Based on arXiv:2112.10238v1 [hep-th]
with Robert Konik, Rob Pisarski and Alexei Tsvelik

When cold, dense quarks are not a Fermi liquid

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ASU Theoretical Physics Colloquium – January 26, 2022

What is QCD?

$$S=\int d^{D+1}x \frac{1}{8\pi g^2} Tr_{\sigma} F_{\mu\nu} F^{\mu\nu} + \bar{q}_{f,\sigma} \gamma^{\mu} \left(i\delta_{\sigma\sigma'} \partial_{\mu} + A_{\mu}^{\sigma\sigma'} \right) q_{f,\sigma'} + m^{(f)} \bar{q}_{f,\sigma} q_{f,\sigma'}$$

Gluon kinetic term

Quark kinetic term

Quark-gluon interaction

Quark mass

$$F_{\mu\nu}=\partial_{mu}A_{\nu}-\partial_{nu}A_{\mu}+[A_{\mu},A_{\nu}]$$

$\sigma:1..N_c$ (# of colors)

$f:1..N_f$ (# of flavors)

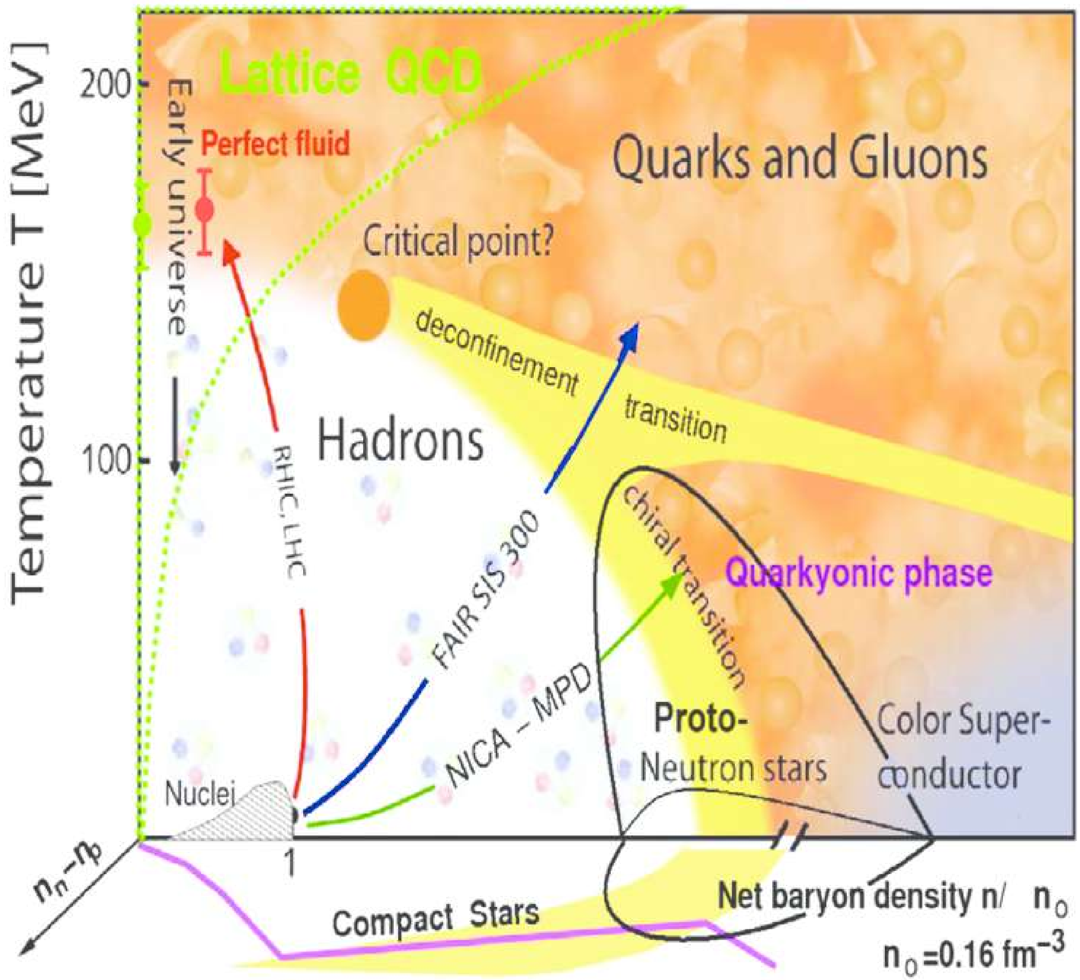
Real world: $N_c=3$

$$N_f=6:(u,d,s,c,t,b)$$

Low temperature:

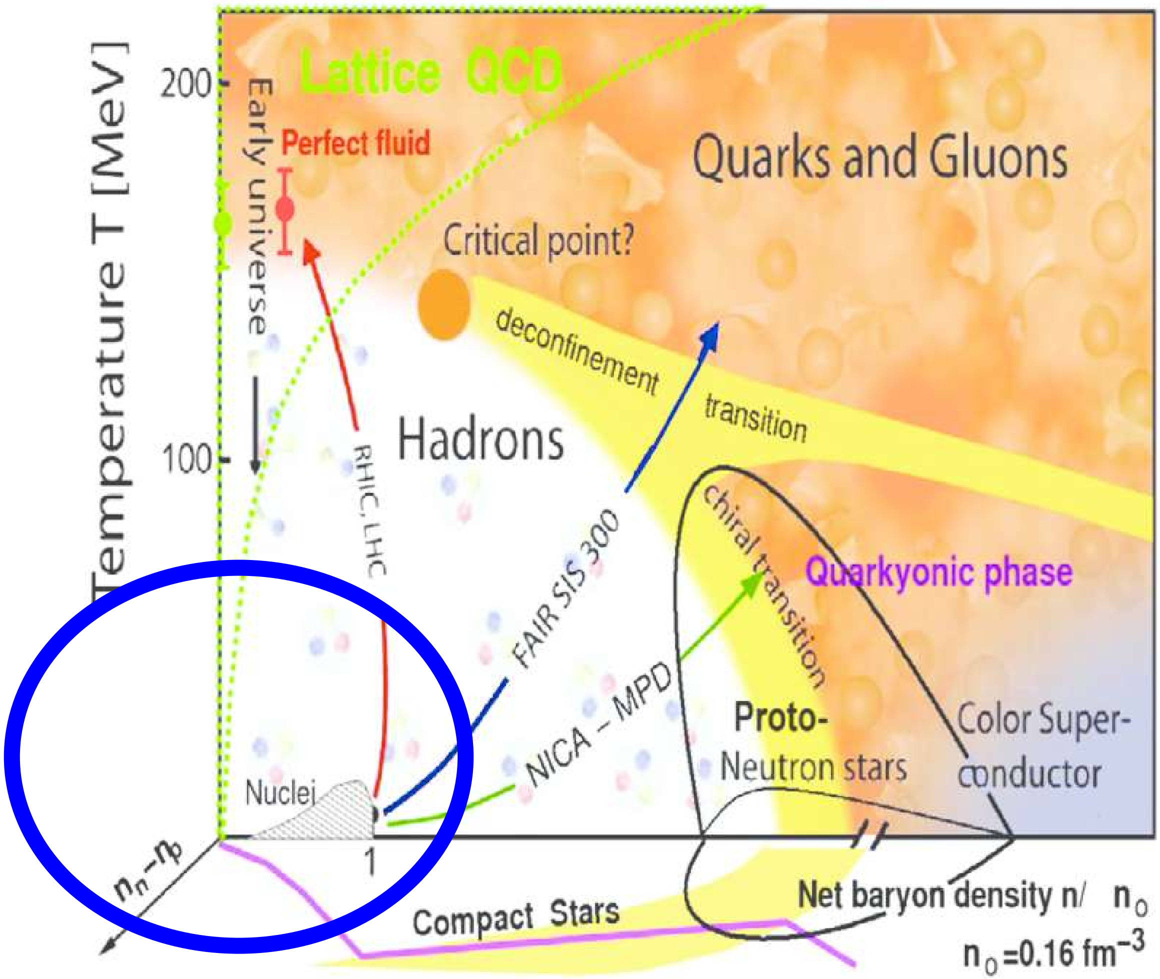
$$N_f=2:(u,d)$$

Phases of QCD



L. McLerran, *Nucl.Phys.Proc.Suppl.* **195**:275-280,2009

QCD at small (zero) density



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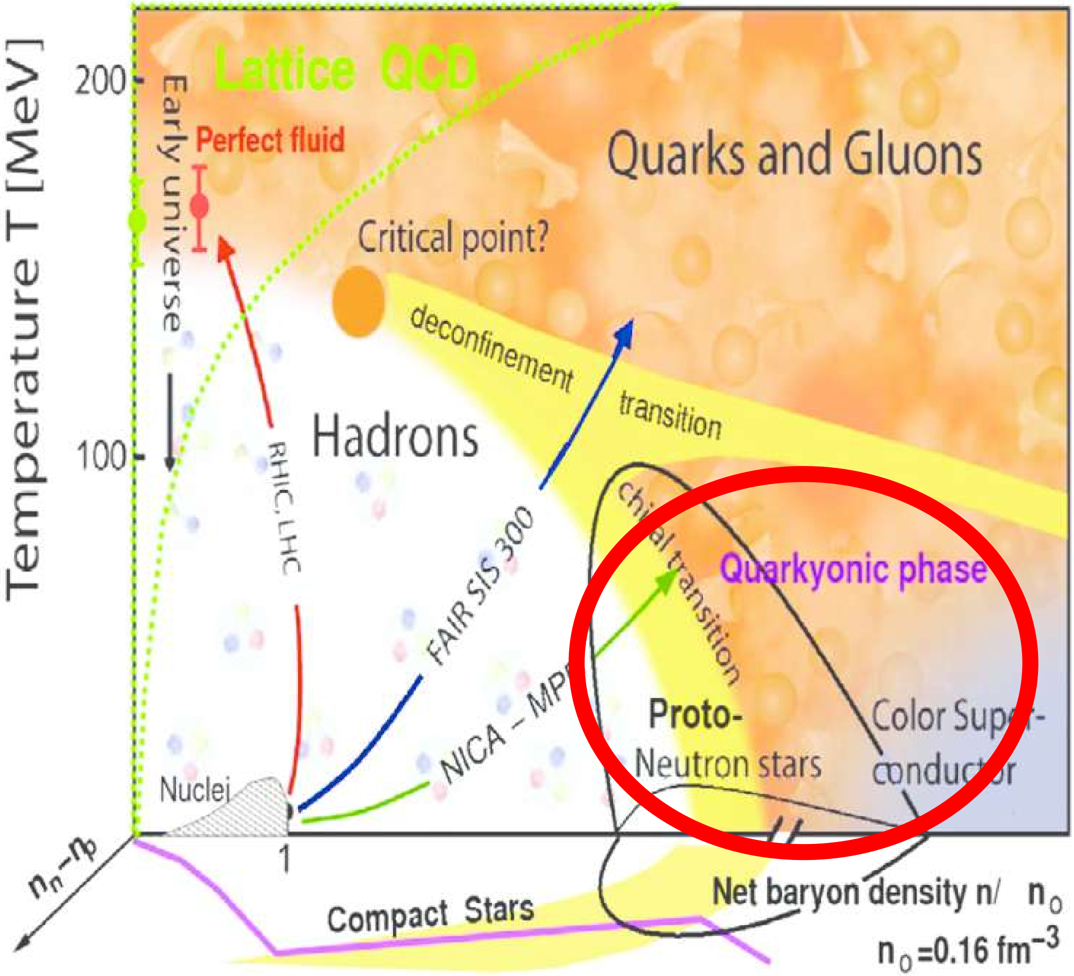
Phases of QCD



L. McLerran, *Nucl.Phys.Proc.Suppl.* **195**:275-280,2009

Regimes of finite-mu QCD phase diagram (T<μ)

- Nuclear matter

  Density: $0 < n < {\color{red}x}\,{\color{blue}n_0}$

  Some O(1) multiplier, x>1

  Saturation density for nuclei

- Color superconductor

  Density: $n_{pert} < n$

  No confinement: „electric” (timelike) gluons suffer Debye screening

- Quarkyonic

  Density: $x\,n_0 < n < n_{pert}$

  free energy is like that of (interacting) quarks and gluons, but quasi-particles **near** the Fermi surface are confined.

