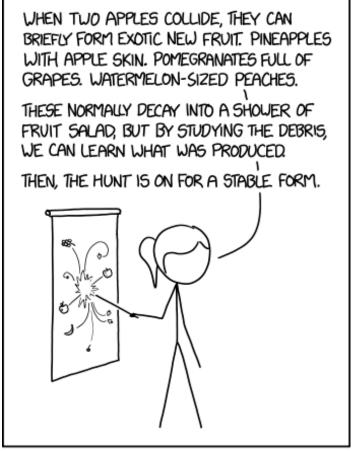
Hadron Renaissance: The Heavy-Quark Exotics



HOW NEW TYPES OF FRUIT ARE DEVELOPED

Richard Lebed



The Online Theoretical Physics Colloquium May, 2023

Outline

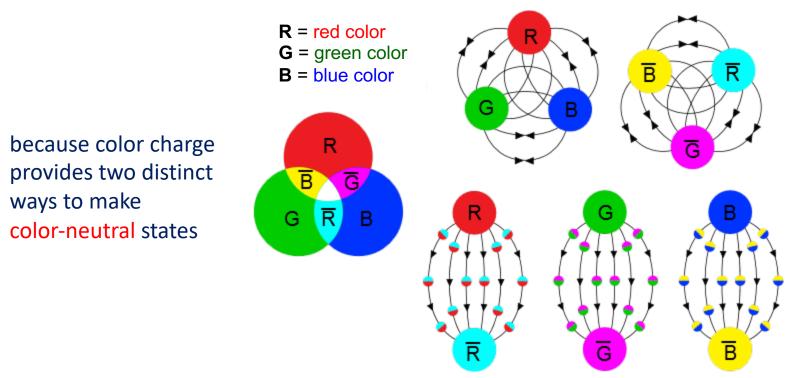
- 1) QCD, quarks, gluons, and hadrons: conventional and exotic
- 2) Bound states: hydrogen, positronium, charmonium
- 3) Discovery of the exotic hadrons X, Y, Z, P_c
- 4) How are the tetraquarks X, Y, Z and pentaquarks P_c assembled?
- 5) A diquark-based model for the X, Y, Z, P_c
- 6) Conclusions

This is a very special colloquium audience! For you, I don't need to define...

- QCD
- Quarks
- Gluons
- Color charge and SU(3) color
- Color confinement
- Hadrons
- Mesons
- Baryons

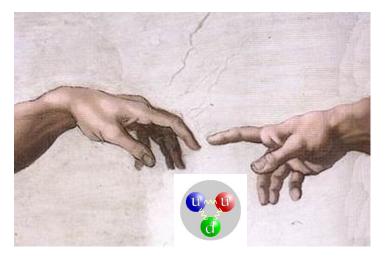
But let me remind you of one simple fact:

Color confinement requires hadrons to be color neutral, and the two most elementary ways to do this with quarks in the color-3 of $SU(3)_{color}$ are $q\bar{q}$ mesons, or qqq baryons



Mesons and baryons are *necessary*

- Create yourself a universe containing q (and \overline{q}) carrying SU(3)_{color} charge (in the fundamental triplet 3 [or $\overline{3}$])
- Demand color confinement and let the universe cool down
- Anyplace with a mixture of q and q
 condenses to mesons, anyplace with an excess of q condenses to baryons, anything more complicated can fall apart into them



But they are not the only possible hadrons

- Any other possible hadron type is called **exotic**
- The SU(3)_{color} rule for forming color-neutral states is simple: (# of q) - (# of q

) = 0 mod 3, & any number of g except one by itself
- *gg, ggg,* ... (*glueball*)
- $q\bar{q}g$, $q\bar{q}gg$, ... (hybrid meson)
- $q\bar{q}q\bar{q}$, $q\bar{q}q\bar{q}q\bar{q}q\bar{q}$, ... (tetraquark, hexaquark, ...)
- $qqqq\overline{q}$, $qqqqqq\overline{q}$, ... (pentaquark, octoquark, ...)
- *qqqqqq*, ... (*dibaryon*, ...)
- Even Gell-Mann and Zweig, in their foundational **1964** quark-model papers, knew about the multiquark possibilities

"So what took them so long?"

- Exotics can mix with ordinary hadrons with the same quantum numbers
 - Especially true for hadrons made solely from the light-quark flavors,
 u (up), d (down), s (strange)
- Weak experimental signals often either disappear with higher statistics, or are never confirmed by other experiments
- A seemingly strong signal for a new particle, even one confirmed by multiple experiments, can turn out to be due to entirely different physics
 - *e.g.*, in the early 2000's, a famous pentaquark candidate $\Theta^+(1540)$ turned out **not** to be an *s*-channel *K*-*N* compound resonance ($K^0 = \bar{s}d, p = uud$), but the result of an unfortunate choice of kinematical *cuts* on the data and *t*-channel exchanges
- ...So when the breakthrough finally came in **2003**, it was not instantly accepted by everyone

