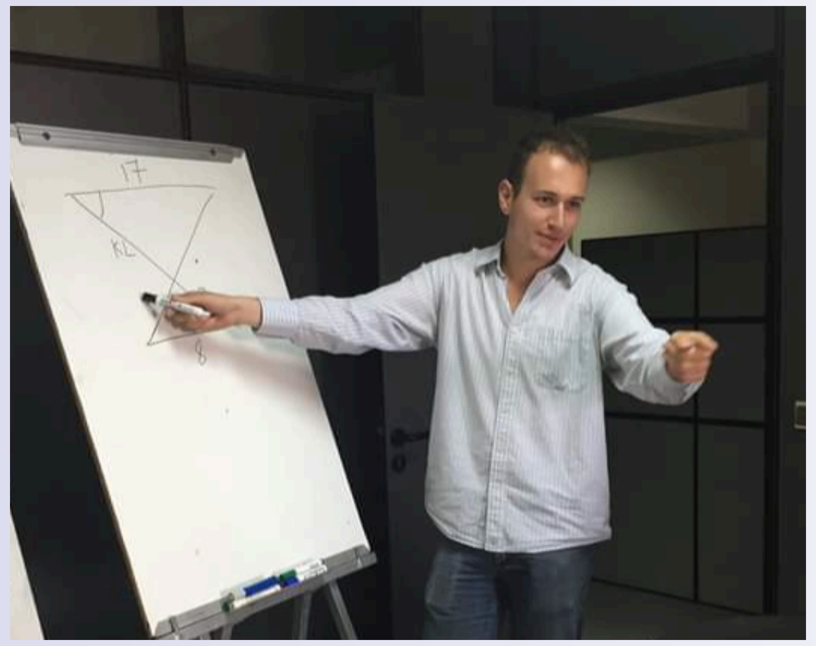
# *Thermalization Hadronization & Entanglement*



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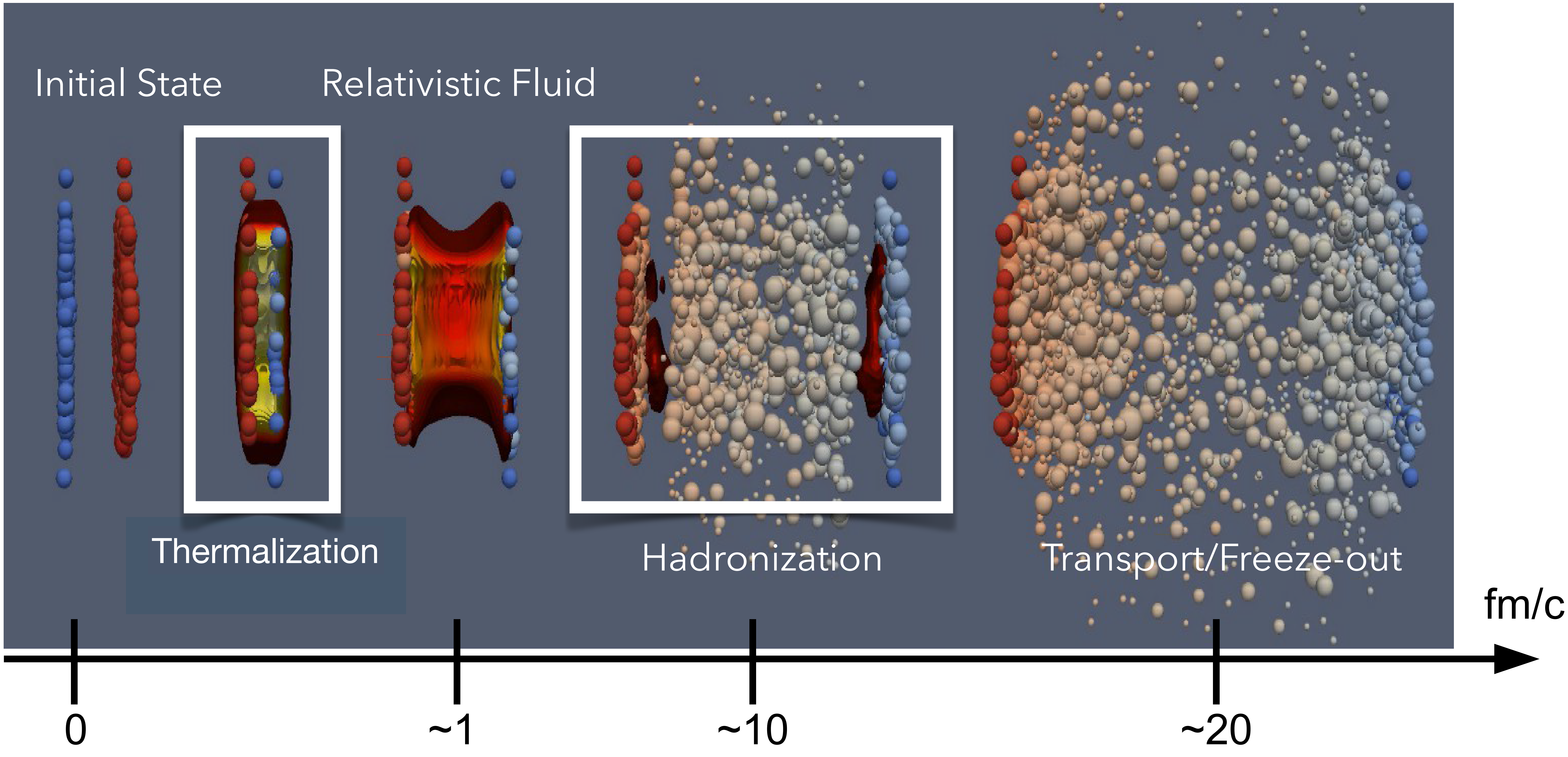
BM & AS, arXiv:2211.16265

