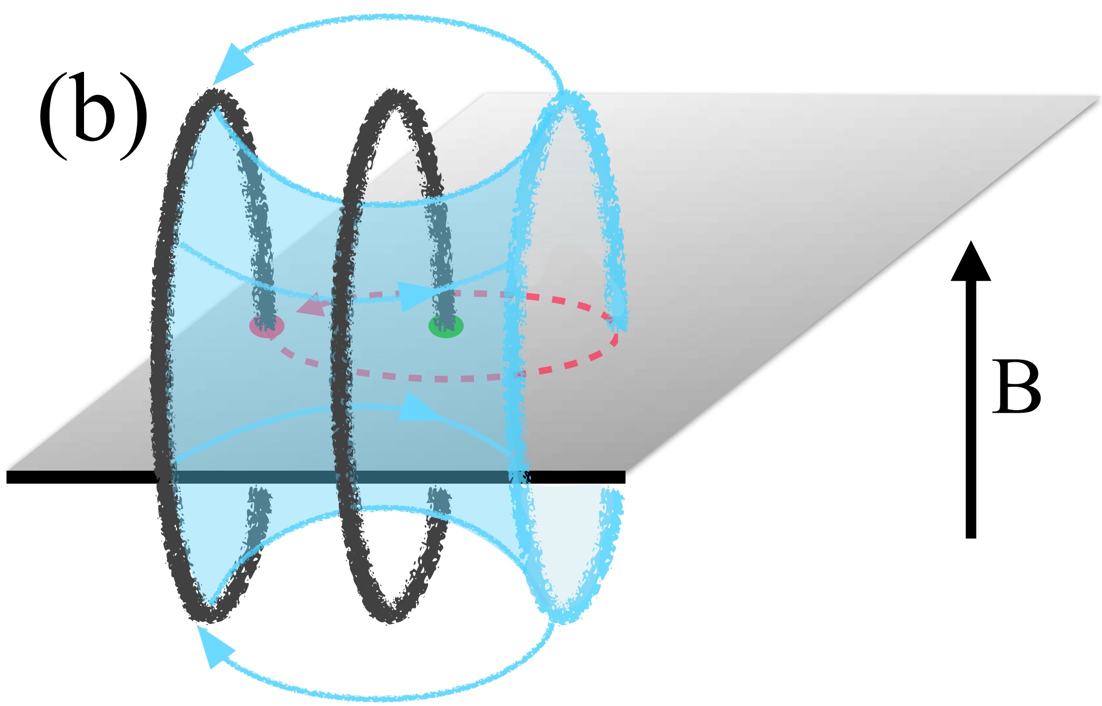
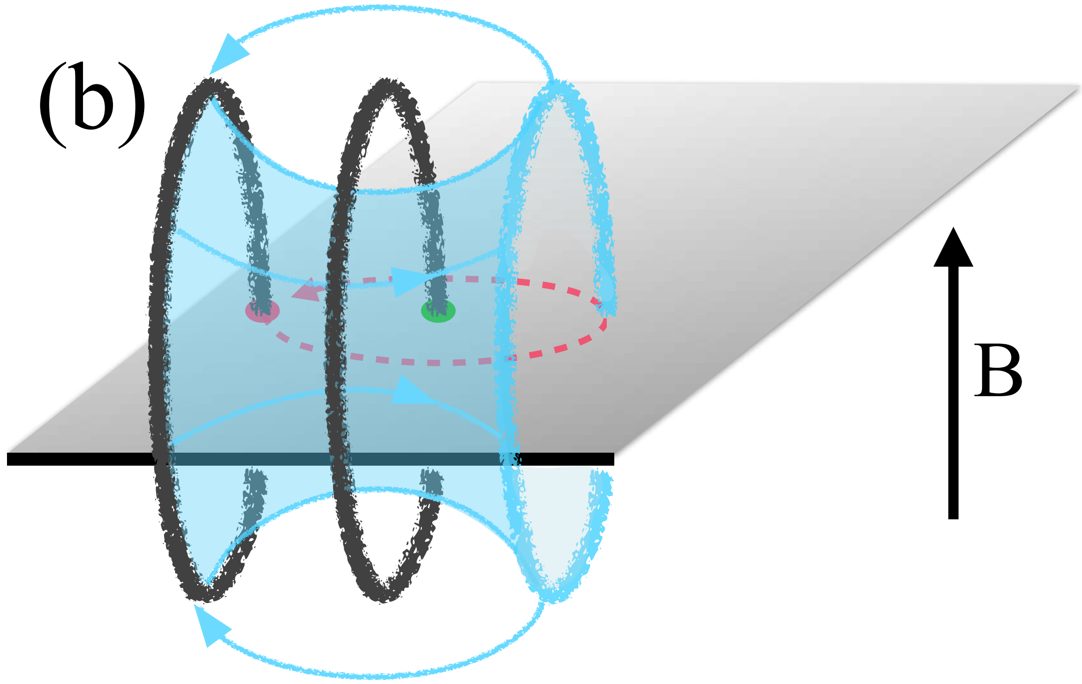


Wang & Levin 3-loop braiding

- Two such charges will have semion exchange statistics:

$$\theta = 2\pi^2 \sigma_{xy} = \frac{\pi}{2}$$

Vortex condensation in FFLO state



- Two such charges will have semion exchange statistics.

- This means that, inserting a dislocation in a crystal with condensed vortices will cost an energy $O(L^2)$

- This implies broken translational symmetry.

