Anomalous spin-rotation coupling at finite temperature

Matteo Buzzegoli

Iowa State University
of Science and Technology

Arizona S.U. Theoretical Physics Colloquium (remote)
August 18 2021

Outline

Based on [MB & D. E. Kharzeev, Phys. Rev. D 103 (2021)]

- Spin-rotation coupling: evidence and Equivalence Principle (EP)
- Breaking of EP at finite temperature
- Anomalous spin-rotation coupling
- Radiative corrections to the Axial Vortical Effect

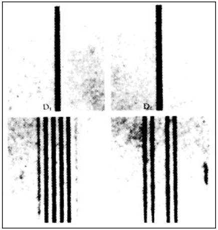
See also O. Teryaev, "QCD, gravity and inertia" in this colloquium series

Magnetic Moment

Particle with mass m charge e and spin $\mathbf{S}$

$$\boldsymbol{\mu}_{\rm B} = -g_B \frac{e}{2m}\mathbf{S}$$

Zeeman effect $\quad H = H_0 - \boldsymbol{\mu}_{\rm B} \cdot \mathbf{B}$



Larmor frequency $\quad \omega = g_{\rm B} \frac{eB}{2mc}$

Magnetic Moment g-factor: $\quad g_{\rm B}$

Spin-Rotation Coupling or Gravitomagnetic Moment g_Ω

$$H = H_0 - g_\Omega \boldsymbol{\Omega} \cdot \mathbf{S} \qquad \boldsymbol{\Omega}\text{: Rotation}$$

Dirac equation in non-inertial frame [F. Hehl and W. Ni, Phys. Rev. D 42 (1990)]

$$\frac{\mathrm{d}e_\alpha}{\mathrm{d}\tau} = \omega \cdot e_\alpha, \quad \omega^{\mu\nu} = \frac{a^\mu u^\nu - a^\nu u^\mu}{c^2} + \epsilon^{\mu\nu\rho\sigma} u_\rho \frac{\Omega_\sigma}{c}, \quad D_\mu = \partial_\mu + \Gamma_\mu$$

the Dirac equation becomes

$$\mathrm{i}\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

$$H = \gamma^0 mc^2 + c\gamma^0 \boldsymbol{\gamma} \cdot \mathbf{p} - \boldsymbol{\Omega}(\mathbf{L} + \mathbf{S}) + \gamma^0 m(\mathbf{a} \cdot \mathbf{x}) + \\ + \frac{1}{2c} \left\{ (\mathbf{a} \cdot \mathbf{x}), (\mathbf{p} \cdot \gamma^0 \boldsymbol{\gamma}) \right\}$$

$\boldsymbol{\Omega} \cdot \mathbf{S}$: Gravitomagnetic Moment $\quad$ $g_\Omega = 1$

In “actual” gravity spin and rotation couple in the same way

[C.G. de Oliveira and J. Tiomno, Nuovo Cimento 24 (1962)]

Direct evidence for Spin-Rotation Coupling

Experiment: [B. Venema et al, PRL (1992)]
Interpretation: [B. Mashhoon, Physics Letters A (1995)]

$$^{199}\text{Hg}(I = \tfrac{1}{2}) \qquad ^{201}\text{Hg}(I = \tfrac{3}{2})$$

$$H^i_{\text{int}} = -g^i_{\text{B}} \mu_{\text{N}} \mathbf{B} \cdot \mathbf{S} - g_\Omega \hbar \boldsymbol{\omega}_{\text{Earth}} \cdot \mathbf{S}$$

Zeeman splitting: parallel ΔE_+, anti-parallel ΔE_-

$$\frac{\Delta E^{201}_+}{\Delta E^{199}_+} - \frac{\Delta E^{201}_-}{\Delta E^{199}_-} \simeq 2 \left(1 - \frac{g^{201}_{\text{B}}}{g^{199}_{\text{B}}}\right) \frac{1}{g^{199}_{\text{B}}} g_\Omega \frac{\hbar \omega_{\text{Earth}}}{\mu_{\text{N}} B}$$

From the Data we deduce [Obukhov, Silenko, Teryaev, Int.J. of Modern Physics (2016)]

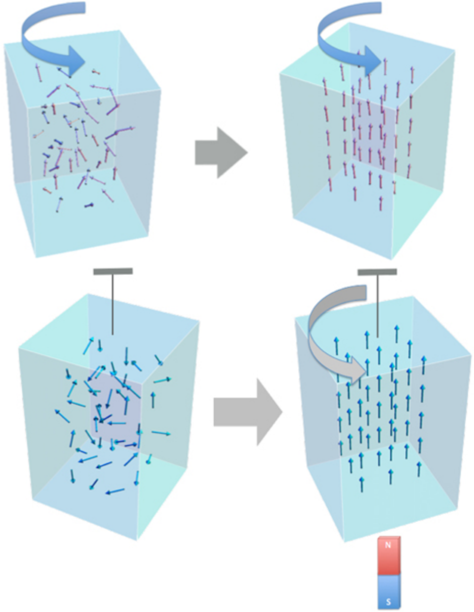
$$\left| \left[g_\Omega \left(^{201}\text{Hg} \right) - 1 \right] + 0.369 \left[g_\Omega \left(^{199}\text{Hg} \right) - 1 \right] \right| < 0.042 \quad (95\%\text{C.L.})$$

Indirect evidence for Spin-Rotation Coupling

Barnett effect:

$$\Delta E = \mathbf{J} \cdot \mathbf{\Omega} = \boldsymbol{\mu} \cdot \mathbf{B}_{\text{Eff}}$$
$$= \gamma \mathbf{J} \cdot \mathbf{B}_{\text{Eff}}$$
$$\mathbf{M} = \chi_B \mathbf{B}_{\text{Eff}} = \frac{\chi_B}{\gamma} \mathbf{\Omega}$$