How to describe a bound state

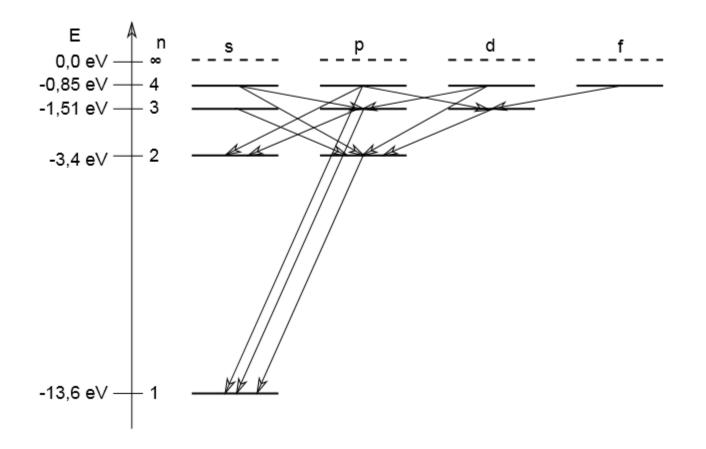
• In nonrelativistic quantum mechanics, finding the bound states of two particles (reduced mass μ) is straightforward: Specify the potential energy V(r) of their interaction, and solve the Schrödinger equation

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(\boldsymbol{r}) + V(\boldsymbol{r})\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r})$$

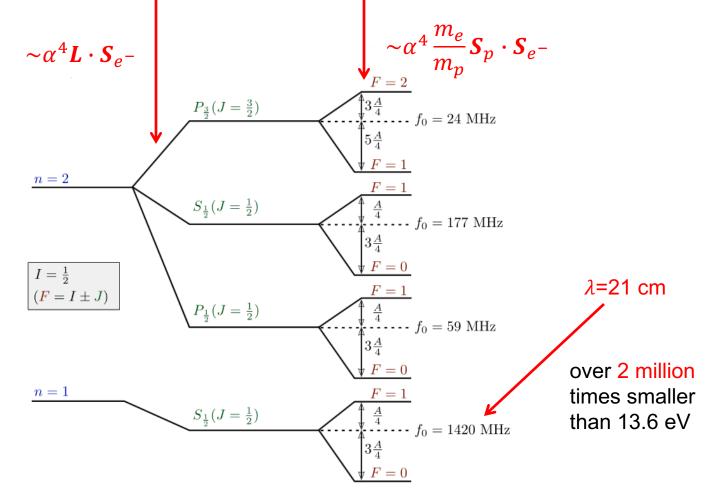
- For hydrogen, the potential is almost entirely due to the Coulomb force: $V(r) = -\frac{\alpha \hbar c}{r}$
- In fact, this potential is so simple, and the Schrödinger equation made from it has such a high degree of symmetry [SO(4)], that its energy eigenvalues are highly degenerate:

$$E_{nlm} = -\frac{\mu c^2 \alpha^2}{2n^2}, \qquad \begin{cases} n = 1, 2, 3, \dots \\ l = 0, 1, 2, \dots, n-1 \\ m = -l, -l+1, \dots, l-1, l \end{cases}$$

Hydrogen spectrum

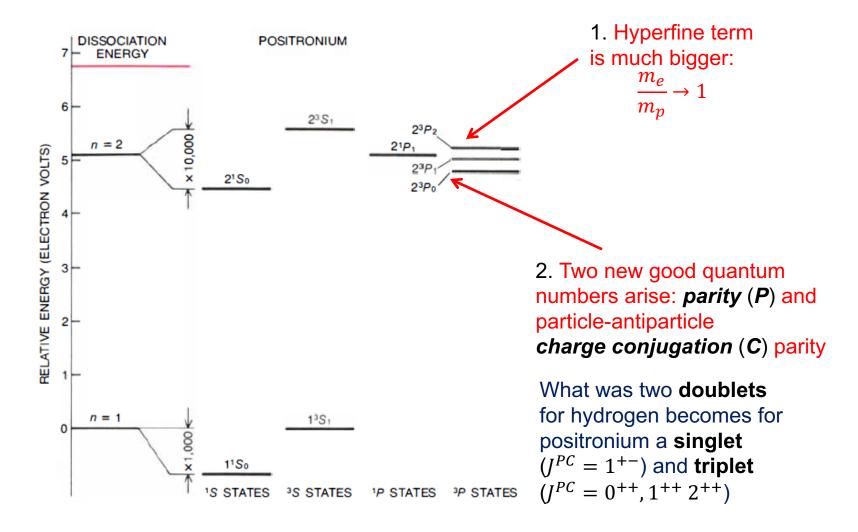


Hydrogen: fine and hyperfine structure perturbations lift the degeneracies



Positronium (e^-e^+)

Figure from E. Bloom and G. Feldman, "Quarkonium," Scientific American, May, 1982

