Small-x Contribution to the Proton Spin Puzzle

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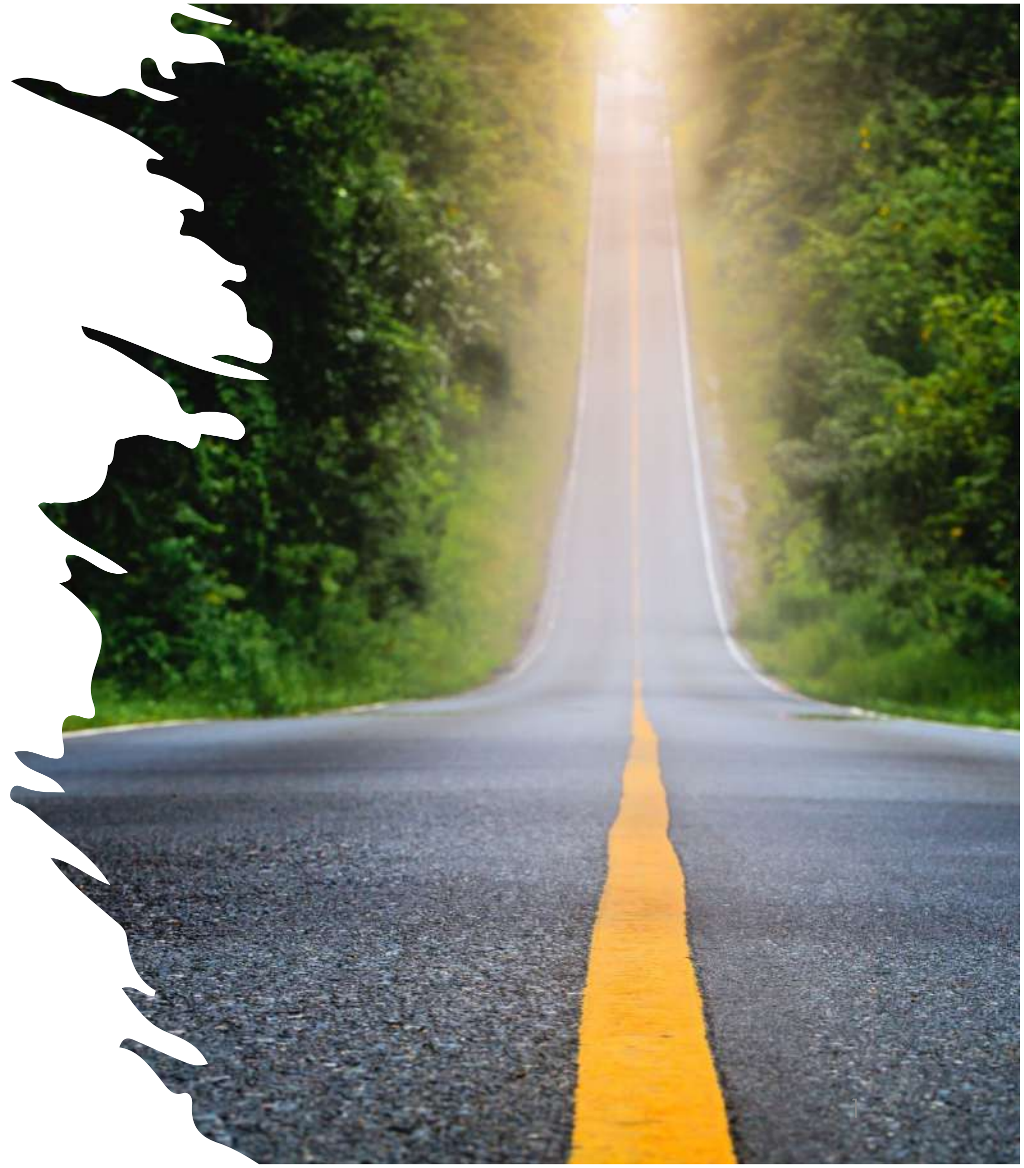
BASED ON WORK DONE WITH DAN PITONYAK AND MATT SIEVERT (2015-2018, 2021), FLORIAN COUGOULIC (2019-2020), JOSH TAWABUTR (2020-2021), ANDREY TARASOV (2021), DANIEL ADAMIAK, WALLY MELNITCHOUK, NOBUO SATO (2021)



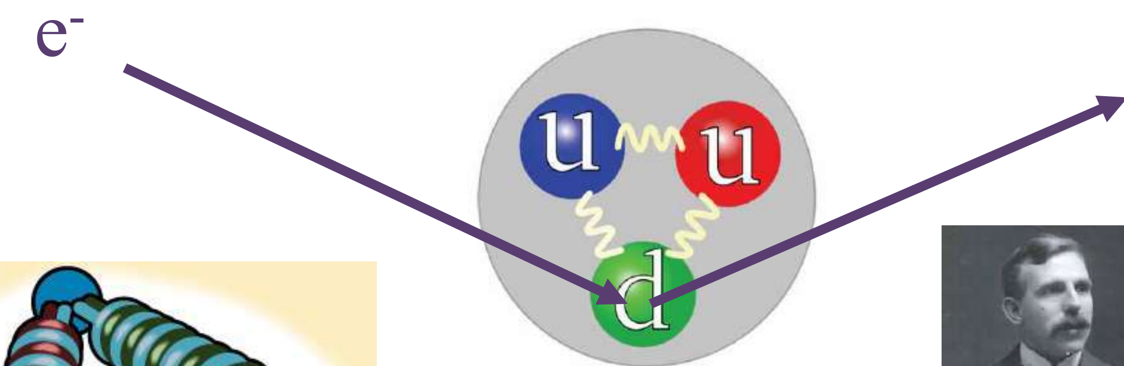
Outline

- Deep Inelastic Scattering (DIS) and the Electron-Ion Collider (EIC)
- Introduction to the proton spin puzzle
- Quark helicity at small x:
  - Quark helicity distribution at small x & polarized dipoles
  - Small-x evolution equations for quark helicity
- Solving the new evolution equations:
  - Small-x asymptotics of quark helicity
  - Phenomenology: first fit of small-x polarized DIS data using evolution in x
- Gluon helicity, quark and gluon OAM at small x: results
- Conclusions and Outlook

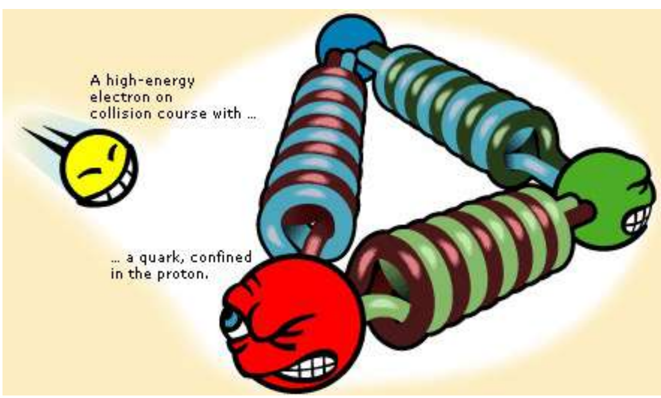
Deep Inelastic Scattering (DIS) and the Electron-Ion Collider (EIC)

Deep Inelastic Scattering

- One can probe the structure of a proton by shooting electrons at it at high energies, observing the electron recoil and proton breakup: this is called Deep Inelastic Scattering (DIS).



proton



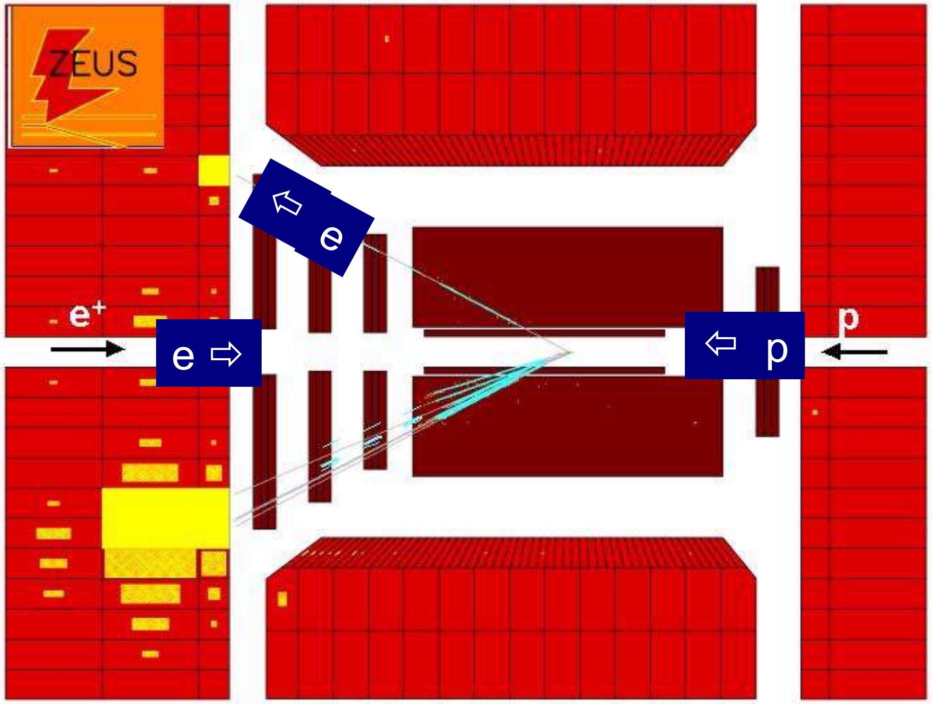
cf. Rutherford et al shooting alpha particles at a gold foil (1909)

DIS Movie

Deep Inelastic Scattering

Here is a typical deep inelastic positron-proton scattering event.

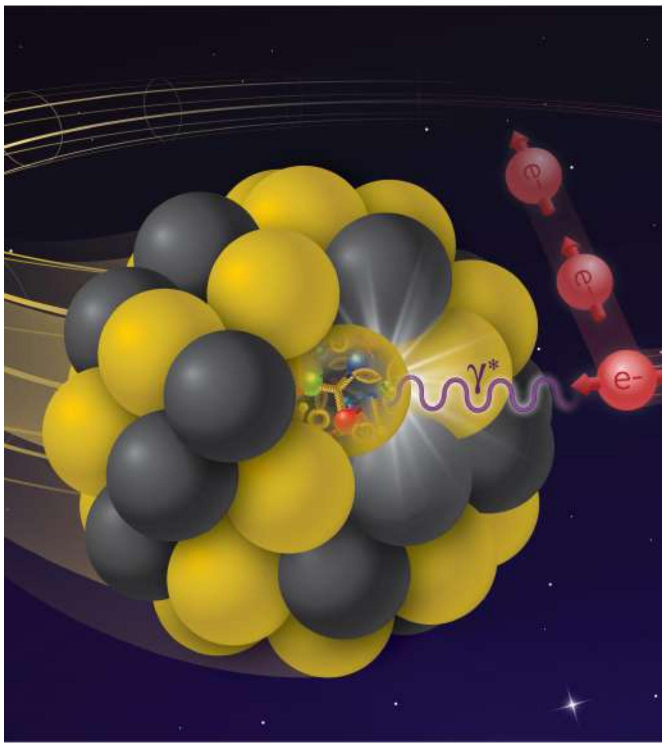
(ZEUS experiment at HERA collider, DESY lab in Hamburg, Germany.)



DIS Experiments

- DIS experiments were done at SLAC, CERN, and HERA.
- There are presently DIS experiments running at Jefferson Lab and at CERN (COMPASS).
- In Europe there is a proposal to build LHeC after LHC.
- In China there is also an EIC proposal being considered.
- US DOE approved future construction of the US Electron-Ion Collider (EIC) in January 2020. (DOE granted EIC project CD0 in December 2019 and CD1 in July 2021.)

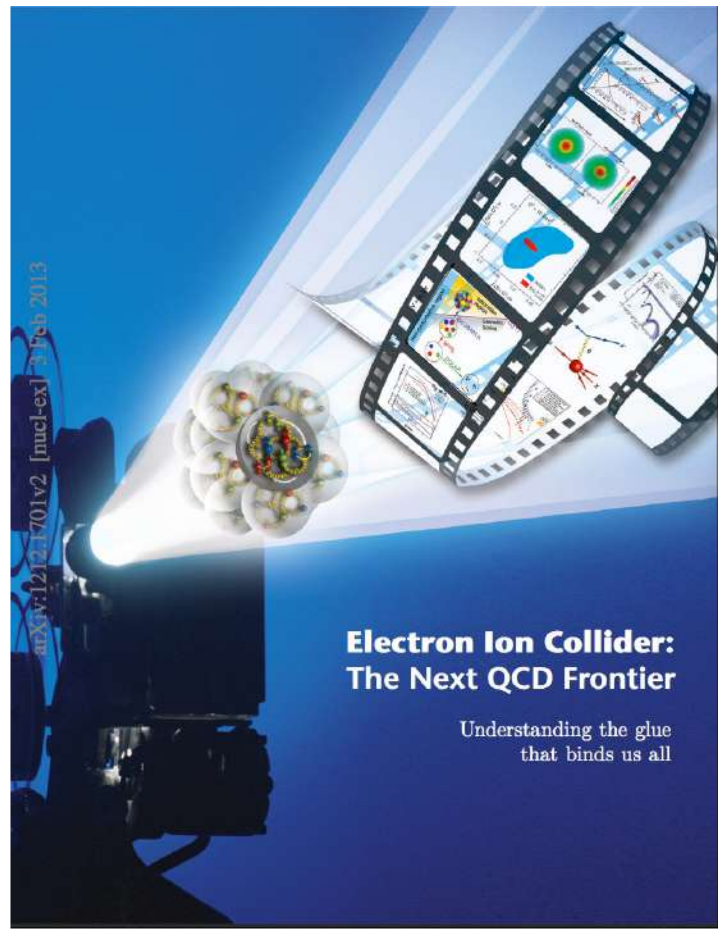
Electron-Ion Collider



- Initial discussions about the EIC started back in the mid- to late-1990s.
- There was a number of workshops in the late 1990s and in 2000s dedicated to EIC physics.

Electron-Ion Collider (EIC) White Paper

- EIC WP was finished in late 2012 + 2nd edition in 2014
- A several-year effort by a 19-member steering committee + 58 co-authors
- arXiv:1212.1701 [nucl-ex]
- At the time EIC could be sited either at Brookhaven or at Jefferson Lab.

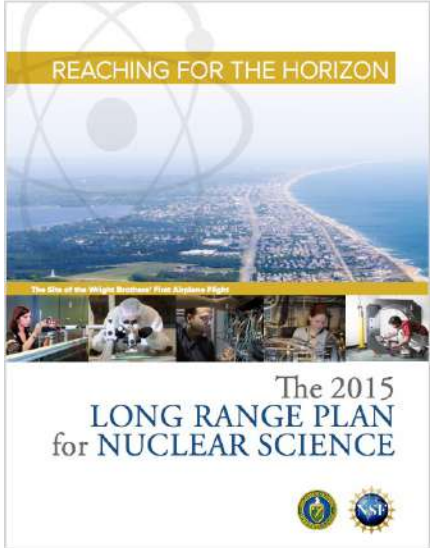


2015 Nuclear Physics Long Range Plan

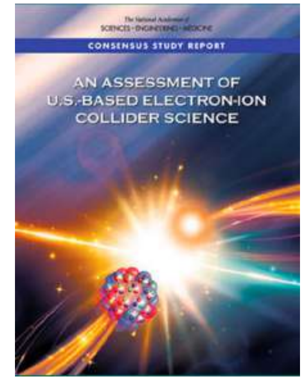
RECOMMENDATION III

Gluons, the carriers of the strong force, bind the quarks together inside nucleons and nuclei and generate nearly all of the visible mass in the universe. Despite their importance, fundamental questions remain about the role of gluons in nucleons and nuclei. These questions can only be answered with a powerful new electron ion collider (EIC), providing unprecedented precision and versatility. The realization of this instrument is enabled by recent advances in accelerator technology.

We recommend a high-energy high-luminosity polarized EIC as the highest priority for new facility construction following the completion of FRIB.



