What is quark matter?

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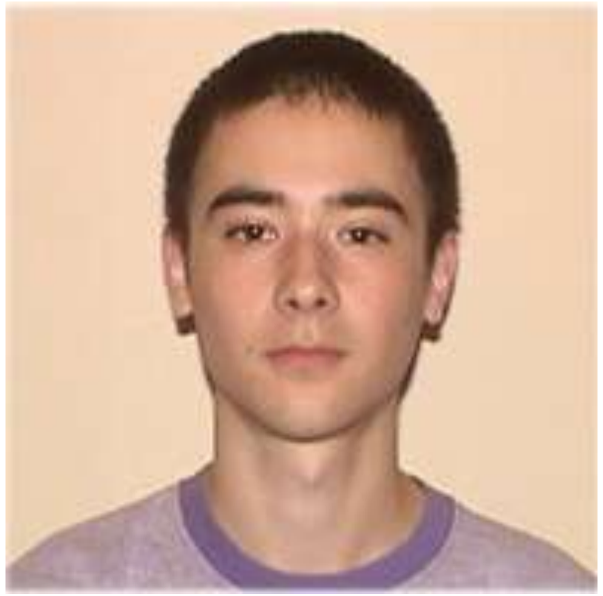
Based on 1808.04827, 2007.08539, with



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What is quark matter?

Question may sound

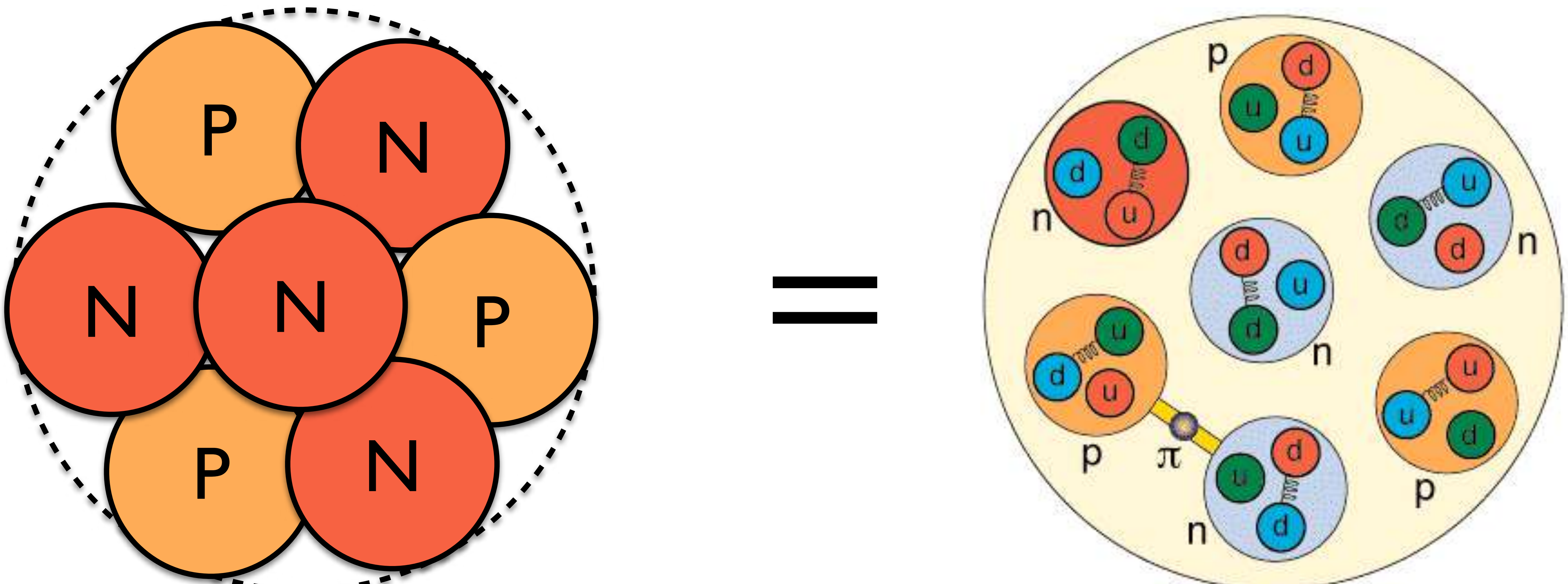
- Interesting
- Boring
- Easy
- Super difficult
- Only for people obsessed with formal QFT
- *Some combination of above options…*

My goal: convince you that “what is quark matter?” is

- **A hard question**
  - But it can be answered!
- **An interesting question**
  - Answer is interesting even if you’re not a formal QFT person!

What is quark matter?

Obvious answer: quark matter = matter made out of quarks.



E. Swanson, U. Pitt

This answer is useless: *everything* is made out of quarks.

What is quark matter?

Quark matter = matter which is best described in terms of quarks, rather than baryons. (?)

- That is, quarks are the natural degrees of freedom

Raises several issues:

- Where would we expect such a system to occur physically?
- “definition” sounds *intrinsically fuzzy*.
  - Is it just a matter of taste whether to use quarks or hadrons to describe matter?
- How can you actually tell that you have “quark matter”?
- Why are these question interesting?
- …

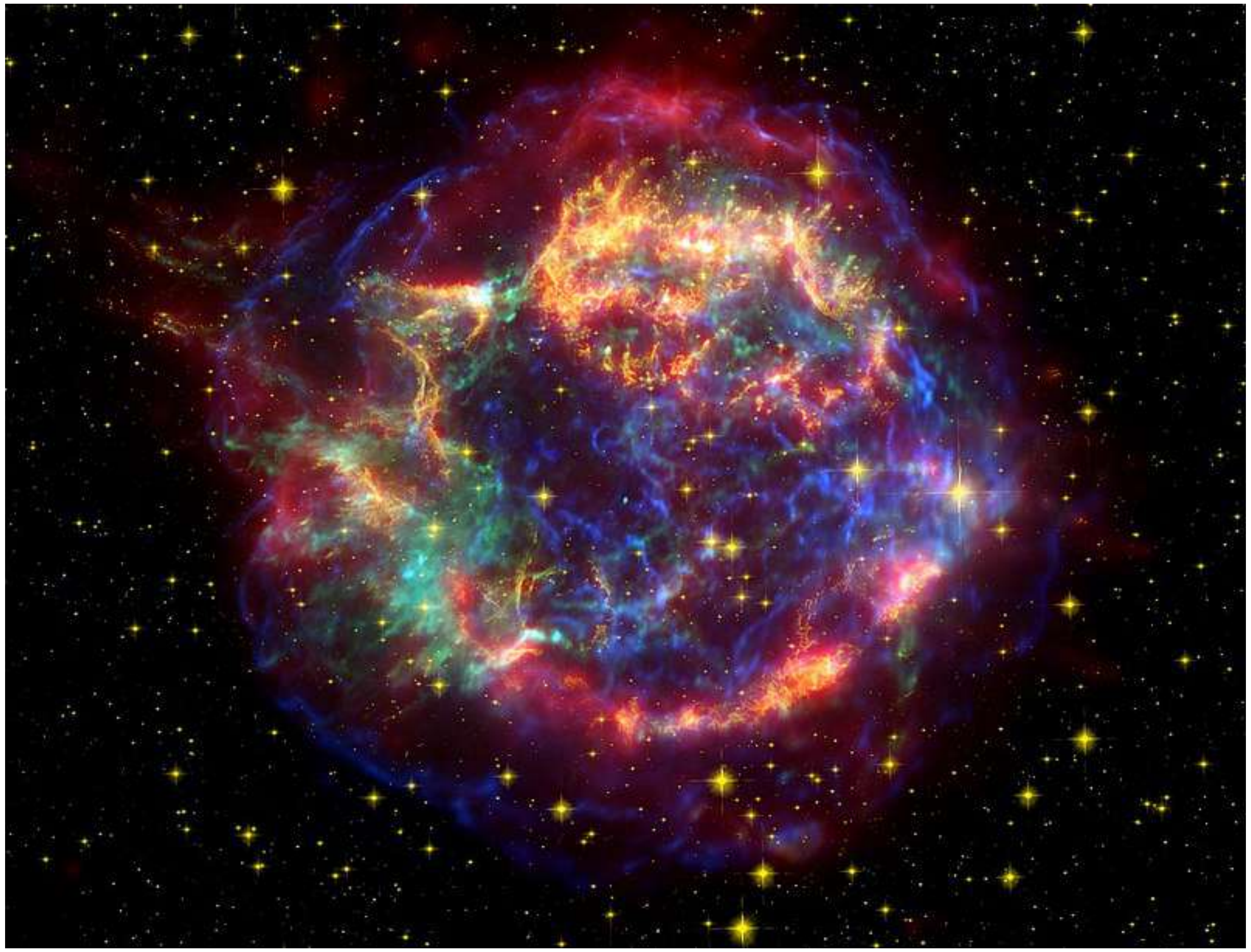
High-density matter

'Normal' nuclear matter = matter in large atomic nuclei

- Density ~ 1 nucleon/fm^3.

What about higher densities?

