The Golden Age of Chirality and Quantum Mechanics

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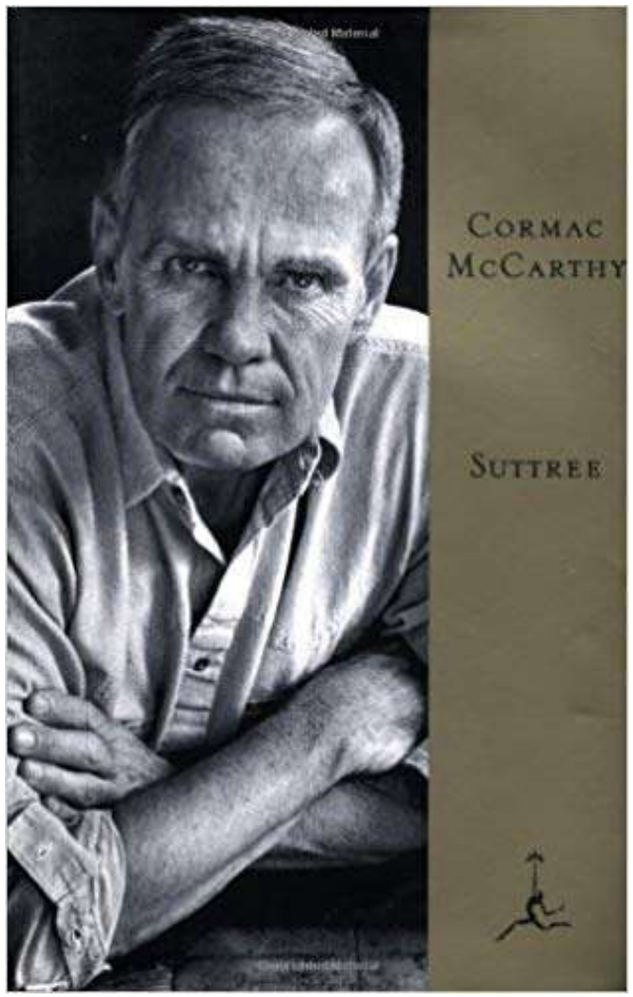
Theoretical Physics Colloquium, Arizona State University, April 8th, 2020

Outline

- Introduction
- Anomalies
- Theory of anomalous transport
- Realizations
  - QGP
  - Weyl semi-metals
  - Optics
- Summary and Outlook

Chiral fluid workshop in Santa Fe, NM, (2018):



"On the right temple a mauve halfmoon. Suttree turned and lay staring at the ceiling, touching a like mark on his own left temple gently with his fingertips. The ordinary of the second son. Mirror image. Gauche carbon."

"Gray vines coiled leftward in the northern hemisphere, what winds them shapes the dogwhelk's shell."

"A dextrocardiac, said the smiling doctor. Your heart's in the right place."

Bryan Giemza: "Mirror Image, Asymmetry, Chirality and Suttree",
Special Issue of the European Journal of American Studies:
Cormac McCarthy Between Worlds

"For now, suffice it to say that we may be in something of a golden age of chirality, from Breaking Bad to Nobel Prize-winning areas of scientific enquiry."

林良 (Lin Liang) 1424-1500

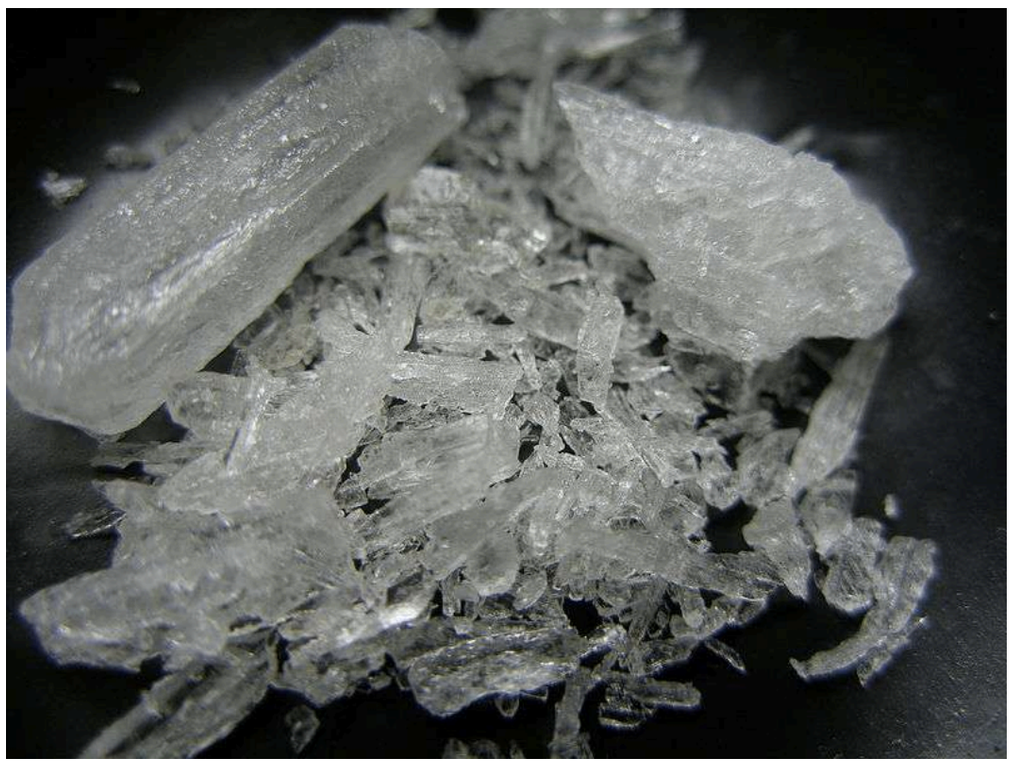
Imperial painter during Ming Dynasty

"~~Two~~ Chiral Eagles"

Breaking Bad to Nobel Prize

CC license, Wikipedia

Levomethamphetamin



Dextromethamphetamin
"crystal meth"

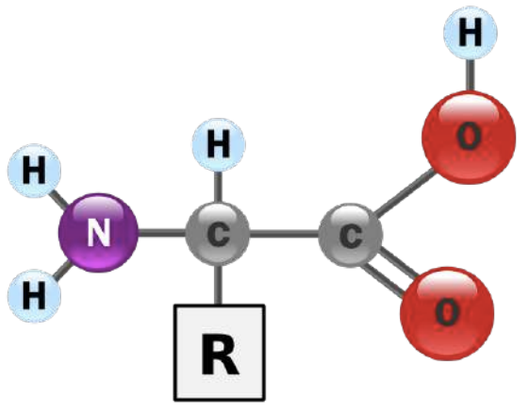
Nobel Prize in Chemistry 2016: Bernard L. Feringa

"for the design and synthesis of (chiral) molecular machines." [Nobel committee]

…"chiral electromagnetic radiation to generate enantioselectivity"… [from Wikipedia]

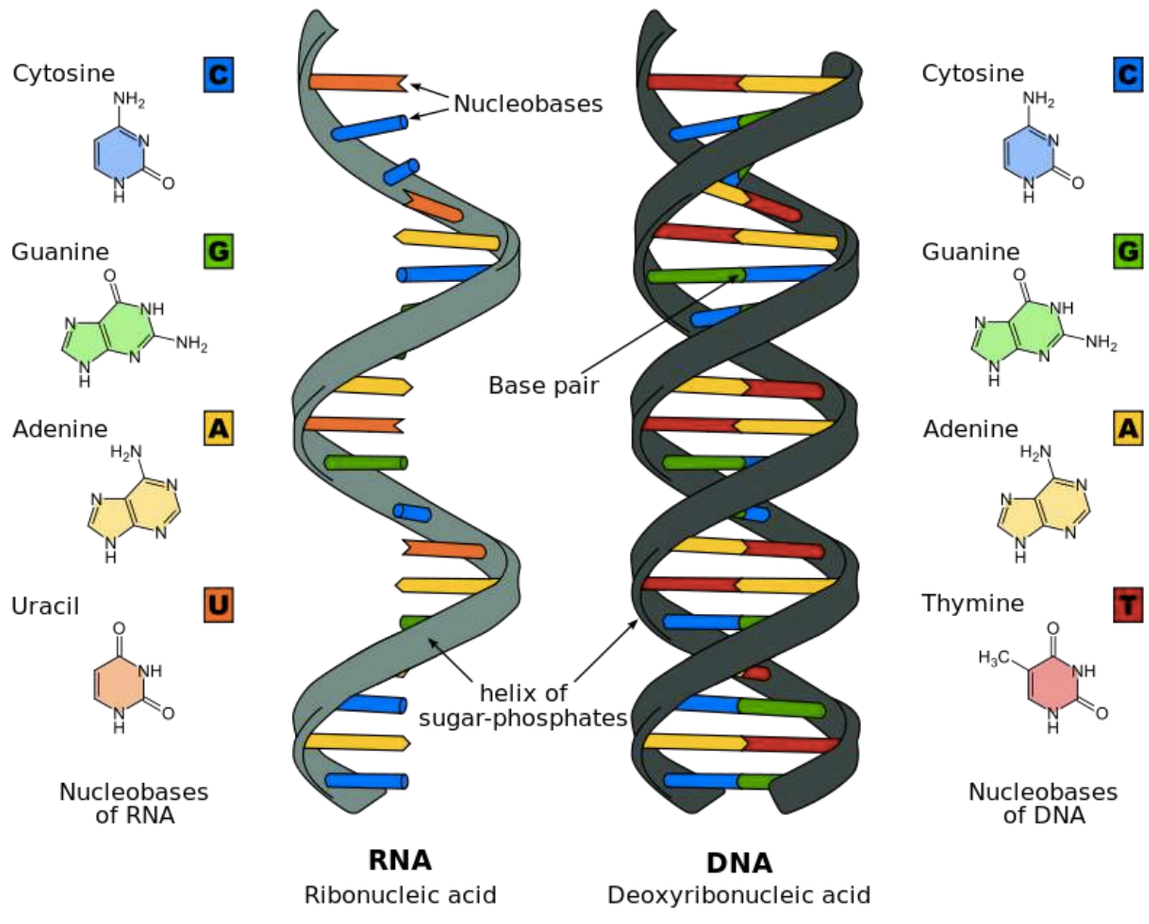
→ "Zilch"

Homochirality of life



Amino acid, all L-isomers

Nucleic acid,
all R-isomers

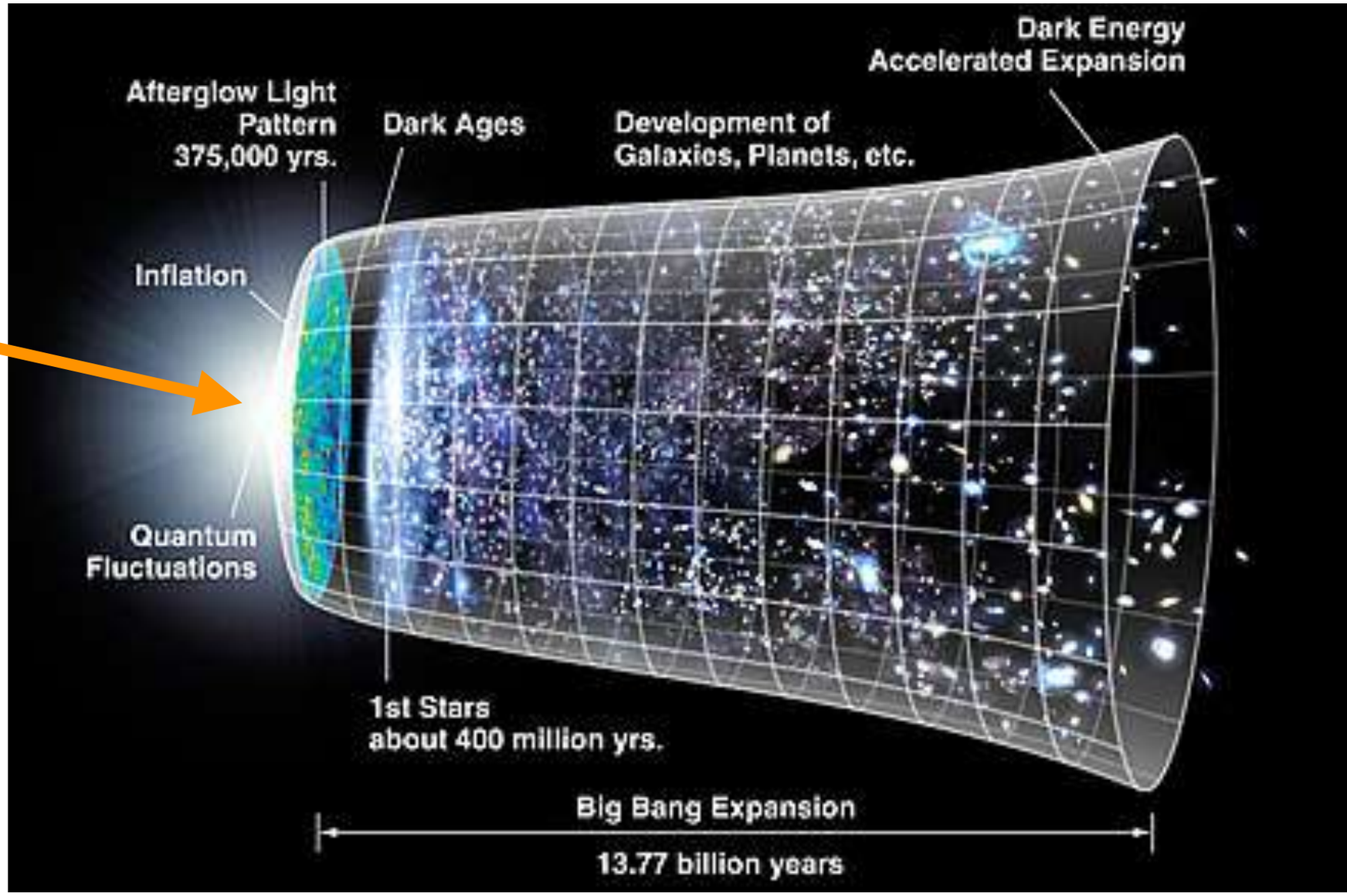


Physics can do better than that:

Golden age
of chirality starts
somewhere here

GUTs,
Standard Model:
Chiral gauge theories

(Type IIB,
Heterotic strings…)

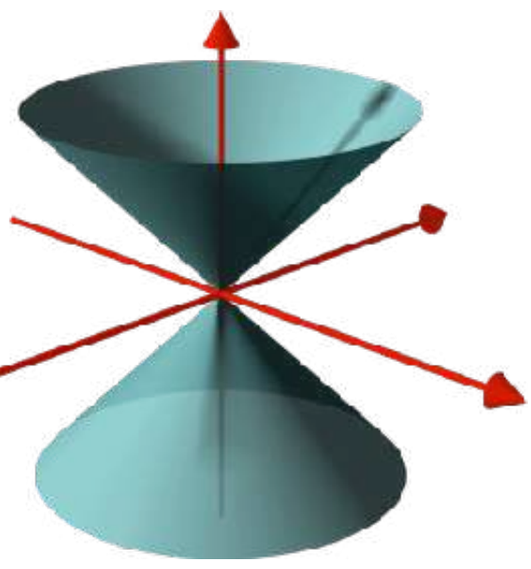


This talk: chiral *states* (not theories),
anomalies and *applications*

Anomalies

Anomaly: Symmetry is not compatible with quantum theory!

Weyl fermions $H = \pm \vec{\sigma} \vec{p}$



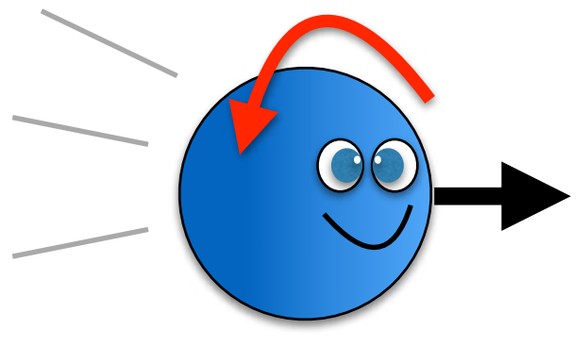
γ

π^0

γ

$$\rho_5 = \Psi^\dagger \gamma_5 \Psi$$

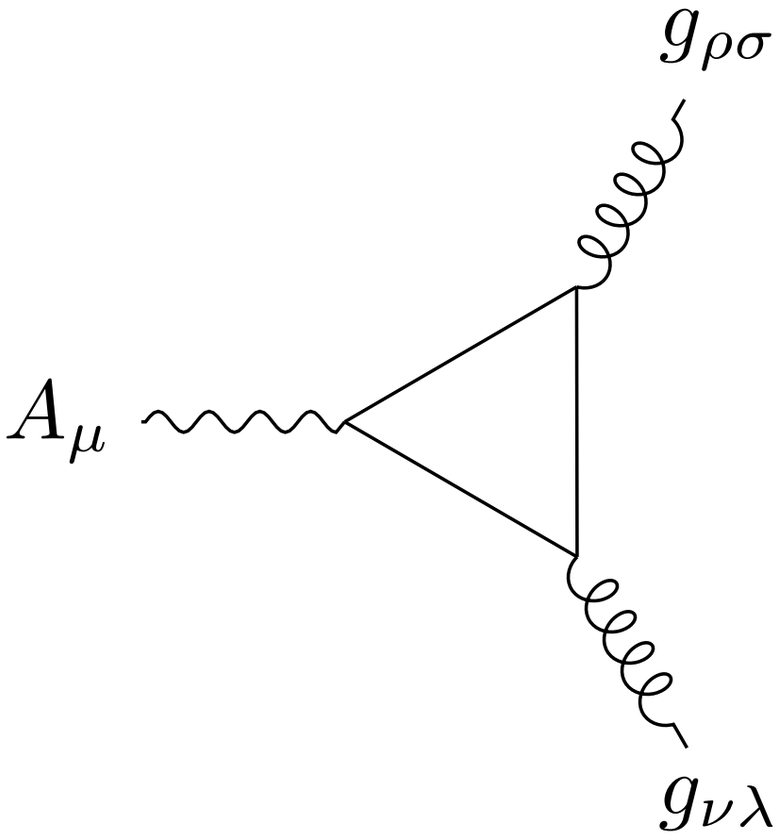
$$\frac{d}{dt}\rho_5 = \frac{1}{2\pi^2}\vec{E}\cdot\vec{B}$$



[Adler], [Bell, Jackiw] 1969

Anomalies

Gravitational contribution to Chiral Anomaly:



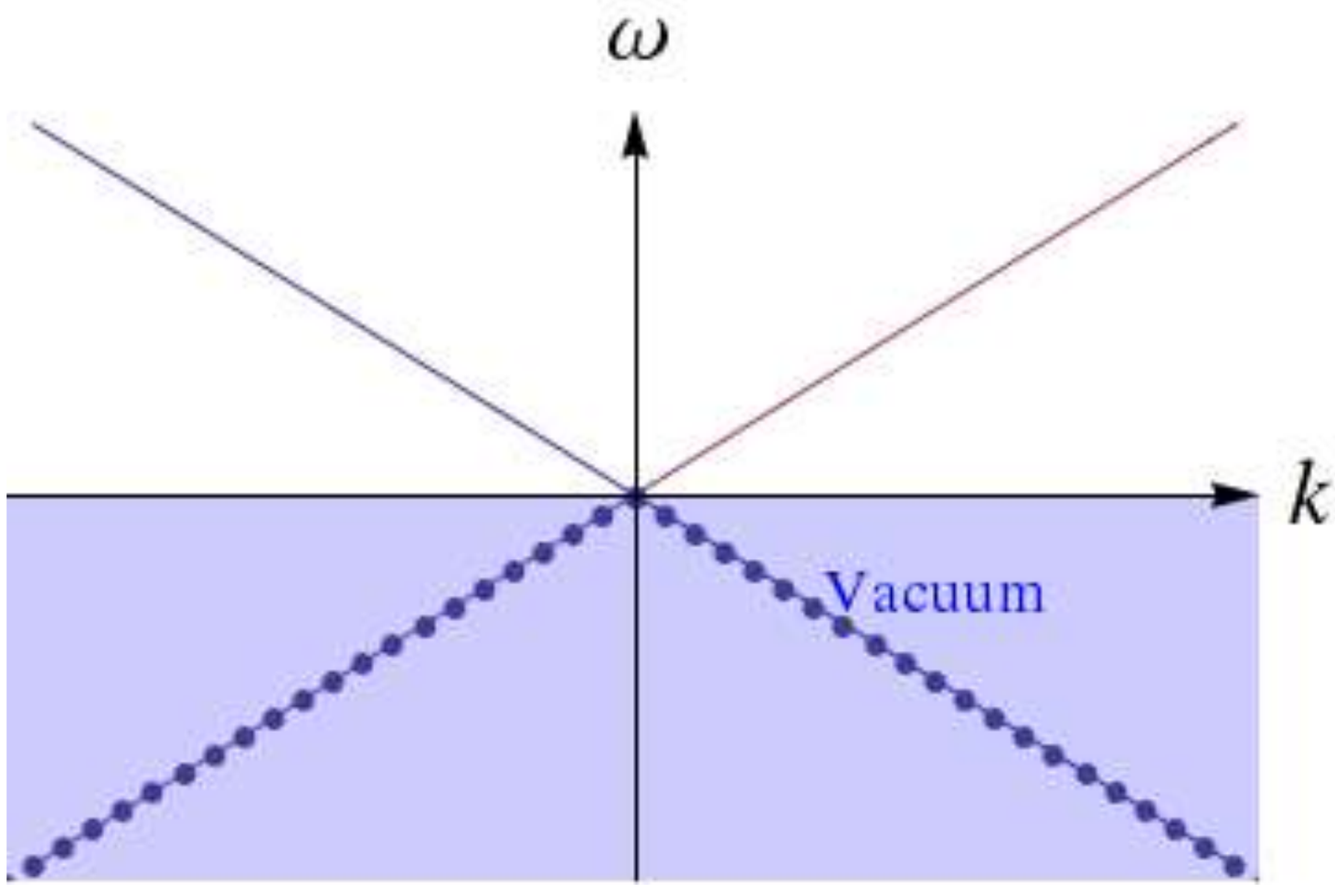
$$D_\mu J_5^\mu = \frac{1}{384\pi^2}\epsilon^{\mu\nu\rho\lambda}\mathcal{R}^a{}_{b\mu\nu}\mathcal{R}^b{}_{a\rho\lambda}$$

- QM: cannot conserve energy-momentum tensor and axial current as operators
- Has a priori nothing to do with gravity: property of QFT in flat space!
- Metric = classical sources for energy-momentum tensor
- If dynamical metric: decay of neutral pion into gravitons

[Kimura] 1969, [Delbourgo, Salam] 1972, [Freund, Eguchi] 1976

Magnetic Field

Spectral Flow Axial Anomaly



[Miransky, Shokovy, *Phys.Rept.* 576 (2015) 1-209]

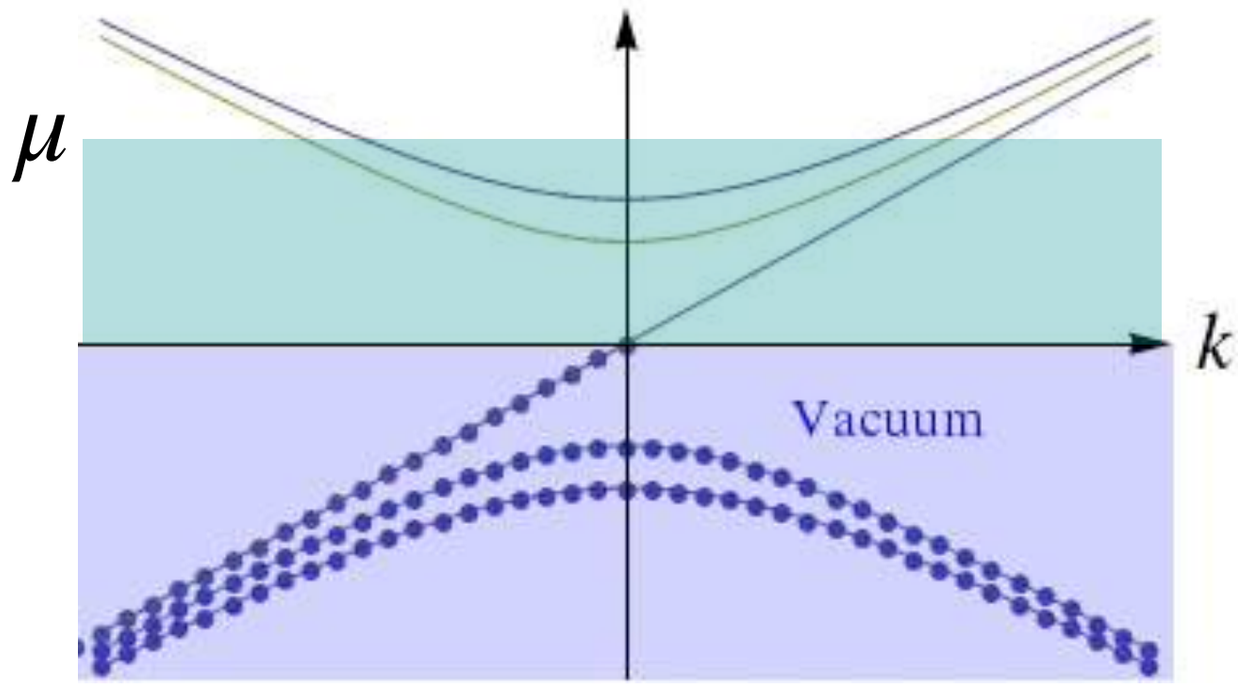
$$\omega_0 = k_z \quad , \quad \omega = \pm\sqrt{2neB + k_z^2} \quad , \quad n = 1, 2, ...$$

Chiral magnetic effect

Chiral Fermions in magnetic field:

Landau - levels

$$J = \frac{B}{2\pi}\int_0^{\mu}\frac{dk}{2\pi} = \frac{\mu}{4\pi^2}B$$



Many fermion species

$$J_a = d_{abc}\frac{\mu_b}{4\pi^2}B_c \qquad d_{abc} = \sum_r q_a^r q_b^r q_c^r - \sum_l q_a^l q_b^l q_c^l$$

Anomaly coefficient !