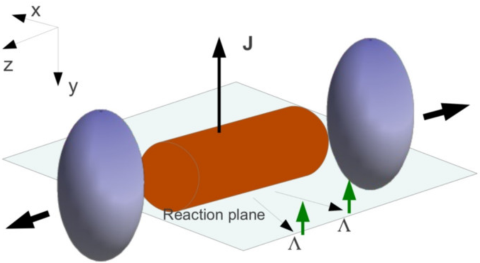
Einstein-de-Haas Effect:



[M. Matsuo, J.Ieda, S. Maekawa, Frontiers in Physics (2015)]

Heavy-ion collisions

Peripheral collisions ⇒ Angular momentum ⇒ Global polarization w.r.t. reaction plane



- Polarization estimated at quark level by spin-orbit coupling

  [Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301]

- By local thermodynamic equilibrium of the spin degrees of freedom

  [F. Becattini, F. Piccinini, Ann. Phys. 323 (2008) 2452; F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906]

Spin polarization

[L. Landau and L. Lifshitz, Statistical Physics (1980); A. Vilenkin, Phys. Rev. D. 21 (1980)]

$$\widehat{\rho} = \frac{1}{Z} \mathrm{e}^{-\beta H} = \frac{1}{Z} \mathrm{e}^{-\beta(H_0 - \mathbf{\Omega} \cdot \mathbf{J})}$$

Quantum field statistical mechanics [Zubarev, (1979); Van Weert (1982); F. Becattini, L. Bucciantini, E. Grossi, L. Tinti, Eur. Phys. J. C 75 (2015)]

$$\widehat{\rho} = \frac{1}{Z} \exp \left\{ -\beta \left[u \cdot \widehat{P} - \frac{1}{2} \omega : \widehat{J} \right] \right\}$$

$$\omega^{\mu\nu} = a^\mu u^\nu - a^\nu u^\mu + \epsilon^{\mu\nu\rho\sigma} u_\rho \Omega_\sigma$$

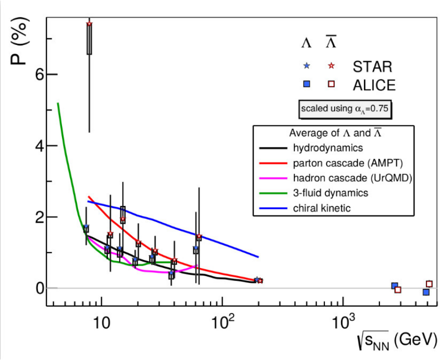
Spin polarization of a Dirac particle

[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)]

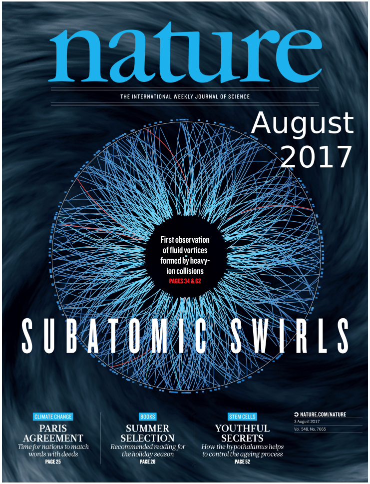
$$S^\mu(k) = -\frac{1}{8m} \epsilon^{\mu\rho\sigma\tau} k_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot k \, n_\mathrm{F} \left(1 - n_\mathrm{F}\right) \beta \omega_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot k \, n_\mathrm{F}}$$

Global polarization (Lambda Polarization)

[STAR Collaboration, Global Lambda hyperon polarization in nuclear collisions, Nature 548 6265, 2017]



[F. Becattini, M. Lisa, Ann. Rev. Part. Nucl. Sc. 70 (2020)]



Anomalous Magnetic Moment

Can the spin-rotation coupling be affected by interactions just like the magnetic moment?

$$\mathcal{L}_{\text{Int}} = J^\mu A_\mu$$

$$\langle p', s'|\widehat{J}^\mu(0)|p, s\rangle = \bar{u}(p', s')\left\{\frac{P^\mu}{2m} + \frac{\mathrm{i}\sigma^{\mu\nu}q_\nu}{2m}\left[1 + F_2(0)\right]\right\}u(p, s) + \mathcal{O}(q^2)$$

The first form factor is protected by charge conservation
g_B is instead anomalous

$$g_B = 2(1 + F_2(0)) = 2\left(1 + \frac{\alpha}{2\pi} - \frac{1}{18}\frac{e^2T^2}{m^2}\right), \quad T \ll m$$

[Schwinger (1948), Fujimoto and Jae (1982), Peressutti and Skagerstam (1982)]

Anomalous Gravitomagnetic Moment

On the contrary,

Gravitomagnetic moment is protected by Einstein Equivalence principle

$$g_{\Omega} = 1$$

Equivalence Principle

motion of a particle in gravity field viewed from an inertial reference frame	=	motion of the particle observed from a frame with acceleration

Larmor's theorem in electrodynamics

motion of a particle of charge q and mass m in a magnetic field $\mathbf{B}$ viewed from an inertial reference frame	=	motion of the particle observed from a frame rotating with frequency $\omega_L = \frac{q\mathbf{B}}{2mc}$

Gravitational Larmor's theorem [B. Mashhoon, Phys. Lett. A 173 (1972)]