# Relativistic heavy ion collision: Stages



# Modeling the different stages

- We have good and demonstrably rigorous (in some limit) descriptions of the three extended stages:
  - Initial state: Color glass condensate
  - QGP stage: Relativistic viscous hydrodynamics
  - Late hadron stage: Boltzmann equation transport

- The two intervening stages are “messy” from a quantum field theory standpoint:
  - Pre-equilibrium and thermalization
  - Hadronization of the QGP

- Use other powerful tools to help describe these stages
  - Holography based on AdS/CFT duality

# Relativistic heavy ion collisions: Scales

Heavy ion collisions are characterized by three distinct scales:

- Scale of initial energy deposition:
  - Gluon saturation scale $Q_s$ in the CGC model
  - Initial temperature $T_0$ at thermalization
- Intrinsic QCD scale:
  - $\Lambda_{QCD} \approx 200$ MeV
  - Pseudocritical temperature $T_c \approx 155$ MeV
- Nuclear radius ($A \approx 200$):
  - $R = r_0 A^{1/3} \approx 5 - 10$ fm $= (20 - 40 \text{ MeV})^{-1}$

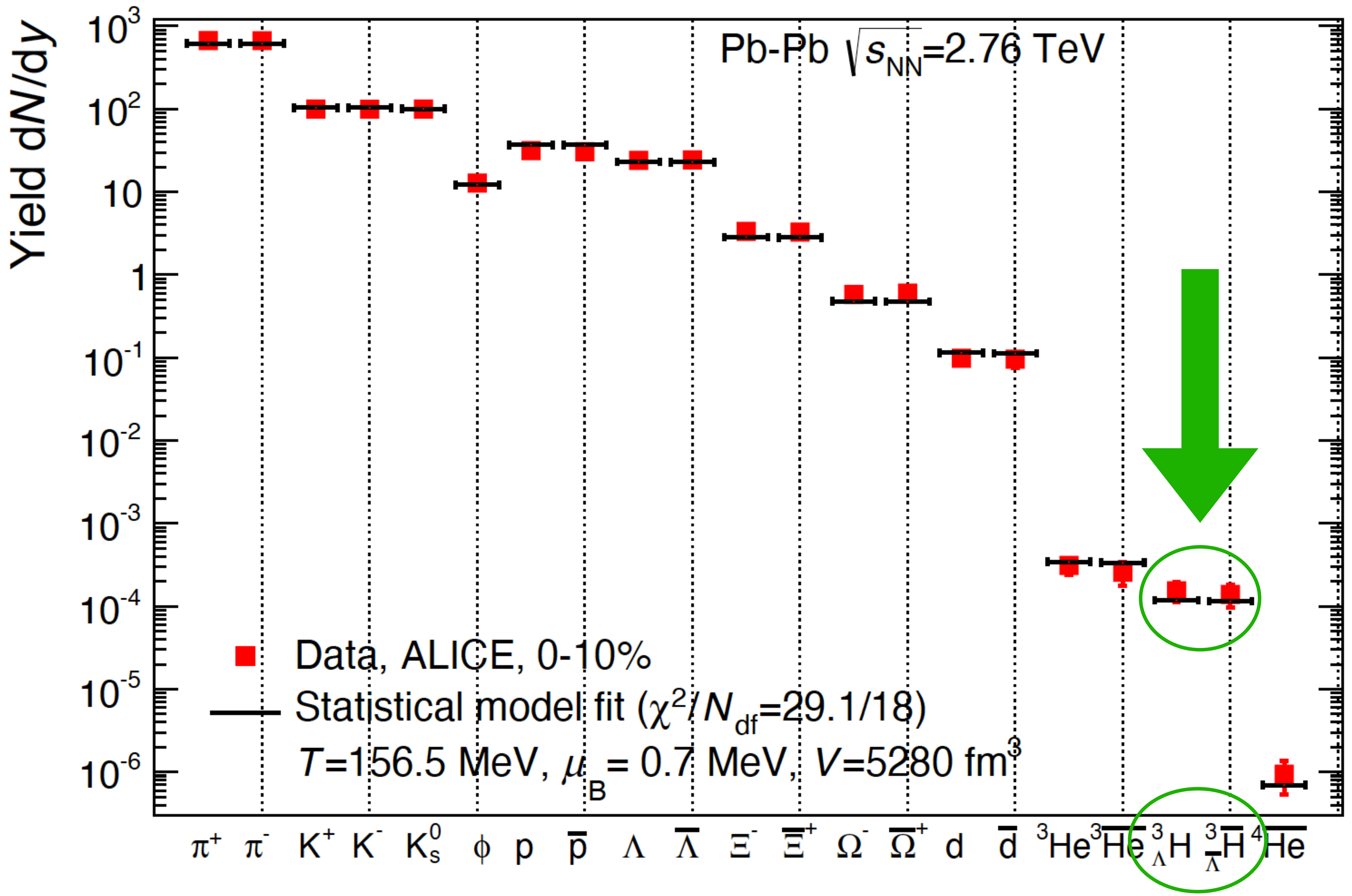
For $Q_s \gg \Lambda_{QCD} \gg R^{-1}$ a theoretically well justified description is possible.

When two scales are comparable, the physics may be interesting but is difficult to control.

Physics near $\Lambda_{QCD}$ requires a nonperturbative approach, e.g. lattice QCD or AdS/CFT.

# Thermalization

# Thermal particle yield conundrum




Thermal model of particle emission

$$\frac{dN}{dy} \propto \exp\left(-\frac{M_i c^2 - \mu_B A_i}{T}\right)$$

The hyper-triton (pnΛ) is the lightest hyper-nucleus and **very** weakly bound: $B_\Lambda \approx -0.4$ MeV and has a radius $R_{rms} > 10$ fm.

