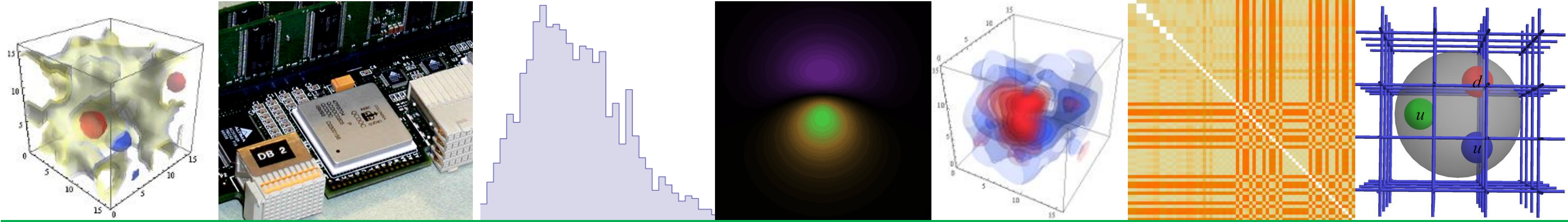


Mapping the Parton Distributions of Hadrons with Lattice QCD

HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

NSF

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

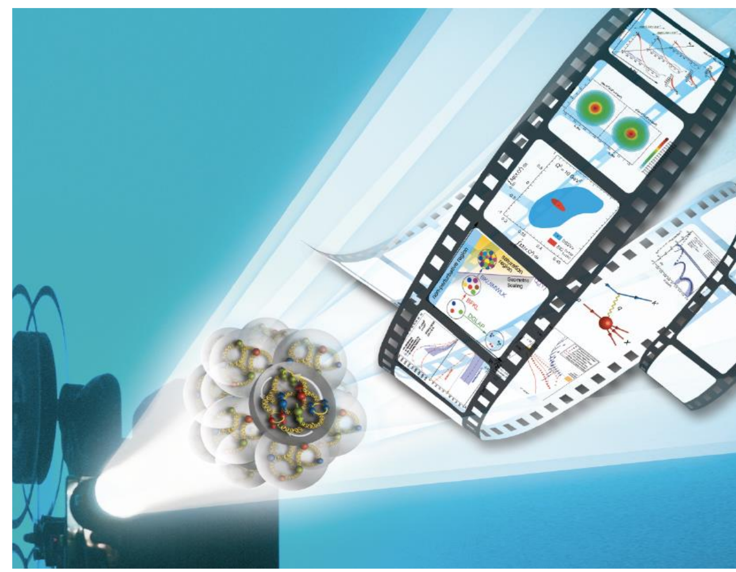
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701



Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

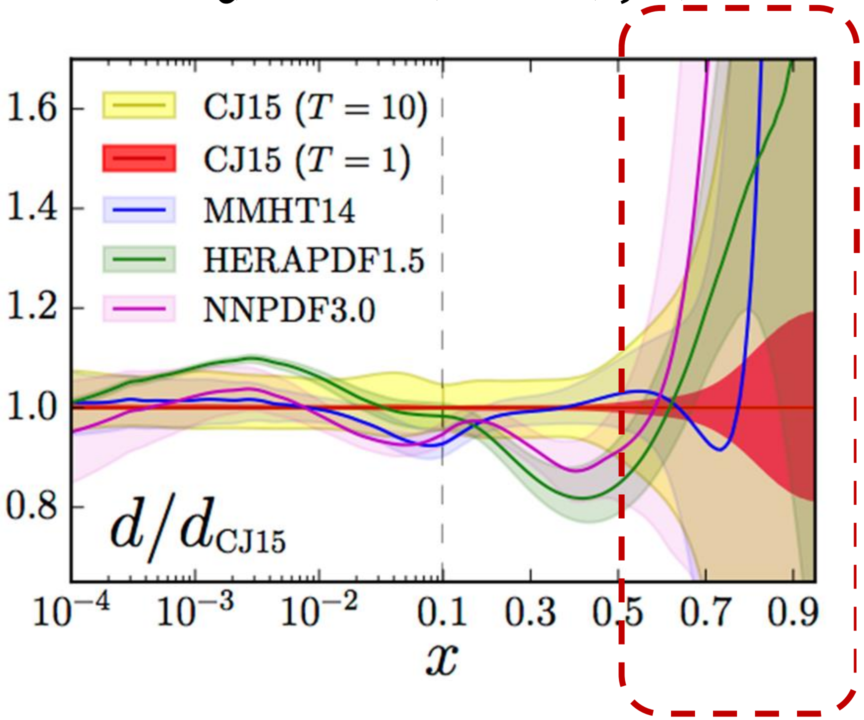
- Assumptions imposed

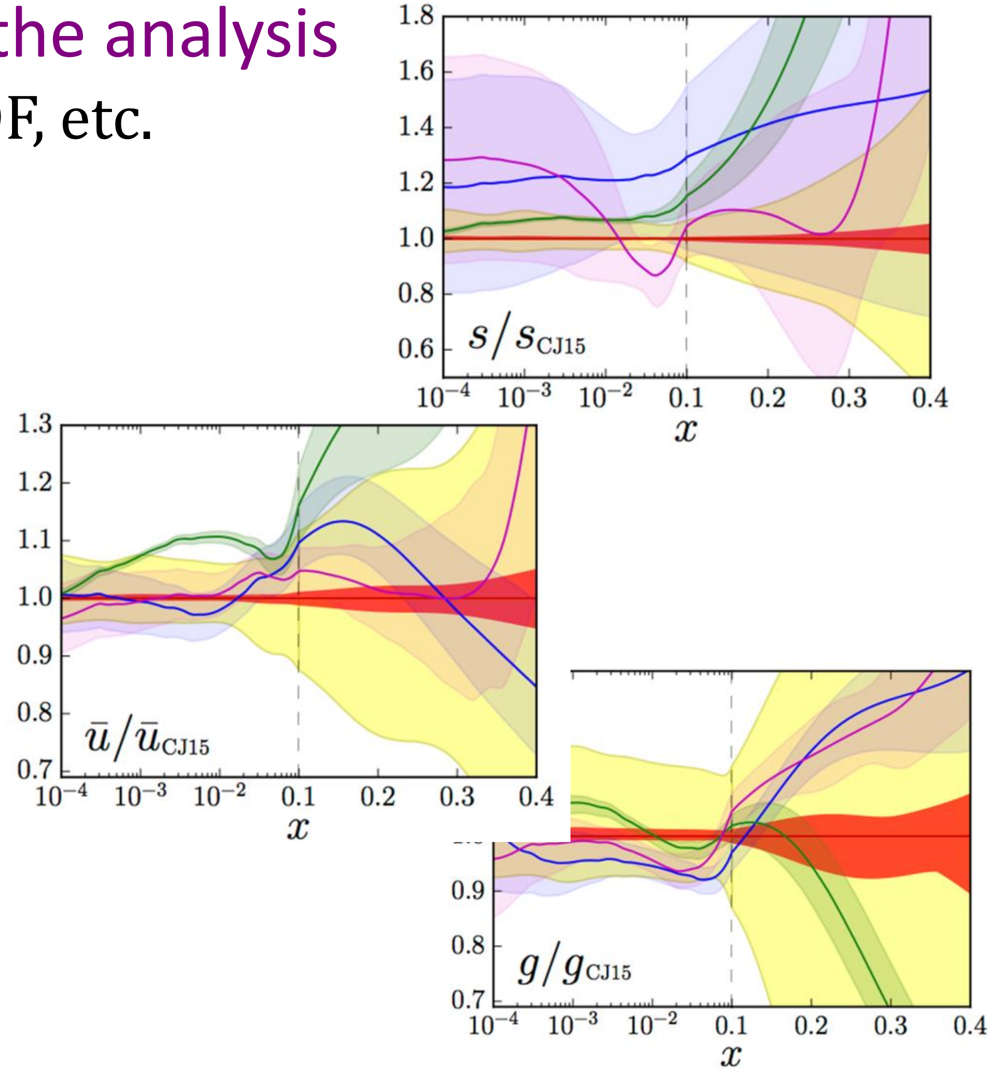
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.





CTEQ-JLAB
https://www.jlab.org/theory/cj/

Outline

§ Consumer's Guide to Lattice Structure Calculations

- **Nucleon** structure with controlled systematics in the physical limit ($m_\pi \to m_\pi^{\mathrm{phys}}, a \to 0, L \to \infty$)
- PDF Moments

§ *x*-dependent Hadron Structure

- Recent Lattice PDFs Progress
- Applications to Generalized Parton Distributions
- Future Prospects and Challenges

Biased selected results, highlighting work done by MSU students/postdocs

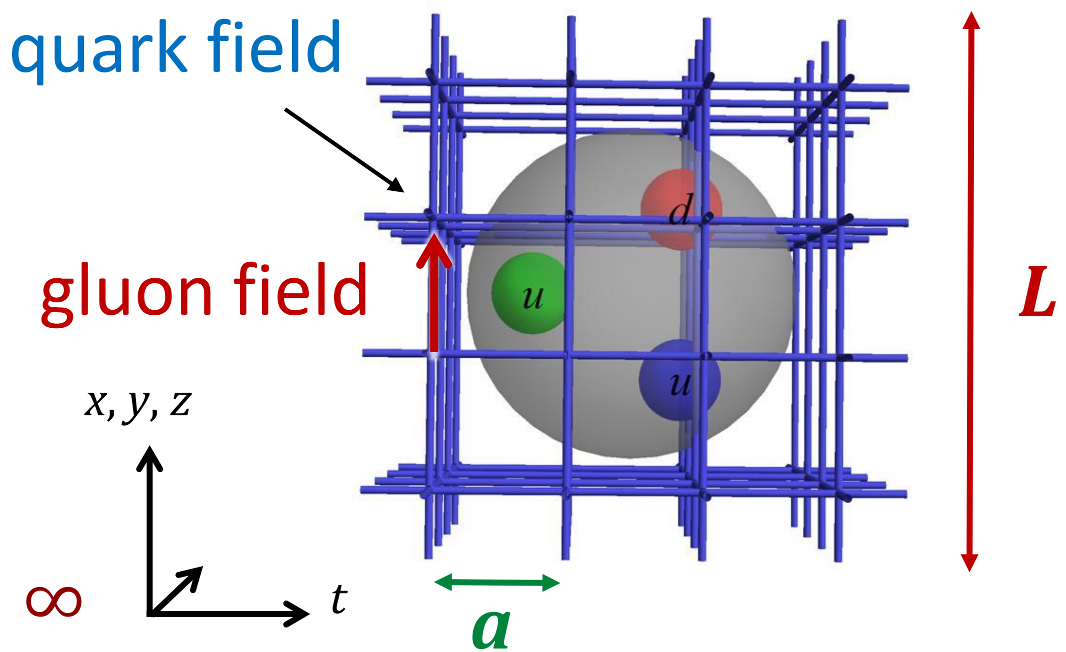
What is Lattice QCD?

- § Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- § Physical observables are calculated from the path integral

$$\langle 0|O(\bar\psi,\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar\psi\,\mathcal{D}\psi\, e^{iS(\bar\psi,\psi,A)}\,O\left(\bar\psi,\psi,A\right)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



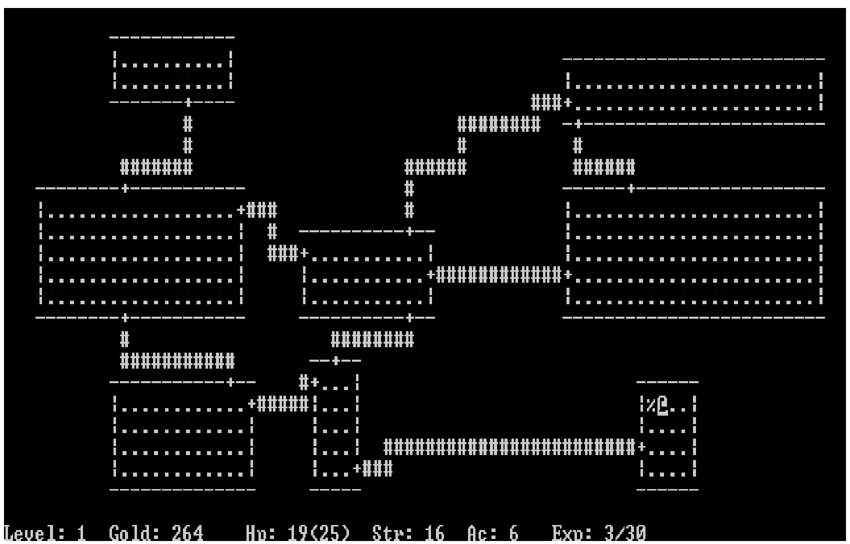
- § Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven’t we solved QCD yet?
- Progress is limited by computational resources

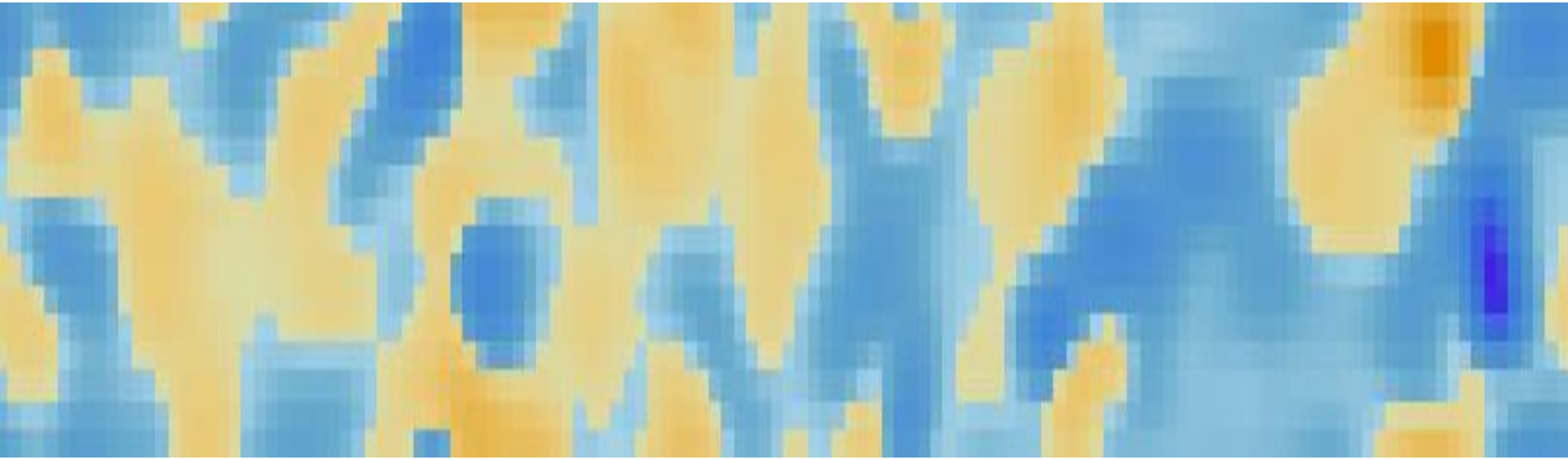
1980s



Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

Nucleon Matrix Elements

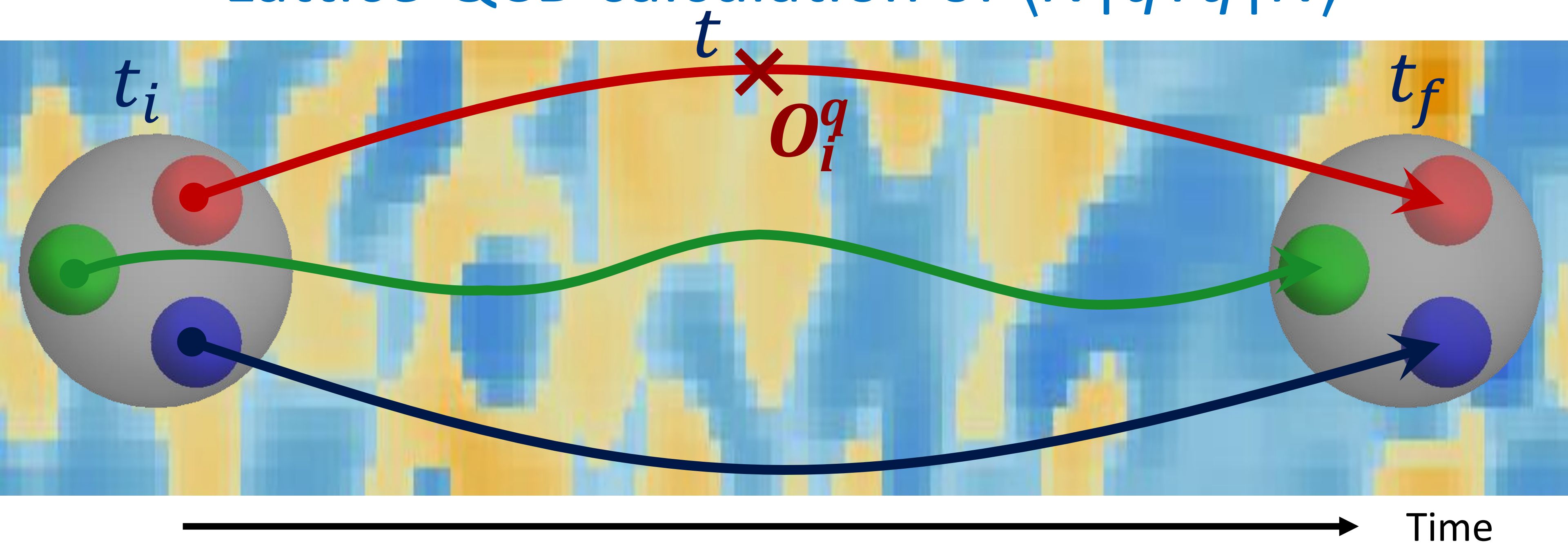


§ Pick a QCD vacuum

- Gauge/fermion actions, flavor (2, 2+1, 2+1+1), m_π, a, L, …

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

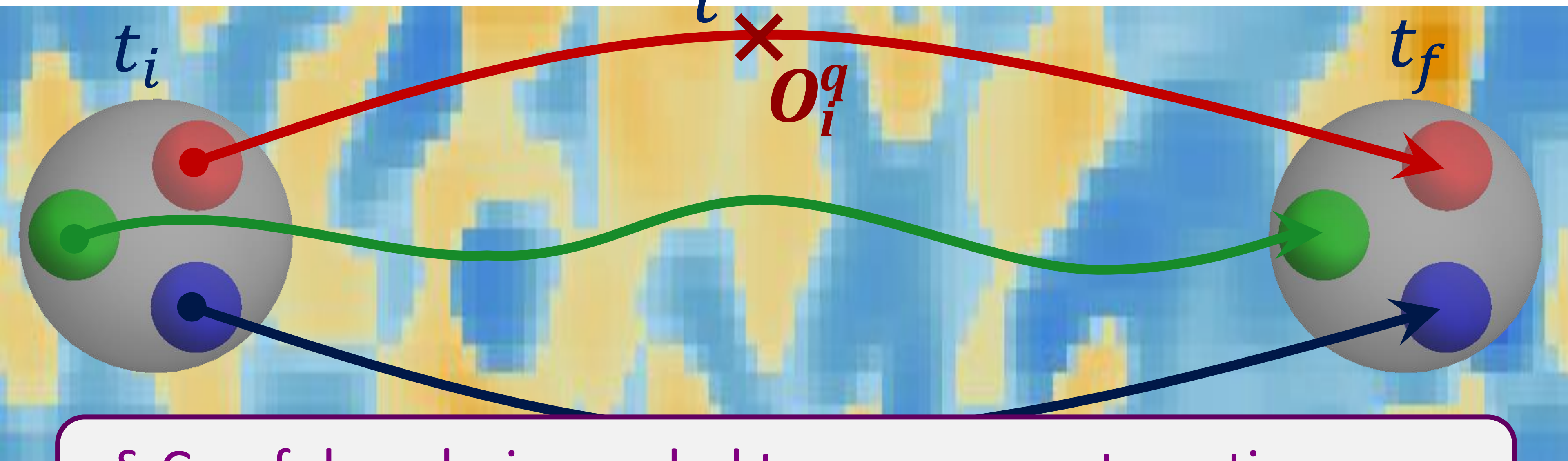


§ Construct correlators (hadronic observables)

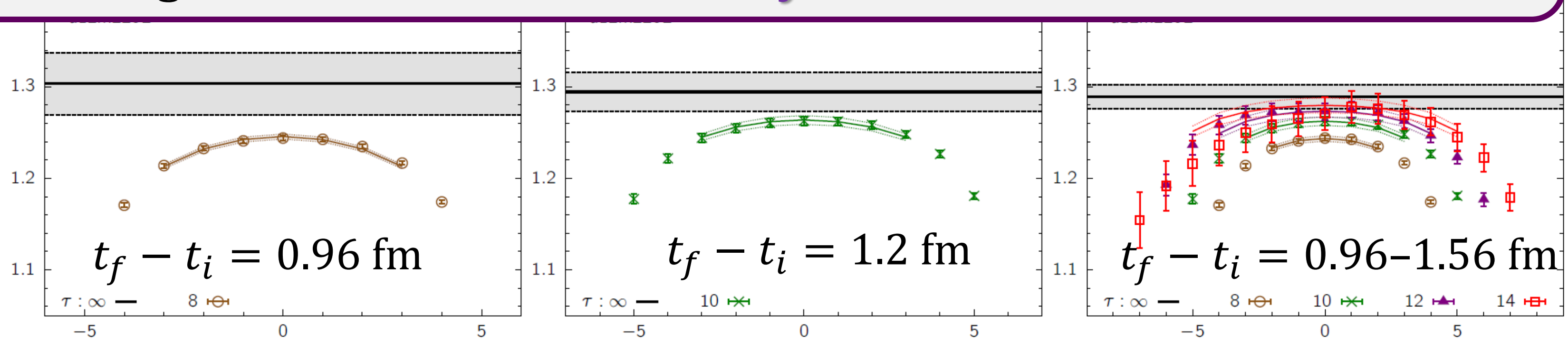
- Requires "quark propagator"
  Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

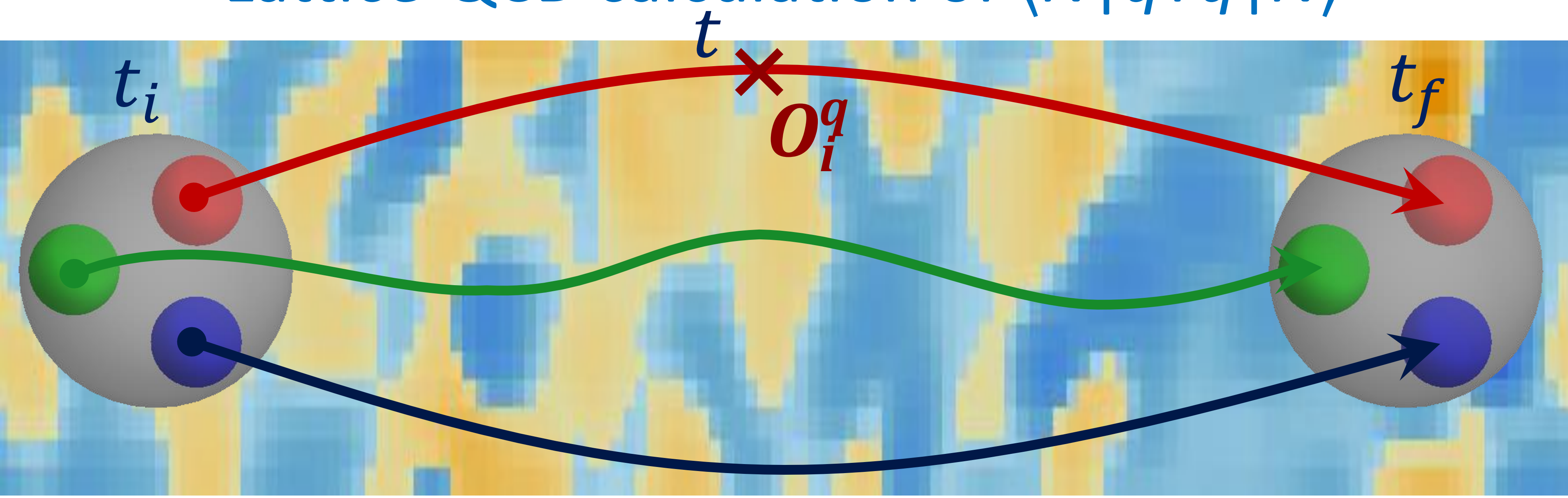


- § Careful analysis needed to remove systematics
  - Wrong results if **excited-state systematic** is not under control



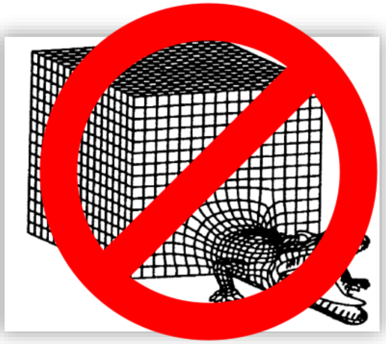
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



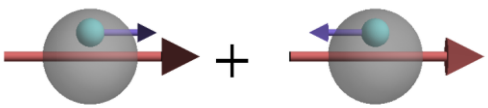
§ Systematic uncertainty (nonzero *a*, finite *L*, etc.)