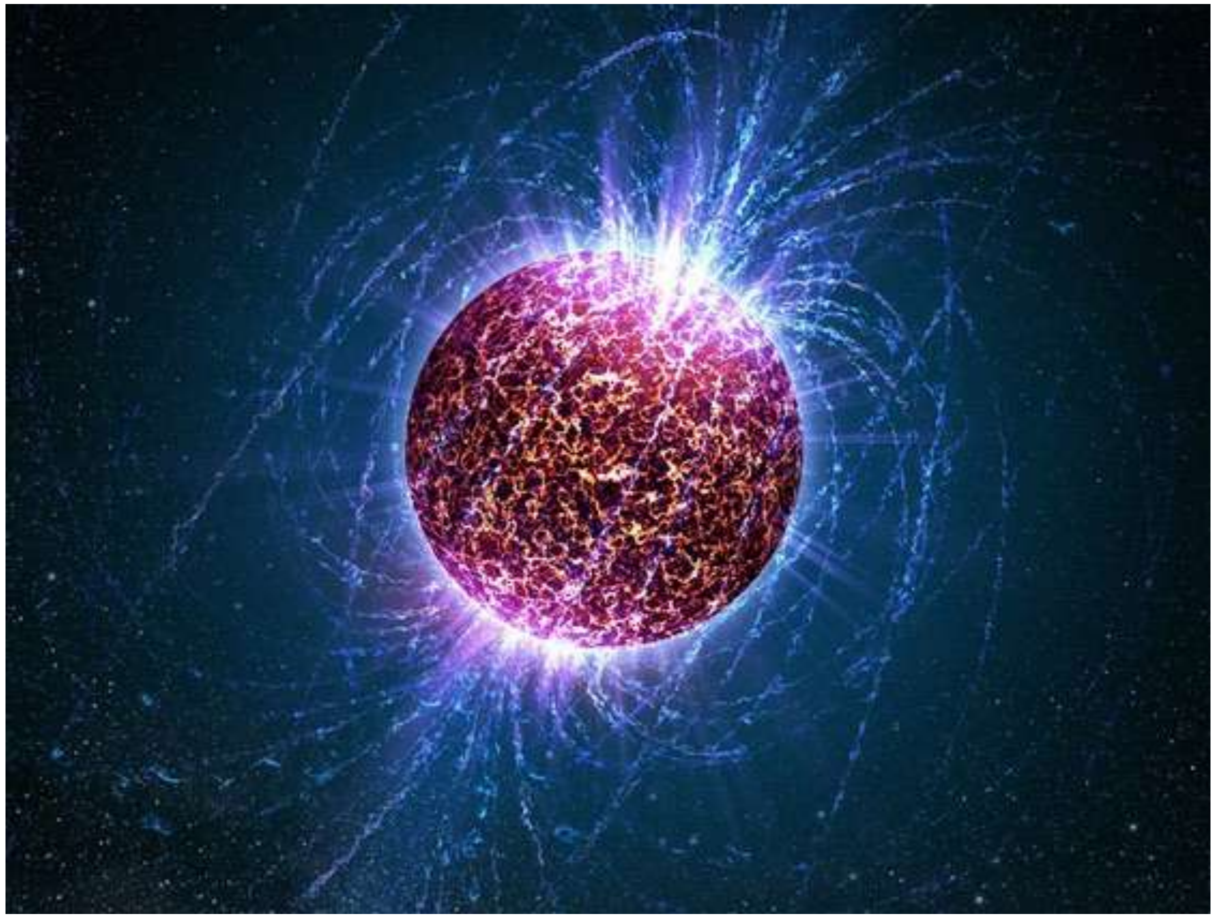
- Produced in core-collapse supernova explosions



Cassiopeia A, NASA/JPL

Neutron star remnant = a couple M_{sun} packed into ~ 20 km diameter!

Neutron stars



Casey Reed/Penn State University

Protons capture electrons, turn into neutrons.

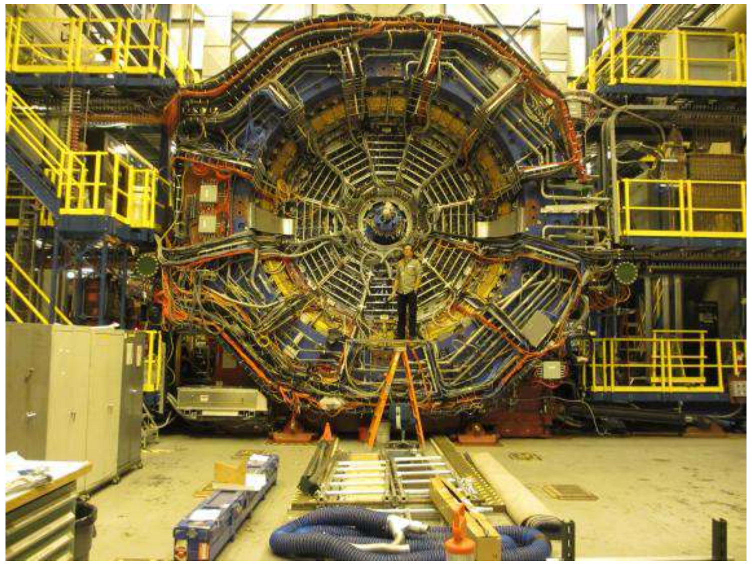
- neutron star cores = highest-density matter in known universe.
- Normal nuclear matter has baryon number density $n \sim n_0 \equiv 0.16\,\mathrm{fm}^{-3}$
  - In cores of neutron stars expect to have $n \gg n_0$.

Phase diagram of matter

What is the temperature-density phase diagram of matter governed by strong interactions — QCD?

Experiments and numerics tell us a lot about high temperature and low density.

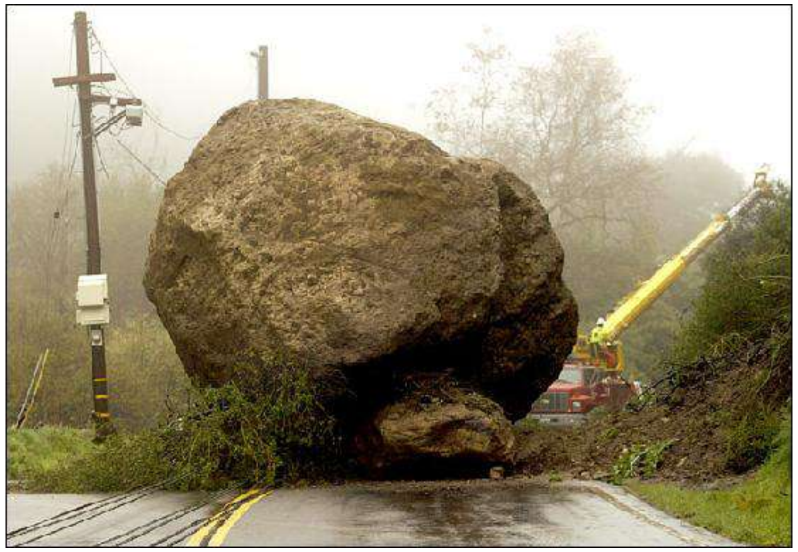
- Heavy ion collision experiments
- Numerical lattice gauge theory calculations



STAR detector, MIT

High density, low temperature is much harder.

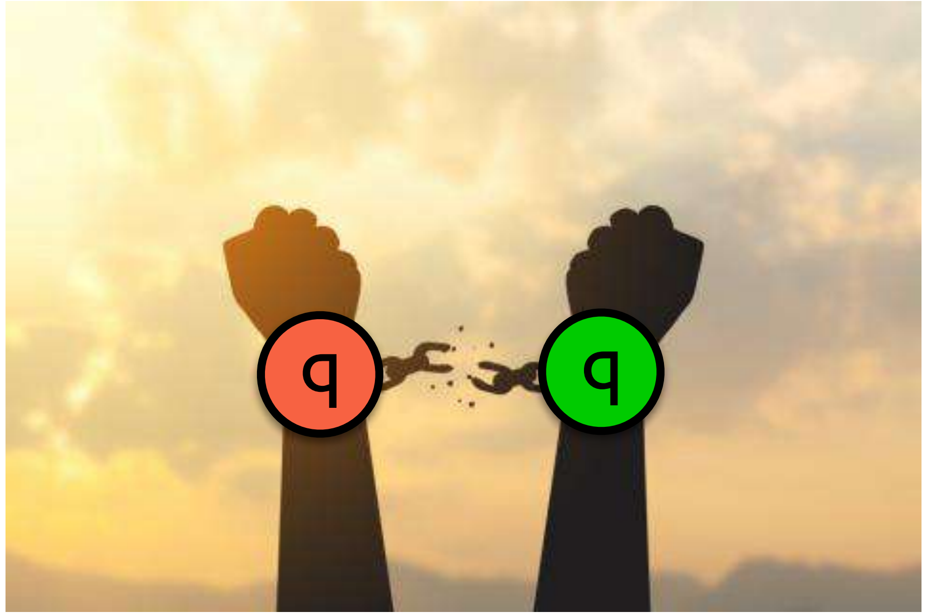
- No direct experiments, no lattice due to sign problem.
- Generally, rely on models without controlled error bars…



Asymptotic freedom

For sufficiently large densities, quarks definitely become the right degrees of freedom.

- Typical quark-quark interactions involve large momentum transfer at high densities
- Asymptotic freedom implies that the interactions become small
- So quarks act like free particles!
    - Naively, they are deconfined…

