# Early time gluon fields in relativistic heavy ion collisions 

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## outline:

1. introduction
first phase produced in a heavy ion collision

- called a glasma

2. motivation
provides initial conditions for subsequent hydro phase
3. structure of the calculation

ColourGlassCondensate effective field theory approach
4. results:
4.1 isotropization
4.2 azimuthal asymmetries
4.3 angular momentum
4.4 momentum broadening of hard probes
5. conclusions

## introduction

drawing of stages of a heavy ion collision


CGC = high energy density largely gluonic matter

- associated with wavefunction of a high energy hadron
- initial state in high energy hadronic collisions after collision CGC fields are transformed into glasma fields
- initially longitudinal color electric and magnetic fields
space-time diagram

collision axis is the $z$-axis
$\rightarrow$ incoming nuclei move along the $x^{ \pm}=(t \pm z) / \sqrt{2}$ axes collision at the origin post-collision region is the forward light cone


## motivation

goal: describe early time ( $\tau \leq 1 \mathrm{fm}$ ) dynamics of HIC

- evolution of system during this early stage not well understood
- importance: initial conditions for subsequent hydro evolution more generally: want to understand transition between early-time dynamics $\longrightarrow$ hydro phase

1. microscropic theory of non-abelian gauge fields
= far from equilibrium
2. macroscopic effective theory

- based on universal conservation laws
- valid close to equilibrium

MEC, Czajka, Mrówczyński
arXiv:2012.03042; 2105.05327; 2112.0681; 2202.00357

## Colour Glass Condensate (CGC) effective theory

method is based on a separation of scales between

1. valence partons with large nucleon momentum fraction ( $x$ )
2. gluon fields with small $x$ and large occupation numbers

- gluons are in the saturation regime
- distributions are controlled by the saturation scale $Q_{s}$ dynamics of gluon fields determined from classical YM equation
$\rightarrow$ source provided by the valence partons


## method - notation

light-cone coordinates $x^{ \pm}=(t \pm z) / \sqrt{2}$
Milne coordinates $\tau=\sqrt{2 x^{+} x^{-}}=\sqrt{t^{2}-z^{2}}$ and $\eta=\ln \left(x^{+} / x^{-}\right) / 2=\ln ((t+z) /(t-z))$.

$$
\begin{aligned}
\text { gauge: } & A_{\text {milne }}^{\mu}=\theta(\tau)\left(0, \alpha\left(\tau, \vec{x}_{\perp}\right), \vec{\alpha}_{\perp}\left(\tau, \vec{x}_{\perp}\right)\right) \\
\text { sources: } & J^{\mu}(x)=J_{1}^{\mu}(x)+J_{2}^{\mu}(x) \\
& J_{1}^{\mu}(x)=\delta^{\mu+} g \rho_{1}\left(x^{-}, \vec{x}_{\perp}\right) \text { and } J_{2}^{\mu}(x)=\delta^{\mu-} g \rho_{2}\left(x^{+}, \vec{x}_{\perp}\right)
\end{aligned}
$$

ansatz:

$$
\begin{aligned}
& A^{+}(x)=\Theta\left(x^{+}\right) \Theta\left(x^{-}\right) x^{+} \alpha\left(\tau, \vec{x}_{\perp}\right) \\
& A^{-}(x)=-\Theta\left(x^{+}\right) \Theta\left(x^{-}\right) x^{-} \alpha\left(\tau, \vec{x}_{\perp}\right) \\
& A^{i}(x)=\Theta\left(x^{+}\right) \Theta\left(x^{-}\right) \alpha_{\perp}^{i}\left(\tau, \vec{x}_{\perp}\right)+\Theta\left(-x^{+}\right) \Theta\left(x^{-}\right) \beta_{1}^{i}\left(x^{-}, \vec{x}_{\perp}\right)+\Theta\left(x^{+}\right) \Theta\left(-x^{-}\right) \beta_{2}^{i}\left(x^{+}, \vec{x}_{\perp}\right)
\end{aligned}
$$


step 1: solve YM equation in the pre-collision region

$$
\left[D_{\mu}, F^{\mu \nu}\right]=J^{\nu} \quad \text { with } F_{\mu \nu}=\frac{i}{g}\left[D_{\mu}, D_{\nu}\right] \text { and } D_{\mu}=\partial_{\mu}-i g A_{\mu}
$$