- Nonperturbative renormalization
  e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



Moments of PDFs

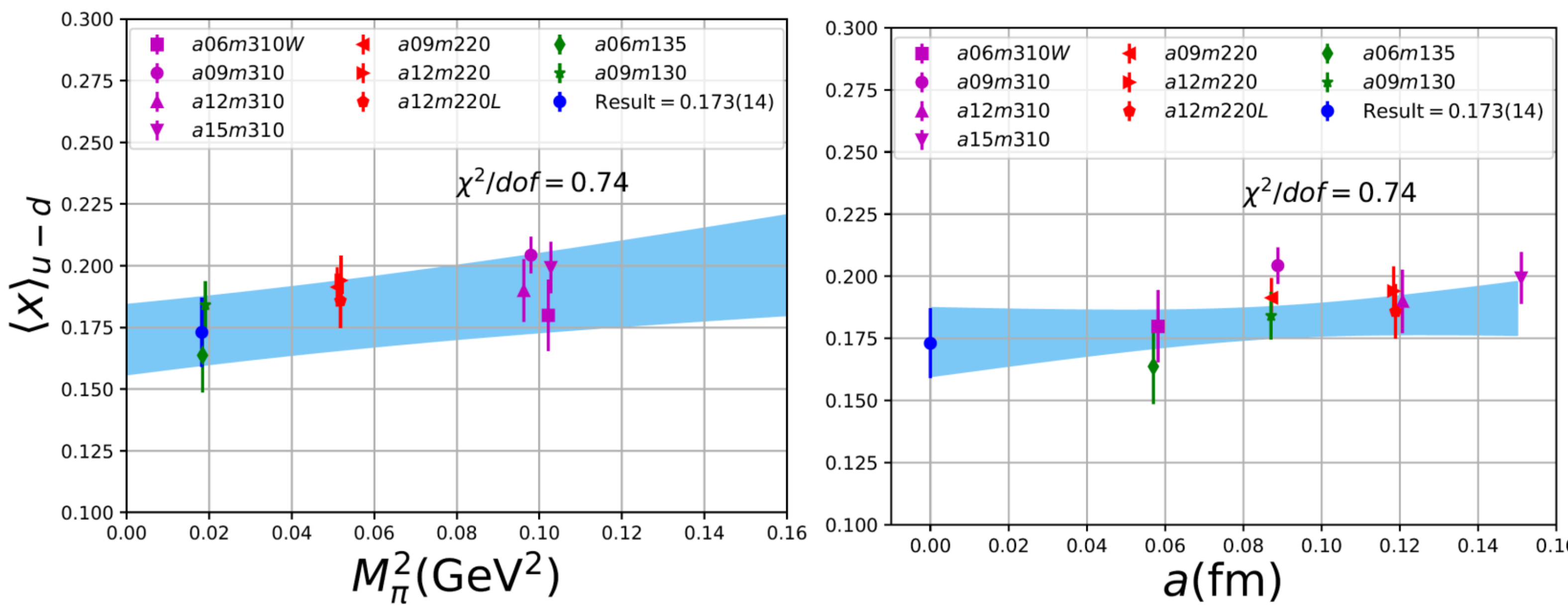
- First moments are most commonly done



- State-of-the art example
  - Extrapolate to the physical limit

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

Santanu Mondal et al (PNDME collaboration), 2005.13779



- Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

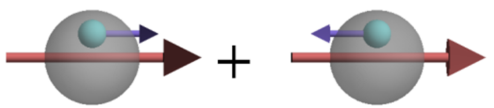
Moments of PDFs

§ PDG-like rating system or average

§ LatticePDF Workshop

- Lattice representatives came together and devised a rating system

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



§ Lattice QCD/global fit status

LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.171(18)	
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★		0.173(14)(07)	0.161(18)
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.180(25)(^{+14}_{-6})$	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★		0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.427(92)	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.482(69)(48)	0.411(8)
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■		0.47(4)(11)	

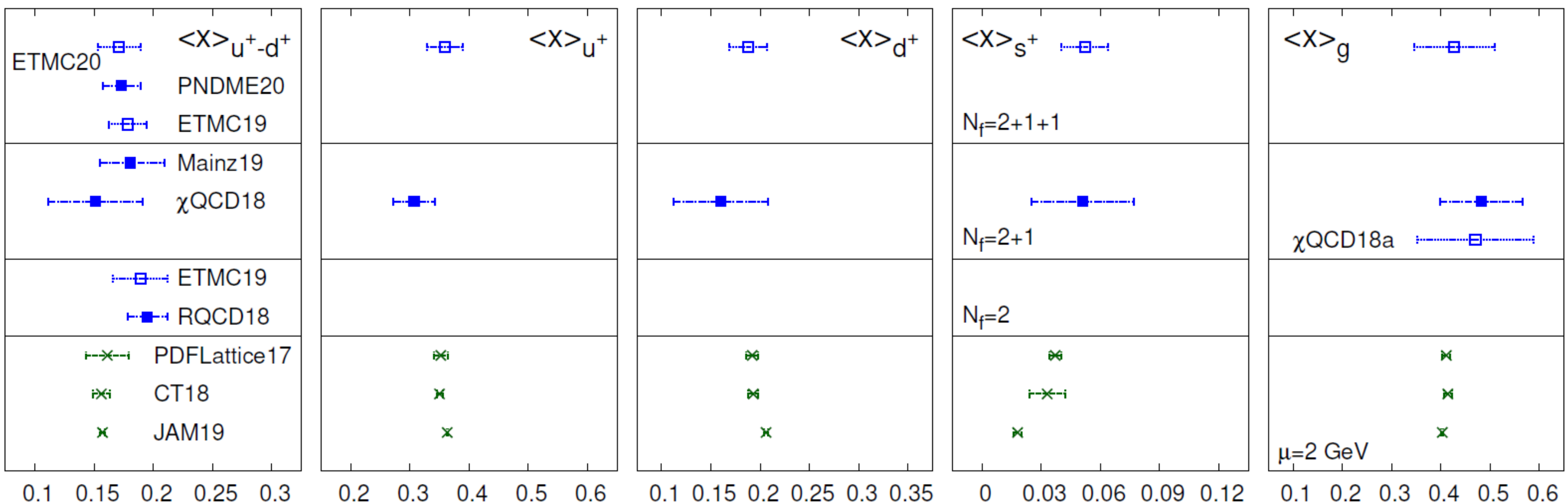
** No quenching effects are seen.

Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

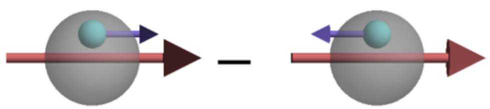
LatticePDF Report, 1711.07916, 2006.08636



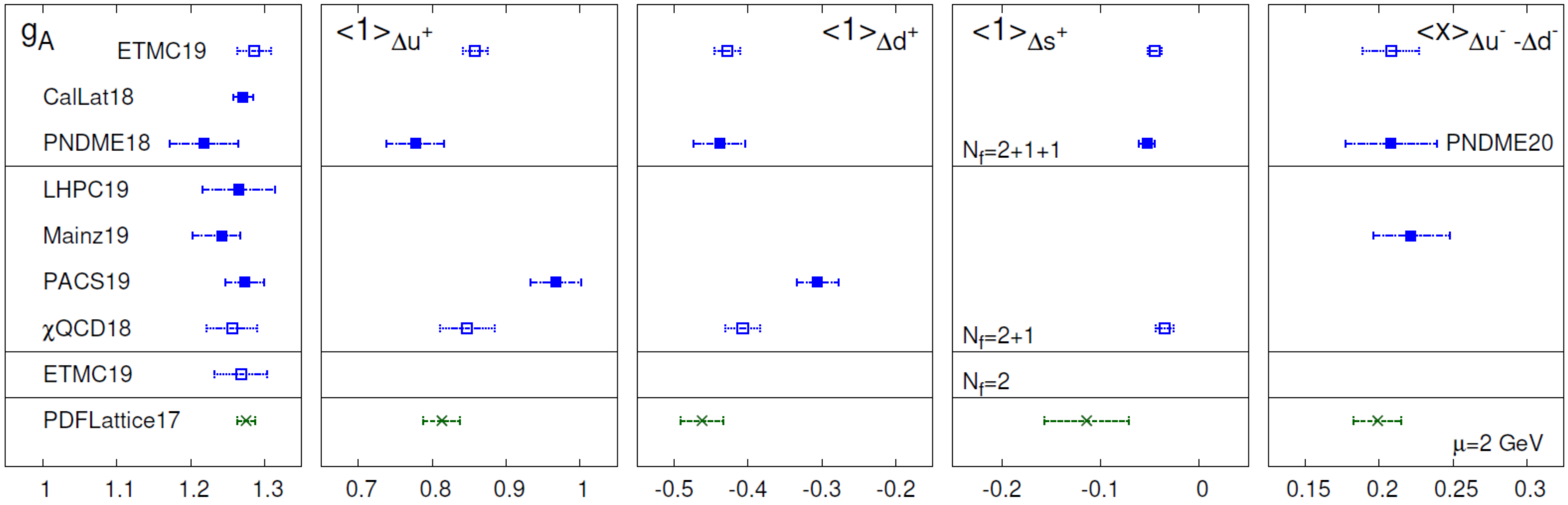
Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Recent lattice QCD/global fit status

$$\langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



LatticePDF Report, 1711.07916,2006.08636



Moments of PDFs

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$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

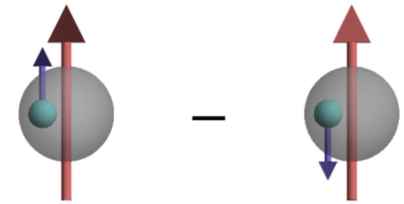
LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	
g_T	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	
$\langle 1\rangle_{\delta u^-}$	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	
$\langle 1\rangle_{\delta d^-}$	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	
$\langle 1\rangle_{\delta s^-}$	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

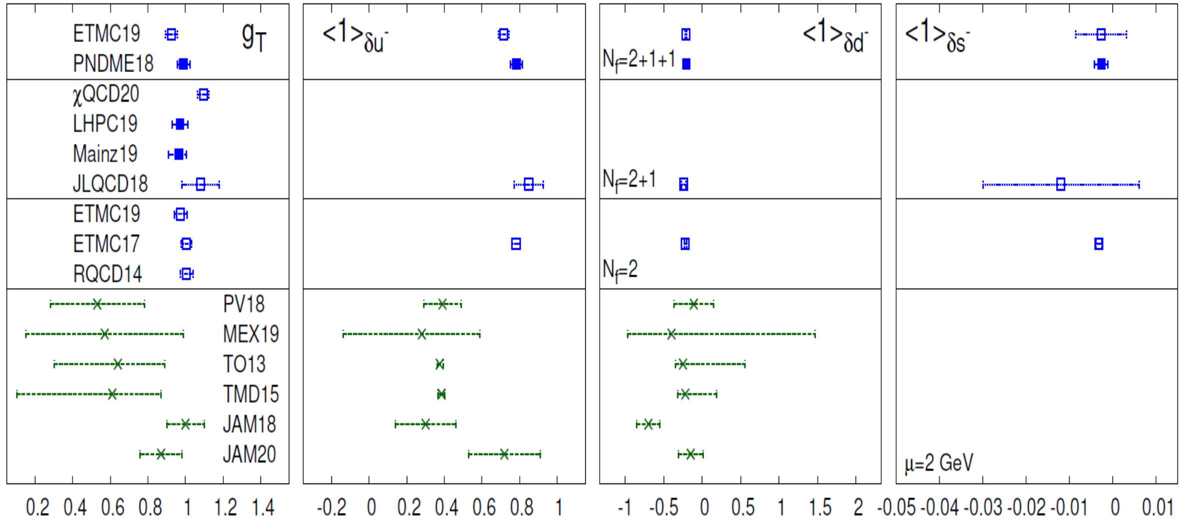
Moments of PDFs

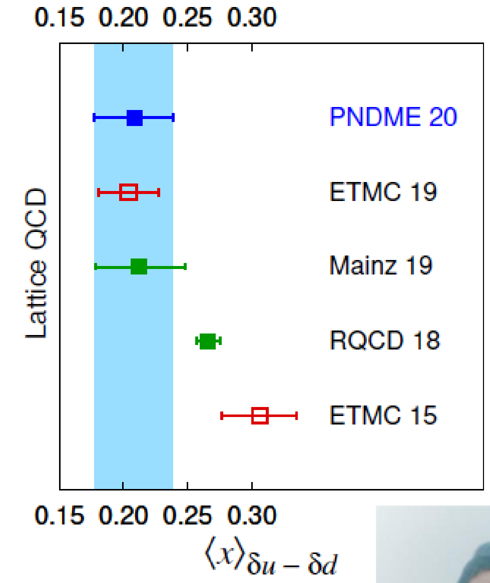
- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636



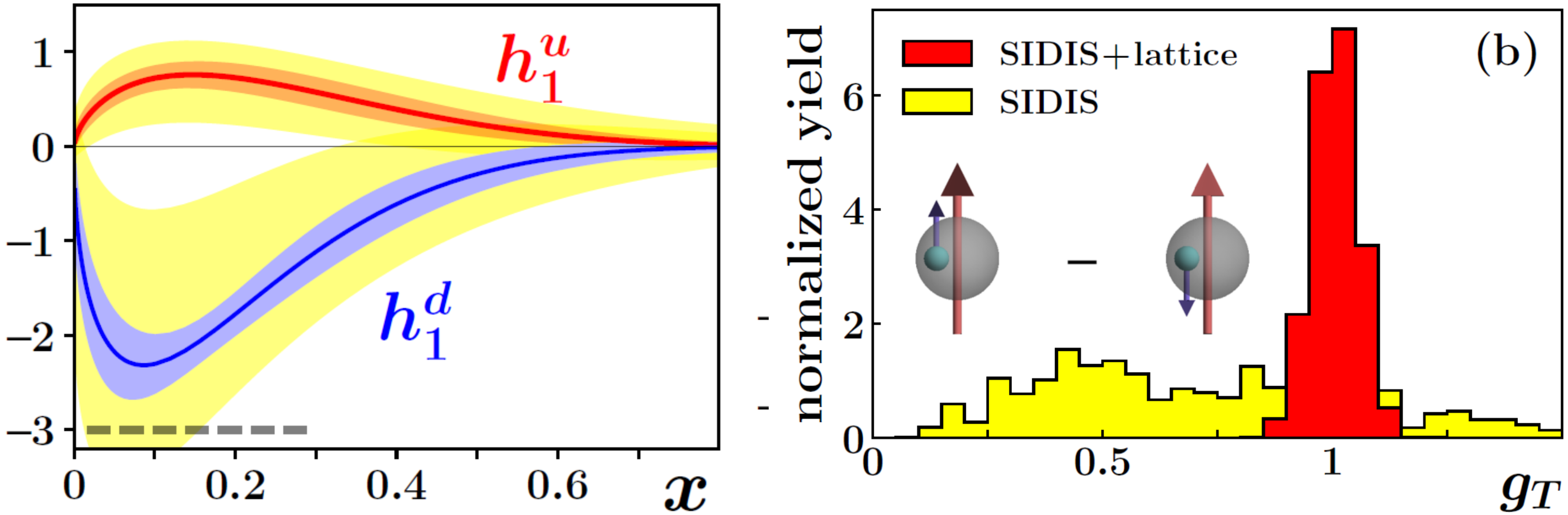


S. Mondal et al
2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global-analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

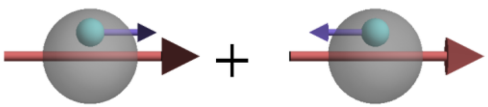


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Structure on the Lattice

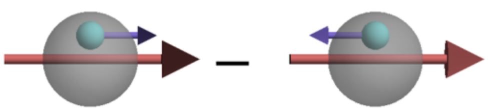
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



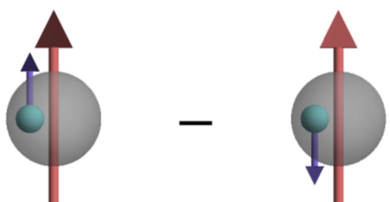
spin-averaged/unpolarized

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

PDFs on the Lattice

§ Limited to the lowest few moments

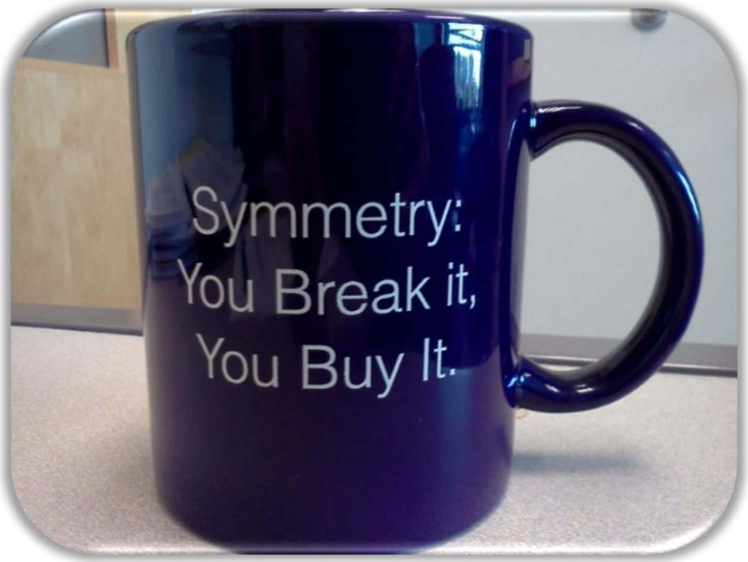
- For higher moments, all ops mix with lower-dimension ops
- Novel proposals to overcome this problem

§ Relative error grows in higher moments

- Calculation would be costly
- Hard to separate valence contrib. from sea

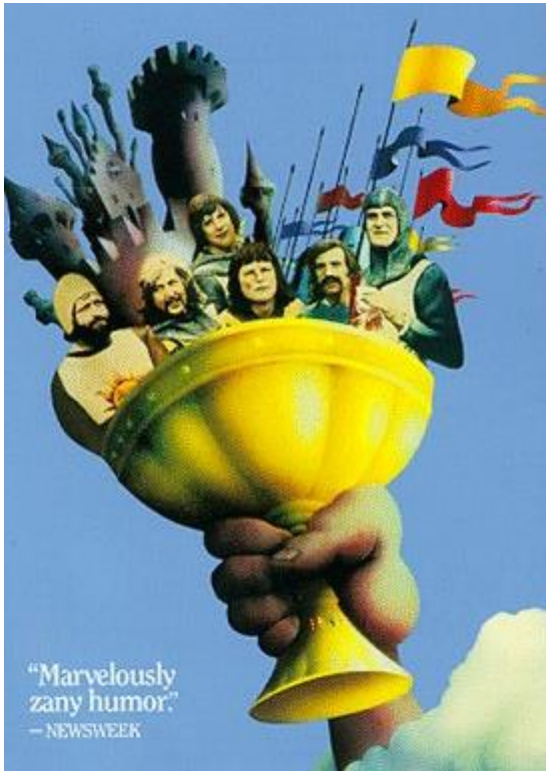
W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505



Beyond Traditional Moments?

