Pushing the limits of hydrodynamics

Pavel Kovtun,
University of Victoria

Theoretical Physics Colloquium at ASU
October 7, 2020

Hydrodynamics is an established field with a venerable history and many applications.

However, today I will not talk about applications.

Rather, I would like to highlight some foundational questions that only came to light in the last few years.

Hydrodynamics: a theoretical physicist's perspective

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What is hydrodynamics?

Set of equations that tell you how stuff flows. “Stuff” can be water, air, a cold atomic cloud, hot primodial matter in early Universe, electron fluid in a solid, etc.

As a student, you open a book with “hydrodynamics” or “fluid dynamics” in the title. You often see derivations, approximations, and applications all mixed together.

If, as a student, you are also learning about vector calculus and partial differential eq-s at the same time, it can be hard to see the big picture.

But the big picture of hydrodynamics
is in fact quite simple

Conserved quantities

Fundamentally, hydrodynamics is a macroscopic theory of things that can not disappear, i.e. are conserved.

$$\frac{\partial}{\partial t}\rho_a = -\nabla \cdot \mathbf{j}_a$$

density of some conserved quantity a

flux of the same conserved quantity a

a = energy, momentum, number of particles,…

Constitutive relations

One eq-n $\partial_t \rho = -\nabla \cdot \mathbf{j}$, four unknown functions: ρ and $\mathbf{j}$. If we assume $\mathbf{j} = \mathbf{j}[\rho]$, then have eq-n for ρ only, can solve!

More generally, take some useful quantities γ, (temperature, velocity,…), express $\rho_a = \rho_a(\gamma),\ \mathbf{j}_a = \mathbf{j}_a(\gamma)$, then get eq-s for γ_a only, can solve!

Example: a = energy, $\gamma = T$ = temperature, then:

$\rho_\epsilon = \epsilon(T)$, constitutive relation $\mathbf{j}_\epsilon = -\kappa\, \boldsymbol{\nabla} T$,

$T = T_0 + \delta T$, get diffusion equation for δT :

$$\frac{\partial}{\partial t}\delta T = -D\nabla^2 \delta T$$

$D = \kappa/\epsilon'(T_0)$

Summary of hydrodynamics

Conservarion laws: $\frac{\partial}{\partial t}\rho_a = -\nabla \cdot \mathbf{j}_a$

Constitutive relations (γ = temperature, fluid velocity,…) :

$$\rho_a = \rho_a(\gamma, \nabla\gamma, \nabla^2\gamma, \dots),$$

$$\mathbf{j}_a = \mathbf{j}_a(\gamma, \nabla\gamma, \nabla^2\gamma, \dots)$$

The procedure is the same, whether the fundamental constituents are classical or quantum, relativistic or not, normal fluid or superfluid, magnetic fields present or not, fluid is chiral or not.

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Perfect fluids (Euler equations)

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Viscous fluids (Navier-Stokes equations)

Summary of hydrodynamics

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2-nd order fluids (Burnett equations)

Questions to ask about every theory

Every theory in physics is only approximately “correct”, limited by its domain of applicability.

Whenever we write down any equations that attempt to describe physical phenomena, we have to answer:

1. Do the equations make physical sense?
2. Can we improve the equations to capture more physics?
3. What kind of physics is beyond our equations?

What will follow are three stories, one for each question.

First story:

Do hydrodynamic equations even make sense?

Relativistic things

Say, you are a student in subatomic physics or astro-physics, and you want to learn about relativistic Navier-Stokes eq-s: quark-gluon plasma, neutron star mergers

Open “Fluid Mechanics” by Landau and Lifshitz:

some hydrodynamic equations

Open “Gravitation and Cosmology” by Weinberg*:

some hydrodynamic equations

And… these equations look very different!

*Formulation of hydrodynamics due to Eckart (1940)

The equations look different, so what?

Let's shut up and calculate. As a simple example, solve for linear perturbations of the thermal equilibrium state.

Both Landau-Lifshitz' and Eckart's equations predict that:

a) thermal equilibrium does not exist

b) things propagate faster than light

Hiscock, Lindblom, 1984

Hiscock, Lindblom, 1987

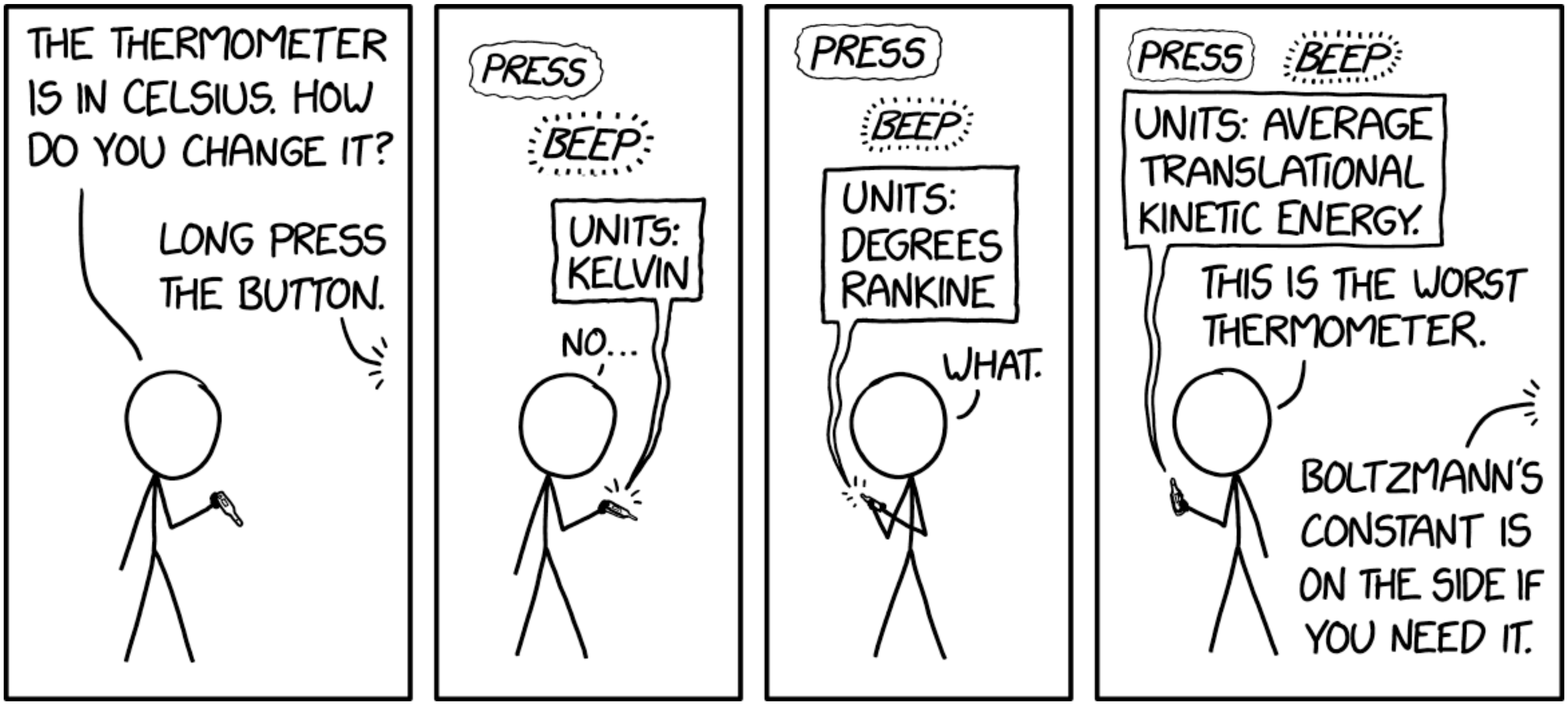
Sad but true: The equations you find in the classic text-books make no physical sense!

This has led to the belief that Navier-Stokes eq-s can not be unified with Einstein's relativity, and have to be aban-doned in a relativistic setting.

Other exotic theories have been proposed in the 1970s to replace the Navier-Stokes, and the field has moved on…

But why do the theories of Landau-Lifshitz and Eckart differ in the first place?