$\rho_{1}\left(x^{+}, \vec{x}_{\perp}\right) \rightarrow \beta_{1}^{i}\left(x^{-}, \vec{x}_{\perp}\right)$ and $\rho_{2}\left(x^{+}, \vec{x}_{\perp}\right) \rightarrow \beta_{2}^{i}\left(x^{+}, \vec{x}_{\perp}\right)$
for the first ion:

$$
\begin{aligned}
& \beta_{1}^{i}\left(x^{-}, \vec{x}_{\perp}\right)=\frac{i}{g} U_{1}^{\dagger}\left(x^{-}, \vec{x}_{\perp}\right) \partial^{i} U_{1}\left(x^{-}, \vec{x}_{\perp}\right) \\
& U_{1}\left(x^{-}, \vec{x}_{\perp}\right)=\mathcal{P} \exp \left[i g \int_{-\infty}^{x^{-}} d z^{-} \Lambda_{1}\left(z^{-}, \vec{x}_{\perp}\right)\right] \\
& \Lambda_{1}\left(x^{-}, \vec{x}_{\perp}\right)=\frac{1}{2 \pi} \int d^{2} z_{\perp} K_{0}\left(m\left(\vec{x}_{\perp}-\vec{z}_{\perp}\right)\right) \rho_{1}\left(x^{-}, \vec{z}_{\perp}\right)
\end{aligned}
$$

$K_{0}$ is a modified Bessel function similar expression for second ion physics:

1. $\rho_{1}\left(x^{-}, \vec{x}_{\perp}\right)$ is independent of the light-cone time $x^{+}$

- the static approximation

2. small width across light cone will be taken to 0
step 2: boundary conditions

$$
\begin{aligned}
& \alpha_{\perp}^{i}\left(0, \vec{x}_{\perp}\right)=\alpha_{\perp}^{i(0)}\left(\vec{x}_{\perp}\right)=\lim _{w \rightarrow 0}\left(\beta_{1}^{i}\left(x^{-}, \vec{x}_{\perp}\right)+\beta_{2}^{i}\left(x^{+}, \vec{x}_{\perp}\right)\right) \\
& \alpha\left(0, \vec{x}_{\perp}\right)=\alpha^{(0)}\left(\vec{x}_{\perp}\right)=-\frac{i g}{2} \lim _{w \rightarrow 0}\left[\beta_{1}^{i}\left(x^{-}, \vec{x}_{\perp}\right), \beta_{2}^{i}\left(x^{+}, \vec{x}_{\perp}\right)\right]
\end{aligned}
$$

step 3: glasma fields (at early times) with proper time expansion

$$
\alpha\left(\tau, \vec{x}_{\perp}\right)=\alpha\left(0, \vec{x}_{\perp}\right)+\tau \alpha^{(1)}\left(\vec{x}_{\perp}\right)+\tau^{2} \alpha^{(2)}\left(\vec{x}_{\perp}\right)+\cdots
$$

and similarly for $\alpha_{\perp}^{i}\left(\tau, \vec{x}_{\perp}\right) \ldots$ (dimensionless small parameter is $\left.\tilde{\tau}=\tau Q_{s}\right)$ coefs of expansion: require vector potential satisfies sourceless YM eqn

$$
\left[D_{\mu}, F^{\mu \nu}\right]=0 \text { with } F_{\mu \nu}=\frac{i}{g}\left[D_{\mu}, D_{\nu}\right] \quad \text { and } \quad D_{\mu}=\partial_{\mu}-i g A_{\mu}
$$