National Academies Review



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A Domestic Electron Ion Collider Would Unlock Scientific Mysteries of Atomic Nuclei, Maintain U.S. Leadership in Accelerator Science, New Report Says

News Release | **July 24, 2018**

WASHINGTON – The science questions that could be answered by an electron ion collider (EIC) – a very large-scale particle accelerator – are significant to advancing our understanding of the atomic nuclei that make up all visible matter in the universe, says a new report by the National Academies of Sciences, Engineering, and Medicine. Beyond its impact on nuclear science, the advances made possible by an EIC could have far-reaching benefits to the nation's science- and technology-driven economy as well as to maintaining U.S. leadership in nuclear physics and in collider and accelerator technologies.

The National Academies were asked by the U.S. Department of Energy (DOE) to examine the scientific importance of an EIC, as well as the international implications of building domestic EIC facility. The committee that conducted the study and wrote the report concluded that the science that could be addressed by an EIC is compelling and would provide long-elusive answers on the nature of matter. An EIC would allow scientists to investigate where quarks and gluons, the tiny particles that make up neutrons and protons, are located inside protons and neutrons, how they move, and how they interact together. While the famous Higgs mechanism explains the masses of the quarks, the most significant portion of the mass of a proton or neutron comes from its gluons and their interactions. Crucial questions that an EIC would answer include the origin of the mass of atomic nuclei, the origin of spin of neutrons and protons – a fundamental property that makes magnetic resonance imaging (MRI) possible, how gluons hold nuclei together, and whether emergent forms of matter made of dense gluons exist.

The report says a new EIC accelerator facility would have capabilities beyond all previous electron scattering machines in the U.S., Europe, and Asia. High energies and luminosities – the measure of the rate at which particle collisions occur – are required to achieve the fine resolution needed, and to reach such intensities and energy levels requires a collider where beams of electrons smash into beams of protons or heavier ions. Comparing all existing and proposed accelerator facilities around the world, the committee concluded that an EIC with high energy and luminosity, and highly polarized electron and ion beams, would be unique and in a position to greatly further our understanding of visible matter.

DOE Announcement on EIC

EIC is to be built at BNL

January 9, 2020

WASHINGTON, D.C. – Today, the **U.S. Department of Energy (DOE)** announced the selection of Brookhaven National Laboratory in Upton, NY, as the site for a planned major new nuclear physics research facility.

The Electron Ion Collider (EIC), to be designed and constructed over ten years at an estimated cost between $1.6 and $2.6 billion, will smash electrons into protons and heavier atomic nuclei in an effort to penetrate the mysteries of the "strong force" that binds the atomic nucleus together.

"The EIC promises to keep America in the forefront of nuclear physics research and particle accelerator technology, critical components of overall U.S. leadership in science," said **U.S. Secretary of Energy Dan Brouillette**. "This facility will deepen our understanding of nature and is expected to be the source of insights ultimately leading to new technology and innovation."

"America is in the golden age of innovation, and we are eager to take this next step with EIC. The EIC will not only ensure U.S. leadership in nuclear physics, but the technology developed for EIC will also support potential tremendous breakthroughs impacting human health, national competiveness, and national security," said **Under Secretary for Science Paul Dabbar**. "We look forward to our continued world-leading scientific discoveries in conjunction with our international partners."

The EIC's high luminosity and highly polarized beams will push the frontiers of particle accelerator science and technology and provide unprecedented insights into the building blocks and forces that hold atomic nuclei together.

Design and construction of an EIC was recommended by the National Research Council of the National Academies of Science, noting that such a facility "would maintain U.S. leadership in nuclear physics" and "help to maintain scientific leadership more broadly." Plans for an EIC were also endorsed by the federal Nuclear Science Advisory Committee.

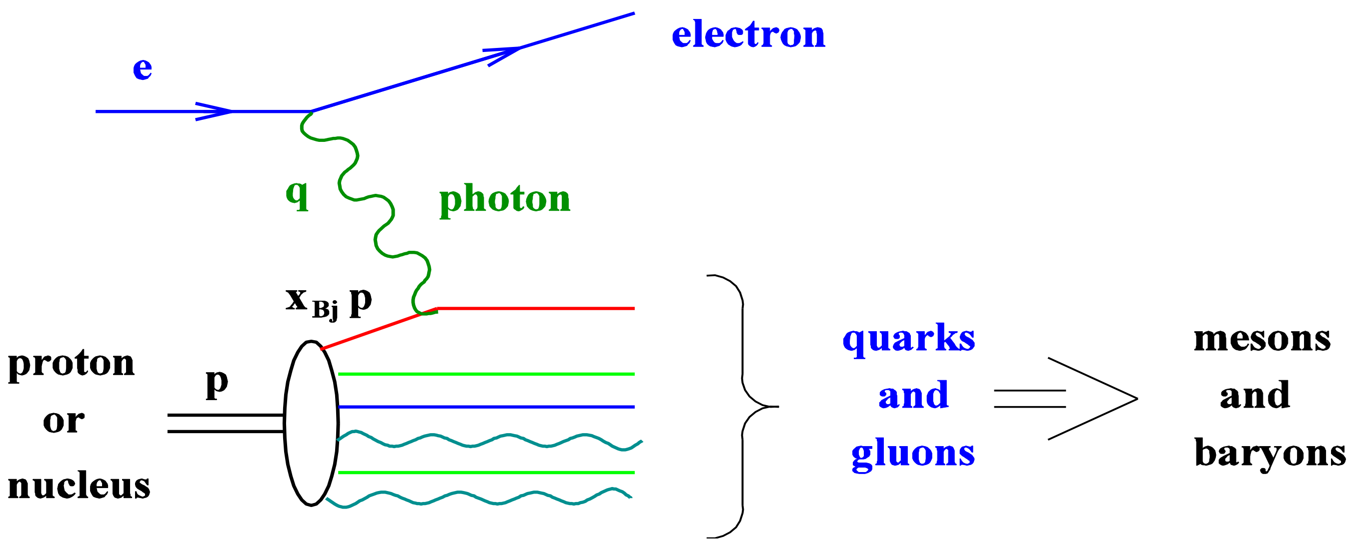
Secretary Brouillette approved Critical Decision-0, "Approve Mission Need," for the EIC on December 19, 2019.

July 2021: CD1 granted

QCD at EIC Physics Topics

- Spin and Nucleon Structure
  - Spin of a nucleon
  - Transverse momentum distributions (TMDs)
  - Spatial imaging of quarks and gluons (GPDs)
- QCD Physics in a Nucleus
  - High gluon densities and saturation
  - Quarks and Gluons in the Nucleus
  - Connections to p+A, A+A, and cosmic ray physics

Kinematics of DIS



- Photon carries 4-momentum q_μ , its virtuality is

$$Q^2 = -q_\mu \, q^\mu$$

- Photon hits a quark in the proton carrying momentum $x_{Bj} p$ with p being the proton's momentum. Parameter x_{Bj} is the Bjorken x variable.

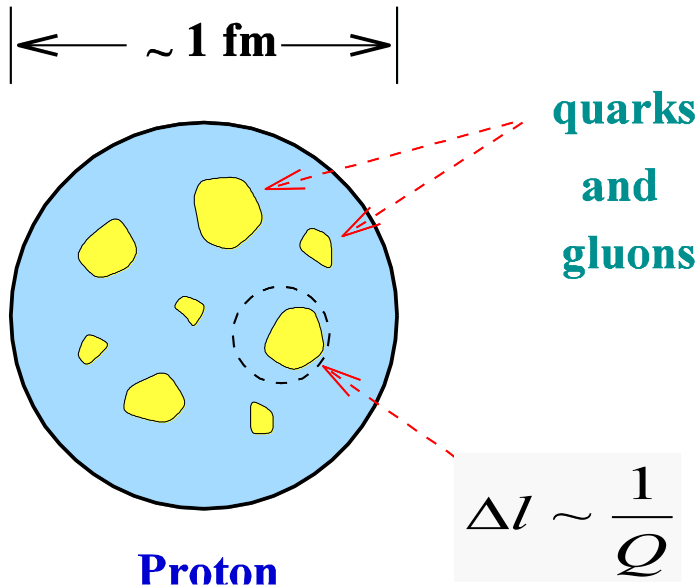
Physical Meaning of Q

Uncertainty principle teaches us that

$$\Delta p \Delta l \approx \hbar$$

which means that the photon probes the proton at the distances of the order (ħ=1)

$$\Delta l \sim \frac{1}{Q}$$

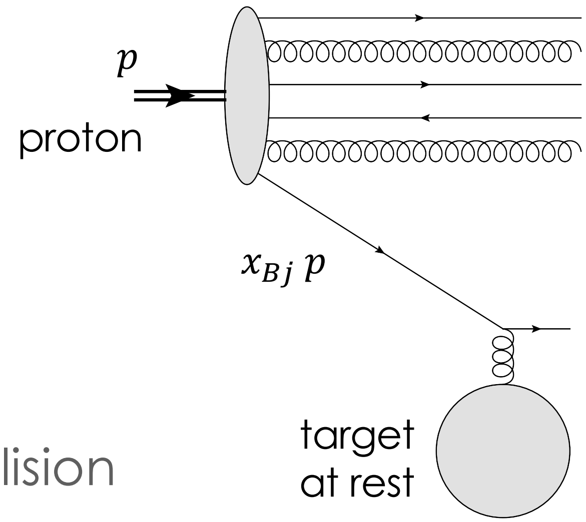


Large Momentum Q = Short Distances Probed

Physical Meaning of Bjorken x

The quarks and gluons that interact with the target have their typical momenta on the order of the typical momentum in the target,

$$x_{Bj}\, p \approx q \approx m.$$



Then the energy of the collision

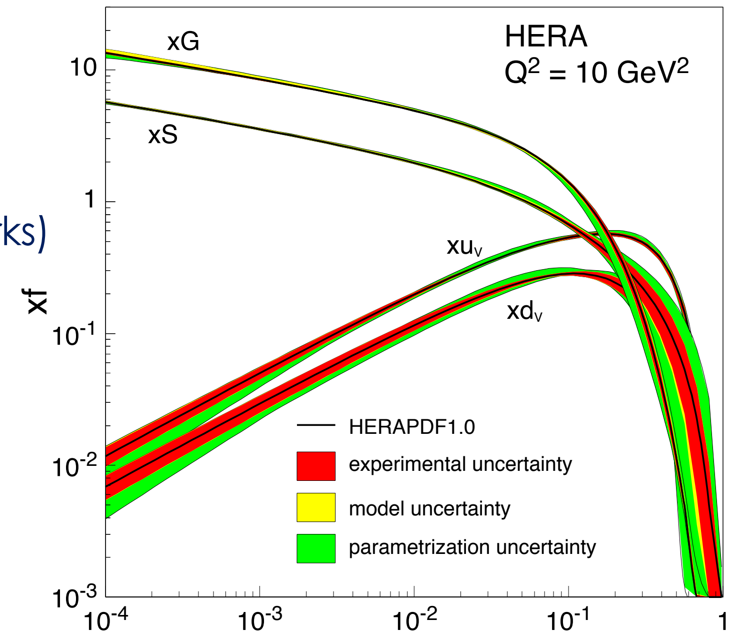
$$E \sim p \sim \frac{1}{x_{Bj}}$$

High Energy = Small x

Gluons and Quarks at Small-x

- There is a large number of small-x gluons (and sea quarks) in a proton:

x (# gluons or quarks)



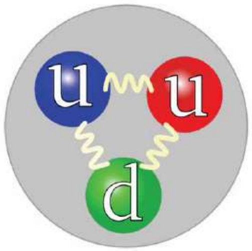
x =fraction of the proton momentum they carry

- $G(x, Q^2)$, $q(x, Q^2)$ = gluon and quark number densities / parton distribution functions (q=u,d, or S for sea).

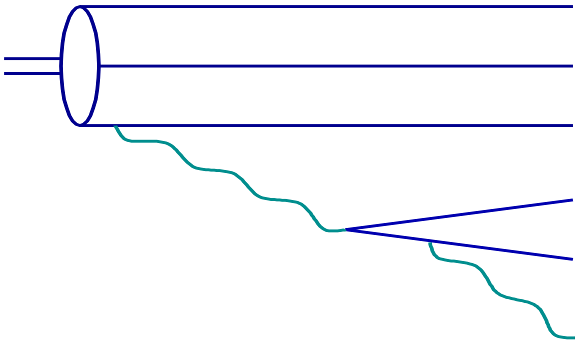
Gluons and Quarks in the Proton

⇨ There are many quarks, anti-quarks and gluons at small-x !

⇨ How do we reconcile this result with the picture of the proton made up of three valence quarks?

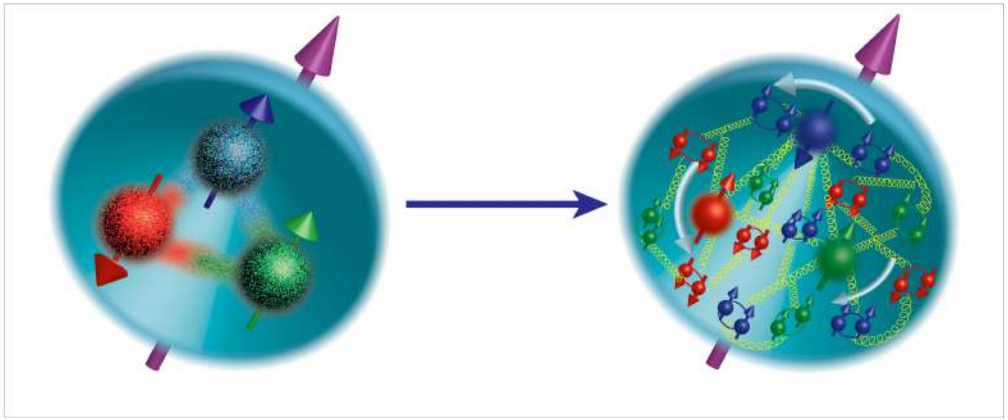


⇨ Qualitatively we understand that these extra (sea) quarks and gluons are emitted by the original three valence quarks in the proton.



Proton Spin Puzzle:
an Introduction

Proton Spin



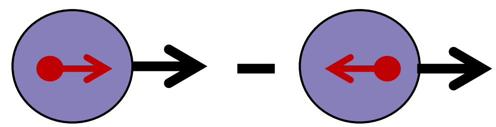
Our understanding of nucleon spin structure has evolved:

- In the 1980's the proton spin was thought of as a sum of constituent quark spins (left panel)

- Currently we believe that the proton spin is a sum of the spins of valence and sea quarks and of gluons, along with the orbital angular momenta of quarks and gluons (right panel)

Helicity Distributions

- To quantify the contributions of quarks and gluons to the proton spin on defines helicity distribution functions: number of quarks/gluons with spin parallel to the proton momentum minus the number of quarks/gluons with the spin opposite to the proton momentum:



- The helicity parton distributions are

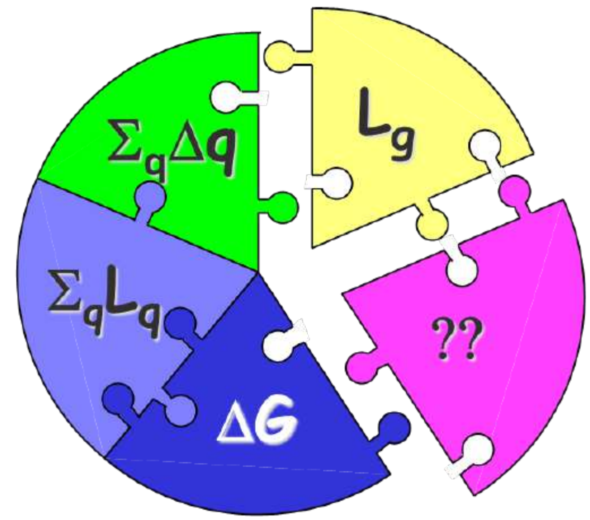
$$\Delta f(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$$

with the flavor-singlet quark helicity distribution

$$\Delta\Sigma \equiv \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}$$

and $\Delta G(x, Q^2)$ the gluon helicity distribution.