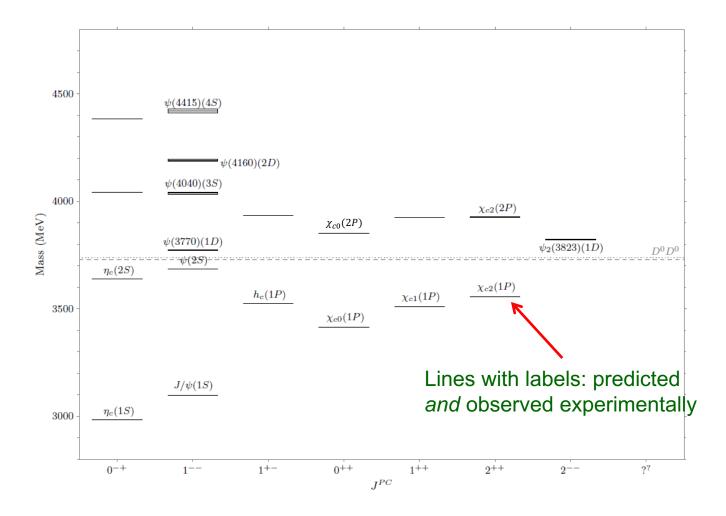


Quarkonium

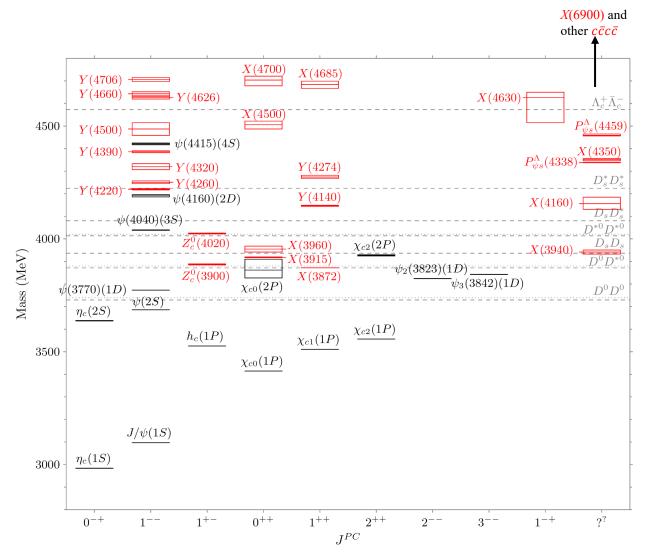
- Can try the same tricks for bound states of a quark and antiquark, except...
 - QCD is much *stronger* than QED [$\alpha \simeq 1/137$, $\alpha_{\rm QCD} = O(1)$]
 - QCD is much "*stickier*" than QED [gluons couple to both quarks & gluons, while photons couple to e^{\mp} but **not** to other photons]
- These two facts likely explain confinement
 - Indeed, the QCD analogue does have a ``dissociation energy'' threshold, but due to confinement, bound states can still appear above it
- The system is quite relativistic for light quarks *u*, *d*, *s*
 - But it should be nonrelativistic for heavy quarks c (charm), b (bottom)
- So model the strong "Coulomb" force and confining force:

$$V(r) = -\frac{a}{r} + br$$
 Cornell potential

What the charmonium system $c\bar{c}$ should look like



What the charmonium system $c\bar{c}$ really looks like



For decades, hadronic spectroscopy was the core of high-energy physics

- 1947: Discovery of π^{\pm} , K^{\pm} , K^{0}
- 1950–1965: The hadron zoo; strangeness; the Eightfold Way; the quark model; color charge
- 1974: Charmonium; evidence for asymptotic freedom & QCD
- 1977: Bottomonium; 3rd generation of quarks needed for *CP* violation
- 1983: First full reconstruction of *B* meson decays
- 1983: *W* & *Z* bosons.

Look for top quark! Look for Higgs! Look for BSM!!

• 1983– Hadron spectroscopy: Fill out the quark-model multiplets



The Breit-Wigner resonance

All resonances in physics (for damped mechanical oscillators, LRC circuits, elementary particles with short lifetimes) mean essentially the same thing:

 a large enhancement (*peak*) of the system response in a particular range

of measured energy in the form of a Lorentzian distribution

$$|f(E)|^2 \propto \frac{1}{(E^2 - M^2)^2 + M^2 \Gamma^2}$$

• In the case of quantum mechanics, the amplitude *f* is

$$f(E) \propto \frac{1}{(E^2 - M^2) + iM\Gamma}$$

(Breit-Wigner amplitude)

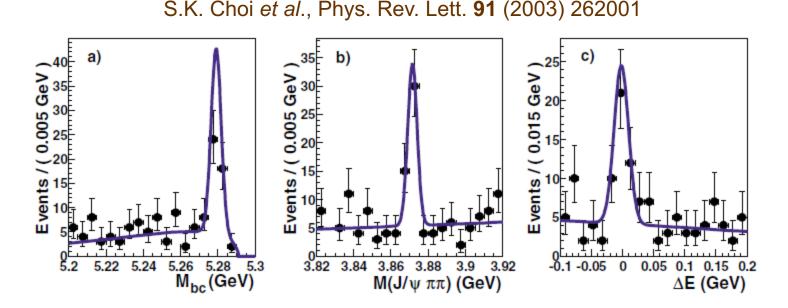
One finds that in increasing E through the peak, the phase angle of f(E) increases from 0 to 2π, *i.e.*, it forms a *loop* in the complex plane
 → evidence of a true resonance, not just a "bump" in the amplitude

When does one declare a particle "confirmed"?

- Threshold adopted for unambiguous *observation*: The signal is 5 *standard deviations*, or 5σ, away from being the result of random statistical fluctuations that one expects from the data if the effect or particle does not exist
 - The chance of such a random fluctuation is 1 in 3.5 million
 - History: This standard was first applied to the discovery of the heaviest quark flavor, *t* (top) at Fermilab in 1995
- The threshold for unambiguous *discovery confirmation*:
 - At least two experiments must observe a particle at 5σ , either at two distinct facilities or through two distinct physical processes
 - Both of the two big Fermilab experiments, DØ and CDF, had to observe the top quark before it was declared "discovered"

In 2003...