$\rightarrow \alpha^{(n)}\left(\vec{x}_{\perp}\right)$ and $\vec{\alpha}_{\perp}^{(n)}\left(\vec{x}_{\perp}\right)$ in terms of $\alpha\left(0, \vec{x}_{\perp}\right)$ and $\vec{\alpha}_{\perp}\left(0, \vec{x}_{\perp}\right)$
R. J. Fries, J. I. Kapusta and Y. Li, Nucl. Phys. A 774, 861 (2006).
summary of method:

$$
\underbrace{\rho^{n}\left(x^{ \pm}, \vec{x}_{\perp}\right)}_{\text {valence parton sources }} \rightarrow \underbrace{\beta^{n}\left(x^{ \pm}, \vec{x}_{\perp}\right)}_{\text {CGC pre-collision fields }} \rightarrow \underbrace{\alpha\left(0, \vec{x}_{\perp}\right)}_{\text {initial glasma fields (boost invariant) }} \rightarrow \underbrace{\alpha\left(\tau, \vec{x}_{\perp}\right)}_{\text {glasma fields }}
$$

next: colour charge distributions are not known

- assume Gaussian distribution of colour charges in each nucleus
- a product of sources is replace by its average over this distro an average over a Gaussian distribution of independent random variables
$\rightarrow$ sum over the averages of all possible pairs (Wick's theorem)
idea of CGC: local fluctuations $\propto$ surface colour charge density $\mu$

$$
\left\langle\rho_{1}\left(x^{-}, \vec{x}_{\perp}\right) \rho_{1}\left(y^{-}, \vec{y}_{\perp}\right)\right\rangle \propto g^{2} \mu_{1}\left(\vec{x}_{\perp}\right) \delta\left(x^{-}-y^{-}\right) \delta^{2}\left(\vec{x}_{\perp}-\vec{y}_{\perp}\right)
$$

analogous expressions for the second ion
glasma graph approximation $\longrightarrow$
result for correlator of 2 potentials: $\left(\vec{R}=\frac{1}{2}\left(\vec{x}_{\perp}+\vec{y}_{\perp}\right), \vec{r}=\vec{x}_{\perp}-\vec{y}_{\perp}\right)$

$$
\begin{aligned}
& \delta_{a b} B^{i j}\left(\vec{x}_{\perp}, \vec{y}_{\perp}\right) \equiv \lim _{w \rightarrow 0}\left\langle\beta_{a}^{i}\left(x^{-}, \vec{x}_{\perp}\right) \beta_{b}^{j}\left(y^{-}, \vec{y}_{\perp}\right)\right\rangle \\
& \lim _{r \rightarrow 0} B^{i j}\left(\vec{x}_{\perp}, \vec{y}_{\perp}\right)=\delta^{i j} g^{2} \frac{\mu(\vec{R})}{8 \pi}\left(\ln \left(\frac{Q_{s}^{2}}{m^{2}}+1\right)-\frac{Q_{s}^{2}}{Q_{s}^{2}+m^{2}}\right)+\cdots
\end{aligned}
$$

infra-red regulator $m \sim \Lambda_{\mathrm{QCD}} \sim 0.2 \mathrm{GeV}$ ultra-violet regulator $=$ saturation scale $=Q_{s}=2 \mathrm{GeV}$
dots indicate we have kept terms to 2 nd order in grad expansion of $\mu(\vec{R})$
J. Jalilian-Marian, A. Kovner, L. McLerran, H. Weigert, Phys. Rev. D 55, 5414 (1997)
H. Fujii, K. Fukushima, Y. Hidaka, Phys. Rev. C 79, 024909 (2009)
G. Chen, R. Fries, J. Kapusta, Y. Li, Phys. Rev. C 92, 064912 (2015)

## surface charge density $\mu$

must specify the form of the surface colour charge density $\mu\left(\vec{x}_{\perp}\right)$