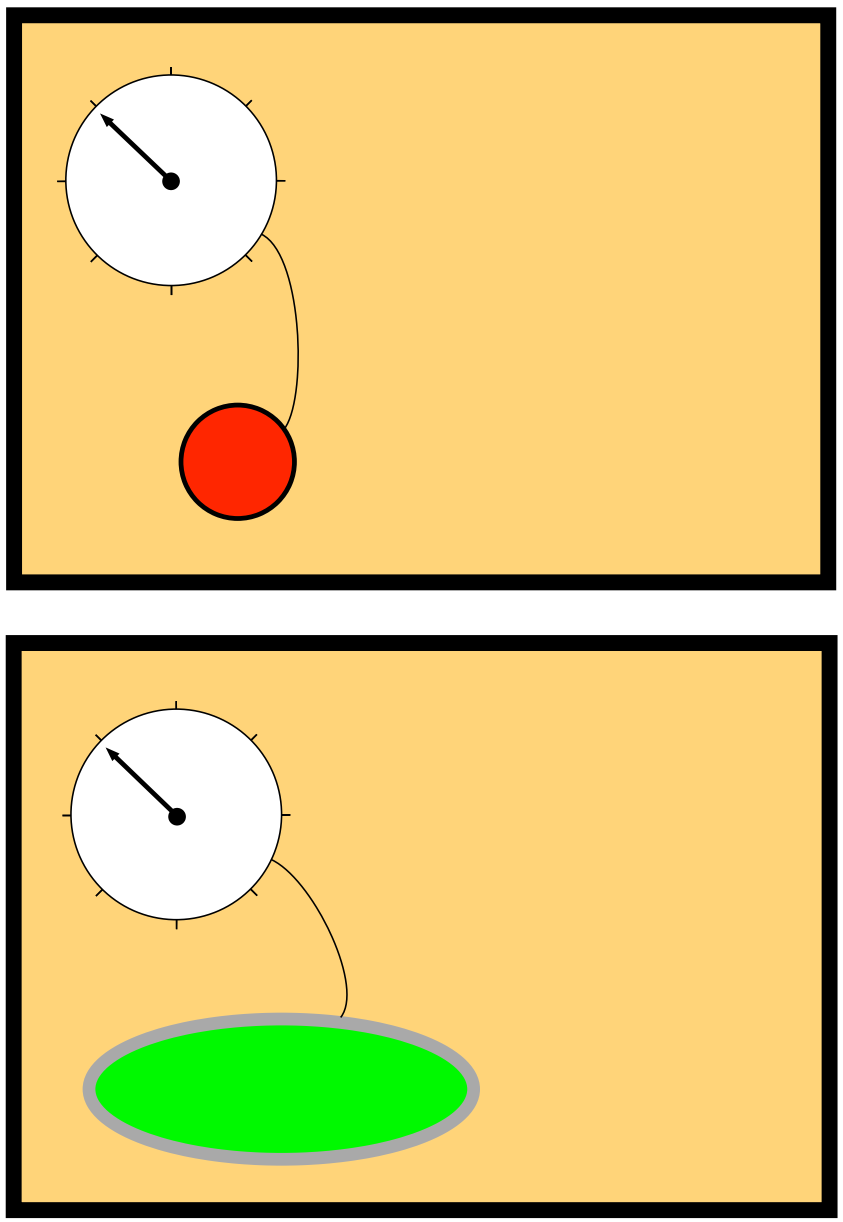
To understand why, let's first talk about temperature



https://xkcd.com/2292/

Example: Temperature

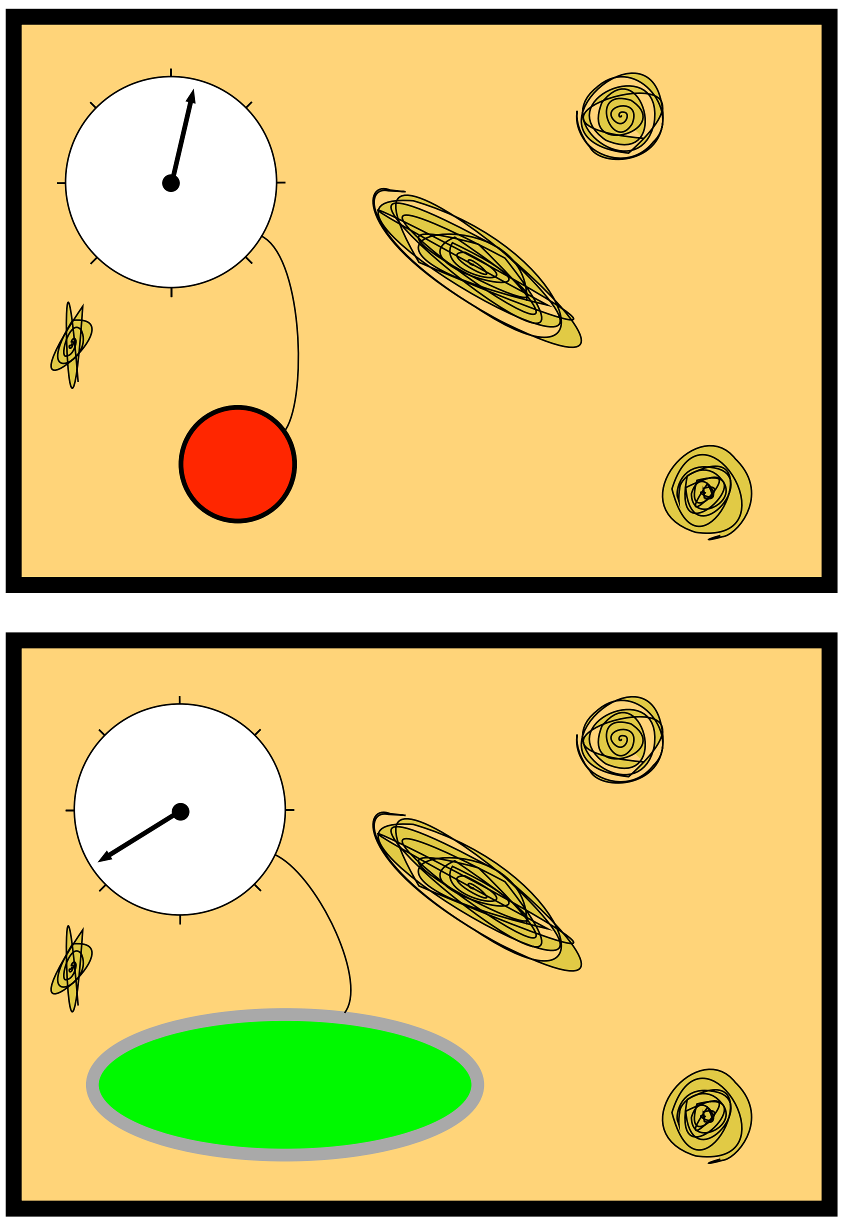
Temperature is something that is only unambiguously defined in equilibrium. By definition, T is the quantity that is measured by a thermometer.



In identical equilibrium states, two different (but properly calibrated) thermometers will show the same temperature.

Example: Temperature

Temperature is something that is only unambiguously defined in equilibrium. By definition, T is the quantity that is measured by a thermometer.



But in *identical* non-equilibrium states, the same two thermo-meters will show *different* temperatures!

Non-equilibrium conventions

So there is arbitrariness in what one means by "fluid temperature": one's choice of thermometer is a convention.

Same arbitrariness in what one means by "fluid velocity": one's choice of velocimeter is a convention.

Landau-Lifshitz's version of Navier-Stokes uses one convention, Eckart's version of Navier-Stokes uses another.

Note: there is no such thing as "the" Navier-Stokes eq-s until you specify *your arbitrarily chosen* convention.

Important:

Different conventions give rise to different, *mathematically inequivalent*, Navier-Stokes equations. These conventions have real consequences.

This is because the Navier-Stokes eq-s only give a crude approximation of a real fluid. The difference between the conventions is hidden in the crudeness of this approximation*.

Landau-Lifshitz and Eckart adopt different conventions for Navier-Stokes, but both are bad, and both lead to non-sensical predictions.

*Using an analogy with quantum field theory, the choices of Landau-Lifshitz and Eckart are analogous to adopting UV regulators which violate unitarity.

What's wrong with the classics?

Both Landau-Lifshitz and Eckart define T by:

Exact non-equilibrium energy density

=

$\epsilon(T)$ given by the equation of state

This means: as the local energy density changes, the thermometer adjusts its temperature *instantaneously*.

Such thermometers violate relativity, and lead to super-luminal propagation in relativistic fluid dynamics.

But wait! Why don't you choose a thermometer/velocimeter that does not react instantaneously and respects relativity? I.e. can you just adopt a sensible convention?

Yes, you can!

With good conventions*, the equilibrium is stable, signals propagate slower than light, the eq-s are mathematically well-posed, and can be coupled to Einstein's eq-s.

Bemfica, Disconzi, Noronha, arXiv:1708.06255, arXiv:1907.12695

PK, arXiv:1907.08191, Hoult, PK, arXiv:2004.04102

*Using an analogy with quantum field theory, these are analogous to UV regulators which preserve unitarity.

Bottomline:

Unifying Navier-Stokes with relativity was thought to be impossible for many decades. But: you just need to choose a physically sensible thermometer/velocimeter, and then the Navier-Stokes eq-s are happily unified with relativity. Still waiting to be solved!

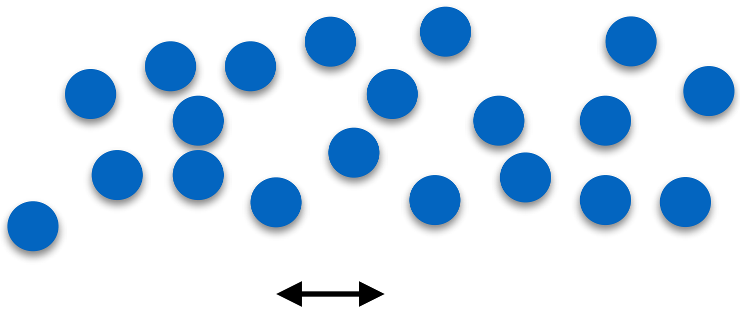
For students:

If you are reading the classics, and they don't make sense, rederive everything using your own way. You may discover something new!