(formal equivalence between Lorentz and Coriolis force, Gravitoelectromagnetism)

motion of a particle of mass m in a gravity field with $\mathbf{g} = g^{0i}$ viewed from an inertial reference frame	=	motion of the particle observed from a frame rotating with frequency $\omega_L = \frac{q_g \mathbf{B}_g}{2mc}$ where

$$q_g = 2m, \quad \mathbf{B}_g = \frac{1}{2}\boldsymbol{\nabla} \times \mathbf{g}$$

$$\implies \boldsymbol{\mu}_\Omega = \frac{q_g}{2mc}\mathbf{S} = \frac{1}{c}\mathbf{S} \implies g_\Omega = 1$$

Equivalence Principle in QFT

[Cho and Dass, PRD 14 (1976)]

Lorentz invariance of the local coupling of gravitational field to Energy Momentum Tensor (EMT)

$$\mathcal{L}_{\rm int} = \frac{1}{2} h_{\mu\nu} \widehat{T}^{\mu\nu} \quad h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

+ Conservation of EMT, implies

$$\langle p' | \widehat{T}^{\mu\nu}(0) | p \rangle = \frac{1}{P^0} \left[P^\mu P^\nu + \frac{\mathrm{i}}{4} \left(J^{\mu\alpha} q_\alpha P^\nu + J^{\nu\alpha} q_\alpha P_\mu \right) + O(q^2) \right]$$

at small $q = p' - p$, where

$$P^\mu = \int \mathrm{d}^3 x \, t^{0\mu}(x), \quad J^{\mu\nu} = \int \mathrm{d}^3 x \left(x^\mu t^{0\nu} - x^\nu t^{0\mu} \right)$$

are conserved quantities.

Equivalence Principle in QFT

[Kobzarev and Okun, Zh. Eksperim. i Teor. Fiz. (1962)]

For the Dirac field

$$\langle p', s'|\widehat{T}^{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s') \left[f_1(q^2)\frac{P^\mu P^\nu}{m} + f_2(q^2)\frac{\mathrm{i}\sigma^{(\mu\alpha} q_\alpha P^{\nu)}}{2m} + O(q^2) \right] u(p, s)$$

$$f_1(q^2 = 0) = 1$$

$$f_2(q^2 = 0) = 1 \implies g_\Omega = 1$$

Breaking of Einstein Equivalence Principle

The Einstein Equivalence Principle (EEP) forbids the appearance of an anomalous spin-rotation coupling $g_\Omega \equiv 1$.

[Kobzarev and Okun, Zh. Eksperim. i Teor. Fiz. (1962); Cho and Dass, PRD 14 (1976);
de Oliveira and Tiomno, Nuovo Cim. 24 (1962); Teryaev, Front. Phys. (2016)]

EEP premises do not hold in the presence of a medium
The presence of a thermal bath breaks the Lorentz invariance of the vacuum
$\Rightarrow$ Breaking of EEP is possible at finite temperature

$$\text{QED:} \qquad \frac{m_{\text{Inertial}}}{m_{\text{Gravitational}}} = 1 + \frac{e^2}{3}\frac{T^2}{m^2}$$

[Donoghue, Holstein, and Robinett, PRD 30 (1984) and Gen. Rel. Grav. 17, 207 (1985);
Mitra, Nieves, and Pal, PRD 64 (2001)]

Breaking of EEP at finite temperature

[Donoghue, Holstein, and Robinett, PRD 30 (1984) and Gen. Rel. Grav. 17, 207 (1985)]

- Phase-space mass m_p
  u is the velocity of thermal bath

$$S(p) = \frac{1}{\not{p} - m - \Sigma(p)}, \quad \Sigma(p) = a\not{p} + b\not{u} + c = \Sigma(p)\Big|_{T=0} + \Sigma^\beta(p)$$

Finding the pole: $m_p^2 = \omega^2 - \mathbf{p}^2 \simeq m^2 + \frac{1}{2}\left(\mathrm{tr}\left[\not{p}\Sigma^\beta(p)\right] + m\,\mathrm{tr}\left[\Sigma^\beta(p)\right]\right)$

$$\text{QED: } m_p^2 - m^2 = \begin{cases} \frac{e^2T^2}{6} & T \ll m, \\ \frac{e^2T^2}{8} & T \gg m \end{cases}$$

- Inertial mass m_I

$$(\not{p} - m)\,\psi = e\langle p', s'|\widehat{J}^\mu(0)|p, s\rangle A_\mu \psi \qquad \to \mathrm{i}\frac{\partial}{\partial t}\psi = H\psi$$

$$\mathbf{A} = 0, \mathbf{E} = -\boldsymbol{\nabla}\phi, \quad \mathbf{a} = [H, [H, \mathbf{r}]] = \frac{e\mathbf{E}}{m_p} \implies \boxed{m_I = m_p}$$

- Gravitational mass m_g

Breaking of EEP at finite temperature

[Mitra, Nieves, and Pal, PRD 64 (2001)]

- Gravitational mass m_g

  Scattering theory in linearized gravity $\widehat{H}_{\text{int}} = \frac{1}{2}\int \mathrm{d}^3x\, \widehat{T}_{\mu\nu} h^{\mu\nu}$

$$\mathcal{A} = -\frac{\mathrm{i}(2\pi)\delta(p\cdot u - p'\cdot u)}{\sqrt{Z_2(p)Z_2(p')}}\frac{1}{2}\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle h^{\mu\nu}(q = p' - p)$$