- 2-dimensional projection of a Woods-Saxon potential
$\mu\left(\vec{x}_{\perp}\right)=\left(\frac{A}{207}\right)^{1 / 3} \frac{\bar{\mu}}{2 a \log \left(1+e^{R_{A} / a}\right)} \int_{-\infty}^{\infty} d z \frac{1}{1+\exp \left[\left(\sqrt{\left(\vec{x}_{\perp}\right)^{2}+z^{2}}-R_{A}\right) / a\right]}$
$R_{A}$ and $a=$ radius and skin thickness of nucleus mass number $A$ $r_{0}=1.25 \mathrm{fm}, a=0.5 \mathrm{fm} \rightarrow$ when $A=207$ gives
$R_{A}=r_{0} A^{1 / 3}=7.4 \mathrm{fm}$
normalization: $\mu(\overrightarrow{0})=\bar{\mu}=Q_{s}^{2} / g^{4}$ lead nucleus
$g^{2} \sqrt{\bar{\mu}}=$ McLerran-Venugopalan (MV) scale
proportional to $Q_{s}$ - exact value not determined with CGC approach
** numerical results for $\mathcal{E} \ldots$ are order of magnitude estimates ratios of different elements of the energy momentum tensor
$\rightarrow$ will have much weaker dependence on the MV scale.


## gradient expansion

the parameter that we assume small is $\delta=\frac{\left|\nabla^{i} \mu(\vec{R})\right|}{m \mu(\vec{R})}$

derivatives are appreciable only in a very small region at the edges
non-zero impact parameter (non-central collisions)
expand $\mu_{1 / 2}\left(\vec{z}_{\perp}\right)$ around ave coord $\vec{R} \mp \vec{b} / 2$

## summary of method:

YM eqn with average over gaussian distributed valence sources
$\rightarrow$ correlators of pre-collision fields
$\rightarrow$ glasma field correlators (b. conds, sourceless YM eqn, $\tau$ exp)
$\rightarrow$ correlators of glasma chromodynamic $\vec{E}$ and $\vec{B}$ fields
$\Rightarrow$ observables
we work to order $\tau^{5}$ or $\tau^{6}$ and study

1. isotropization of transverse/longitudinal pressures
2. azimuthal momentum distribution and spatial eccentricity
3. angular momentum
4. momentum broadening of hard probes
comment: many numerical approaches to study initial dynamics our method is fully analytic

- allows control over different approximations and sources of errors
- can be systematically extended
- it has limitations (classical / no fluctuations of positions of nucleons)


## isotropization

at $\tau=0^{+}$the energy-momentum tensor has the diagonal form

$$
T(\tau=0)=\left(\begin{array}{cccc}
\mathcal{E}_{0} & 0 & 0 & 0 \\
0 & -\mathcal{E}_{0} & 0 & 0 \\
0 & 0 & \mathcal{E}_{0} & 0 \\
0 & 0 & 0 & \mathcal{E}_{0}
\end{array}\right)
$$

$\rightarrow$ the longitudinal pressure is large and negative

- system is far from equilibrium
if the system approaches equilibrium as it evolves:
- the longitudinal pressure must grow
- transverse pressure must decrease ( $T_{\mu \nu}$ is traceless)
to compare longitudinal and transverse pressures $\left(\tilde{\tau} \equiv \tau Q_{s}\right)$

$$
A_{T L} \equiv \frac{3\left(p_{T}-p_{L}\right)}{2 p_{T}+p_{L}}
$$

J. Jankowski, S. Kamata, M. Martinez and M. Spaliński, Phys. Rev. D 104, 074012 (2021).
in equilibrium ( $p_{L}=p_{T}=\mathcal{E} / 3$ ) $\longrightarrow A_{T L}=0$

$R=5 \mathrm{fm}, \eta=0$ and $b=0$.
faster isotropization with smaller impact parameter
$\rightarrow$ increased region of overlap

reaction plane defined by collision axis and impact parameter $\phi=0$ is in reaction plane
$\phi=\pi / 2$ is perpendicular to reaction plane

$A_{T L}$ at order $\tau^{6}$
$\tau=0.04 \mathrm{fm}$ (left panel)
$\tau=0.045 \mathrm{fm}$ (centre panel)
$\tau=0.05 \mathrm{fm}$ (right panel)
the axes show $R_{x}$ and $R_{y}$ in fm

## dependence on confinement and saturation scales

the correlator $\left\langle\beta_{a}^{i}\left(x^{-}, \vec{x}_{\perp}\right) \beta_{b}^{j}\left(y^{-}, \vec{y}_{\perp}\right)\right\rangle$
depends on two regulators: $m$ (infra-red) and $Q_{s}$ (ultra-violet)