The Belle Collaboration at KEK [Japan] found evidence for a new particle at a mass of 3872 MeV ($\approx 4m_p$), decaying to $J/\psi \pi^+\pi^-$



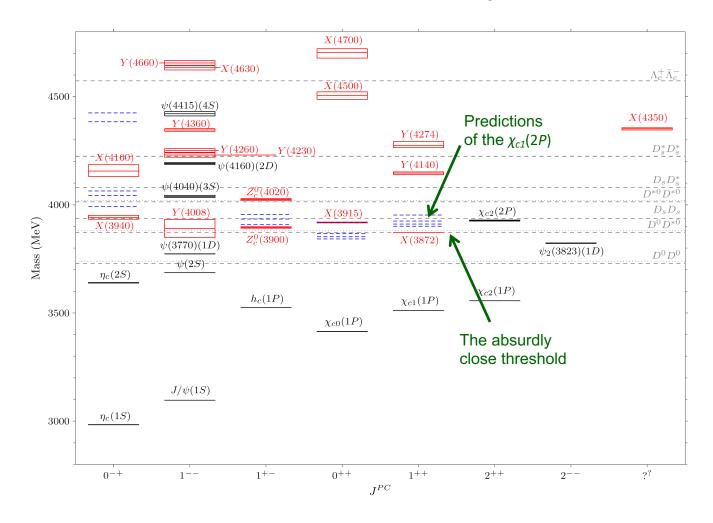
Looking for new particles wasn't even their primary goal! That was to measure *CP* violation in the *B*-meson system, which to that point had been seen only in neutral *K* decays

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X = Unknown

- X(3872) believed to contain cc since in same mass range as charmonium
 & it always decays into a final state containing cc (charmoniumlike)
- Has been confirmed at BABAR (SLAC), CDF, DØ (Fermilab), LHCb, CMS, ATLAS, COMPASS (CERN), BESIII (Beijing)
- $J^{PC} = 1^{++}$, but not believed to be ordinary $c\bar{c}$: Mass is many 10's of MeV below nearest $\bar{c}c$ candidate with these quantum numbers, $\chi_{c1}(2P)$
- Believed to be principally a $(c\bar{c}u\bar{u})$ tetraquark
 - $m_{X(3872)} = 3871.65 \pm 0.06 \text{ MeV}$
 - Note: $m_{X(3872)} m_{D^{*0}} m_{D^0} = -0.04 \pm 0.09 \text{ MeV}$ Leads to endless speculation that X(3872) is a $D\overline{D}^*$ hadronic molecule $(D^0 = c\overline{u}, \overline{D}^{*0} = \overline{c}u)$
 - Width: $\Gamma_{X(3872)} = 1.19 \pm 0.21$ MeV (Very small for a hadron this heavy!)

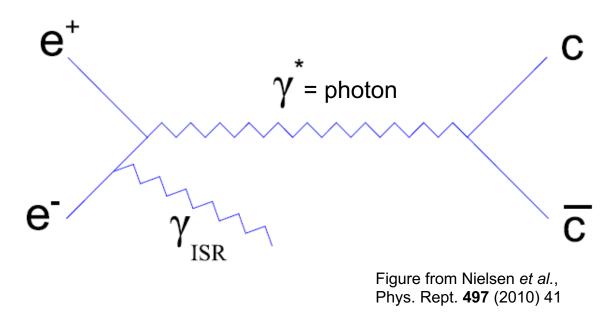
The Peculiar X(3872), the first *tetraquark*



...And in 2005: **Y**

BABAR Collaboration (B. Aubert et al., PRL 95, 142001 [2005])

Charmoniumlike states started to show up in initial-state radiation (ISR) e^+e^- annihilation:



Such states necessarily have $J^{PC} = 1^{--}$ (same quantum numbers as the photon), and are called "Y"

The first one discovered (2005) is named Y(4260)

...And in 2013: **Z**

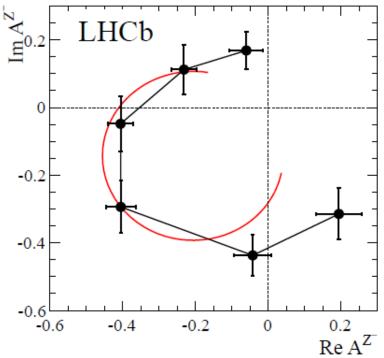
BESIII Collaboration [Beijing] (M. Ablikim *et al.*, PRL **110**, 252001 [2013]), Belle Collaboration [Japan] (Z. Liu *et al.*, PRL **110**, 252002 [2013])

- A charged charmoniumlike resonance is observed in $Y(4260) \rightarrow \pi^{-}(\pi^{+}J/\psi)$
- Minimal possible quark content: $c\bar{c}u\bar{d}$: No question that it has four valence quarks
- Now called $Z_c^+(3900)$, $J^P = 1^+$
- The first manifestly exotic state ever confirmed beyond 5σ by two experiments
 - [not counting the $\Theta^+(1540)$]
- What if all these states are not really states, but rather brilliant forgeries, like the $\Theta^+(1540)$?

...And in 2014: Resonance

LHCb Collaboration (R. Aaij et al., PRL 112, 222002 [2014])

- The first charged charmoniumlike exotic, now called Z_c+(4430), was actually first seen by Belle in 2008 (PRL 100, 142001 [2008]) and confirmed by them in papers from 2009 and 2013
- LHCb not only confirmed the $Z_c^+(4430)$ at 13.9 σ , and showed $J^P = 1^+$, but for the first time plotted the full complex production amplitude and showed that it obeys the proper phase-shift looping behavior of a Breit-Wigner **resonance**
- Welcome to the Age of the Third Hadron