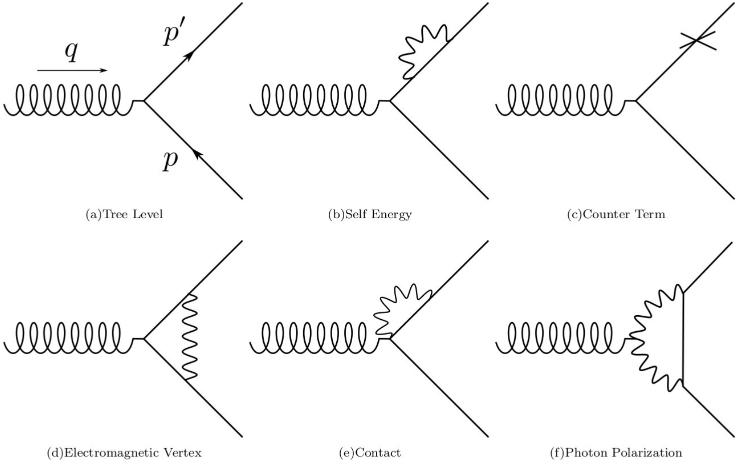
$$h^{\mu\nu}(\mathbf{q}) = 2\phi_g(\mathbf{q})(2u^\mu u^\nu - \eta^{\mu\nu})$$

  Comparing with the amplitude in the Born approximation

$$\mathcal{A} = -\mathrm{i}m_g\phi_g(\mathbf{q}), \qquad \text{from } V(\mathbf{x}) = m_g\phi_g(\mathbf{x})$$

  we obtain m_g

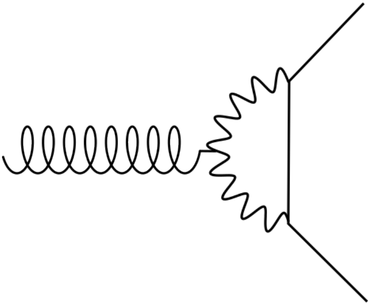
Renormalization at finite temperature



$$\left.\frac{\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle}{\sqrt{Z_2(p)Z_2(p')}}\right|_{p'=p} = \frac{p_\mu p_\nu - \frac{e^2T^2}{6}u_\mu u_\nu}{\sqrt{p^2+m_p^2}}, \quad \frac{m_I}{m_g} = 1 + \frac{e^2}{3}\frac{T^2}{m^2}, \quad T \ll m$$

[Donoghue, Holstein, and Robinett, PRD 30 (1984)]

Can the gravitomagnetic moment be anomalous?



The thermal bath provides the rotated photons

$$\widehat{a}_{\mathrm{B}}^{\dagger}\widehat{a}_{\mathrm{B}}(p)|0\rangle = n_{\mathrm{B}}(E)|0\rangle$$

[MB & D. E. Kharzeev, Phys. Rev. D 103 (2021)]

$$\widehat{H}_{\text{int}} = \frac{1}{2}\int \mathrm{d}^3x\, \widehat{T}_{\mu\nu} h^{\mu\nu}$$

Amplitude

$$\mathcal{A} = -\mathrm{i}(2\pi)\delta(p\cdot u - p'\cdot u)\frac{1}{2}\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle h^{\mu\nu}(p'-p, \boldsymbol{\Omega})$$

$$h_{\mu\nu} = \begin{pmatrix} -(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & 0 & 0 & 0 \\ -x\Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \simeq \begin{pmatrix} 0 & y\Omega & -x\Omega & 0 \\ y\Omega & 0 & 0 & 0 \\ -x\Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\Omega^2)$$

to compare with the spin-rotation coupling amplitude

$$V = -g_\Omega \mathbf{S}\cdot\boldsymbol{\Omega} \Rightarrow \mathcal{A} = -\mathrm{i}(2\pi)\delta(p_0 - p_0')\left[-g_\Omega\, \xi'^\dagger \frac{\boldsymbol{\sigma}}{2}\xi\cdot\boldsymbol{\Omega}\right]$$

Additional form factors appear at finite temperature

$$P = p' + p,\ q = p' - p, \quad \omega_P = P \cdot u,\ P_s = \sqrt{\omega_P^2 - P^2}$$

$$l^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho q_\sigma, \quad \hat{l}^\mu = \frac{l^\mu}{\sqrt{-l^2}}$$

$$\langle p', s' | \widehat{T}_{\mu\nu}(0) | p, s \rangle = \bar{u}(p', s') \Big\{ I_{P\gamma}(P,q) \left(P_\mu \gamma_\nu + P_\nu \gamma_\mu \right) + I_{u\gamma}(P,q) \left(u_\mu \gamma_\nu + u_\nu \gamma_\mu \right) + \\ + I_{Pl}(P,q) \hat{\not{l}} \left(P_\mu \hat{l}_\nu + P_\nu \hat{l}_\mu \right) + I_{ul}(P,q) \hat{\not{l}} \left(u_\mu \hat{l}_\nu + u_\nu \hat{l}_\mu \right) \Big\} u(p,s) + \cdots$$

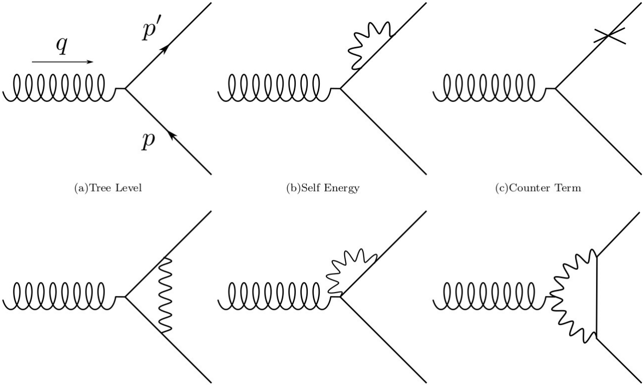
Eventually, we find

$$g_\Omega = \lim_{\substack{q \to 0 \\ P_s \to 0}} 4 \left(I_{P\gamma}(P,q) + \frac{I_{u\gamma}(P,q)}{\omega_P} - I_{Pl}(P,q) - \frac{I_{ul}(P,q)}{\omega_P} \right)$$