Can't solve QCD at low-T, high-μ

- N_c=3 difficult to probe
  - Lattice: can't go beyond μ_{quark}~T
  - Quantum computers (of the future)
- Strategy: look at two easier limits
  - **$N_c \to \infty$, $N_c \gg N_f$** Expansion in $1/N_c$
  - **N_c=2** Lattice: quark determinant real, no sign problem

Plan

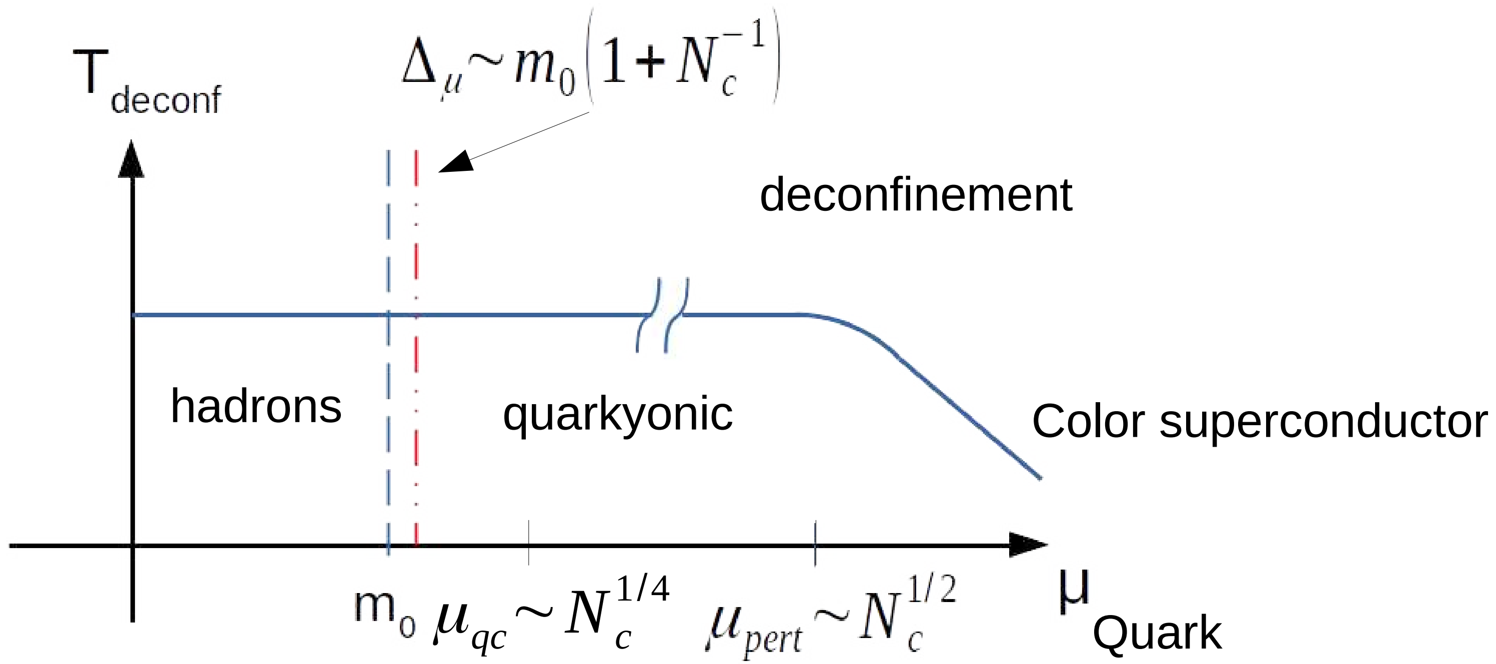
1. Quarkyonic matter and its anisotropy
2. What is a Luttinger liquid?
3. QCD in 2D
4. Some insights on neutron star cooling

1 Quarkyonic matter and its anisotropy

Quarkyonic matter, $N_c \gg N_f$

- N_c colors, N_f flavors
- Debye mass: $$m_D^2 \sim g^2 \left(N_c \frac{T^2}{3} + N_f \frac{\mu^2}{2\pi^2} \right)$$
- Theory **confines** until $$\mu > \mu_{pert} \sim \left(\frac{N_c}{N_f} \right)^{\frac{1}{2}} T_\chi$$

$$m_0 = \frac{m_{Nucleon}}{N_c}$$



Quarkyonic matter, $N_c >> N_f$

- Pressure: $p \sim \underbrace{N_c^2 T^2}_{\text{gluons}} + \underbrace{N_c N_f \mu^4}_{\text{quarks}}$ Balance: $\mu_{\pi q} \sim \left(\frac{N_c}{N_f} \right)^{\frac{1}{4}}$

- 4 regimes

	$\mu : m_0 \rightarrow \mu_N = m_0 \left(1 + \frac{\#}{N_c} \right)$	Nuclear matter
Quarkyonic	$\mu_N \rightarrow \mu_{\pi q}$	Chiral symm. restored, π/K condensation (meson chiral spirals)
Quarkyonic	$\mu_{\pi q} \rightarrow \mu_{pert}$	Quark chiral spirals
	$\mu_{pert} \rightarrow \infty$	Perturbative QCD

Quarkyonic matter, $N_c \gg N_f$

- Pressure: $p \sim \underbrace{N_c^2 T^2}_{\text{gluons}} + \underbrace{N_c N_f \mu^4}_{\text{quarks}}$ Balance: $\mu_{\pi q} \sim \left(\frac{N_c}{N_f} \right)^{\frac{1}{4}}$

- 4 regimes

Quarkyonic

$\mu_N \rightarrow \mu_{\pi q}$	Chiral symm. restored, π/K condensation (meson chiral spirals)
$\mu_{\pi q} \rightarrow \mu_{pert}$	Quark chiral spirals

Or quantum π liquid? See also:
Pisarski, Tsvelik, Valgushev, Phys. Rev. D **102**, 016015 (2020)

Quarkyonic matter, N_c>>N_f

gluons quarks

- Pressure: $p \sim N_c^2 T^2 + N_c N_f \mu^4$ Balance: $\mu_{\pi q} \sim \left(\frac{N_c}{N_f} \right)^{\frac{1}{4}}$

- 4 regimes

Region with "moat" spectrum may be **much** larger than basin for Critical End-Point!
Pisarski, Rennecke, Phys. Rev. Lett. **127**, 152302 (2021)

Quarkyonic

$\mu_N \rightarrow \mu_{\pi q}$ Chiral symm. restored, π/K condensation (meson chiral spirals)

$\mu_{\pi q} \rightarrow \mu_{pert}$ Quark chiral spirals

Or quantum π liquid? See also:
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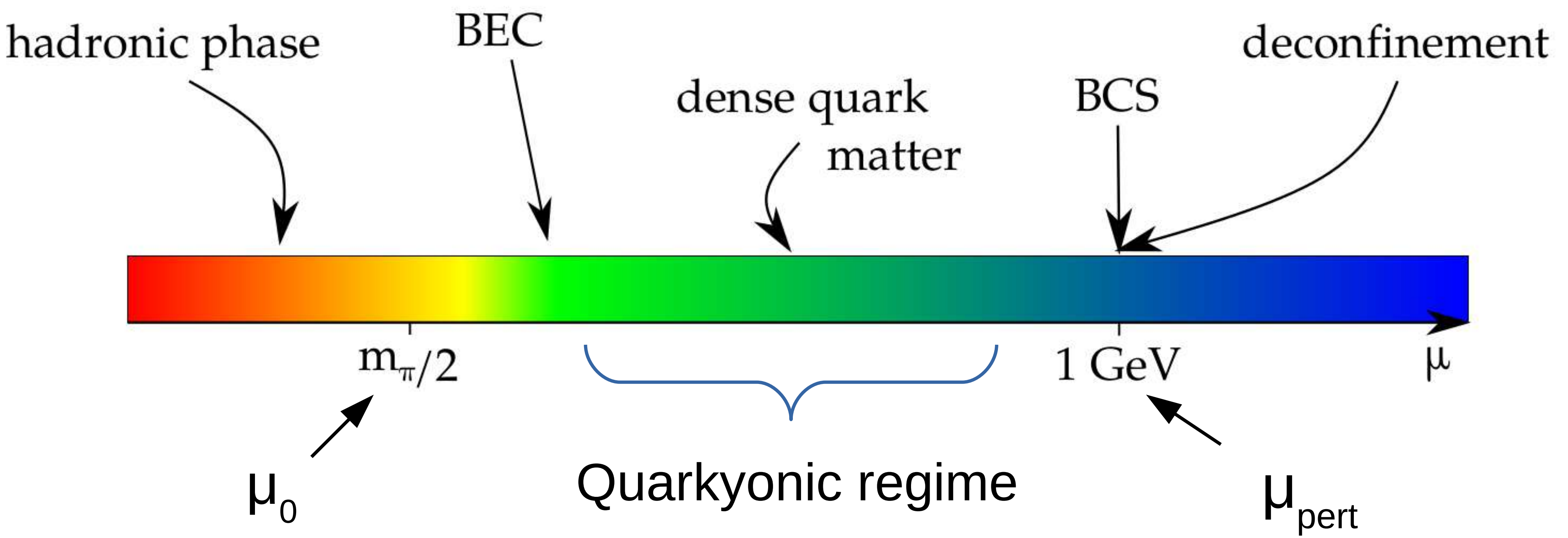
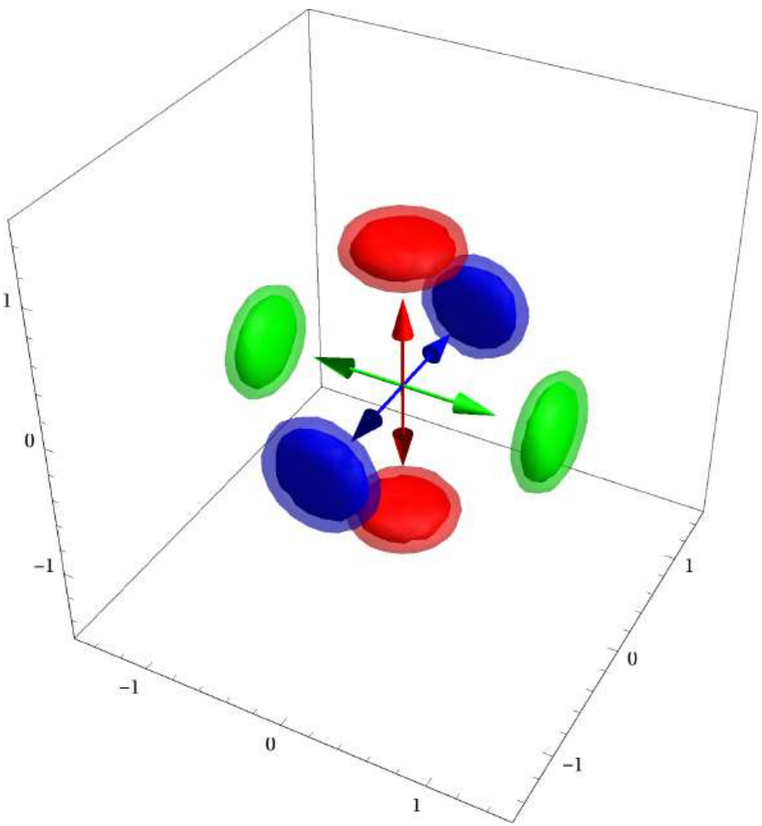
Is N_c=3 close to N_c=∞?

