Chiral anomalous processes in magnetospheres of compact stars

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September 29, 2021

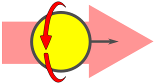
Chirality

Spin state and momentum are independent quantities for non-relativistic electrons

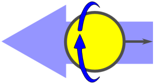
In the ultrarelativistic limit, the Weyl equation

$$\mathcal{H}_W = \pm c\boldsymbol{\sigma}\cdot\mathbf{k}$$

implies that spin is completely locked to momentum



Right-handed (R) fermion



Left-handed (L) fermion

Electric and chiral currents $\mathbf{j} = \mathbf{j}_R + \mathbf{j}_L, \quad \mathbf{j}_5 = \mathbf{j}_R - \mathbf{j}_L$

Chiral magnetic effect

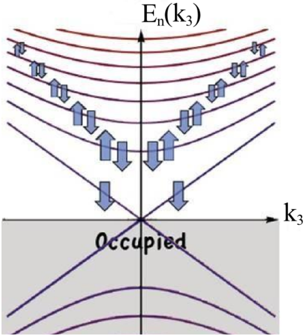


Figure: Landau levels

In a magnetic field, LLL states are completely spin polarized. Right- and left-handed electrons propagate in the opposite directions $\rightarrow$ an imbalance (quantified by μ_5) results in an electric current $\mathbf{j} = e^2\mu_5\mathbf{B}/(2\pi^2)$. This is the chiral magnetic effect (CME).

Chiral charge nonconservation

Parallel electric and magnetic fields break a balance between the Fermi surfaces of the right- and left-handed fermions

$$\dot{\mathbf{k}}_R = e\mathbf{E} \quad \to \quad \frac{dN_R}{dt\,dz} = \frac{e\mathbf{E}}{2\pi}, \tag{1}$$

$$\frac{dN_R}{dx\,dy} = \frac{eB}{2\pi} \quad \to \quad \frac{dN_R}{dt\,dV} = \frac{e^2(\mathbf{E}\cdot\mathbf{B})}{(2\pi)^2}, \tag{2}$$

$$\frac{dN_L}{dt\,dV} = -\frac{dN_R}{dt\,dV} \tag{3}$$

Chirality generation

Therefore, the chiral charge is not conserved (the chiral anomaly) [S. L. Adler, Phys. Rev. **177**, 2426, (1969); J. S. Bell and R. Jackiw, Nuovo Cim. A **60**, 47, (1969)]

$$\dot{q}_5 \equiv \dot{n}_R - \dot{n}_L = \frac{e^2(\mathbf{E}\cdot\mathbf{B})}{2\pi^2} \tag{4}$$

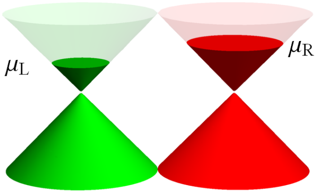


Figure: Chiral imbalance $\mu_5 = (\mu_R - \mu_L)/2 \neq 0$ induced by the chiral anomaly

Many-body chiral fermion systems

Chiral matter is realized in the following physical systems:

- Ultrarelativistic primordial plasma in early Universe
- Quark-gluon plasma in heavy-ion collisions
- Electron quasiparticles in Dirac and Weyl semimetals
- Degenerate electrons in compact stars
- Relativistic jets in black holes and neutron stars

Is there a chiral asymmetry in relativistic jets?

Chirality production in proto-neutron stars

Electron capture (core collapse 10^6 km$\rightarrow$10 km)

$$p \quad + \quad e_L^- \quad \rightarrow \quad n \quad + \quad \nu_L^e \tag{5}$$

is a weak interaction process where left-handed electrons are captured by protons producing $\mu_5 \neq 0$.
A. Ohnishi and N. Yamamoto, arXiv:1402.4760 suggested that magnetic field of neutron stars can be generated due to the chiral magnetic instability. Maxwell's equations

$$\nabla \times \mathbf{B} = \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}, \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{6}$$

with the CME and Ohm's currents

$$\mathbf{j} = \frac{\mu_5 \mathbf{B}}{2\pi^2} + \sigma \mathbf{E} \tag{7}$$

Chiral magnetic instability

result in

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma}\nabla^2 \mathbf{B} + \frac{2\alpha\mu_5}{\pi\sigma}\nabla \times \mathbf{B} \tag{8}$$

For the field $\mathbf{B}_{\pm} \sim (\hat{x} \pm \hat{y})e^{i(kz-\omega t)}$, modes with $0 < k < 2k_*$ are unstable

$$B_k(t) = B_k(0)e^{tk(2k_*-k)/\sigma}, \qquad k_* = \frac{\alpha\mu_5}{\pi} \tag{9}$$

The maximally unstable mode occurs for $k = k_*$

Chirality generation

Using the increase of neutron density due to the electron capture

$$\Delta n_n \sim 0.1\,\text{fm}^{-3}, \tag{10}$$

Ohnishi and Yamamoto estimated the generated chiral chemical potential

$$\mu_5 \approx 200\,\text{MeV} \tag{11}$$

that may give enormous magnetic field $B \sim 10^{18}$ G. Still electrons have nonzero mass that may hinder the chiral charge generation. Since the electron mass $m_e = 0.51$ MeV is much less than the electron chemical potential $\mu_e \approx 100$ MeV, it was argued in arXiv:1402.4760 that the electron mass effects can be neglected.