[Vilenkin, 80's], [Alekseev, Cheianov, Froehlich]
[Shaposhnikov, Giovannini][Fukushima, Kharzeev, Warringa]

Transport & Anomalies

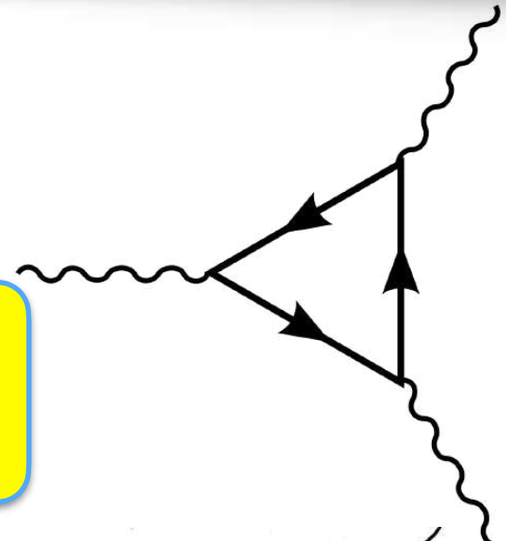
Anomaly causes dissipationless currents

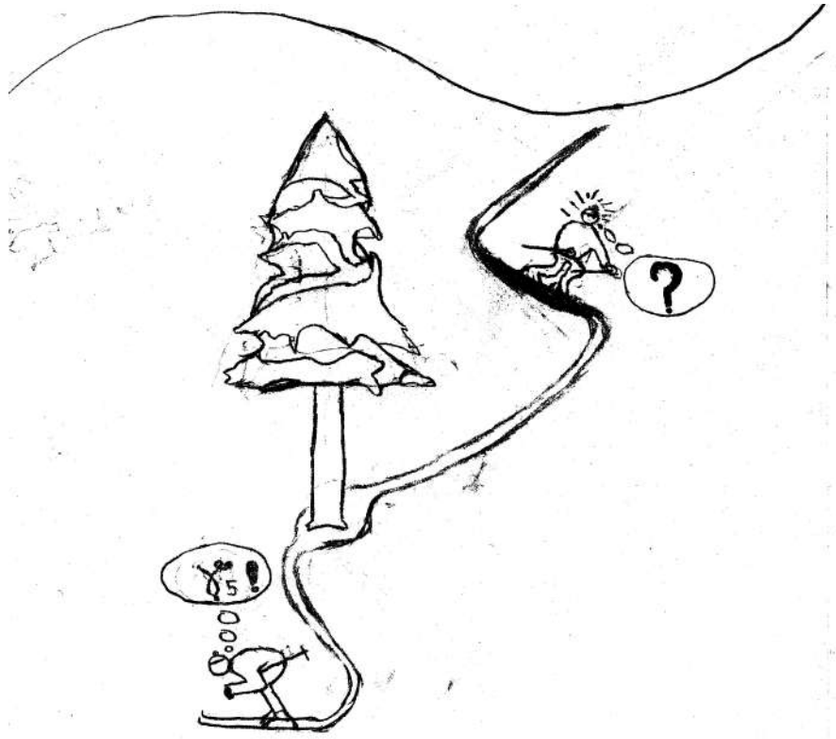
- Chiral Magnetic Effect

$$\vec{J}_{R,L} = \pm \frac{\mu}{4\pi^2} \vec{B}$$

- Chiral Vortical Effect

$$\vec{J}_{R,L} = \pm \left(\frac{\mu^2}{4\pi^2} + \frac{1}{12} T^2 \right) \vec{\omega}$$





- Dissipationless Currents!
- No Entropy generation!
- No quantum corrections!

[Vilenkin],[Froehlich, Chaianov], [Fukushima,Kharzeev,Warringa] ,
[Erdmenger et al.][Batthacharya et al.],
[K.L., Megias, Melgar, Pena-Benitez], [K.L., Megias, Pena-Benitez], [Son, Surowka],
[Stephanov,Yee], [Copetti, Fernandez-Pendas, K.L., E. Megias]
Nonrenormalization: [Golkar, Son], [Hou, Liu, Ren]

Anomalous effective action

$$\delta_\lambda \Gamma = \mathcal{A}_\lambda$$

Non-local

local

But anomaly can be written as local in 5 dimensions:

$$\delta_\lambda \int_{\mathcal{M}} A \wedge F \wedge F = \int_{\partial \mathcal{M}} \lambda F \wedge F$$

$$\int_{\mathcal{M}} d\lambda\, R \wedge R = \int_{\partial \mathcal{M}} \lambda \left(R^{(4)} \wedge R^{(4)} + D(K \wedge DK) \right)$$

[Haehl, Loganayagam,Rangamani], [Jensen,Loganaygam,Yarom],
[di Pietro, Komargodski], [Banerjee, Batthacharya, Battacharyya, Jain, Minwalla, Sharma],
[Mañes, Megias, Valle, Vazquez-Mozo]

Anomaly induced currents

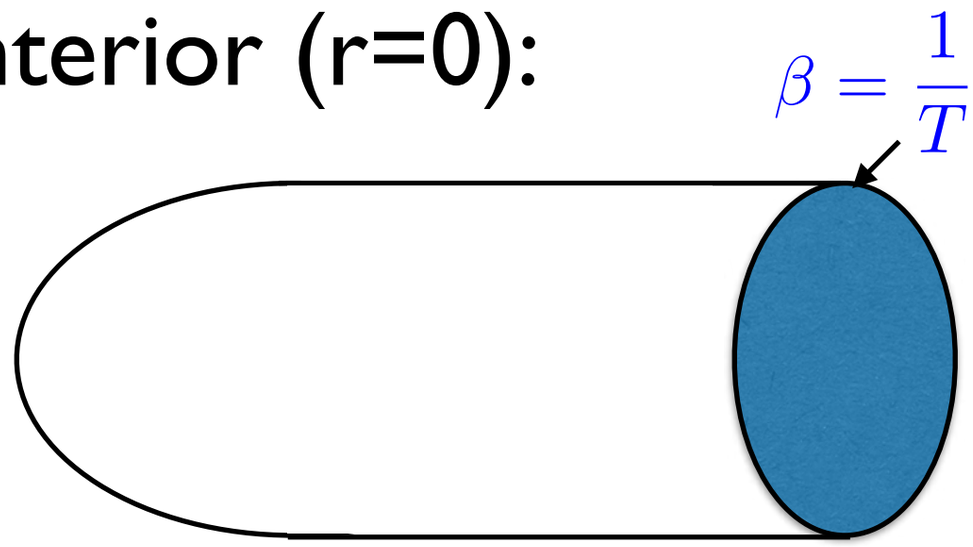
Thermal equilibrium = constraint on topology

Finite T Euclidean: $\partial M = \mathbb{S}^1 \otimes \mathbb{R}^3$

$$ds^2 = dr^2 + f(r)^2 d\tau^2 + g(r)^2 d\vec{x}^2$$

Smooth geometry in the interior (r=0):

$$f(0) = 0$$
$$f'(0) = 2\pi T$$



Thermal equilibrium = 5D black hole !

[Gibbons, Hawking]

Effective Action

Chiral Magnetic Effect, U(1)^3 anomaly

$$\delta\Gamma_{CS} = 3\int_{\mathcal{M}} \delta A \wedge F \wedge F + 2\int_{\partial\mathcal{M}} \delta A \wedge A \wedge F$$

"Covariant" current

"Bardeen-Zumino" current

[Bardeen, Zumino] '84

$$F = B dx \wedge dy + F_{0r} dt dr$$

$$A_r = 0$$

$$A_0 = A_0(r) \quad , \quad A_0|_\partial = \mu \quad , \quad A_0(0) = 0 \qquad \partial_r B = 0$$

$$\delta A = dz$$

$$J_{cov} = 6\mu B$$

$$J_{BZ} = -2\mu B$$

Normalisation: $\frac{1}{24\pi^2}$

One chiral fermion $J_{\text{total}} = \frac{\mu}{6\pi^2} B$

Effective Action

CME proper: V-A theory

$$\Gamma = \int_{\mathcal{M}} A \wedge F_V \wedge F_V$$

$$\delta\Gamma = 2\int_{\mathcal{M}} \delta V \wedge F_V \wedge F_A + 2\int_{\partial\mathcal{M}} \delta V \wedge A \wedge F_V$$

CME: $J_V = 2\mu_A B - 2\mu_A B = 0$

CSE: $J_A = 2\mu_V B$

Bloch theorem: Exactly conserved currents have to vanish in exact equilibrium

[Gynther, K.L., Pena-Benitez, Rebhan], [Kharzeev], [Yamamoto], [Franz, Vazifeh]

Anomaly induced currents

Calculate current due to rotation from CS action in slowly rotating black hole

$$ds^2 = dr^2 - f(r)^2[dt - (\vec{\omega} \times \vec{x}) \cdot d\vec{x}]^2 + g(r)^2 d\vec{x}^2$$

$$\delta\Gamma_{CS} = \int \delta A \wedge R \wedge R = \int \delta A_\mu \langle J^\mu \rangle_{\text{non-local}}$$

$$\vec{J} = 4f'(0)^2\vec{\omega} = 16\pi^2 T^2 \vec{\omega}$$

Fixed by Topology!

[Megias, K.L., Pena-Benitez], [Megias, Melgar, K.L., Pena-Benitez]
[Loganayagam, Jensen, Yarom], [de Pietro, Komargodski], [Stone, Kim],

Transport & Anomalies

Luttinger: Theory of thermal transport 1964

"*..if the gravitational field did not exist one could invent it for the purpose of this paper…*"

Gravity ⟷ Thermal transport

$$\vec{\nabla}\Phi_g \equiv \frac{-\vec{\nabla}T}{T}$$

21st century:

Quantum
Gravity

⟷

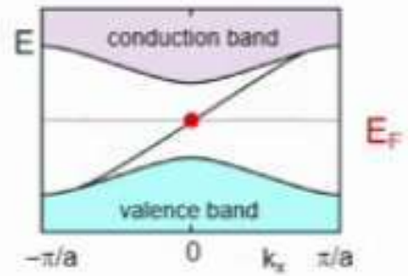
Quantum
Thermal transport

Charles Kane

Christopher H. Browne Distinguished Professor of Physics

Chiral Edge States

Single-particle edge spectrum : one-way propagating single particle states

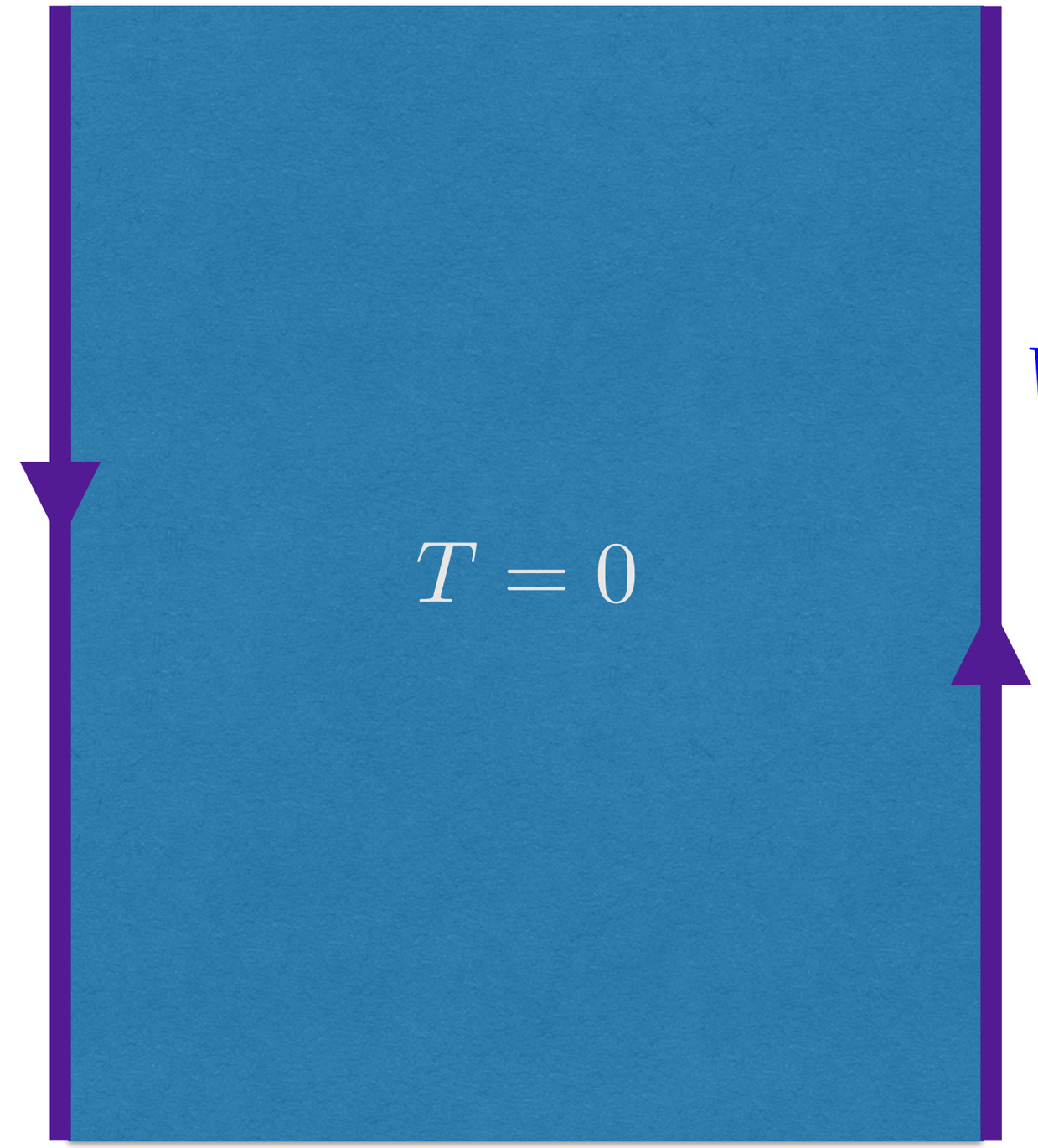


Many-body edge spectrum : "chiral Fermi liquid"

- Free Dirac fermion conformal field theory $H = -iv\psi^\dagger \partial_x \psi$
- Quantized electrical conductance $G = \nu \frac{e^2}{h}$ $\nu = 1$
  (chiral anomaly)
- Quantized thermal conductance: $\kappa = c\frac{\pi^2}{3}\frac{k_B^2}{h}T$ $c = 1$ chiral central charge
  (gravitational anomaly)

- Benjamin Franklin Medal, 2015
- Elected to National Academy of Sciences, 2014
- Lindback Award for Distinguished Teaching, University of Pennsylvania, 2014
- Physics Frontiers Prize, Fundamental Physics Prize Foundation, 2013
- P.A.M Dirac Medal and Prize, 2012
- Simons Investigator grant, 2012
- Oliver Buckley Prize, 2012
- Condensed Matter Europhysics Prize, 2010
- Fellow of the American Physical Society 2006-
- IBM Predoctoral Fellowship 1988-1989
- National Science Foundation Graduate Fellow 1985-1988

Thermal Hall effect



$$W_{\text{bulk}} = c_g \int_{\text{bulk}} (\Gamma d\Gamma + \frac{2}{3}\Gamma^3)$$

$$W_{\text{boundary}} = -c_g \int_{\mathcal{BH}} (\Gamma d\Gamma + \frac{2}{3}\Gamma^3)$$

Anomaly free:

$$\partial(\text{bulk}) = \partial(\mathcal{BH})$$

Only boundary current:

$$J_E^{\perp} = 16 c_g \pi^2 T \Delta T$$

Gobal anomaly

[Golkar, Sethi], [Chowdhyry, David], [Glorioso, Liu, Rajagopal]

Compactify on $S_T \times S^1 \times S^2$

Magnetic flux

$$Z \to e^{i2\pi na} Z$$

$$S_{\text{eff}} = \frac{i}{48\pi} \int d^4x \, \vec{A}_g . d\mathcal{A}$$

- No global anomaly if a is integer: "fractional" part of CVE
- Gravitinos ?

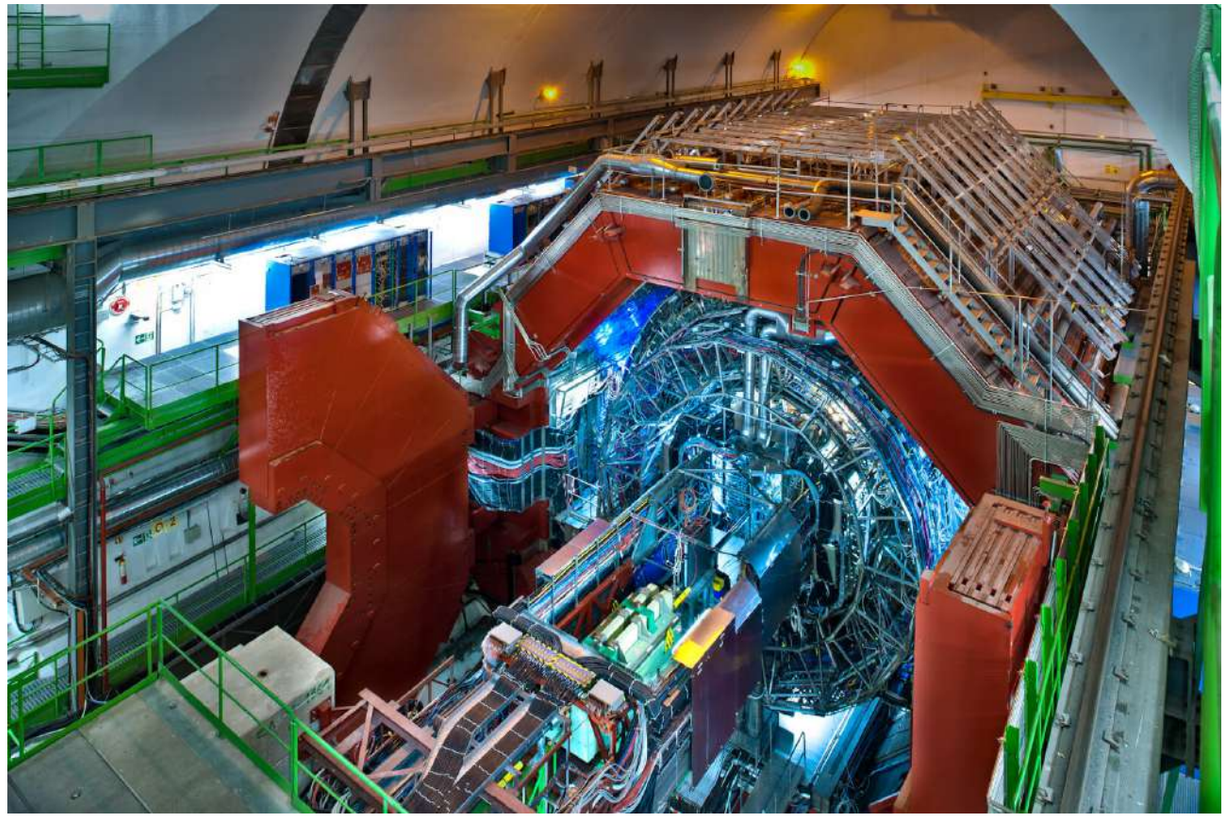
[Loganayagam]

Applications

- Quark Gluon Plasma
- Weyl Semi-Metals
- Light

RHIC, Brookhaven

LHC, Geneva

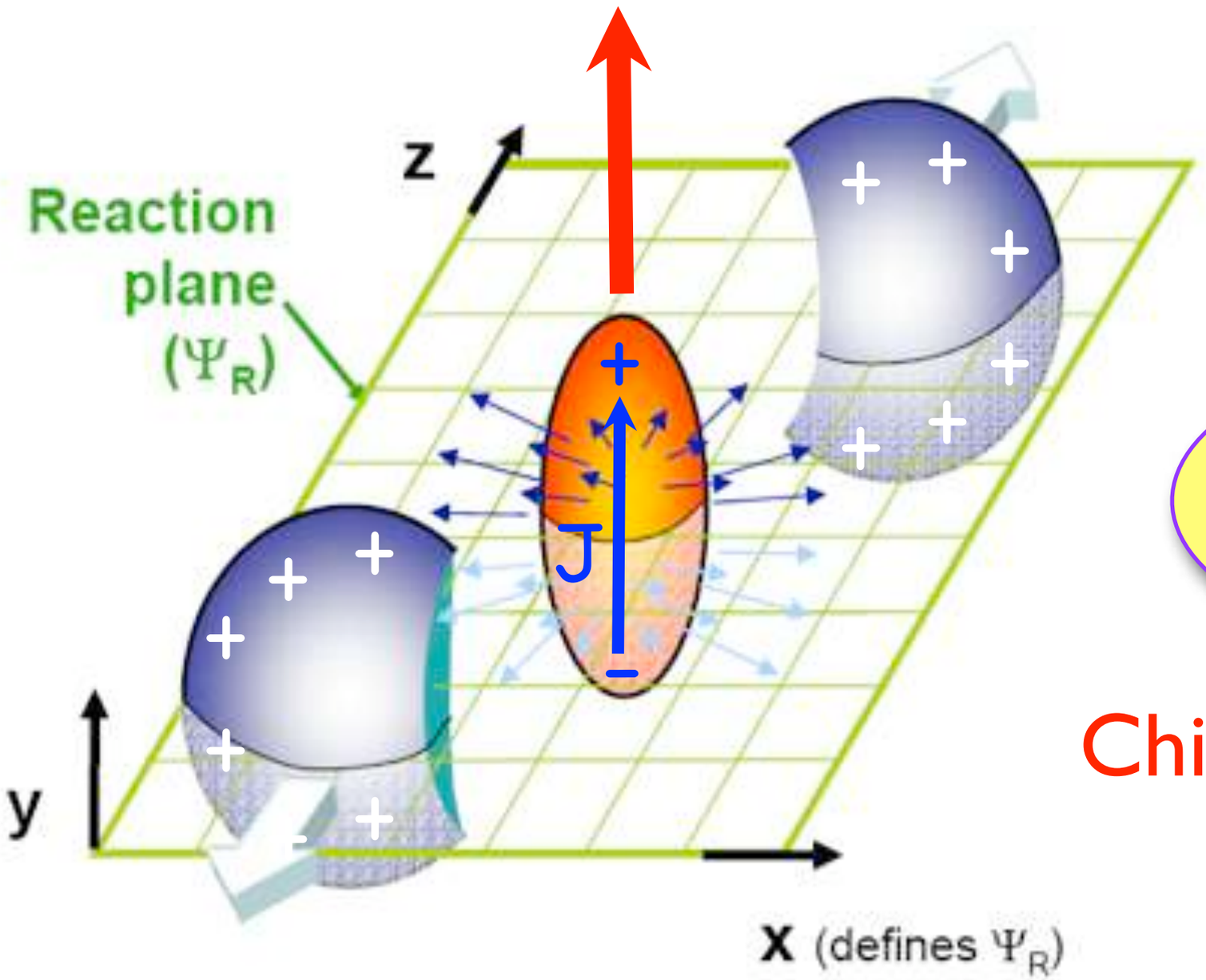


Quark gluon plasma

strongest Magnetic field in the Universe 10^{15}T!!!