...And in 2015: **P**_c

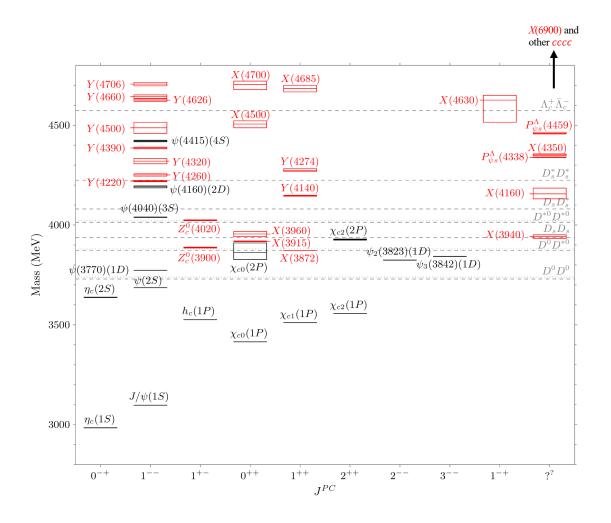
LHCb Collaboration [R. Aaij et al., PRL 115 (2015) 072001]

- The first two *baryonic* charmoniumlike exotics, $P_c^+(4450)$, $P_c^+(4380)$
- Decay to $J/\psi + p \rightarrow Valence$ structure $c\bar{c}uud$: **Pentaquarks**!
- $m_1 = 4380 \pm 8 \pm 29$ MeV, $\Gamma_1 = 205 \pm 18 \pm 86$ MeV, 9σ significance
- $m_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV, $\Gamma_2 = 39 \pm 5 \pm 19$ MeV, 12σ significance
- Preferred J^P • $\mathsf{Im}\,\mathsf{A}^{P_c}$ (a) (b) 0.1 assignments: 0.05 $(3/2^{-}, 5/2^{+}) >$ $(3/2^+, 5/2^-) >$ -0.05 P_c(4450) $(5/2^+, 3/2^-)$ P_c(4380) -0.1 Welcome to -0.15 the Age of the -0.2 -0.25 **Fourth Hadron** -0.3 LHCb 0.05 0.1 0.15 -0.1 -0.05 -0.05 0 0 0.05 0.1 0.15 0.25 0.3 0.2

Re A^P

Re A^P

Neutral charmoniumlike system, May 2023



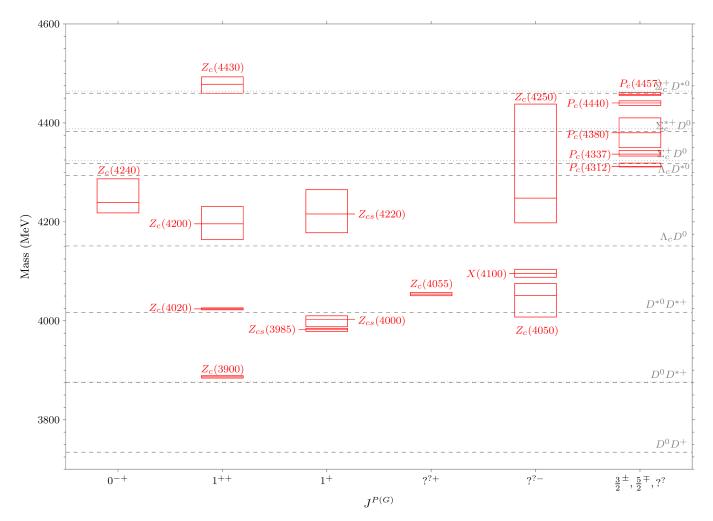
Several of the states are quite close to di-hadron thresholds

Most prominent example: $m_{X(3872)} - m_{D^0} - m_{D^{*0}}$ $= -40 \pm 90 \text{ keV}$

cf. the deuteron: $m_d - m_p - m_n$ = -2.2452(2) MeV

But many are *not* close to thresholds! *e.g.*, the $1^{--}Y$ states

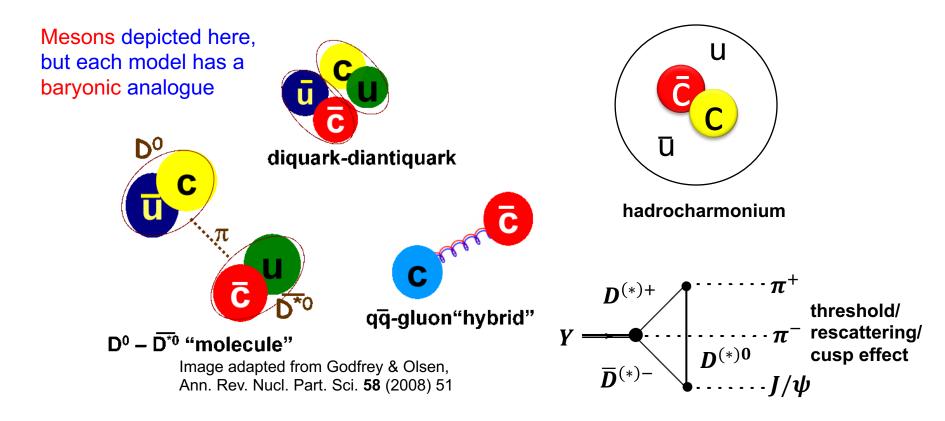
Charmonium: May 2023 Charged sector



Heavy-quark exotics census: May 2023

- 64 observed heavy-quark exotics, both tetraquarks and pentaquarks
 - 49 in the charmonium sector (neutral & charged, including open-strange)
 - 5 in the (much less explored) bottomonium sector
 - 4 with a single c quark (and an s, a u, and a d)
 - 1 with a single *b* quark (and an *s*, a *u*, and a *d*)
 - 4 with all c and \overline{c} quarks
 - 1 with two *c* quarks
- My naïve count estimates over 100 more exotics are waiting to be discovered

How are exotics assembled?