How can this particle be emitted with a thermal yield from a fireball of temperature $T$ = 156 MeV ?

Does this observation question the validity of the entire thermalization picture, because it is preposterous?

# Thermalization and Quantum Mechanics

Experience: Interacting systems thermalize and attain a state of maximal entropy

Quantum mechanics: Isolated quantum systems evolve unitarily. If they are formed in a pure state, they remain in a pure quantum state, and their von Neumann entropy $S = \mathrm{Tr}(\rho \ln \rho) = 0$.

Resolution: Entanglement. Different parts of the system (or the system and a heat bath) become quantum mechanically entangled. Entanglement entropy: $S_A = -\mathrm{Tr}_A(\rho_A \ln \rho_A) > 0$ with $\rho_A = \mathrm{Tr}_B(\rho_{AB})$.

Example: "Thermofield double" state of two copies *A* and *B* of a quantum mechanical system.

$$|\Psi\rangle = Z(\beta)^{-1/2} \sum_E e^{-\beta E/2} |\psi_E^{(A)}\rangle |\psi_E^{(B)}\rangle$$

$$\rho_A = \mathrm{Tr}_B\left(|\Psi\rangle\langle\Psi|\right) = Z(\beta)^{-1} \sum_E e^{-\beta E} |\psi^{(A)}\rangle\langle\psi_E^{(A)}|$$

Thermal ensemble for subsystem *A*.

Generalization: W. Cottrell et al., *How to build a thermofield double?*, arXiv:1811.11528 [hep-th]

# Eigenstate Thermalization Hypothesis (ETH)

Observables (operators) are only sensitive to certain aspects of a complex quantum system. This suggests that matrix elements of most observables in stationary states (energy eigenstates) of an isolated quantum system may exhibit “thermal“ properties.