Proton Helicity Sum Rule



- Helicity sum rule (Jaffe&Manohar, 1989):

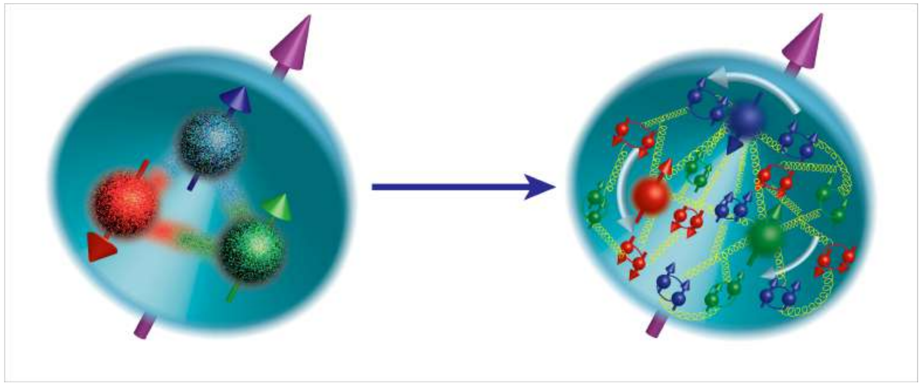
$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

with the net quark and gluon spin

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

- L_q and L_g are the quark and gluon orbital angular momenta (OAM)

Constituent Quark Model Expectation



- In the constituent quark model of the proton, the picture is simple: two spin-up quarks and one spin-down quark, as shown in the left panel.

- This predicts that all of the proton spin is carried by the quarks, such that

$$S_q = \frac{1}{2}$$

- Done?

Proton Spin Puzzle

- The spin puzzle began when the EMC collaboration measured the proton g_1 structure function ca 1988. Their data resulted in

$$S_q \approx 0.05$$

- It appears (constituent) quarks do not carry all of the proton spin (which would have corresponded to $S_q = 1/2$).

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

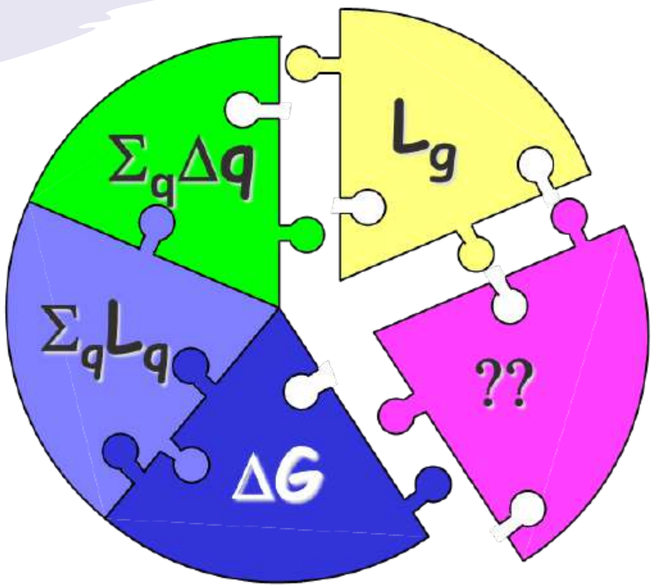
- Missing spin can be
  - Carried by gluons
  - In the orbital angular momenta of quarks and gluons
  - At small x:

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

Can't integrate down to zero, use x_{min} instead!

  - Or all of the above!

Current Knowledge of Proton Spin



- The proton spin carried by the quarks is estimated to be (for $0.001 < x < 1$)

$$S_q(Q^2 = 10\,\mathrm{GeV}^2) \approx 0.15 \div 0.20$$

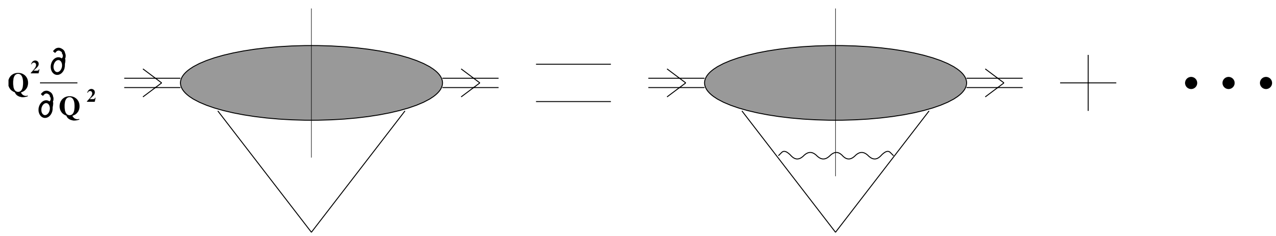
- The proton spin carried by the gluons is (for $0.05 < x < 1$, STAR+COMPASS+HERMES+…)

$$S_G(Q^2 = 10\,\mathrm{GeV}^2) \approx 0.13 \div 0.26$$

- Unfortunately, the uncertainties are large. Note also that the x-ranges are limited, with more spin (positive or negative) possible at small x.


The DGLAP Equation


The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation is a renormalization group equation describing variation of parton distributions with Q^2. Diagrammatically we can represent it as follows:



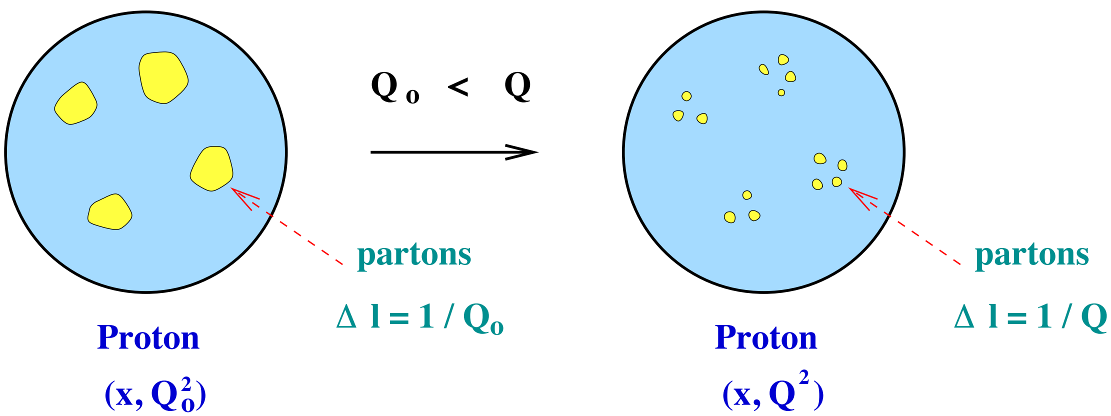
- For the helicity-dependent case the equations read

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} \Delta\Sigma(x,Q^2) \\ \Delta G(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta P_{qq}(z) & \Delta P_{qG}(z) \\ \Delta P_{Gq}(z) & \Delta P_{GG}(z) \end{pmatrix} \begin{pmatrix} \Delta\Sigma(\frac{x}{z},Q^2) \\ \Delta G(\frac{x}{z},Q^2) \end{pmatrix}$$

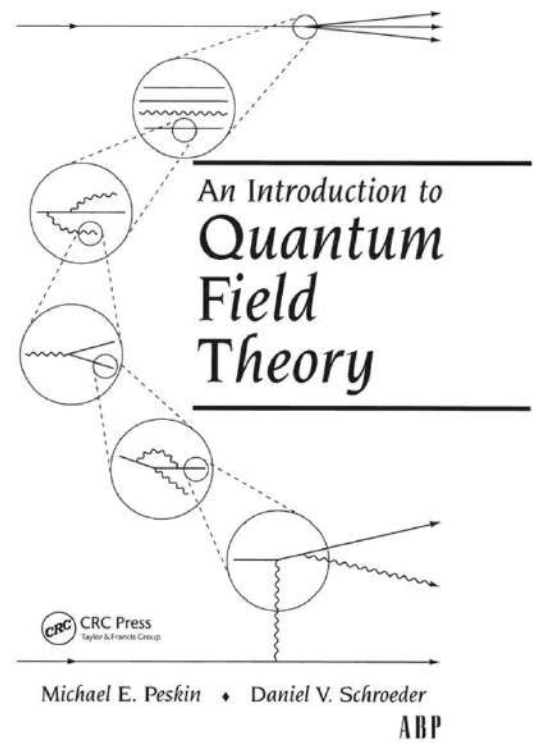
(ΔP_{ij} are the spin-dependent splitting functions).

DGLAP Evolution: the Physics Meaning

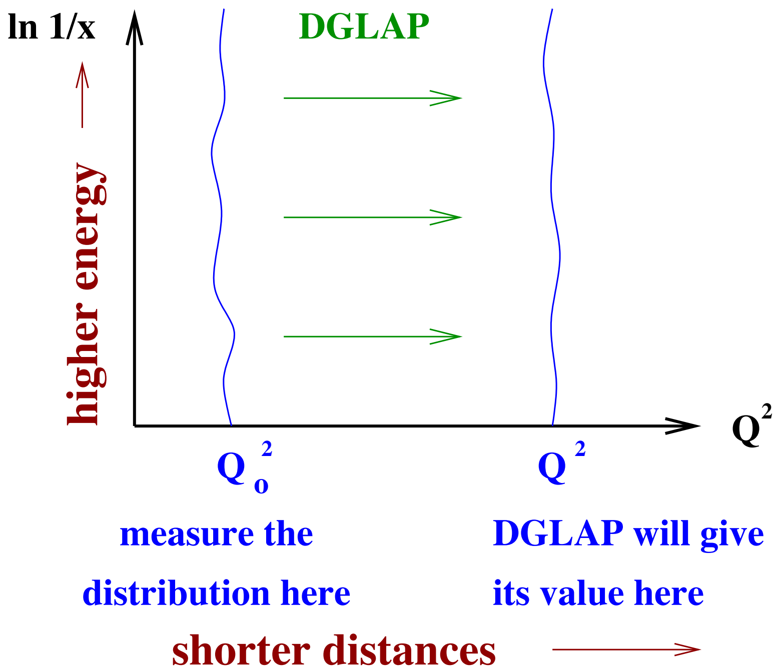
As we increase the resolution (decrease 1/Q), we “see” more partons:



Indeed, this RG flow is on the cover of Peskin& Schroeder:



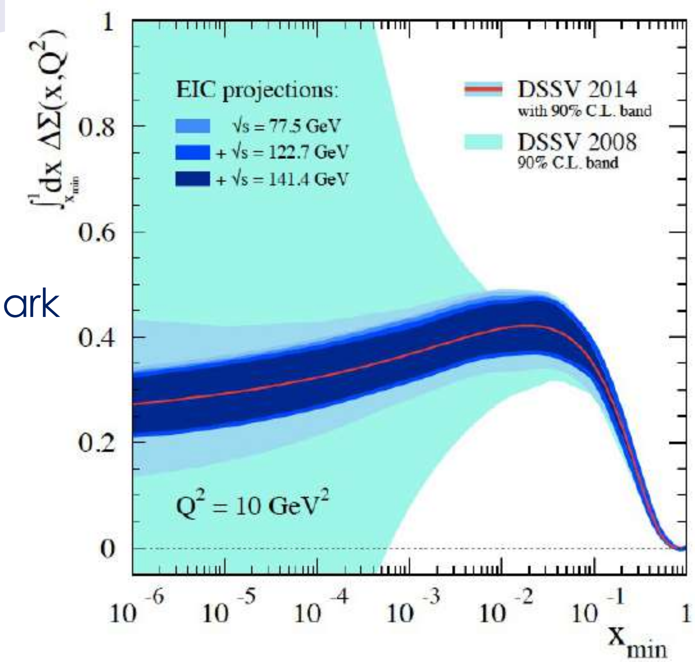
How DGLAP works



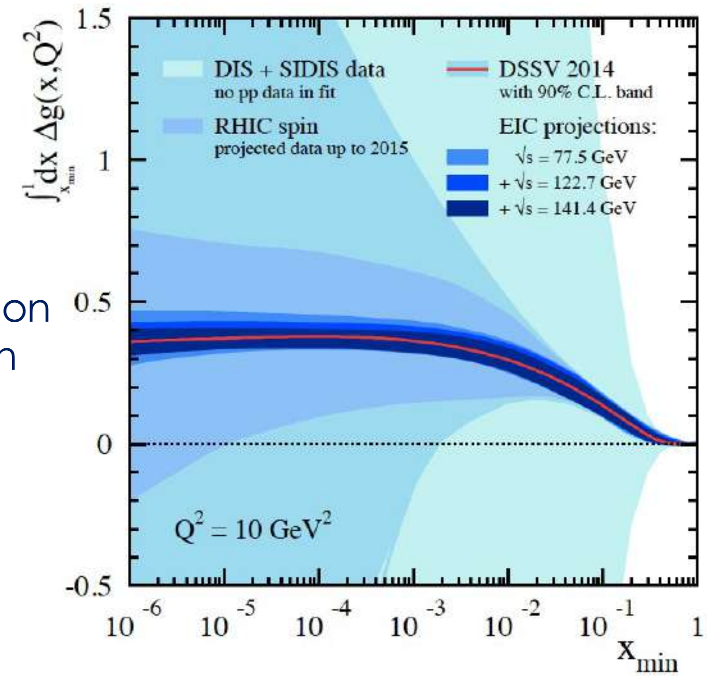
- One still needs to measure the PDFs at the initial scale Q_0^2 (at all values of x). Then the DGLAP equation would predict the PDFs at higher Q^2.
- In practice one parametrizes the x-dependence of PDFs at Q_0^2 and varies the parameters until they fit the data at all available Q^2.
- Problem/feature: DGLAP equation does not quite predict the x-dependence of PDFs. The x-dependence is strongly affected by the initial conditions at Q_0^2.
- Consequence: DGLAP **cannot predict** how much spin there is at small x!

How much spin is there at small x?

2 x (quark spin)



gluon spin



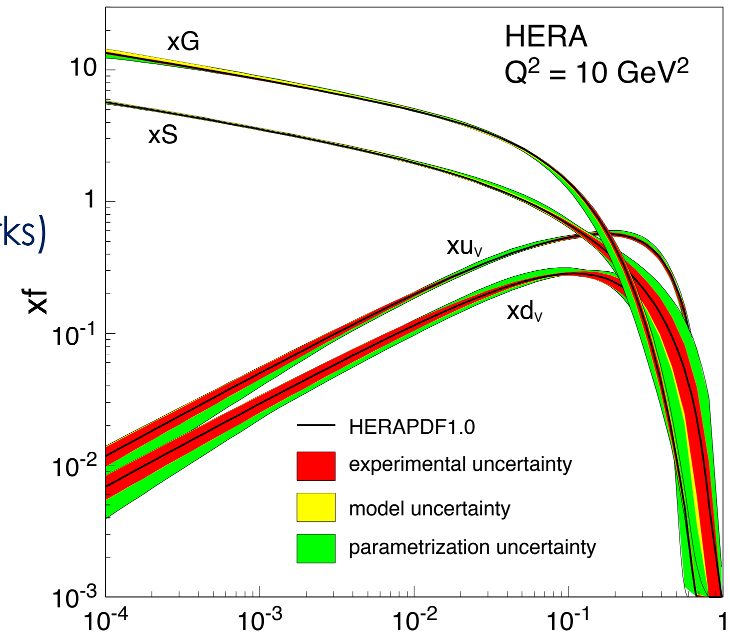
- E. Aschenauer et al, arXiv:1509.06489 **[hep-ph]**, (DSSV = de Florian, Sassot, Stratmann, Vogelsang, DGLAP-based helicity PDF extraction from data)

- Uncertainties are very large at small x! (EIC may reduce them.)

Gluons and Quarks at Small-x

- There is a large number of small-x gluons (and sea quarks) in a proton:

x (# gluons or quarks)



x =fraction of the proton momentum they carry

- $G(x, Q^2)$, $q(x, Q^2)$ = gluon and quark number densities / parton distribution functions (q=u,d, or S for sea).

Our goal

- The goal is to constrain theoretically the amount of proton spin and OAM coming from small x.

- Any existing and future experiment probes the helicity distributions and OAM down to some x_{min} .

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx \, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx \, \Delta G(x, Q^2)$$

$$L_{q+\bar{q}}(Q^2) = \int_0^1 dx \, L_{q+\bar{q}}(x, Q^2) \qquad L_G(Q^2) = \int_0^1 dx \, L_G(x, Q^2)$$

- At very small x (for the proton), saturation sets in: that region likely carries a negligible amount of proton spin. But what happens at larger (but still small) x?