[MB & D. E. Kharzeev, Phys. Rev. D 103 (2021)]

Anomalous Gravitomagnetic Moment (AGM)

QED Renormalization



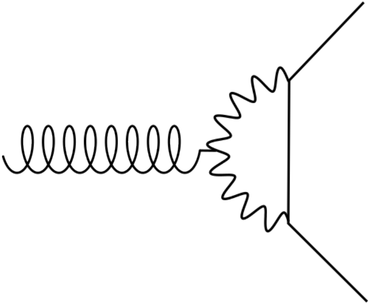
(a)Tree Level (b)Self Energy (c)Counter Term

(d)Electromagnetic Vertex (e)Contact (f)Photon Polarization

$$g_\Omega - 1 = \begin{cases} -\dfrac{1}{6}\dfrac{e^2T^2}{m^2} & T \ll m \\ -\dfrac{5}{36}\dfrac{e^2T^2}{m^2} & T \gg m,\ m > eT \end{cases}$$

[MB & D. E. Kharzeev, Phys. Rev. D 103 (2021)]

Contribution from Photon Polarization



$$g_\Omega^{\mathrm{P}} = \begin{cases} 0 & T \ll m \\ \dfrac{1}{18}\dfrac{e^2T^2}{m^2} & T \gg m,\ m > eT \end{cases}$$

Order of magnitude

QED, Electron, $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

$$\frac{m_I}{m_g} \sim |\Delta g_\Omega| \sim \frac{e^2}{6}\frac{T^2}{m_e^2} = \begin{cases} 4\times 10^{-17} & T = 300\,\mathrm{K} \\ 0.2 & T = 1.7\times 10^{10}\,\mathrm{K} = 1.5\,\mathrm{MeV},\ m_e < T < m_e/e \end{cases}$$

Eötvös type-experiment, max precision 10^{-15}

Heavy-ion collisions

$T \simeq 175 - 300$ MeV, $gT \simeq 270 - 450$ MeV

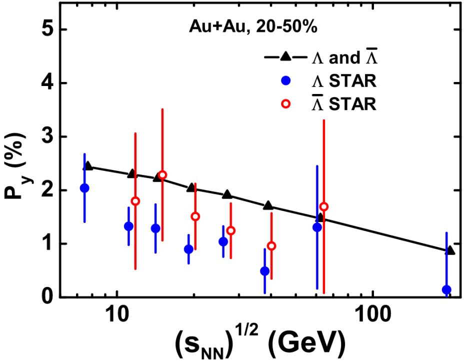
Constituent strange quark mass $m_s \simeq 400$ MeV $\implies T < m_s$

$$g_\Omega^{\mathrm{QCD}} - 1 = -\frac{N_c^2 - 1}{2}\frac{1}{6}\frac{g^2T^2}{m_s^2} \sim -0.3 \div 0.8$$

Detect AGM with spin polarization?

Quark recombination: $P_\Lambda \simeq P_s$

[Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94 (2005); J.-H. Gao, Z.-T. Liang, Q. Wang, and X.-N. Wang, Lect. Notes Phys. 987 (2021)]



[Y. Sun and C. M. Ko, Phys. Rev. C 96 (2017)]

With AGM: $\Delta P_s/P_s \propto -g^2T^2/m_s^2$

We expect the AGM to further reduce polarization as energy increase.

Caveat: Too simplistic model

Connection between the AVE and the AGM

Phenomenology

We want to establish how the AGM affects the Axial Vortical Effect (AVE)

Quantum effects in relativistic fluids

For free massless Dirac field

Electric current:

$$\mathbf{j} = \frac{\mu_{\rm A}}{2\pi^2}\,\mathbf{B} + \frac{\mu\,\mu_{\rm A}}{\pi^2}\,\boldsymbol{\Omega}$$

Chiral Magnetic Effect — Chiral Vortical Effect

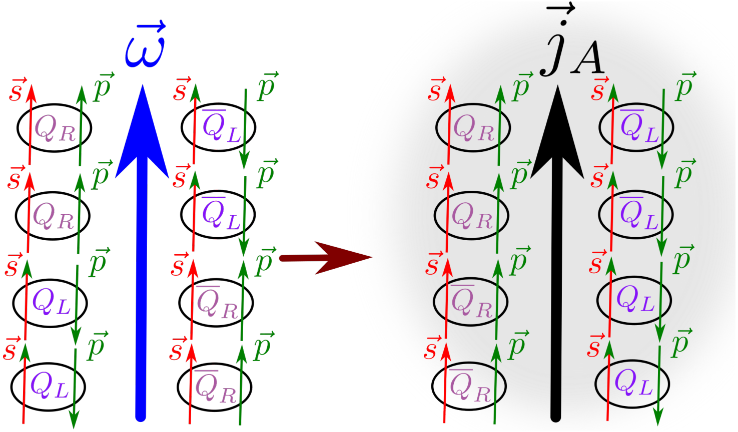
Axial current:

$$\mathbf{j}_{\rm A} = \frac{\mu}{2\pi^2}\,\mathbf{B} + \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} + \frac{\mu_{\rm A}^2}{2\pi^2}\right)\boldsymbol{\Omega}$$

Axial Vortical Effect

$\mathbf{B}$: Magnetic field, $\boldsymbol{\Omega}$: Rotation, T Temperature,
μ: Electric Chemical potential, $\mu_{\rm A}$: Chiral Chemical potential