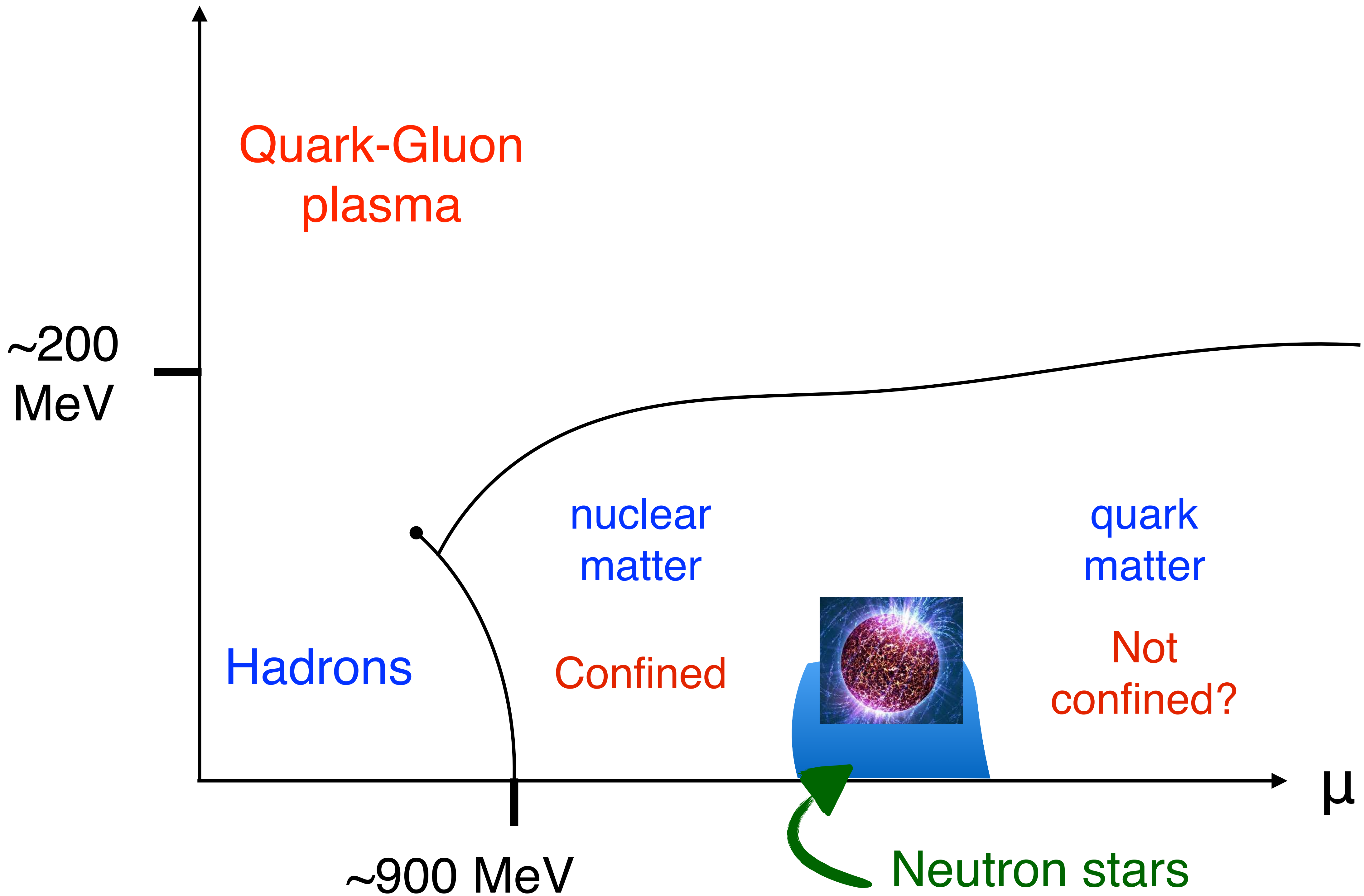


myersdavid. com

Cartoon QCD phase diagram

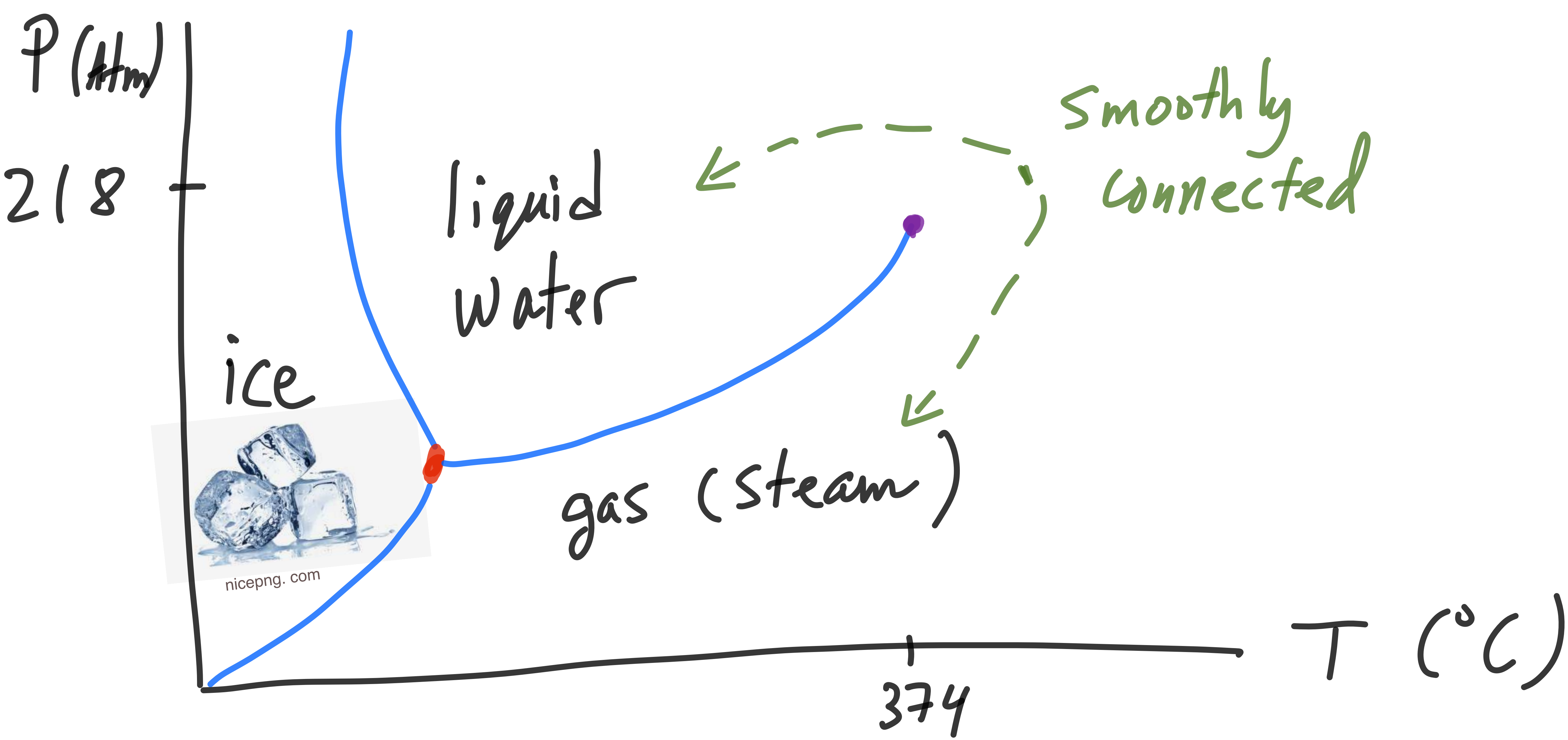
$N_c = N_f = 3$

common quark mass $m > 0$



Is quark matter a distinct phase of matter?

Is the difference between nuclear matter and quark matter like the difference between liquid water and steam, or difference between liquid water and ice?



Who cares?

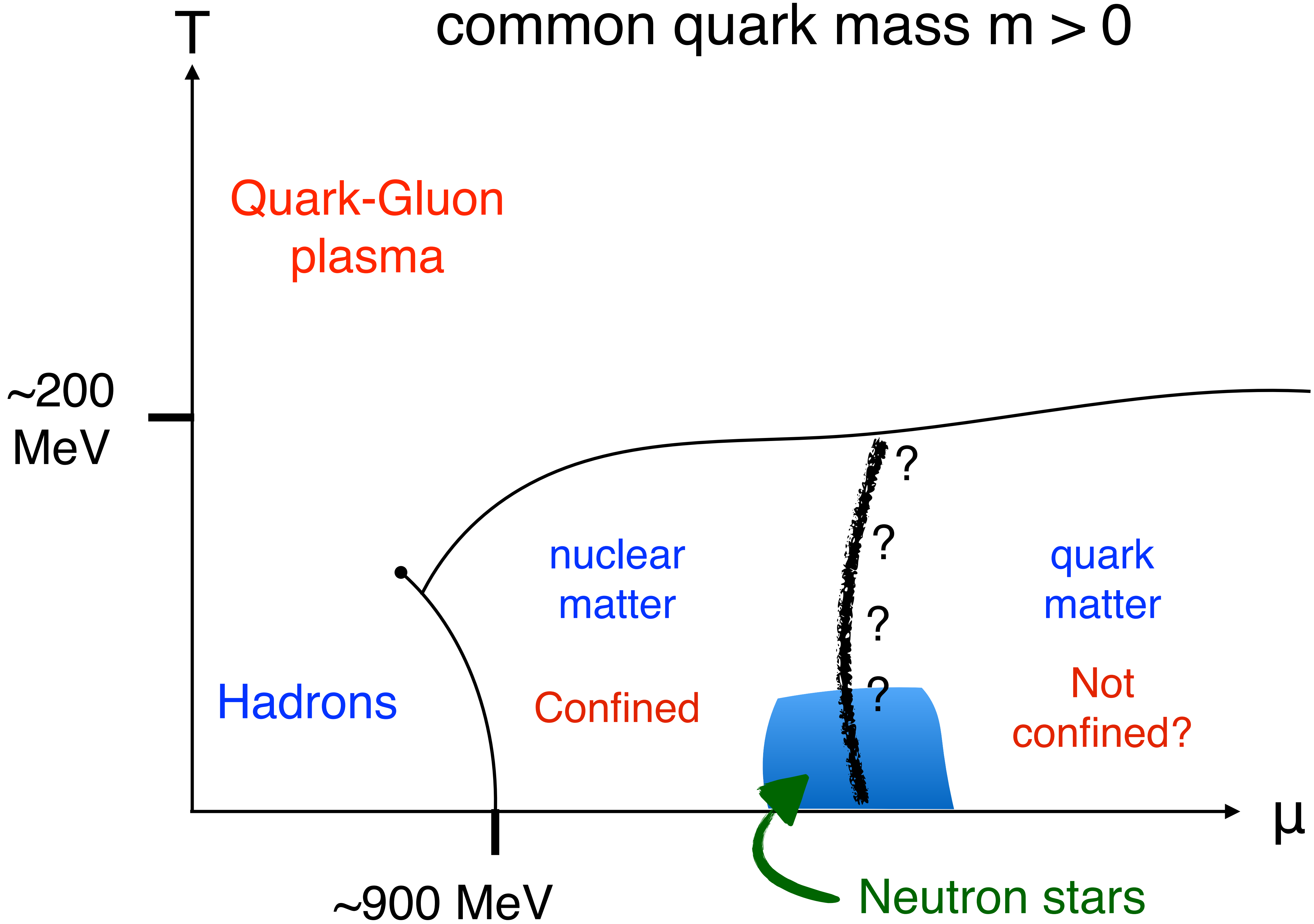
Suppose we understand QFT well enough to say that quark matter is a well-defined phase of matter, distinct from others.

- Then there *must be* a phase transition as density is increased!
  - Model-independent prediction!
- By construction, the transition would be in a physically interesting place where we can’t do systematic calculations.

Makes it worth thinking about “what is quark matter”!

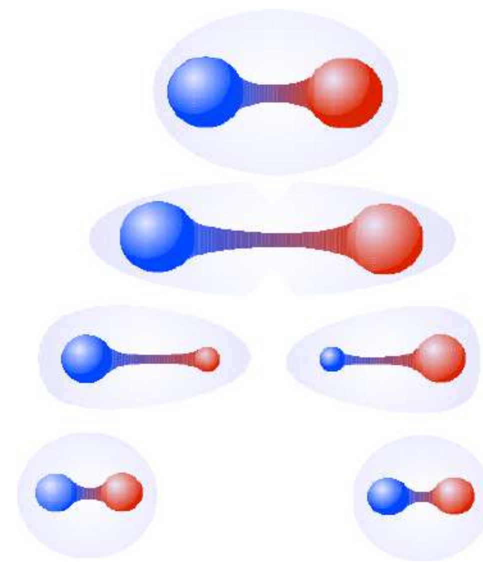
Cartoon QCD phase diagram

$N_c = N_f = 3$
common quark mass $m > 0$



Confinement

In QCD, quarks are in the “fundamental representation”.