- § Longstanding obstacle!
- § Holy grail of structure calculations
- § Applies to many structure quantities:
  - Generalized parton distributions (GPDs)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes…
  - Wigner distribution



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

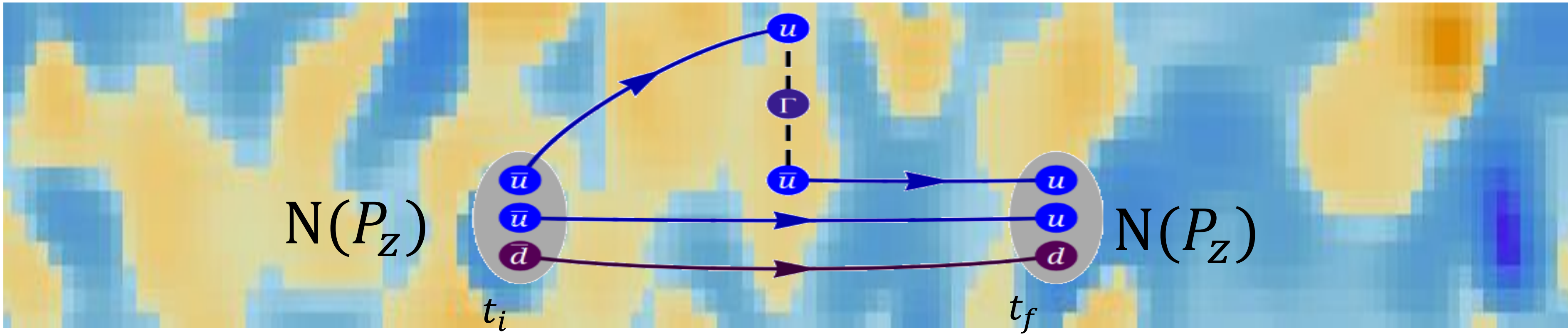
They carry secret plans to build the most powerful

Bjorken-x Dependent Hadron Structure

Biased selected results, highlighting work done by MSU students/postdocs

Lattice Parton Method

- § Large-momentum effective theory (LaMET)/Quasi-PDF
  (X. Ji, 2013; See 2004.03543 for review)



- § Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0)\right| P \right\rangle$$

- § Recover true distribution (take Pz → ∞ limit)

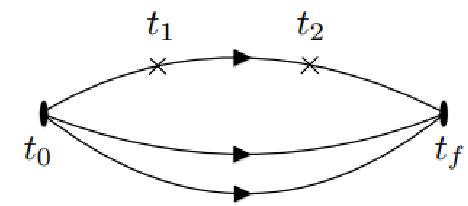
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2},\frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$

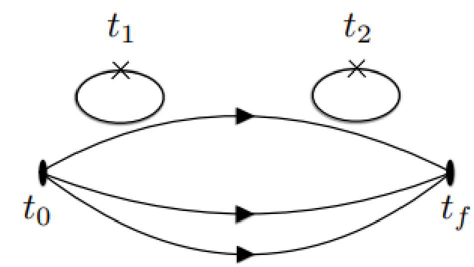
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Method

§ Short-distance factorization (SDF)

- Pseudo-PDF method (A. Radyushkin, 2017)
- Hadronic tensor currents
  (Liu et al., hep-ph/9806491, … 1603.07352)
- Lattice cross-section method (LCS)
  (Y Ma and J. Qiu, 2014, 2017)
- Euclidean correlation functions
  (RQCD, 1709.04325)
- Compton amplitude approach (QCDSF, 1703.01153)





Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

Lattice Parton Method

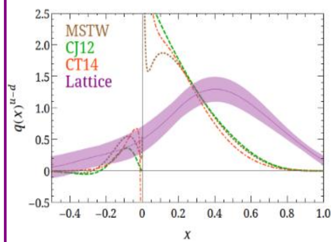
§ Differences and similarity

- Large momentum is needed in the lattice calculations in all methods to reach small-x region
  - Current projects focus on $x \in [0.3, 0.8]$ (for 2-GeV boosted hadron)
- Kernel is a complicated object;
  mostly current calculations used up to one-loop level
- SDF suffers Inverse problem to extract the wanted distribution
- LaMET requires to reach large Wilson-line displacement

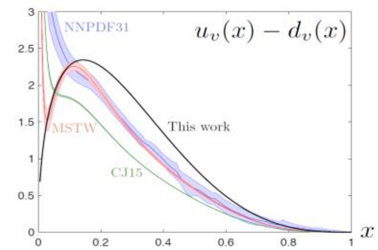
Lattice Parton Calculations

§ Rapid developments!

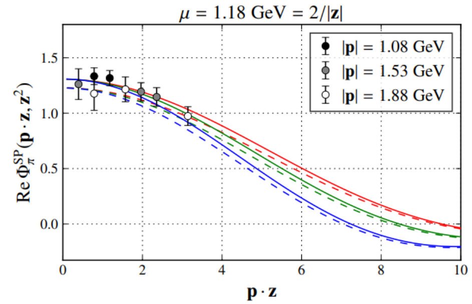
First unpol. PDF lattice calculation (2013)



First lattice pseudo-PDFs (2017)

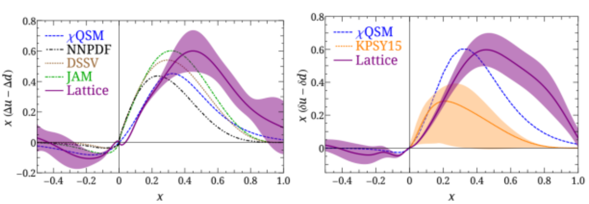


Euclidean correlation functions (2018)

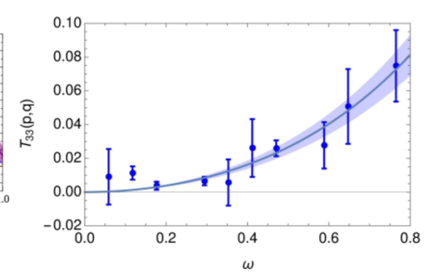


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

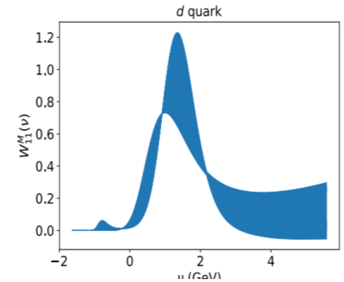
Pol. PDFs and mass corrections (2016)



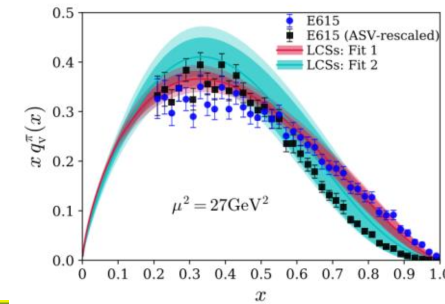
Compton amplitude (2017)



Hadronic tensor (2019)

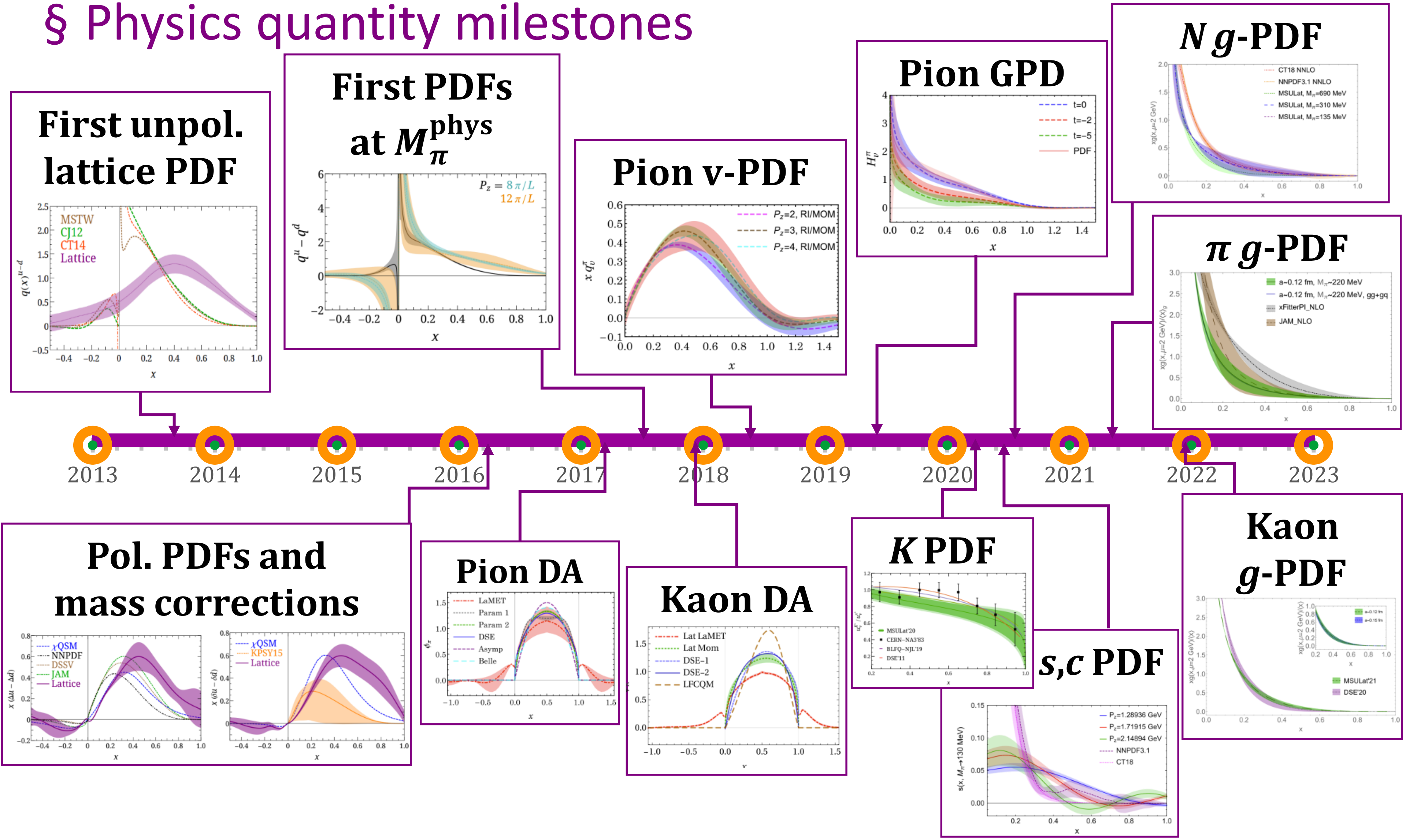


LCS (2020)



Lattice Parton Calculations

§ Physics quantity milestones

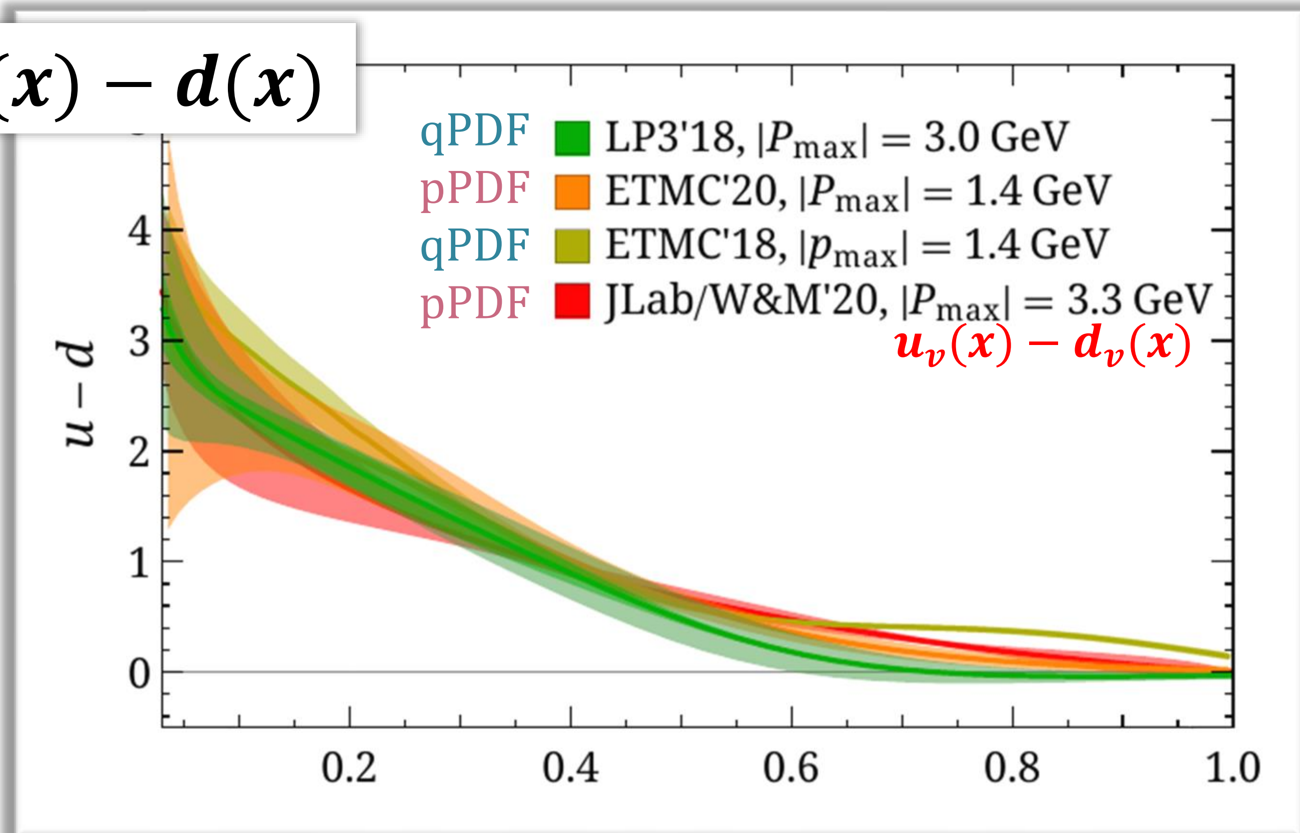


Physical Pion Mass Results

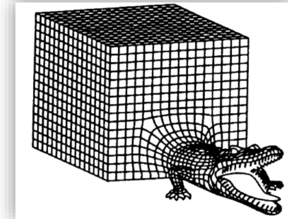
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$u(x) - d(x)$



2006.08636, PDFLattice2019 report



Finite volume,
Discretization,
…

Physical Pion Mass Results

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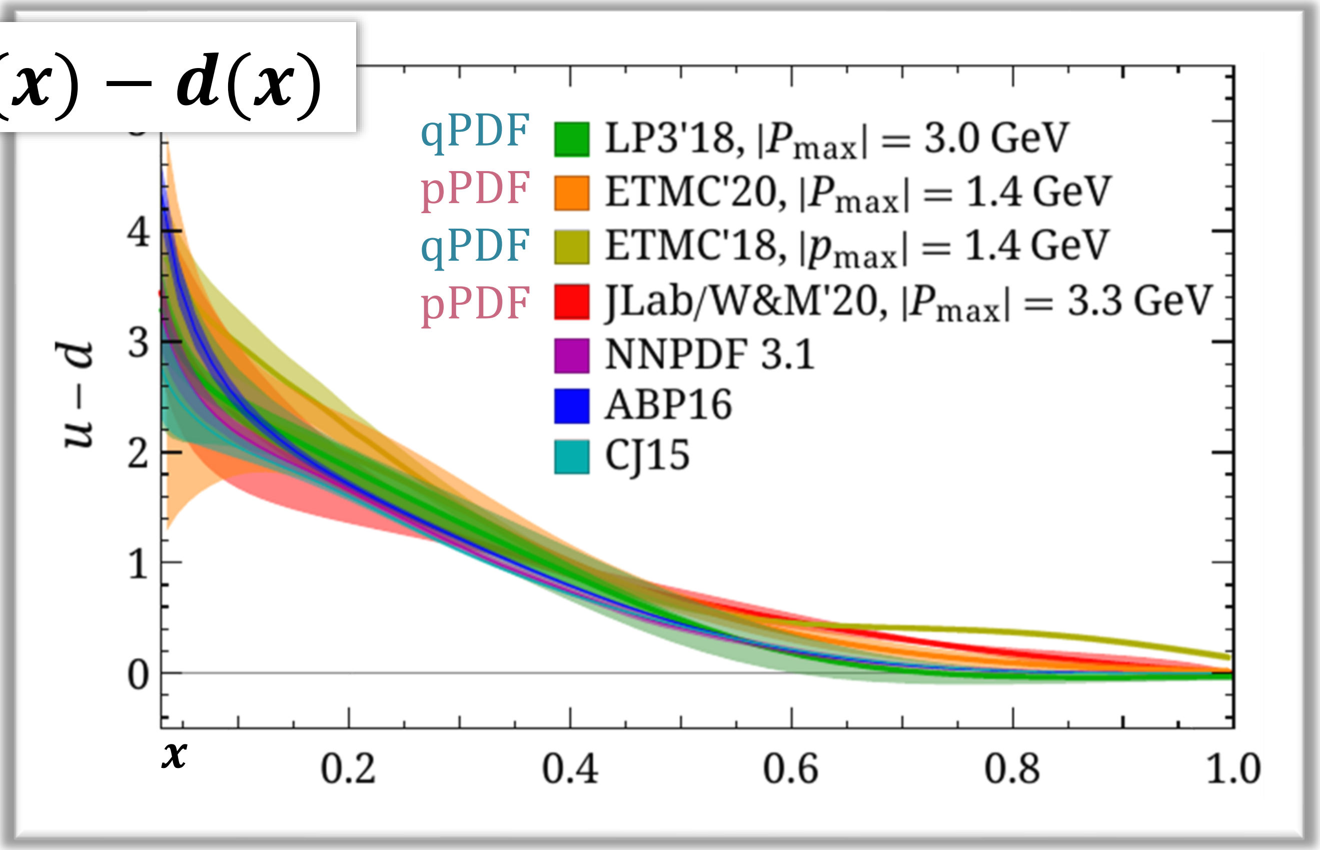
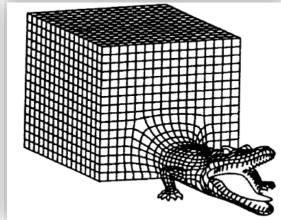




Finite volume,
Discretization,
…

2006.08636, PDFLattice2019 report

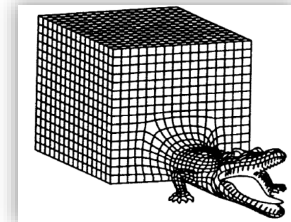
Polarized PDFs

§ Summary of physical pion mass results

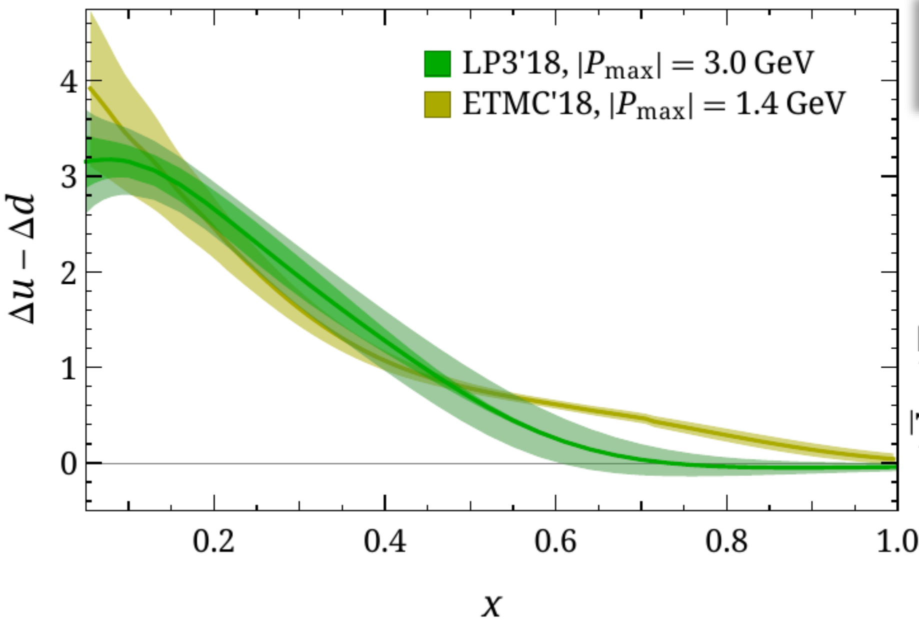
- Quasi-PDF method

Helicity
long. polarized

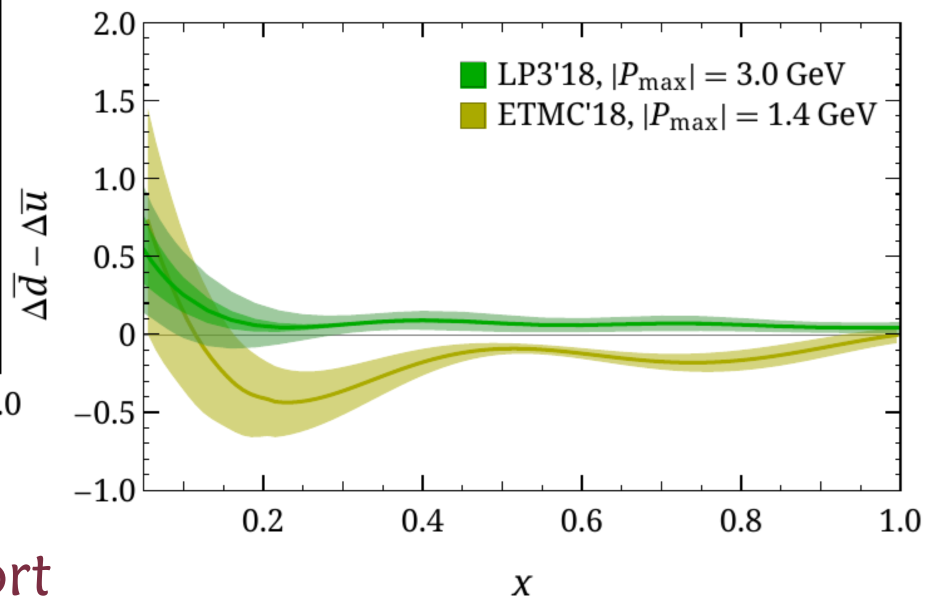
$$\Delta u(x) - \Delta d(x)$$



Finite volume,
Discretization,
...