"Each of the interpretations provides a natural explanation of parts of the data, but neither explains all of the data. It is quite possible that both kinds of structures appear in Nature. It may also be the case that certain states are superpositions of the compact and molecular configurations." —Karliner, Santopinto, *et al.* [Snowmass, "Substructure of Multiquark Hadrons", 2203.16583]

Trouble with the dynamical pictures

- Hybrids
 - Only usable for neutral states; then what are the Z's?
 - Only produces certain quantum numbers (like $J^{PC} = 1^{++}$) easily
- Diquark and hadrocharmonium pictures
 - What stabilizes the states against instantly segregating into meson pairs?
 - Diquark models tend to overpredict the number of bound states
 - Why wouldn't hadrocharmonium *always* decay into charmonium, instead of *DD*?
- Threshold effects
 - Might be able to generate some resonances on its own, but >60 of them? And certainly not ones as narrow as X(3872) ($\Gamma < 1.2$ MeV)

The hadron molecular picture

 A number of XYZ (and P_c) states are suspiciously close to hadron thresholds

- *e.g.*, recall $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.04 \pm 0.09 \text{ MeV}$

- So we theorists have *many hundreds* of papers analyzing the *XYZ* states as di-meson molecules
- But not all of them are close to thresholds!

- *e.g.*, *Z*(4430) and many of the *Y* states are prime examples

And even X(3872), which due to its tiny binding energy should be *huge* (~ 10 fm), has substantial decays to J/ψ, which is only ~ 0.4 fm in radius

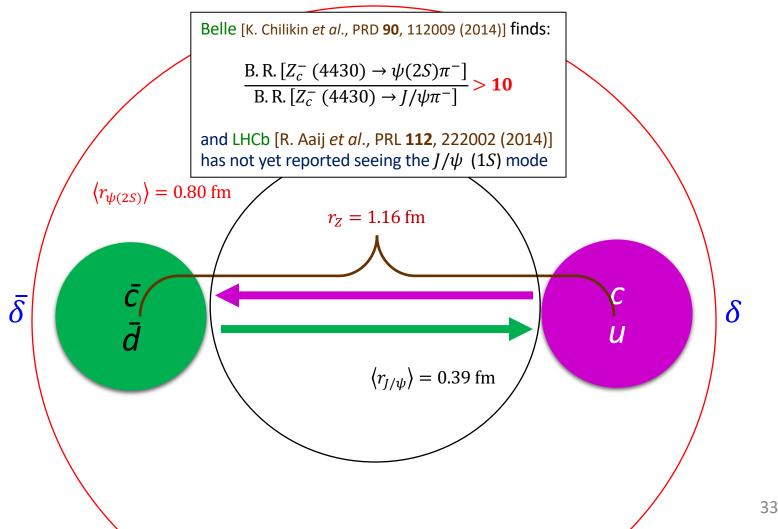
It is entirely possible...

- ...that no single structure accommodates all of these exotic states
- Some could be *molecules*, some could be *hybrids*, some could be *kinematical effects*, or *quantum-mechanical mixtures* of these...
- But what these static pictures fail take into account is the full complexity of QCD dynamics for rather short-lived states
- Alternative: diquark attraction in a dynamical setting

Amazing (well-known) fact about color:

- Short-distance QCD: 3 ⊗ 3 → 3 (attractive diquark coupling) is *fully one-half* as strong as 3 ⊗ 3 → 1 (confining attraction) (*i.e.*, diquark attraction nearly as strong as confining attraction)
- Exact SU(2) analogue: Just as one computes a spin-spin coupling, $\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{2} \left[(\vec{s}_1 + \vec{s}_2)^2 - \vec{s}_1^2 - \vec{s}_2^2 \right],$ for particles in representations 1 & 2 combined into the representation 1+2: If $s_1, s_2 = \operatorname{spin} \frac{1}{2}$, and $\vec{s}_1 + \vec{s}_2 = \left[\operatorname{spin} 0; -\frac{3}{4} \right];$ $\left[\operatorname{spin} 1; +\frac{1}{4} \right]$
- So let us use the diquark quasiparticle δ as the core constituent

Diquarks allow large, but still strongly bound, states



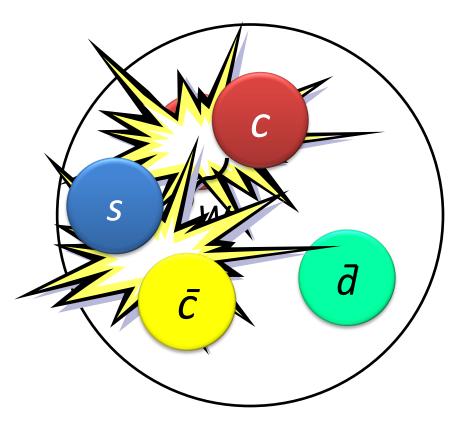
The dynamical diquark picture

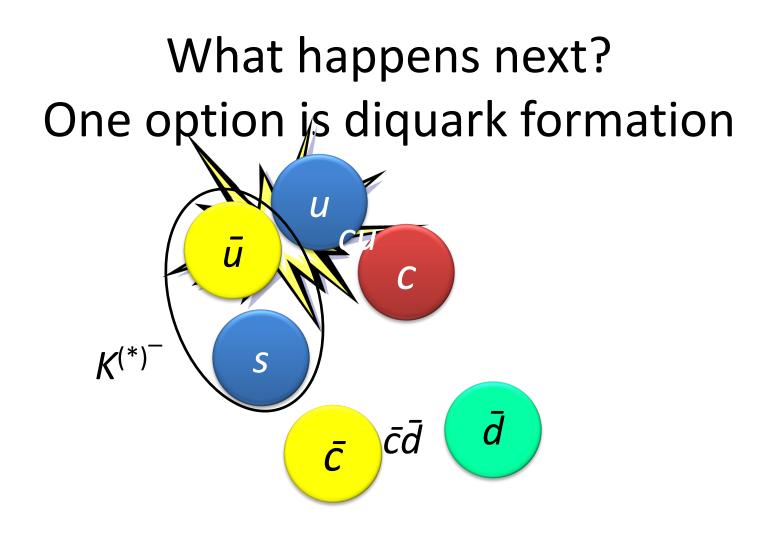
Brodsky, Hwang, RFL [PRL 113, 112001 (2014)]