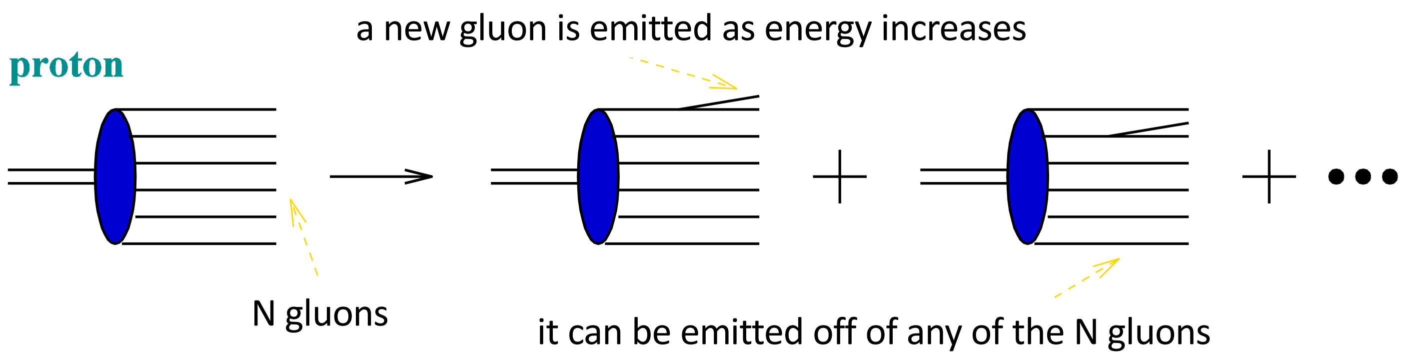
Philosophy of our approach

- DGLAP equation evolves in Q^2, it does not evolve in x.
- Hence, DGLAP-based analyses (DSSV, NNPDF, standard JAM) cannot predict the x-dependence of PDFs.
- If we want to predict helicity PDFs at small x, we need a different evolution equation evolving in x.
- Such helicity evolution equations were constructed by D. Pitonyak, M. Sievert, and YK, 2015-2018 using an approach similar to the BFKL/BK/JIMWLK evolution (applied to the unpolarized distributions).


The BFKL Equation

Balitsky, Fadin, Kuraev, Lipatov ‘78


Start with N gluons in the proton’s wave function. As we increase the energy a new gluon can be emitted by either one of the N gluons. The number of newly emitted particles is proportional to N.

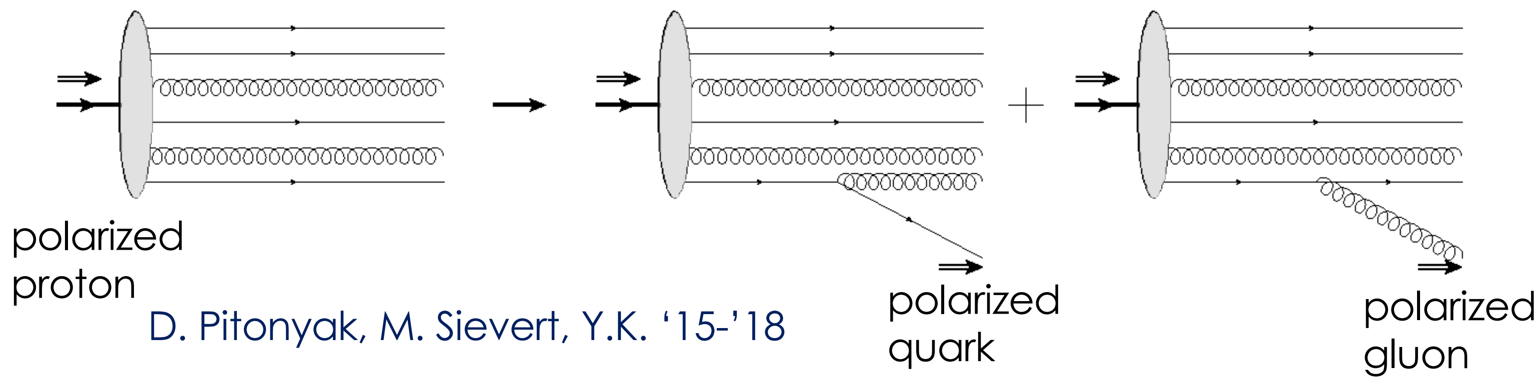


The BFKL equation for the number of gluons N reads:

$$\frac{\partial}{\partial \ln(1/x)} N(x,Q^2) = \alpha_S K_{BFKL} \otimes N(x,Q^2)$$

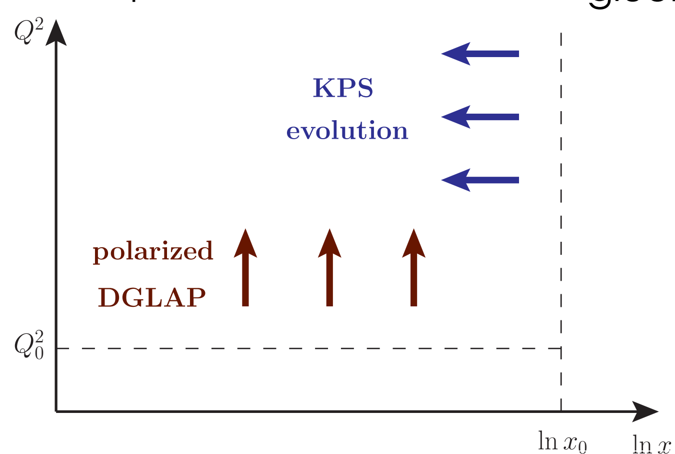
Helicity Evolution at Small x

- To understand how much of the proton's spin is at small x one can construct a helicity analogue of the BFKL equation:



D. Pitonyak, M. Sievert, Y.K. '15-'18

- This new helicity evolution equation is subtle, since it must keep track of both quark and gluon helicities. (Other small-x evolution equations, BFKL/BK/JIMWLK, only have gluons at leading order.)



Quark Helicity at Small x

YK, D. Pitonyak, M. Sievert, arXiv:1511.06737 [hep-ph], arXiv:1610.06197 [hep-ph];
YK, M. Sievert, arXiv:1505.01176 [hep-ph], arXiv:1808.09010 [hep-ph].

Dipole picture of DIS

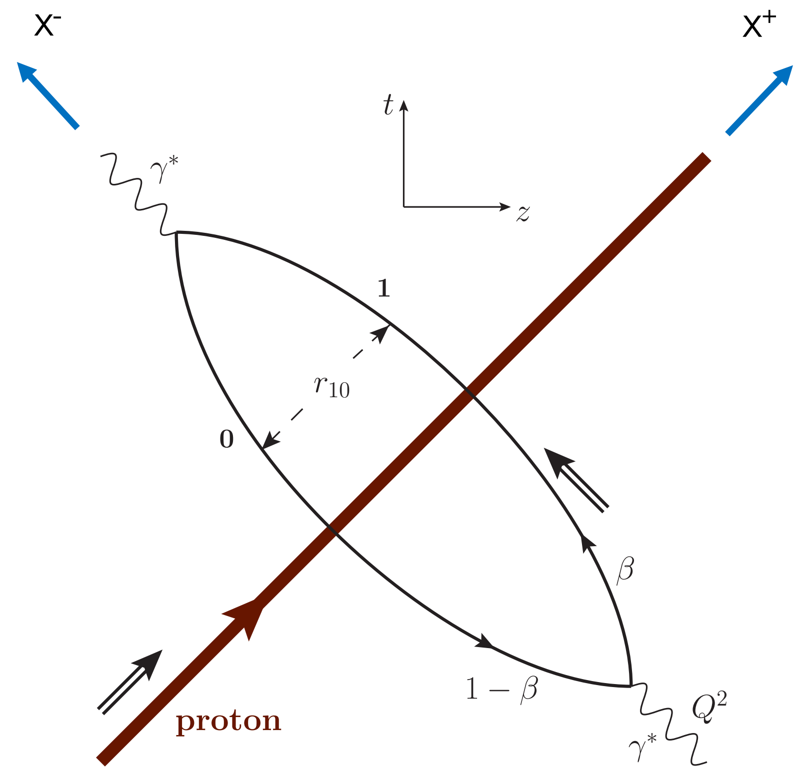
$$W^{\mu\nu} = \frac{1}{4\pi M_p} \int d^4x \, e^{iq\cdot x} \, \langle P | j^\mu(x) \, j^\nu(0) | P \rangle$$

Large q^- → large x^- separation

$$q^\mu = \left(\frac{Q^2}{2q^-}, q^-, 0_\perp \right)$$

Same is true for PDFs: small x means large x^- spead

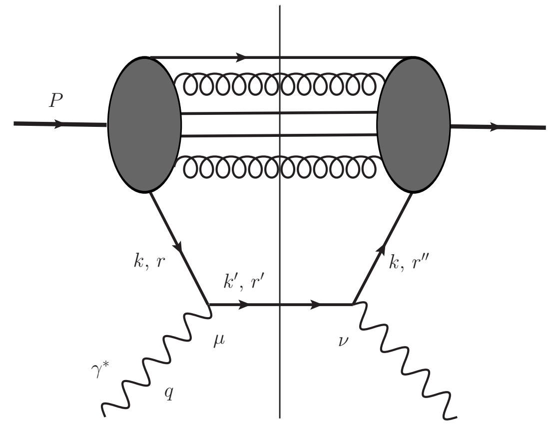
$$q(x, Q^2) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dx^- \, e^{ixP^+x^-} \, \langle P | \bar{q}(x^-) \gamma^+ \mathcal{U} \, q(0) | P \rangle$$

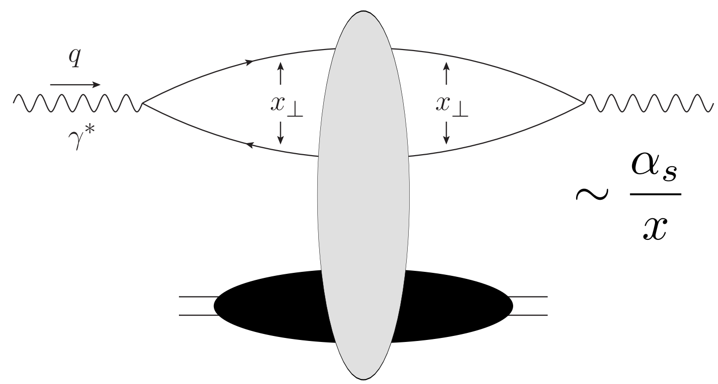


Dipole picture of DIS

- At small x, the dominant contribution to DIS structure functions does not come from the handbag diagram.

- Instead, the dominant terms comes from the dipole picture of DIS, where the virtual photon splits into a quark-antiquark pair, which then interacts with the target.



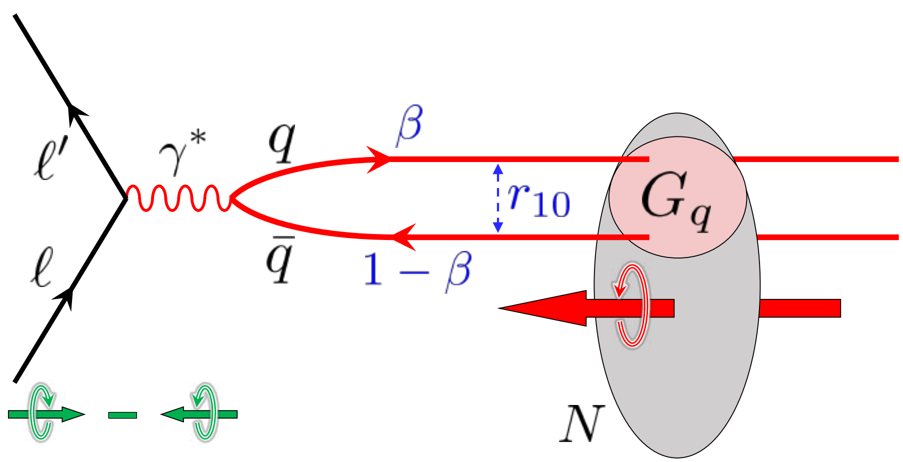


$$\sim \frac{\alpha_s}{x}$$

Quark Helicity Distribution at Small x

- One can show that the quark helicity PDF ($\Delta\Sigma$) at small-x can be expressed in terms of the polarized dipole amplitude:

$$\Delta\Sigma(x, Q^2) \sim G(r_{10}^2, \beta)$$

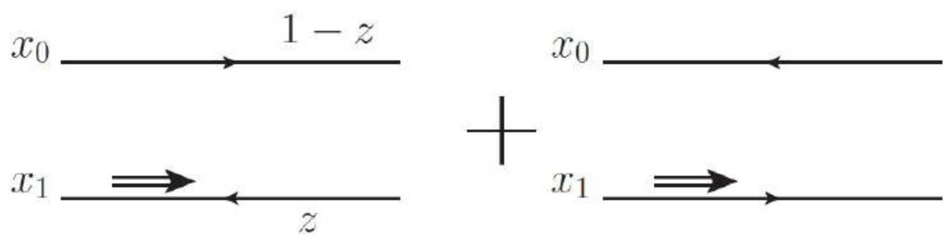


β = longitudinal momentum fraction (aka z);
r_{10} = (transverse) dipole size (aka x_{10})

$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

Polarized Dipole: non-eikonal small-x physics

- All flavor-singlet small-x helicity observables depend on one object, “polarized dipole amplitude”:



$$G_{10}(z) \equiv \frac{1}{2N_c} \mathrm{Re} \left\langle\!\!\left\langle \mathrm{T\,tr} \left[V_{\underline{0}} V_{\underline{1}}^{pol\,\dagger} \right] + \mathrm{T\,tr} \left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger} \right] \right\rangle\!\!\right\rangle (z)$$

unpolarized quark

polarized quark: eikonal propagation, non-eikonal spin-dependent interaction

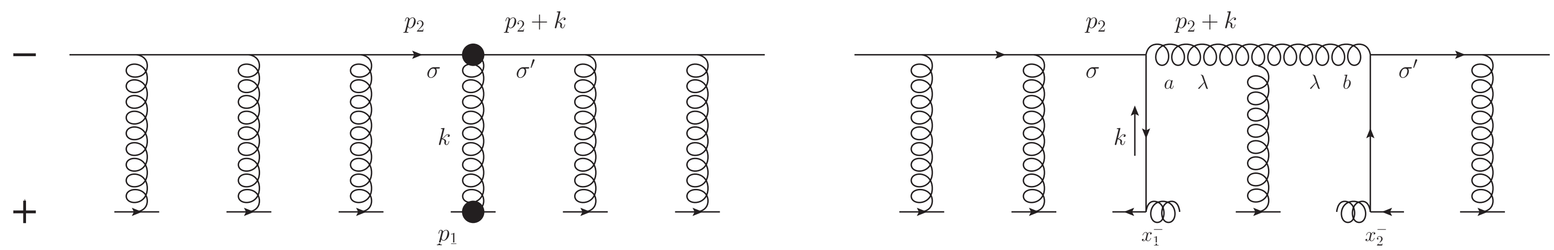
$$V_{\underline{x}} = \mathcal{P} \exp \left[ig \int\limits_{-\infty}^{\infty} dx^- A^+(0^+, x^-, \underline{x}) \right]$$

- Double brackets denote an object with energy suppression scaled out:

$$\left\langle\!\!\left\langle \mathcal{O} \right\rangle\!\!\right\rangle (z) \equiv zs \left\langle \mathcal{O} \right\rangle (z)$$

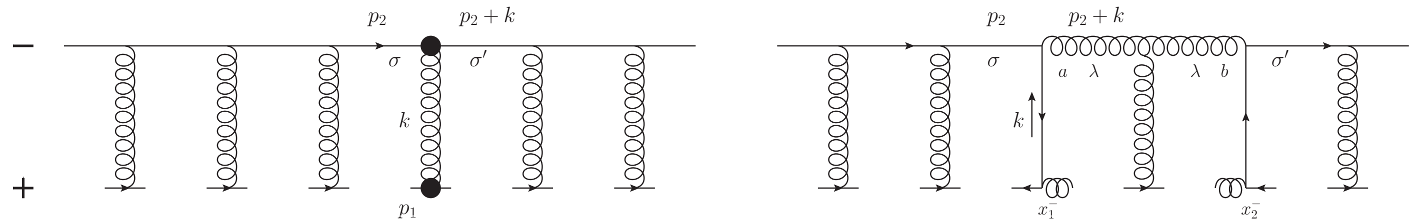
Polarized fundamental "Wilson line"

- To complete the definition of the polarized dipole amplitude, we need to construct the definition of the polarized "Wilson line" V^{pol}, which is the leading helicity-dependent contribution for the quark scattering amplitude on a longitudinally-polarized target proton.