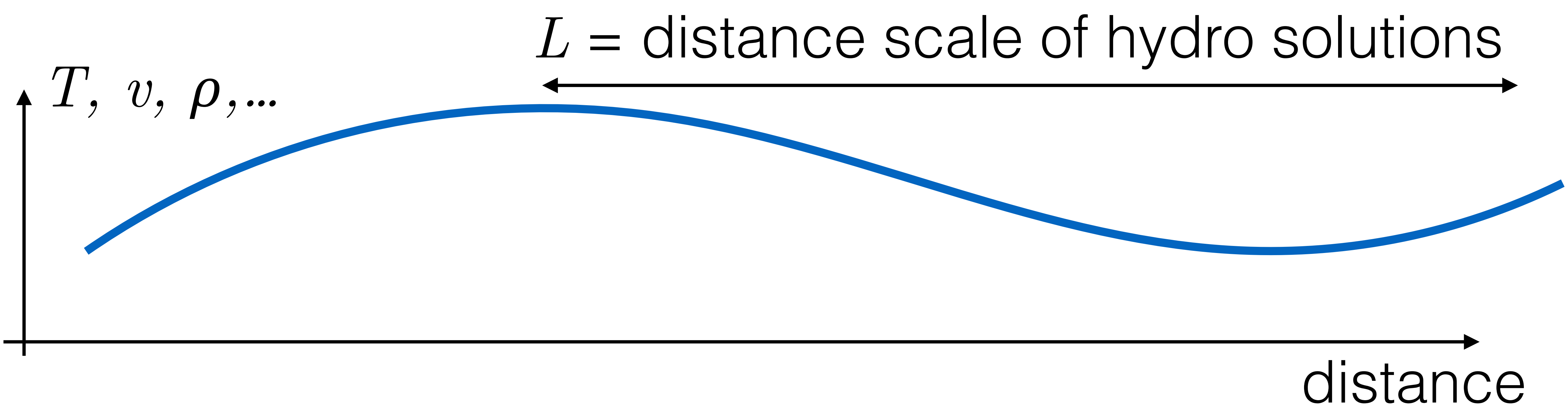
Second story:

Are there limits to improving the hydrodynamic equations?

A first look at the limitations of hydrodynamics

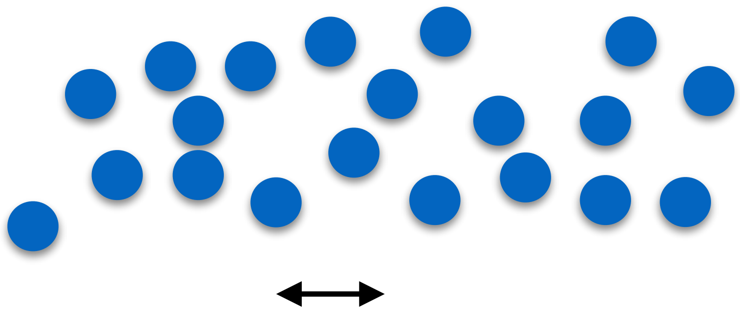


ℓ = typical microscopic distance scale



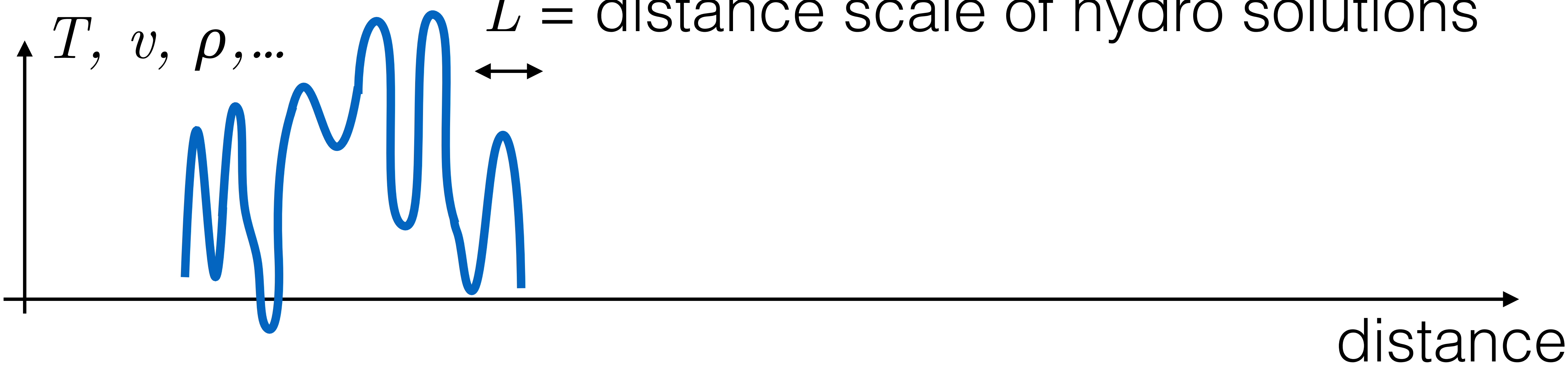
Hydrodynamics probably OK for $L \gg \ell$, small derivatives

A first look at the limitations of hydrodynamics



ℓ = typical microscopic distance scale

L = distance scale of hydro solutions



Hydrodynamics probably <u>not OK</u> for $L \sim \ell$, large derivatives

Hydrodynamics as an expansion in derivatives

Conservarion laws: $\frac{\partial}{\partial t}\rho_a = -\nabla \cdot \mathbf{j}_a$

Constitutive relations (γ = temperature, fluid velocity,…) :

$$\rho_a = \rho^{(0)}(\gamma) + \rho^{(1)}(\nabla\gamma) + \rho^{(2)}(\nabla^2\gamma, (\nabla\gamma)^2) + \dots$$

$$\mathbf{j}_a = \mathbf{j}^{(0)}(\gamma) + \mathbf{j}^{(1)}(\nabla\gamma) + \mathbf{j}^{(2)}(\nabla^2\gamma, (\nabla\gamma)^2) + \dots$$

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Perfect fluids: the most imperfect model of fluids. Can not flow through a pipe, sound propagates forever, diffusion does not exist

Hydrodynamics as an expansion in derivatives

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Navier-Stokes fluids: an improvement of the imperfect "perfect fluids", introduce dissipation and restore common sense.

Hydrodynamics as an expansion in derivatives

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Second-order fluids: an improvement of Navier-Stokes fluids, etc etc

Can we keep improving forever? Let's say we generate an infinite series in the gradients.

Does this series converge (as in $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$), or does it diverge (as in $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$) ?

If converges: Hydrodynamics can be systematically improved to include more transport phenomena. 😀

If diverges: Hydrodynamics is not supposed to work… then why does it? Is hydrodynamics a fluke? 😕

What exactly is the expansion?

For a plane wave, $e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}$, derivatives are $\frac{\partial}{\partial \mathbf{x}} = i\mathbf{k}$, so the hydrodynamic expansion is the expansion in powers of $\mathbf{k}$.

Example: sound wave $\omega_{\rm sound}(k) = \pm v_s k - i\Gamma k^2 + \ldots$

Do the series $\omega(k) = ak + bk^2 + ck^3 + dk^4 + \ldots$
converge or diverge in hydrodynamics?

Not clear how to answer in general, let's look at examples.

Analytic examples

There are fluids which can be studied analytically using "holography". These fluids are similar to the quark-gluon plasma produced in nuclear collisions.

In the simplest solvable examples* we find that $\omega_{\rm sound}(k)$ is an analytic function of k, convergent for $|k| < k_c$, with

$$k_c = \sqrt{2}(2\pi)\frac{k_{\rm B}T}{\hbar c}$$

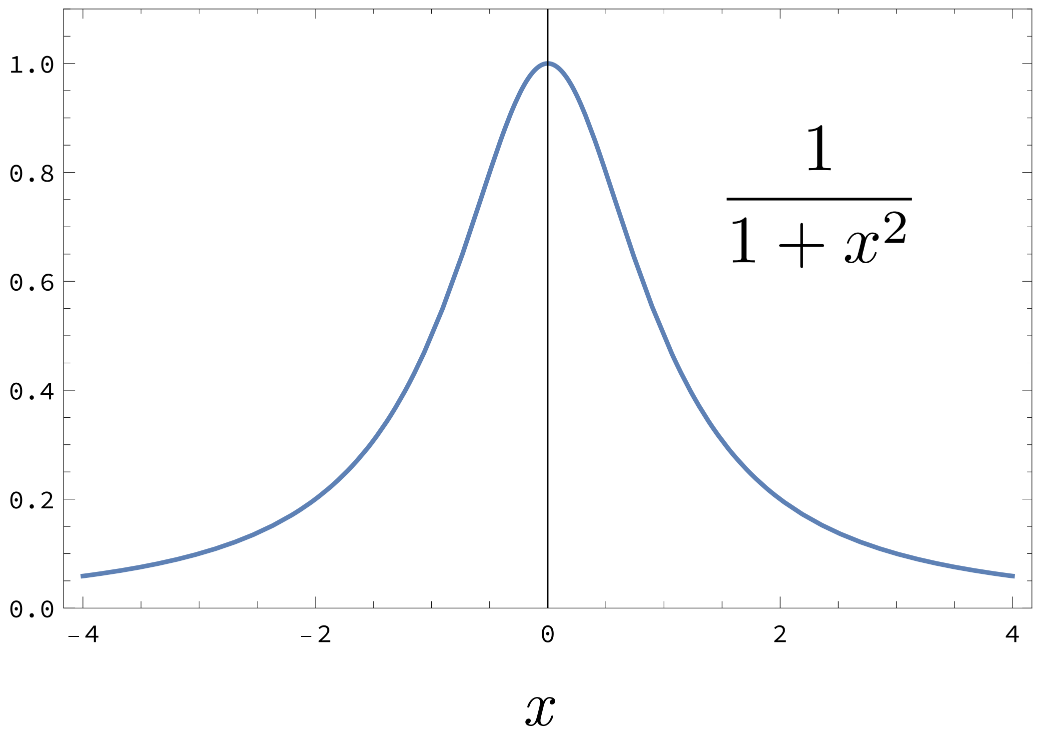
Grozdanov, PK, Starinets, Tadić, arXiv:1904.01018

The convergence is important. Gives one hope that hydrodynamics is improvable.