- Bornyakov et al, 1808.06466
- **N_c=2**, m_π=740
- Baryons are bosons, Bose condensate (BEC)



Quarkyonic matter (I): $\mu \in [\mu_N, \mu_{\pi q}]$



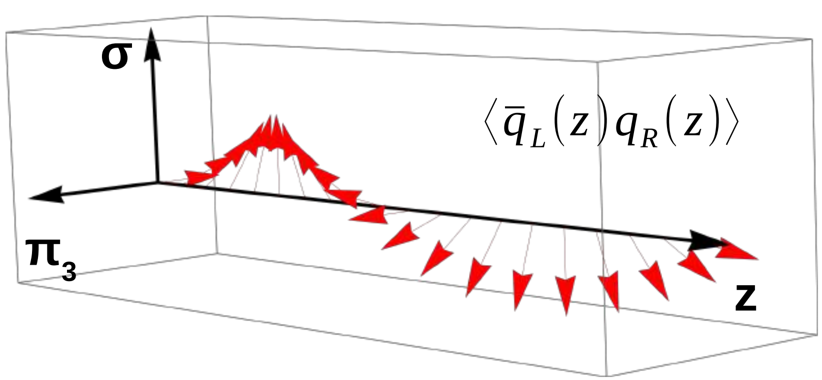
Fermi surface covered in patches, width ~ Λ_{QCD}

Quarks (hadrons) near Fermi surface: Scatter back and forth between 2M patches

Each pair of patches → density wave

Patch vector **Q**

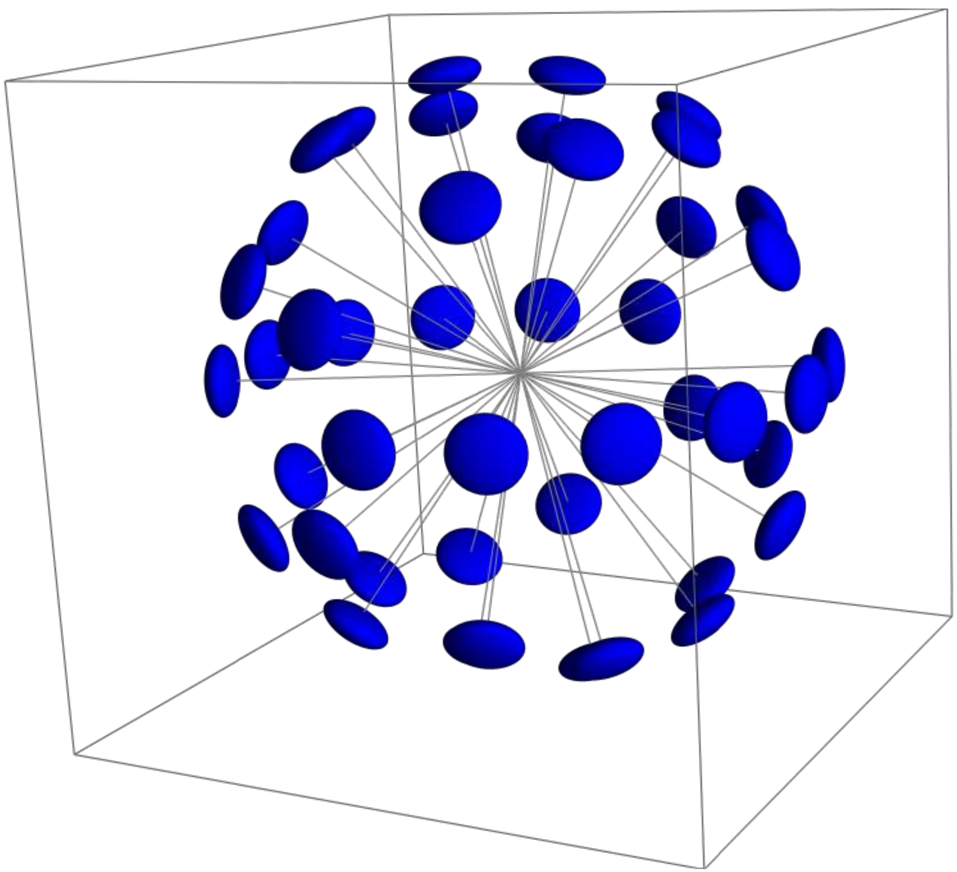
Anisotropic!



π condensation:
Overhauser, *Phys. Rev. Lett.* ***4****, 415 (1960)*
Migdal, *Soviet Physics JETP* ***36****, 1052 (1973)*

K condensation:
Kaplan, Nelson, *Phys. Lett. B* ***175****, 57 (1986)*

Quarkyonic matter (II): $\mu \in [\mu_{\pi q}, \mu_{pert}]$



Effective model: U(1) and SU(2N_f) fields for each pair of patches, ~ WZNW

$$\mathcal{F} = \mathcal{F}_{U(1)} + \mathcal{F}_{SU(2N_f)} + V \qquad \boldsymbol{q} = \boldsymbol{Q}/|\boldsymbol{Q}|$$

longitudinal transverse

$$\mathcal{F}_{U(1)} = \frac{1}{2}\sum_{\boldsymbol{Q}} \left[\lambda_1 (\boldsymbol{q}\cdot\nabla\phi_Q)^2 + \lambda_2 [(\boldsymbol{q}\times\nabla)^2 \phi_q]^2\right]$$

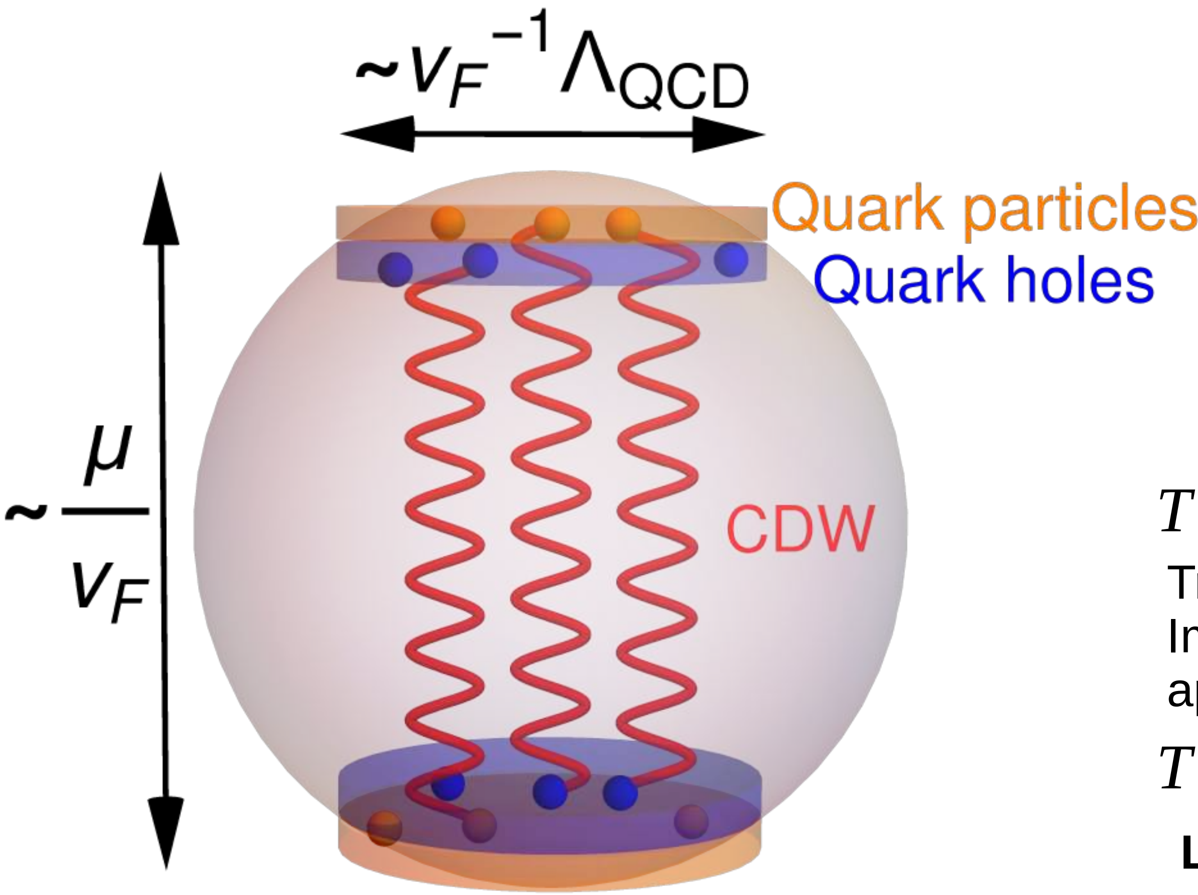
$$\mathcal{F}_{SU(2N_f)} = \frac{1}{2}\sum_{\boldsymbol{Q}} \left\{\lambda_1 Tr(\boldsymbol{q}\cdot\nabla G_Q)(\boldsymbol{q}\cdot\nabla G_Q^\dagger) + \lambda_2 Tr\left(G_Q^\dagger [\boldsymbol{Q}\times\nabla]^2 G_Q\right)^2\right\}$$

T. Kojo, R. D. Pisarski, and A. M. Tsvelik (2010)
R. D. Pisarski, V. V. Skokov, and A. M. Tsvelik (2018)

Low-dimensional behavior

Linearized meson dispersion relation

$$\omega_{\mathbf{Q}}(\mathbf{q})^2 = \left(\frac{v_F}{2k_F}\right)^2 \left\{ \left[\mathbf{Q}\cdot(\mathbf{q}-\mathbf{Q})\right]^2 + \alpha \left(\frac{\left[(\mathbf{Q}\times\mathbf{q})^2\right]^2}{4k_F^2} \right) \right\}$$



String tension $\sigma \sim \Lambda_{QCD}$

 # patches $\mathcal{N} \sim k_F^2/\sigma$

Transverse energy scale $W \sim \sigma/k_F$

Patch size: σ/v_F

$T < W$:

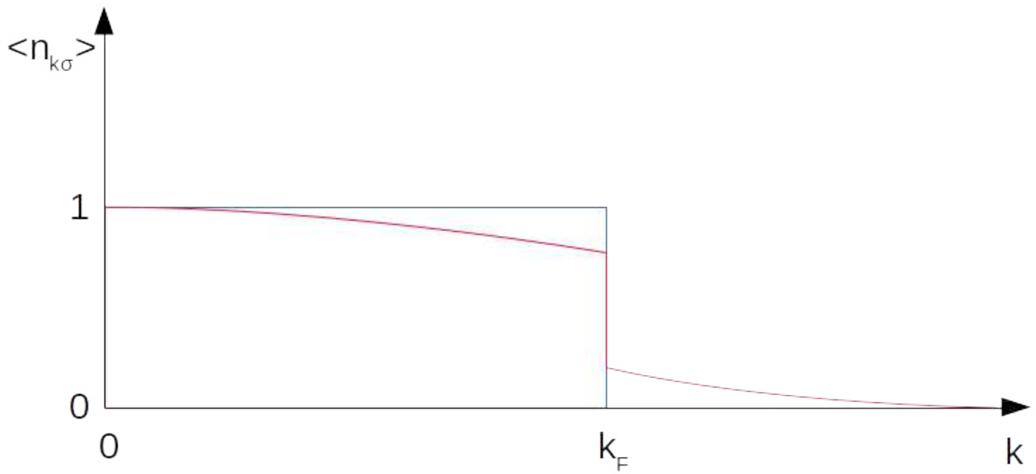
Transverse (anisotropic) mesons Important; bosonization not applicable

$T > W$:

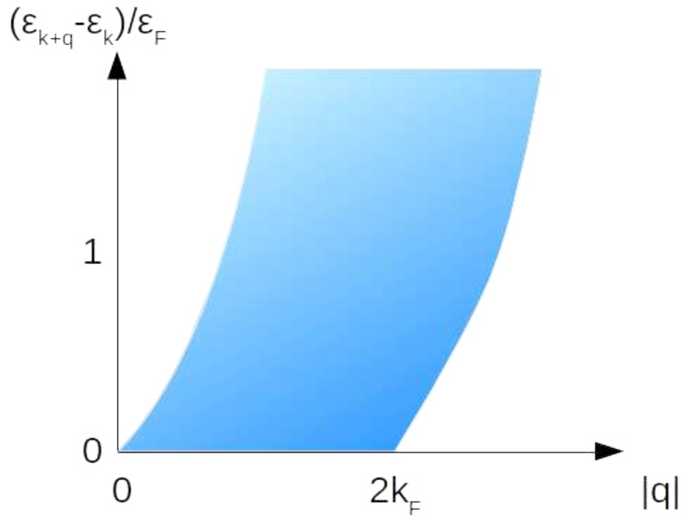
Luttinger liquid

2 What is a Luttinger liquid?

Fermi liquid



Occupation number wrt. momentum



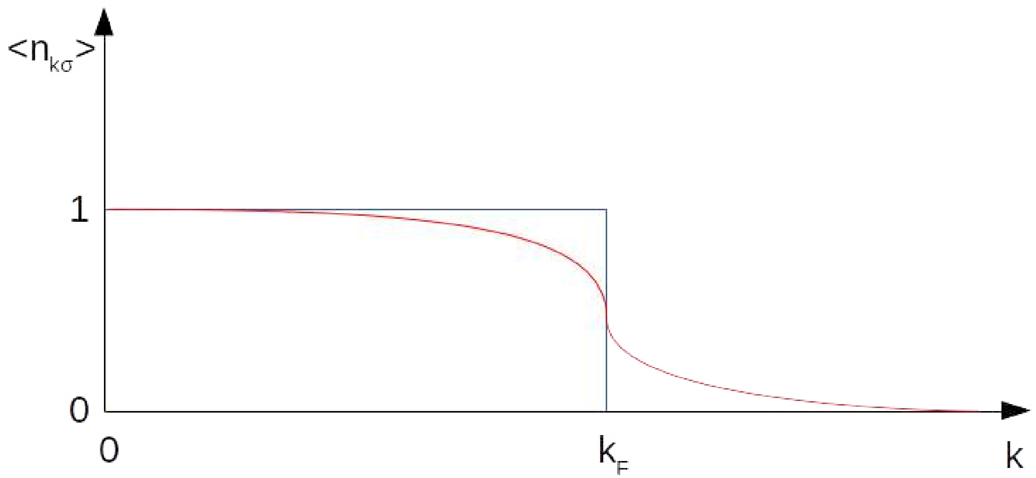
Particle-hole excitation spectrum

Well-defined quasiparticles with singular lifetime on the Fermi surface

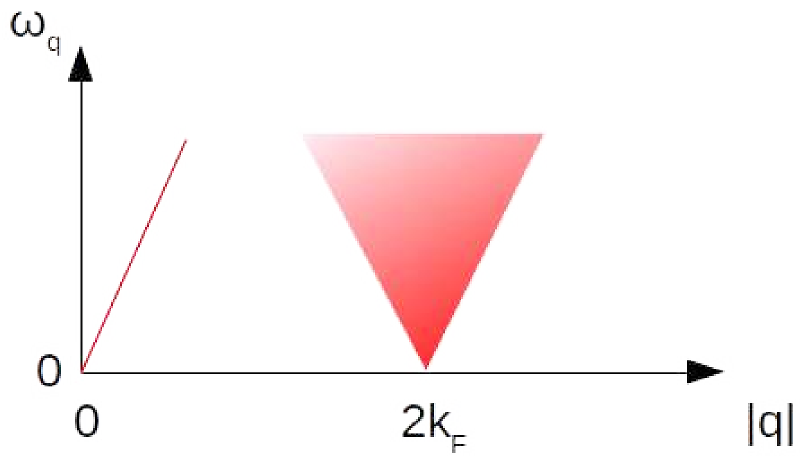
$$G(\boldsymbol{k},\omega)\approx\frac{Z_{\boldsymbol{k}}}{\omega-\epsilon_{\boldsymbol{k}}+\mu\pm i\,\Gamma_{\boldsymbol{k}}} \qquad A(\boldsymbol{k},\omega)\sim\Im\,G(\boldsymbol{k},\omega)\sim\delta(\omega-(\epsilon_{\boldsymbol{k}}-\mu))$$

At Fermi surface

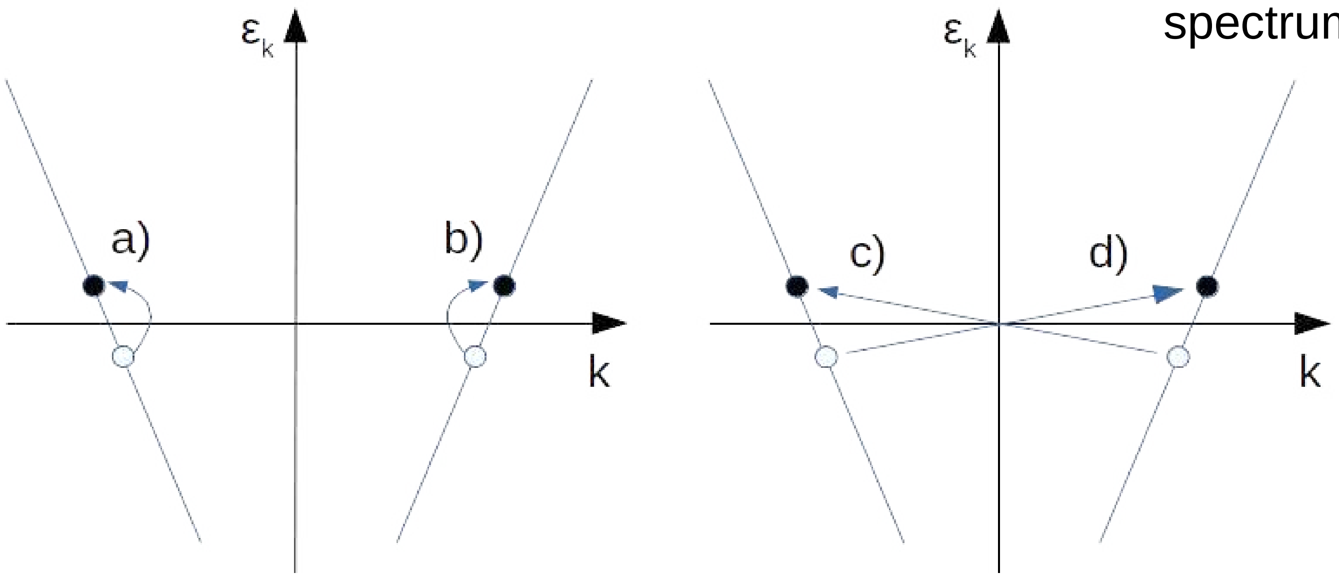
Luttinger liquid (1+1d)



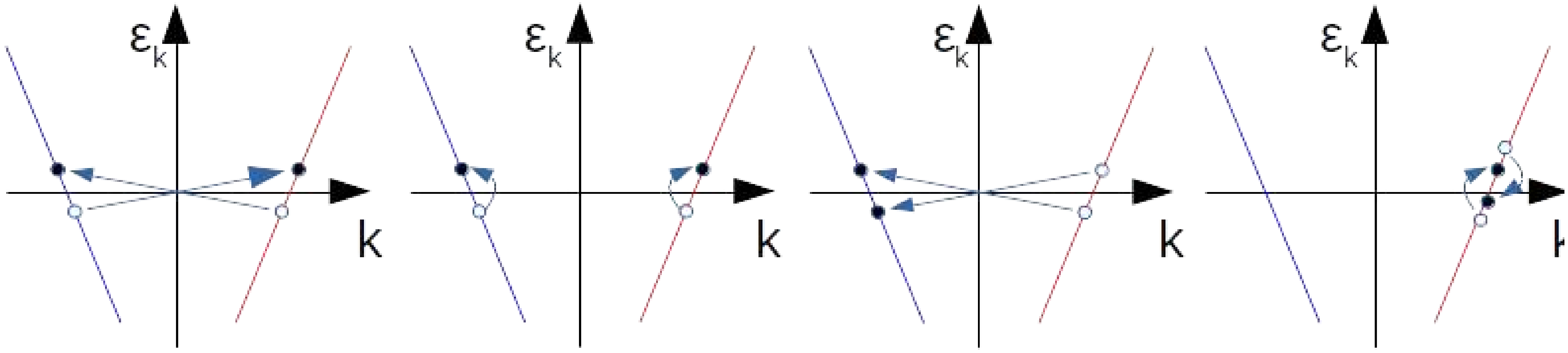
Occupation number wrt. momentum



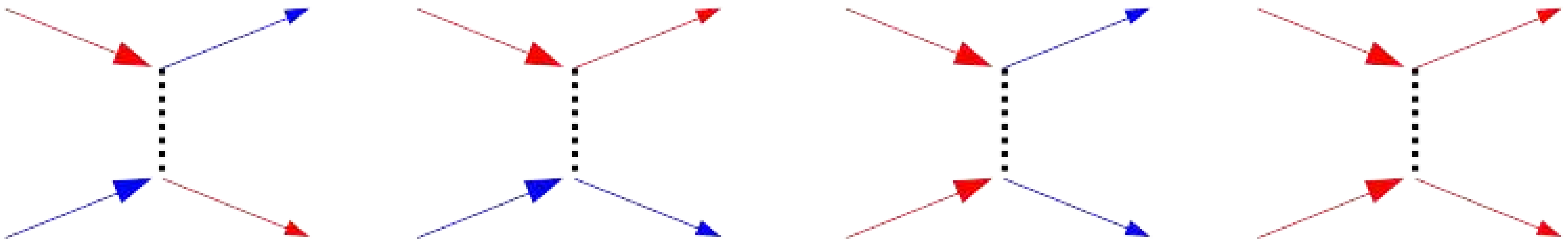
Particle-hole excitation spectrum



Luttinger liquid



g_1 g_2 g_3 g_4



backward | forward I | „Umklapp” | forward II

(Only differs from *forward I* for spinful particles)

(needs lattice)

Bosonization

Dictionary between massless fermions and massless bosons (1+1D)

$$\psi_R = \frac{1}{\sqrt{2\pi}} \xi_R \exp\left(i\sqrt{4\pi}\varphi\right)$$

$$\psi_L = \frac{1}{\sqrt{2\pi}} \xi_L \exp\left(-i\sqrt{4\pi}\bar{\varphi}\right)$$

Chiral bosonic field

Klein factors

$K = \frac{1}{1}$

Bosonized Lagrangian of *interacting* Luttinger liquid:

Couplings from previous slide

$$L = \frac{K}{2}\left[v^{-1}\left(\partial_\tau \phi\right)^2 + v\left(\partial_x \phi\right)^2\right]$$

$$K = \left(\frac{1+g_4-g_2}{1+g_4+g_2}\right)^{\frac{1}{2}}$$

$$v = v_0\left(\left(1+g_4\right)^2 - g_2^2\right)^{1/2}$$

Compactified massless boson

$$\phi \equiv \phi + 2\pi$$

Luttinger liquid as a non-Fermi liquid

- Fermionic quasiparticles incoherent (cuts instead of poles in Green's function)
- Critical phase (non-trivial fix point of RG) but correlators follow non-universal power-law behavior

$$G_{\pm}(x,t) \sim \frac{e^{\pm i k_F x}}{\sqrt{x \mp v t}} \left(x^2 - v^2 t^2\right)^{-2\Delta} \qquad \Delta = \frac{(K-1)^2}{16K}$$

3 QCD in 2D

QCD$_{1+1}$: Abelian bosonization

$$\psi_{R,f,\sigma} = \frac{1}{\sqrt{2\pi}} \xi_{f,\sigma} \exp\left(i\sqrt{4\pi}\varphi_{f,\sigma}\right)$$

$$\psi_{L,f,\sigma} = \frac{1}{\sqrt{2\pi}} \xi_{f,\sigma} \exp\left(-i\sqrt{4\pi}\bar{\varphi}_{f,\sigma}\right)$$

$$\sigma \in \{1,\ldots,N_c\}$$

$$f \in \{1,\ldots,N_f\}$$

$\mathcal{H}_1 = ¿$

Chiral bosonic field

Klein factors

QCD Hamiltonian density, $N_f=1$ (Baluni, 1980):

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$$

$$\mathcal{H}_0 = \frac{1}{2}\sum_i \left\{\pi_i^2 + (\partial\phi_i)^2 + 2m\Lambda\left[1-\cos\left(\sqrt{4\pi}\phi_i\right)\right]\right\}$$

$$\mathcal{H}_1 = \frac{g^2}{8\pi N_c}\sum_{i,k}(\phi_i-\phi_k)^2 + \sqrt{\pi}\Lambda^2\sum_{i,k}\left\{1-\int_0^1 d\gamma \cos\sqrt{4\pi}\gamma(\phi_i-\phi_k)\right\}$$

Nonabelian bosonization

$$\int d^2x \sum_{f=1}^{N_f} \sum_{\sigma=1}^{N_c} \bar{q}_{f\sigma} \gamma^\mu \partial_\mu q_{f\sigma} \longleftrightarrow \underbrace{N_c S[g]}_{\substack{SU_{N_c}(N_f) \\ \text{WZNW}}} + \underbrace{N_f S[h]}_{\substack{SU_{N_f}(N_c) \\ \text{WZNW}}} + \underbrace{\int d^2x \frac{1}{2} \partial_\mu \phi \partial^\mu \phi}_{\text{U(1) boson}}$$

WZNW = Wess-Zumino-Novikov-Witten CFT

$$S[g] = \underbrace{\frac{1}{8\pi} \int d^2x\, Tr\left(\partial_\mu g\, \partial^\mu g^{-1}\right)}_{\text{Non-linear } \sigma\text{-model}} + \underbrace{\frac{1}{12\pi} \int_B d^3y\, \epsilon^{ijk}\, Tr\left(g^{-1}\partial_i g\right)\left(g^{-1}\partial_j g\right)\left(g^{-1}\partial_k g\right)}_{\text{Topological WZ term}}$$

1+1D QCD in vacuum

Complicated for $N_f>1$

Bosonization (abelian/**nonabelian**)

Strong coupling, low-energy effective theory derived from first principles:

$$S_{eff}=\int \frac{1}{2}(\partial_\mu \phi)^2+W[SU_{N_c}(N_f):g]+\frac{\tilde{m}}{2\pi}:e^{i\sqrt{\frac{4\pi}{N_c N_f}}\phi}Tr_f g+H.c.:$$