- At the leading order we can either exchange one non-eikonal *t*-channel gluon (with quark-gluon vertices denoted by blobs above) to transfer polarization between the projectile and the target, or two *t*-channel quarks, as shown above.

Polarized fundamental "Wilson line"



- In the end one arrives at (KPS '17; YK, Sievert, '18; cf. Chirilli '18; Altinoluk et al, '20)

$$V_{\underline{x}}^{pol} = \frac{igp_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

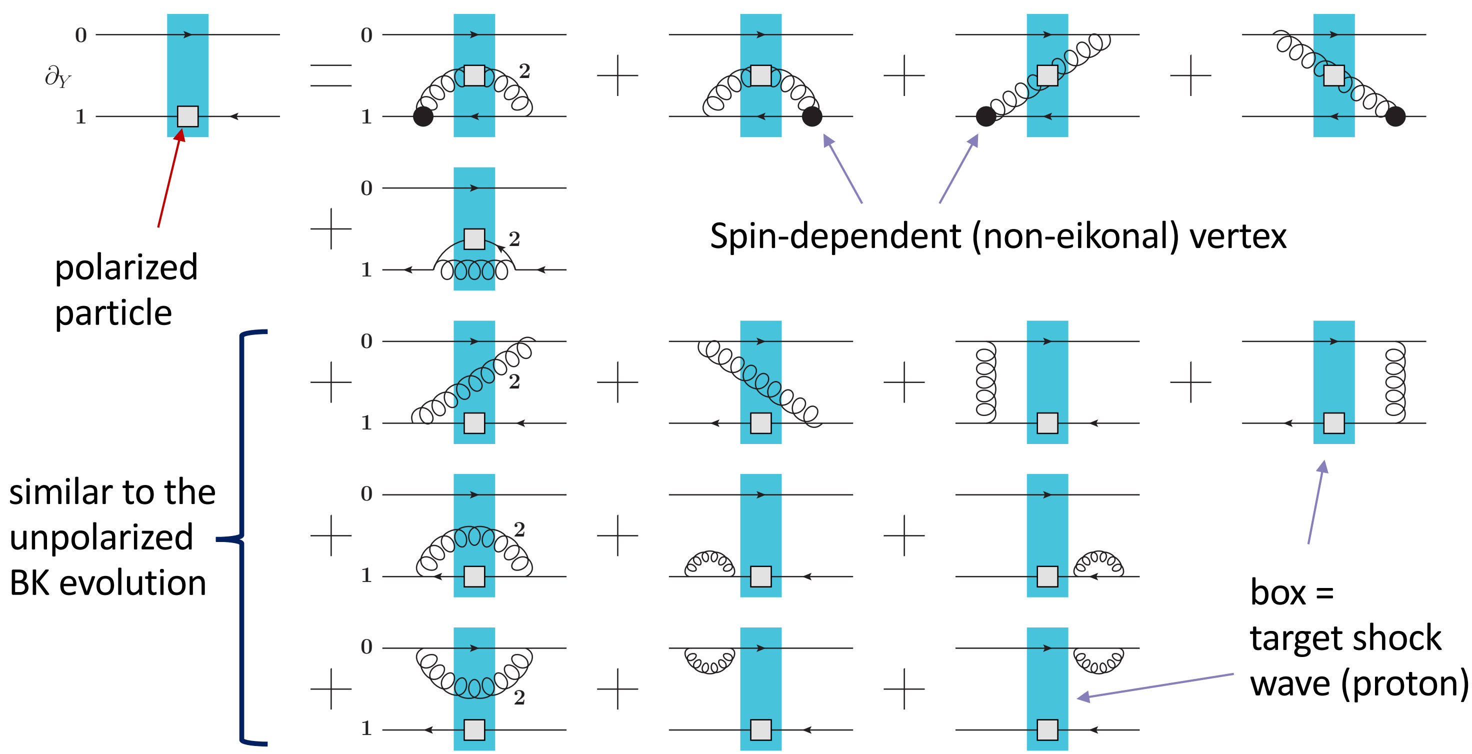
$$- \frac{g^2 \, p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2} \gamma^+ \gamma^5\right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty].$$

- We have employed an adjoint light-cone Wilson line $U_{\underline{x}}[b^-, a^-] = \mathcal{P} \exp \left[ig \int\limits_{a^-}^{b^-} dx^- \, \mathcal{A}^+(x^+ = 0, x^-, \underline{x}) \right]$
- Note the simple physical meaning of the first term:

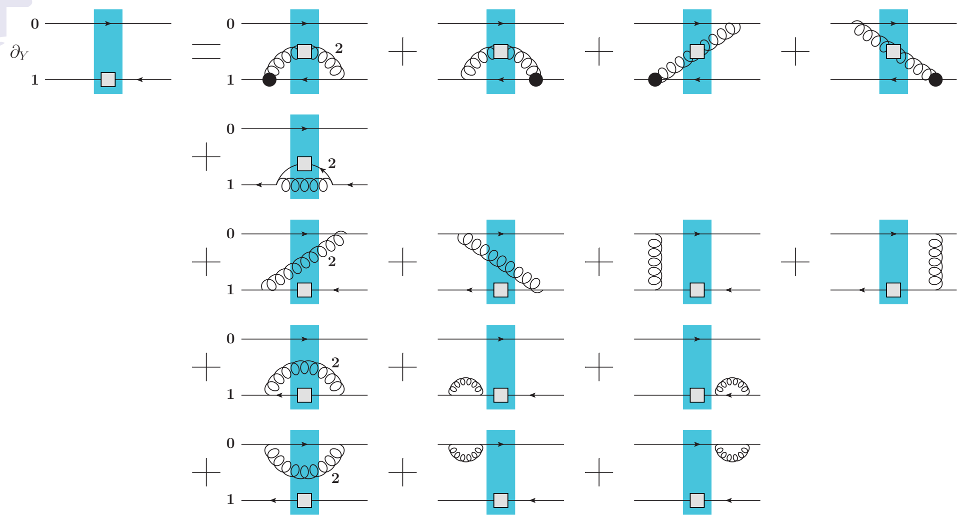
$$-\vec{\mu} \cdot \vec{B} = -\mu_z \, B_z = \mu_z \, F^{12}$$

Evolution for Polarized Quark Dipole

One can construct an evolution equation for the polarized dipole:



Evolution for Polarized Quark Dipole



$$\langle\langle \ldots \rangle\rangle = \frac{1}{z\, s} \langle \ldots \rangle$$

$$\rho'^{\,2} = \frac{1}{z'\, s}$$

$$\frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr} \left[V_0^{unp} \, V_1^{pol \, \dagger} \right] \right\rangle\!\!\right\rangle (z) = \frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr} \left[V_0^{unp} \, V_1^{pol \, \dagger} \right] \right\rangle\!\!\right\rangle_0 (z) + \frac{\alpha_s}{2\pi^2} \int\limits_{z_i}^{z} \frac{dz'}{z'} \int\limits_{\rho'^2} \frac{d^2 x_2}{x_{21}^2}$$

$$\times \Bigg\{ \theta(x_{10} - x_{21}) \, \frac{2}{N_c} \left\langle\!\!\left\langle \mathrm{tr} \left[t^b \, V_0^{unp} \, t^a \, V_1^{unp \, \dagger} \right] \, U_2^{pol \, ba} \right\rangle\!\!\right\rangle (z')$$

$$+ \, \theta(x_{10}^2 z - x_{21}^2 z') \, \frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr} \left[t^b \, V_0^{unp} \, t^a \, V_2^{pol \, \dagger} \right] \, U_1^{unp \, ba} \right\rangle\!\!\right\rangle (z')$$

$$+ \, \theta(x_{10} - x_{21}) \, \frac{1}{N_c} \left[\left\langle\!\!\left\langle \mathrm{tr} \left[V_0^{unp} \, V_2^{unp \, \dagger} \right] \, \mathrm{tr} \left[V_2^{unp} \, V_1^{pol \, \dagger} \right] \right\rangle\!\!\right\rangle (z') - N_c \left\langle\!\!\left\langle \mathrm{tr} \left[V_0^{unp} \, V_1^{pol \, \dagger} \right] \right\rangle\!\!\right\rangle (z') \right] \Bigg\}$$

Equation does not close!

Large-N_c Evolution

Resummation parameter: $\alpha_s \, \ln^2 \frac{1}{x}$

Double-logarithmic approximation (DLA)

- In the strict DLA limit and at large N_c, we get (here Γ is an auxiliary function we call the 'neighbor dipole amplitude') (KPS '15)

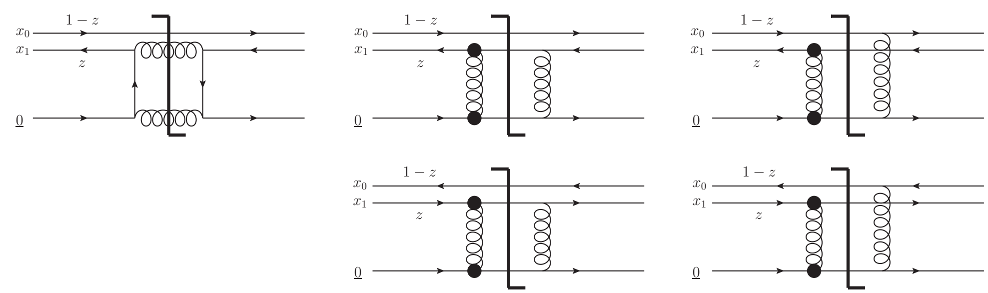
$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s \, N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \, \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3\, G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = \Gamma^{(0)}(x_{10}^2, x_{21}^2, z') + \frac{\alpha_s \, N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left\{x_{10}^2, x_{21}^2 \frac{z'}{z''}\right\}} \frac{dx_{32}^2}{x_{32}^2} \, \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3\, G(x_{32}^2, z'')\right]$$

- The initial conditions are given by the Born-level graphs

$$\Gamma^{(0)}(x_{10}^2, x_{21}^2, z) = G^{(0)}(x_{10}^2, z)$$

$$G^{(0)}(x_{10}^2, z) = \frac{\alpha_s^2 C_F}{N_c} \pi \left[C_F \ln \frac{zs}{\Lambda^2} - 2\, \ln(zs\, x_{10}^2)\right]$$

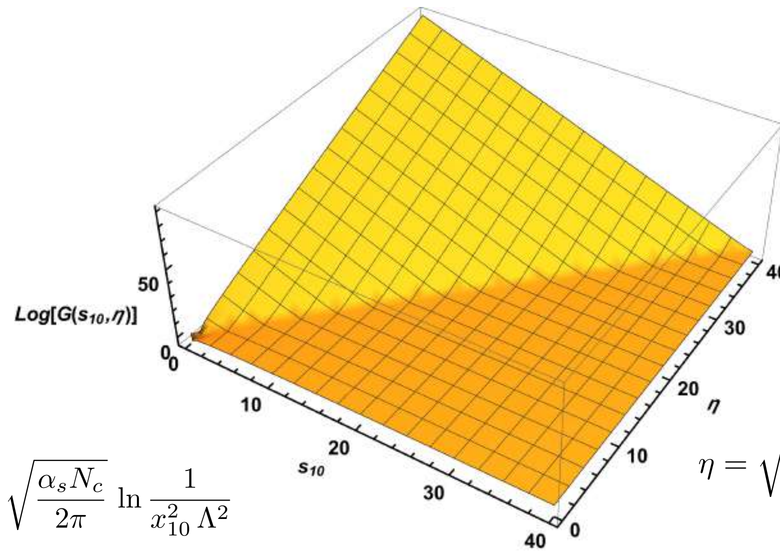


Quark Helicity at Small x: Asymptotics and Phenomenology

YK, D. Pitonyak, M. Sievert, arXiv:1610.06188 [hep-ph], arXiv:1703.05809 [hep-ph];
D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert, YK, 2102.06159 [hep-ph].

Quark Helicity at Small x

- These equations can be solved both numerically and analytically. (KPS '16-'17)



$$s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{x_{10}^2 \Lambda^2} \qquad \eta = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{zs}{\Lambda^2}$$

- The small-x asymptotics of quark helicity is (at large N_c)

$$\Delta q(x, Q^2) \sim \left(\frac{1}{x} \right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$g_1(x,Q^2) = \frac{1}{2}\sum_f e_f^2\left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

(Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK, 2102.06159 [hep-ph] = JAMsmallx)

Small-x Polarized DIS Data

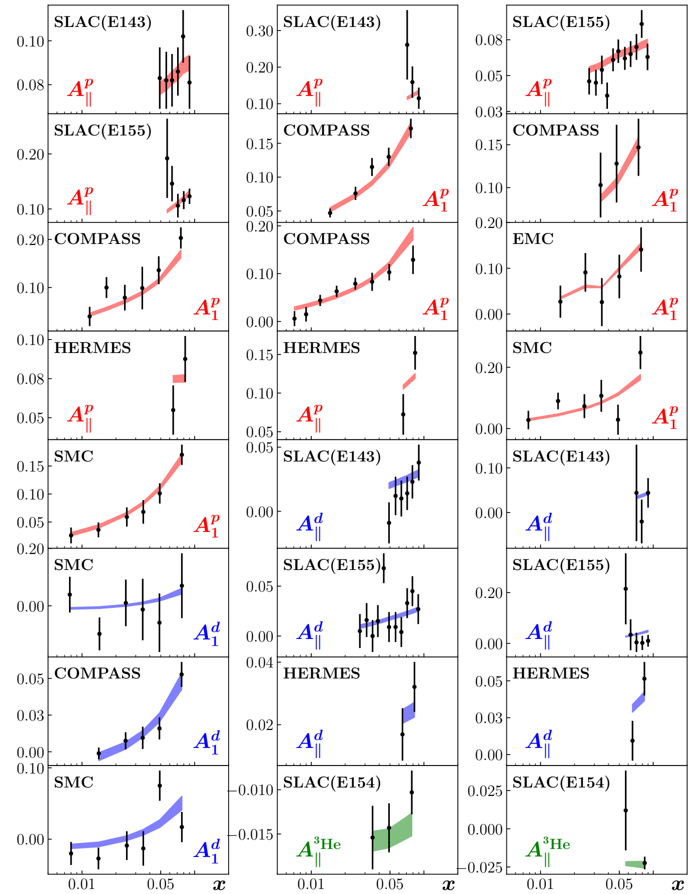
$$A_1 \sim A_{\|} = \frac{\sigma_{+-} - \sigma_{++}}{\sigma_{+-} + \sigma_{++}} \sim \frac{g_1}{F_1}$$

- We have analyzed all existing world polarized DIS data with x<0.1=x_0 , $Q^2 > m_c^2$ (122 data points) using the large-N_C KPS evolution with the Born-inspired initial conditions (8 parameters for 2 flavors, 11 parameters for 3 flavors).

$$G^{(0)}(x_{10}^2, z) \propto a_q \ln\frac{zs}{\Lambda^2} + b_q \ln\frac{1}{x_{10}^2\Lambda^2} + c_q$$

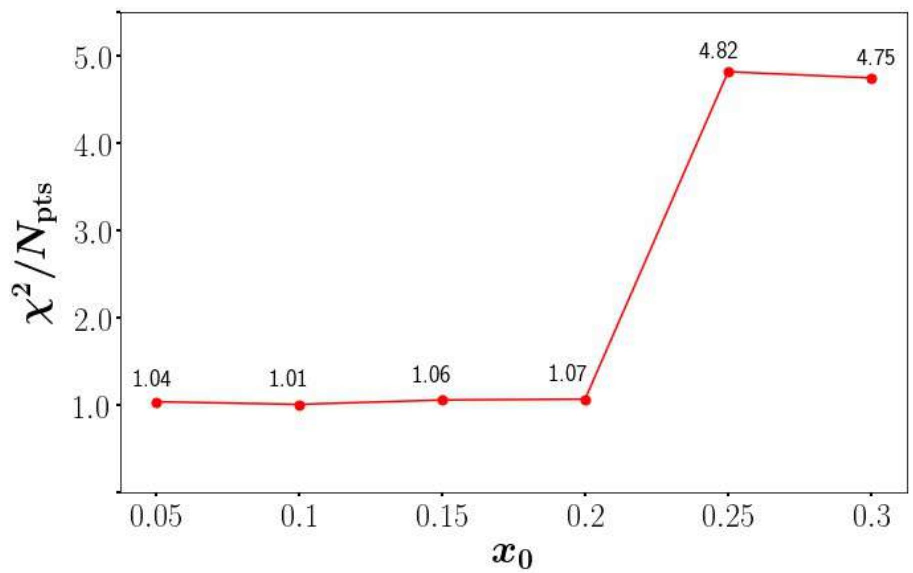
- It worked well, with χ^2/N_{pts} =1.01 (cf. JAM16: χ^2/N_{pts} =1.07)

- Small-x evolution starts at x_0=0.1 ! (cf. x_0=0.01 for unpolarized BK/JIMWLK evolution) Our approach fails at larger x as expected (x_0=0.3 gives χ^2/N_{pts} =4.75).