Massless Dirac fermions



Spin-rotation $\rightarrow \langle \mathbf{s} \rangle \propto \boldsymbol{\omega}$, $\quad \rightarrow \mathbf{j}_\mathrm{A} = n_\mathrm{R}\mathbf{v}_\mathrm{R} - n_\mathrm{L}\mathbf{v}_\mathrm{L} \propto \boldsymbol{\omega}$

Thermodynamics and Symmetries

Symmetries of the density operator imply vanishing thermal expectation values

$$\widehat{\rho} = \frac{1}{Z} \exp\left\{ -\beta \left[u \cdot \widehat{P} - \frac{1}{2}\omega : \widehat{J} - \mu \widehat{Q} \right] \right\}$$

$$\omega^{\mu\nu} = a^\mu u^\nu - a^\nu u^\mu + \epsilon^{\mu\nu\rho\sigma} u_\rho \Omega_\sigma$$

Rotation R and C, P, T symmetries of the density operator need to be broken

	$\widehat{Q}$	Ω	$\mathbf{B}$
R	✓	×	×
C	×	✓	×
P	✓	×	✓
T	✓	✓	×

$$\mathbf{j}_{\mathrm{A}} = \mathrm{tr}\left[\widehat{\rho}\, \widehat{\mathbf{j}}_{\mathrm{A}} \right] = \sigma_B \mathbf{B} + \sigma_\omega \boldsymbol{\omega}$$

Axial Vortical Effect (AVE)

Non interacting Dirac field

[Vilenkin, PRD 20 (1979); Landsteiner, Megias, Melgar, and Pena-Benitez JHEP 09 (2011); Gao, Liang, Pu, Wang, and Wang, PRL 109 (2012)]

Spin $\frac{1}{2}$ Dirac particle at thermal equilibrium in a rotating medium

$$\text{Axial current: } \widehat{j}^\mu_{\rm A} = \Psi\gamma^\mu\gamma^5\Psi$$

$$\langle\widehat{j}^\mu_{\rm A}\rangle = W^{\rm A}\frac{\omega^\mu}{T}$$

$$W^{\rm A}_{\text{Non-Int}} = \frac{1}{2\pi^2\beta}\int \mathrm{d}k\,\frac{\varepsilon_k^2+k^2}{\varepsilon_k}n_{\rm F}(\varepsilon_k)$$

[MB, Grossi, Becattini, JHEP 10 (2017); MB, Lect. Notes Phys. 987 (2021)]

$$\text{Massless field: } W^{\rm A}_{\text{Non-Int}} = \frac{T^3}{6}$$

[review: Kharzeev, Liao, Voloshin, and Wang, Prog. Part. Nucl. Phys. 88 (2016)]

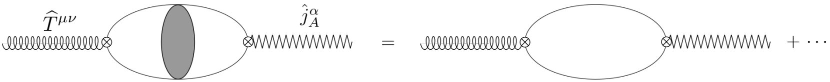
Axial Vortical Effect (AVE)

Conductivity W^{A}

$$\text{Kubo formula: } W^{\mathrm{A}} = \langle\langle \widehat{J}^{3} \widehat{j}_{\mathrm{A}}^{3} \rangle\rangle$$
$$= 2 \int_0^\beta \frac{\mathrm{d}\tau}{\beta} \int \mathrm{d}^3 x \, x^1 \langle \widehat{T}_{\mathrm{B}}^{02}(-\mathrm{i}\tau, \mathbf{x}) \, \widehat{j}_{\mathrm{A}}^{3}(0) \rangle_{\mathrm{T,c}}$$

$$\langle \widehat{O} \rangle_{\mathrm{T,c}} = \mathrm{tr}\left[\frac{1}{Z} \mathrm{e}^{-\frac{\widehat{H}}{T}} \, \widehat{O} \right]$$

Diagrams



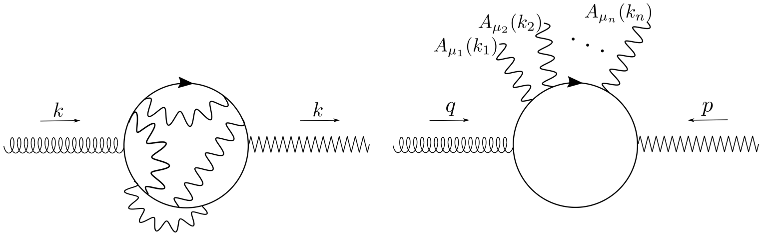
Similarly, for the Chiral Vortical Effect (CVE) $W^{\mathrm{V}} = \langle\langle \widehat{J}^{3} \widehat{j}_{\mathrm{V}}^{3} \rangle\rangle$

Chiral Vortical Effect (CVE)

QED Radiative corrections

[De-Fu Hou, Hui Liu, and Hai-cang Ren, Phys. Rev. D86 (2012)]

[Siavash Golkar and Dam T. Son, JHEP 02 (2015)]



Since $\partial_\mu \widehat{j}^\mu_{\mathrm{V}} = 0$, then for gauge-invariance

$$p^i \Gamma^{(n)}_{ij}(p, q, k_1, \ldots, k_n) = 0, \quad q^i \Gamma^{(n)}_{ij}(p, q, k_1, \ldots, k_n) = 0$$

$\Longrightarrow$ Only the free theory diagram contributes.

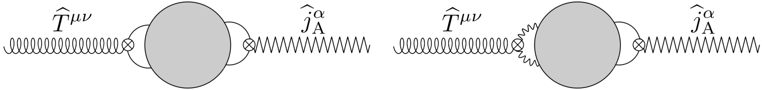
This argument holds for the CVE but not for the AVE.