Role of mass

Chirality flip rate due to electron's mass [D. Grabowska, D.B. Kaplan, and S. Reddy, Phys. Rev. D **91**, 085035 (2015)] equals

$$\Gamma_m \approx \frac{\alpha^2 m_e^2}{3\pi\mu_e} \approx 1.4 \times 10^{-8}\,\text{MeV} \tag{12}$$

Evolution of chiral charge density is governed by

$$\frac{\partial n_5}{\partial t} = n_e \Gamma_w - n_5 \Gamma_m \tag{13}$$

During core collapse the electron fraction changes $\delta Y_e \approx 0.4$ in the free fall time $t_{ff} = 0.1\,s$ giving the chirality production rate per electron

$$\Gamma_w = \frac{\dot{Y}_e}{Y_e} \sim 1\,s^{-1} \sim 6.6 \times 10^{-22}\,\text{MeV} \tag{14}$$

Generated chiral chemical potential

For the steady state solution, the chiral charge density

$$n_5 = n_e \frac{\Gamma_w}{\Gamma_m} \sim 10^{-14} n_e \tag{15}$$

and the chemical potential

$$\mu_5 = \frac{\pi^2 n_5}{\mu_e^2} \sim 10^{-14} \mu_e \tag{16}$$

are very small.
Nonzero mass strongly (!) hinders the generation of chiral asymmetry.

Jets of active galactic nuclei

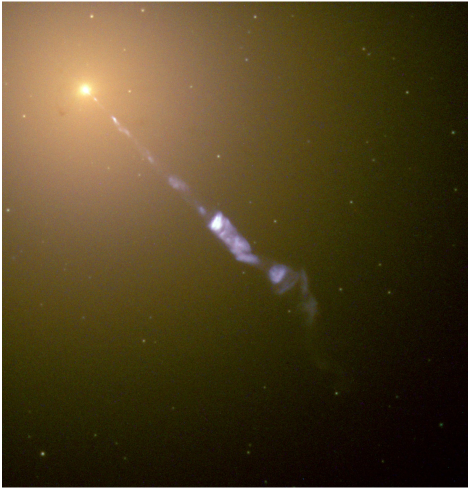


Figure: M87 jet, $\gamma \approx 6$, radio lobes stretch up to 80 kiloparsecs

Central engine

Figure: Central engine is supermassive black hole $M = 6.5 \times 10^9\, M_\odot$ with $R_s = 120\,\text{AU}$

Magnetic field $B \simeq 10^4$ G, time scale $t_0 = R_s/c \sim 10^5\ s$ is very large (macroscopic) in view of $R_s \sim 10^{13}\ m$

Chiral charge generation

For most optimistic $E \sim B$, the chiral charge density due to the chiral anomaly equals naively

$$n_5^{\rm naive} \simeq \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}\,t_0 \sim 10^4\,\mathrm{MeV}^3 \tag{17}$$

The inclusion of the chirality flip changes the situation dramatically with $\Gamma_m = \alpha^2 m_e^2/(3\pi T)$ and $T = 1\,\mathrm{MeV}$

$$n_5 = \frac{e^2}{2\pi^2\Gamma_m}\mathbf{E}\cdot\mathbf{B} \sim 10^{-17}\mathrm{MeV}^3, \qquad \mu_5 \sim 10^{-17}\mathrm{MeV} \tag{18}$$

leading to negligible chiral chemical potential

Magnetars

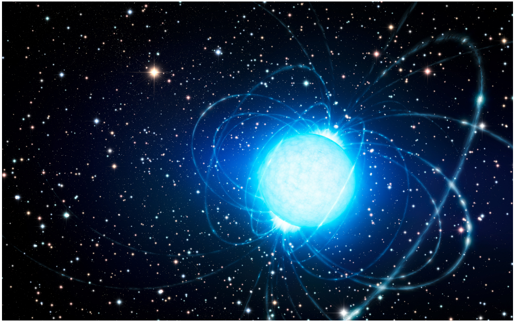


Figure: Artist's conception of a magnetar

$\mathbf{B} \sim 10^{11} - 10^{13}$ G (radio pulsars), $\mathbf{B} \sim 10^{14} - 10^{15}$ G (magnetars). About 30 magnetars are known in the Milky Way and are observed as soft gamma-repeaters or anomalous X-ray pulsars. Magnetar SGR 1935+2154 has been associated with fast radio burst

Fast radio bursts

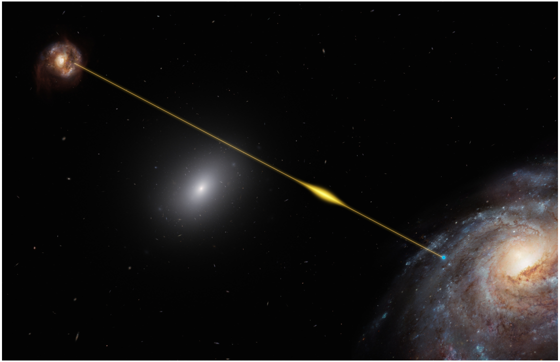


Figure: Artist's conception of FRB 181112 reaching the Earth

A fast radio burst is a transient radio pulse in the millisecond range with typical frequency 1.4 GHz and releasing on average as much energy as the Sun in 3 days