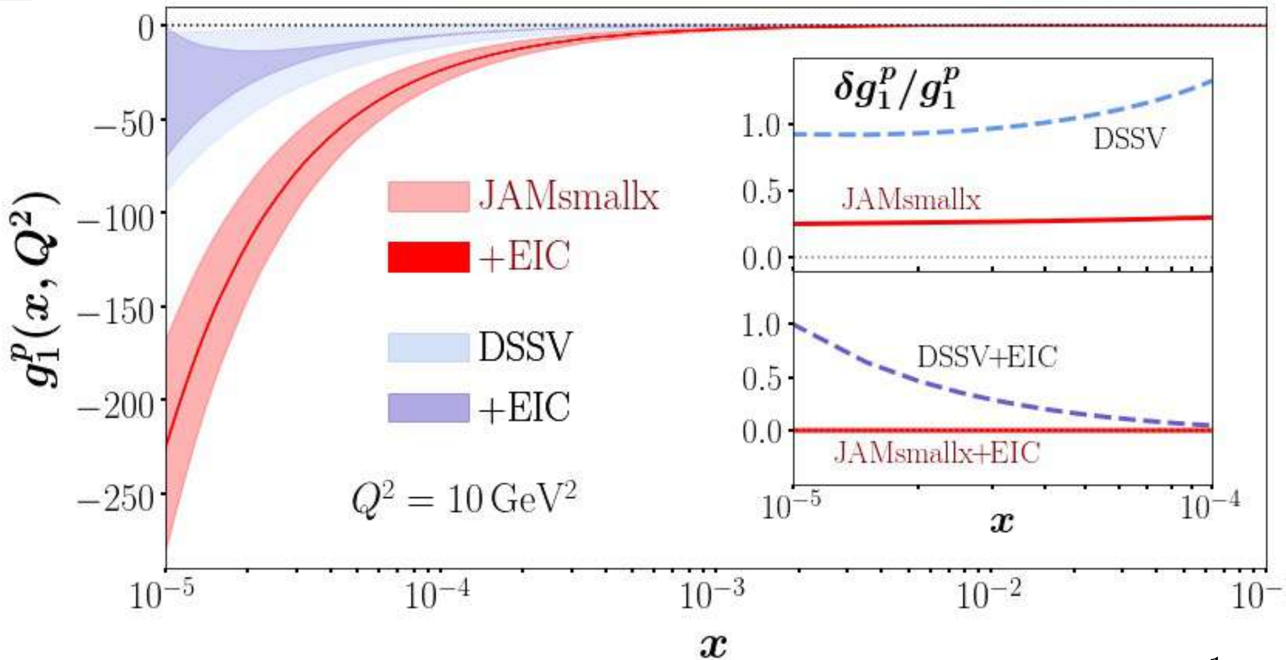


Where to start small-x evolution

- The evolution starts at $x=x_0$, and continues toward smaller x.
- The quality of our fit rapidly deteriorates for $x_0>0.2$, as expected from a small-x approach.
- In unpolarized BK/JIMWLK evolution, typically $x_0=0.01$, so the fact that our fit works up to such a high x_0 is quite remarkable.



Prediction for g_1 structure function

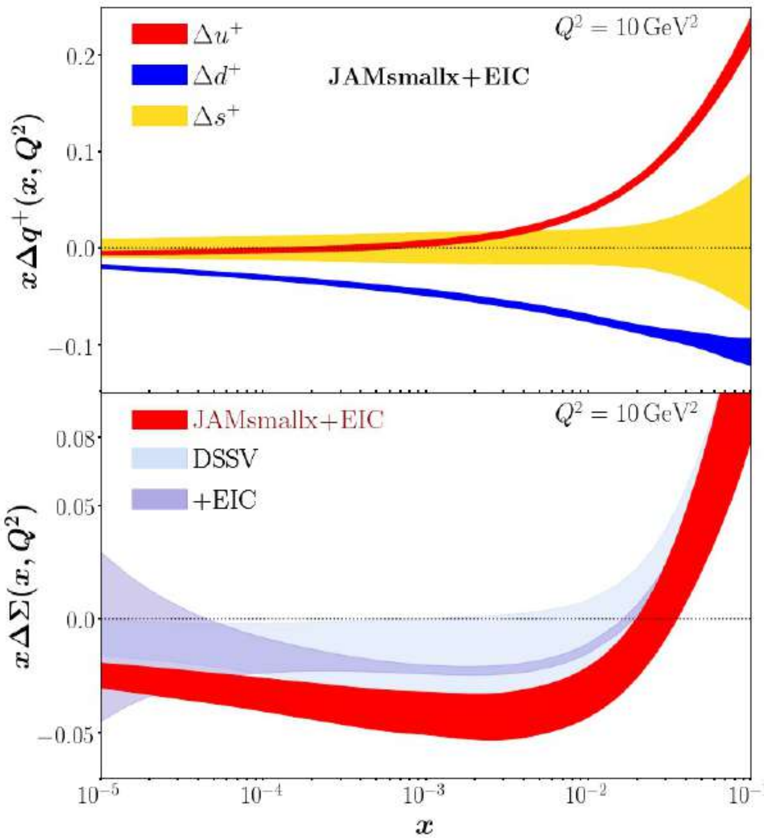


g_1 structure function ≈ spin-dependent part of σ^{e+p}

$$g_1(x, Q^2) = \frac{1}{2} \sum_f e_f^2 \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

Thick band: 1σ CL; thin band: impact of EIC data. With the EIC pseudo-data we have 1096 data points.

Predictions for helicity PDFs



Our (red) error band does not explode in the unmeasured region. We can **predict** spin at small x.

D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert & YK, 2102.06159 [hep-ph], in the JAM Collaboration framework.

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$

$$\Delta\Sigma(x, Q^2) = \sum_f \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

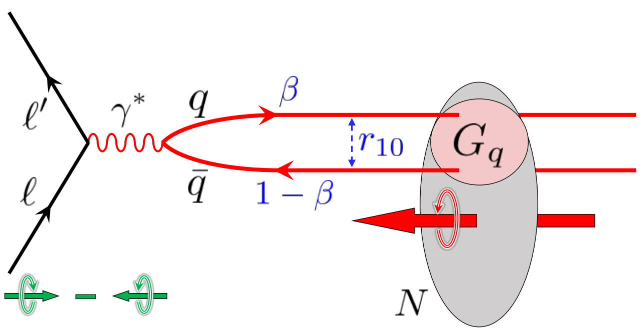
If we plug in $\alpha_S = 0.25$ we get $\alpha_h^q = 0.80$

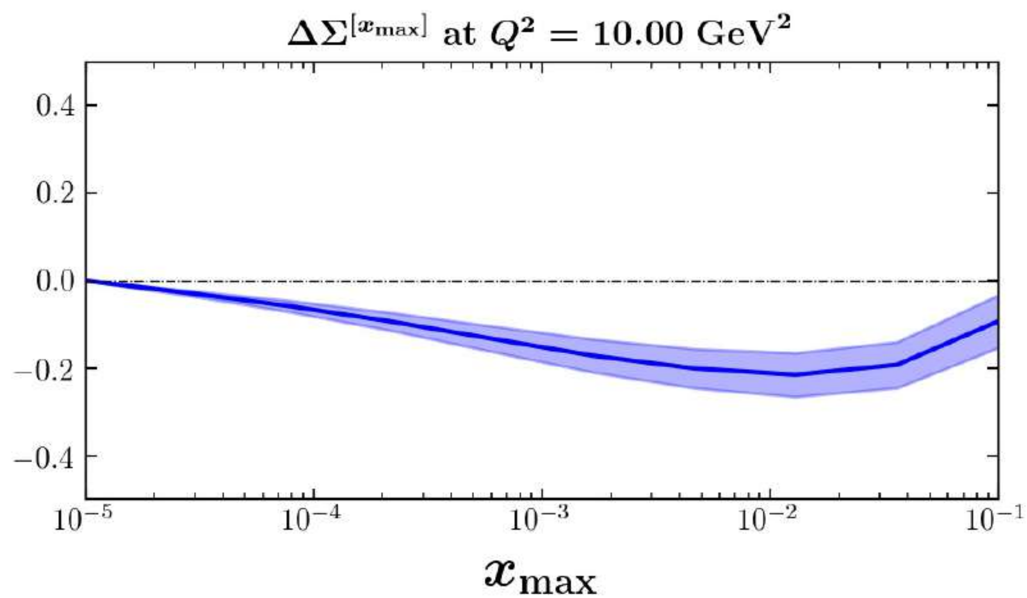
$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

Small-x quarks impact on the proton spin

- Potentially negative 10-20% of the proton spin may be carried by small-x quarks (JAMsmallx, preliminary):

$$\Delta\Sigma^{[x_{max}]}(Q^2) = \int_{10^{-5}}^{x_{max}} dx\, \Delta\Sigma(x, Q^2)$$





Speculation on a path to resolving the spin puzzle

- Above we discussed quark helicity at small x. Let's add the orbital angular momentum (OAM) (Hatta & Yang, '18; YK '19):

$$\frac{1}{2}\,\Delta\Sigma(x,Q^2) + L_{q+\bar{q}}(x,Q^2) = -\frac{1}{2}\,\Delta\Sigma(x,Q^2)$$

- So, the net quark (1/2) helicity+OAM = (-1/2) helicity.
- For x<0.001 we thus expect (preliminary!)

$$\left[\frac{1}{2}\Delta\Sigma + L_{q+\bar{q}}\right]_{Q^2=10\,\text{GeV}^2,\ x<0.001} \approx -\frac{1}{2}\,(-0.2) = 0.1$$

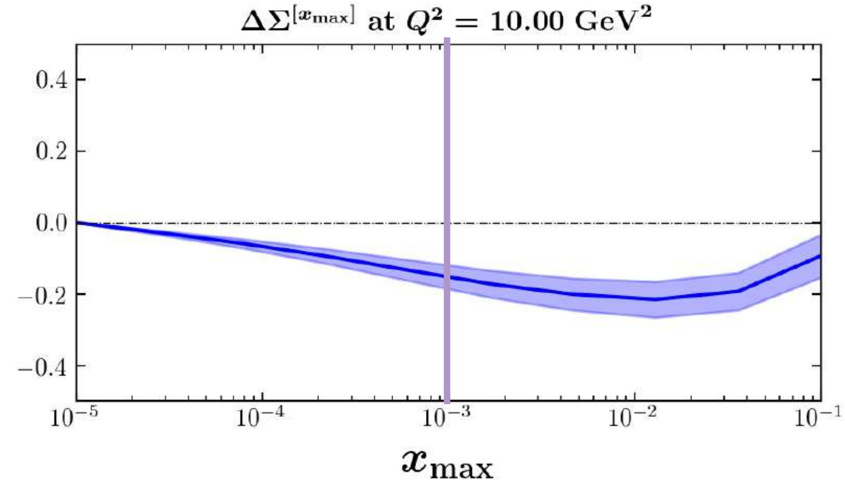
- Add to this the larger-x numbers

$$S_q(Q^2 = 10\,\text{GeV}^2, x > 0.001) \approx 0.18$$
$$S_G(Q^2 = 10\,\text{GeV}^2, x > 0.05) \approx 0.2$$

- We get

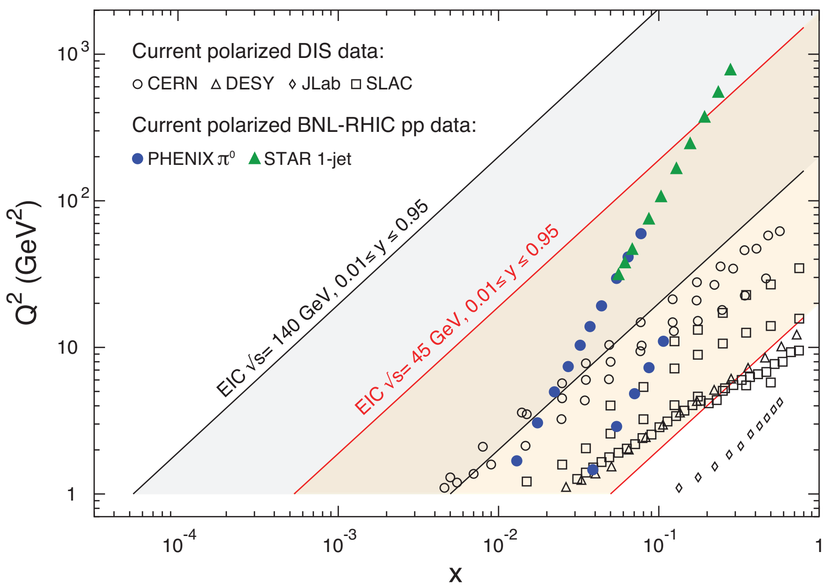
$$0.18 + 0.2 + 0.1 = 0.48$$

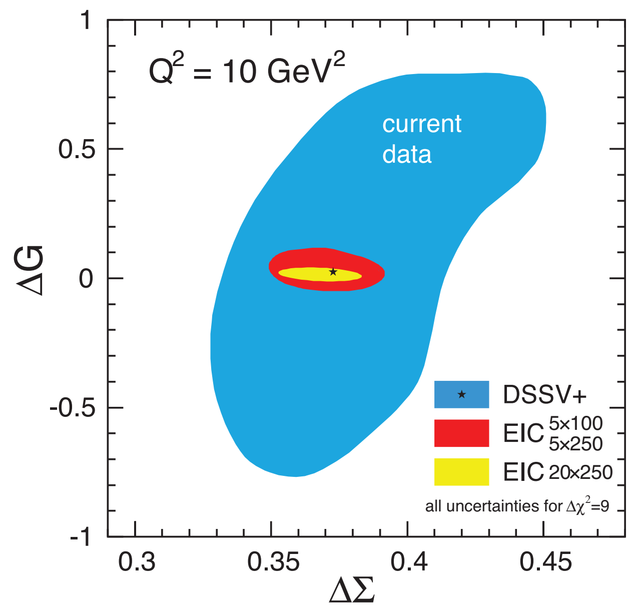
JAMsmallx, preliminary, Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, YK



EIC & Spin Puzzle

- Parton helicity distributions are sensitive to low-x physics.
- EIC would have an unprecedented low-x reach for a polarized DIS experiment, allowing to pinpoint the values of quark and gluon contributions to proton's spin:





- ΔG and $\Delta\Sigma$ are integrated over x in the $0.001 < x < 1$ interval.

Beyond Quark Helicity at Small x and at Large N_c

YK, D. Pitonyak, M. Sievert, arXiv:1706.04236 [nucl-th];
YK, F. Cougoulic, arXiv:1910.04268 [hep-ph], arXiv:2005.14688 [hep-ph];
YK, Y. Tawabutr, arXiv:2005.07285 [hep-ph];
A. Tarasov, Y. Tawabutr, YK, 2104.11765 [hep-ph].