This concept is encoded in a generic form of the matrix elements of an observable $\mathscr{A}$ in the energy basis:

$$A_{\alpha\beta} = \langle E_\alpha | \mathscr{A} | E_\beta \rangle = A(E)\,\delta_{\alpha\beta} + e^{-S(E)/2} f(E,\omega)\,R_{\alpha\beta}$$

where $E = (E_\alpha + E_\beta)/2,\ \omega = E_\alpha - E_\beta$ and $R_{\alpha\beta}$ is a normalized random matrix; $f(E,\omega)$ is the spectral function, and $S(E)$ is the usual microcanonical entropy defined in statistical physics.

Off-diagonal matrix elements are statistically suppressed. Diagonal matrix elements yield the thermal average of the observable:

$$\langle \mathscr{A} \rangle_T = Z(\beta)^{-1} \int \frac{dE}{E}\, e^{S(E)-\beta E}\, A(E) + O(e^{-S/2})$$

Thermal fluctuations are related to quantum fluctuations: $\langle \mathscr{A}^2 \rangle_T - \langle \mathscr{A} \rangle_T^2$ .

The autocorrelation function $\langle \mathscr{A}(t+\tau)\mathscr{A}(t) \rangle_T$ is related to the spectral function $f(E,\omega)$.

# Questions about ETH

- Which systems exhibit ETH behavior?
  - Chaotic quantum systems characterized by exponentially growing out-of-time correlators (OTOC), e.g.:

$$C(t) = \langle \Psi \,|\, [x(t), p(0)]^2 \,|\, \Psi \rangle \sim \hbar^2 e^{2\lambda t}$$

  $\lambda \le 2\pi T$ is called quantum Lyapunov exponent.

- Over which frequency range $\Delta\omega$ does ETH behavior manifest itself — equivalent to the question on what time scale thermal behavior becomes established?
  - The answer varies for different observables.
  - The generic scale is the Thouless energy $E_{\rm Th} = D/L^2$
  - For some observables $\Delta\omega$ is parametrically larger: $E_{\rm Th}/L^d \sim D/L^{2+d}$ (Dymarsky, 1804:08626)

- Heavy ion collisions: $E_{\rm Th} \sim D/R^2 \sim (\pi T R^2)^{-1} \sim 2\ {\rm MeV}^{-1} \sim (100\ {\rm fm/c})^{-1}$

# What we (don't) know

- ETH behavior can be demonstrated in
  - Finite systems, i.e. tensor networks, where exact (classical) numerical solutions are possible (typically $< 20$ degrees of freedom)
  - Certain chaotic matrix models, such as the SYK model
  - Certain models of (1+1) dimensional gravity and (1+1)-dim CFTs

- In such theories, exact or asymptotically exact expressions for the spectral density $\langle \rho(E) \rangle$ and its correlations $\langle \rho(E)\rho(E+\omega) \rangle$ can be found, and long-time behavior can be determined.

- In gravity theories or theories with a holographic dual, $\langle \rho(E)\rho(E+\omega) \rangle \neq \langle \rho(E) \rangle \langle \rho(E+\omega) \rangle$ because of complex saddle points of the action with higher-genus topologies, so-called "Euclidean wormholes" (see, e.g., P. Saad et al., 1806.06840). This leads to intriguing questions about the relationship between QFTs and their holographic duals (see, e.g., C. Johnson, 2201.11942).

# Entanglement

# Quantum entanglement

Best known for bipartite systems (A∪B). A and B are entangled when $\rho_{A\cup B} \neq \rho_A \rho_B$ .

Entanglement entropy:

$$S_E = -\,\mathrm{Tr}\,(\rho_A \ln \rho_A) = -\,\mathrm{Tr}\,(\rho_B \ln \rho_B) \quad \text{with } \rho_A = \mathrm{Tr}_B(\rho_{A\cup B}),\ \rho_B = \mathrm{Tr}_A(\rho_{A\cup B})$$

Thermal entropy in the traditional sense = entanglement entropy between a system and the heat bath.