SU(N_f) **Wess-Zumino-Novikov-Witten** model at level N_c

G: SU(N_f) matrix

Φ: U(1) compact boson field

See e.g. Frishman-Sonnenschein, 1993

1+1D analogue of Skyrme model, but derived from QCD Lagrangian

1+1 QCD spectrum

Strong coupling limit equivalent to bosonized NJL model

See also: Azaria et al., Phys. Rev. D **94**, 045003 (2016)

Spectrum:
Mesons → fluctuations of Φ and g
Baryons → solitons

$$J_\mu = \sum_{f=1}^{N_f} \sum_{\sigma=1}^{N_c} \bar{\psi}_{f\sigma} \gamma_\mu \psi_{f\sigma}$$

$$\sim - \epsilon_{\mu\nu} \partial_\nu \phi$$

Baryon number = U(1) (topological) charge

Coupling to baryon number = shift of U(1) boson:

$$H \rightarrow H - \mu \int dx\, J_0(x)$$

$$\left(\frac{4\pi}{N_c N_f}\right)^{\frac{1}{2}} \phi \rightarrow 2 k_F x + \left(\frac{4\pi}{N_c N_f}\right)^{\frac{1}{2}} \phi \qquad k_F = \mu \frac{1}{N_c N_f}$$

Strange metal in 1+1D: Luttinger liquid

Large-k_F effective Lagrangian:

$$L_{eff,\mu} = \frac{\widetilde{K}}{2}\left[v^{-1}(\partial_\tau \phi)^2 + v(\partial_x \phi)^2\right] + W\left(SU_{N_c}(N_f), \widetilde{G}\right)$$

Luttinger parameter

$$\widetilde{K} \equiv \widetilde{K}(N_c, N_f, \mu)$$

Fermi velocity

$$v \equiv v(N_c, N_f, \mu)$$

Coherent excitations are gapless **bosons**

Nf=1: Integrability

$$S_{SG}=\int d^2x\,\frac{1}{2}(\partial_\mu\phi)^2-\frac{\widetilde{m}}{2\pi}\cos\left(\sqrt{\frac{4\pi}{N_c}}\,\phi+2k_F x\right)$$

Higher spin conserved charges: symmetries include momentum-dependent translations

Special to 1+1, circumvents Coleman-Mandula

Scattering:

1. Purely elastic
2. Factorises
3. Elastic $2\rightarrow 2$ phases provide complete description

Rapidity parametrization: θ

$$E=m\cosh\theta \qquad\qquad P=m\sinh\theta$$

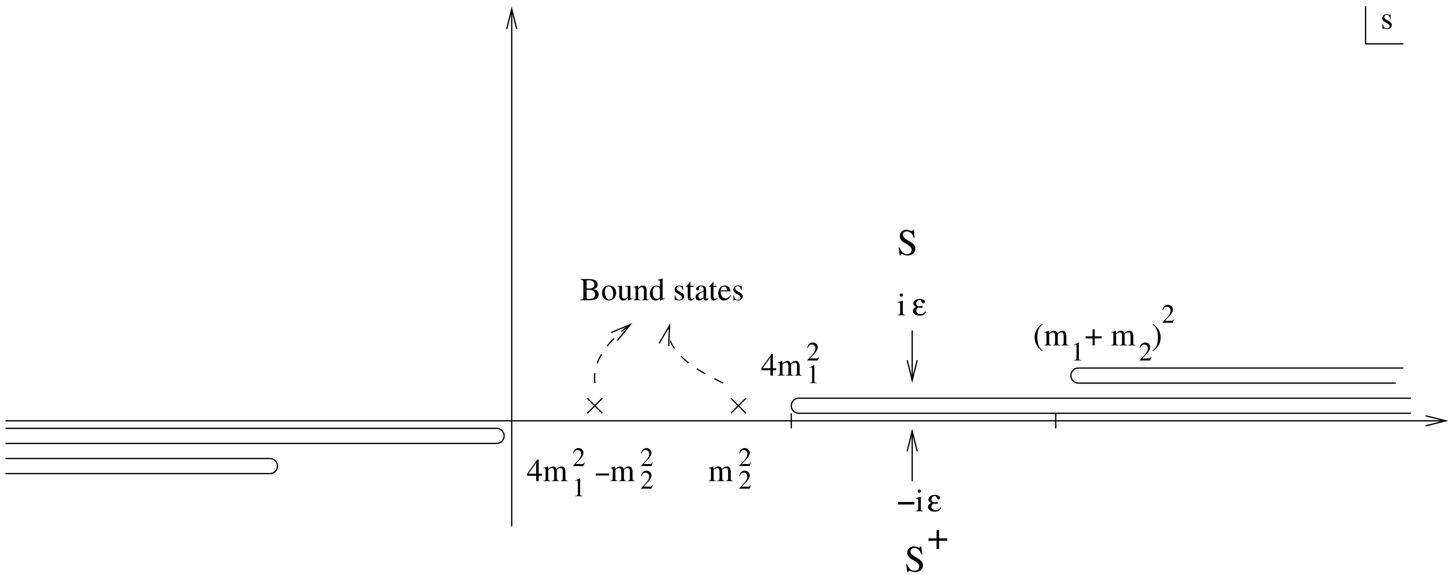
S-matrix elements

$$s=(p1+p2)^2 \qquad t=(p1-p3)^2 \qquad u=(p1-p4)^2$$

$$\langle p_3, p_4|S|p_1, p_2\rangle=(2\pi)^2\,\delta^2(p_1+p_2-p_3-p_4)\,S(s,t,u)$$

$s+t+u=\sum_{i=1}^{4} m_i^2$ Purely elastic: $p_1=p_4$ $S(s,t,u)\equiv S(s)$

Analytic structure of 2→2 elastic S-matrix from optical theorem, general non-integrable case



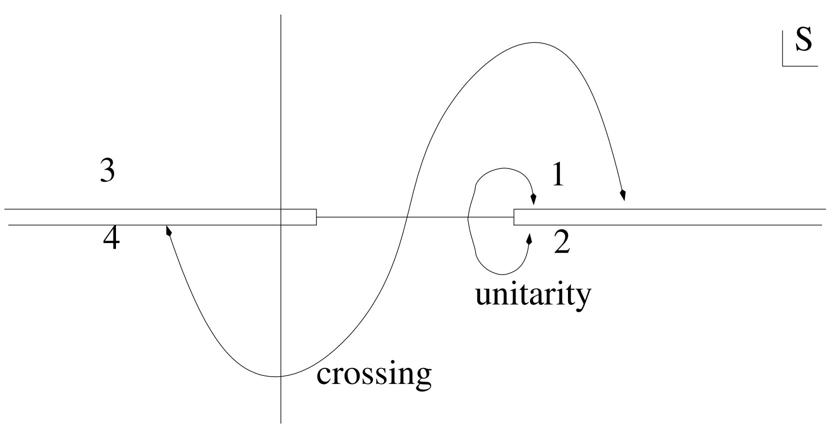
S-matrix structure

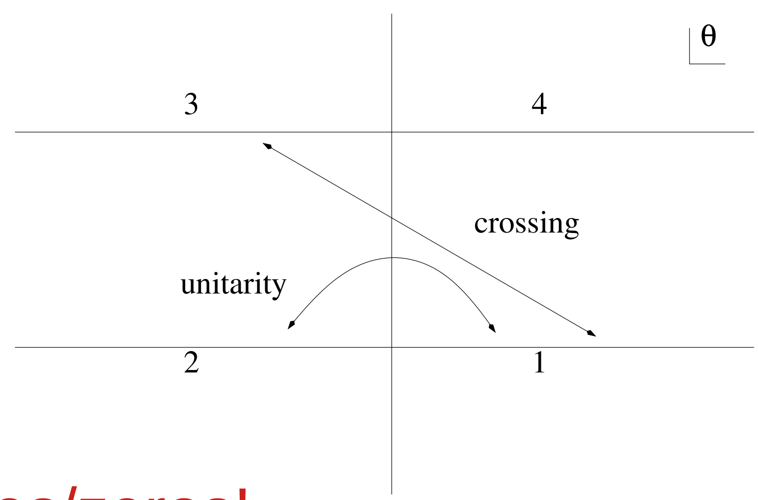
Integrability: no particle creation/decay on-shell

S-matrix analytic structure: **only 1 pair of cuts** on s-plane $s \geq (m_1 + m_2)^2$ $s \leq (m_1 - m_2)^2$

S: real analytic function $S_{ij}^{kl}(s^*) = [S_{ij}^{kl}(s)]^*$

Transform cuts away: rapidity parametrization $s(\theta) = m_1^2 + m_2^2 + 2 m_1 m_2 \cosh\theta$

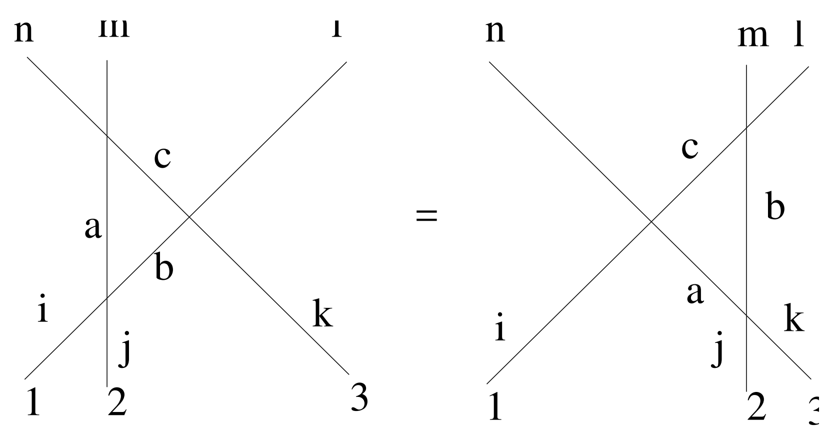


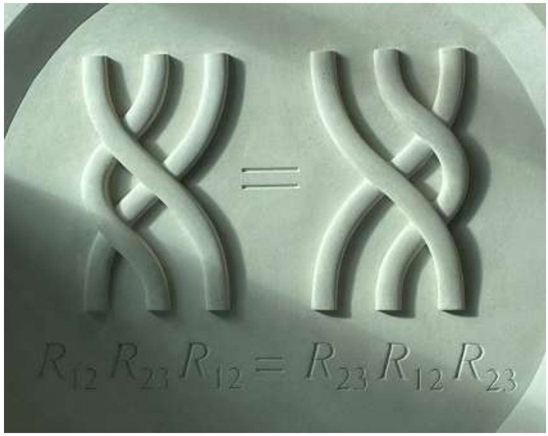


$S(\theta)$: No cuts, only poles/zeros!

Function equations for S(θ)

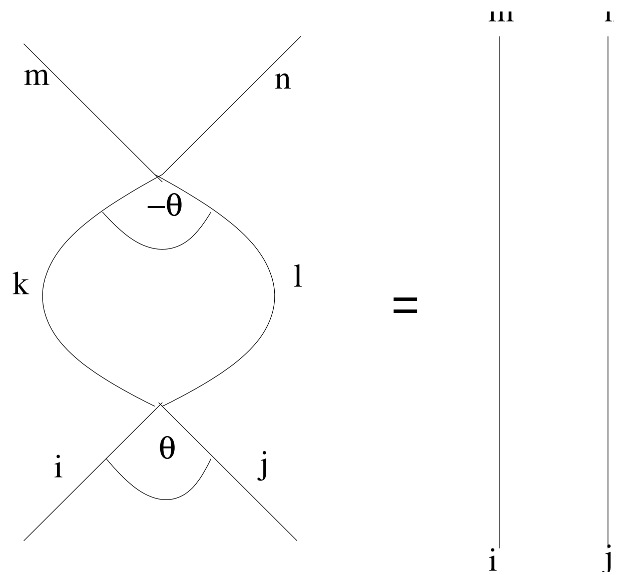
Yang-Baxter equations (hallmark of integrability)





$$S_{ij}^{ab}(\theta_{12})\,S_{bk}^{cl}(\theta_{13})\,S_{ac}^{nm}(\theta_{23})=S_{jk}^{ab}(\theta_{23})\,S_{ia}^{nc}(\theta_{13})\,S_{cb}^{ml}(\theta_{12})$$