(QHE: 10 T)

(T ~ 10^{12} K)

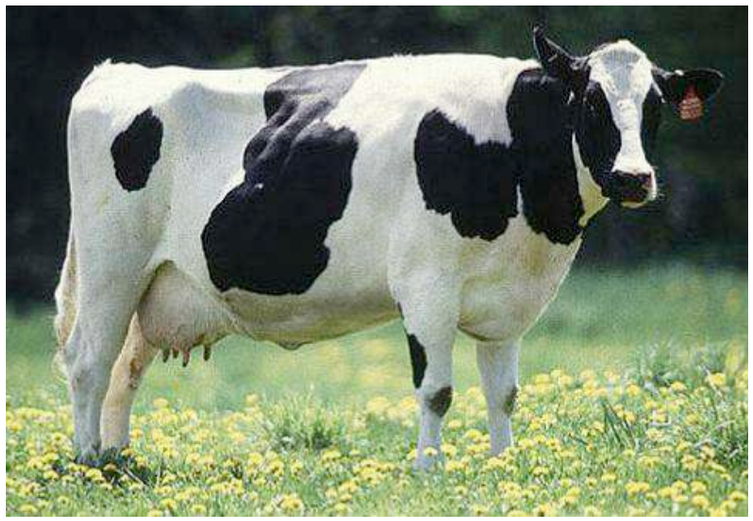


$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$

Chiral Magnetic Effect

[Fukushima, Kharzeev, McLarren]
[Fukushima, Kharzeev, Warringa]

AdS/CFT = 5D geometry



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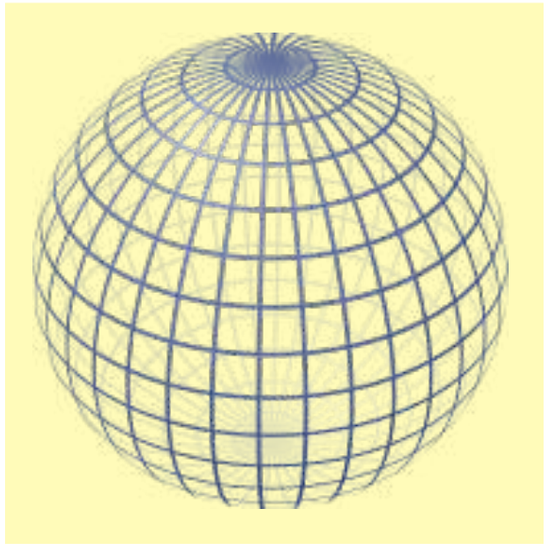

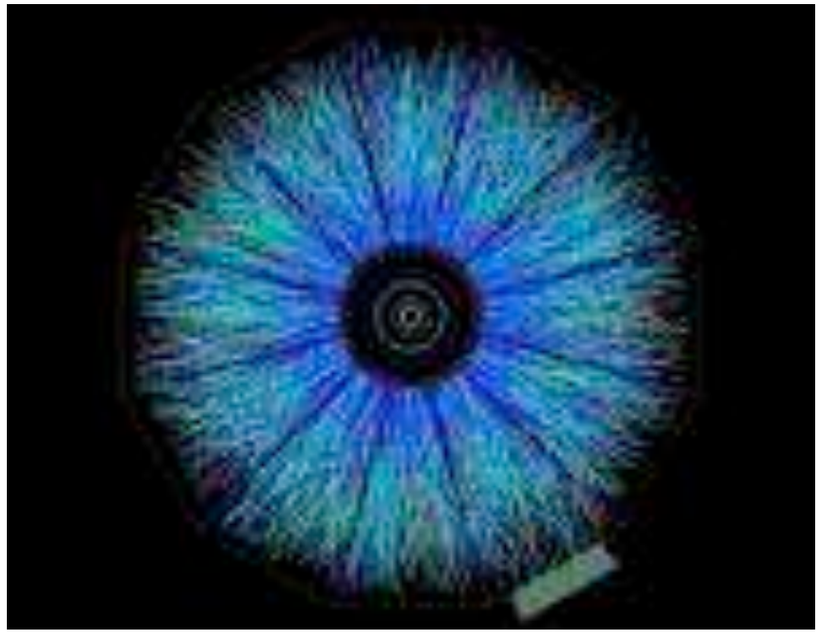

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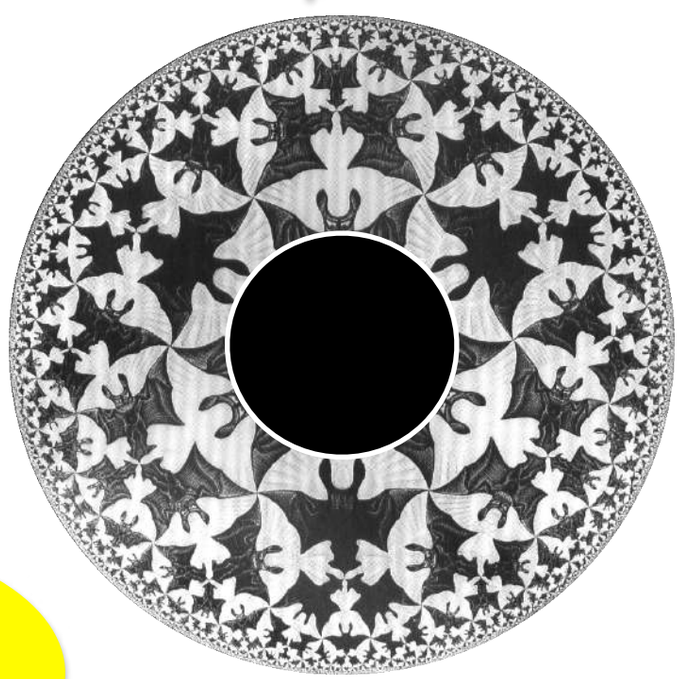


$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Policastro, Son, Starinets]

Far from equilibrium CME

[K.L., E. Lopez, G. Milans del Bosch]
[K.L. , J. Fernandez-Pendas]
[Ongoing work with S. Tejero-Morales and J. Ghosh]

AdS + infalling null dust = AdS Vaidya metric

$$ds^2 = -f(r,v)dv^2 + 2drdv + r^2 d\vec{x}^2$$

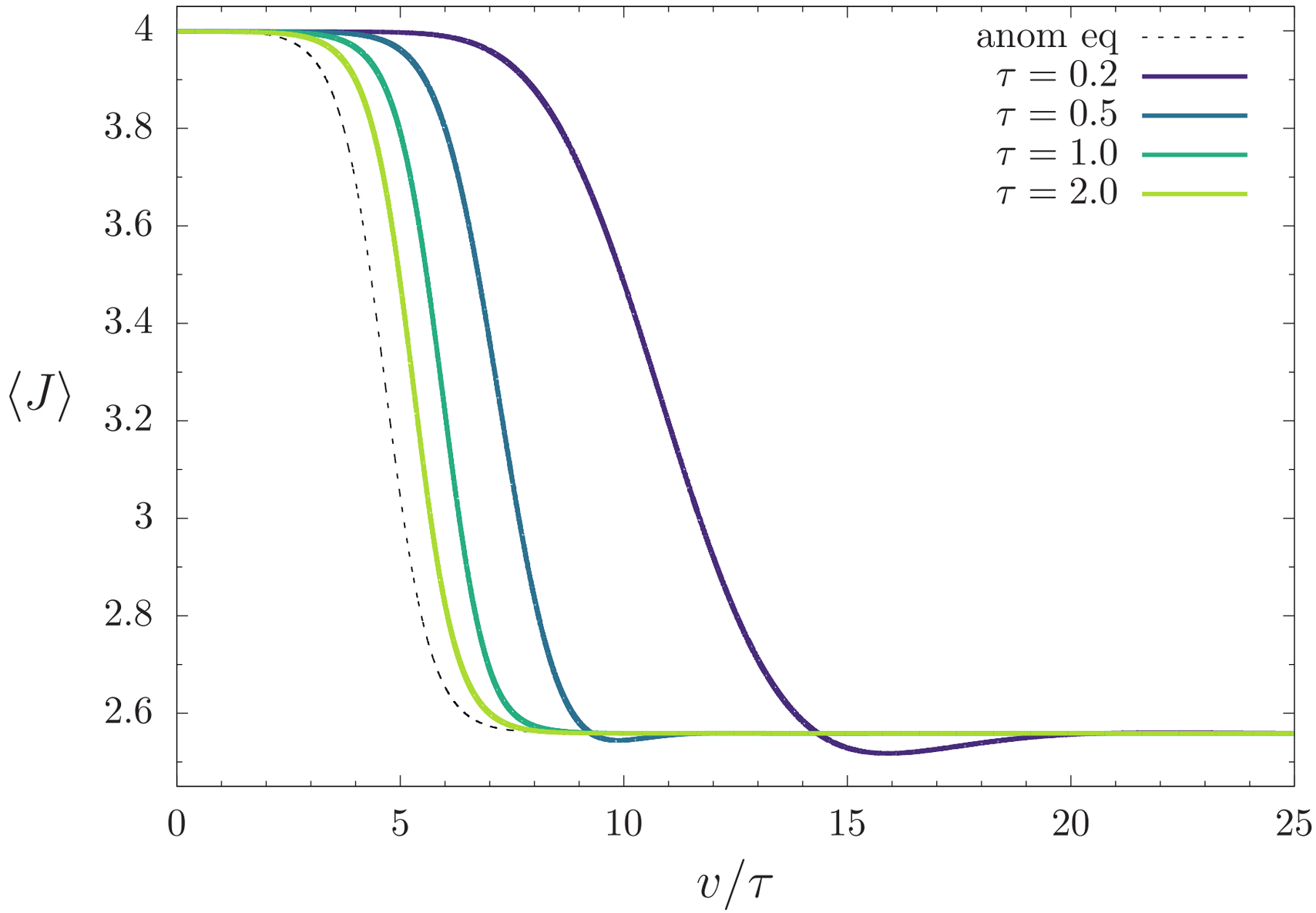
$$f(r,v) = r^2\left(1 - \frac{2m(v)}{r^4} + \frac{q_5(v)^2}{12r^6}\right)$$

[Lin, Yee], [Ammon, Grieninger, Jimenez-Alba, Malcedo], [Cartwright, Kaminski], [Cartwritht]

Editors' Suggestion

Out-of-equilibrium chiral magnetic effect and momentum relaxation in holography

Jorge Fernández-Pendás* and Karl Landsteiner†



CME @ RHIC

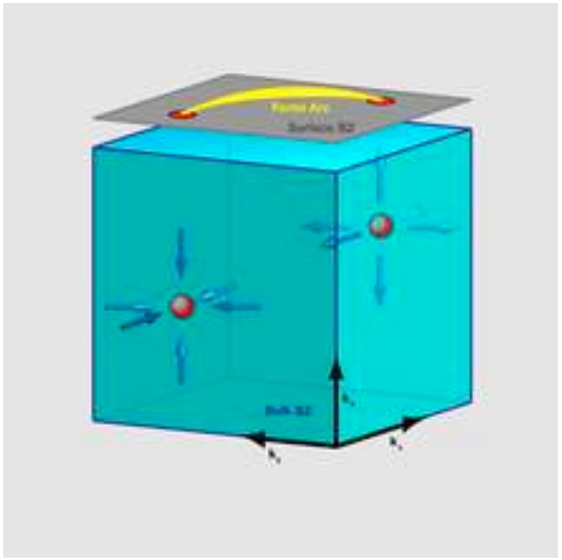
vs

CME @ LHC

???

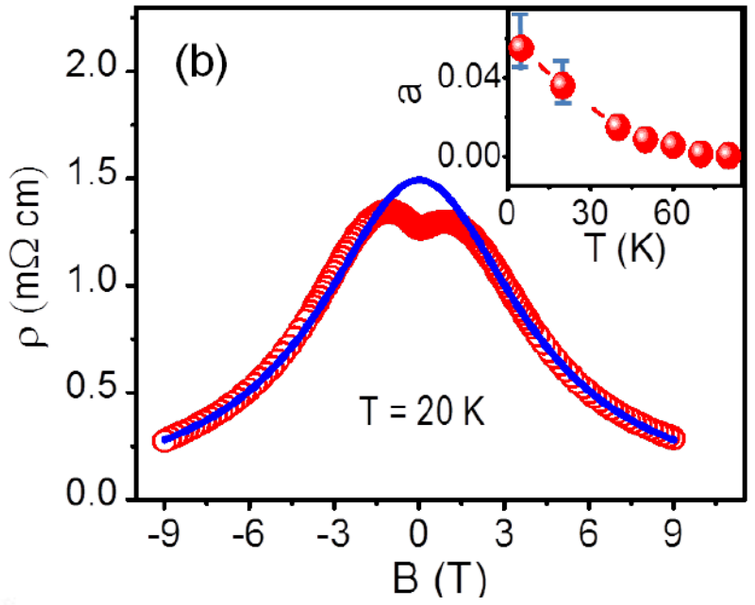
Final verdict: isobar run results? When?