$$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$$



2006.08636, PDFLattice2019 report

Polarized PDFs

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- Quasi-PDF method

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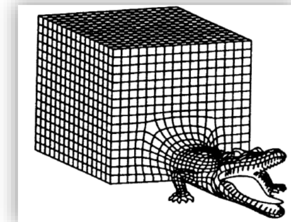
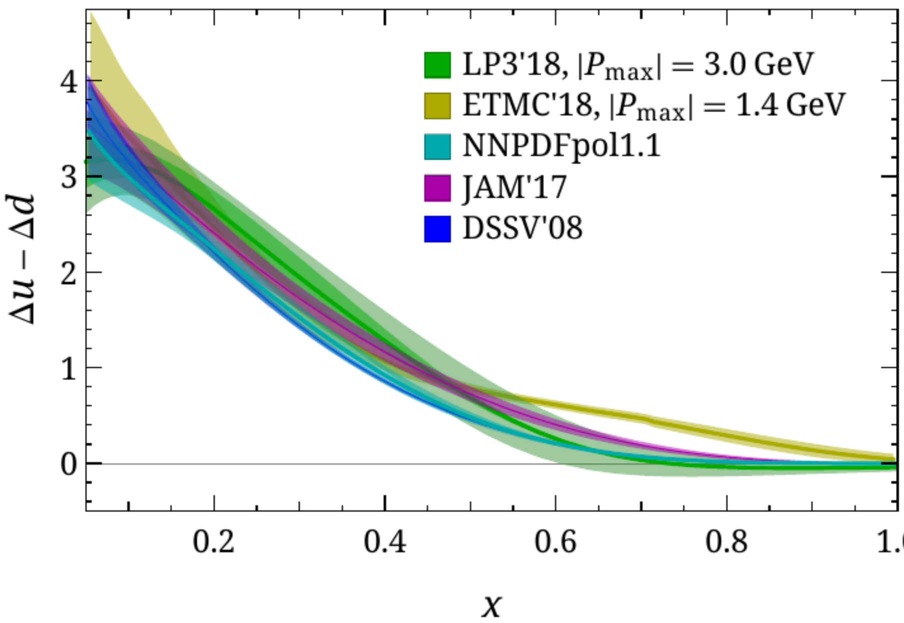
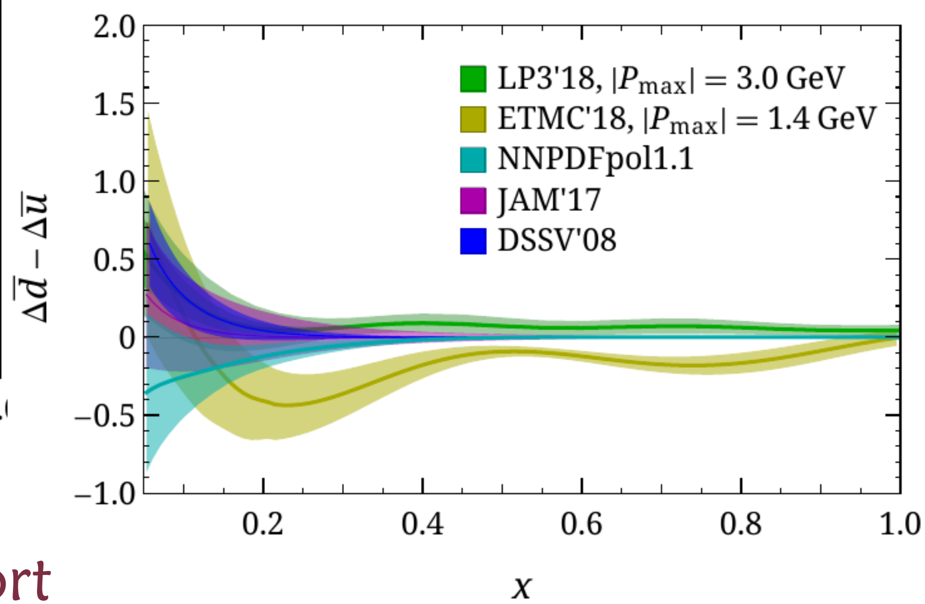


Finite volume, Discretization, …

$$\Delta u(x) - \Delta d(x)$$



$$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$$



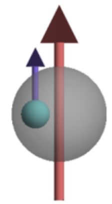
2006.08636, PDFLattice2019 report

Transversity

§ Summary of physical pion mass results

➢ Quasi-PDF method

$\delta u(x) - \delta d(x)$



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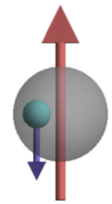
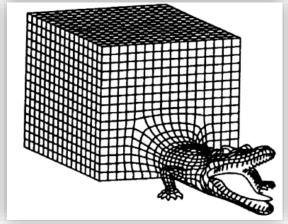
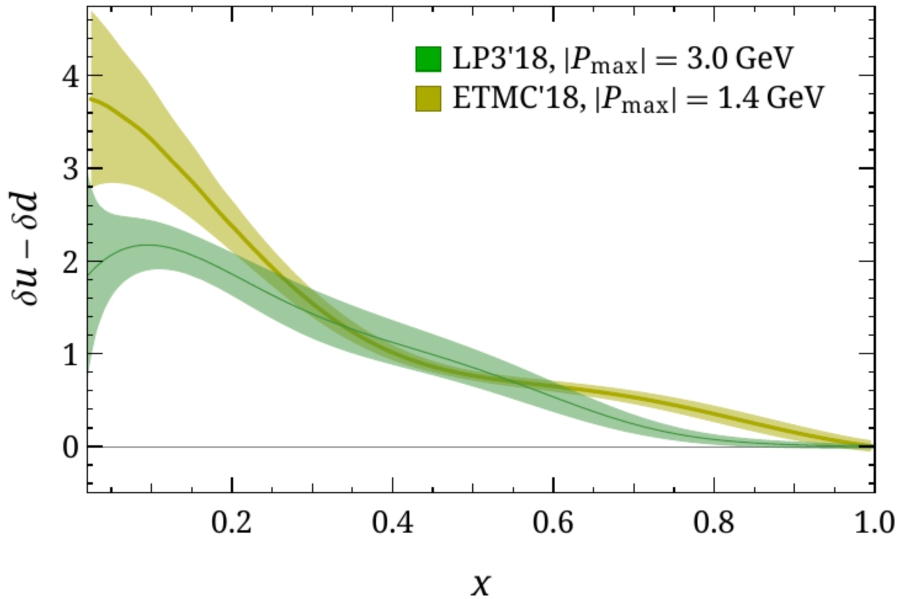


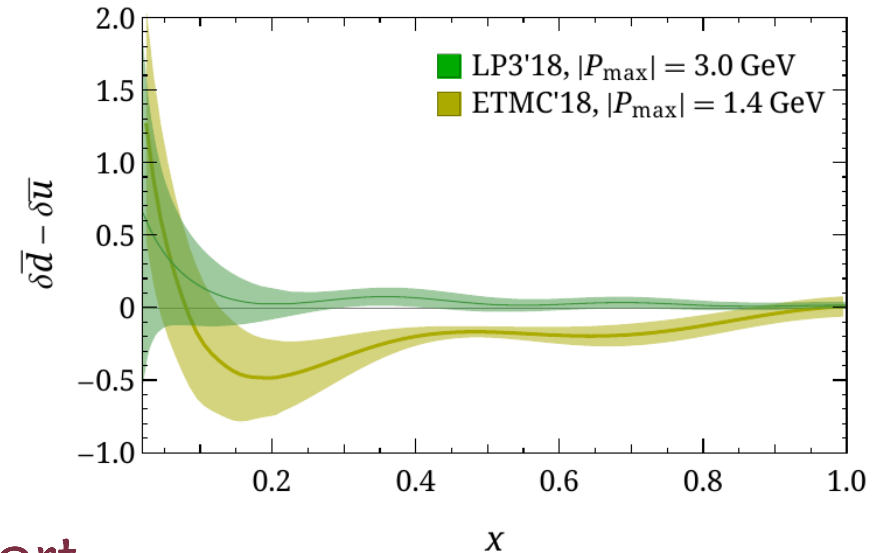
Transversity



Finite volume,
Discretization,
…

$\delta \bar{d}(x) - \delta \bar{u}(x)$





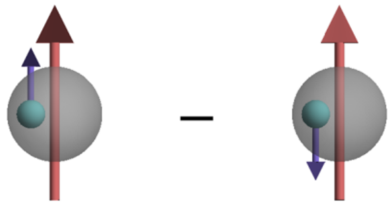
2006.08636, PDFLattice2019 report

Transversity

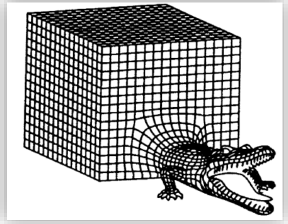
§ Summary of physical pion mass results

- Quasi-PDF method

$$\delta u(x) - \delta d(x)$$

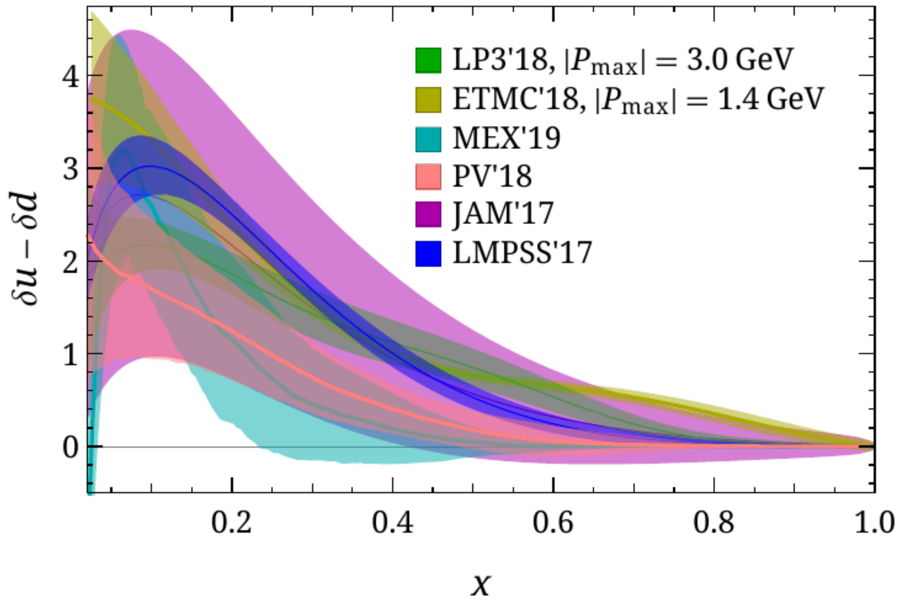


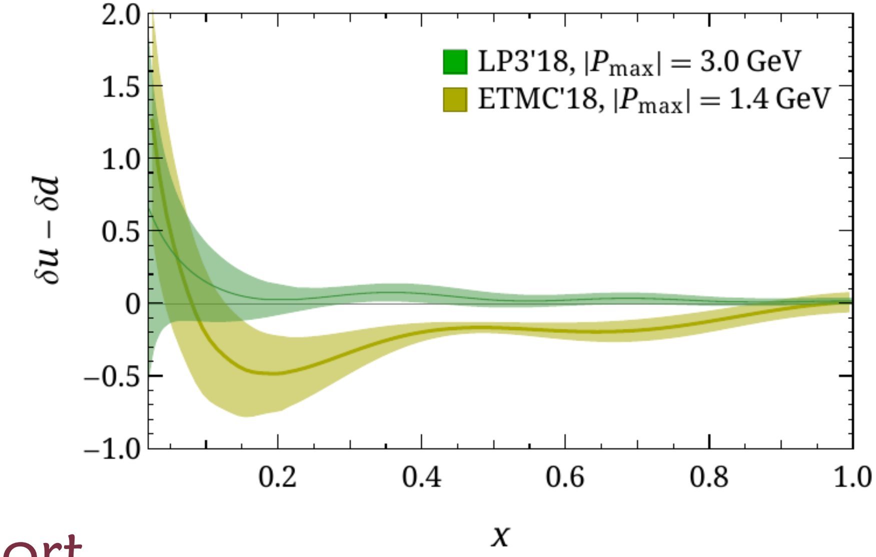
Transversity



Finite volume, Discretization, …

$$\delta \bar{d}(x) - \delta \bar{u}(x)$$





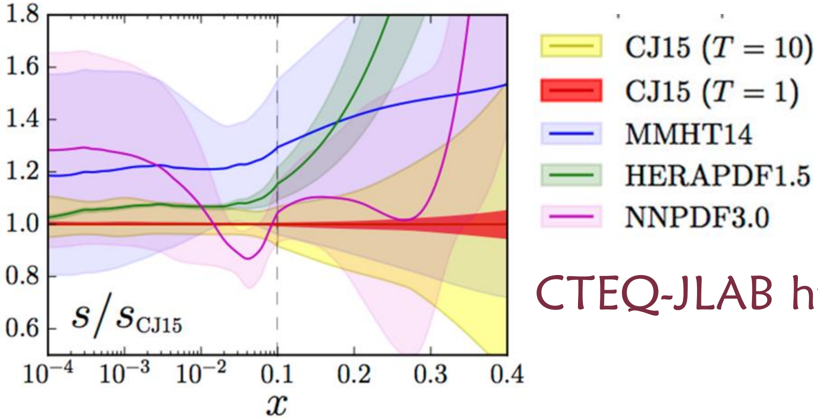
2006.08636, PDFLattice2019 report

Flavor-Dependent PDFs

Biased selected results, highlighting work done by MSU students/postdocs

First Lattice Strange PDF

§ Large uncertainties in global PDFs



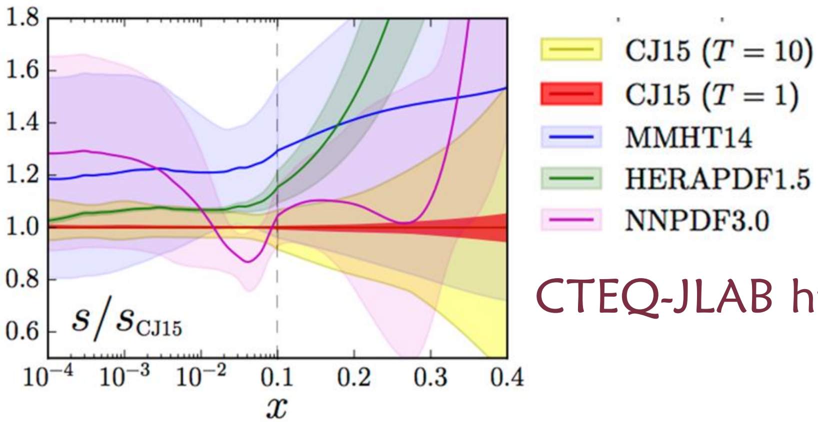
- Assumptions imposed due to lack of precision data

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

CTEQ-JLAB https://www.jlab.org/theory/cj/

First Lattice Strange PDF

§ Large uncertainties in global PDFs



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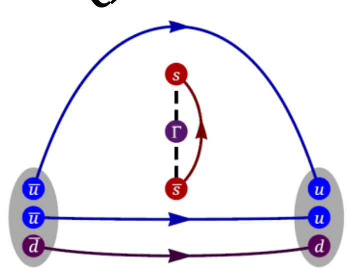
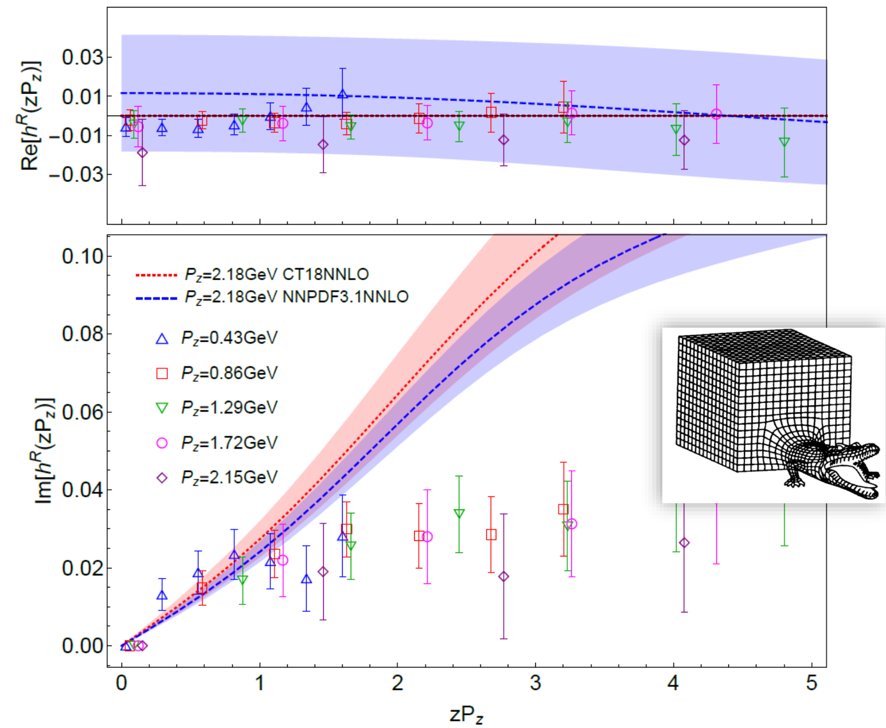
CTEQ-JLAB https://www.jlab.org/theory/cj/

§ Results by MSULat/quasi-PDF method 2005.12015, Zhang, Lin, Yoon

- Clover on 2+1+1 HISQ, $a \approx 0.12$ fm extrapolated to $M_\pi \approx 140$ MeV





First Lattice Strange PDF

$$h^R(z,\mu^R,p_z^R,P_z)=\int_{-\infty}^{\infty}dx e^{ixzPz}\int_{-1}^{1}\frac{dy}{|y|}C(\frac{x}{y},\frac{\mu R}{\mu},\frac{\mu}{yP^z},\frac{p_z^R}{yP^z})q(y,\mu=2\text{ GeV})$$

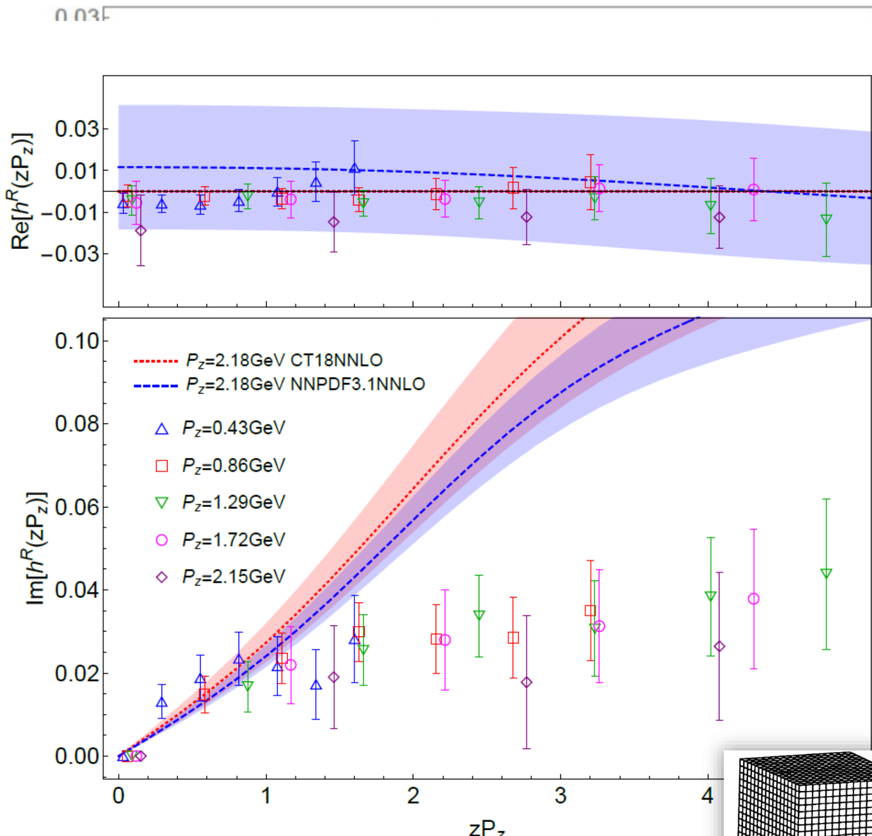
$\mathrm{Re}[h(z)]\propto$

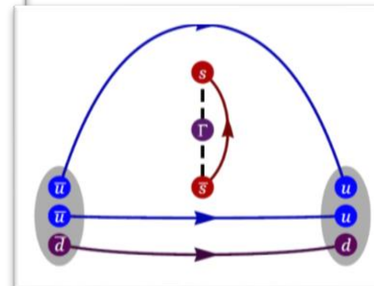
$$\int dx\,(s(x)-\bar{s}(x))\cos(xzP_z)$$

$\mathrm{Im}[h(z)]\propto$

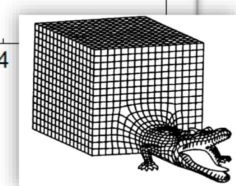
$$\int dx\,(s(x)+\bar{s}(x))\sin(xzP_z)$$

- symmetric $s-\bar{s}$ distribution.
- smaller momentum fraction.