Photo (C) Gabriel Cuomo, Simons Center



Unitarity

$$S_{ij}^{kl}(\theta)\,S_{kl}^{mn}(-\theta)=\delta_i^m\,\delta_j^n$$

Crossing

$$S_{ij}^{kl}(\theta)=S_{i\bar{l}}^{k\bar{j}}(i\pi-\theta)$$

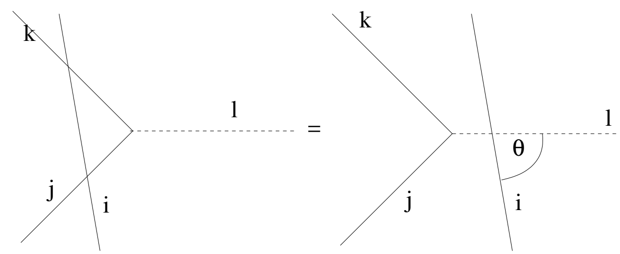
Exact S-matrices

Name the parameters: rapidity, **parameter nu, relate to usual sine-Gordon beta**

Exact S-matrices from maximal analycity, unitarity, crossing, YBE

$$S_{ss} = -\exp\left[\int_0^\infty \frac{dt}{t}\,\frac{\sinh(\nu-2)t}{\sinh(t)\cosh((\nu-1)t)}\sinh\left(\frac{2t\,\theta(\nu-1)}{i\pi}\right)\right]$$

S-matrix bootstrap:



$$S_{sn}(\theta) = \frac{\sinh\theta + i\cos\frac{n\pi}{2(\nu-1)}}{\sinh\theta - i\cos\frac{n\pi}{2(\nu-1)}}\prod_{k=1}^{n-1}\frac{\sin^2\left(\frac{n-2k}{4(\nu-1)}\pi - \frac{\pi}{4} + i\frac{\theta}{2}\right)}{\sin^2\left(\frac{n-2k}{4(\nu-1)}\pi - \frac{\pi}{4} - i\frac{\theta}{2}\right)}$$

A.B. Zamolodchikov, Commun. math. Phys. **55**, 183—186 (1977)
Karowski et al., Phys. Lett. **67B**, 321 (1977)

Exact S-matrices

$$E=\sum_j m_{r_j}\cosh\theta_j\,, \qquad P=\sum_j m_{r_j}\sinh\theta_j\,,$$

Sine-Gordon: generally non-diagonal due to $s\bar{s}\rightarrow\bar{s}s$ process

$N_c\in\mathbb{Z}:$ Diagonal points

$$S_{ss}(\theta)=S_{s\bar{s}}(\theta)=S_{\bar{s}s}(\theta)=S_{\bar{s}\bar{s}}(\theta)$$

$$S_{sn}(\theta)=S_{ns}(\theta)=S_{\bar{s}n}(\theta)=S_{n\bar{s}}(\theta)$$

Nf=1 spectrum

μ=0 spectrum from bootstrap

Soliton, antisoliton: mass m_s fermions (baryon)

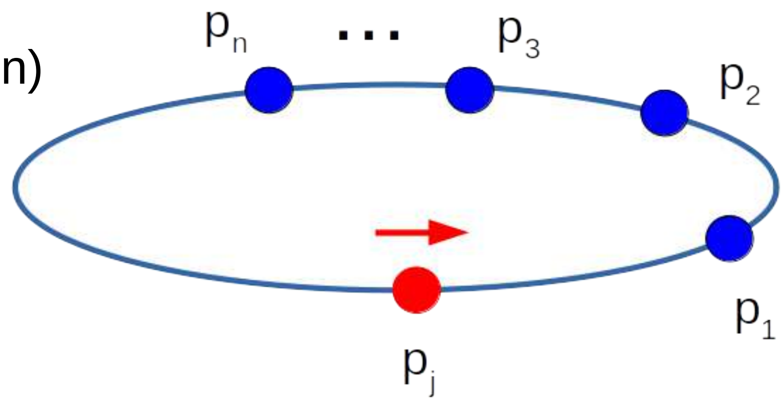
Breathers: soliton-antisoliton bosonic excited states (mesons)

$$m_n = 2 m_s \sin\left(\frac{\pi n}{2} \frac{1}{2N_c - 1}\right), \quad n = 1, \dots, N_c + 2 = \nu - 1$$

Asymptotic Bethe Ansatz

First quantized picture: N_{tot} particles on a circle of size L

Pointlike interactions (~p-dependent delta-interaction)



Quantization condition for momentum p_j:

$$e^{i p_j L} \prod_{k \neq j} S_{r_j r_k}(\theta_j - \theta_k) = 1$$

Take the log → Bethe-Yang equations

$$m_{r_j} L \sinh \theta_j + \sum_{k \neq j} \delta_{r_j r_k}(\theta_j - \theta_k) = 2\pi n_j \qquad n_j \in \mathbb{Z}$$

Phase shift:

$$\delta_{r_j r_k}(\theta) = -i \log S_{r_j r_k}(\theta)$$

Thermodynamic Bethe Ansatz

Thermodynamic limit: $N_{tot} \to \infty$ particles,
Continuum formulation of BY equations

$$\rho_s(\theta) = L^{-1} \frac{dN}{d\theta} \qquad n_j = L \int_0^{\theta_j} \rho_s(\theta') d\theta'$$

$$m_s L \sinh\theta + L \int_{-B}^{B} d\theta' \rho_s(\theta') \delta_{ss}(\theta - \theta') = 2\pi L \int_0^{\theta} \rho_s(\theta') d\theta'$$

Introduce the integral kernel (symmetric linear integral operator):

$$\mathscr{K}_{ss}(\theta) = \frac{-1}{2\pi} \partial_\theta \delta_{ss}(\theta)$$

The continuum BY can also be written as

$$\frac{m_s}{2\pi} \cosh\theta = \rho_s(\theta) + \int_{-B}^{B} d\theta' \rho_s(\theta') \mathscr{K}_{ss}(\theta - \theta')$$

GS energy

Finite-B ground state energy

Soliton density

One-soliton energy

$$E_0 = \sum_j \left(m_s \cosh \theta_j - \mu \right) = L \int_{-B}^{B} \rho_s(\theta) \left(m_s \cosh \theta - \mu \right) d\theta$$

$\left(1 - \mathscr{K}_{ss}\right)^{-1}$ symmetric op: by introducing the dispersion

$$\epsilon_s(\theta) = \left(1 - \mathscr{K}_{ss}\right)^{-1} \left[m_s \cosh \theta - \mu \right]$$

GS energy can be written equivalently as

$$E_0 = L m_s \int_{-B}^{B} \frac{d\theta}{2\pi} \cosh \theta \, \epsilon_s(\theta)$$

Dressed charge, excitations...

Same calculation for the U(1) charge...

U(1) charge in the ground state

Dressed charge

$$Q_0 = L\int_{-B}^{B}\rho_s(\theta)\,d\theta = L\,m_s\int_{-B}^{B}\frac{d\theta}{2\pi}\cosh\theta\,\zeta(\theta) \qquad \zeta(\theta) = (1-\mathcal{K}_{ss})^{-1}(1)$$

How to get one-particle excitation energies?

BYE for **1-particle (of type r) excited state** above soliton sea

$$2\pi n_j = m_s L\sinh\theta_j + \delta_{rs}(\theta_j-\theta_0) + \sum_{k\neq j}\delta_{ss}(\theta_j-\theta_k) \qquad n_j\in\mathbb{Z}$$

Continuum formulation of BYE

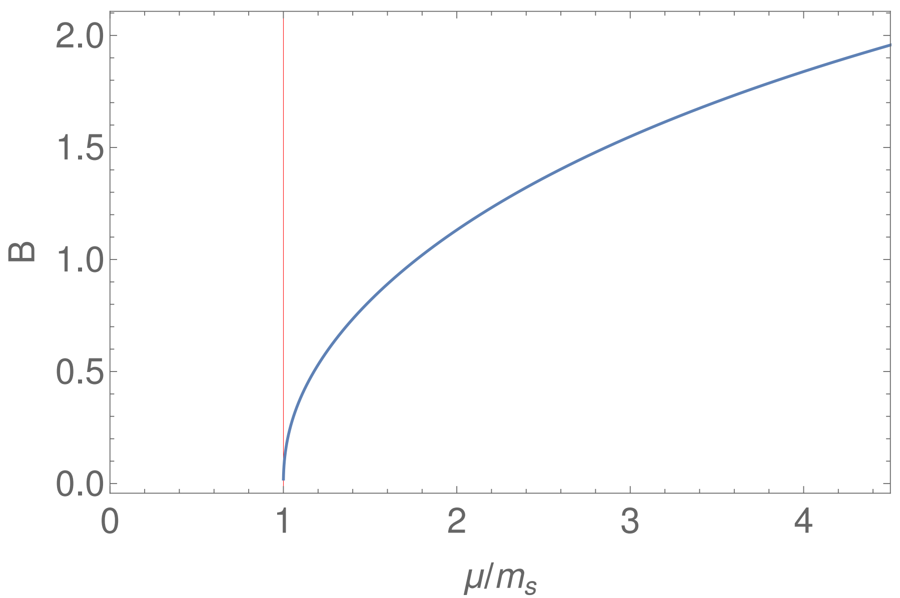
$$\frac{m_s}{2\pi}\cosh\theta = \tilde{\rho}(\theta) - \frac{1}{2\pi L}\partial_\theta\delta_{rs}(\theta-\theta_0) + \int_{-B}^{B}d\theta'\,\tilde{\rho}(\theta')\,\mathcal{K}_{ss}(\theta-\theta')$$

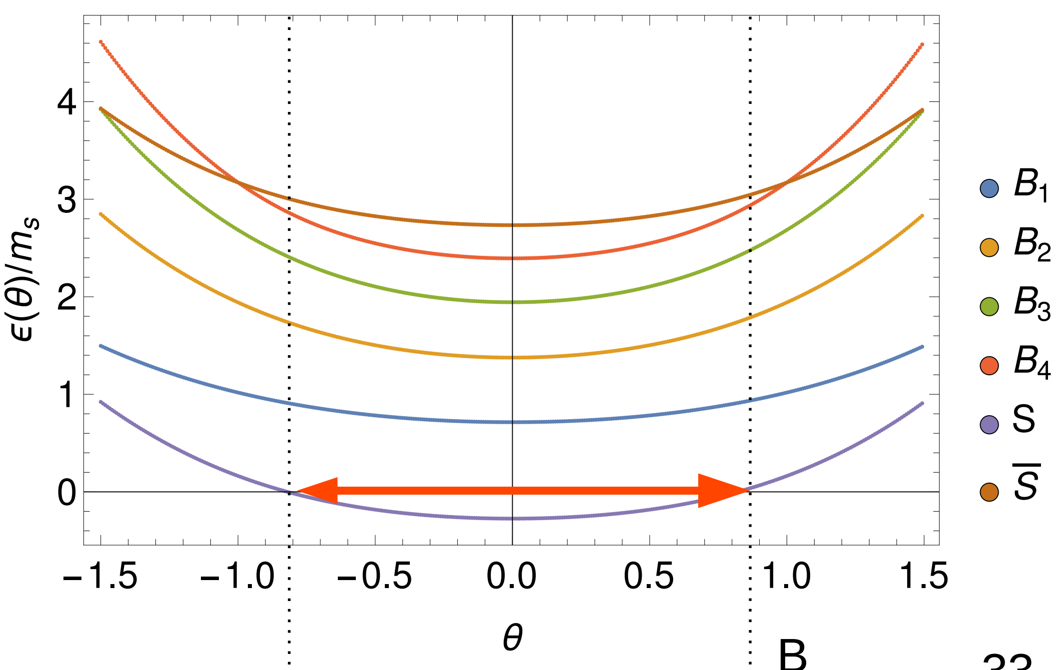
Thermodynamic Bethe Ansatz

$$\epsilon_s(\theta,\mu)+\int_{-B}^{B}\mathcal{K}_{ss}(\theta-\theta')\,\epsilon_s(\theta',\mu)\,d\theta'=m_s\cosh\theta-\mu$$

$$\epsilon_{\bar{s}}(\theta,\mu)+\int_{-B}^{B}\mathcal{K}_{\bar{s}s}(\theta-\theta')\,\epsilon_s(\theta',\mu)\,d\theta'=m_s\cosh\theta+\mu$$

$$\epsilon_n(\theta,\mu)+\int_{-B}^{B}\mathcal{K}_{ns}(\theta-\theta')\,\epsilon_s(\theta',\mu)\,d\theta'=m_n\cosh\theta$$