Magnetospheres of compact stars

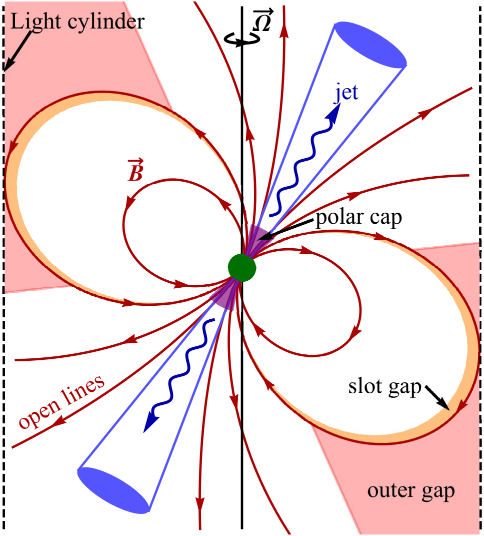


Figure: Magnetosphere of a compact star

General properties

- Vacuum model with charges on the compact star's surface and vacuum outside
- Corotating plasma model with the Goldreich–Julian charge density $\rho = \mathrm{div}\mathbf{E} \approx -2\,\boldsymbol{\Omega}\cdot\mathbf{B}$
- Consistency with the Faraday's law implies the necessity of transient gap regions with $\mathbf{E}\cdot\mathbf{B} \neq 0$

Physical characteristics

- Chirality production is possible in the gap region, where $\mathbf{E} \cdot \mathbf{B} \neq 0$
- $B \sim 1/r^3 \to$ only the polar cap region is of interest for chirality production
- Gap height $h = 3.6\,\mathrm{m}$, voltage drop across the gap $10^{12}\,\mathrm{V}$ that gives electric field $eE_{||} = 2.1 \times 10^{-7}\, m_e^2$

Chirality production

For magnetar with $B = 10^{15}\,\mathrm{G}$ and plasma temperature $T = 1\,\mathrm{MeV}$, we find that the steady state solution to

$$\frac{\partial n_5}{\partial t} = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} - \Gamma_m n_5 \qquad (19)$$

leads to a sizeable chiral charge density and chiral chemical potential

$$n_5 = \frac{e^2 EB}{2\pi^2\Gamma_m} \approx (0.1\,\mathrm{MeV})^3, \qquad \mu_5 \approx \frac{3n_5}{T^2} \approx 3.5\times 10^{-3}\,\mathrm{MeV} \qquad (20)$$

giving $k_* = \alpha\mu_5/\pi = 8\,\mathrm{eV}$

Dynamics in gap region

- Still electric field $E_{||} = 2.1 \times 10^{-7}\, m_e^2/e$ is much smaller than the Schwinger electric field $E_c = m_e^2/e$
- Gap is an intermittent phenomenon [D.B. Melrose and R. Yuen, Pulsar electrodynamics, J. Plasma Phys. **82**, 635820202 (2016)]
- As $E_{||}$ grows in the charge starvation region, it could lead to avalanches induced by a photon flux
- Gap region opening and closing is a dynamical process $\rightarrow$ particle-in-cell simulations are necessary
- Our proposition is to include the evolution equation for the chiral charge density and the CME current in these simulations

What should be clarified?

- Chirality and electron-positron pair production induced by energetic photons
- The rate of chirality flip Γ_m in a superstrong ($|eB| \gg m_e^2$) magnetic field
- Joint evolution of chiral imbalance and magnetic fields
- Inverse magnetic cascade and its observational consequences for electromagnetic emission (relevance for fast radio bursts?)

Inverse magnetic cascade

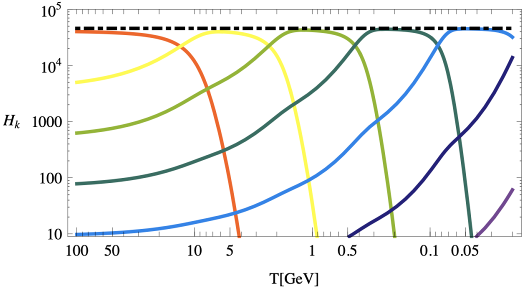


Figure: Transfer of helicity from shorter to longer modes (red to blue)

Magnetic field helicity evolves in chirally asymmetric primordial plasma in the form of inverse cascade [A. Boyarsky, J. Frohlich, and O. Ruchayskiy, Phys. Rev. Lett. **108**, 031301 (2012)]

Conclusions

- Chirality generation is possible in polar caps of magnetars due to the chiral anomaly
- Spinodal instability due to the CME leads to strong helical electromagnetic field modes
- Observational features could be polarized electromagnetic radiation (possibly relevant for fast radio bursts)

Thank you for attention!