Rui Zhang (MSU)

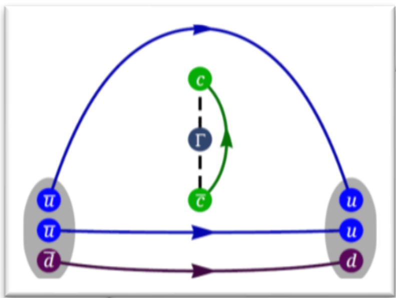


Slide by Rui Zhang @ DNP2020

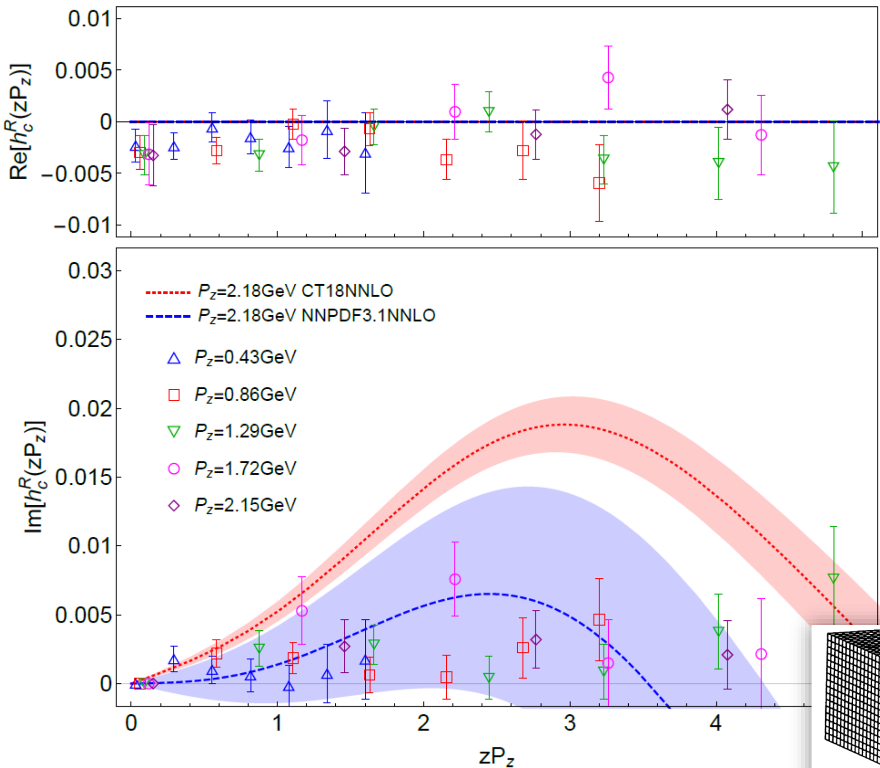
First Lattice Charm PDF

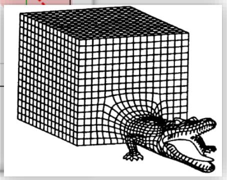
- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method

2005.12015, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF





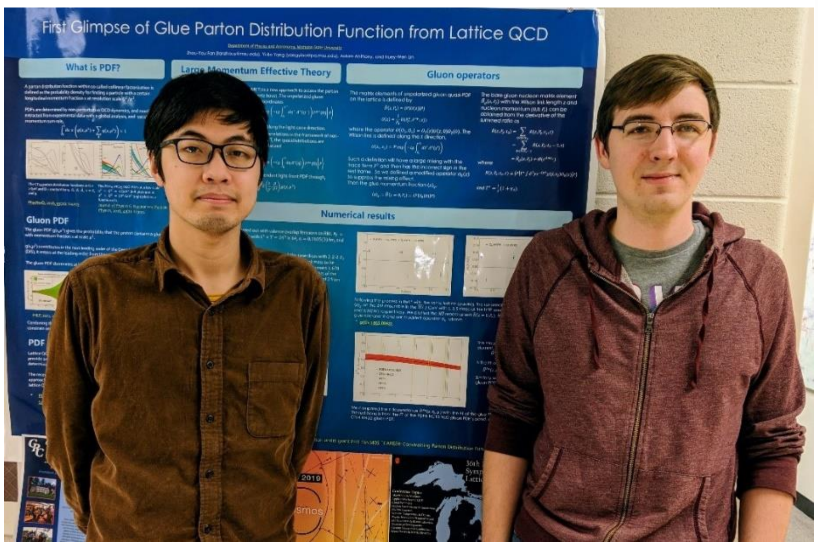
Rui Zhang (MSU)

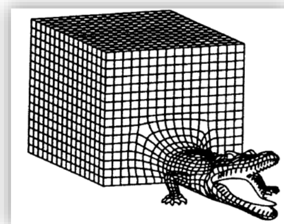
Gluon PDF

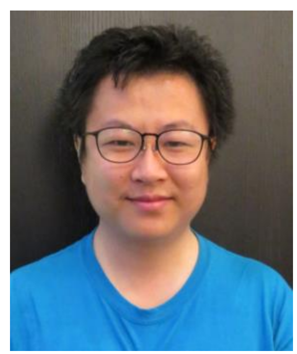
§ Pioneering first glimpse into gluon PDF using LaMET

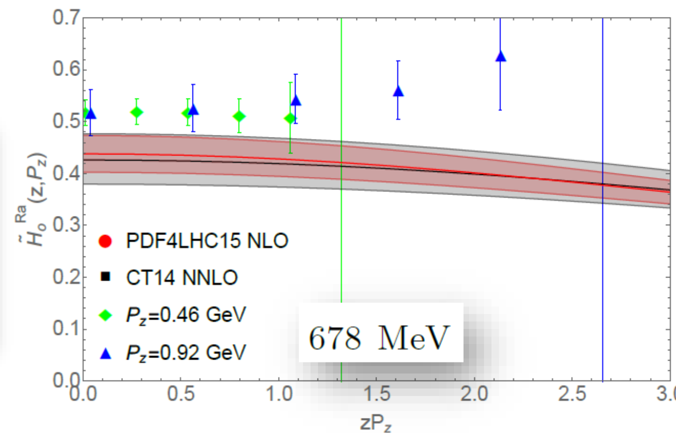
- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved

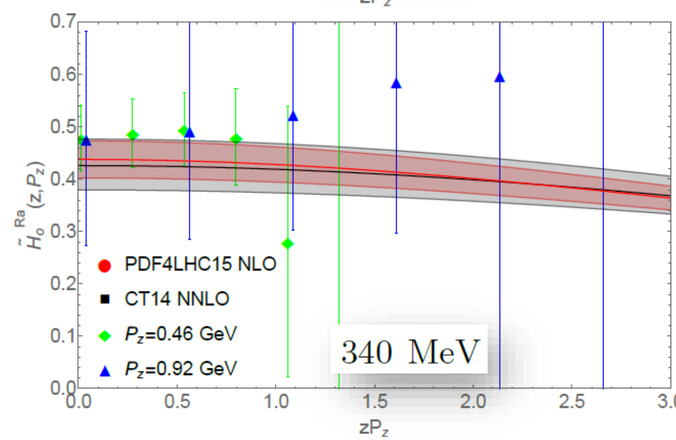
Fan et al, Phys.Rev.Lett. 121, 242001 (2018)











G: Zhouyou Fan G: Adam Antony P: Yi-Bo Yang

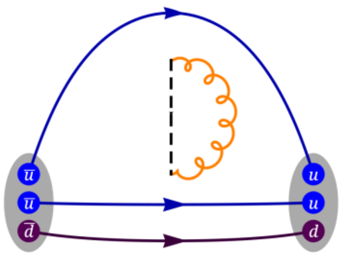
iCER@MSU is crucial for earlier code development and completion of this work

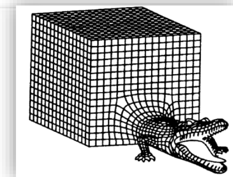
Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

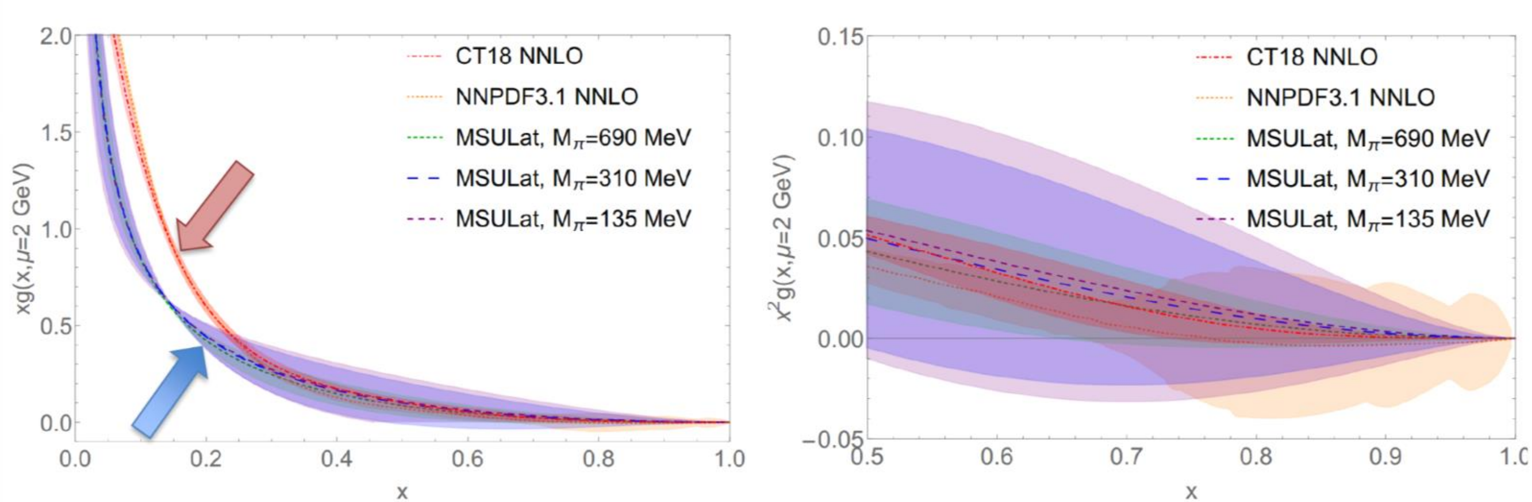
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

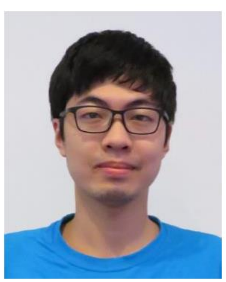
Z. Fan. et al (MSULat), 2007.16113





The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\mathrm{MS}}$ scheme.



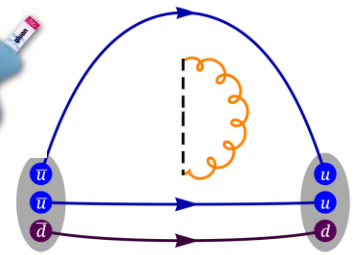


Zhouyou Fan (MSU)

Slide by Zhouyou Fan@DNP2020

Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF



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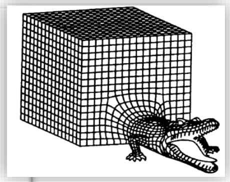
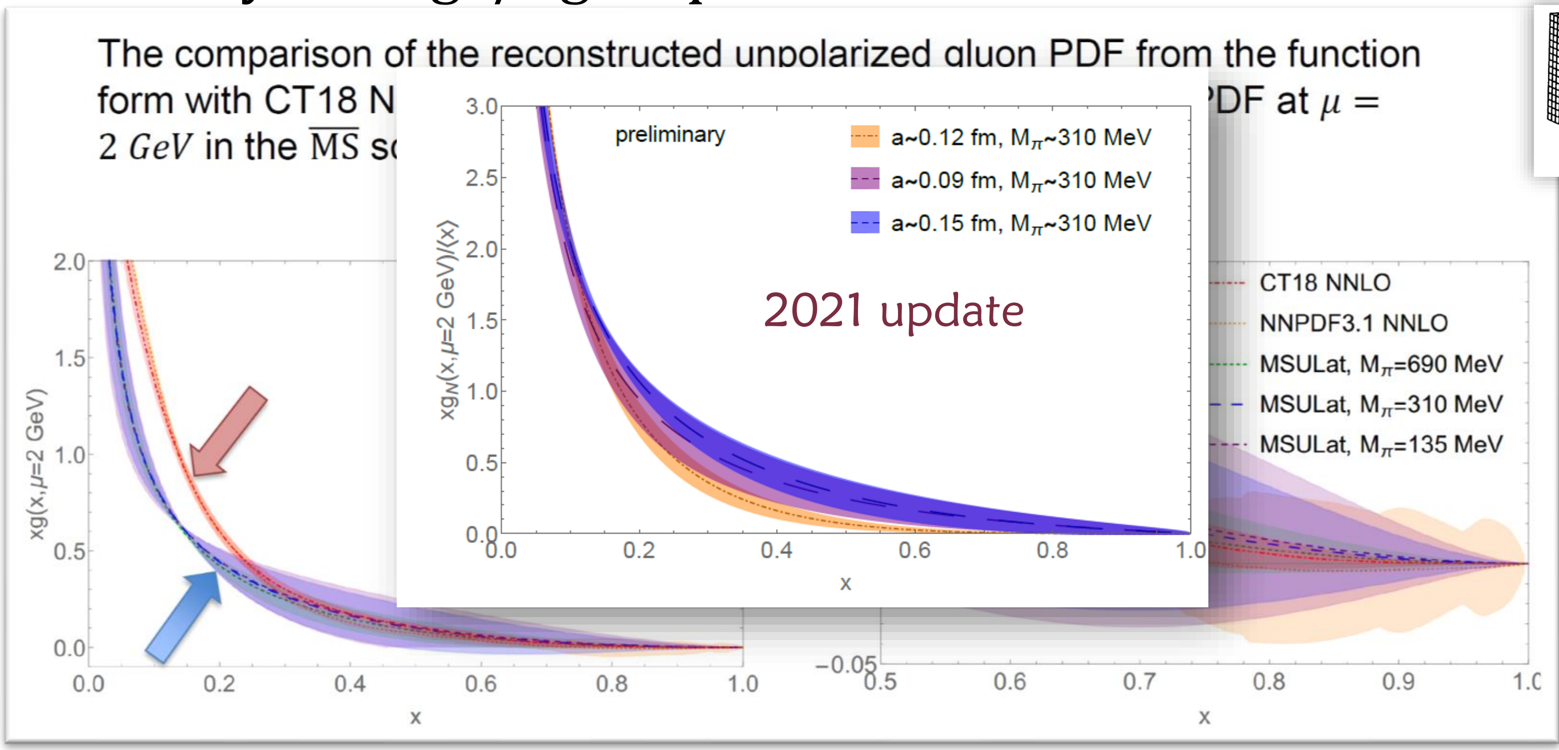
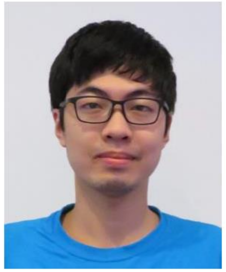




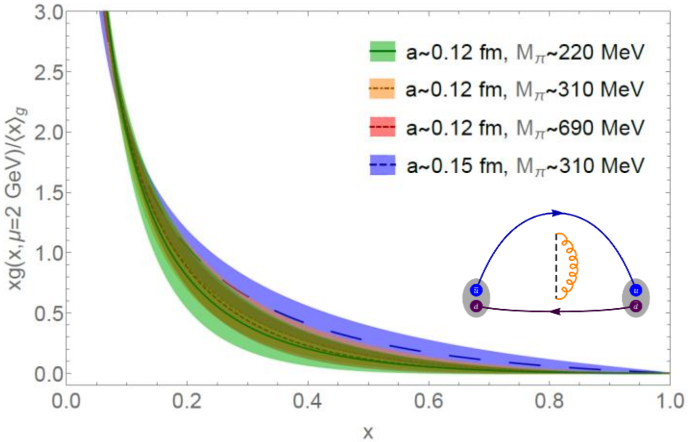


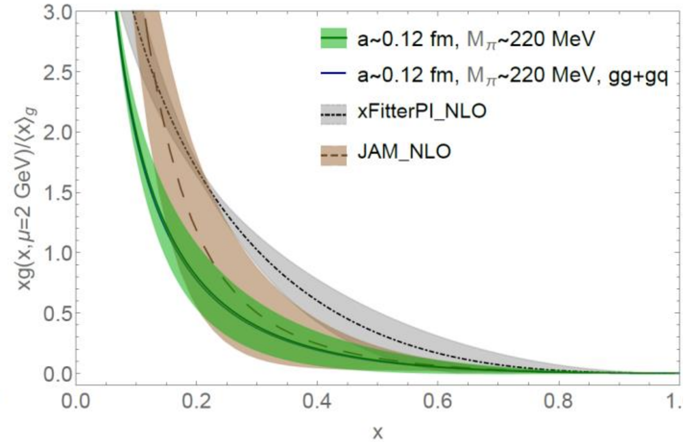
Slide by Zhouyou Fan @ DNP 2020

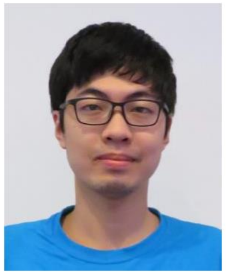
G: Zhouyou Fan

Meson Gluon PDFs

§ First pion and kaon gluon PDFs using pseudo-PDF

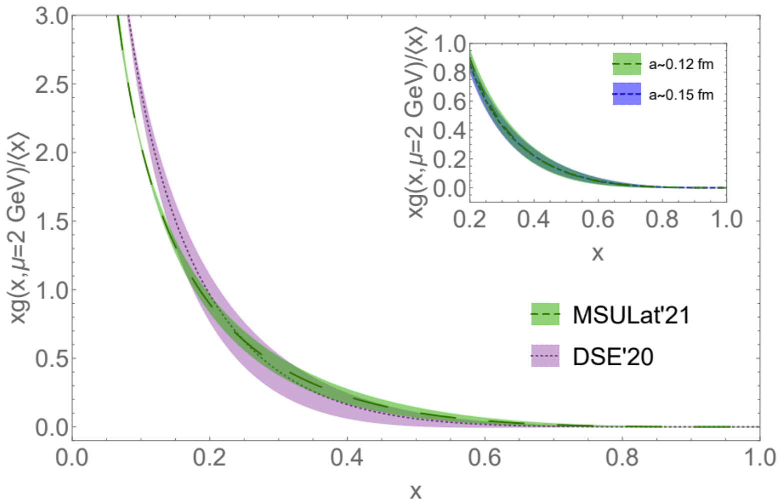






Zhouyou Fan (MSU)

2007.16113, 2104.06372, Fan et al (MSULat)

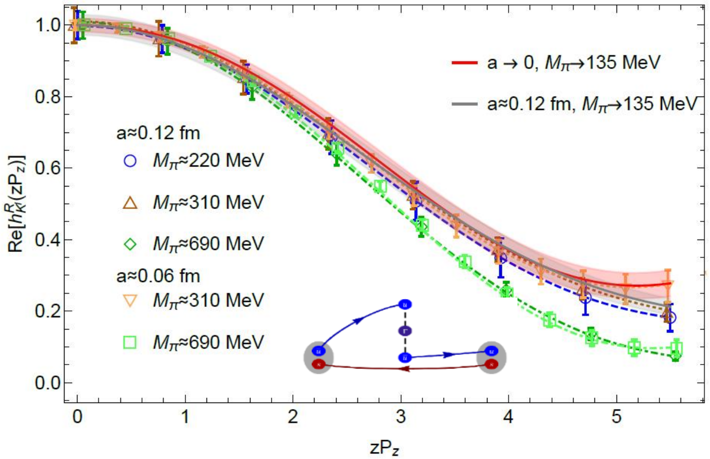


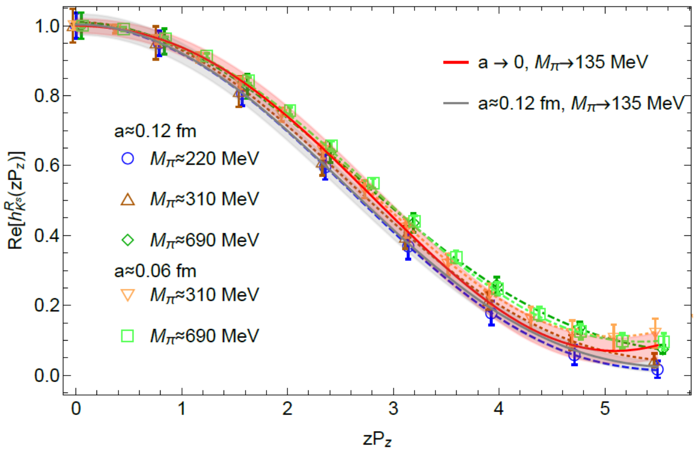
Alejandro Salas-Chavira (MSU)

2112.03124 , Salas-Chavira et al (MSULat)

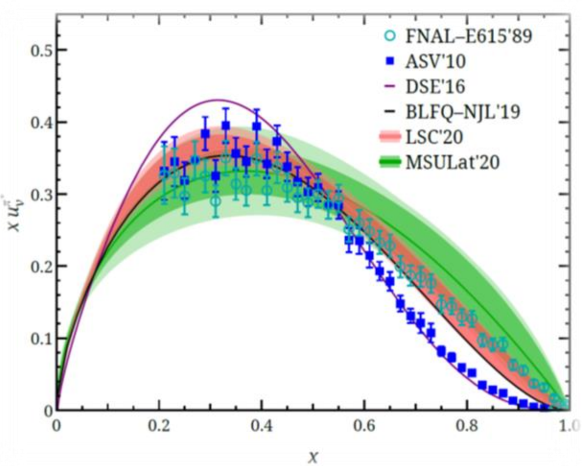
Meson PDFs

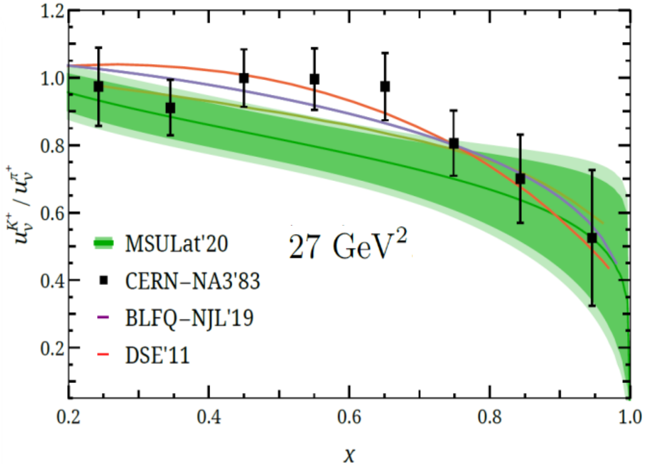
§ Valence-quark PDFs of Pion/Kaon using quasi-PDF in the continuum limit

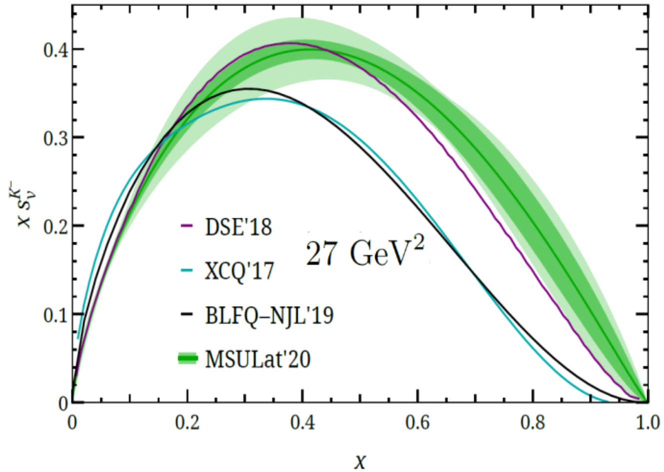




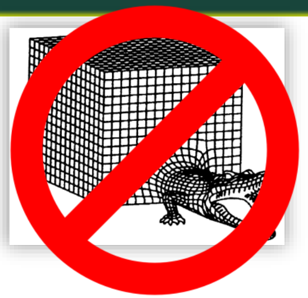
MSULat, 2003.14128





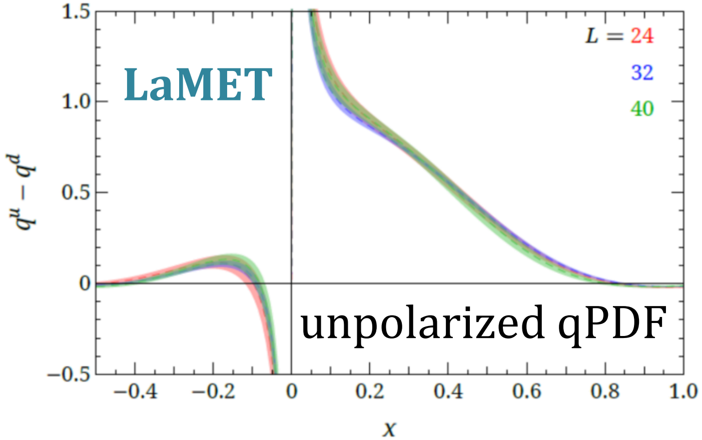


Systematics Study



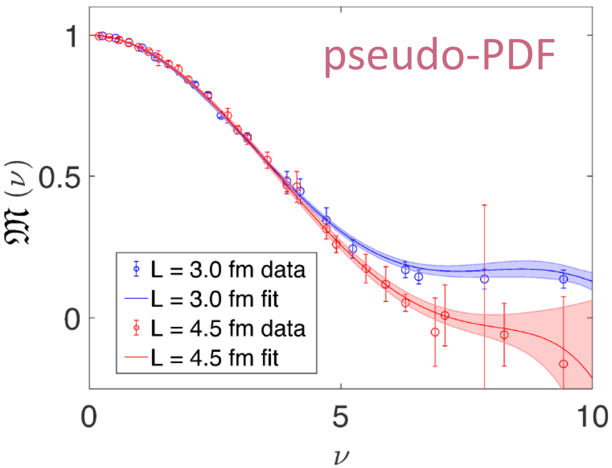
§ Finite-volume study

$M_\pi \approx$ **220** MeV (2+1+1f)
$L \approx 2.9, 3.8, 4.8$ fm



MSULat, PRD 100 (2019) 7, 074502

$M_\pi \approx$ **415** MeV (2+1f)
$L \approx 3, 4.5$ fm



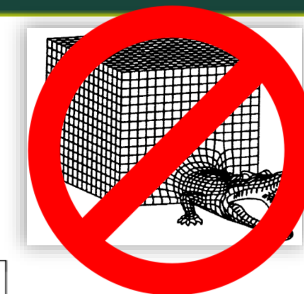
HadStruc, JHEP 12 (2019) 081

§ Lattice artifacts are sensitive to the simulated QCD vacuum

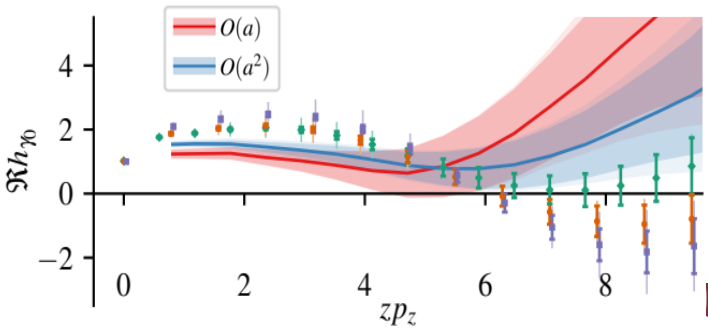
- Each group will have to check their own systematics carefully