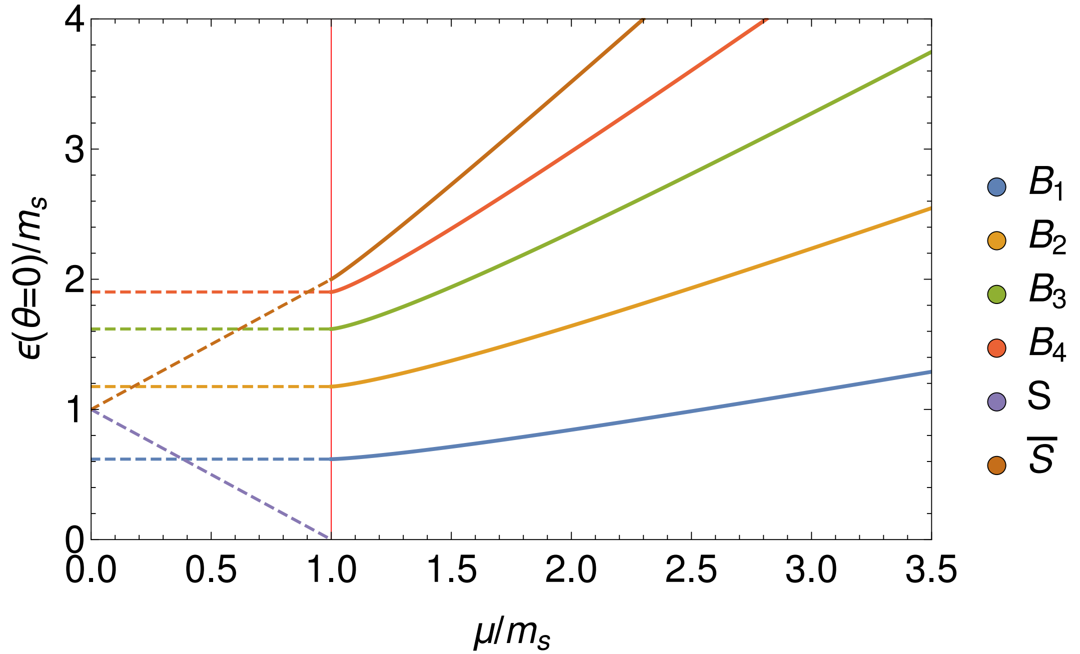




Spectrum

$\mu > m_s$

$\epsilon(\theta) < 0$ excitations part of the „Fermi sea”



Luttinger parameters

$$S_{eff,\mu} = \frac{\widetilde{K}}{2}\left[v^{-1}(\partial_\tau \phi)^2 + v(\partial_x \phi)^2\right] + W\left(SU_{N_c}(N_f), \widetilde{G}\right)$$

Dressed charge

Excess charge due to extra soliton:

$$Q_1 - Q_0 = 1 + \int_{-B}^{B} D_{\theta_0}(\theta)\, d\theta \;\; = 1 - \int_{-B}^{B} \mathscr{K}_{ss}(\theta - \theta_0)\, \zeta(\theta) \equiv \zeta(\theta_0)$$

Luttinger parameter from dressed charge:

$$\widetilde{K}(\mu) = \zeta^2(B)$$

Group velocity of excitations at the edge of the Fermi sea

$$v_F = \left.\frac{\partial \epsilon_s(\theta, \mu)}{\partial \theta}\right|_{\theta = B} \frac{1}{2\pi \rho_s(B)}$$

Nf>1: mass expansion

Dynamic U(1) susceptibility

$$\chi(q,i\omega)=\int_{-\infty}^{\infty} dx\,d\tau\, e^{iqx+i\omega\tau} T_\tau\left[\langle 0|J_0(x,\tau)J_0(0,0)|0\rangle\right]$$

For q << $2k_F$:

$$\chi(q,i\omega)=\frac{v K_c}{\pi}\frac{q^2}{\omega^2+v^2q^2}$$

$$\chi(q,i\omega)=\chi_0(q,i\omega)+\left(\frac{\tilde{m}}{4\pi}\right)^2\chi_1(q,i\omega)+\dots$$

Calculated explicitly (conformal perturbation theory)

PT vs. TBA (N_c=3, N_f=1)

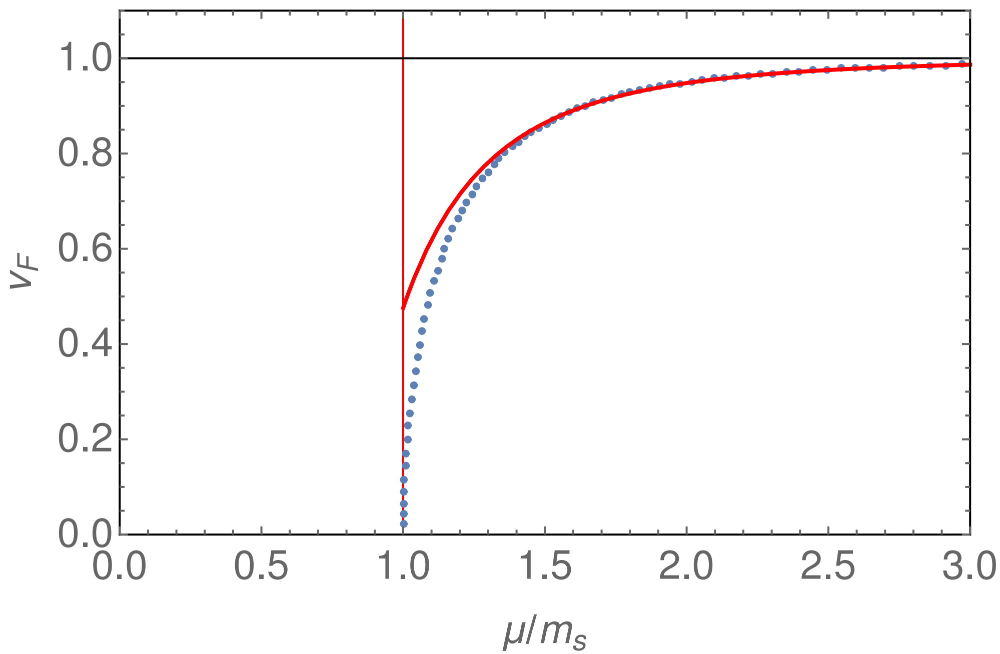
$$v = 1 + \left(\frac{\tilde{m}}{4\pi}\right)^2 v_1 + \ldots$$

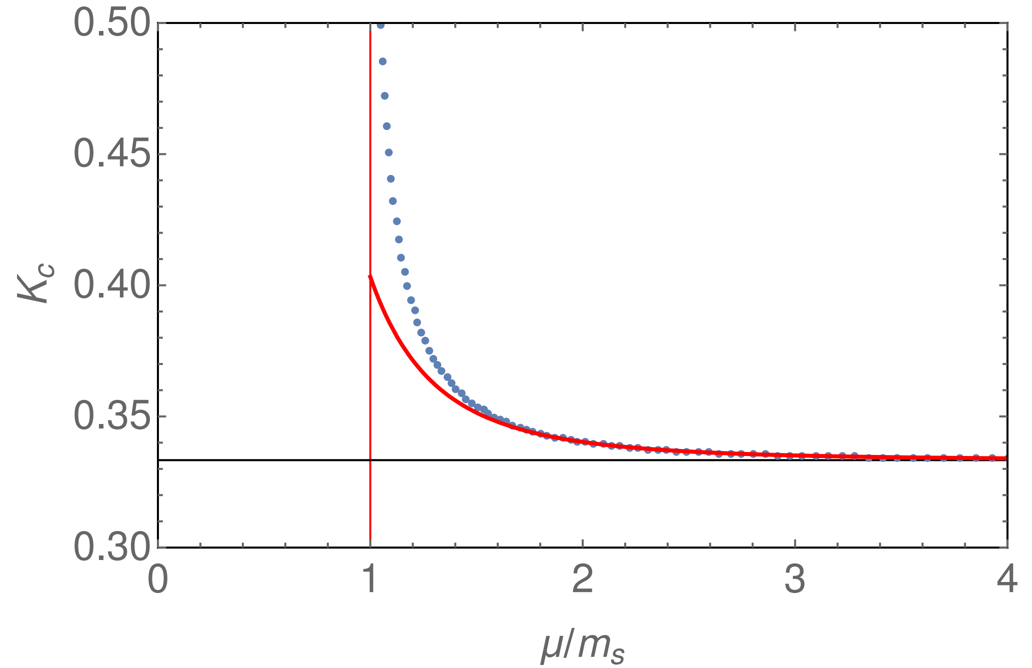
$$\alpha = \frac{1 + N_c N_f}{N_c^2 + N_c N_f}$$

$$K_c = K_0 \left(1 + \left(\frac{\tilde{m}}{4\pi}\right)^2 K_1 + \ldots\right)$$

$$v_1 = -k_F^{-4+2\alpha} \frac{\pi(-1+\alpha)(-2+\alpha)}{4}$$

$$K_1 = k_F^{-4+2\alpha} \frac{\pi(-1+\alpha)^2}{4}$$





Baryon correlators in 1+1D

(right-moving) nucleon field

$$n_R^{\alpha\beta\gamma} = \epsilon^{abc} R_{a\alpha} R_{b\beta} L_{c\gamma} \sim \exp\left[i\sqrt{\frac{2\pi}{3}}(2\phi - \bar{\phi}) \right] \mathcal{F}_{2/5}^{(1)} \bar{\mathcal{F}}_{3/20}^{(1/2)}$$

(right-moving) delta field

$$\Delta_R^{\alpha\beta\gamma} = \epsilon^{abc} R_{a\alpha} R_{b\beta} R_{c\gamma} \sim \exp\left[3i\sqrt{\frac{2\pi}{3}}\phi \right] \mathcal{F}_{3/4}^{(3/2)}$$

$$\langle n_R(\tau, x) n_R^\dagger(0,0) \rangle = Z_n e^{ik_F x} \left[\frac{(\tau v_F + ix)(\tau v_{fl} + ix)}{(\tau v_F - ix)(\tau v_{fl} - ix)} \right]^{\frac{1}{4}} \left(\frac{\tau_0^2}{\tau^2 + x^2/v_F^2} \right)^{\frac{3}{8}\left(\tilde{K} + \frac{1}{9\tilde{K}}\right)} \left(\frac{\tau_0^2}{\tau^2 + x^2/v_{fl}^2} \right)^{\frac{11}{20}}$$

$$\langle n_R(\tau, x) n_R^\dagger(0,0) \rangle = Z_n e^{3ik_F x} \left[\frac{(\tau v_F + ix)(\tau v_{fl} + ix)}{(\tau v_F - ix)(\tau v_{fl} - ix)} \right]^{\frac{3}{4}} \left(\frac{\tau_0^2}{\tau^2 + x^2/v_F^2} \right)^{\frac{3}{8}\left(\tilde{K} + \frac{1}{\tilde{K}}\right)} \left(\frac{\tau_0^2}{\tau^2 + x^2/v_{fl}^2} \right)^{\frac{3}{4}}$$

Correlation functions in 1+1D

- Quarks confined, color sector gapped, power-law correlations in baryon correlators
- Correlators depend on Luttinger parameters + velocities
- Baryons „incoherent”: not usual poles in Green’s function; spectral function is not a delta function on Fermi surface: lifetime ~ 1/E
- Bosonic excitations are coherent

4 Some insights on neutron star cooling

Recap: regimes in QCD_{3+1}

$$m_0 = \frac{m_{Nucleon}}{N_c}$$

$$m_0 \rightarrow \mu_N = m_0 \left(1 + \frac{\#}{N_c}\right)$$

Nuclear matter
Fermi sea of nucleons
Nucleon superfluidity/superconductivity, gapped Fermi Surface

$$\mu_N \rightarrow \mu_{\pi q}$$

Quarkyonic (I.)
π/K condensation (meson chiral spirals)
gapped Fermi surface, no goldstones due to anisotropy

$$\mu_{\pi q} \rightarrow \mu_{pert}$$

Quarkyonic (II.)
quarkyonic condensates/quantum pi liquid. NON-Fermi liquid, no baryons about the Fermi sea, but only bosons.

$$\mu_{pert} \rightarrow \infty$$

Perturbative regime
color superconductivity, gapped Fermi surface.

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quarkyonic condensates/quantum pi liquid. NON-Fermi liquid, no baryons about the Fermi sea, but only bosons.