- Heavy quarks provide nucleation points for diquark formation
- BUT the pairs are not in a static configuration; they are created with a lot of relative energy, and rapidly separate from each other
- Diquarks are not color neutral!
 They cannot, due to confinement, separate asymptotically far
- Diquark-antidiquark pair remain strongly connected by color flux tube \rightarrow tetraquark $(Qq)_{\overline{3}}(\overline{Q}\overline{q})_3 = \delta\overline{\delta}$
- Same color-triplet mechanism supports pentaquark formation, using a triquark: $[Q_3(\bar{q}_1\bar{q}_2)_3]_{\overline{3}}(\bar{Q}\bar{q})_3$ {RFL [PLB **749**, 454 (2015)]}
- They must decay into ordinary hadrons via large-r tails of conventional meson (or baryon) wave functions, which suppresses decay widths to make them observably narrow

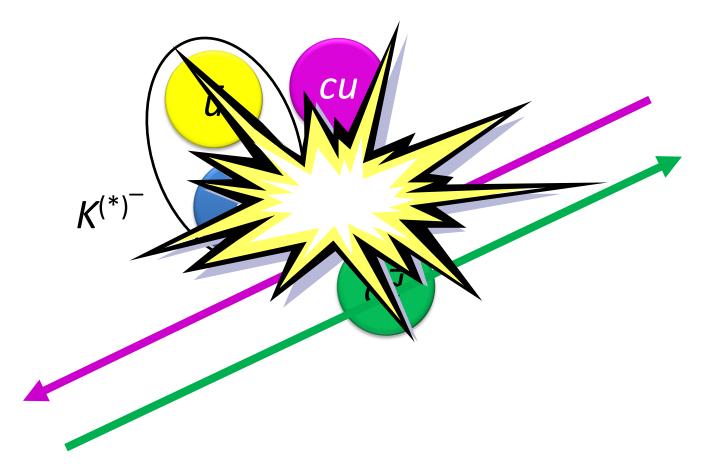
Nonleptonic \overline{B}^0 meson decay

B.R.~22% (Branching Ratio = probability)





What happens next? One option is diquark formation



The dynamical diquark model:

RFL [PRD 96, 116003 (2017)]

- Exotic eigenstate: configuration in which KE of heavy di-(tri-)quarks converts into potential energy of color flux tube
- Two heavy, slow sources connected by light degrees of freedom (d.o.f.)? That's the adiabatic approximation → ordinary Schrödinger equation
- At energies where only one interaction potential function is important (*i.e.*, away from level-crossing thresholds), can use the single-channel approximation
- Together, form the Born-Oppenheimer (BO) approximation
- BO potentials created by light d.o.f. are same ones computed in lattice simulations of heavy-quark hybrids, labeled by axial quantum numbers such as in the potentials Σ_g^+ , Π_u^- , etc.

First numerical results of the model

[Giron, RFL, Peterson, JHEP **05** (2019) 061]

	BO states	Potential	M (GeV) m_δ		$\left< 1/r \right>^{-1}$	$\langle r angle$ (fm)
_	$\Sigma_g^+(1S)$	JKM	$\overline{3.8711}$	1.8747	0.27202	0.36485
Fixed to		CPRRW	3.8721	1.8535	0.27519	0.36904
X(3872)		BGS	3.8718	1.9402	0.21347	0.30268
	$\Sigma_g^+(2S)$	JKM	4.4430	1.8747	0.42698	0.69081
Right atop		CPRRW	4.4410	1.8535	0.43057	0.69640
<i>Z_c</i> (4430)		BGS	4.4674	1.9402	0.42621	0.69756
	$\Sigma_q^+(1P)$	JKM	4.2457	1.8747	0.48968	0.56601
Right atop	5	CPRRW	4.2435	1.8535	0.49379	0.57067
Y(4220) Right atop Y(4660)	$\Sigma_g^+(2P)$	BGS	4.3471	1.9402	0.48361	0.56787
		JKM	4.7128	1.8747	0.62445	0.84285
	5	CPRRW	4.7092	1.8535	0.62911	0.84913
		BGS	4.7416	1.9402	0.65333	0.89663
	$\Sigma_{g}^{+}(1D)$	JKM	4.5318	1.8747	0.66414	0.73132
Right atop	3	CPRRW	4.5282	1.8535	0.66921	0.73668
X(4500)	$\Sigma_a^+(2D)$	BGS	4.6151	1.9402	0.69780	0.77323
		JKM	4.9476	1.8747	0.78634	0.98022
	5	CPRRW	4.9431	1.8535	0.79199	0.98697
		BGS	4.9486	1.9402	0.84597	1.0645
	$\Pi_{u}^{+}(1P) \&$	JKM	4.9156	1.8747	0.44931	0.56950
	$\Sigma_u^-(1P)$	CPRRW	4.8786	1.8535	0.44614	0.56438