Systematics Study

§ Lattice discretization study examples

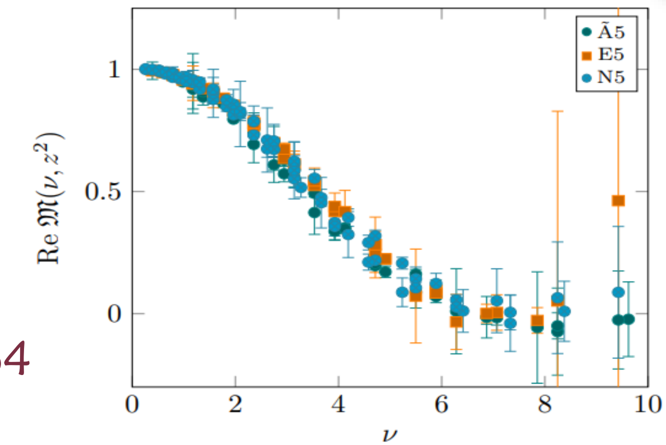


$M_\pi \approx \mathbf{370}$ MeV (2+1+1f)
$a \approx [0.064, 0.093]$ fm



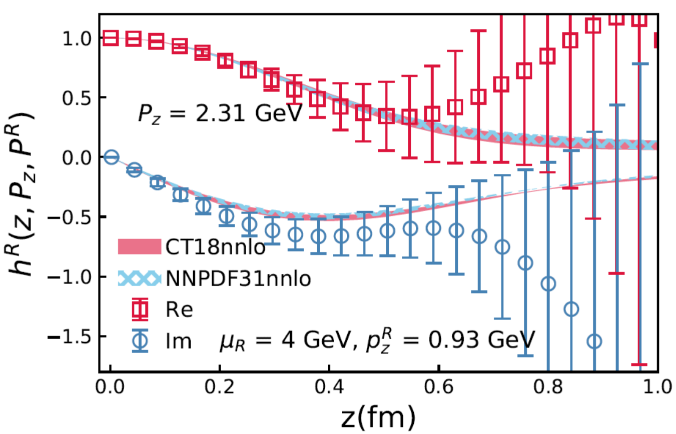
ETMC, 2011.00964

$M_\pi \approx \mathbf{440}$ MeV (2f)
$a \approx [0.048, 0.075]$ fm



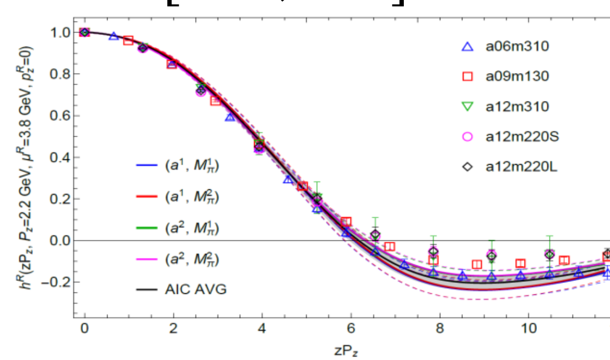
HadStruc, 2105.13313

$M_\pi \approx \mathbf{310}$ MeV (2+1+1f)
$a \approx 0.042$ fm



BNL/MSU, 2005.12015

$M_\pi \approx [\mathbf{135}, \mathbf{310}]$ MeV (2+1+1f)
$a \approx [0.06, 0.12]$ fm

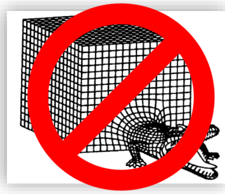


MSULat, 2011.14971

First Continuum PDF

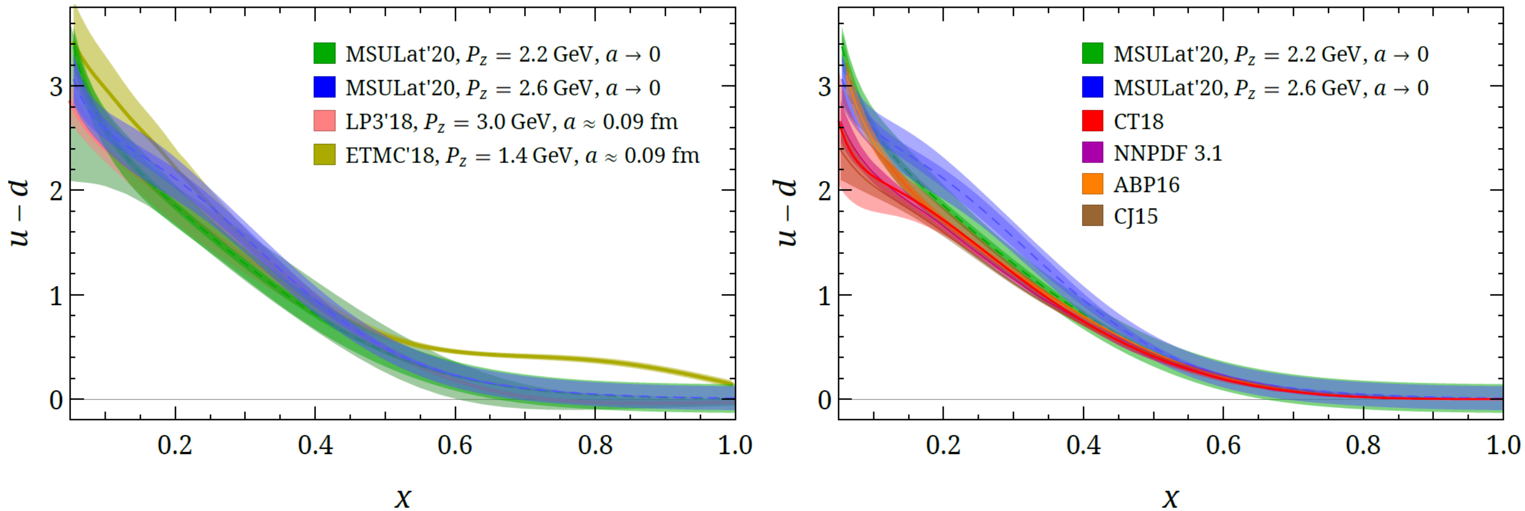
§ Nucleon PDFs using quasi-PDFs in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV

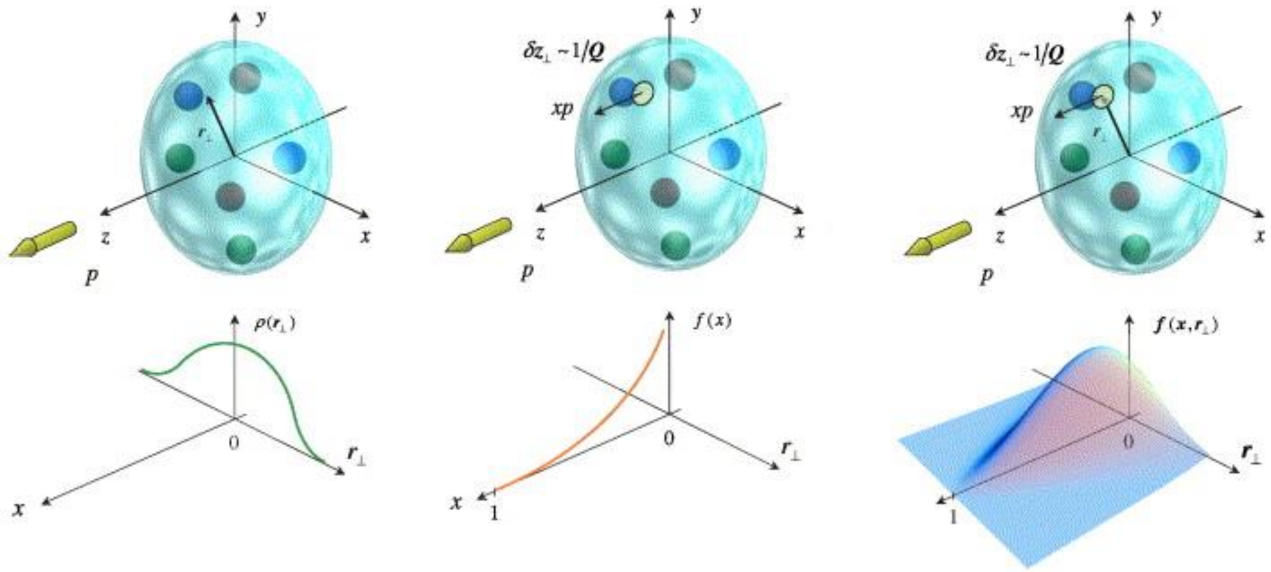


2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit



Bjorken-x Dependent GPDs



Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

Nucleon Tomography

- Assuming we live in the Marvel Universe
  - The special quantum tunnel allows us to shrink to the size particle to sub-nucleon scale ($< 10^{-15}$m)
- What would it look like to travel inside the nucleon?

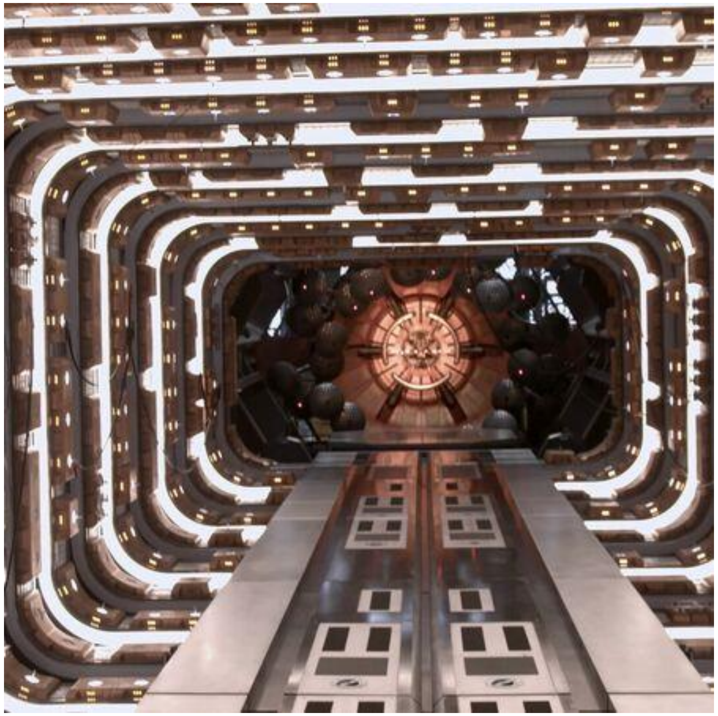
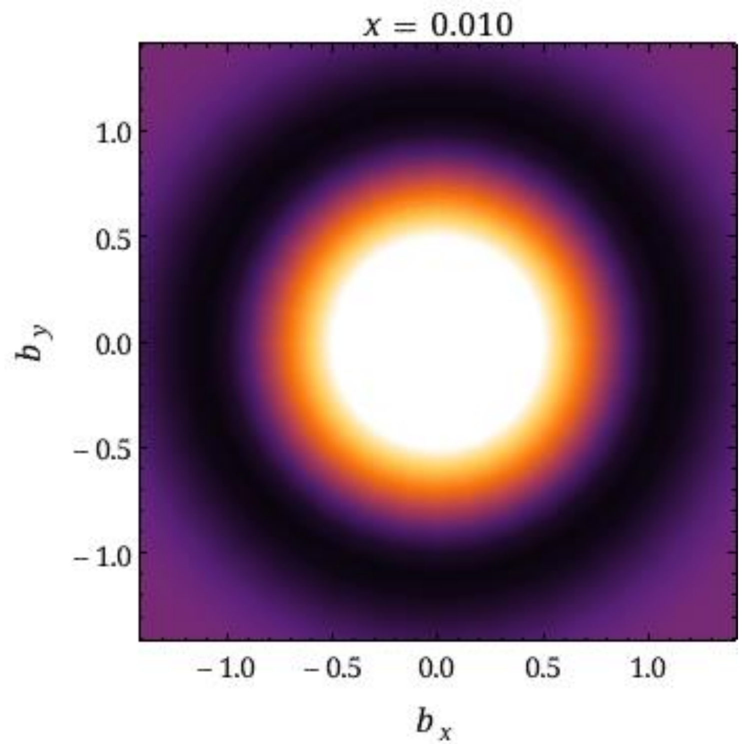


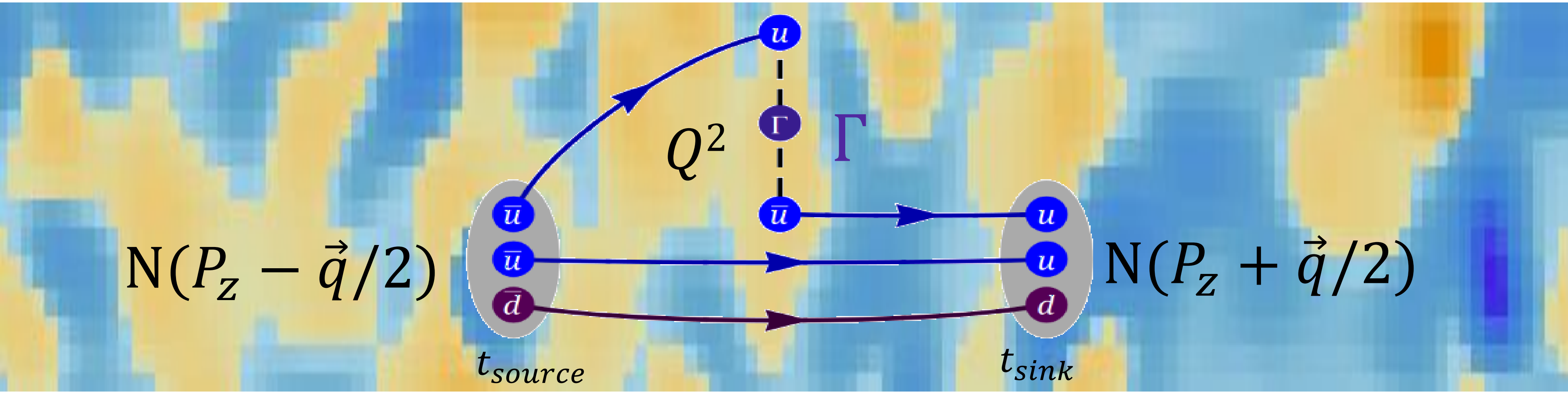
Image credit: Marvel Studios



Thanks to Cottrell Scholar Award from RCSA

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)



$$\tilde{F}(x, \tilde{\xi}, t, \bar{P}_Z)$$

$$= \frac{\bar{P}_z}{\bar{P}_0} \int \frac{dz}{4\pi} e^{ixz\bar{P}_Z} \langle P' | \tilde{O}_{\gamma_0}(z) | P \rangle = \frac{\bar{u}(P')}{2\bar{P}^0} \left(H(x, \tilde{\xi}, t, \bar{P}_Z) \gamma^0 + E(x, \tilde{\xi}, t, \bar{P}_Z) \frac{i\sigma^{0\mu}\Delta_\mu}{2M} \right) u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

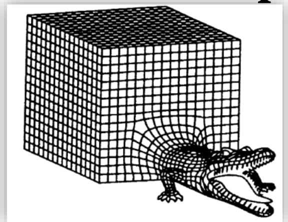
First Lattice GPDs

§ Pioneering first glimpse into pion GPD using LaMET

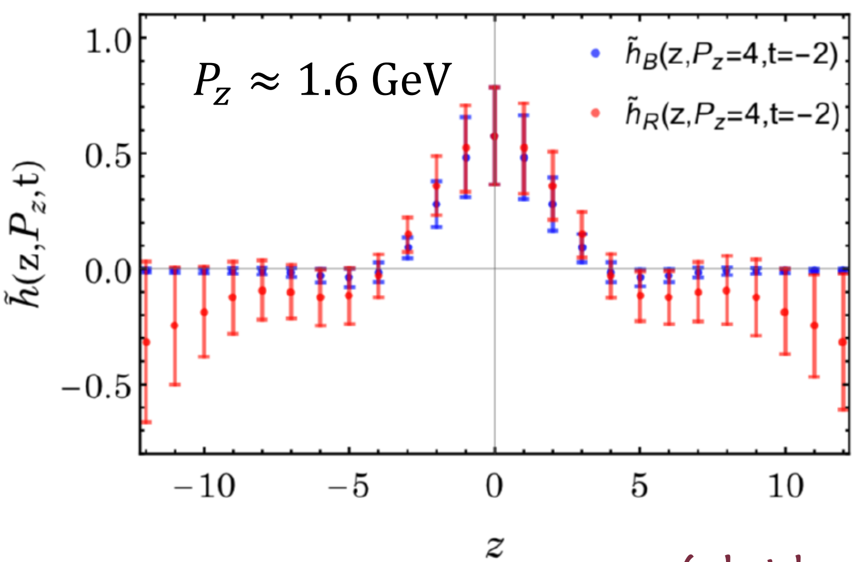
- Lattice details: clover/HISQ, 0.12fm, **310-MeV** pion mass

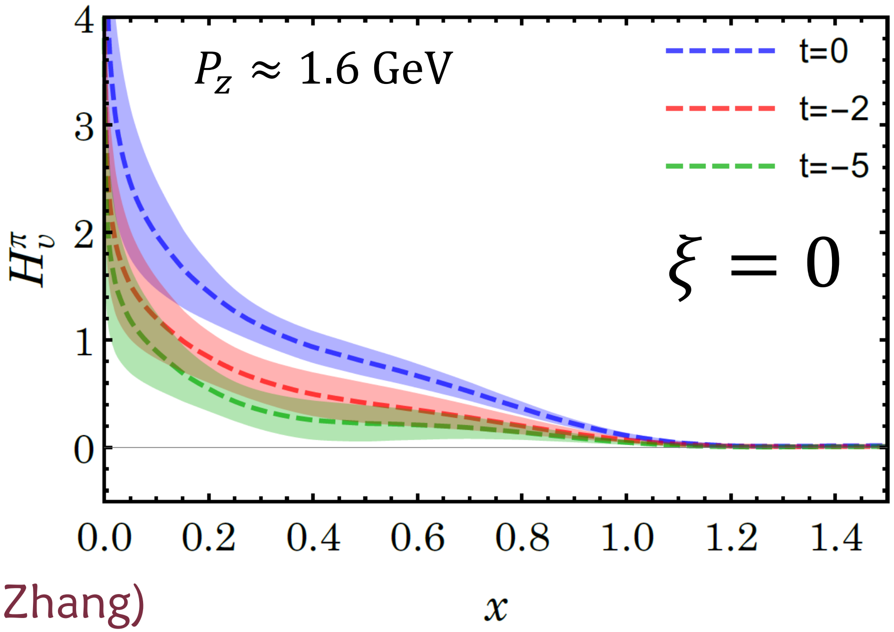
  $P_z \approx 1.3, 1.6$ GeV

  J. Chen, HL, J. Zhang, 1904.12376



$$H_q^{\pi}(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



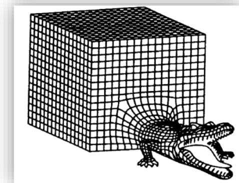


(plot by J. Zhang)

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

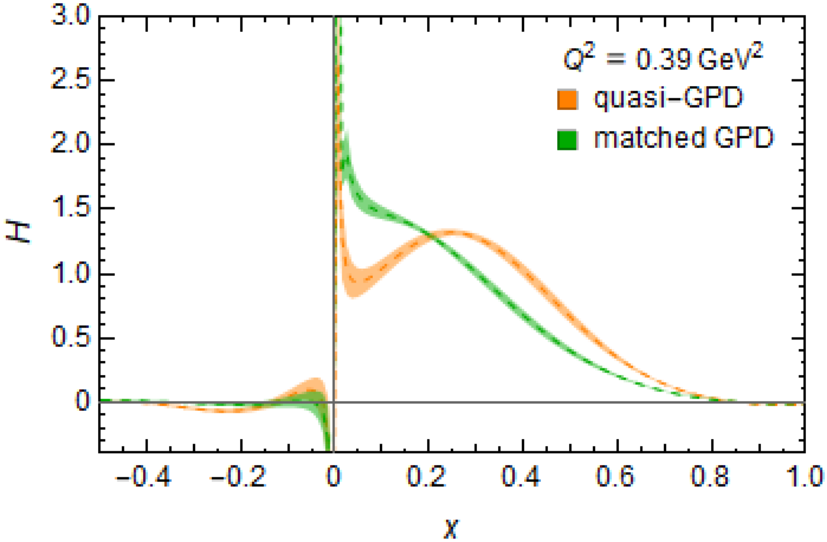
- Lattice details: clover/2+1+1 HISQ
  0.09fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