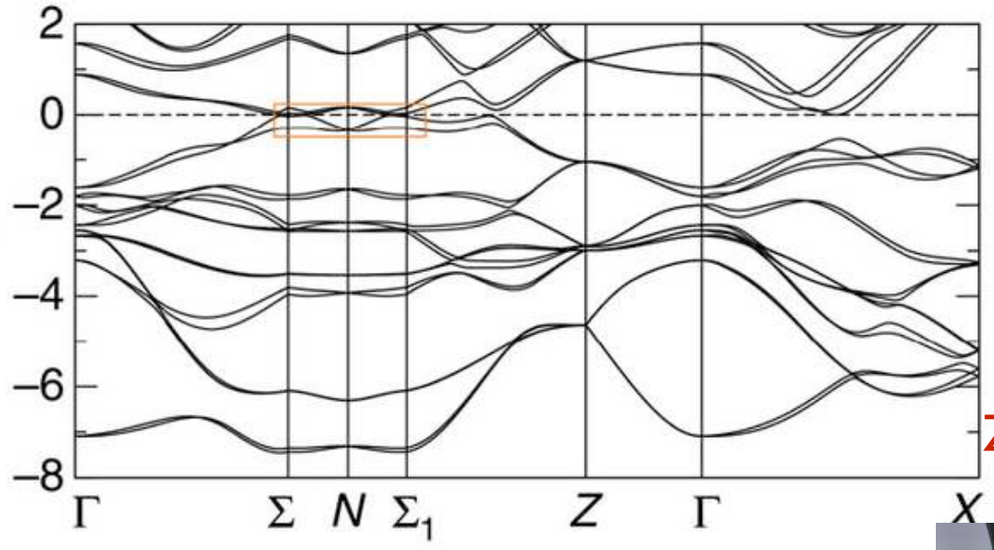
Weyl Semi-Metal



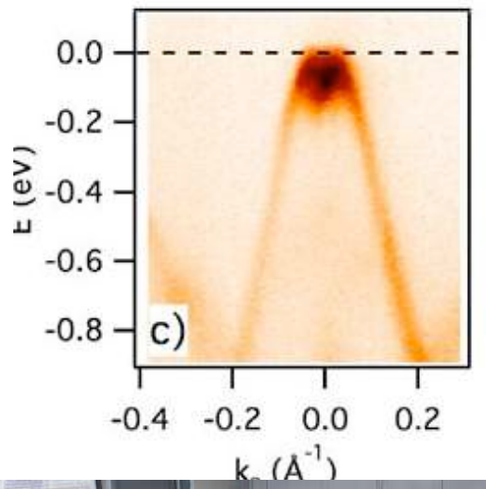
Wikipedia

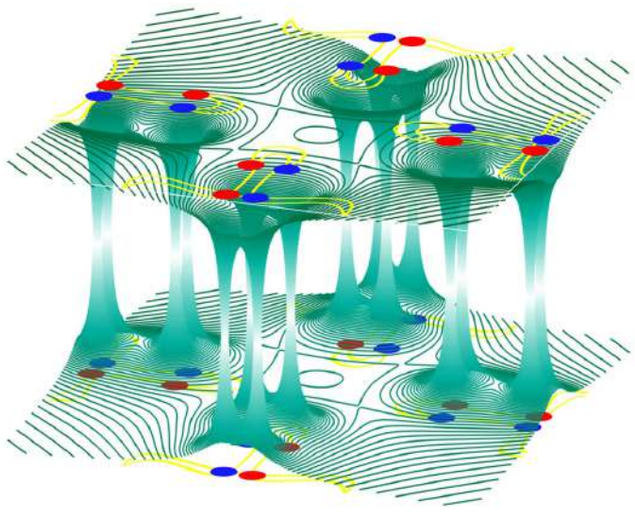
physicsworld TOP 10 BREAKTHROUGH 2015





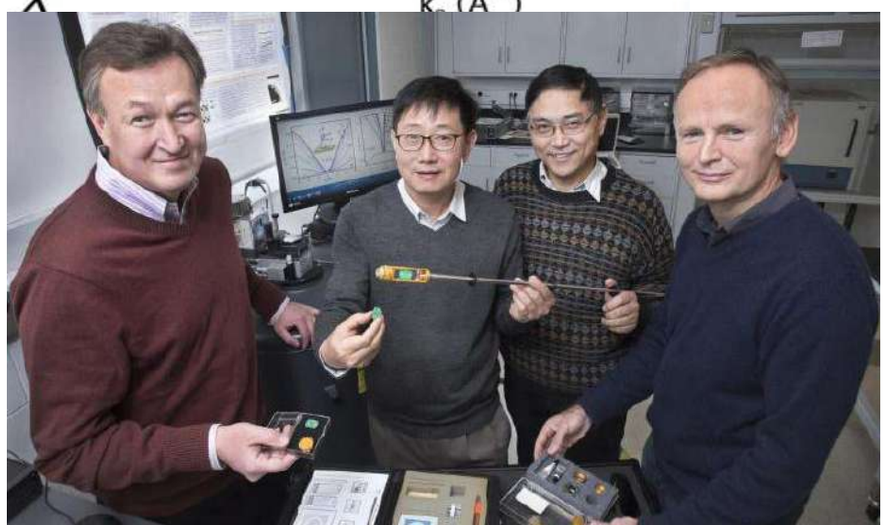
Zr_5Te





TaAs

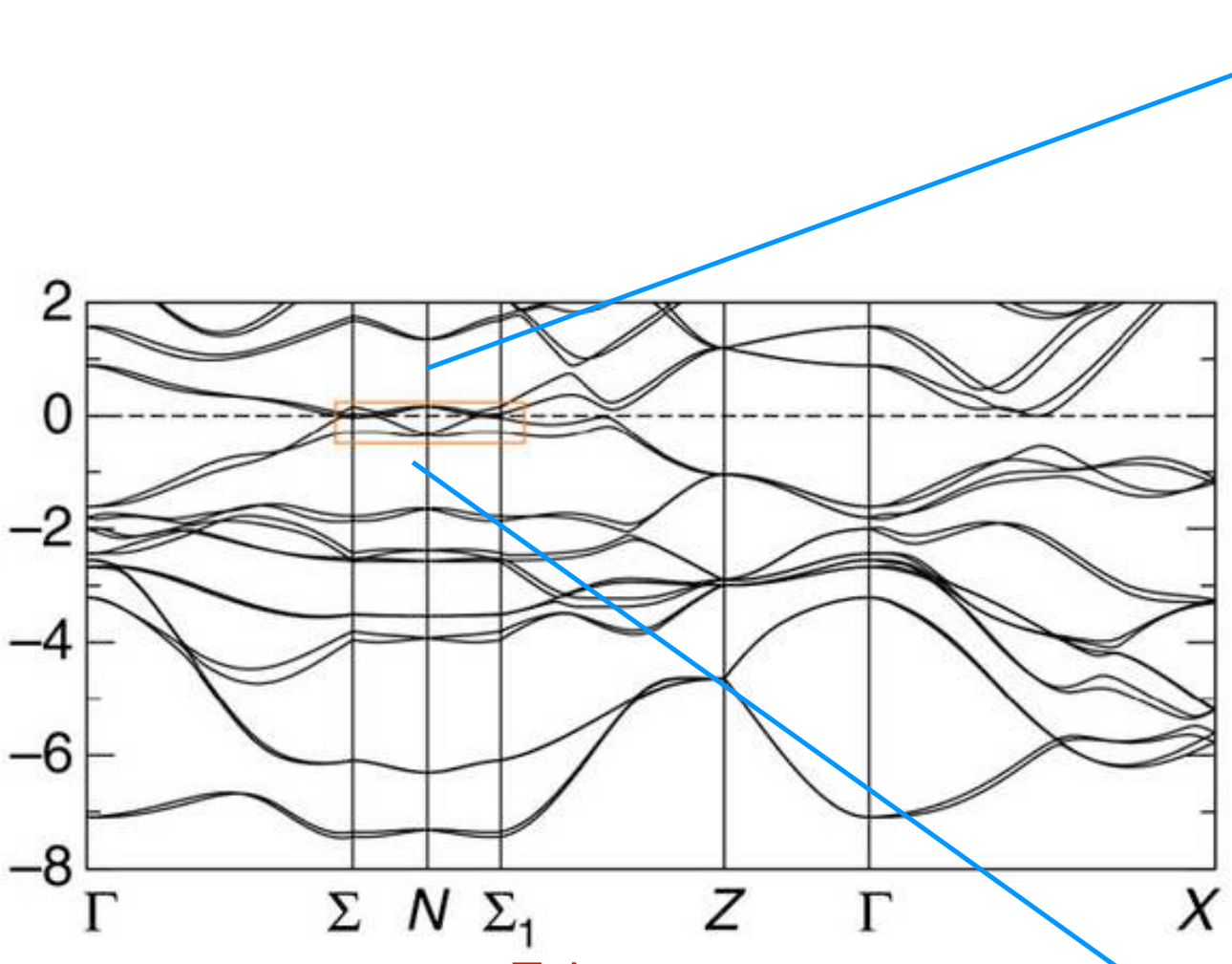
[Huang, Xu, Belopolski,Hasan] Nature Comm.



Hiroyuki Inoue, András Gyenis, Zhijun Wang, Jian Li, Seong Woo Oh, Shan Jiang,
Ni Ni, B. Andrei Bernevig,and Ali Yazdani, was published in the March 11, 2016 issue of the journal *Science*