- physically related to confinement / saturation scales
$\rightarrow$ constraints on how to choose them
we used: $m=0.2 \mathrm{GeV}$ and $Q_{s}=2.0 \mathrm{Gev}$ - standard choices
- want results $\approx$ independent of these numbers
- especially since the two scales are pretty close together
$A_{T L}$ at order $\tau^{6}$ as a function of time
3 different values of $Q_{s}$ with $m=0.2 \mathrm{GeV}$ (left)
3 different values of $m$ with $Q_{s}=2.0 \mathrm{GeV}$ (right)
$R=5 \mathrm{fm}, b=0$ and $\eta=0$ at order $\tau^{6}$
$\Rightarrow$ dependence on these scales is weak




## Radial Flow

transverse momentum flow vector $=T_{i 0}$ (trans. Poynting vector) radial flow of the expanding glasma $=$ radial projection $P \equiv \hat{R}_{i} T_{i 0}$ $\phi=\pi / 2$ is perpendicular to the reaction plane


- at lowest order $P$ increases linearly with time
- including higher order contros $\rightarrow P$ slows as system expands
- order $\tau^{5}$ shows flattening up to about $\tilde{\tau}=0.5$
$\rightarrow$ indicates the limit of validity of the $\tau$ expansion


## Azimuthal asymmetry

in a non-central collision - initial spatial asymmetry relativistic collision $\rightarrow$ spatial asymmetries rapidly decrease $\rightarrow$ anisotropic momentum flow can develop only in the first $\mathrm{fm} / \mathrm{c}$

- sensitive to system properties very early in its evolution
- provides direct information about the early stages of the system

$$
\begin{aligned}
& \varphi\left(\vec{x}_{\perp}\right)=\tan ^{-1}\left(\frac{T^{0 y}\left(\vec{x}_{\perp}\right)}{T^{0 x}\left(\vec{x}_{\perp}\right)}\right) \\
& W\left(\vec{x}_{\perp}\right) \equiv \sqrt{\left(T^{0 x}\left(\vec{x}_{\perp}\right)\right)^{2}+\left(T^{0 y}\left(\vec{x}_{\perp}\right)\right)^{2}} \\
& P(\phi) \equiv \frac{1}{\Omega} \int d^{2} \vec{x}_{\perp} \delta\left(\phi-\varphi\left(\vec{x}_{\perp}\right)\right) W\left(\vec{x}_{\perp}\right), \quad \Omega \equiv \int d^{2} \vec{x}_{\perp} W\left(\vec{x}_{\perp}\right) \\
& P(\phi)=\frac{1}{2 \pi}\left(1+2 \sum_{n=1}^{\infty} v_{n} \cos (n \phi)\right) \\
& v_{n}=\int_{0}^{2 \pi} d \phi \cos (n \phi) P(\phi)
\end{aligned}
$$




used $\eta=0.5$ and $\tau=0.04 \mathrm{fm}$.
$v_{2}$ and $v_{3}$ are $\sim$ experimental values
$v_{1}$ is bigger than expected
note: usually assumed anisotropy develops mostly during hydro evolution $\rightarrow$ our results for all three Fourier coefficients are large
comment:
experimentally: impact parameter / reaction plane not known exactly our calculation does not correspond exactly to what is measured

- spatial deviations from azimuthal symmetry

$$
\varepsilon_{n}=-\frac{\int d^{2} \vec{R}|\vec{R}| \cos (n \phi) \mathcal{E}(\vec{R})}{\int d^{2} \vec{R}|\vec{R}| \mathcal{E}(\vec{R})} \text { with } \phi=\tan ^{-1}\left(R_{y} / R_{x}\right)
$$

where $\mathcal{E}(\vec{R})$ denotes the energy density

$\tau=0.04 \mathrm{fm}$ and $\eta=0 \quad$ [normalized to 1 at $b=0.5 \mathrm{fm}$ ]
$\rightarrow$ correlation btwn spatial asymmetry introduced by the initial geometry and anisotropy of azimuthal momentum distribution
these correlations $\sim$ characteristic of onset of hydrodynamic behaviour