* $\mathcal{N}$=4 supersymmetric Yang-Mills theory in 3+1 dimensions and its cousins.

So… why is there a critical value $|k|=k_c$ in $\omega(k)$?

Example from basic math: function f(x) = 1/(1+x²)



Function is perfectly smooth for all $-\infty<|x|<\infty$. But the small-x Taylor expansion only converges for $|x|<1$. To understand why, take x complex.

Similarly, $\omega(k)$ is a smooth function at real k. To understand why it only converges for $|k|<k_c$, we must take k complex.

Complex ω and k

In classical physics, dispersion relations $\omega=\omega(k)$ come about by solving $F(\omega,k^2)=0$, where F is determined by the equations (hydrodynamics, Maxwell's eq-s in matter, etc).

Example: diffusion equation $\partial_t\,\delta T + D\nabla^2\delta T = 0$ gives

$$F(\omega,k^2) = -i\omega - Dk^2 = 0.$$

$\delta T \sim e^{i\,\mathbf{k}\cdot\mathbf{x} - i\omega t}$

Example: sound waves in a viscous fluid give

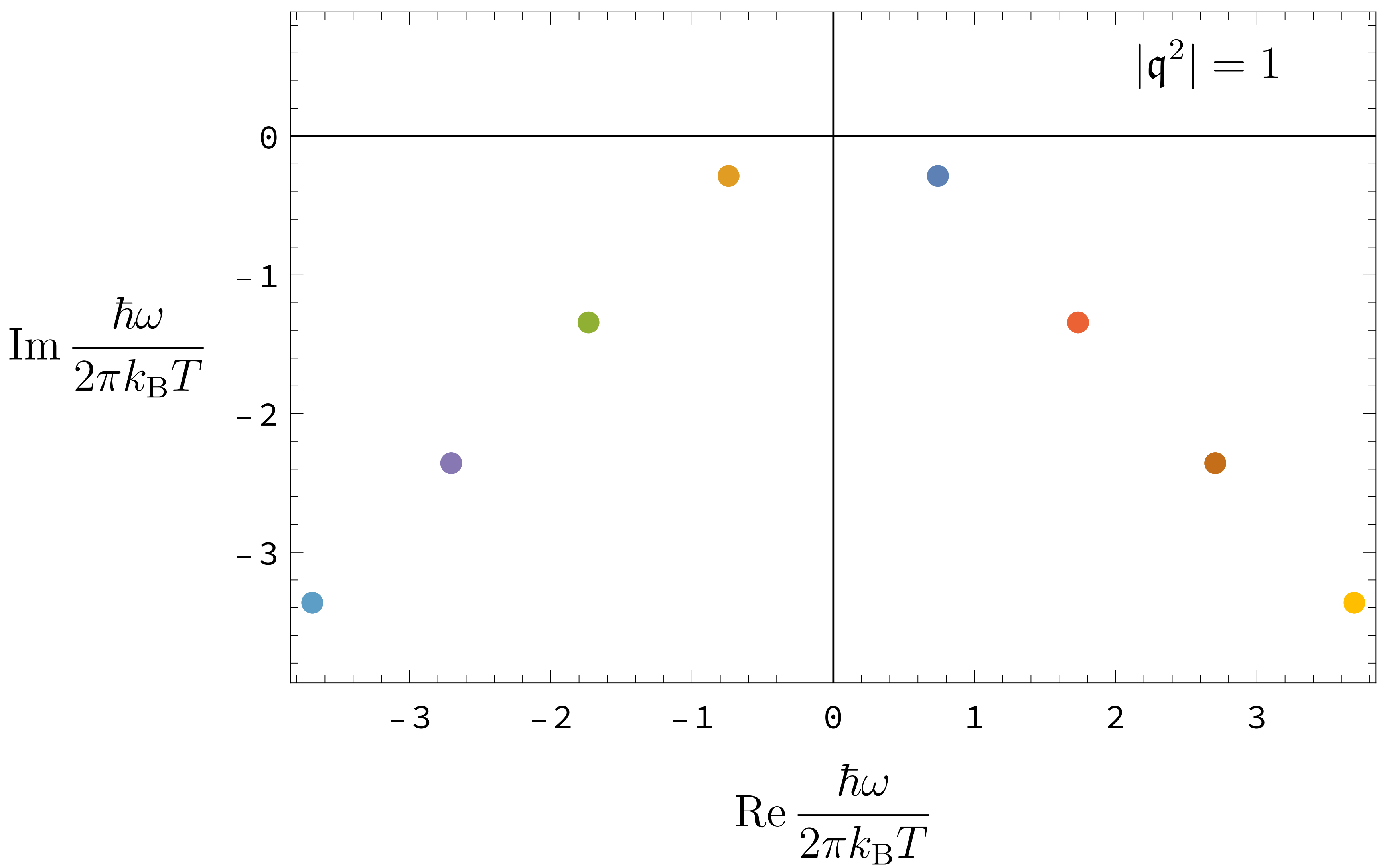
$$F(\omega,k^2) = \omega^2 - v_s^2\,k^2 + 2i\Gamma\omega k^2 = 0.$$

Take ω real, then $F(\omega,k^2) = 0$ gives $k(\omega)$, in general complex.
Imaginary part of k ⇒ damping length/penetration depth

Take k real, then $F(\omega,k^2) = 0$ gives $\omega(k)$, in general complex.
Imaginary part of ω ⇒ relaxation time

But what if *both* ω and k are complex?

Oscillation modes of a fluid, real k



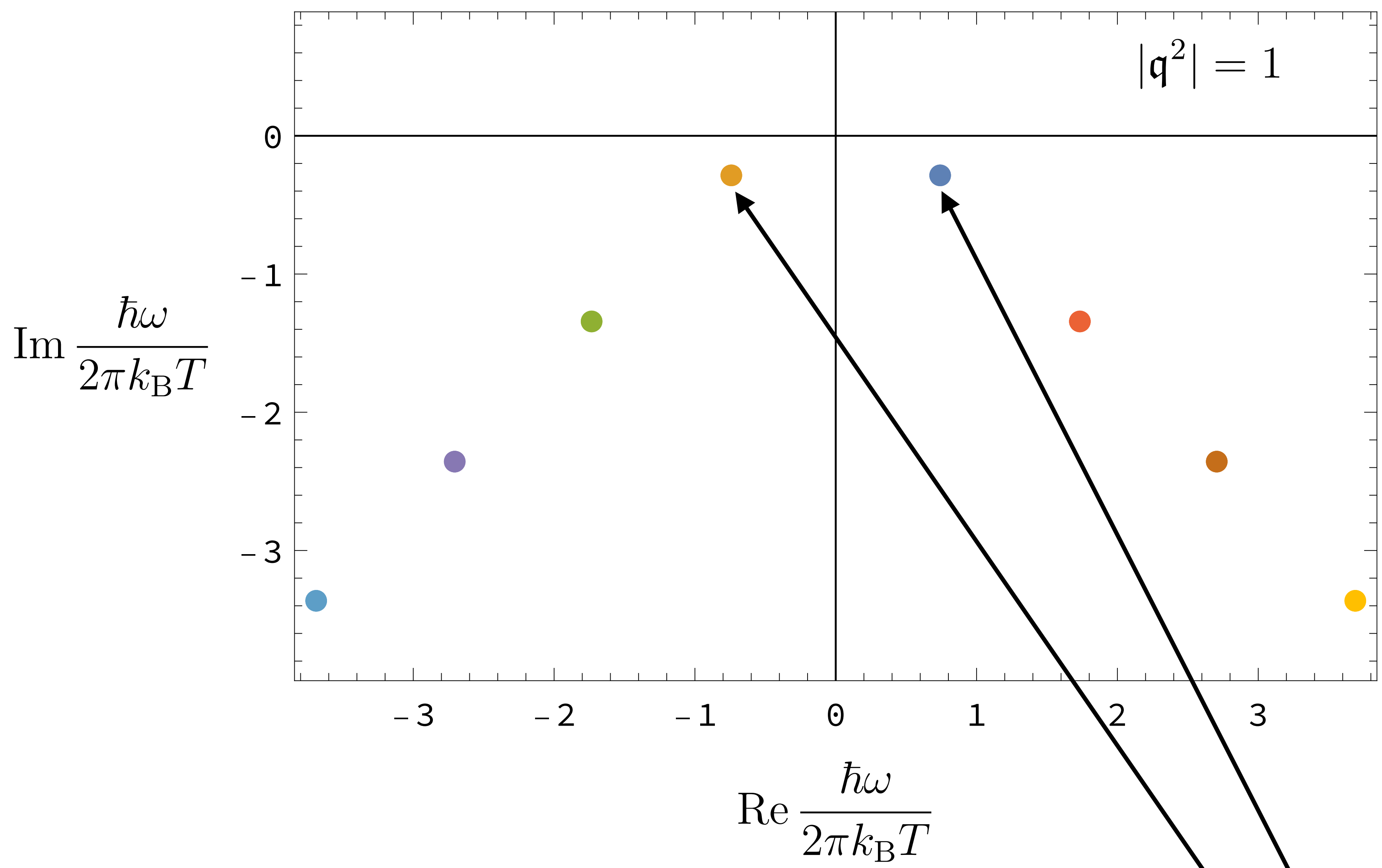
Solutions to $F(\omega,k^2)=0^*$ in the plane of complex ω

$$\mathfrak{q} \equiv \frac{\hbar c\, k}{2\pi k_B T}$$

PK, Starinets, arXiv:hep-th/0506184

* Poles of the exact retarded Green's function of the energy density.