Dynamical diquark model, fine structure & isospin Giron, RFL, Peterson [JHEP **01**, 124 (2020)]

- With only a few known exotics in each multiplet, need to identify the most physically important perturbation Hamiltonian operators
- *e.g.*, the multiplet $(cq)(\overline{cq}) \Sigma_g^+(1S)$ contains 6 I = 0 and 6 I = 1 states, and we know only $X(3872) [I = 0], Z_c(3900), Z_c(4020) [I = 1]$
- Fixes 2 operators, taken to be:
 (1) quark spin-spin coupling within each diquark, and
 (2) isospin/spin exchange between diquarks (analogous to π exchange)
- Naturally predicts X(3872) to be lightest narrow state in multiplet
- Naturally predicts $Z_c(3900)$ to decay preferentially to J/ψ ($s_{c\bar{c}} = 1$) and $Z_c(4020)$ to h_c ($s_{c\bar{c}} = 0$), as is experimentally observed

If the model is any good, it must also apply to other flavor sectors

Using the same Hamiltonian operators (plus spin-orbit & tensor for orbitally excited states), apply to:

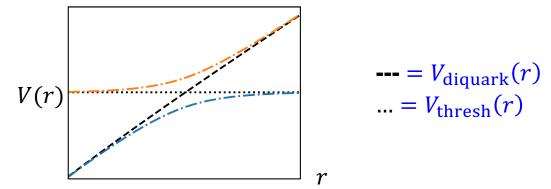
- the $(cq)(\overline{cq})$ negative-parity states {Giron, RFL [PRD **101**, 074032 (2020)]}
- the $(bq)(\bar{b}\bar{q})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
- the $(cs)(c\bar{s})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
- the $(cc)(c\bar{c})$ sector {Giron, RFL [PRD **102**, 074003 (2020)]}
- the $(cq)(c\bar{s})$ sector {Giron, RFL, Martinez [PRD **104**, 054001 (2021)]}
- the $(cu)(\bar{c}ud) \& (cs)(\bar{c}ud)$ pentaquarks {Giron, RFL [PRD 104, 114028 (2021)]}

But what about the closeness of some exotics to di-hadron thresholds?

- $m_{X(3872)} m_{D^0} m_{D^{*0}} = -40 \pm 90 \text{ keV} \text{ cannot be an accident!}$
- This binding energy is much smaller than expected for "conventional" hadron molecule—more likely a threshold rescattering effect [coupling to near-on-shell particles leads to an enhanced amplitude]
- A great deal of theory work has been performed to explain some exotics as purely threshold effects, but not every threshold seems to have a prominent associated state

Diabatic corrections

- But what if both types of interaction potentials are present (diquark-antidiquark and di-hadron threshold)?
- This is a well-known problem in atomic physics: One must perform a coupled-channel calculation to find mixed-configuration eigenstates near level crossing, where the adiabatic approximation fails
- Rigorous method to incorporate these effects: diabatic approach



Diabatic framework first results

RFL & Martinez [PRD 106 (2022) 7, 074007]

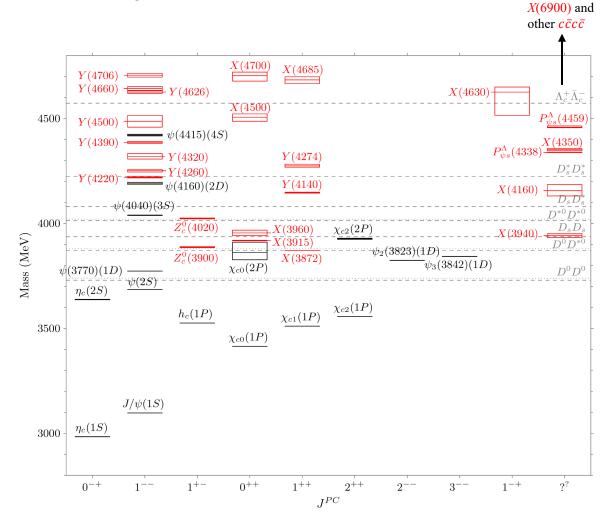
• It is not at all unnatural for a diquark $(\delta \overline{\delta})$ state near a threshold to acquire a very large di-hadron component, while others do not:

J^{PC}	E (MeV)	$\delta \overline{\delta}$	$D\bar{D}^*$	$D_s \bar{D}_s$	$D^* \bar{D}^*$	$D_s^* \bar{D}_s^*$	$\langle r \rangle$ (fm)	$\langle r^2 \rangle^{1/2} ({ m fm})$
0^{++}	3905.4	63.0%		27.4%	8.4%	1.2%	0.596	0.605
1^{++}	3871.5	8.6%	91.4%				4.974	5.459
2++	3922.3	83.1%		1.5%	13.9%	1.5%	0.443	0.497

Conclusions

- The past several years have provided confirmation of the existence of the *tetraquark* and observation of the *pentaquark*, the third and fourth classes of hadron
- Over 60 such states (X, Y, Z, P_c) have thus far been observed
- All of the popular physical pictures for describing their structure seem to suffer some inadequacies
- We developed a dynamical model based on diquark-antidiquark (or diquark-triquark) pairs rapidly separating until forced to hadronize due to confinement
- Then, which particles the X, Y, Z, P_c states like to decay into, their spectrum, and even proximity to thresholds, can be understood
- With my students Giron, Peterson, Gens, & Martinez, this picture has blossomed in the past 4 years into a full predictive model

It's Still Quite Early! This ``Map'' of New Particles in 2023



...Might Be As Incomplete As This Map of the New World in 1540