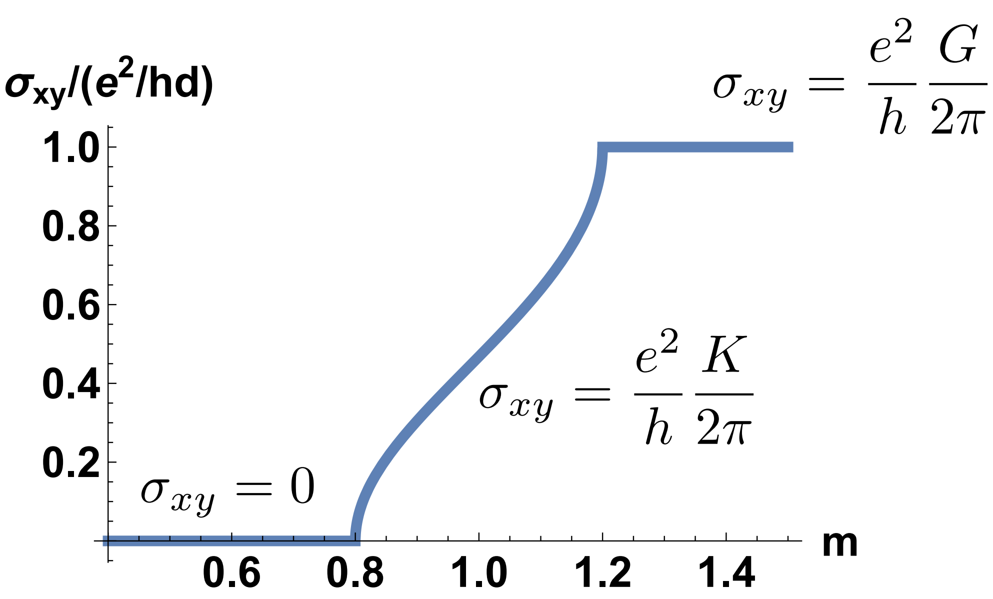
Vortex condensation in FFLO state

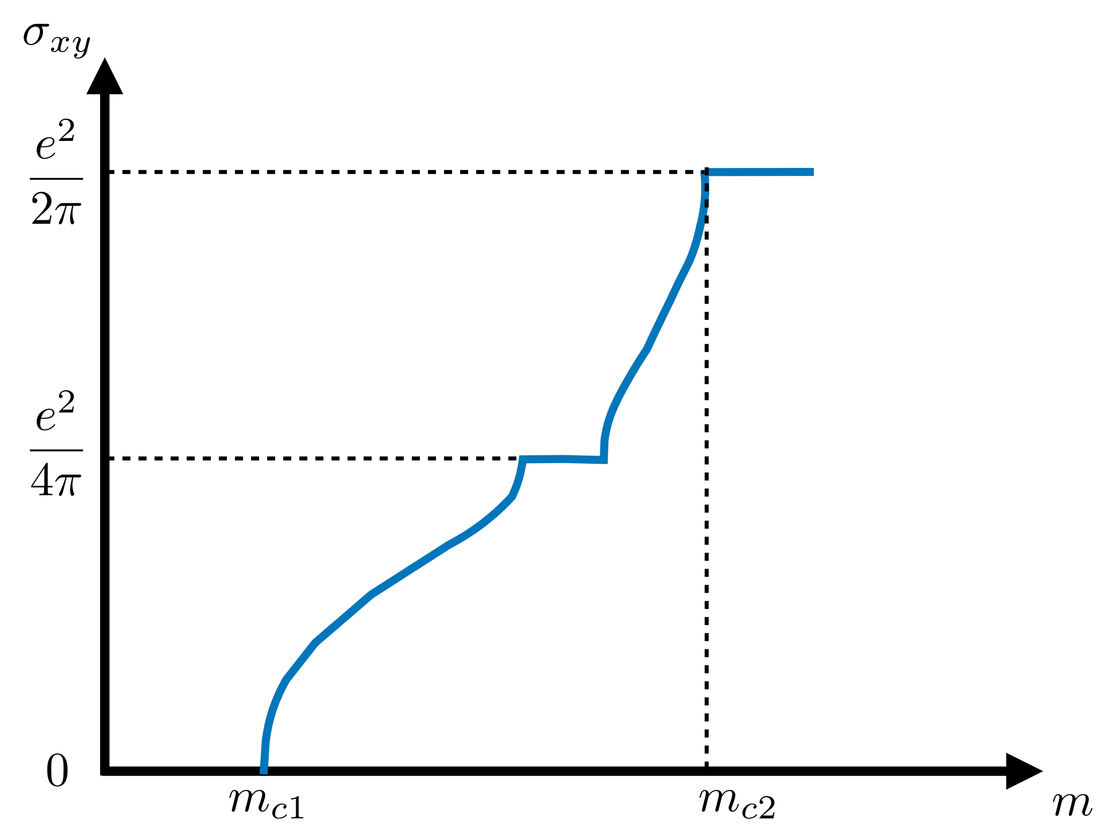
- Following the same logic, quadruple vortices have bosonic statistics and thus may be condensed without breaking any symmetries.

- This is an insulating state that preserves the chiral anomaly and does not break any symmetries.

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi} \qquad \kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Nontrivial generalization of FQHE to 3D





- In the presence of interactions, smooth evolution of the Hall conductivity with the magnetization in a Weyl semimetal may be interrupted by a half-quantized plateau.

Conclusions

- Both insulators and metals may be topological.
- Topological metal is a metal, whose Fermi surface breaks up into disconnected sheets, each enclosing a “magnetic monopole in momentum space”, or Weyl node.
- These nodes are topological objects and lead to observable phenomena: Fermi arc surface states, giant anisotropic magnetoresistance and dissipationless transport, non-Drude optical conductivity, 3D fractional quantum Hall effect, etc.