Oscillation modes of a fluid, real k



Solutions to $F(\omega,k^2)=0^*$ in the plane of complex ω

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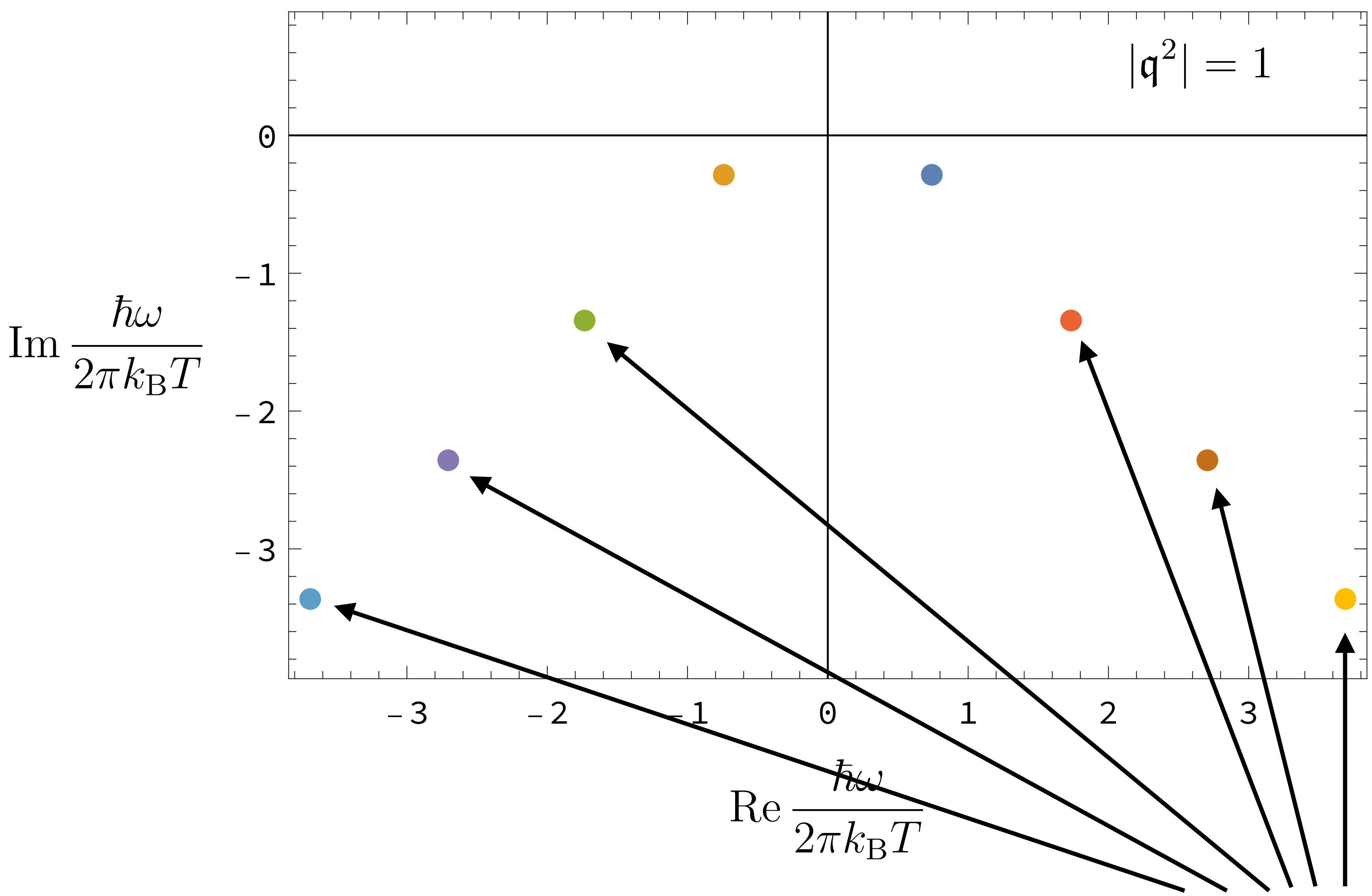
PK, Starinets, arXiv:hep-th/0506184

Sound waves, $\omega(k) = \pm v_s k - i\Gamma k^2 + \ldots$
Macroscopic, classical, approach the origin as $k \to 0$.

* Poles of the exact retarded Green's function of the energy density.

Oscillation modes of a fluid, real k



Solutions to F(ω,k²)=0* in the plane of complex ω

$$\mathfrak{q} \equiv \frac{\hbar c\, k}{2\pi k_B T}$$

PK, Starinets, arXiv:hep-th/0506184

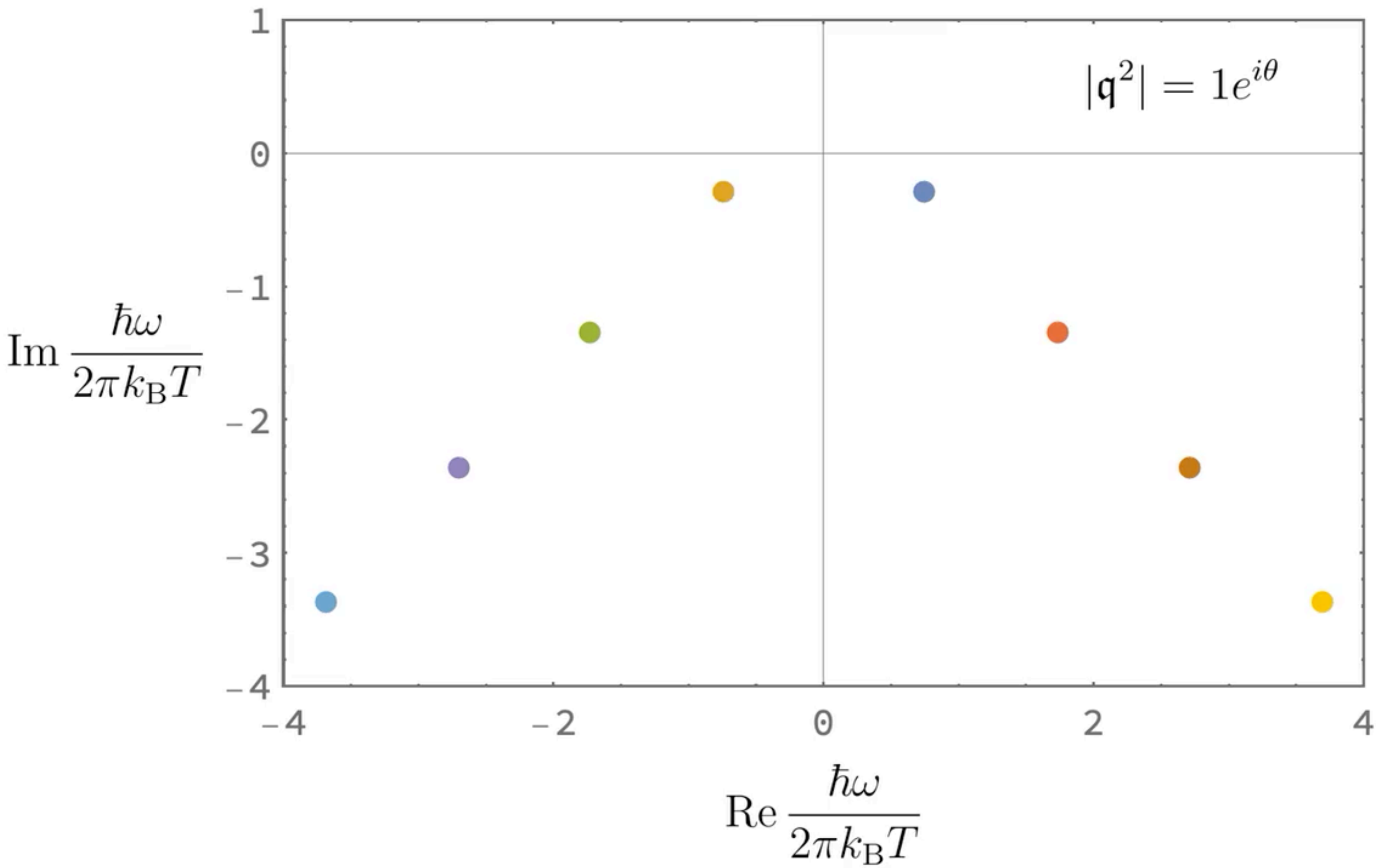
microscopic oscillation modes, no classical interpretation, stay away from the origin as $k \to 0$.

* Poles of the exact retarded Green's function of the energy density.