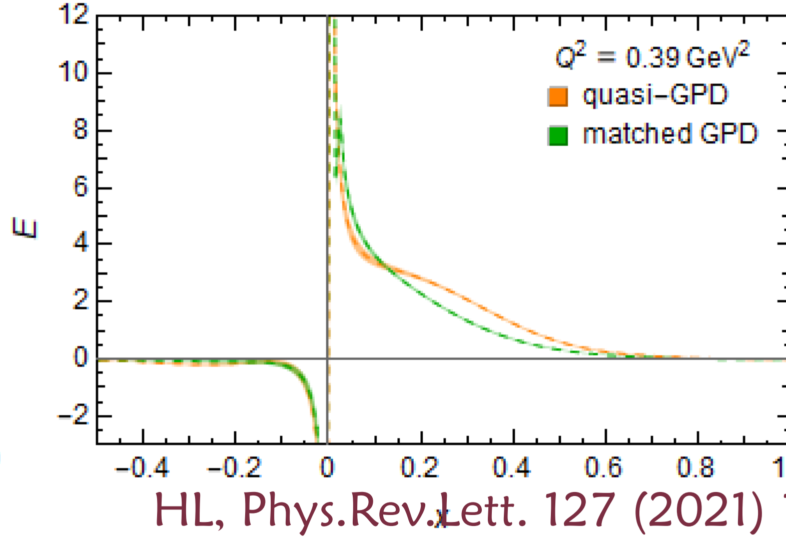


finite-volume, discretization, …

$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\tilde{H}(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + \tilde{E}(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$



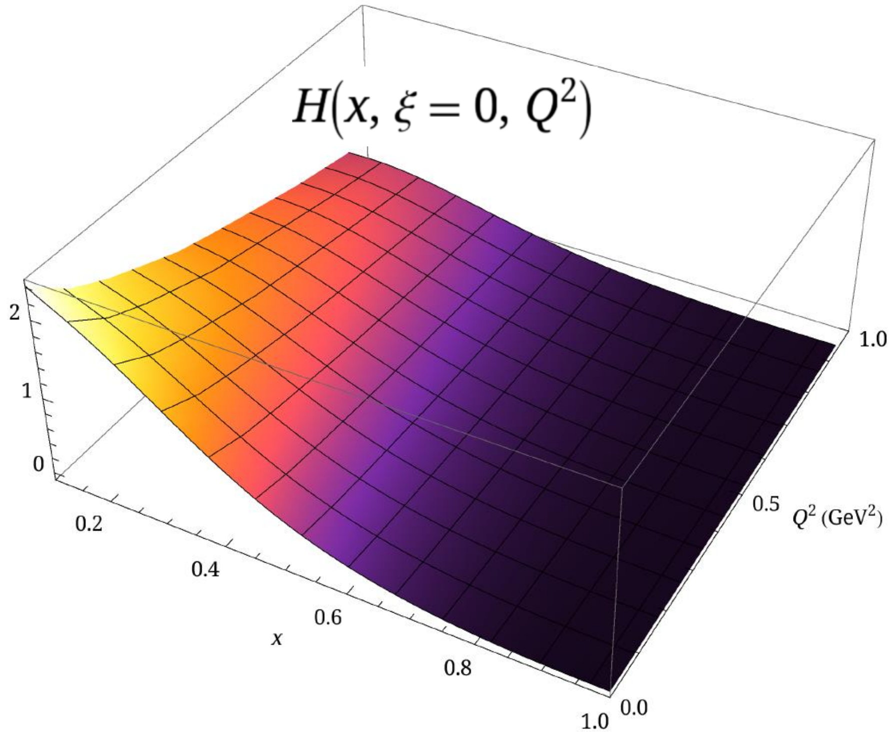


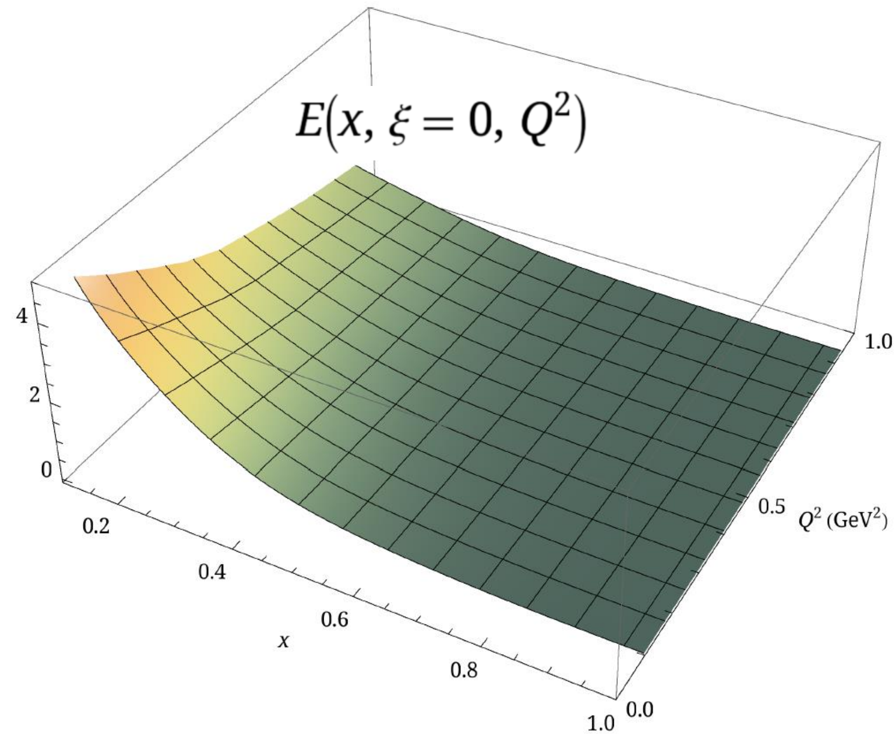
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results





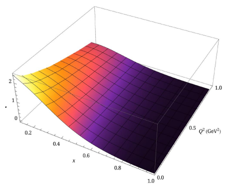
HL (MSULat), 2008.12474, to appear on PRL

Isovector Nucleon GPDs

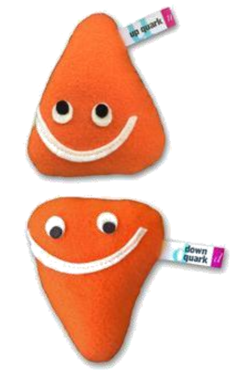
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

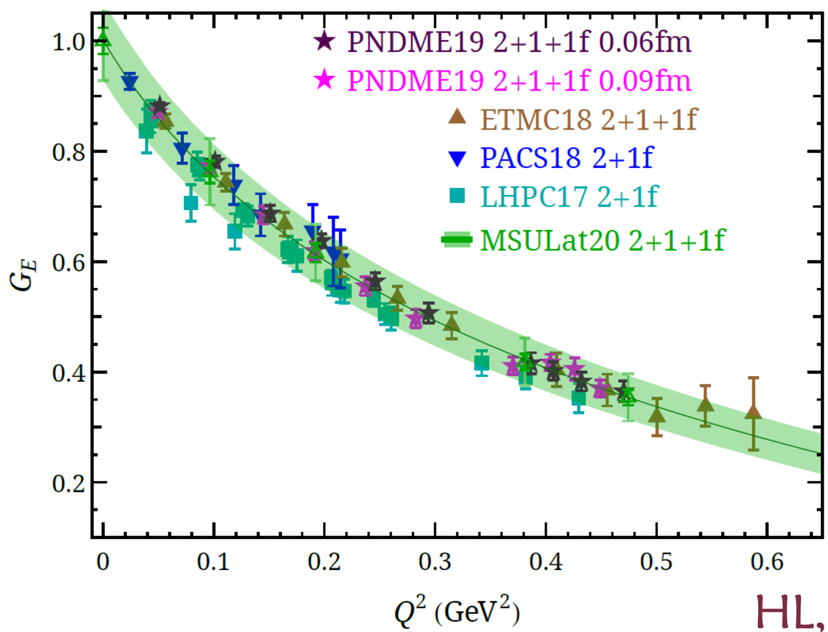
$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$



$n = 1$



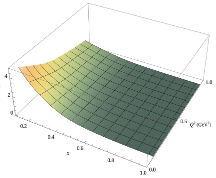
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon GPDs

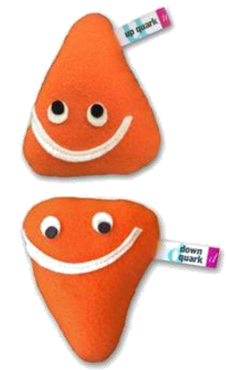
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

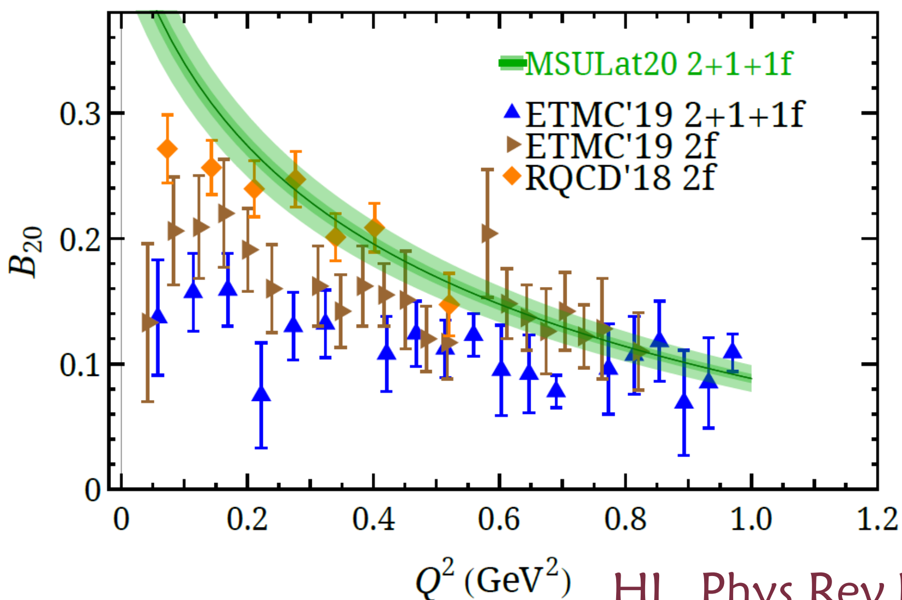
$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n \text{ even}}$$



$n = 2$



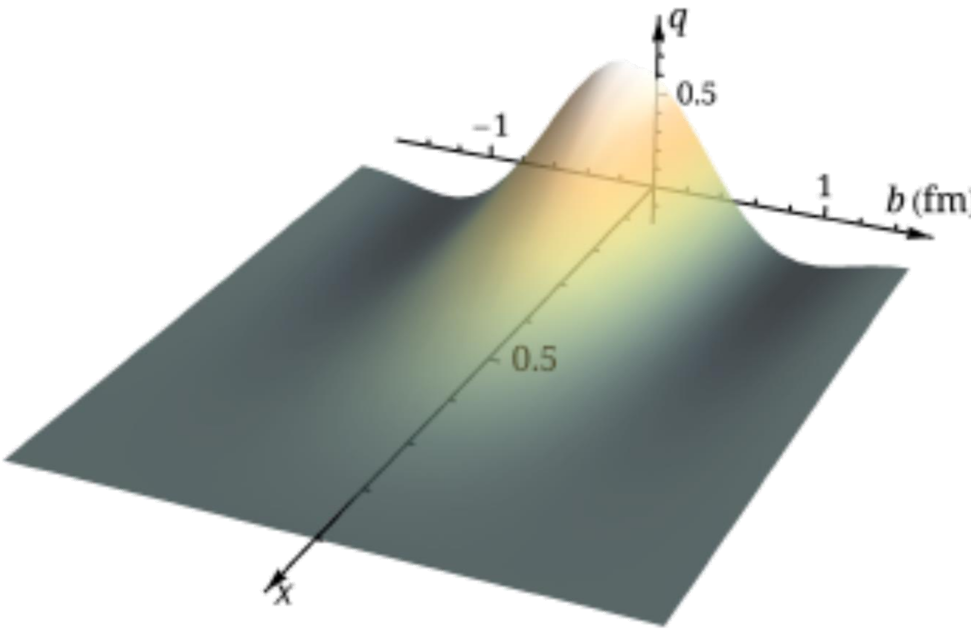
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$



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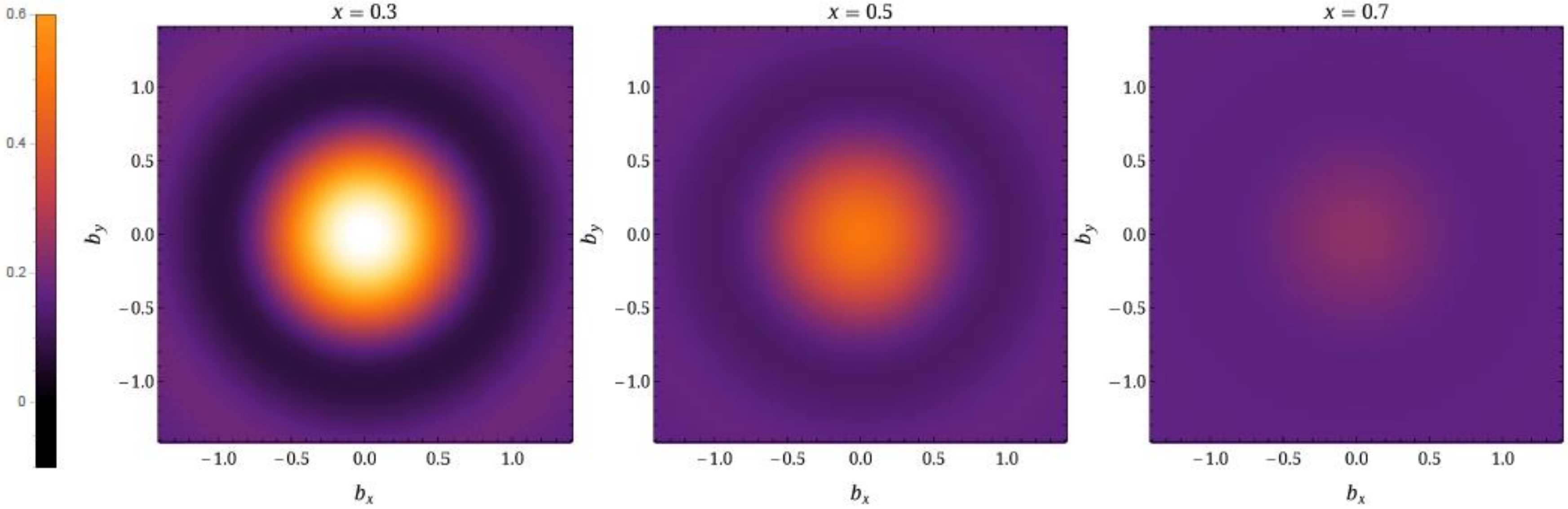
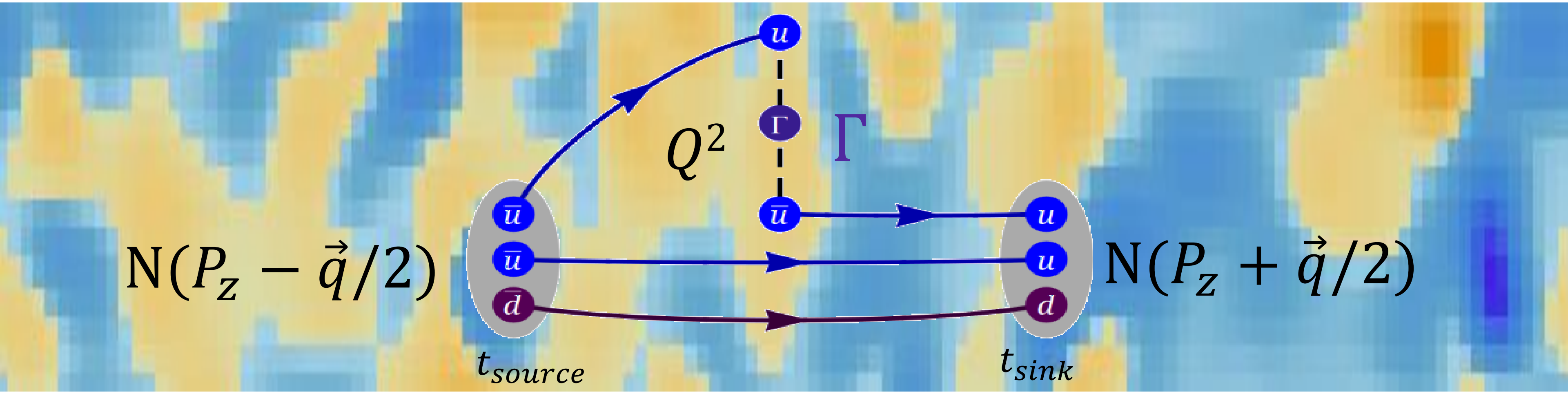


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Generalized Parton Distributions

- On the lattice, one needs to calculate the following (nucleon example)



$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z)$$
$$= \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_5\gamma_z}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\tilde{H}(x,\tilde{\xi},t,\bar{P}_z)\gamma_5\gamma_z + \tilde{E}(x,\tilde{\xi},t,\bar{P}_z)\frac{i\gamma_5\Delta_z}{2M}\right)u(P'')$$

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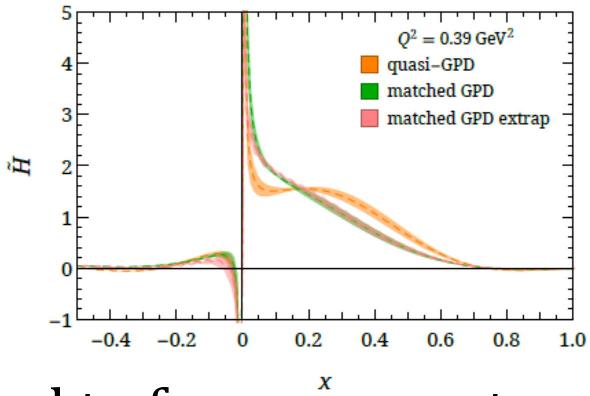
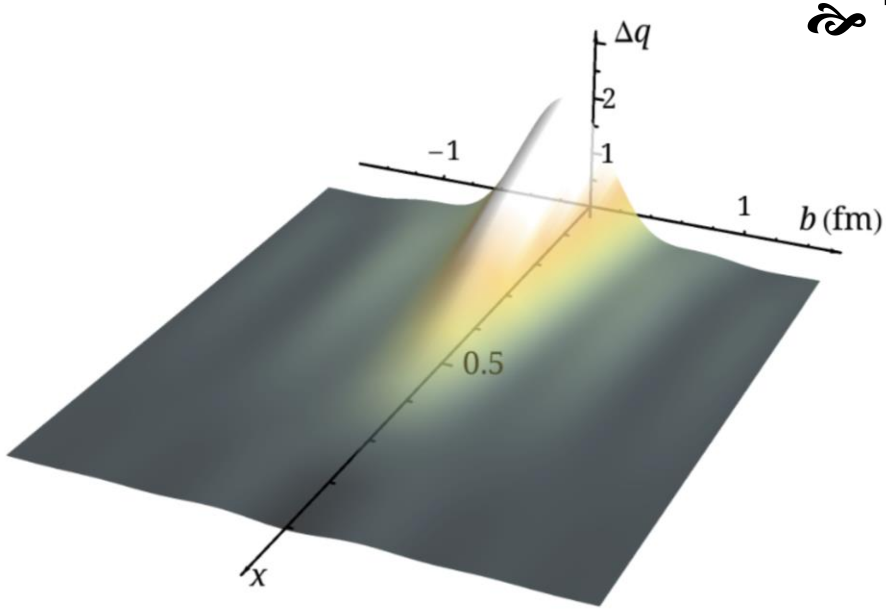
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

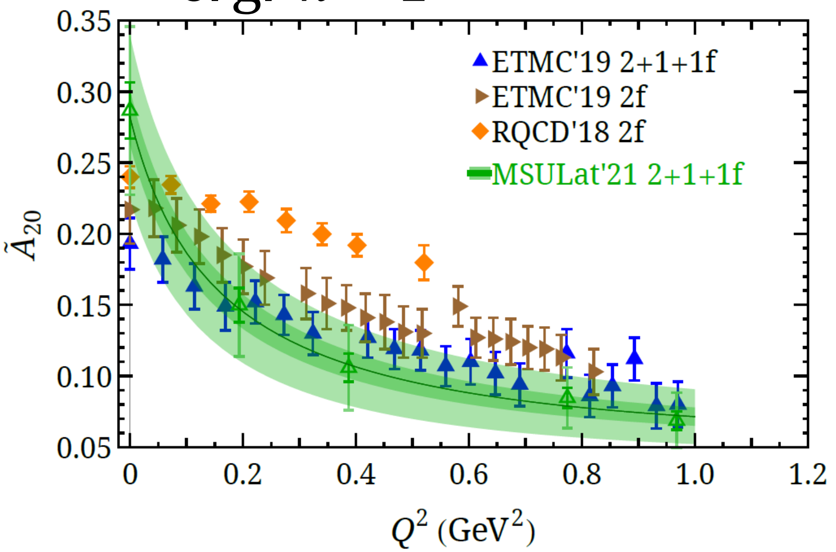
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821





- Take the integral to form moments;
  e.g. $n = 2$



Future Prospects & Challenges

Challenges

- § Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- § Renormalization of linear divergence
  - Wilson-line ops have linear divergences that must be subtracted
- § Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- § Inverse problems PDF extraction in SDF
  - Remove the model/preconditioner-choice dependence
- § Reaching long-range correlations in LaMET
  - For small-x physics, new methods for calculating longer-range correlations must be developed

2202.07193

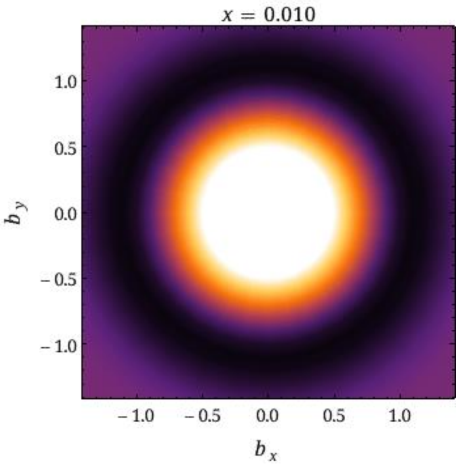
Challenges

- Wanted lattice calculations in the next few years for isovector nucleon PDFs
  - a = 0.05 fm (corresponding to $a^{-1} \approx$ 4 GeV)
  - $M_\pi \approx$ 139 MeV with at least M_πL = 3
    (L = 4.5 fm , L/a ≈ 90)
  - We need nucleon momenta of P ≈ 2.6 GeV
- Flavor-dependent PDFs more challenging to overcome

2202.07193

Summary

- § Exciting era using LQCD to study hadron structure
  - Well-studied systematics → precision structures
  - More nucleon matrix elements with physical pion masses
- § Overcoming longstanding limitations of moment method
  - Bjorken-x dependence of parton distributions are widely studied with LaMET and its variants
  - More study of systematics planned for the near future
  - Start to address neglected disconnected contributions obtaining flavor-dependent quantities



- § Stay tuned for more updates from LQCD

Summit @ORNL
IC@LANL

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices
The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405 & RCSA Cottrell Scholar Award