## angular momentum

define tensor $M^{\mu \nu \lambda}=T^{\mu \nu} R^{\lambda}-T^{\mu \lambda} R^{\nu}$ with $R^{\mu}=(\tau, \eta, \vec{R})$
$\nabla_{\mu} M^{\mu \nu \lambda}=0 \rightarrow$ conserved charges $J^{\nu \lambda}=\int_{\Sigma} d^{3} y \sqrt{|\gamma|} n_{\mu} M^{\mu \nu \lambda}$

- $n^{\mu}$ is a unit vector perpendicular to the hypersurface $\Sigma$
- $\gamma$ is the induced metric on this hypersurface
- $d^{3} y$ is the corresponding volume element
$n^{\mu}=(1,0,0,0)$ in Milne coordinates
$\rightarrow J^{\nu \lambda}$ defined on a hypersurface of constant $\tau$
Pauli-Lubanski vector: $L_{\mu}=-\frac{1}{2} \epsilon_{\mu \alpha \beta \gamma} J^{\alpha \beta} u^{\gamma}$
result: angular momentum per unit rapidity (symmetric collision)

$$
\frac{d L^{y}}{d \eta}=-\tau^{2} \int d^{2} \vec{R} R^{x} T^{0 z}
$$

result:

ions moving in $+/-z$ dirns displaced in $+/-x$ dirns $\rightarrow L_{y}$ is negative
warning: dominant contro to $\vec{L}$ from regions farthest from collision centre $=$ regions gradient expansion is least trusted $\rightarrow$ error bars large
comparison:
$L_{y} \sim 10^{5}$ at RHIC energies for initial system of colliding ions
J. H. Gao, S. W. Chen, W. t. Deng, Z. T. Liang, Q. Wang and X. N. Wang


- even larger at LHC energies
F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C 77, 024906 (2008).
idea: initial rapid rotation of glasma
$\rightarrow$ could be observed via polarization of final state hadrons
- large $\vec{L} \&$ spin-orbit coupling $\rightarrow$ alignment of spins with $\vec{L}$
many experimental searches for this polarization
- effect of a few percent observed at RHIC
- at LHC result consistent with zero
- difficult to measure . . .
F. Becattini, M.A. Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020).
these results are consistent our calculation:
glasma carries only tiny imprint of the $\vec{L}$ of the intial state $\rightarrow$ majority of the angular momentum is carried by valence quarks


## hard probes - momentum broadening

idea:
hard probes produced via hard interactions at earliest phase of HIC

- propagate through the evolving medium
- suppression of high- $p_{T}$ probes (jet quenching)
$\Rightarrow$ signal of formation of QGP
- deconfined state of matter $=$ significant braking of hard partons heavy quarks:
- rare constituents of the quark-gluon plasma
- external and clean probes of the medium

EL and MB of hard probes / equilibrium plasma studied extensvely

- contro from pre-equilibrium phases has been largely ignored however, see for example:
Ruggieri, Das et al, Phys. Rev. D 98, 094024 (2018)
Boguslavski, Kurkela, Lappi, Peuron, JHEP 09, 077 (2020)
Ipp, Müller, Schuh, Phys. Lett. B 810, 135810 (2020)
physics: frequent small $\vec{p}$ exchanges btwn probe and glasma fields $\rightarrow$ transport equation in Fokker-Planck form
- describes interactions of hard probe interacting with glasma fields

$$
\left(\mathcal{D}-\nabla_{p}^{\alpha} X^{\alpha \beta}(\vec{v}) \nabla_{p}^{\beta}-\nabla_{p}^{\alpha} Y^{\alpha}(\vec{v})\right) n(t, \vec{x}, \vec{p})=0
$$

notation: $\alpha \in(1,2,3)$
$n(t, \vec{x}, \vec{p})=$ distribution function of heavy quarks
$\vec{v}=\vec{p} / E_{\vec{p}}=\vec{p} / \sqrt{p^{2}+m_{Q}^{2}}=$ velocity of heavy quark
$\mathcal{D} \equiv \frac{\partial}{\partial t}+\vec{v} \cdot \vec{\nabla}=$ material derivative (drift term)
$Y^{\alpha}$ related to collisional energy loss
$X^{\alpha \beta}$ related to momentum broadening
St. Mrówczyński, Eur. Phys. J. A 54, 43 (2018).