Institut de Fisica Corpuscular, Valencia University

Confining color flux tubes break!

Giving a precise definition of ‘confinement’ in theories with fundamental-representation matter is hard.

or impossible?

What is quark matter?

"quark matter = deconfined phase" is (apparently) meaningless

Things getting complicated, so focus on the simplest case:

3 colors, 3 flavors of quarks with identical masses.

- At high densities, effect of physical non-equality of quark masses is suppressed by $\sim (m_{\text{strange}} - m_{\text{light}})/\mu$
- Even this most-symmetric area of parameter space is hard!

Credit: Gary Larson

What is quark matter?

Quark matter = Higgs phase of QCD (?)

- quark Fermi liquid unstable to Bose condensation of quark pairs

$$\langle q_a^i q_b^j \rangle = \phi_{ab}^{ij} \sim \mu^2 \Delta \epsilon^{ijk} \epsilon_{abk} \ , \ \ \Delta \sim \mu e^{-\frac{3\pi^2}{g\sqrt{2}}}$$

up-down
red-blue

'Cooper pairs' = set of three color anti-fundamental Higgs fields Φ

$$\epsilon_{ijk} \epsilon^{abc} \phi_{ab}^{ij} = (\Phi)_k^c$$

Cooper pairs condensation " $\langle \Phi \rangle = 0$ " completely "breaks" $SU(3)_{\rm color}$

"Color-flavor-locked color superconductivity"

Anderson-Higgs-Englert-Brout-… mechanism!

Superfluidity

Cold nuclear matter is a superfluid.

- Bose condensation of baryon pairs
- $U(1)_B$ global symmetry is spontaneously broken to $\mathbb{Z}_2$
- “Confined” superfluid

Cold quark matter is *also* a superfluid.

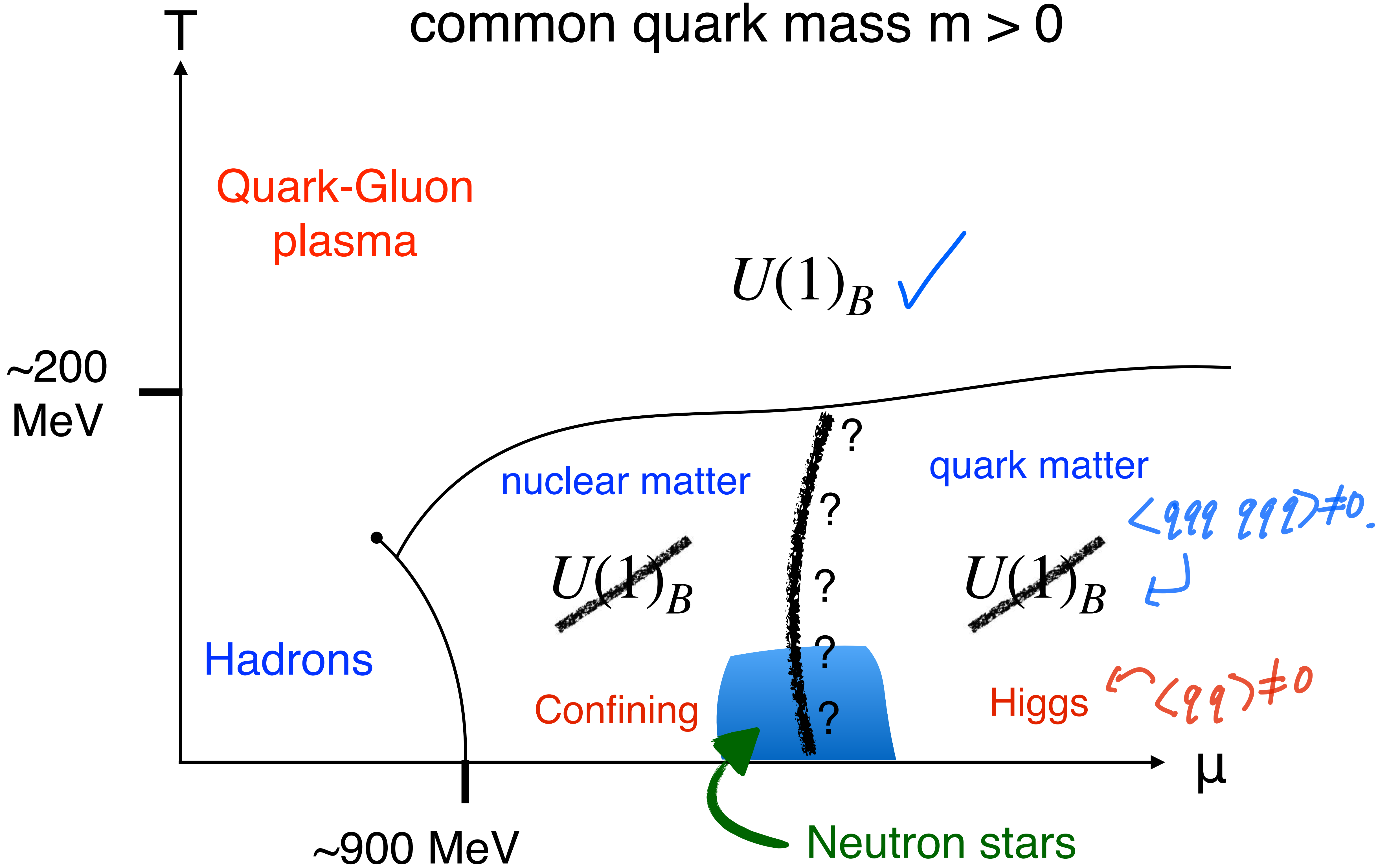
- “$\langle qq \rangle \neq 0$” $\Rightarrow \langle qqq\, qqq \rangle \neq 0$
- $U(1)_B$ global symmetry is spontaneously broken to $\mathbb{Z}_2$
- “Higgs” superfluid

VS

Cartoon QCD phase diagram

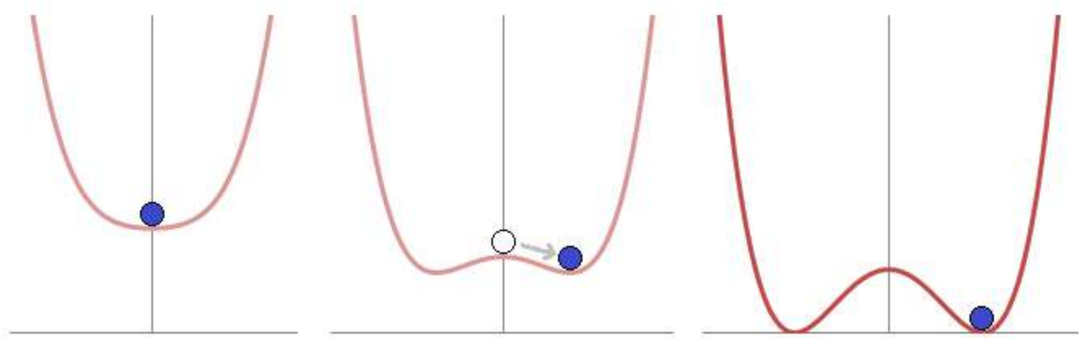
$N_c = N_f = 3$

common quark mass $m > 0$

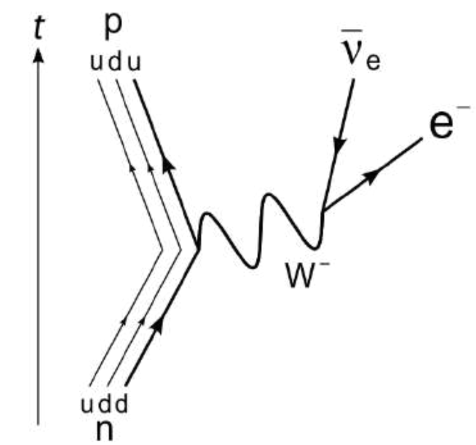


Higgs regime

In high density QCD, $SU(3)_{\mathrm{color}}$ is completely Higgsed. Many other examples: superconductors; weak SU(2); …



Wikimedia



Wikipedia

Anderson-Higgs-… mechanism:

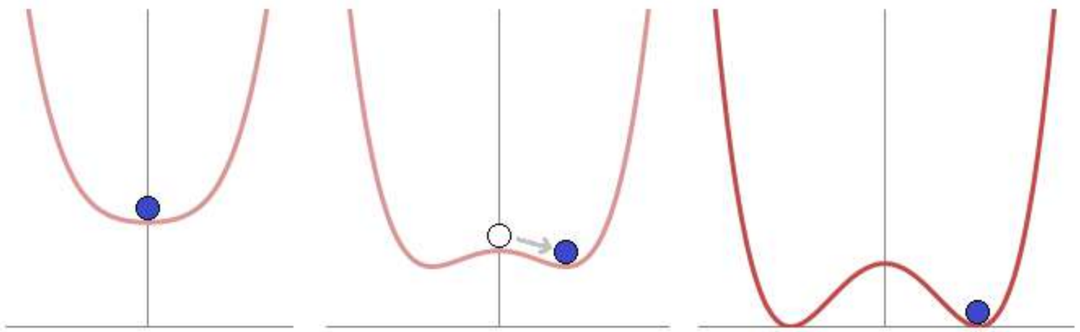
- "broken gauge symmetry", "$\langle \phi \rangle \neq 0$", "physical massive gauge bosons"
  - But $\langle \phi \rangle = 0$ whenever ϕ is not gauge invariant.
  - All the physical states are still secretly 'hadrons'

't Hooft, Elitzur, … 1970s

$$W_\mu \leftrightarrow \epsilon^{ab} \phi_a \left(D_\mu \phi \right)_b$$

Higgs regime

In high density QCD, $SU(3)_{\mathrm{color}}$ is completely Higgsed. Many other examples: superconductors; weak SU(2); …



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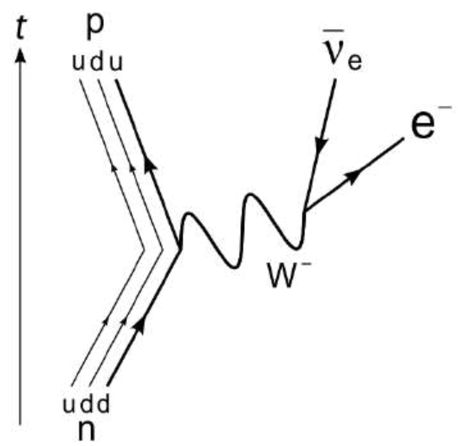


Wikipedia

Giving a precise definition of ‘Higgs phase’ in theories with fundamental-representation matter is also hard.

or impossible?