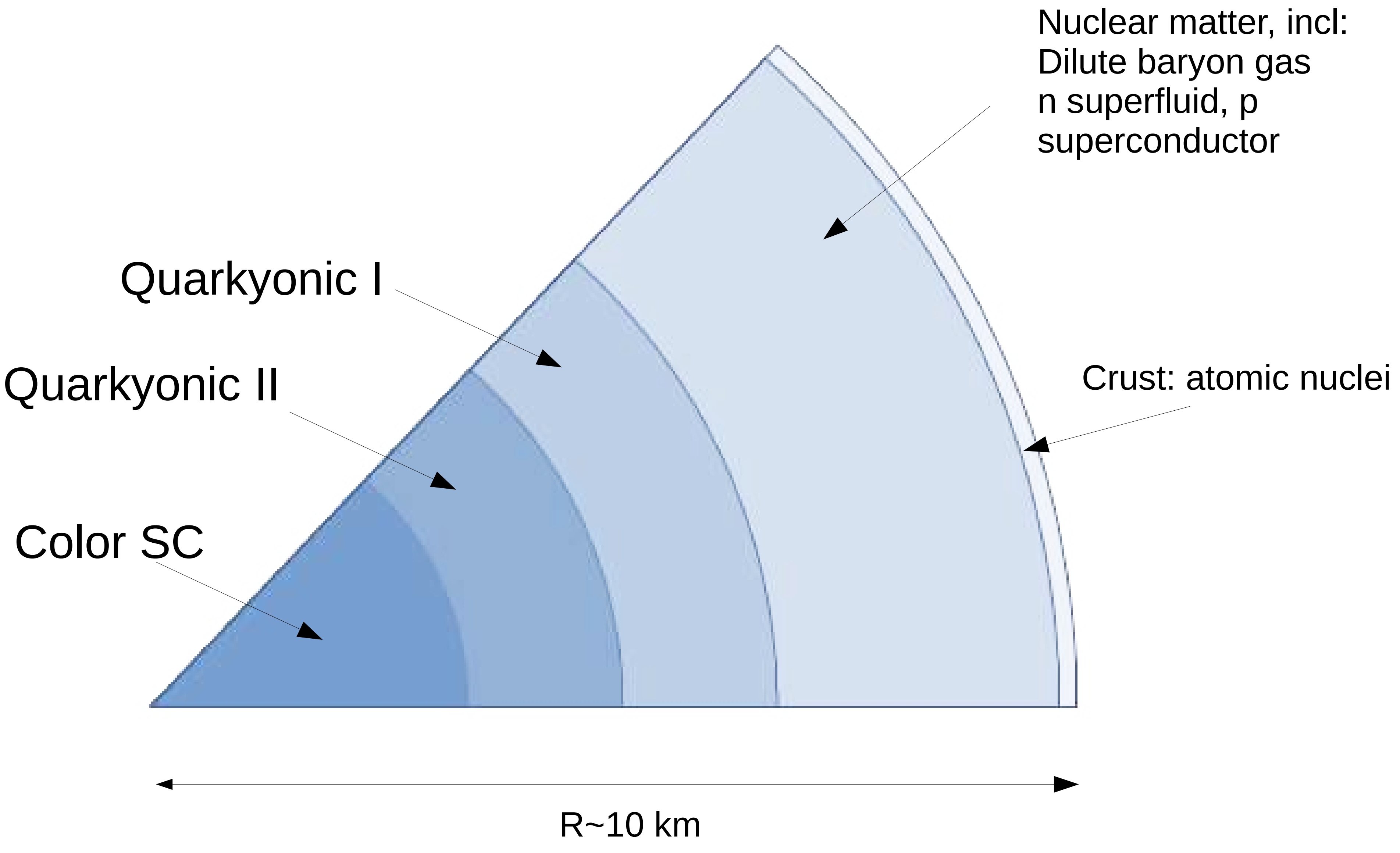
$$\mu_{pert} \rightarrow \infty$$

Perturbative regime

color superconductivity, gapped Fermi surface.

$\mu_{pert} \approx 1$ GeV
Gorda et al., [2103.05658], [2103.07427]
Bornyakov et al, [1808.06466]

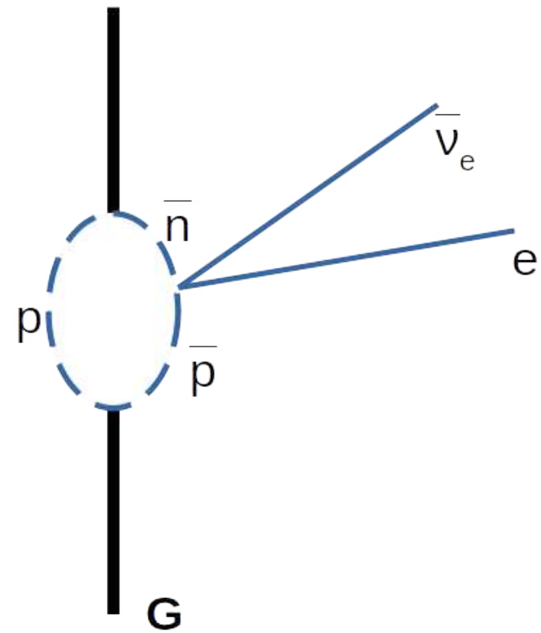
Inside neutron stars



Neutrino emission (in Quarkyonic II phase)

$$\mathcal{L}_W \sim i\, g_W \sum_{\boldsymbol{Q}} \underbrace{\left[\bar{e}\left(\boldsymbol{Q}\cdot\gamma\right)\left(1-\gamma_5\right)\nu_e + \bar{\mu}\left(\boldsymbol{Q}\cdot\gamma\right)\left(1-\gamma_5\right)\nu_\mu\right]}_{\text{Lepton current}} \overbrace{Tr\left[G_Q^\dagger\, \boldsymbol{Q}\, \dot{\nabla}\, G_Q\, \hat{\tau}^\dagger\right]}^{\text{Flavored WZNW current}} + H.c.$$

Patch vectors



Neutron star cooling

by neutrino radiation

See also: A. Schmitt and P. Shternin, Astrophys. Space Sci. Libr. 457, 455 (2018)

ν emissivity: $\varepsilon_\nu(T) = A\,\mu_e\,C(T)\,T^6 g_s^2$

e[-] chemical pot.

Running coupling

$$\frac{\partial F}{\partial t} = -\varepsilon_\nu(T)$$

$$\frac{\partial T}{\partial t} = -A\,\mu_e\left(\frac{\partial \ln S}{\partial T}\right)T^6 g_s^2(T)$$

C~T (Luttinger liquid)

$C \sim T^2$ (anisotropic mesons)

$$T(t) = \frac{T_{<,>}}{(1+t/\tau)^{1/4}}$$

So what is it good for?

- Neutron stars may reach quarkyonic densities
- Qualitatively new transport properties
- Non-Fermi liquid
- Affects cooling by ν emission
- Lots of work left to do

Thank you

More details: arXiv:2112.10238v1 [hep-th]
M.L., Robert Konik, Rob Pisarski and Alexei Tsvelik

Soliton density difference compared to the ground state

$$D_{\theta_0}(\theta)=L\left[\widetilde{\rho}(\theta)-\rho_s(\theta)\right]$$

Operator equation for D:

$$D_{\theta_0}(\theta)=\left(1-\mathscr{K}_{ss}\right)^{-1}\left[-\mathscr{K}_{rs}(\theta-\theta_0)\right]$$

$$\mathscr{K}_{rs}(\theta-\theta_0)=\frac{-1}{2\pi}\partial_\theta\delta_{rs}(\theta-\theta_0) \qquad \mathscr{K}_{ns}(\omega)=\frac{\coth\left[\frac{\pi\omega}{2(\nu-1)}\right]\sinh\left[\frac{\pi n\omega}{2(\nu-1)}\right]}{\cosh\left(\frac{\pi\omega}{2}\right)}$$

Thermodynamic Bethe Ansatz

Operator expression for soliton density

$$\rho_s(\theta) = \left(1 - \mathscr{K}_{ss}\right)^{-1}\left[\frac{m_s}{2\pi}\cosh\theta\right]$$

Fourier transforms of integral kernels given explicitly:

$$\mathscr{K}_{ss}(\omega) = \frac{\sinh\left[\frac{\pi\omega}{2}\left(1 - \frac{1}{\nu - 1}\right)\right]}{2\sinh\left[\frac{\pi\omega}{2(\nu - 1)}\right]\cosh\left(\frac{\pi\omega}{2}\right)}$$

Energy change due to the presence of the extra particle of rapidity θ_0

$$E_1 - E_0 = \epsilon_r(\theta_0)$$

$$\epsilon_r(\theta_0) = m_r \cosh\theta_0 - \mu Q_r + \int_{-B}^{B} D_{\theta_0}(\theta)(m_s \cosh\theta - \mu)\, d\theta$$

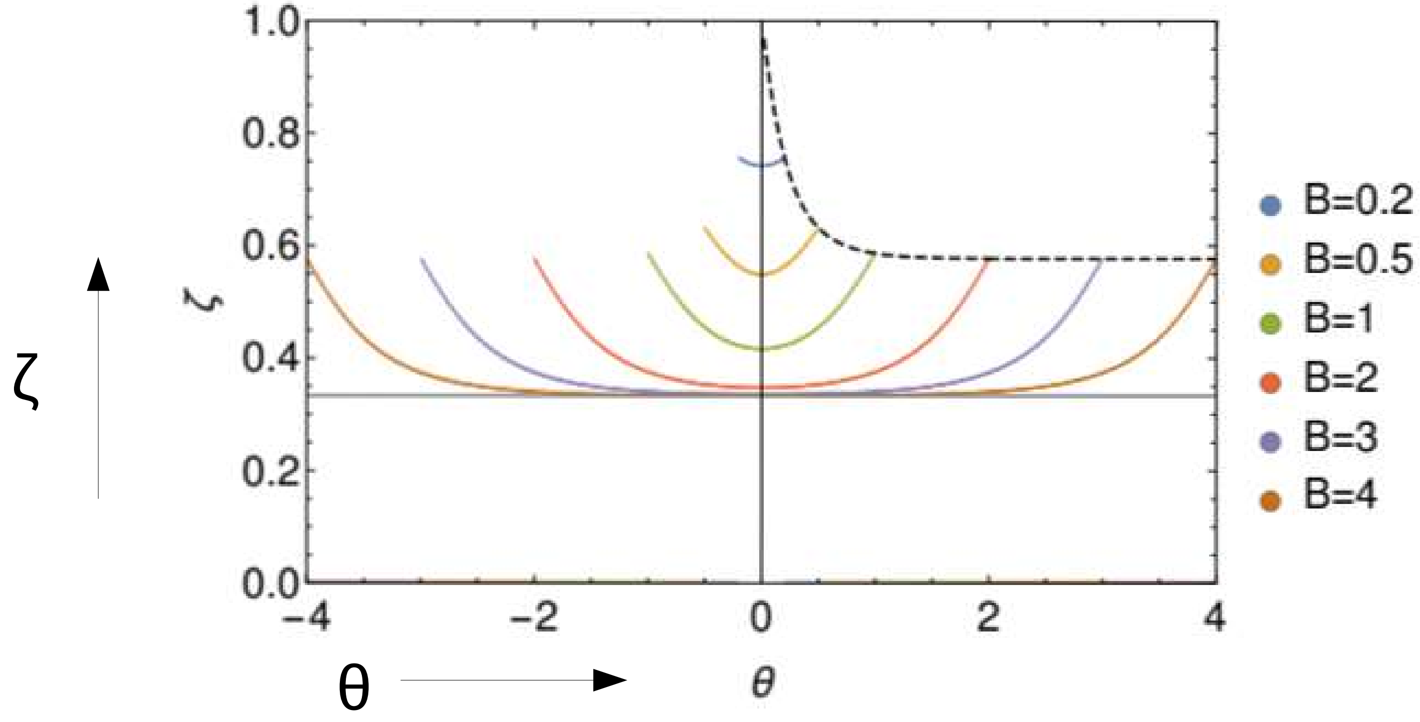
Exploiting the form of D:

$$\epsilon_r(\theta_0) = m_r \cosh\theta_0 - \mu Q_r + \int_{-B}^{B} \mathscr{K}_{rs}(\theta - \theta_0)\, \epsilon_s(\theta)\, d\theta$$

$$Q_s = +1 \qquad Q_{\bar{s}} = -1 \qquad Q_n = 0$$

N_c=3

$$\zeta(\theta)+\int_{-B}^{B}\mathscr{K}_{ss}(\theta-\theta')\,\zeta(\theta')\,d\theta'=1$$



$$\mu\rightarrow\infty:\qquad \zeta(B)=\left[1+\mathscr{K}_{ss}(\omega=0)\right]^{-1/2}=N_c^{-1/2}$$

$$B\rightarrow 0:\qquad \zeta(\mu)=1-2^{3/2}\,\mathscr{K}_{ss}(\theta=0)\,\delta_\mu^{1/2}+O(\delta_\mu)\qquad\qquad \delta_\mu=\frac{\mu}{m_s}-1$$

$$\widetilde{K}(\mu)\rightarrow 1$$