Backup Slides

Other Lattice Progress

- Exploratory study on strange, charm and gluon PDFs
- Many approaches are moving to the NNLO level
  - Expect to see more improved lattice calculations
- Beyond the standard twist-2 collinear PDFs
  - Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
  - Transverse-momentum– dependent distributions (TMDs)
    - Collins-Soper kernel, soft function and wavefunctions
  - Twist-3 PDFs and GPDs

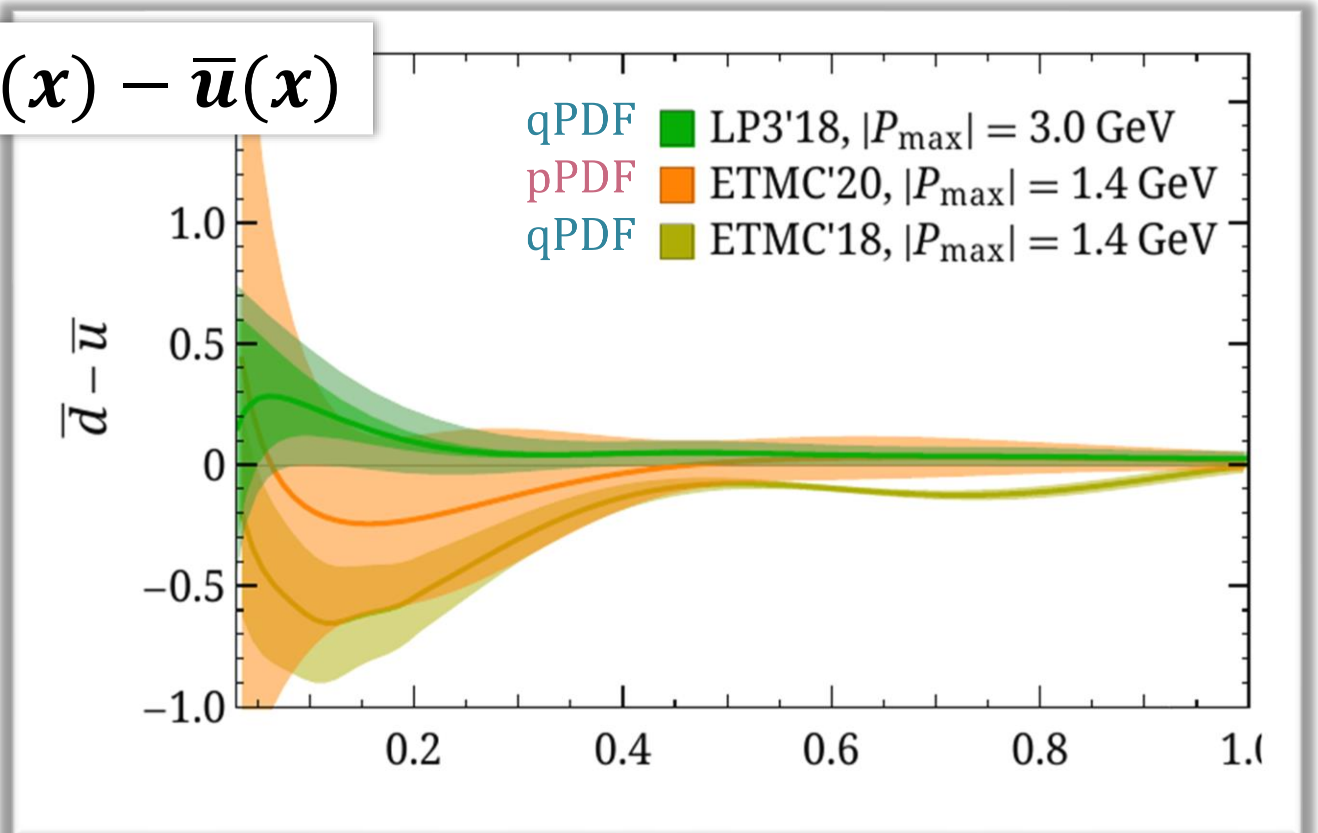
For more details and references, refer to 2202.07193

Physical Pion Mass Results

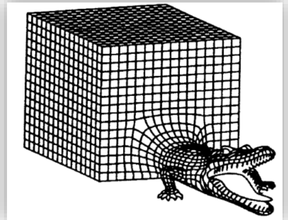
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



2006.08636, PDFLattice2019 report



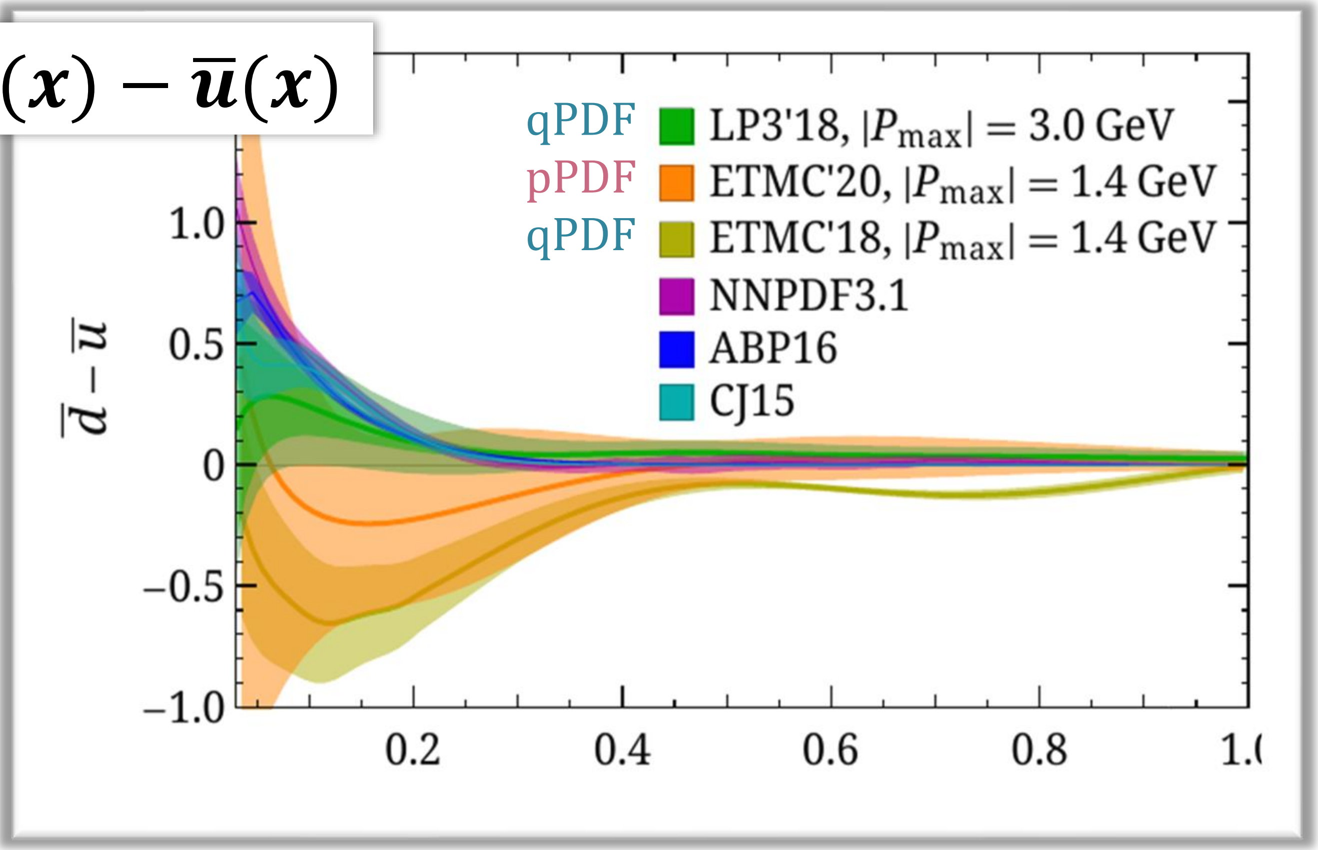
Finite volume,
Discretization,
…

Physical Pion Mass Results

§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



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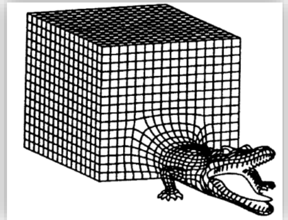
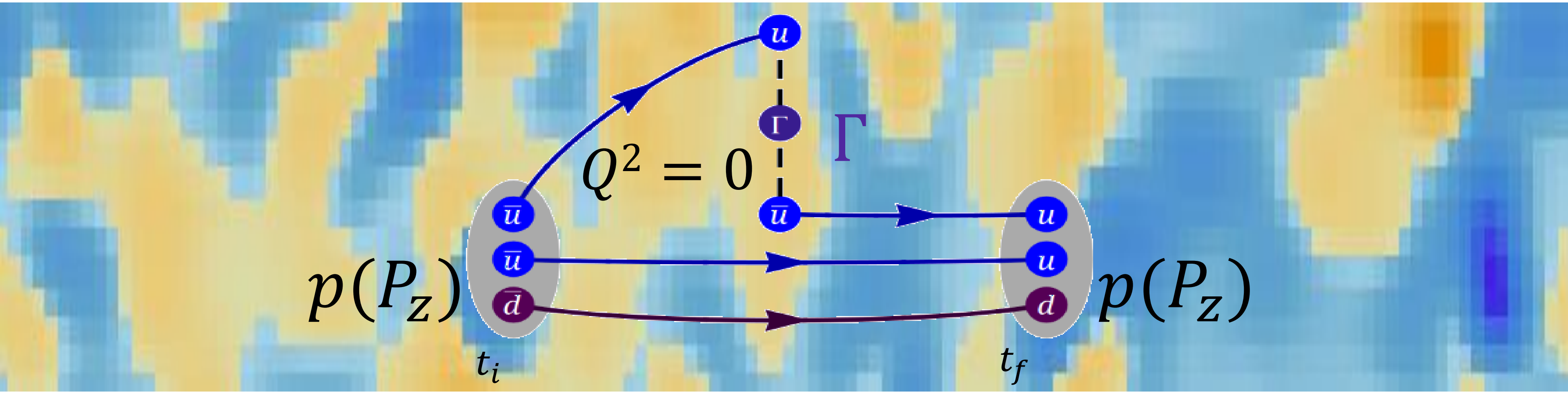


Finite volume,
Discretization,
…

Quasi-PDF vs Pseudo-PDF

§ They both calculate the matrix element $h(z, P_z)$



§ Pseudo-PDF

- No renormalization

$$\mathcal{M}(zP_z, z^2) = \frac{h(z,P_z)}{h(z,0)}$$

- FT zP_z-space to x-space at fixed z^2
  pseudo-PDF $\widetilde{\mathcal{M}}(x, z^2)$

§ Quasi-PDF

- Renormalization and ratios

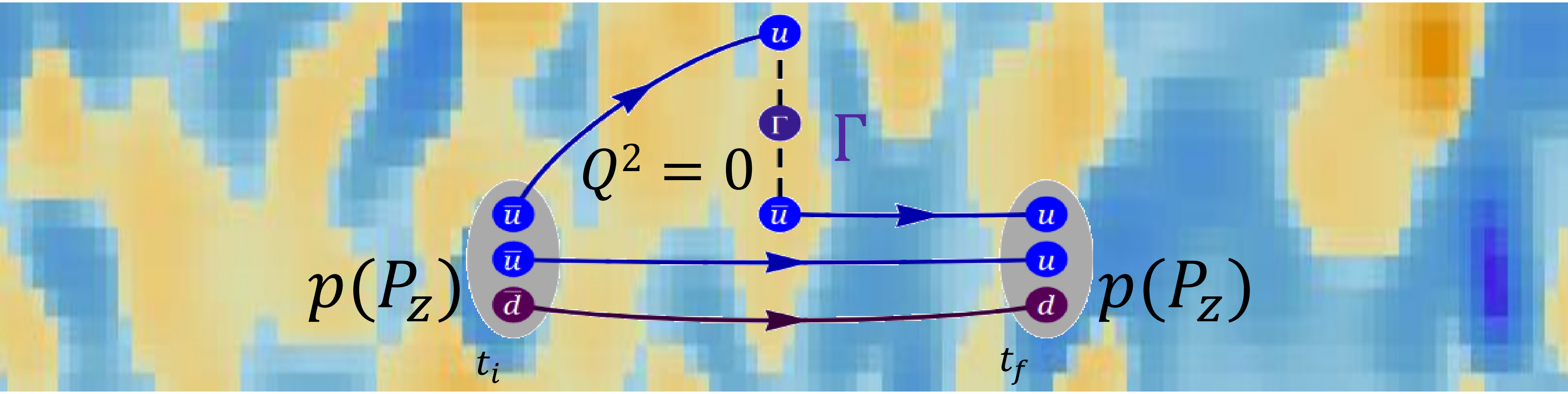
$$h^R(z, P_z, P^R) \text{ or } \frac{h(z,P_z,P^R)}{h(z=0,P_z,P^R)}$$

- FT z-space to x-space at fixed P_z
  quasi-PDF $\tilde{q}(x, P_z, P^R)$

See X. Ji, et al., NPB 964 (2021) and references on newer renormalization proposals

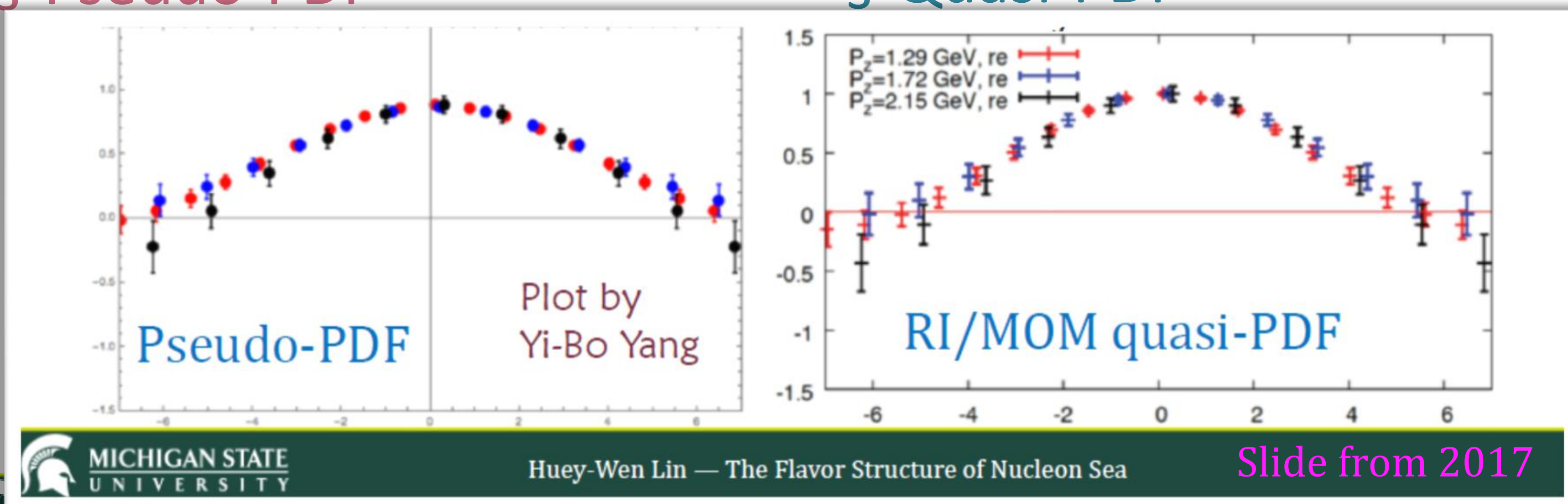
Quasi-PDF vs Pseudo-PDF

- § They both calculate the matrix element $h(z, P_z)$



§ Pseudo-PDF

§ Quasi-PDF

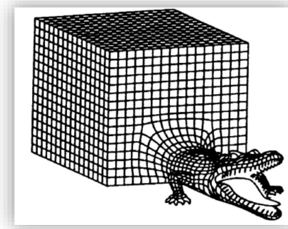


Isovector Nucleon GPDs

§ Pioneering first glimpse into nucleon GPD using quasi-PDFs

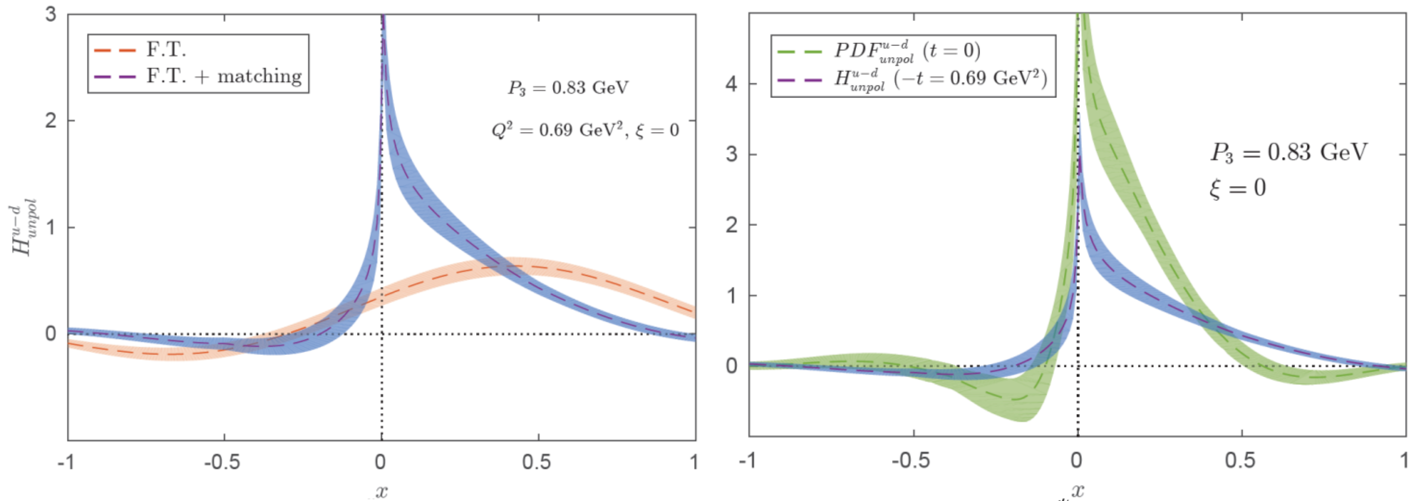
- Lattice details: twisted-mass fermions, 0.09fm, **270-MeV** pion mass, $P_z \approx 0.83$ GeV

$$F(x,\xi,t) = \int \frac{d\zeta^-}{4\pi} e^{-ix\bar{P}^+\zeta^-} \langle P'|O_{\gamma^+}(\zeta^-)|P\rangle = \frac{1}{2\bar{P}^+}\bar{u}(P')\left\{H(x,\xi,t)\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right\}u(P)$$



nucleon $\xi = 0$ isovector results

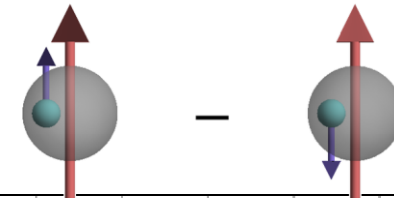
C. Alexandrou, (**ETMC**), 1910.13229 (Lattice 2019 Proceeding)



Moments of PDFs

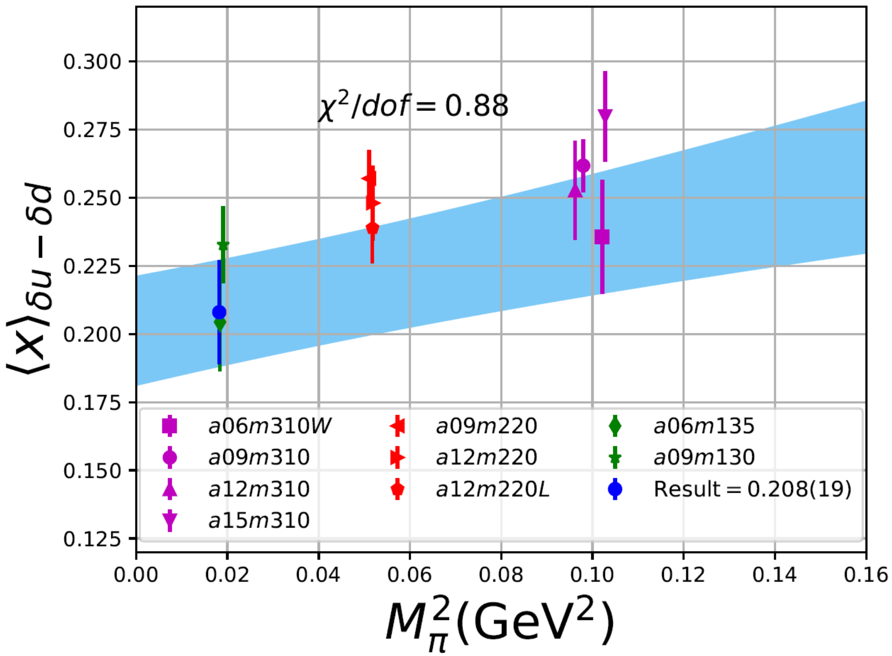
- § Only lowest few moments

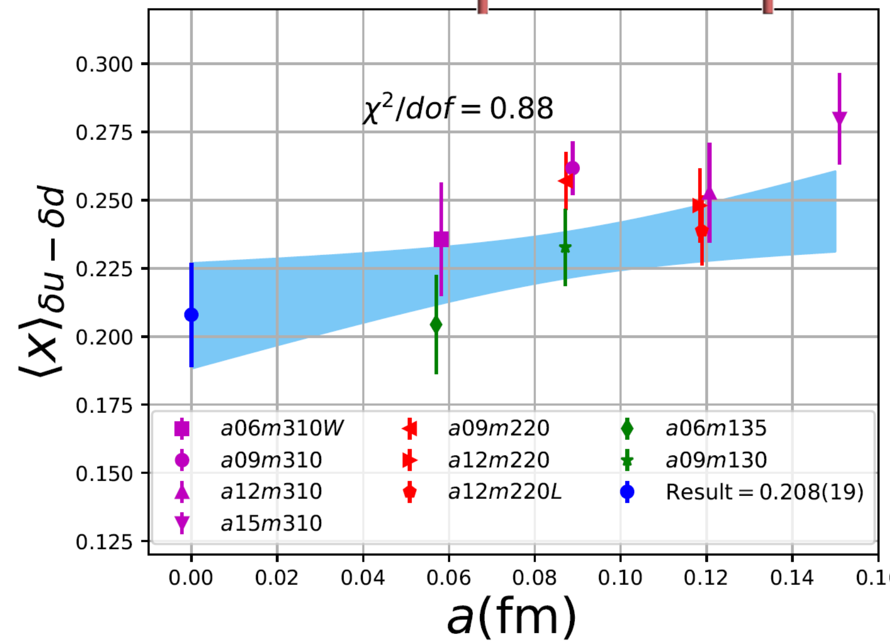
$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



- § State-of-the art example
  - Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779





- § Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…