Small-x Helicity Evolution at large N_c&N_f

- The resulting equations are (KPS '15)

$$Q_{10}(zs) = Q_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left\{ \frac{1}{2}\Gamma_{02,21}^{adj}(z') + \frac{1}{2} G_{21}^{adj}(z') + Q_{12}(z') - \bar{\Gamma}_{01,21}(z') \right\}$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} Q_{21}(z'),$$

$$G_{10}^{adj}(z) = G_{10}^{adj\,(0)}(z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z' s)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma_{10,21}^{adj}(z') + 3\, G_{21}^{adj}(z') \right]$$

$$- \frac{\alpha_s N_f}{2\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z' s)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} \bar{\Gamma}_{02;21}(z'),$$

$$\Gamma_{10,21}^{adj}(z') = \Gamma_{10,21}^{adj\,(0)}(z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z'' s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma_{10,32}^{adj}(z'') + 3\, G_{32}^{adj}(z'') \right]$$

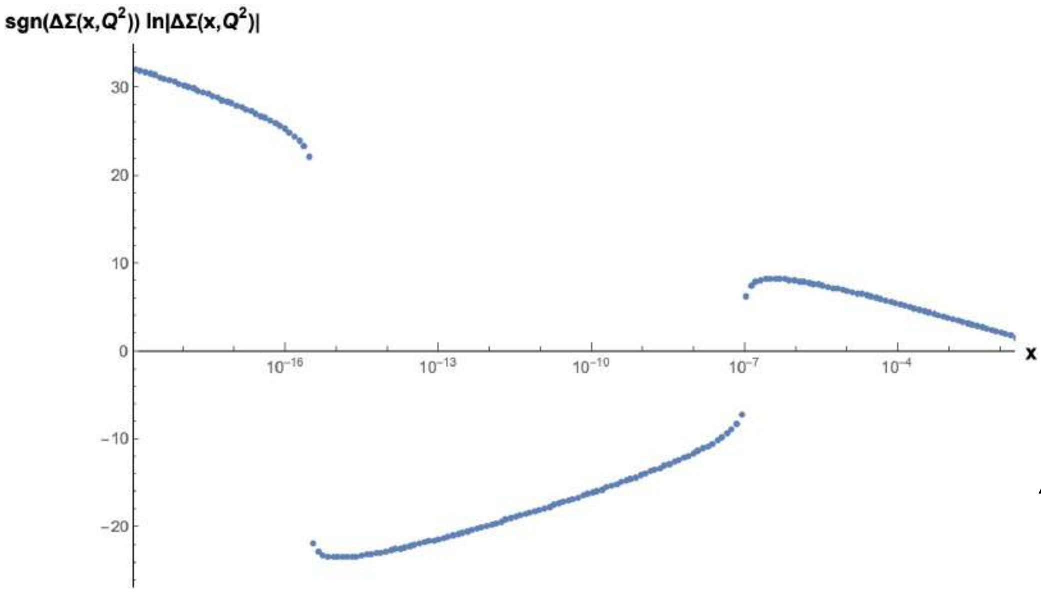
$$- \frac{\alpha_s N_f}{2\pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z'' s)}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2} \bar{\Gamma}_{03;32}(z''),$$

$$\bar{\Gamma}_{10,21}(z') = \bar{\Gamma}_{10,21}^{(0)}(z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left\{ \frac{1}{2}\Gamma_{03,32}^{adj}(z'') + \frac{1}{2} G_{32}^{adj}(z'') + Q_{32}(z'') - \bar{\Gamma}_{01,32}(z \right.$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{x_{21}^2 z/z'} \frac{dx_{32}^2}{x_{32}^2} Q_{32}(z'').$$

Small-x Helicity Asymptotics at large N_c&N_f

- Large N_c&N_f equations can be solved only numerically, due to their complexity. This was done by Y. Tawabutr & YK in arXiv:2005.07285 [hep-ph].
- The solution exhibits an interesting qualitative change compared to large-N_c: it oscillates with ln(1/x)!



$$\alpha_h^q \approx 2.3 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$\omega_q \approx \frac{0.66(N_f/N_c)}{1+0.3795(N_f/N_c)} \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$\Delta\Sigma(x,Q^2)\Big|_{\text{large-}N_c\&N_f} \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \cos\left[\omega_q \ln\frac{1}{x} + \varphi_q\right]$$

Helicity JIMWLK

$$\alpha = A^+, \quad \beta = F^{12}$$

$$\langle \mathcal{O}_{\alpha,\beta,\psi,\bar\psi} \rangle_Y = \frac{\int \mathcal{D}\alpha \mathcal{D}\beta \mathcal{D}\psi \mathcal{D}\bar\psi \ \mathcal{O}_{\alpha,\beta,\psi,\bar\psi} \, \mathcal{W}_Y[\alpha,\beta,\psi,\bar\psi]}{\int \mathcal{D}\alpha \mathcal{D}\beta \mathcal{D}\psi \mathcal{D}\bar\psi \ \mathcal{W}_Y[\alpha,\beta,\psi,\bar\psi]}$$

- To go beyond the large-N_c and large-N_c&N_f limits need to write down a helicity analogue of JIMWLK evolution.
- This has been done recently (F. Cougoulic, YK, arXiv:1910.04268 [hep-ph], arXiv:2005.14688 [hep-ph]):

$$W_\tau[\alpha,\beta,\psi,\bar\psi] = W_\tau^{(0)}[\alpha,\beta,\psi,\bar\psi] + \int d^3\tau' \ \mathcal{K}_h[\tau,\tau'] \cdot W_{\tau'}[\alpha,\beta,\psi,\bar\psi]$$

with $\tau \equiv \{z, z\, X_\perp^2, z\, Y_\perp^2\}$ and the kernel

Life-time ordered!

$$\mathcal{K}_h[\tau,\tau'] = \frac{\alpha_s}{\pi^2} \int d^2 w_\perp \frac{\underline{X}' \cdot \underline{Y}'}{X'^2\, Y'^2} \ \theta^{(3)}(\tau-\tau')\theta\left(z' - \frac{\Lambda^2}{s}\right) \theta\left(X'^2 - \frac{1}{z'\,s}\right) \theta\left(Y'^2 - \frac{1}{z'\,s}\right)$$

$$\times \left\{ U_w^{ba} D^+_{x,a,<} D^+_{y,b,>} - \frac{1}{2}\left(D^+_{x,a,<} D^+_{y,a,<} + D^+_{x,a,>} D^+_{y,a,>}\right)\right.$$ Regular (spin-averaged) JIMWLK

$$+ \frac{1}{2} U_w^{pol,ba} (D^+_{x,a,<} D^\perp_{y,b,>} + D^\perp_{x,a,<} D^+_{y,b,>})$$ Polarized gluon emissions

$$\left. + \left(\frac{1}{2}\gamma^5\gamma^-\right)_{\beta\alpha} \frac{1}{2}\left((V_w^{pol})_{ij} D^{\bar\psi}_{x,j,\alpha,<} D^{\psi}_{y,i,\beta,>} + (V_w^{pol\,\dagger})_{ij} D^{\bar\psi}_{x,j,\alpha,>} D^{\psi}_{y,i,\beta,<}\right)\right\}$$ Polarized quark emissions

Helicity McLerran-Venugopalan Model

- The initial conditions for helicity JIMWLK are given by the helicity MV model, with the weight functional (F. Cougoulic, YK, 2005.14688 [hep-ph])

Standard MV

$$\mathcal{W}^{(0)}[\alpha, \beta, \psi, \bar{\psi}] \propto \exp\left\{-\int \mathrm{d}^2 x_\perp dx^- \ \mathrm{tr}\left[(\nabla_\perp^2 \alpha)^2 \frac{\mu_+^2 + \mu_-^2}{8\mu_+^2 \, \mu_-^2} + \left(\langle p^+ \rangle \beta\right)^2 \frac{\mu_+^2 + \mu_-^2}{2\mu_+^2 \, \mu_-^2} + (\nabla_\perp^2 \alpha) \langle p^+ \rangle \beta \frac{\mu_+^2 - \mu_-^2}{2\mu_+^2 \, \mu_-^2}\right]\right\}$$
$$\times \exp\left\{\int \mathrm{d}^2 x_\perp dx^- \ \langle p^+ \rangle \ \left[\frac{\nu_+^2 + \nu_-^2}{\nu_+^2 \, \nu_-^2} \, \bar{\psi} \frac{1}{2} \gamma^+ \nabla_\perp^2 \psi - \frac{\nu_+^2 - \nu_-^2}{\nu_+^2 \, \nu_-^2} \, \bar{\psi} \frac{1}{2} \gamma^+ \gamma^5 \nabla_\perp^2 \psi\right]\right\}.$$

- Here $\alpha = A^+, \quad \beta = F^{12}$

- The parameters $\mu_\pm, \quad \nu_\pm$ are ~ “saturation scales” of helicity-plus and helicity-minus partons.

Single-logarithmic corrections

- Small-x helicity evolution equations have been constructed very recently to the single-logarithmic order (SLA) + running coupling (Tarasov, Tawabutr, YK, 2104.11765 [hep-ph]).

- DLA: $\alpha_s \ln^2 \frac{1}{x}$

- SLA: $\alpha_s \ln \frac{1}{x}$

- Include exact LO DGLAP splitting functions. Unprecedented precision for small-x helicity evolution.

Asymptotics of helicity PDFs and OAM

- At large N_c we have obtained the following small-x asymptotics:

$$\Delta\Sigma(x,Q^2) \sim \Delta q(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$\Delta G(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^G} \quad \text{with} \quad \alpha_h^G = \frac{13}{4\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 1.88\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

KPS '17, YK '19

$$L_{q+\bar{q}}(x,Q^2) = -\Delta\Sigma(x,Q^2) \sim \left(\frac{1}{x}\right)^{\frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}}},$$

$$L_G(x,Q^2) \sim \Delta G(x,Q^2) \sim \left(\frac{1}{x}\right)^{\frac{13}{4\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

(cf. Y. Hatta & D.-J. Yang, 2018 for the quark OAM equality)

- At large N_c & N_f we have obtained

$$\Delta\Sigma(x,Q^2)\Big|_{\text{large-}N_c\&N_f} \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \cos\left[\omega_q \ln\frac{1}{x} + \varphi_q\right]$$

Y. Tawabutr, YK '20

with

$$\omega_q \approx \frac{0.22 N_f}{1+0.1265 N_f}\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

Conclusions

We have constructed new evolution equations in x for the quark and gluon helicity distributions. We can now predict the x-dependence of helicity PDFs at small x.

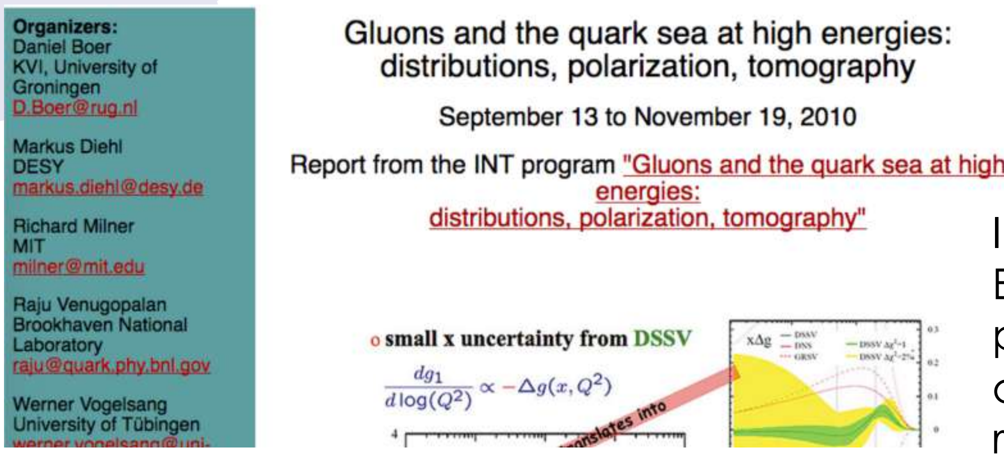
These equations have now been studied at large-N_c and large-N_c&N_f, yielding the small-x asymptotics of $\Delta\Sigma(x, Q^2)$, $\Delta G(x, Q^2)$, and OAM distributions.

First successful fit of polarized world DIS data for x<0.1 done using solely the small-x helicity evolution (JAMsmallx). There is a clear possibility of a significant amount of proton spin to be found at small x, about 20% in the quark sector alone.

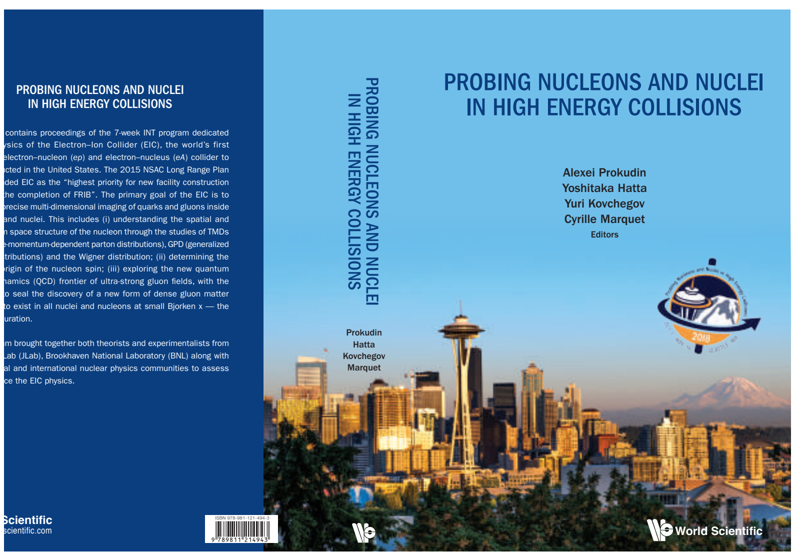
More precise and comprehensive phenomenology to come in the future (helicity+OAM), in preparation for EIC, with the aim of resolving the proton spin puzzle.

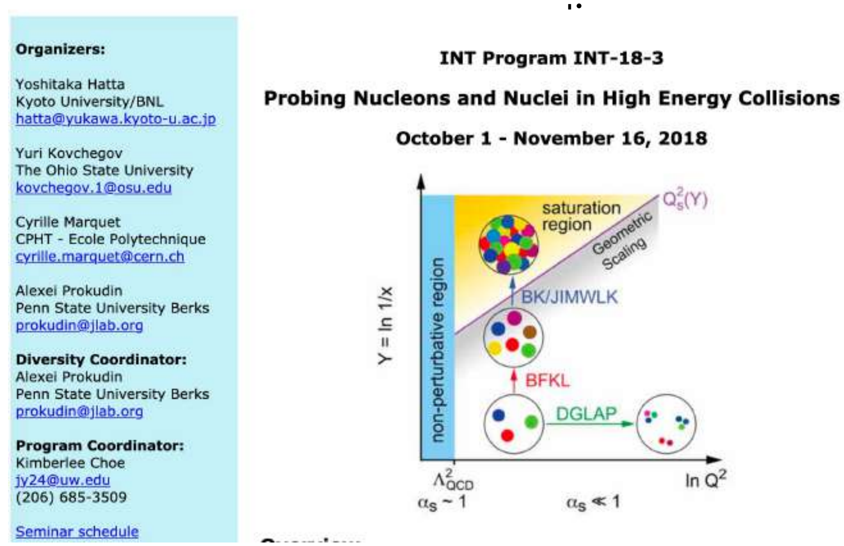
Backup Slides

INT Programs on EIC

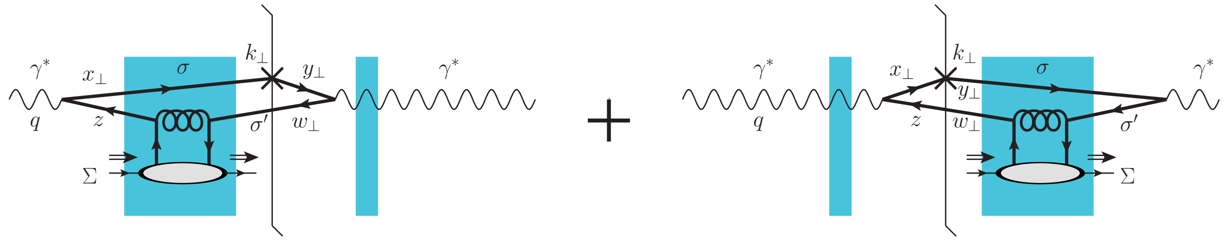


INT held two programs on EIC physics, one in 2010 and another one in 2018: both resulted in published