$$
\begin{aligned}
\hat{q} & =\frac{1}{v}\left(\delta^{\alpha \beta}-\frac{v^{\alpha} v^{\beta}}{v^{2}}\right) \frac{\left\langle\Delta p^{\alpha} \Delta p^{\beta}\right\rangle}{\Delta t} \\
& =\frac{2}{v}\left(\delta^{\alpha \beta}-\frac{v^{\alpha} v^{\beta}}{v^{2}}\right) X^{\alpha \beta}(\vec{v}) \\
X^{\alpha \beta}(\vec{v}) & \equiv \frac{1}{2 N_{c}} \int_{0}^{t} d t^{\prime} \operatorname{Tr}\left[\left\langle\mathcal{F}^{\alpha}(t, \vec{x}) \mathcal{F}^{\beta}\left(t-t^{\prime}, \vec{x}-\vec{v} t^{\prime}\right)\right\rangle\right]
\end{aligned}
$$

colour Lorentz force: $\overrightarrow{\mathcal{F}}(t, \vec{x}) \equiv g(\vec{E}(t, \vec{x})+\vec{v} \times \vec{B}(t, \vec{x}))$

$$
\begin{aligned}
X^{\alpha \beta}(\mathbf{v})= & \frac{g^{2}}{2 N_{c}} \int_{0}^{t} d t^{\prime}\left[\left\langle E_{a}^{\alpha}(t, \vec{x}) E_{a}^{\beta}\left(t-t^{\prime}, \vec{y}\right)\right\rangle+\epsilon^{\beta \gamma \gamma^{\prime}} v^{\gamma}\left\langle E_{a}^{\alpha}(t, \vec{x}) B_{a}^{\gamma^{\prime}}\left(t-t^{\prime}, \vec{y}\right)\right\rangle\right. \\
& \left.+\epsilon^{\alpha \gamma \gamma^{\prime}} v^{\gamma}\left\langle B_{a}^{\gamma^{\prime}}(t, \vec{x}) E_{a}^{\beta}\left(t-t^{\prime}, \vec{y}\right)\right\rangle+\epsilon^{\alpha \gamma \gamma^{\prime}} \epsilon^{\beta \delta \delta^{\prime}} v^{\gamma} v^{\delta}\left\langle B_{a}^{\gamma^{\prime}}(t, \mathbf{x}) B_{a}^{\delta^{\prime}}\left(t-t^{\prime}, \vec{y}\right)\right\rangle\right]
\end{aligned}
$$

where $\vec{y}=\vec{x}-\vec{v} t^{\prime}$.
note: combination of two approaches

1. medium that the hard probe interacts with is a glasma
$\rightarrow$ described with CGC effective theory with proper time expansion ** description is valid only at very early times
2. FP eqn describes interactions of hard probe with glasma fields
** valid at times long enough that collision terms saturate
$\Rightarrow$ conflict btwn assumptions that set these two time scales also:

- FP description requires gradient expansion type approximations
- our CGC approach assumes boost invariance
** can all these conditions can be satisfied simultaneously?
result: $\hat{q}$ as a function of $\tau$ at different orders in the expansion

key: saturation regime appears before $\tau$ expansion breaks down caution:
figure above obtained for $v_{\perp}=v$
calculation works less well when $v_{\|} \neq 0$
reason: at very early times glasma fields represented as longitudinal flux tubes

$\lambda_{\perp}$ can be inferred the 2-point correlator
$\hat{q}$ built up during time probe is in domain of correlated fields at zeroth order this time is determined by
- transverse correlation length
- orientation and magnitude of the probe's velocity
$\rightarrow$ saturation is faster if $v_{\|}=0$
note: probe's velocity also enters through the Lorentz force

fifth and fourth order results for increasing $v_{\|}$
- saturation is less pronounced as $v_{\|}$increases