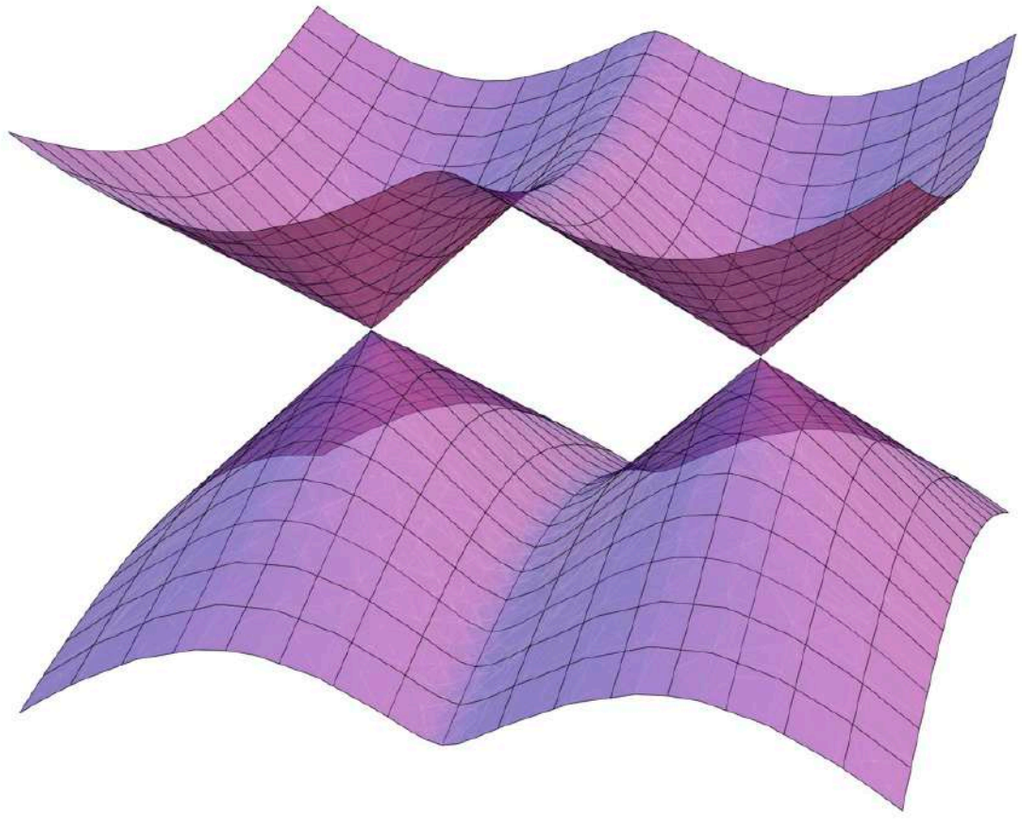
Weyl semi-metal

Linear band touching in Brillouin zone



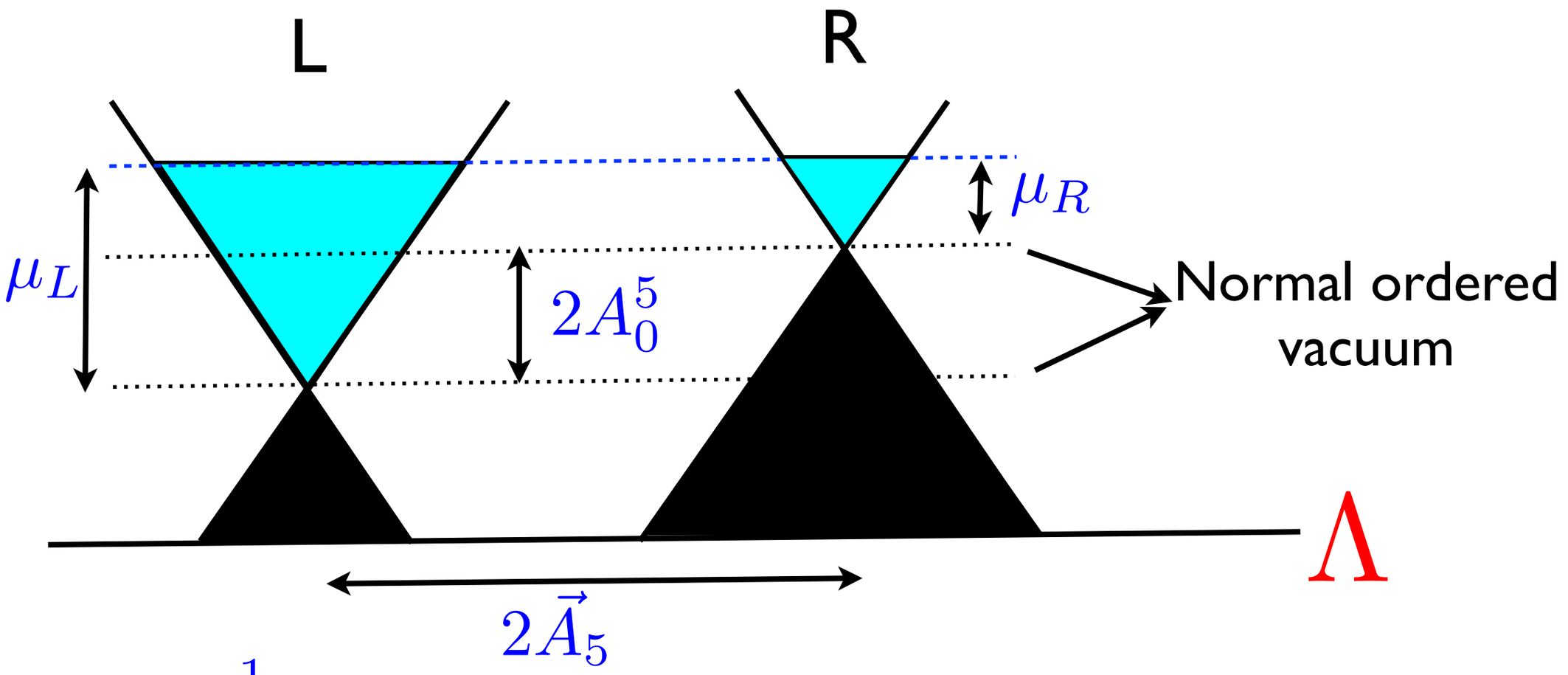
TaAs

ψ_L ψ_R



Chiral fermions have to in pairs R and L ! [Nielsen, Ninomiya],

CME in WSMs



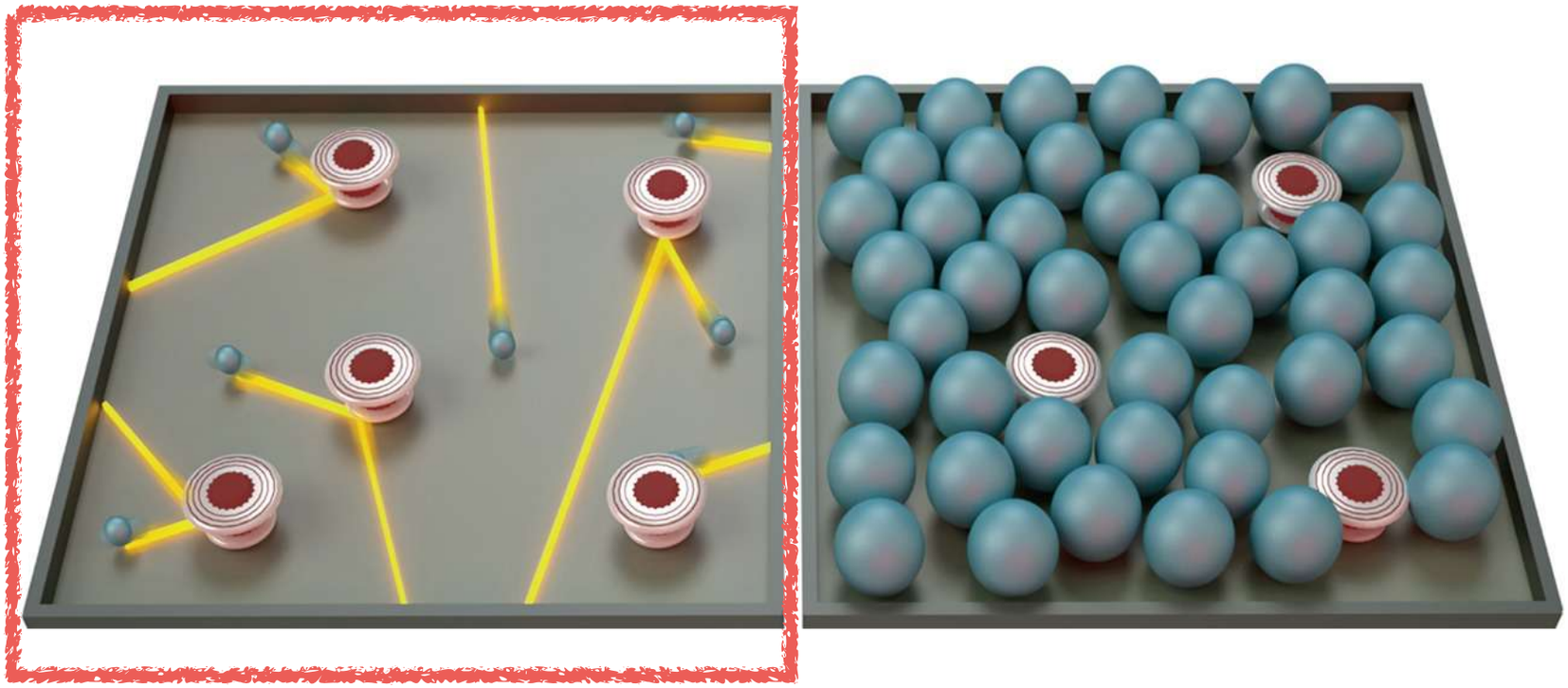
$$\mu_5 = \frac{1}{2}(\mu_L - \mu_R) \qquad \mu = \frac{1}{2}(\mu_R + \mu_L)$$

CME: $\vec{J} = \frac{1}{2\pi^2}\left(\mu_5 - A_0^5\right)\vec{B} = 0$

Covariant and Bardeen-Zumino!

[K.L.],[Gorbar, Miransky, Shovkovy, Sukhachov]

NMR and NTMR in WSM

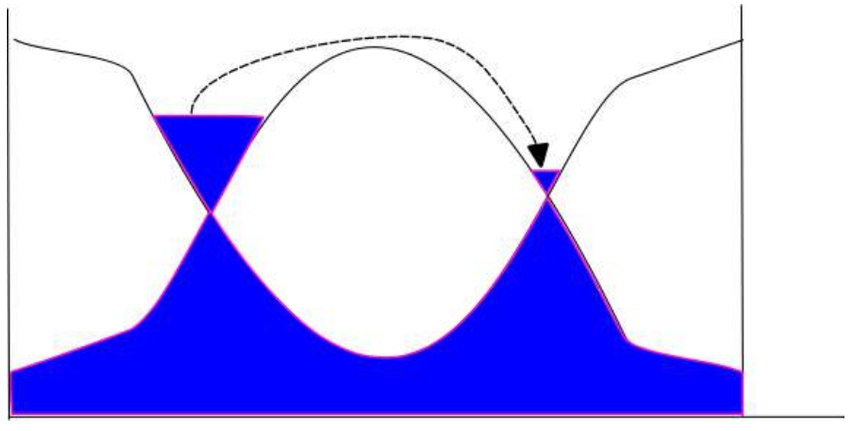


[J. Zaanen, "Electrons go with the flow in exotic materials", *Science* Vol. 351, 6277]

If WSM is not strongly coupled, hierarchy of scattering times

$$\tau_{\text{inner}} < \tau_{\text{inter}} < \tau_{ee}$$

Kills $\vec{P}$ Kills ρ_5, ϵ_5 Is irrelevant



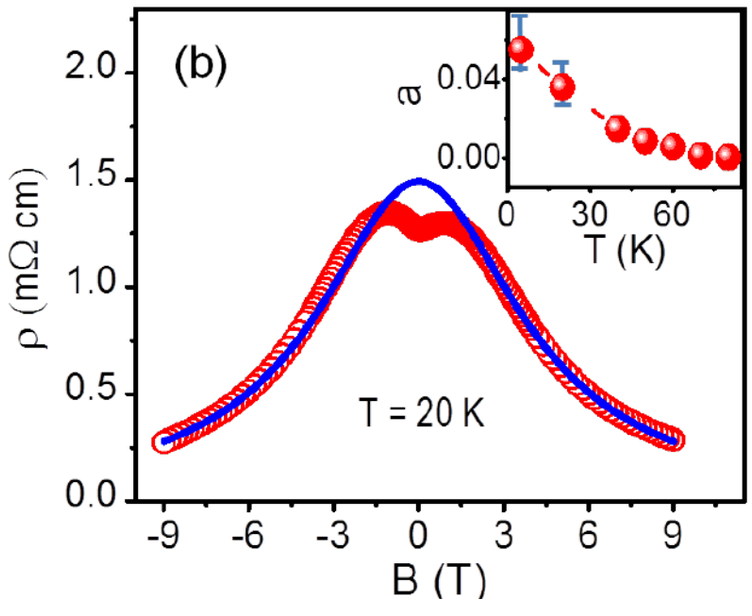
NMR and NTMR in WSM

NMR = **N**egative **M**agneto **R**esistivity
NTMR = **N**egative **T**hermo**M**agneto **R**esistivity

In equilibrium CME vanishes,
Induce non-equilibrium steady state

$$\dot{\rho}_5 = \frac{1}{2\pi^2}\vec{E}\cdot\vec{B} - \frac{1}{\tau_5}\rho_5$$

$$J = \left(\sigma + \frac{\tau_5 B^2}{4\pi^4 \chi_5}\right) E$$



[Spivak, Son], [Nielsen, Ninomiya], [Kharzeev]

NTMR via CME

Coupled charge and energy transport of chiral currents

$$\vec{J}_\epsilon = \left(\frac{a_\chi}{2}\mu^2 + a_g T^2\right)\vec{B}$$

$$\vec{J} = a_\chi \mu \vec{B}$$

$$G_E = \tau_5 \frac{a_\chi^2}{\det(\Xi)}\left(\frac{\partial \epsilon}{\partial T} - \mu\frac{\partial \rho}{\partial T}\right)$$

$$G_T = \tau_5 \frac{2 a_g a_\chi}{\det(\Xi)}\frac{\partial \rho}{\partial T} B^2$$

$$\vec{J} = G_W \vec{E} + G_T \vec{\nabla} T$$

Large B (ultra-quantum limit): $\rho = \frac{|B|}{4\pi^2}\mu$

- G_E linear in B
- G_T vanishes

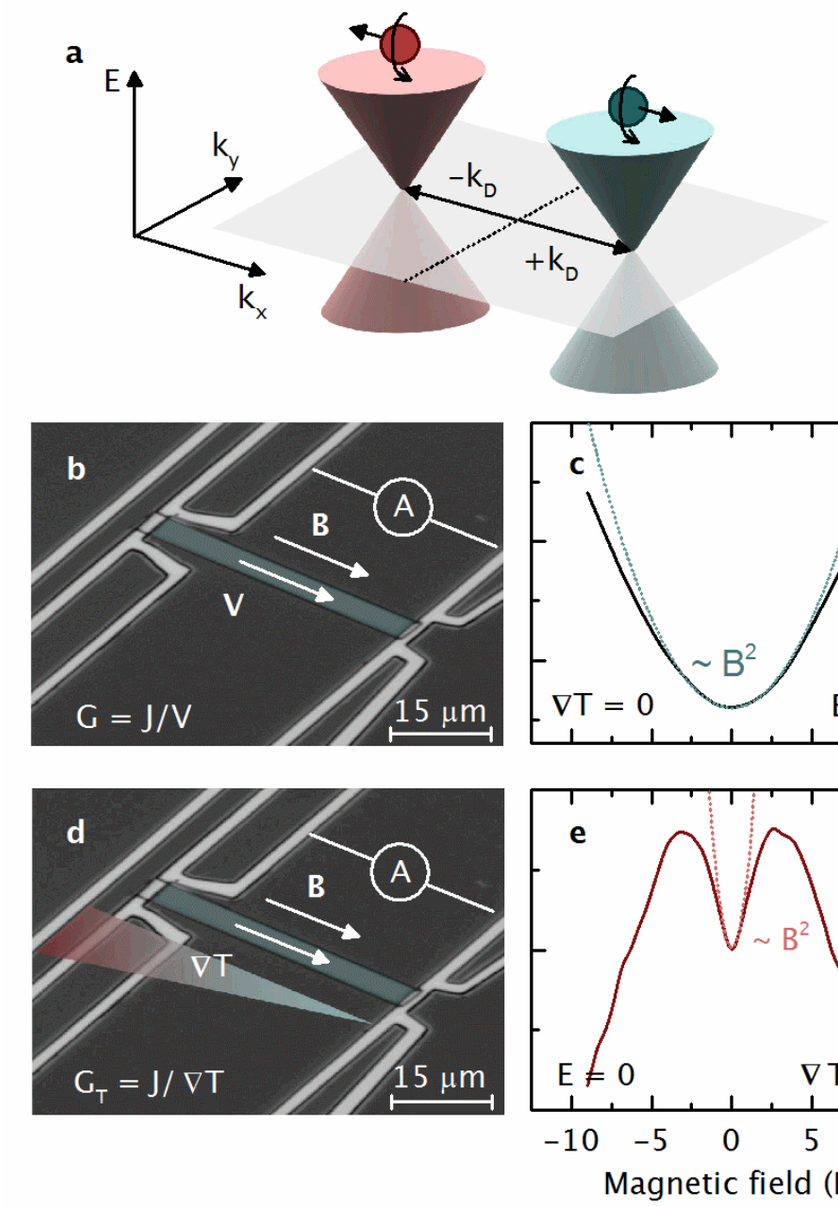
[Spivak, Andreev], [Lundgren, Laurell, Fiete] *kinetic theory*
[Lucas, Davison, Sachdev] *chiral fluids,* [K.L.]