Are “Higgs” and “confining” regimes ever distinct when there’s fundamental-representation matter?

Higgsing vs. Symmetry

When should Higgsing lead to a phase boundary?

- Easy case: if Higgs scalar charged under global symmetry, then Landau and Ginzburg tell us there's a phase boundary

$$\langle \phi \rangle = 0 \;\Rightarrow\; G \,\checkmark \qquad \Big| \qquad \langle \phi \rangle \neq 0 \;\Rightarrow\; \cancel{G}$$

Not very interesting…

Unconnected to confinement vs. Higgs per se: standard spontaneous global symmetry breaking!

- Not relevant to QCD: nuclear matter and quark matter have **identical** global symmetry breaking pattern.

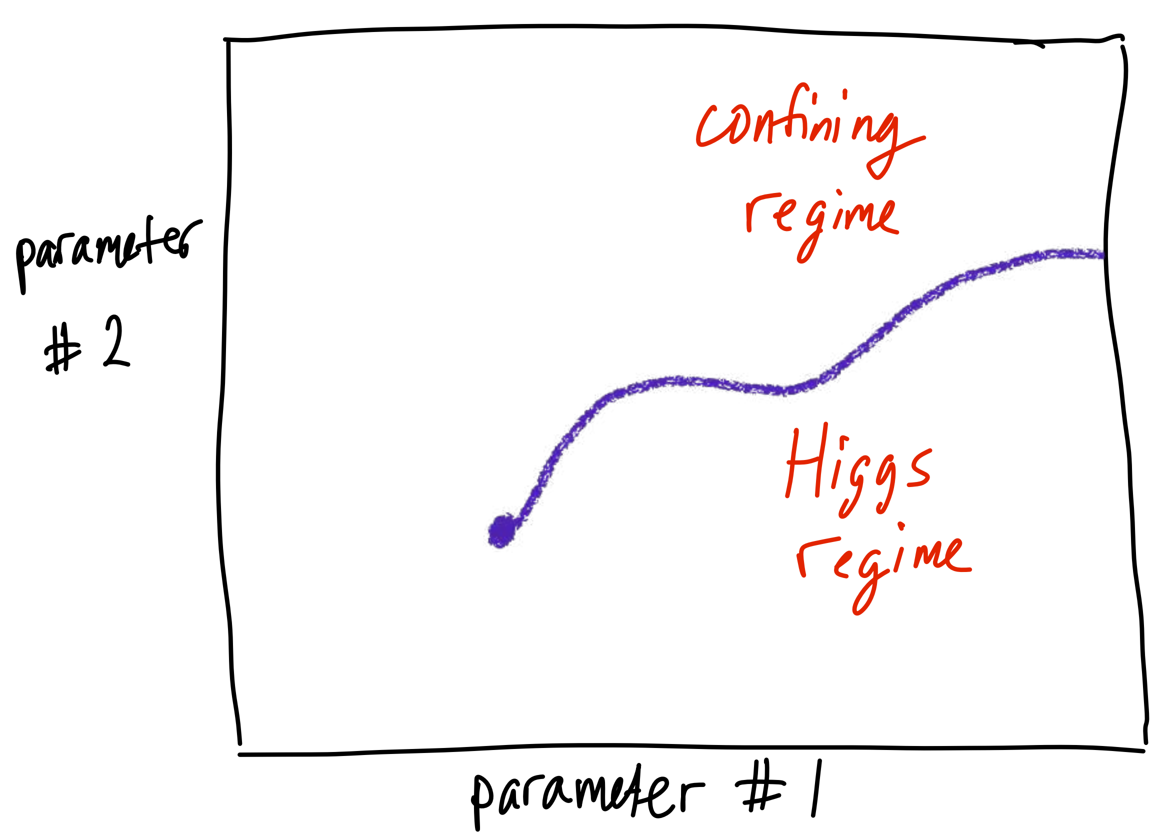
Famous result

’t Hooft; Osterwalder,Seiler; Fradkin, Shenker; Banks, Rabinovici
Late 1970s

To avoid a boring phase boundary, assume Higgs scalars are **not** charged under global symmetry.

Fradkin-Shenker-… theorem:

In specific models, Higgs and confining regimes of QFTs with fund. matter are smoothly connected.



Famous result

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To avoid a boring phase boundary, assume Higgs scalars are **not** charged under global symmetry.

Fradkin-Shenker-… theorem:

In specific models, Higgs and confining regimes of QFTs with fund. matter are smoothly connected.

No gauge-invariant order parameters $\Rightarrow$ no phase boundary.

- Basic idea seems general!

Folk theorem: no non-trivial phase boundaries separating Higgs from confinement in general.

Standard view

Schafer, Wilczek
1998

Quark-Hadron Continuity conjecture: no phase boundary separating nuclear matter and quark matter in SU(3) flavor limit

Thomas
Schafer

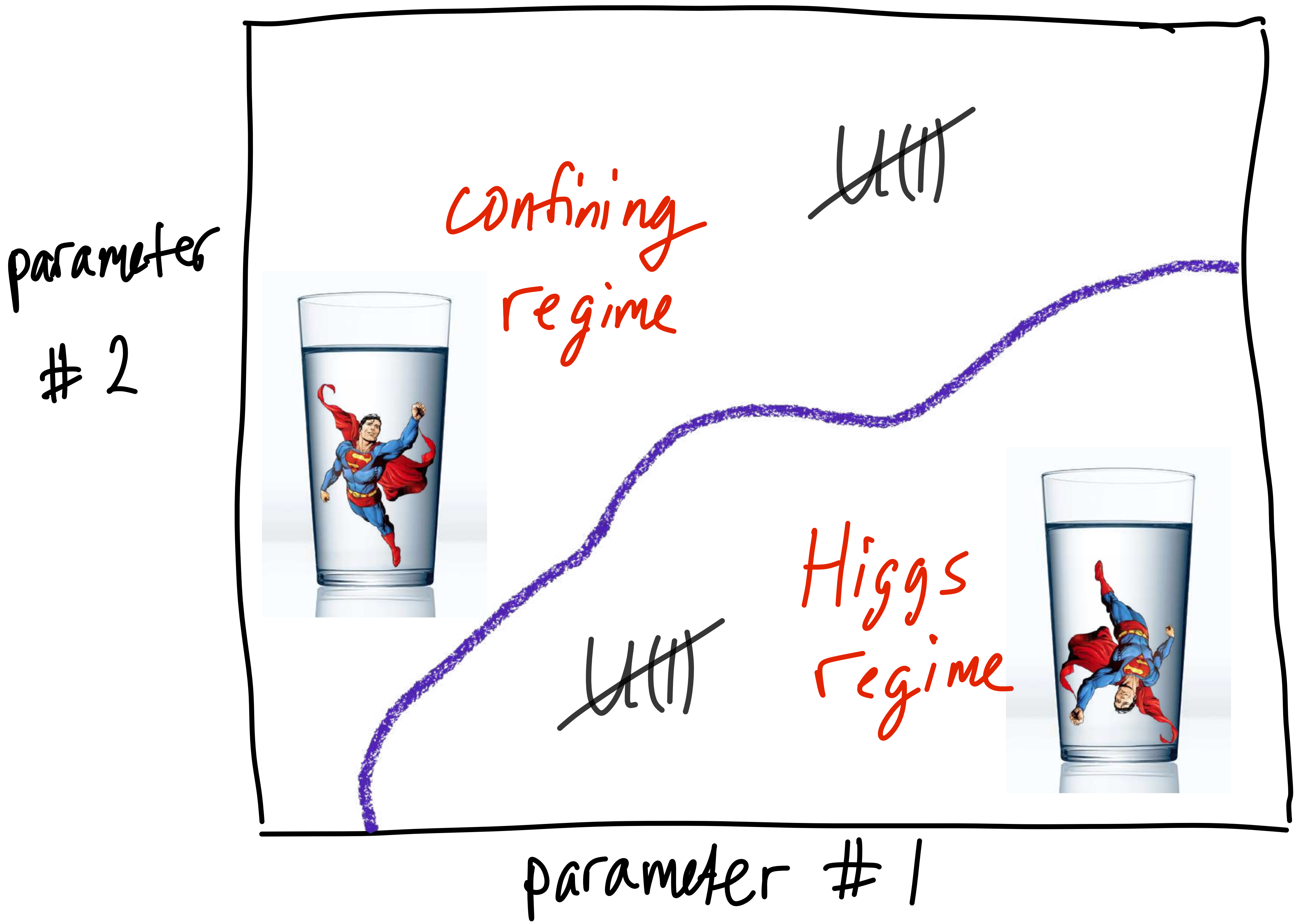
Frank
Wilczek

Proved that there is no Landau paradigm reason for a phase transition; appealed to Fradkin-Shenker result.

Our claim

AC, Sen, Yaffe 2018;
AC, Jacobson, Sen, Yaffe 2020.

Actually, there is reason to expect a phase boundary!



Our claim

AC, Sen, Yaffe 2018;
AC, Jacobson, Sen, Yaffe 2020.

$\exists$ non-trivial phase boundaries between $U(1)$-broken confining and Higgs regimes in some gauge theories with fundamental matter.

- Higgsed and confining regimes are identical within the Landau paradigm, but Higgs charged under global $U(1)$ symmetry.
- Higgsing & symmetry breaking connection is model dependent; not connected in QCD!

New gauge-invariant order parameter
$\Rightarrow$ phase boundary

Strong evidence against the “quark-hadron continuity” conjecture

$U(1)$-broken Higgs and confining phases

Are the $U(1)$-broken regimes distinct?

- Identical within Landau-Ginzburg paradigm
  - Not distinguished by any local order parameters
- No reason to expect any standard 'topological order'
  - No mass gap. No useful higher-form symmetries, no extra ground states in genus g > 0, …

Yet these regimes are in fact distinct! We need a new order parameter to see it.