Oscillation modes of a fluid, complex k

Now take k to be complex, $|\mathfrak{q}^2| = 1e^{i\theta}$, and vary θ from $0 \to 2\pi$.

$$\mathfrak{q} \equiv \frac{\hbar c\, k}{2\pi k_B T}$$

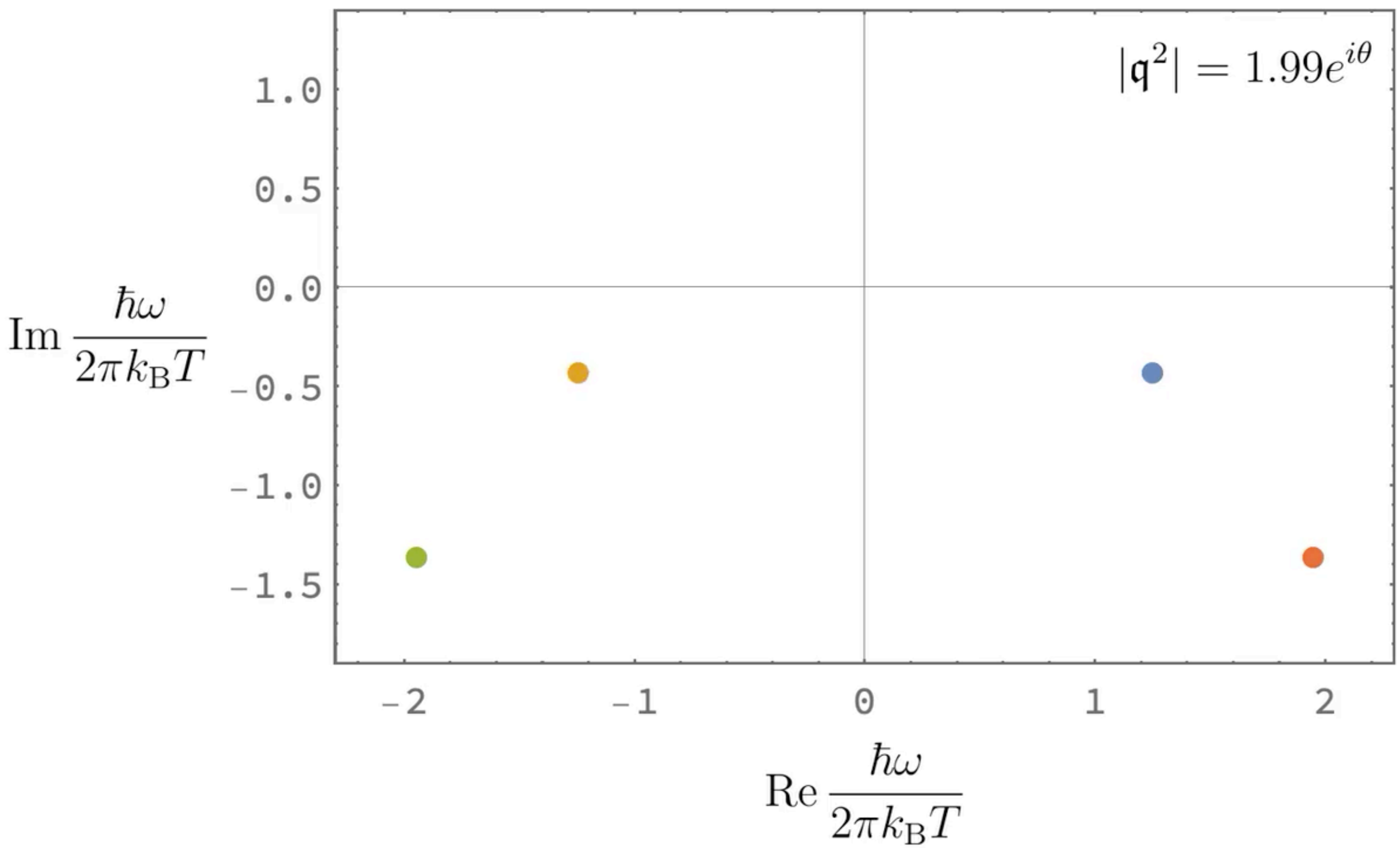


Sound modes (blue and gold) swap places, but remain sound modes when θ becomes 2π.

Oscillation modes of a fluid, complex k

Now take k to be complex, $|\mathfrak{q}^2| = 1.99\, e^{i\theta}$, and vary θ from $0 \rightarrow 2\pi$.

$$\mathfrak{q} \equiv \frac{\hbar c\, k}{2\pi k_{\rm B} T}$$

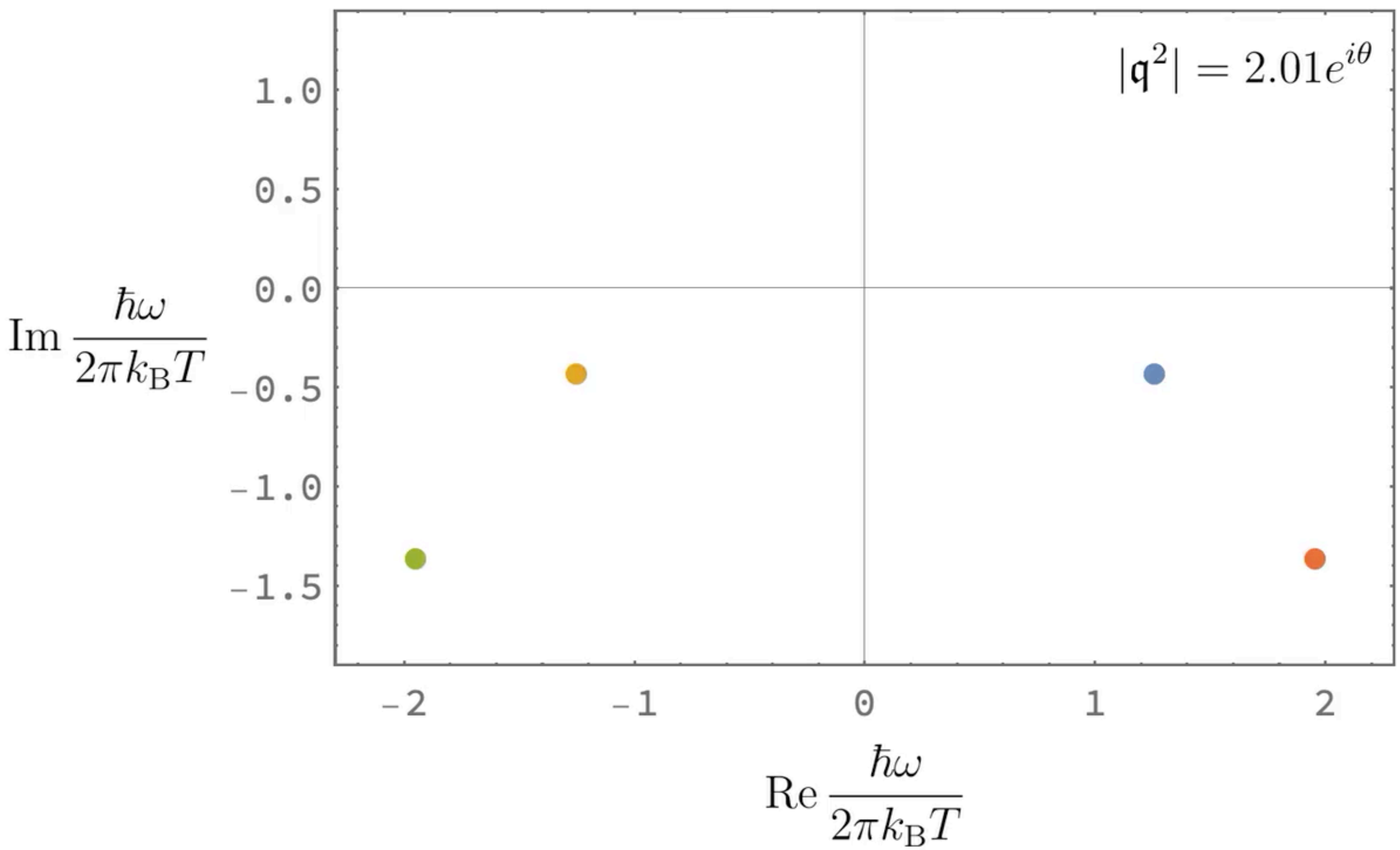


Sound modes (blue and gold) swap places, but remain sound modes when θ becomes 2π.

Oscillation modes of a fluid, complex k

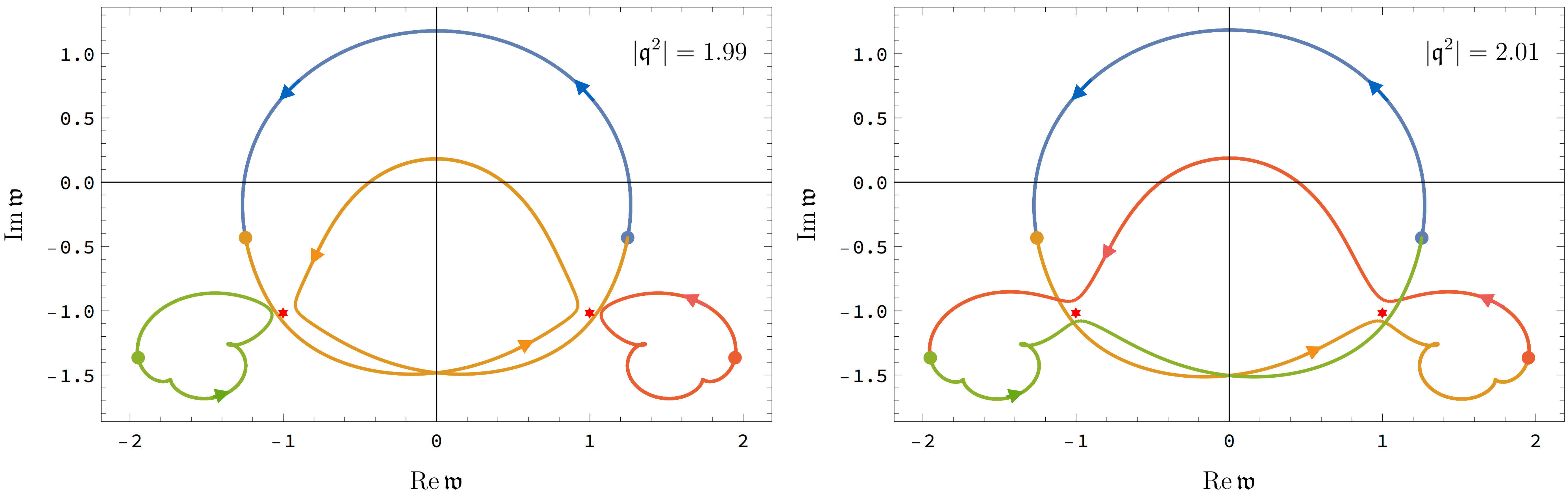
Now take k to be complex, $|\mathfrak{q}^2| = 2.01\, e^{i\theta}$, and vary θ from $0 \to 2\pi$.

$$\mathfrak{q} \equiv \frac{\hbar c\, k}{2\pi k_{\mathrm{B}} T}$$



Sound mode (gold) becomes one of the non-classical modes when θ becomes 2π!