## impact of the glasma on jet quenching

radiative Eloss/length of probe traversing medium of length $L$ $\propto$ total accumulated transverse momentum broadening

$$
\Delta p_{T}^{2}=\int_{0}^{L} d t \hat{q}(t)
$$

our calculation gives $\hat{q}_{\text {max }}=6 \mathrm{GeV}^{2} / \mathrm{fm}$
compare with equilibrium values:
hard quark of $p_{T}>40 \mathrm{GeV} \longrightarrow 2<\hat{q} / T^{3}<4$

- inferred from experimental data S. Cao et al. [JETSCAPE], Phys. Rev. C 104, 024905 (2021).
$\hat{q}=3 T^{3}$ and $450>T>150 \mathrm{MeV} \rightarrow(0.05<\hat{q}<1.0) \mathrm{GeV}^{2} / \mathrm{fm}$
$\Rightarrow$ equilibrium value of $\hat{q}$ is much smaller
but $\tau_{\text {life }}$ of pre-equilibrium phase $<1 \mathrm{fm} / \mathrm{c}$
$\rightarrow$ contro of pre-equilibrium phase to jet quenching usually ignored
schematic representation of the time dependence of $\hat{q}$


1. rapid growth to $\hat{q}_{\max } \approx 6 \mathrm{GeV}^{2} / \mathrm{fm}$ at $t_{\text {max }} \approx 0.06 \mathrm{fm}$

- this is a rough description of our result
- no saturation region because of time scales

2. decrease from $t_{\max } \rightarrow t_{0}$ - not captured by our calculation

- is reproduced by the simulations
A. Ipp, D. I. Müller and D. Schuh, Phys. Lett. B 810, 135810 (2020)

$$
\left.\rightarrow \Delta p_{T}^{2}\right|^{\text {non }-\mathrm{eq}}=\int_{0}^{t_{0}} d t \hat{q}(t)=\frac{1}{2} \hat{q}_{\max } t_{0}+\frac{1}{2} \hat{q}_{0}\left(t_{0}-t_{\max }\right)
$$

3. assume hydro evolution from $t_{0}$

- using 1d boost invariant hydrodynamics
$\hat{q}=3 T^{3}$ with $T=T_{0}\left(\frac{t_{0}}{t}\right)^{1 / 3}$ and

$$
\left.\Delta p_{T}^{2}\right|^{\mathrm{eq}}=\int_{t_{0}}^{L} d t \hat{q}(t)=3 T_{0}^{3} t_{0} \ln \frac{L}{t_{0}}
$$

values:
$t_{0}=0.6 \mathrm{fm}, T_{0}=0.45 \mathrm{GeV}$
C. Shen, U. Heinz, P. Huovinen and H. Song, Phys. Rev. C 84, 044903 (2011)
$L=10 \mathrm{fm}$

$$
\text { result: } \frac{\Delta p_{T}^{2}[\text { non-equib }]}{\Delta p_{T}^{2}[\text { equib }]} \approx 0.93
$$

rough estimate that depends on values of parameters chosen - but result is not very sensitive to values of shape of peak
$\Rightarrow$ glasma plays an important role in jet quenching

## scale dependence


$Q_{s}$ between 1.9 (bottom) and 2.1 (top) GeV with ratio $Q_{s} / m$ fixed - one sees that the dependence is fairly weak

## boost and translation invariance

calculation of $\hat{q}$ is formulated in Minkowski space

- assumes at least approximate translation invariance
but used correlators of electric and magnetic fields obtained from an boost invariant ansatz for the vector potential
check of consistency:
previous result was $z=\eta=0$ (red line) $\hat{q}$ as a function of $\tau$ for three values of $\eta$



## conclusions

1. developed an efficient method to calculate correlators of electric and magnetic fields using a CGC approach and a proper time expansion
2. 6th order $\tau$ expansion can be trusted to about $\tau=0.05 \mathrm{fm}$
3. correlation btwn elliptic flow coef $v_{2} /$ spatial eccentricity - spatial asymmetry introduced by initial geometry is effectively transmitted to azimuthal distribution of gluon momentum field
$\rightsquigarrow$ indication of the onset of hydrodynamics.
4. most of the angular momentum of the intial system not transmitted to glasma

- contradicts picture of a rapidly rotating initial glasma state

5. glasma plays an important role in jet quenching