It can't be local - but there's a natural non-local one!

Superfluids have vortex excitations

$U(1)_G$ breaking $\Rightarrow$
- vanishing gap due to NG boson
- vortices

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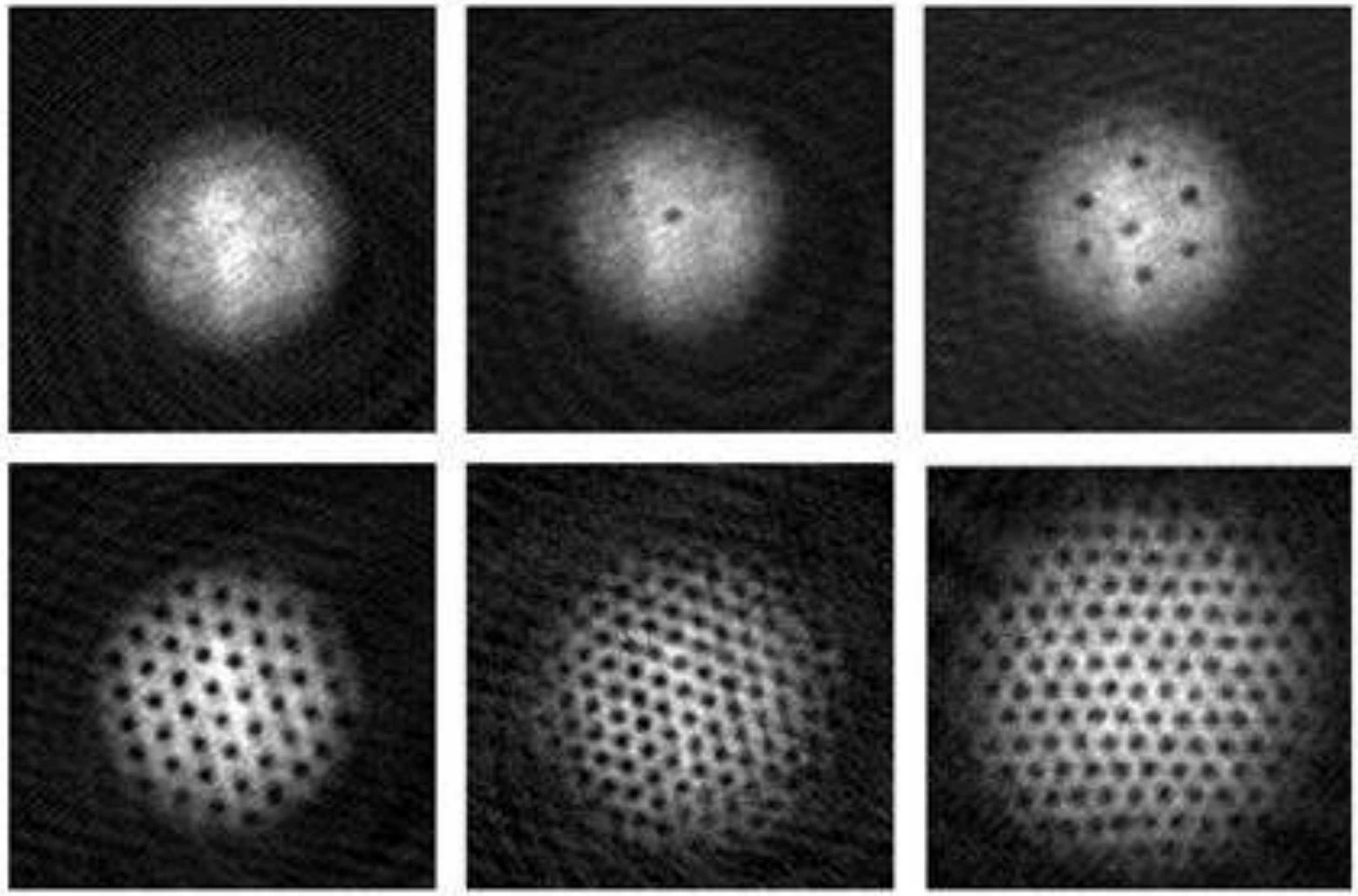

From work of Engels, Cornell, et al, early 2000s

Univ. of Washington

Vortices appear due to rotation - and neutron stars rotate, so they have a huge number of vortices!

Superfluids have vortex excitations

$U(1)_G$ breaking $\Rightarrow$

- vanishing gap due to NG boson
- vortex excitations

$$w = \frac{1}{2\pi}\int_\Gamma \text{super-current} \in \mathbb{Z}$$

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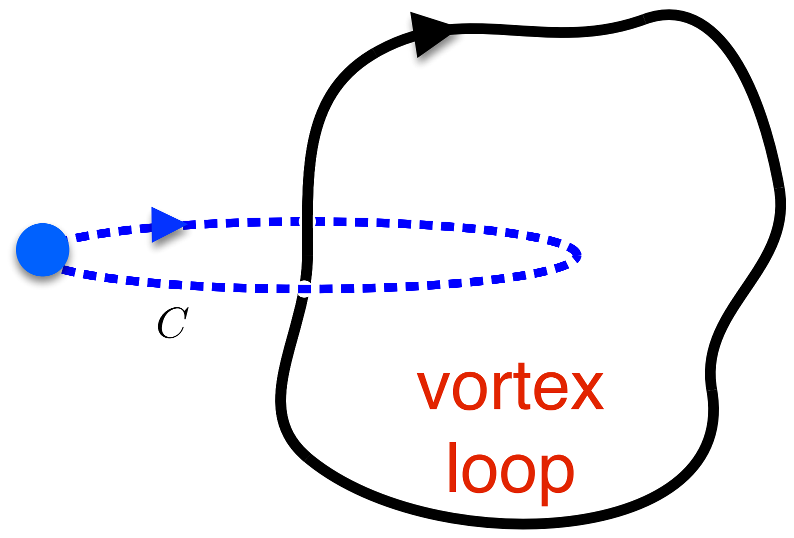
How do gauge-charged particles interact with vortices?

Aharonov-Bohm phase

In general, charged particles can pick up a phase moving around a vortex:



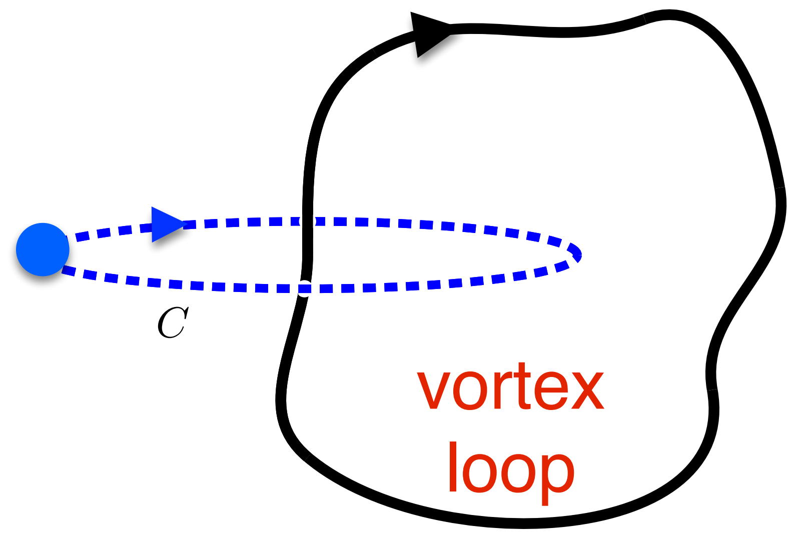
$$\langle \Omega(C) = \mathrm{tr}\, e^{i\int_C A} \rangle \sim e^{i\Phi}\, e^{-mL_C}$$

An Aharonov-Bohm phase along a closed curve C is determined by "magnetic flux" through the surface bounded by C.

- Here the "magnetic flux" is "color-magnetic flux"
  - "color-magnetic flux" isn't gauge invariant. But the Aharonov-Bohm phase is gauge-invariant and hence physical.

Aharonov-Bohm phase

In general, charged particles can pick up a phase moving around a vortex:



$$\langle \Omega(C) = \mathrm{tr}\, e^{i\int_C A} \rangle \sim e^{i\Phi}\, e^{-mL_C}$$

Our order parameter is the Aharonov-Bohm phase!

$$e^{i\Phi} \equiv \lim_{r\to\infty} \frac{\langle \Omega(C) \rangle_{w=1}}{\langle \Omega(C) \rangle}$$

Factors like $\langle \Omega(C) \rangle \sim e^{-mL_C}$ cancel in ratio; only AB phase left!

What is quark matter?

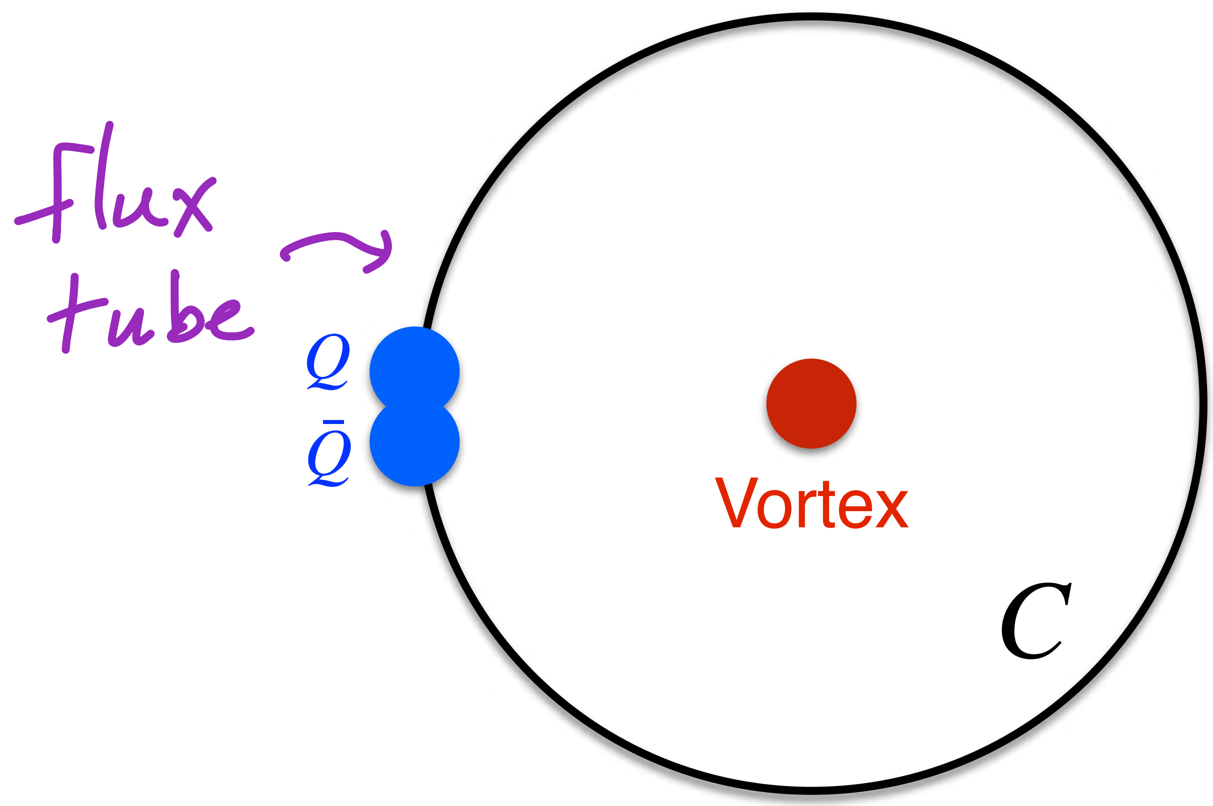
Skipping ahead, I can finally tell you our answer to this question!