Trajectories of the modes at $|\mathfrak{q}^2|$=1.99 and $|\mathfrak{q}^2|$=2.01



At $|\mathfrak{q}^2|=2$, modes collide, and the topology of the trajectories changes.

This is *level-crossing in macroscopic dissipative systems*, when a classical (hydrodynamic) excitation becomes a non-classical excitation.

This determines the convergence of the hydro expansion, and gives the critical wavelength for sound $\lambda_c=2\pi/k_c$,

$$\lambda_c = \frac{1}{\sqrt{2}} \frac{\hbar c}{k_{\mathrm{B}} T}$$

Grozdanov, PK, Starinets, Tadić, arXiv:1904.01018

But there is more to complex momentum…

Complex momentum and chaos

Chaos: How microscopics gives rise to macroscopics

Classical mechanics: Lyapunov exponent λ describes the divergence of phase-space trajectories $\delta X(t) \sim e^{\lambda t}\, \delta X(0)$

A quantum analogue: $C(t) = \langle\, [Q(t),P(0)]^2 \,\rangle \sim e^{2\lambda t}$
$\langle \ldots \rangle$ = thermal equilibrium average

Plane wave $e^{-i\omega t+i\boldsymbol{k}\cdot\boldsymbol{x}}$, let $\omega = i\lambda$, $k = i\lambda/v$, get $e^{\lambda(t-x/v)}$.
In many examples, sound wave dispersion relation $\omega_{\rm sound}$(k) at complex k allows to extract the Lyapunov exponent λ!

Maldacena, Shenker, Stanford, arXiv:1503.01409
Grozdanov, Schalm, Scopelliti, arXiv:1710.00921
Blake, Lee, Liu, arXiv:1801.00010
… many more papers

Bottomline:

In many solvable examples, classical macroscopic excitations and non-classical microscopic excitations are merely different branches of the same multi-valued complex function.

In many solvable examples, classical hydrodynamics at complex momentum knows about the quantum Lyapunov exponent.

Final story:

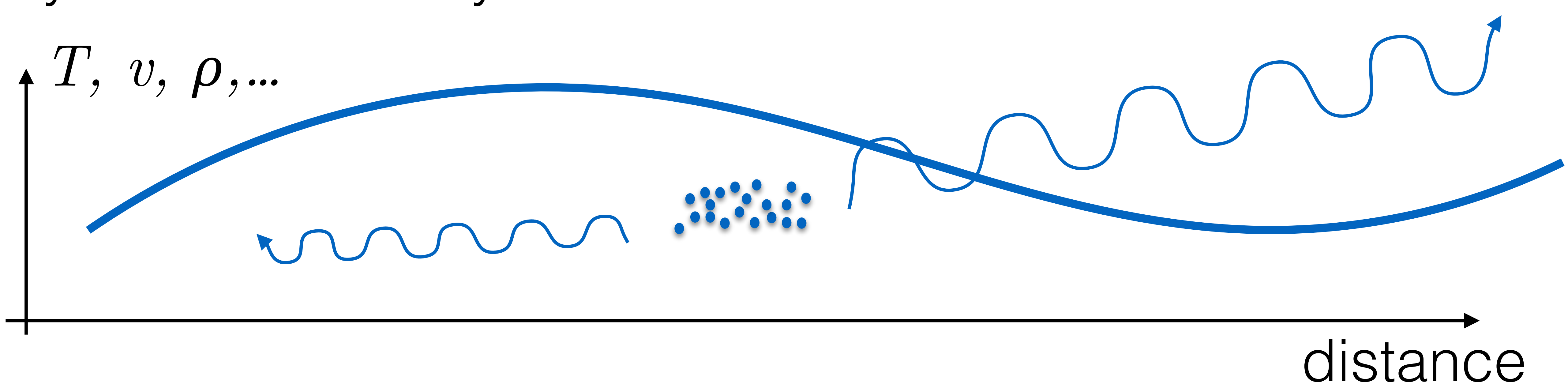
Why everything I said so far is wrong

A second look at the limitations of hydrodynamics

Macroscopic stuff is made out of microscopic stuff.

In a quantum vacuum, virtual particles are constantly produced and absorbed due to quantum fluctuations. Similarly, in a macro-state, virtual sound waves are constantly produced and absorbed due to statistical fluctuations.

These sound waves will back-react on the macroscopic physics because hydro is non-linear and waves interact.



Is the back-reaction relevant?

The back-reaction of quantum fluctuations is responsible for most of the mass of the matter we see in the Universe (Physics Nobel 2004: Gross, Wilczek, Politzer).

Back-reaction of statistical fluctuations is usually not so dramatic, but does modify Navier-Stokes eq-s near liquid-gas critical points, and in two dimensions.

Should I care?

Depends on the questions you ask.

Hydrodynamics to microscopic physics is like classical mechanics to quantum mechanics.

Quantum *equations of motion* look like the classical equations of motion. But if you look at *correlations*, then classical vs quantum predictions can differ dramatically.

Similarly, if you are interested in *macroscopic long-time, long-distance correlations*, classical hydrodynamic eq-s which ignore the back-reaction can lead to dramatically wrong predictions.

For normal fluids

Hydrodynamics = differential equations ❌

Power series $\omega(k) = ak + bk^2 + ck^3 + dk^4 + ...$ ❌

Hydrodynamics → quantum Lyapunov exponents ❌

Transport coef-s determine long-distance correlations ❌

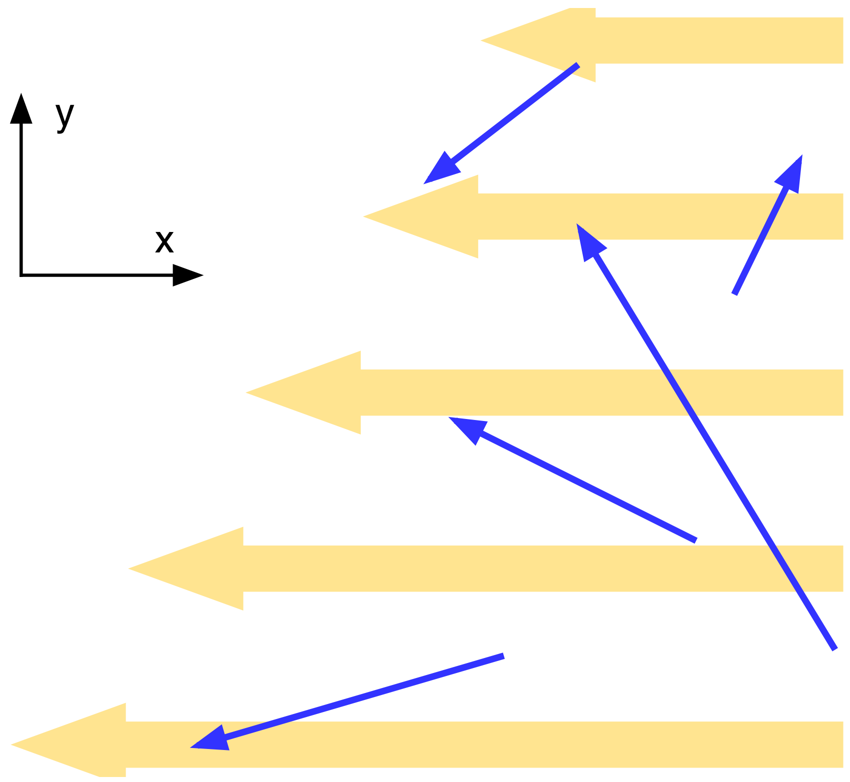
Incompleteness of classical hydrodynamics has been well known in statistical physics, going back to 1970's. Recast hydrodynamics in a language similar to quantum field thry.