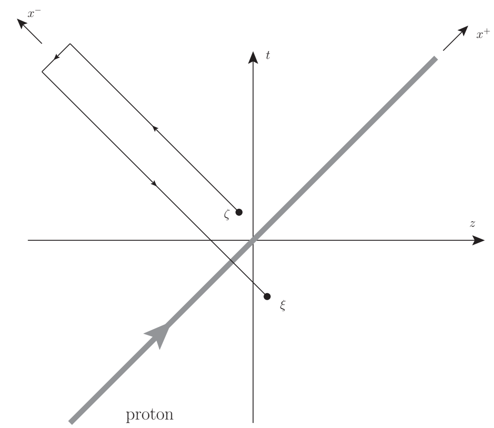




Quark Helicity Observables at Small x



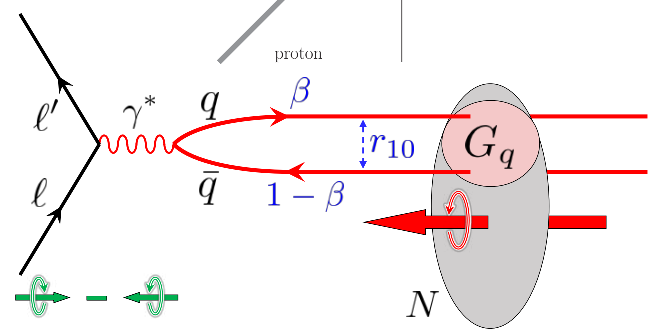
- One can show that the g_1 structure function and quark helicity PDF (Δq) and TMD at small-x can be expressed in terms of the polarized dipole amplitude (flavor singlet case):



$$g_1^S(x,Q^2) = \frac{N_c\, N_f}{2\,\pi^2 \alpha_{EM}} \int\limits_{z_i}^{1} \frac{dz}{z^2(1-z)} \int dx_{01}^2 \left[\frac{1}{2}\sum_{\lambda\sigma\sigma'} |\psi^T_{\lambda\sigma\sigma'}|^2_{(x_{01}^2,z)} + \sum_{\sigma\sigma'} |\psi^L_{\sigma\sigma'}|^2_{(x_{01}^2,z)}\right] G(x_{01}^2,z),$$

$$\Delta q^S(x,Q^2) = \frac{N_c\, N_f}{2\pi^3} \int\limits_{z_i}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\frac{1}{zQ^2}} \frac{dx_{01}^2}{x_{01}^2}\, G(x_{01}^2,z), \quad = \Delta\Sigma(x,Q^2)$$

$$g_{1L}^S(x,k_T^2) = \frac{8\,N_c\,N_f}{(2\pi)^6} \int\limits_{z_i}^{1} \frac{dz}{z} \int d^2x_{01}\, d^2x_{0'1}\, e^{-i\underline{k}\cdot(\underline{x}_{01}-\underline{x}_{0'1})}\, \frac{\underline{x}_{01}\cdot\underline{x}_{0'1}}{x_{01}^2 x_{0'1}^2}\, G(x_{01}^2,z)$$



- Here s is cms energy squared, $z_i = \Lambda^2/s$, $G(x_{01}^2,z) \equiv \int d^2b\, G_{10}(z)$

Polarized Dipole Amplitude

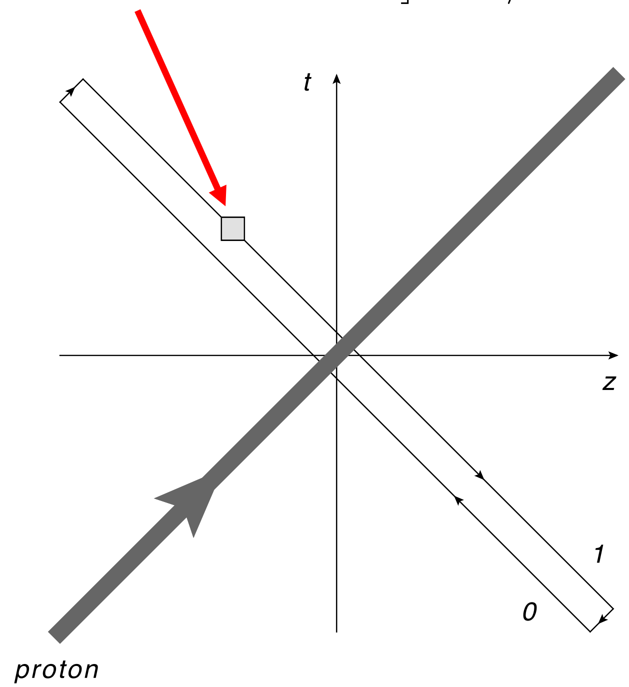
- The polarized dipole amplitude is then defined by

$\sim F^{12}$

$$G_{10}(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \left\langle \mathrm{tr} \left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \underline{\nabla} \times \underline{\tilde{A}}(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty] \right] + \mathrm{c.c.} \right\rangle (z)$$

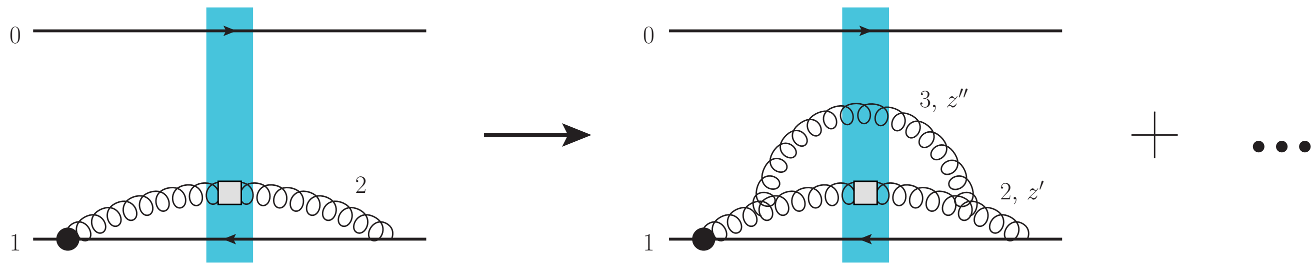
with the standard light-cone Wilson line

$$V_{\underline{x}}[b^-, a^-] = \mathrm{P} \exp \left\{ ig \int\limits_{a^-}^{b^-} dx^- \, A^+(x^-, \underline{x}) \right\}$$



"Neighbor" dipole

- There is a new object in the evolution equation – **the neighbor dipole amplitude**.
- This is specific for the DLA evolution. Gluon emission may happen in one dipole, but, due to transverse distance ordering, may `know' about another dipole:



$$x_{21}^2 \, z' \gg x_{32}^2 \, z''$$

- We denote the evolution in the neighbor dipole 02 by $\Gamma_{02,\,21}(z')$

Resummation Parameter

- For helicity evolution the resummation parameter is different from BFKL, BK or JIMWLK, which resum powers of leading logarithms (LLA)

$$\alpha_s \ \ln(1/x)$$

- Helicity evolution resummation parameter is double-logarithmic (DLA):

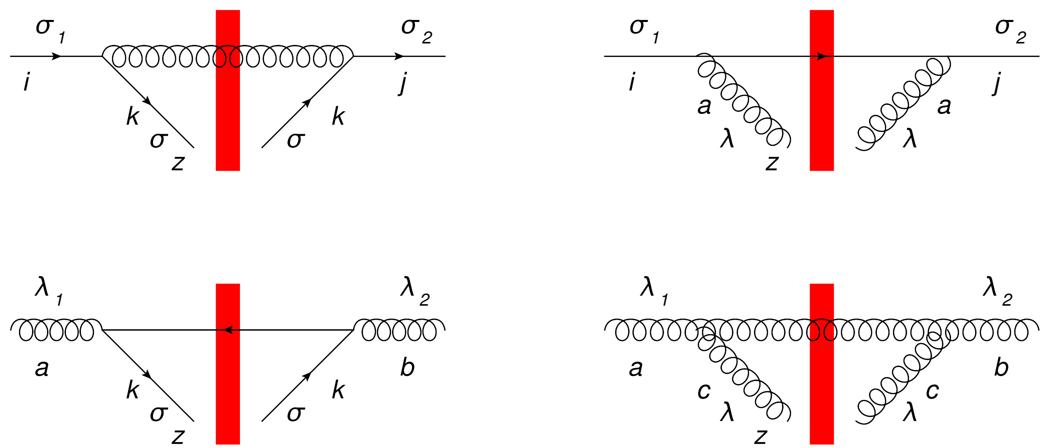
$$\alpha_s \ \ln^2 \frac{1}{x}$$

- The second logarithm of x arises due to transverse momentum (or transverse coordinate) integration being logarithmic both in the UV and IR.

- This was known before: Kirschner and Lipatov '83; Kirschner '84; Bartels, Ermolaev, Ryskin '95, '96; Griffiths and Ross '99; Itakura et al '03; Bartels and Lublinsky '03.

Helicity Evolution Ingredients

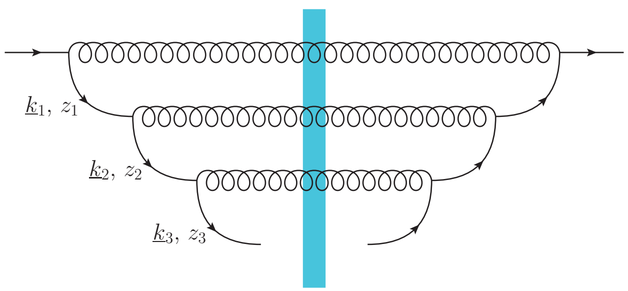
- Unlike the unpolarized evolution, in one step of helicity evolution we may emit a soft gluon or a soft quark (all in $A^-=0$ LC gauge of the projectile):

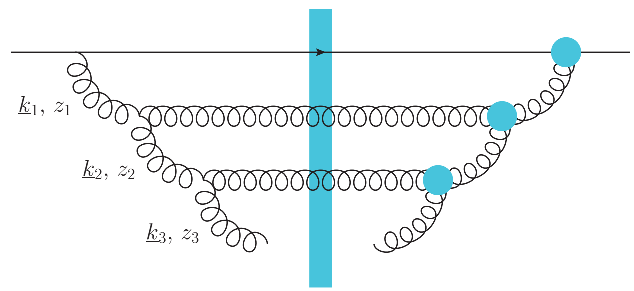


- When emitting gluons, one emission is eikonal, while another one is soft, but non-eikonal, as is needed to transfer polarization down the cascade/ladder.

Helicity Evolution: Ladders

- To get an idea of how the helicity evolution works let us try iterating the splitting kernels by considering ladder diagrams (circles denote non-eikonal gluon vertices):

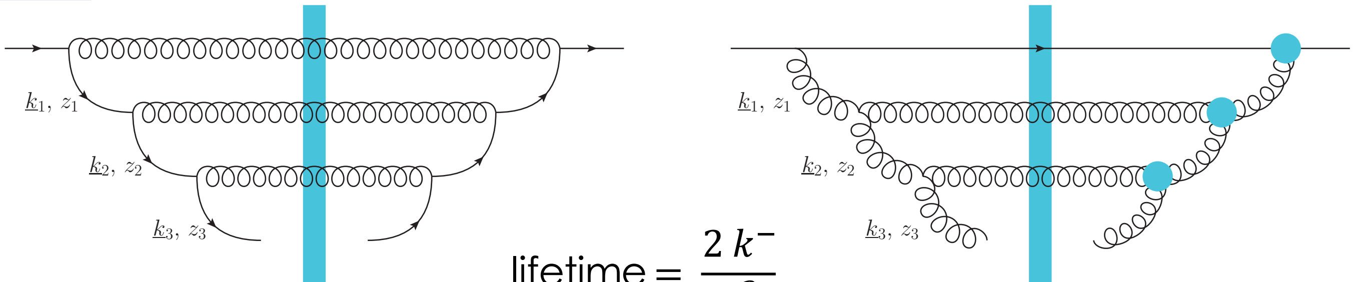




- To get the leading-energy asymptotics we need to order the longitudinal momentum fractions of the quarks and gluons (just like in the unpolarized evolution case) $1 \gg z_1 \gg z_2 \gg z_3 \gg \ldots$

  obtaining a nested integral $$\alpha_s^3 \int\limits_{z_i}^{1} \frac{dz_1}{z_1} \int\limits_{z_i}^{z_1} \frac{dz_2}{z_2} \int\limits_{z_i}^{z_2} \frac{dz_3}{z_3}\, z_3 \otimes \frac{1}{z_3\, s} \sim \frac{1}{s}\, \alpha_s^3\, \ln^3 s$$

Helicity Evolution: Ladders



$$\text{lifetime} = \frac{2\,k^-}{k_\perp^2}$$

- However, these are not all the logs of energy one can get here. Transverse momentum (or distance) integrals have UV and IR divergences, which lead to logs of energy as well.
- If we order gluon/quark lifetimes as (Sudakov-β ordering)

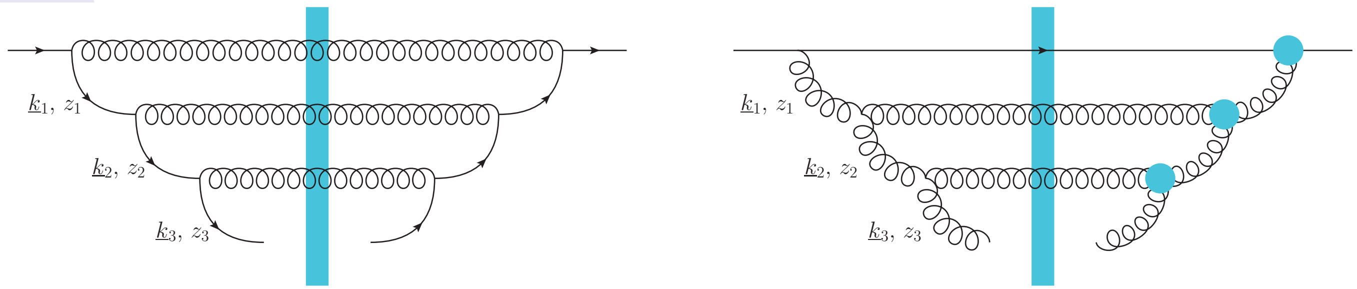
$$\frac{\underline{k}_1^2}{z_1} \ll \frac{\underline{k}_2^2}{z_2} \ll \frac{\underline{k}_3^2}{z_3} \ll \ldots \qquad\qquad z_1\,\underline{x}_1^2 \gg z_2\,\underline{x}_2^2 \gg z_3\,\underline{x}_3^2 \gg \ldots$$

we would get integrals like

$$\int\limits_{1/(z_n\,s)}^{x^2_{n-1,\perp}\,z_{n-1}/z_n} \frac{dx^2_{n,\perp}}{x^2_{n,\perp}}$$

also generating logs of energy.

Helicity Evolution: Ladders



- To summarize, the above ladder diagrams are parametrically of the order

$$\frac{1}{s}\,\alpha_s^3\,\ln^6 s$$

- Note two features:
    - 1/s suppression due to non-eikonal exchange
    - two logs of energy per each power of the coupling!

Gluon OAM: small-x asymptotics

- For the gluon OAM, we arrive at the following relation

$$L_G(x, Q^2) = \left(\frac{\alpha_h^q}{4} \, \ln \frac{Q^2}{\Lambda^2} \right) \Delta G(x, Q^2)$$

where

$$\alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}}$$

- We conclude that

$$L_G(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x} \right)^{\alpha_h^G} \sim \left(\frac{1}{x} \right)^{\frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}}} \sim \left(\frac{1}{x} \right)^{1.88 \sqrt{\frac{\alpha_s \, N_c}{2\pi}}}$$

- Note that with the DLA accuracy we could also simply conclude that

$$|L_G| \ll |\Delta G|$$

Beyond Helicity and OAM

- Similar small-x analysis was applied to transversity (Sievert, YK, 2018), yielding

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp\,NS}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s\, C_F}{\pi}}$$

- and to the quark Sivers function, obtaining the spin-dependent odderon contribution, such that (M. Gabriel Santiago, YK, later this month)

$$f_{1T}^{\perp q}(x, k_T^2) \sim \frac{1}{x} + \text{const}$$

with the non-perturbative accuracy, it seems.