- Turns out that $e^{i\Phi} = 1$ in cold nuclear matter.
- On the other hand, $e^{i\Phi} \neq 1$ in cold quark matter.
  - In flavor-symmetric limit, $e^{i\Phi} = e^{2\pi i/3}$

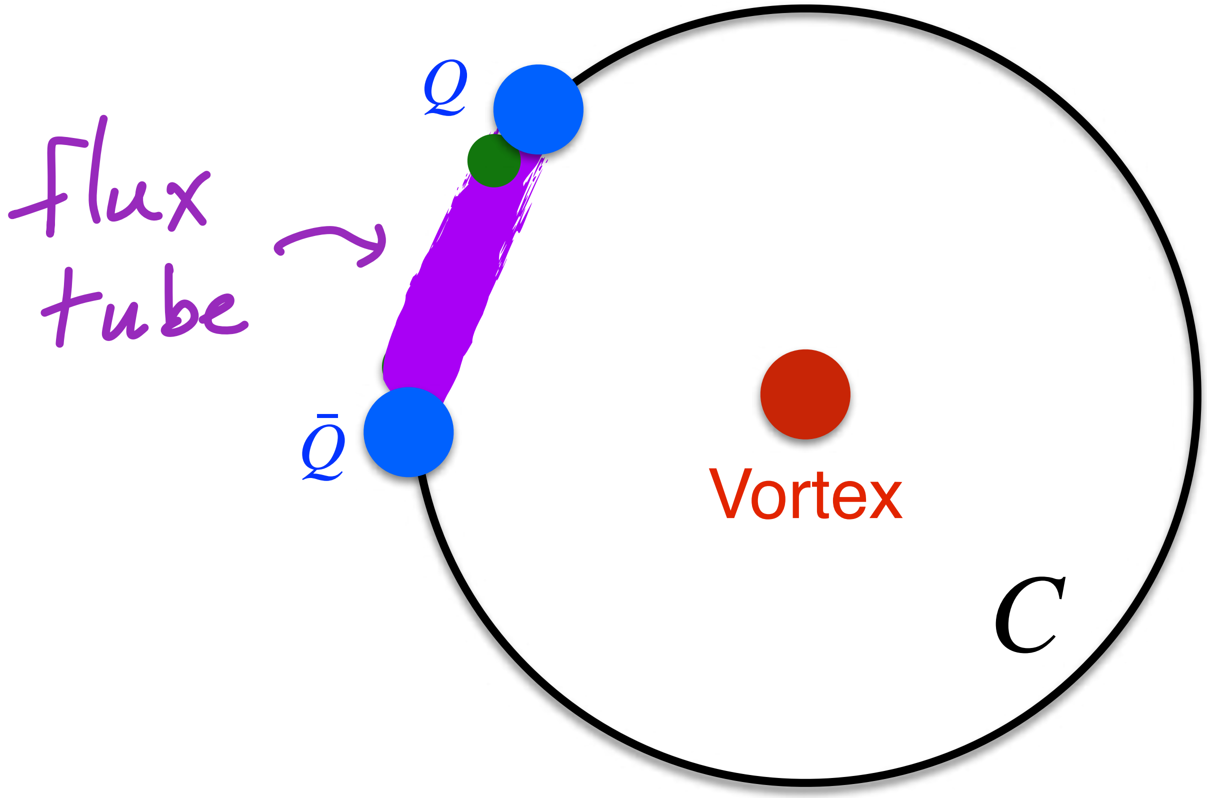
quark matter: phase where superfluid vortices carry non-trivial color Aharonov-Bohm phases.

nuclear matter: phase where these Aharonov-Bohm phases are all trivial (that is: $e^{i\Phi} = 1$).

Aharonov-Bohm phase in confined phase



Aharonov-Bohm phase in confined phase



Wilson loop exponentially dominated by physics close to the curve C, within size of a heavy-light meson.

- Doesn't care whether there's a vortex at center!

$$e^{i\Phi} \equiv \lim_{r\to\infty} \frac{\langle \mathrm{tr}\,\Omega(C)\rangle_{w=1}}{\langle \mathrm{tr}\,\Omega(C)\rangle} = +1$$

Aharonov-Bohm phase in quark matter

Higgs phase is (asymptotically) weakly coupled, but argument is more technical. Two steps:

- Evaluate the Wilson loop in the high-density limit
  - Classical calculation in an appropriate effective field theory
- Show that there are no quantum corrections to the result, so high-density result is exact within the quark matter phase.
  - More involved use of effective field theory.

@jitgo , Twitter

Aharonov-Bohm phase in quark matter

Order parameter field Φ can be written as a 3×3 matrix. Then a straight superfluid vortex with winding w looks like

$$\Phi(r,\theta) = v_\Phi \,\mathrm{diag}\left(f_1(r)\, e^{i(n+w)\theta},\; f_2(r)\, e^{i(m-n)\theta},\; f_3(r)\, e^{-im\theta}\right),$$

$$A_\theta(r) = \frac{a\, h_8(r)}{2\pi r}\, t_8 + \frac{b\, h_3(r)}{2\pi r}\, t_3\,.$$

Minimizing energy with fixed winding w and assuming SU(3) flavor symmetry, the values of a, b are

$$a = -\frac{2\pi}{\sqrt{3}}\,, \quad b = -2\pi\,,$$

$\Rightarrow \frac{1}{3}\langle \mathrm{tr}\, \Omega(C) \rangle_{w=1} = e^{2\pi i/3}$ at tree level.

Aharonov-Bohm phase in quark matter

Integrating out fluctuations generates new terms and renormalizes old terms in quantum effective actions

- Only the ones with two derivatives contribute to log-divergent part of vortex energy which determines $\langle \mathrm{tr}\, \Omega(C) \rangle_{w=1}$

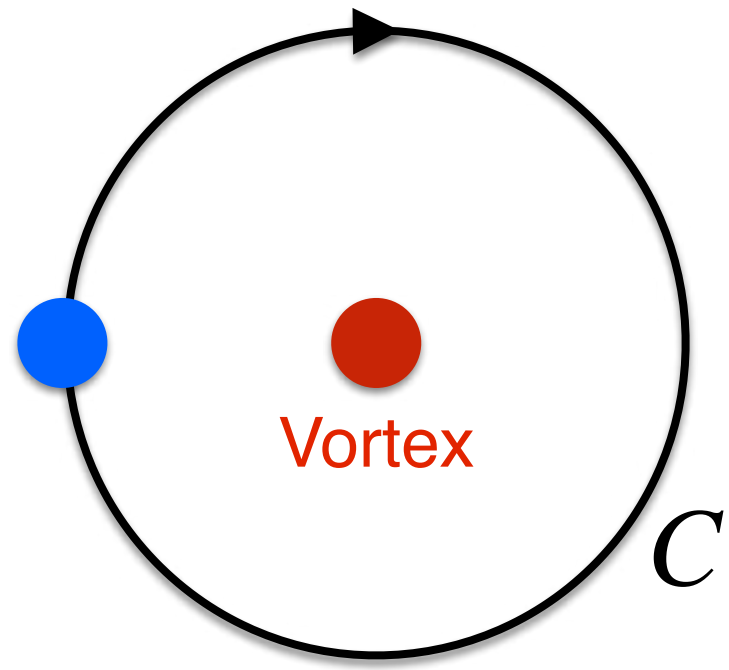
$$S_{\mathrm{eff},\, SU(3)\ \mathrm{holonomy}} = \int d^4x \left\{ \mathrm{tr}\left[f_1(\Phi)(D_\mu \Phi)^\dagger f_2(\Phi)(D^\mu \Phi)\right] + \epsilon^{ABC} \epsilon_{IJK}\, f_3(\Phi)^I{}_A (D_\mu \Phi)^J{}_B (D^\mu \Phi)^K_C \right\} + \mathrm{h.c.}$$

A,B,C: color;
I,J,K: flavor

On $w = 1$ vortex configuration $a = -2\pi/\sqrt{3}$, $b = -2\pi$ are extrema of all terms.

- phase of $\langle \mathrm{tr}\, \Omega(C) \rangle_{w=1}$ remains exactly $2\pi/3$ within Higgs phase in flavor SU(3) limit

Confining phase



So $e^{i\Phi} = 1$ in confining phase

But in Higgs phase it is $e^{i\Phi} = e^{2\pi i/3}$!

AB phase must change non-analytically with density!

But do non-analycities in vortex Aharonov-Bohm phases imply non-analycity in e.g. ground state energy?

Detour to 2+1 dimensions

Super hard to explicitly check AB phase non-analyticity vs. ground state energy non-analyticity in 4d non-Abelian gauge theory.

- Transition occurs at strong coupling, confinement dynamics aren't under analytic control.
- The good news: this is a *question of principle*!
  - If we can find some simpler QFT where the same questions can be asked and answered, then we'll be set.

Idea: confinement in 2+1d "compact" Abelian gauge theories is well-understood analytically!