Axial Vortical Effect (AVE)

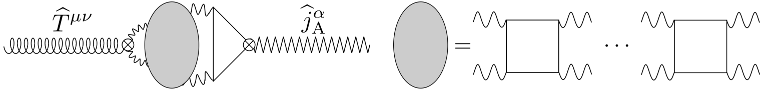
QED Radiative corrections, massless

For massless field

$$\partial_\mu \widehat{j}^\mu_{\mathrm{A}} = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$



Only "gauge" type diagrams connected to anomaly contribute



For massive field

$$\partial_\mu \widehat{j}^\mu_{\mathrm{A}} = 2m\mathrm{i}\bar{\Psi}\gamma^5\Psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

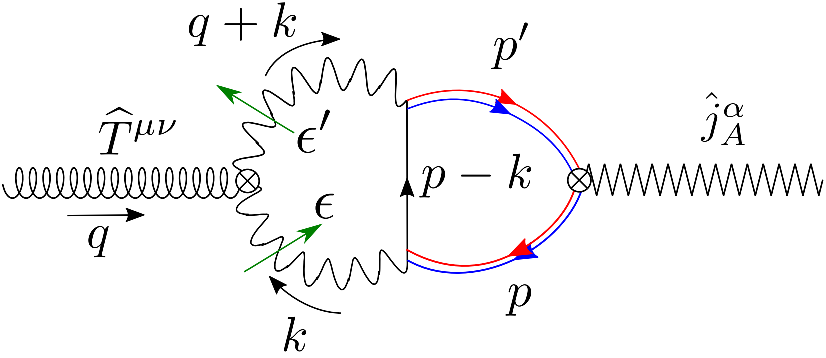
also "fermionic" type diagrams are relevant

Axial Vortical Effect (AVE)

QED Radiative corrections, massless, first loop

$$W^{\mathrm{A}}_{QED,m=0} = \left(\frac{1}{6} + \frac{e^2}{24\pi^2}\right) T^3$$

[D.-F. Hou, H. Liu, and H.-c. Ren, PRD 86 (2012); S. Golkar and D. T. Son, JHEP 02 (2015)]



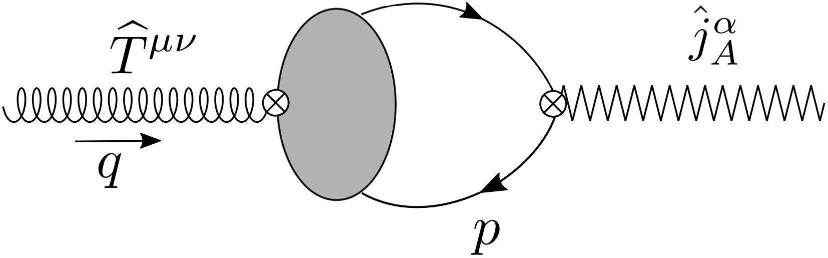
The thermal bath provides the rotated photons

Axial Vortical Effect (AVE)

QED Radiative corrections, massive

For massive field: [MB & D. E. Kharzeev, Phys. Rev. D 103 (2021)]

$$\text{Kubo formula: } W^{\mathrm{A}} = 2\int_0^\beta \frac{\mathrm{d}\tau}{\beta}\int \mathrm{d}^3x\, x^1 \langle \widehat{T}^{02}(-\mathrm{i}\tau, \mathbf{x})\, \widehat{j}^{\,3}_{\mathrm{A}}(0)\rangle_{\mathrm{T,c}}$$



Large mass limit

- the effects of interactions above some energy scale (short distances) are contained in the form factors: $\langle p'|\widehat{T}^{\mu\nu}|p\rangle$ $\langle p'|\widehat{j}^{\,\alpha}_{\mathrm{A}}|p\rangle$;
- the low energy contributions (large distance) are collected into the thermal averages of one-particle states, which correspond to an expansion in $\frac{T}{m}$.

Quantum operators

[Weinberg, QFT Vol I]: Any operator can be written as sums of multi-particle states. For additive operators:

$$\widehat{T}^{\mu\nu}(x) = \frac{1}{2\pi^2}\sum_{\tau,\tau'}\int\frac{\mathrm{d}^3q'}{2\varepsilon_{q'}}\int\frac{\mathrm{d}^3q}{2\varepsilon_q}\widehat{a}^\dagger_{\tau'}(q')\widehat{a}_\tau(q)\mathrm{e}^{\mathrm{i}(q-q')\cdot x}\langle q',\tau'|\widehat{T}^{\mu\nu}(0)|q,\tau\rangle$$
$$+\ \text{anti-particles} + \text{photons}$$

For axial current

$$\widehat{j}^\mu_{\mathrm{A}}(0) = \frac{1}{(2\pi)^3}\sum_{\sigma,\sigma'}\int\frac{\mathrm{d}^3k}{2\varepsilon_k}\frac{\mathrm{d}^3k'}{2\varepsilon_{k'}}\bar{u}_\sigma(k)_{A'}\left(\gamma^\mu\gamma^5\right)_{A'B'}u_{\sigma'}(k')\widehat{a}^\dagger_\sigma(k)_{B'}\widehat{a}_{\sigma'}(k')$$
$$+\ \text{anti-particles}$$

Thermal averages: $\langle\widehat{a}^\dagger_{\tau'}(q')\widehat{a}_{\sigma'}(k')\rangle_T = \delta_{\tau'\sigma'}2\varepsilon_{q'}\delta^3(\mathbf{k}'-\mathbf{q}')n_{\mathrm{F}}(k')$