Alder, Wainwright, Phys. Rev. A 1, 18 (1970)

Martin, Siggia, Rose, Phys. Rev. A 8, 423 (1973)

… huge literature

Example: viscosity



Momentum transfer between layers of fluid,

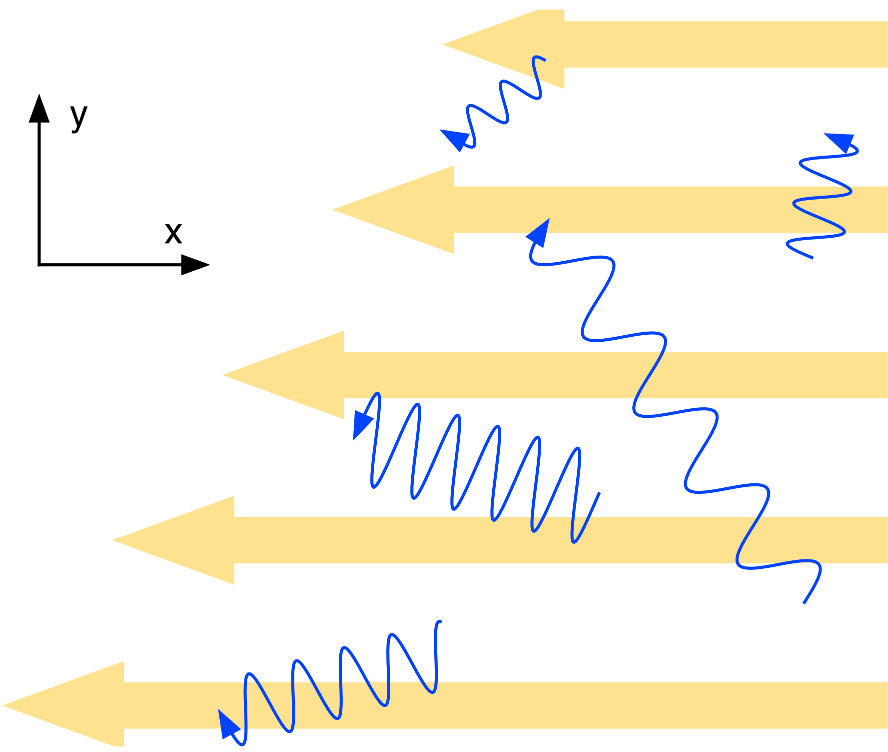
$T_{xy} = \eta\, \partial_y v_x + O(\partial^2)$

Related to correlations of stress:

$$\langle T^{xy} T^{xy} \rangle_{\mathrm{ret.}} = p - i\omega\eta + O(\omega^2)$$

In a gas: $\eta = \rho\, v_{th}\, \ell_{mfp}$

Example: viscosity



Momentum can also be transferred by collective excitations.

Gas of sound waves: $\ell_{\rm mfp} \sim \dfrac{1}{\frac{\eta}{\epsilon+p}\mathbf{k}^2}$

Contribution to viscosity: $$\int^{\Lambda} d^D k \, \frac{T}{\frac{\eta}{\epsilon+p}\mathbf{k}^2}$$

→ $\dfrac{\Lambda T^2}{\eta/s}$ in D=3

→ IR divergent in D=2

Stochastic hydrodynamics

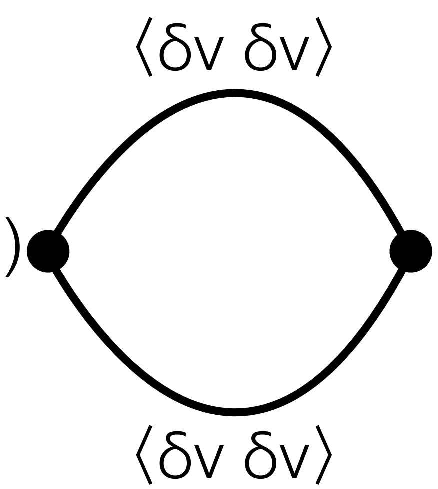
$$T^{ij} = \cdots + (\epsilon + p) v^i v^j + \cdots + \tau^{ij}$$

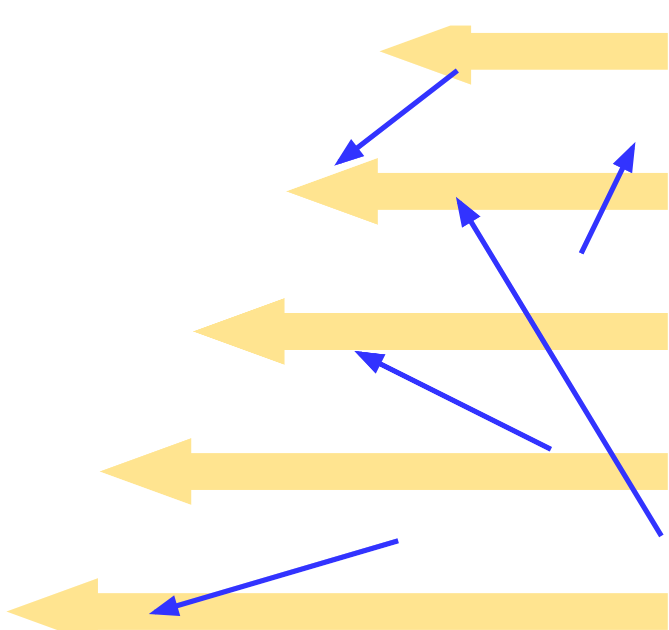
linear

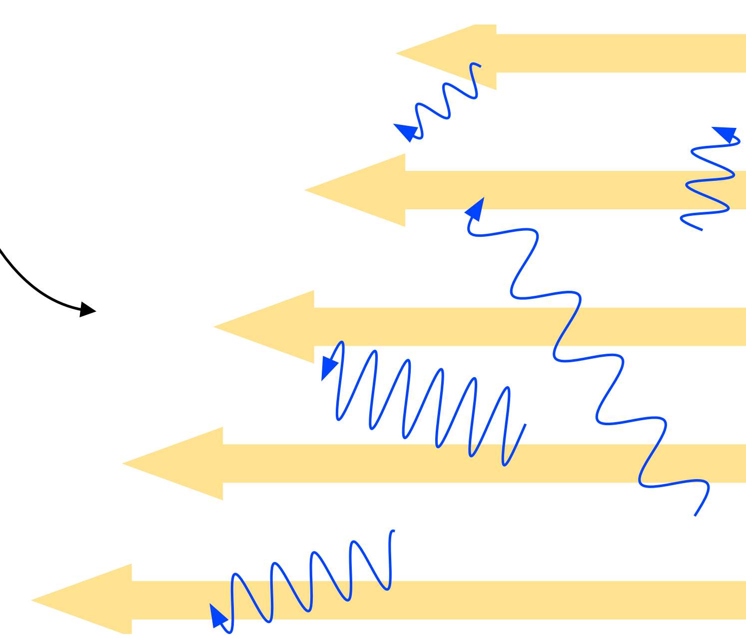
leading non-linear, no derivatives term

higher order

noise

$\langle T_{xy}\, T_{xy} \rangle = 2T\eta + (\epsilon + p)$  $(\epsilon + p)$





Stress correlations in 3+1 dimensions

$$\langle T_{xy} T_{xy} \rangle^R = p + O(\Lambda^3 T) - i\omega \left(\eta + \frac{17 T^2 \Lambda}{120 \pi^2 \eta / s} \right) + O\left(\frac{\omega^{3/2}}{(\eta/s)^{3/2}} \right) + O(\omega^2)$$

0-th order classical — 1-st order classical — 2-nd order classical

correction to p — correction to η — cutoff-independent

PK, Yaffe, hep-th/0303010

PK, Moore, Romatschke, 1104.1586

- This is "one-loop" fluctuation correction to $\langle T^{xy} T^{xy} \rangle_{ret}$. Actual physical viscosity includes all such corrections.
- As expected, small η/s implies large corrections to η/s. Fluctuations are *mandatory* for small-viscosity physics.
- Fluctuations are more important than 2-nd order hydro. IR contributions determined by thermodynamics and η/s.

Bottomline

Using classical hydrodynamics (i.e. hydrodynamics=PDEs) to evaluate correlations as $\omega \to 0$, $k \to 0$ is unreliable,

Short-time statistical fluctuations give rise to universal infrared *late-time* correlations,

The deviations from classical hydrodynamics are more pronounced at small viscosity,

Stochastic hydrodynamics predicts that these infrared correlations are universal, determined only by thermo-dynamics and transport coef-s (viscosity, conductivity etc).

But things are even worse for classical hydrodynamics
(i.e. for thinking of hydro as PDEs)

Nonhydro

Stochastic hydro is not systematic: does not incorporate the derivative expansion, non-Gaussian noise, and the non-linear fluctuation-dissipation theorems.

Can be replaced with effective field thry that does all those things and includes stochastic hydro as a special case.

Glorioso, Liu, arXiv:1805.09331

Because of the noise field, hydro modes have extra interactions that are *not visible* in the constitutive relations.

These interactions give rise to *infrared* correlations that are *not* determined by just thermodynamics and standard transport coef-s (viscosity etc). Need *stochastic transport coef-s* to which classical hydro is completely blind.

Jain, PK, arXiv:2009.01356

Where does this leave us?

A dream

a “grand unified theory” of hydrodynamics that:

- Applies to many different kinds of fluids,
- Ensures that the eq-s are sensible,
- Allows for systematic improvements,
- Correctly predicts macro correlations,
- Connects with quantum chaos,
- Is experimentally verified!

PK, arXiv:1205.5040
Glorioso, Liu, arXiv:1805.09331
…active field of research

Encouragement

As a student, one often tends to think that all interesting questions in physics have already been answered many years ago by the great.

Hydrodynamics is an ancient field by modern standards, and people are still trying to figure out some very basic questions.

If, as a student, you work in a field that is less than 200 years old, do not lose hope: there surely are many fundamental things left to discover.

Thanks for listening!