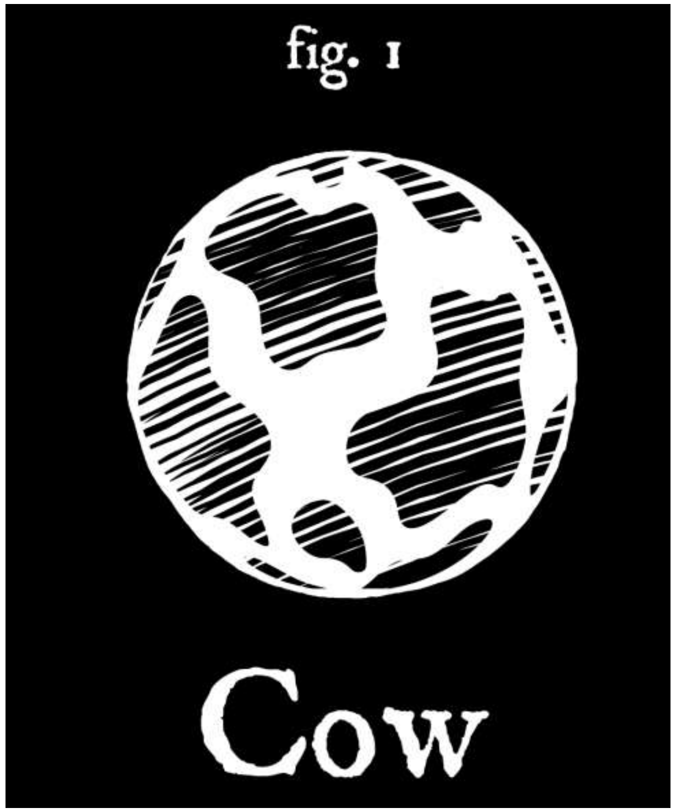
- Electric charge confinement driven by proliferation of magnetic monopole-instantons, described using 3d Abelian duality.

Polyakov 1970s

Detour to 2+1 dimensions

We found a 2+1d Abelian gauge theory with Higgs and confining superfluid regimes.

- Superfluid vortex Aharonov-Bohm phases change non-analytically as one goes from one regime to the other.
- Won't explain details of model for lack of time.



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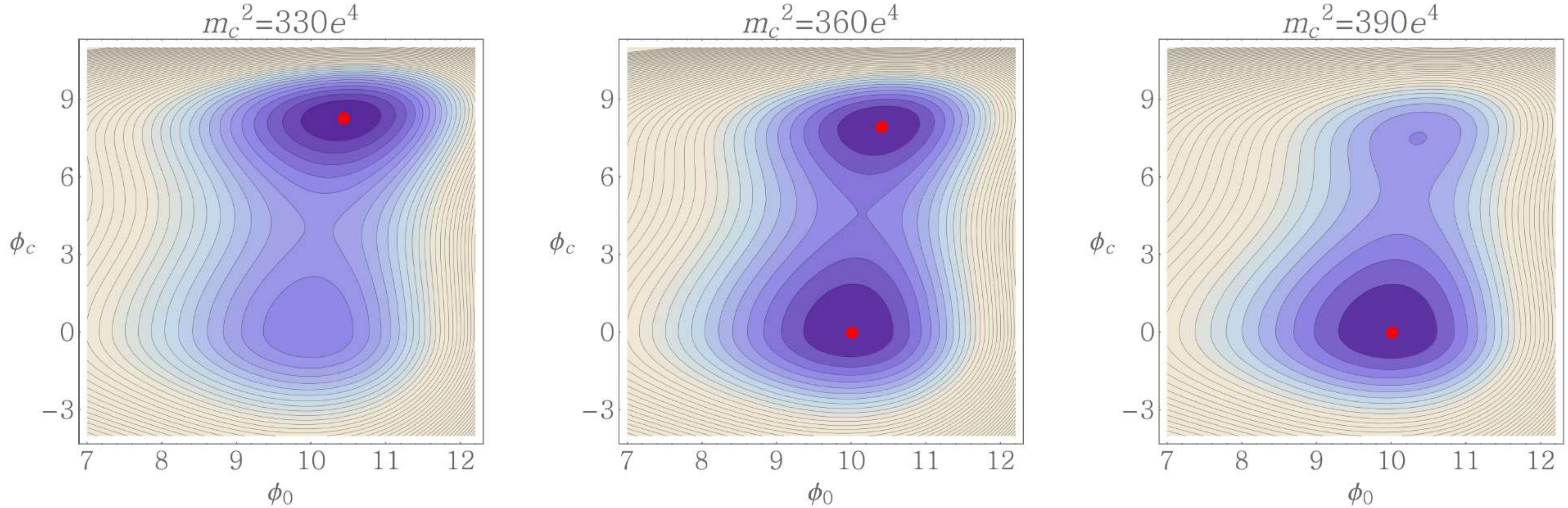
In this model, we can explicitly check correlation between a jump in the Aharonov-Bohm phase with the location of a thermodynamic phase transition.

Higgs-confinement phase transition in 2+1d



Higgs

confining

First-order transition, weak coupling methods reliable.

- Aharonov-Bohm phase jumps precisely at the transition!

kindpng. com

Higgs-confinement phase transition

There's also a more general heuristic argument for a transition.

Superfluid ground state contains some density of vortex loops and, so if the vortex Aharonov-Bohm phase jumps:

$\Rightarrow$ jump in "color-magnetic field" carried by vortex

$\Rightarrow$ jump in energy of vortex loops

$\Rightarrow$ jump in ground state energy = phase transition

Quark matter vs nuclear matter

Considered Aharonov-Bohm phase of superfluid vortices in dense matter.

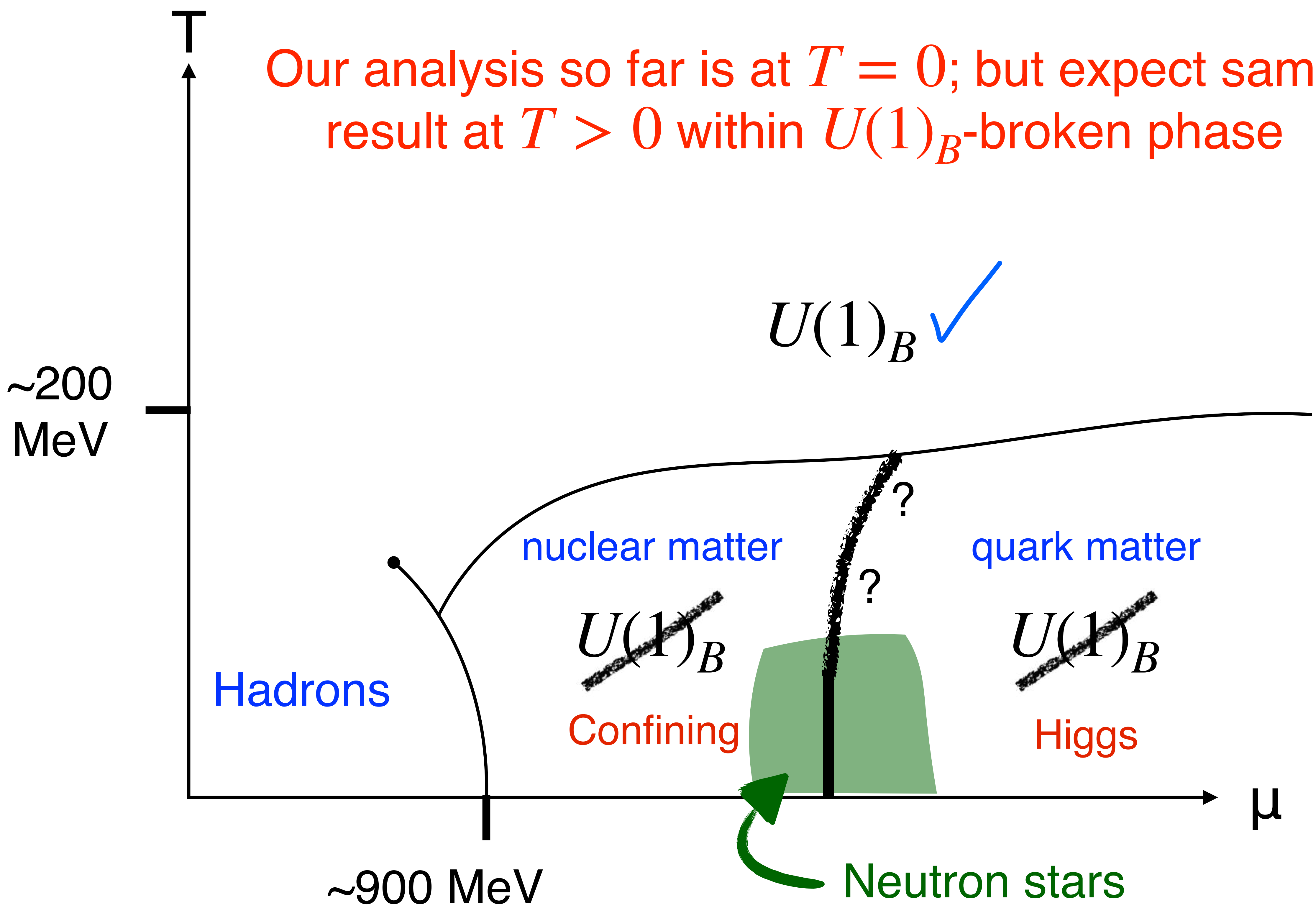
- In cold nuclear matter $e^{i\Phi} = 1$ due to confinement and string breaking.
- But $e^{i\Phi} \neq 1$ in cold quark matter (in SU(3) flavor-symmetric limit, $e^{i\Phi} = e^{2\pi i/3}$)

$\Rightarrow$ Density-driven phase transition between "confined" nuclear matter and "deconfined" quark matter in QCD

Strong evidence against the quark-hadron continuity conjecture

Revised cartoon QCD phase diagram

Our analysis so far is at $T = 0$; but expect same result at $T > 0$ within $U(1)_B$-broken phase



Conclusions

QFT progress:

- $\exists$ phase boundaries separating $U(1)$-breaking confining and Higgs regimes in QFTs with fundamental-representation matter
- Order parameter: Aharonov-Bohm phase of superfluid vortices
- Applies to Abelian and non-Abelian QFTs in 2+1d and 3+1d

$\Rightarrow$ new result on phase structure of QCD

Open issues

Finite T effects? Physics on interfaces? Order of transitions?

Neutron star physics? cond-mat? hep-ph?

How does all this fit into some bigger scheme?

....

Thank you for your attention!