Axial Vortical Effect (AVE) Conductivity W^{A}

At Low Temperature $T \ll m$, we obtain

$$W^{\mathrm{A}}_{\mathrm{LT}} = -\frac{\mathrm{i}}{(2\pi)^3}\frac{1}{8\beta}\int \frac{\mathrm{d}^3 k}{\varepsilon_k^2} 4n'_{\mathrm{F}}(\varepsilon_k - \mu)\left[\frac{\partial}{\partial k'_x} T(k,k')\right]_{k'=k}$$

$$T(k,k') = \mathrm{tr}\left[M^{02}(k,k')\,(\not{k}+m)\,\gamma^3\gamma^5\left(\not{k}'+m\right)\right]$$

$$\bar{u}(k')M^{\mu\nu}(k,k')\,u(k) \equiv \langle k'|\widehat{T}^{\mu\nu}(0)|k\rangle$$

The matrix elements are renormalized at finite temperature

$$\langle p',s'|\widehat{T}_{\mu\nu}(0)|p,s\rangle = \bar{u}(p',s')\Big\{I_{P\gamma}(P,q)\,(P_\mu\gamma_\nu + P_\nu\gamma_\mu) + I_{u\gamma}(P,q)\,(u_\mu\gamma_\nu + u_\nu\gamma_\mu) + \\ + I_{Pl}(P,q)\hat{\not{l}}\left(P_\mu\hat{l}_\nu + P_\nu\hat{l}_\mu\right) + I_{ul}(P,q)\hat{\not{l}}\left(u_\mu\hat{l}_\nu + u_\nu\hat{l}_\mu\right)\Big\}u(p,s) + \cdots$$

The trace recombine the form factors to create the gravitomagnetic moment

$$T(k,k') = \frac{g_\Omega(P,q)}{4}\mathrm{tr}\left[\left(\gamma^0 P^2 + \gamma^2 P^0\right)(\not{k}+m)\,\gamma^3\gamma^5\left(\not{k}'+m\right)\right]$$

Axial Vortical Effect (AVE) Conductivity W^{A}

$$g_\Omega(\varepsilon_k) = 4\left(I_{P\gamma}(\varepsilon_k) + \frac{I_{u\gamma}(\varepsilon_k)}{\varepsilon_k} - I_{Pl}(\varepsilon_k) - \frac{I_{ul}(\varepsilon_k)}{\varepsilon_k}\right)$$

$T \ll m$

$$W^{\mathrm{A}} \simeq \frac{1}{2\pi^2\beta}\int \mathrm{d}k\left[g_\Omega(\varepsilon_k)\frac{\varepsilon_k^2 + k^2}{\varepsilon_k} + g'_\Omega(\varepsilon_k)\left(k^2 + \frac{k^4}{3\varepsilon_k^2}\right)\right] n_{\mathrm{F}}(\varepsilon_k - \mu)$$

Connection between AVE and AGM

Radiative corrections to the AVE $\iff$ $g_\Omega(\varepsilon_k) \neq 1$

$$\Delta W^{\mathrm{A}} = -\frac{1}{6}\frac{e^2T^2}{m^2}W^{\mathrm{A}}_{\mathrm{free}} \simeq -\frac{1}{6}\frac{e^2T^2}{m^2}\frac{(mT)^{3/2}}{\sqrt{2}\pi^{3/2}}\mathrm{e}^{-(m-\mu)/T}.$$

Summary

1. Gravitomagnetic moment of a Dirac massive particle at **thermal** equilibrium is anomalous; AGM for QED:

$$g_\Omega - 1 = \begin{cases} -\frac{1}{6}\frac{e^2T^2}{m^2} & T \ll m \\ -\frac{5}{36}\frac{e^2T^2}{m^2} & T \gg m,\ m > eT \end{cases}$$

2. AGM causes radiative corrections to the axial vortical effect

$$\begin{aligned} W^{\mathrm{A}} =& \frac{1}{2\pi^2\beta}\int \mathrm{d}k \left[g_\Omega(\varepsilon_k)\frac{\varepsilon_k^2 + k^2}{\varepsilon_k} + g'_\Omega(\varepsilon_k)\left(k^2 + \frac{k^4}{3\varepsilon_k^2}\right)\right] n_{\mathrm{F}}(\varepsilon_k - \mu) \\ \simeq& \left(1 - \frac{1}{6}\frac{e^2T^2}{m^2}\right)\frac{(mT)^{3/2}}{\sqrt{2}\pi^{3/2}}\mathrm{e}^{-(m-\mu)/T} \end{aligned}$$

3. Could an AGM be revealed in heavy-ion collision?

Backup

Different definitions of form factors

[Kobzarev and Okun, Zh. Eksperim. i Teor. Fiz. (1962)]

$$\langle p', s'|\widehat{T}^{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s')\left[f_1(q^2)\frac{P^\mu P^\nu}{m} + f_2(q^2)\frac{\mathrm{i}\sigma^{(\mu\alpha}q_\alpha P^{\nu)}}{2m} + O(q^2)\right]u(p, s)$$

$$f_1(q^2 = 0) = 1$$

$$f_2(q^2 = 0) = 1 \implies g_\Omega = 1$$

Or using Gordon Identity

$$\bar{u}(p', s')\gamma^\mu u(p, s) = \bar{u}(p', s')\left[\frac{P^\mu}{2m} + \mathrm{i}\frac{\sigma^{\mu\alpha}q_\alpha}{2m}\right]u(p, s)$$

$$\langle p', s'|\widehat{T}^{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s')\left[I_{P\gamma}(q^2)P^{(\mu}\gamma^{\nu)} + I_{PP}(q^2)\frac{P^\mu P^\nu}{m} + O(q^2)\right]u(p, s)$$

$$I_{P\gamma}(q^2) = f_2(q^2), \quad I_{PP}(q^2) = f_1(q^2) - f_2(q^2)$$