NMR and NTMR in WSMs

Experimental signatures of the mixed axial-gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth, Anna Corinna Niemann, Tobias Meng, Adolfo G. Grushin, Karl Landsteiner, Bernd Gotsmann, Fabian Menges, Marcus Schmidt, Chandra Shekhar, Vicky Sueß, Ruben Huehne, Bernd Rellinghaus, Claudia Felser, Binghai Yan, Kornelius Nielsch

arXiv:1703.10682 (Nature)



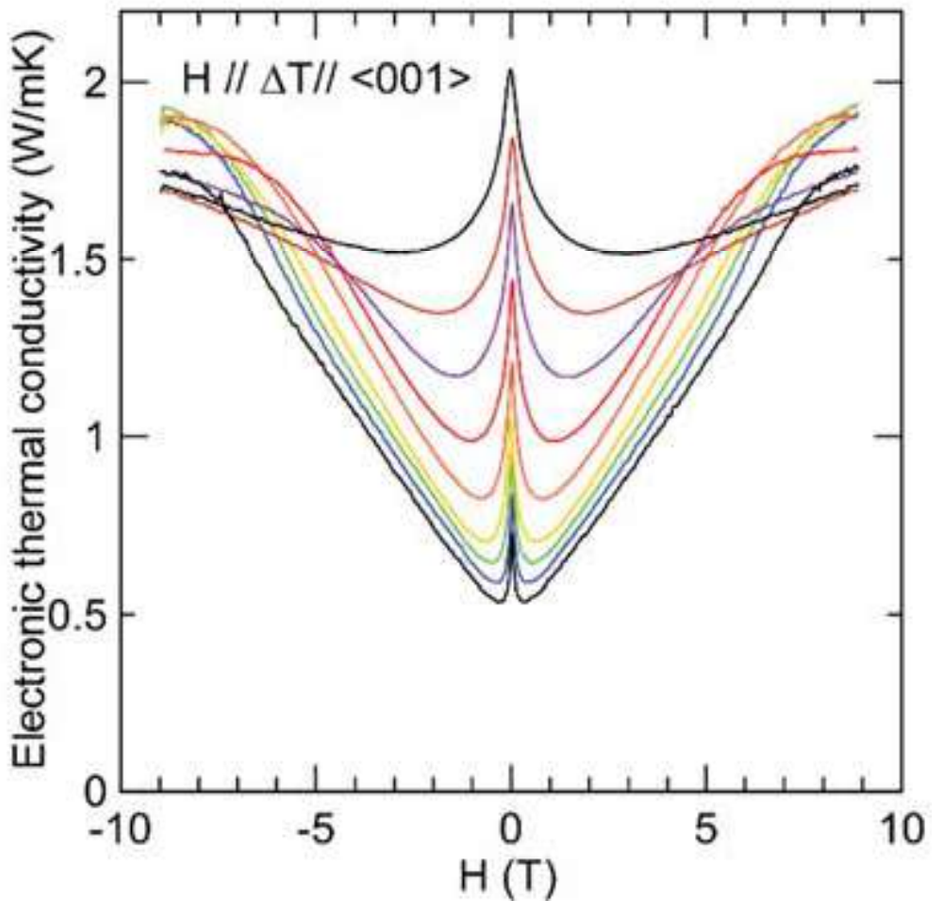
- ☑ Angle dependence
- ☑ NMR and NTMR show B^2 at small B
- ☑ NMR ~ linear for large B field
- ☑ NTMR vanishes for large B field

NbP very difficult material: Doping, T dependence

NMR and NTMR in WSMs

Thermal chiral anomaly in the magnetic-field induced ideal Weyl phase of $Bi_{89}Sb_{11}$

Dung Vu (1), Wenjuan Zhang (2), Cüneyt Şahin (3,4), Michael Flatté (3,4), Nandini Trivedi (2), Joseph P. Heremans (1,2,5)



arXiv:1906.02248 [cond-mat.mtrl-sci]

Quasiparticles
=
Wiedemann Franz

Chiral anomaly
=
Gravitational anomaly

Get rid of quasiparticles!
Hydro, B_5, J_5

Chiral Optics

Helicity of Maxwell theory $J^\mu = \epsilon^{\mu\nu\rho\lambda}\left(A_\nu F_{\rho\lambda} - C_\nu \tilde{F}_{\rho\lambda}\right)$

$$D_\mu J_h^\mu = \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\lambda} R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\lambda}$$

[Dolgov, Kriplovich, Vainstein, Zhakharov], [Agullo, del Rio, Navarro-Salas]

Chirality: local, gauge invariant = Zilch

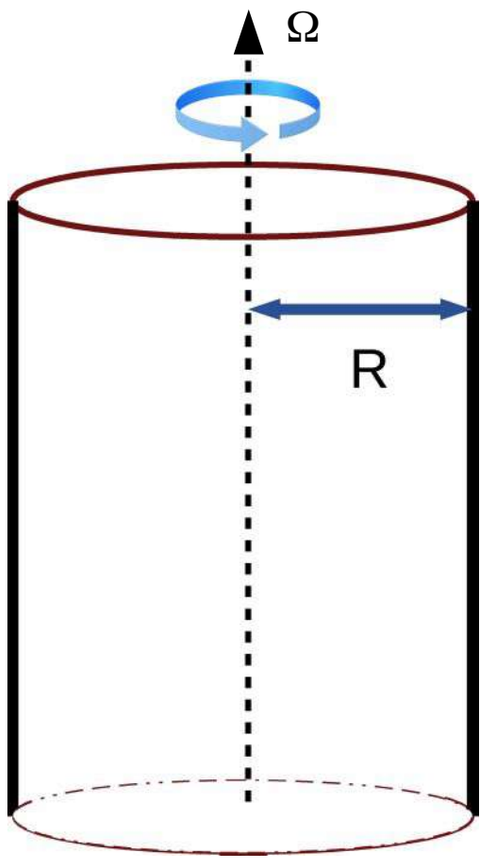
$$Z = \vec{B}\cdot(\vec{\nabla}\times\vec{B}) + \vec{E}\cdot(\vec{\nabla}\times\vec{E})$$
$$J_Z = \vec{E}\times(\vec{\nabla}\times\vec{B}) - \vec{B}\times(\vec{\nabla}\times\vec{E})$$

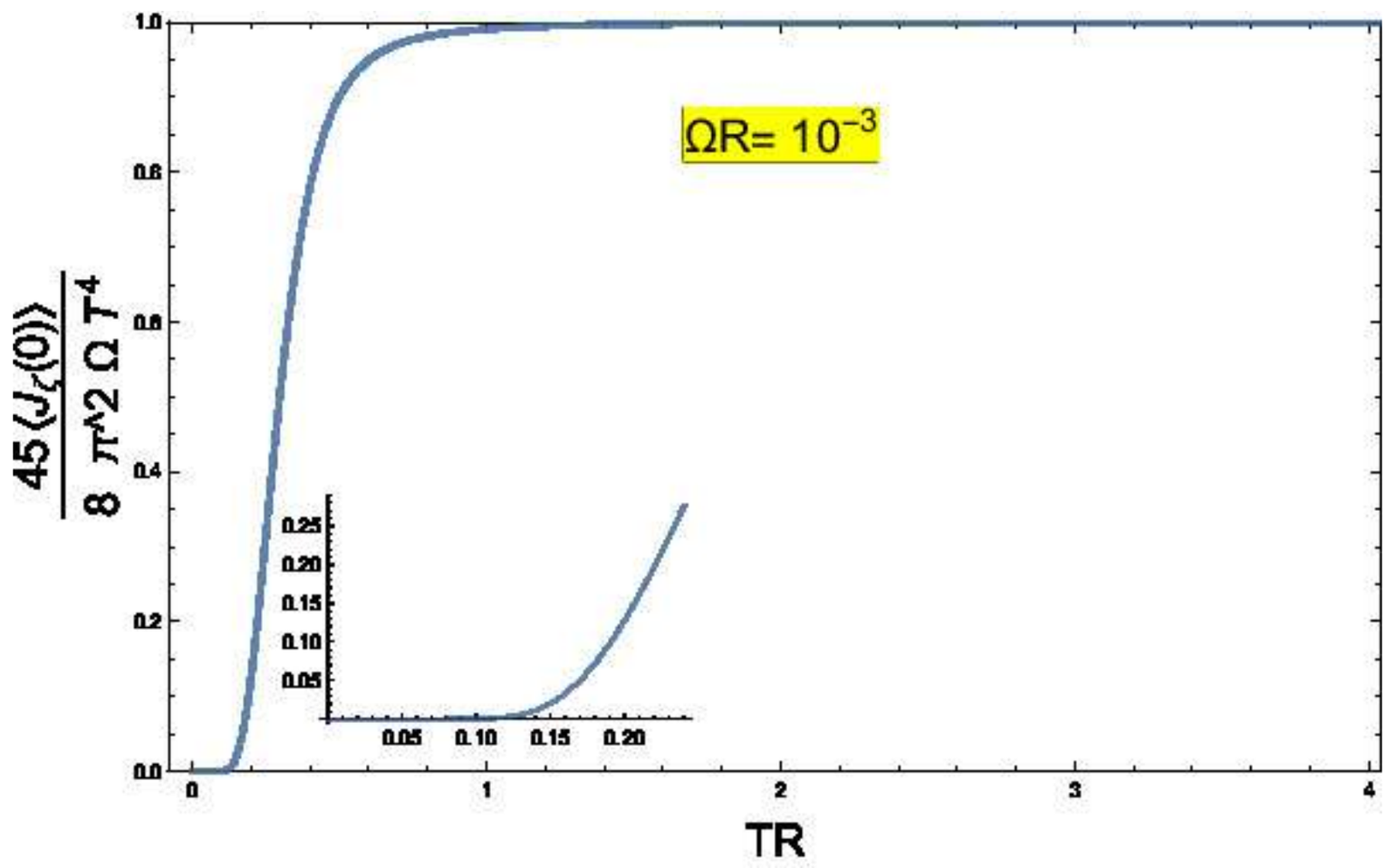
[Lipkin] 1966, [Kibble] 1967, [Cohen, Tang] 2010 !

Zilch vortical effect

$$TR \to \infty$$

$$\vec{J}_Z = \frac{8\pi^2 T^4}{45}\vec{\omega}$$





[Chernodub, Cortijo, K.L.]
[Fernandez-Pendas, Copetti]
[Avkhadiev, Sadofyev]
[N. Yamamoto]

- Universal result at axis of rotation
- Vanishing Poynting vector !

Summary

- Anomalies: rich anomaly induced transport phenomenology
- Axial magnetic fields in WSMs [Cortijo, Ferrreiros, K.L., Vozmediano]
- Chiral magnetic waves [Kharzeev, Yee] [Song, Dai] [Chernodub, Vozmediano]
- Conformal anomaly [Chernodub], [Chernodub, Cortijo, Vozmediano]
- Helicity vs. Chirality [Ambrus], [Ambrus, Chernodub]
- Torsional anomaly (?) [Hughes, Leigh, Parrikar], [Nissinen, Volovik], [Ferreiros et al.], [Huang, Han, Stone]
- Quantum computing with the CME? [Kharzeev, Li]

Wellcome to the anomalous golden age of chirality !