Entanglement between two particles is *maximal* when they form a *Bell pair*, e.g. for spins:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\downarrow\rangle_2 \pm |\downarrow\rangle_1 |\uparrow\rangle_2\right)$$

*Monogamy of entanglement:* One particle cannot be maximally entangled with more than a single other particle, but entanglement can be shared among multiple particles ($\tau$ = “tangle” function):

$$\sum_{k=2}^{N} \tau(\rho_{A_1 A_k}) \leq \tau(\rho_{A_1(A_2\cup\cdots\cup A_N)}) \leq 1$$

# Heavy ion collision: Entanglement

The initial state (two moving nuclei in their ground states) is approximately a pure state.

The internal parton wave functions of each nucleon are approximately pure states but highly entangled states (color singlets, spin/isospin 1/2, baryon number 1).

Parton wave functions of different nucleons are approximately unentangled.

Entanglement among partons from different nucleons must be built up by interactions over the course of the collision.

Longitudinal direction: Entanglement is created during impact, when partons are liberated.

Transverse direction: Entanglement must spread by causal entanglement transport, which occurs at the "butterfly velocity" $v_B$. ($v_B$ is not known for QCD.)

$v_B$ depends on the dimension and the initial state of the system; it is always $v_B \leq c$.

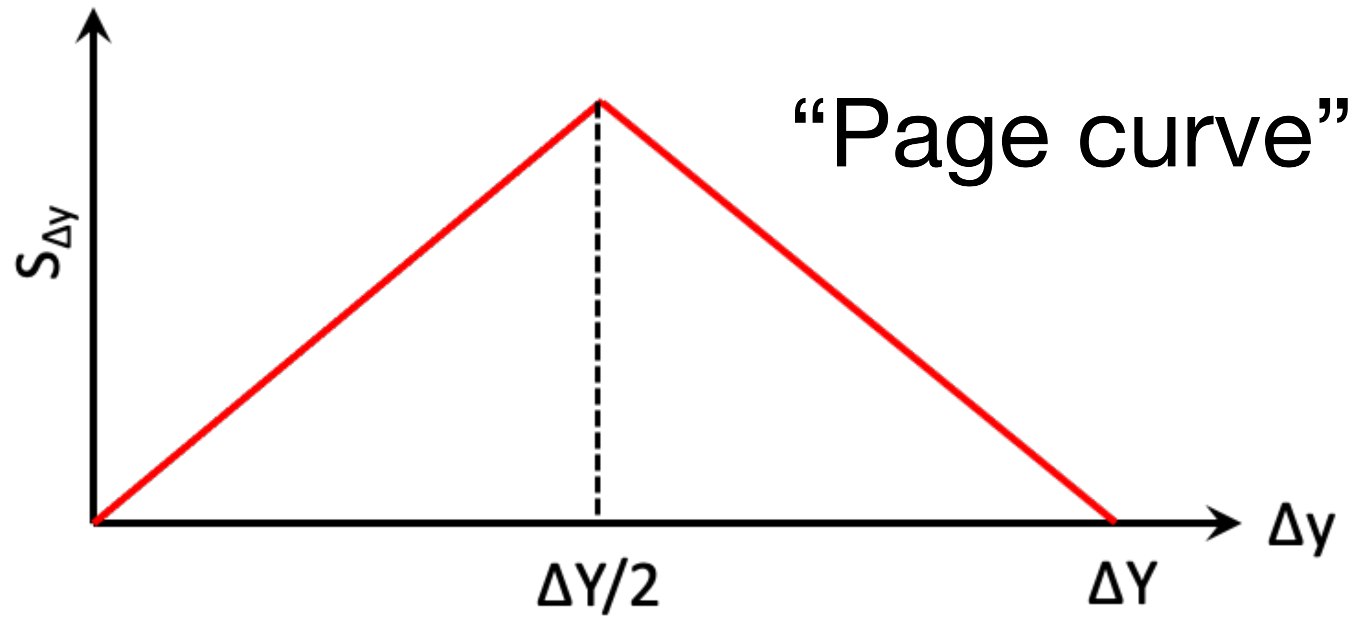
E.g., in strongly coupled *N*=4 SYM: $v_B^{(1+1)} = c, \quad v_B^{(2+1)} = 0.687c, \quad v_B^{(3+1)} = 0.620c$

# Heavy ion collision: Entropy

The particles emitted in a rapidity interval Δy are entangled with particles at other rapidities.

Entanglement entropy: $S_{\Delta y} = -\,\mathrm{Tr}\,(\rho_{\Delta y} \ln \rho_{\Delta y})$ where $\rho_{\Delta y} = \mathrm{Tr}_{y \notin \Delta y}(\rho)$.

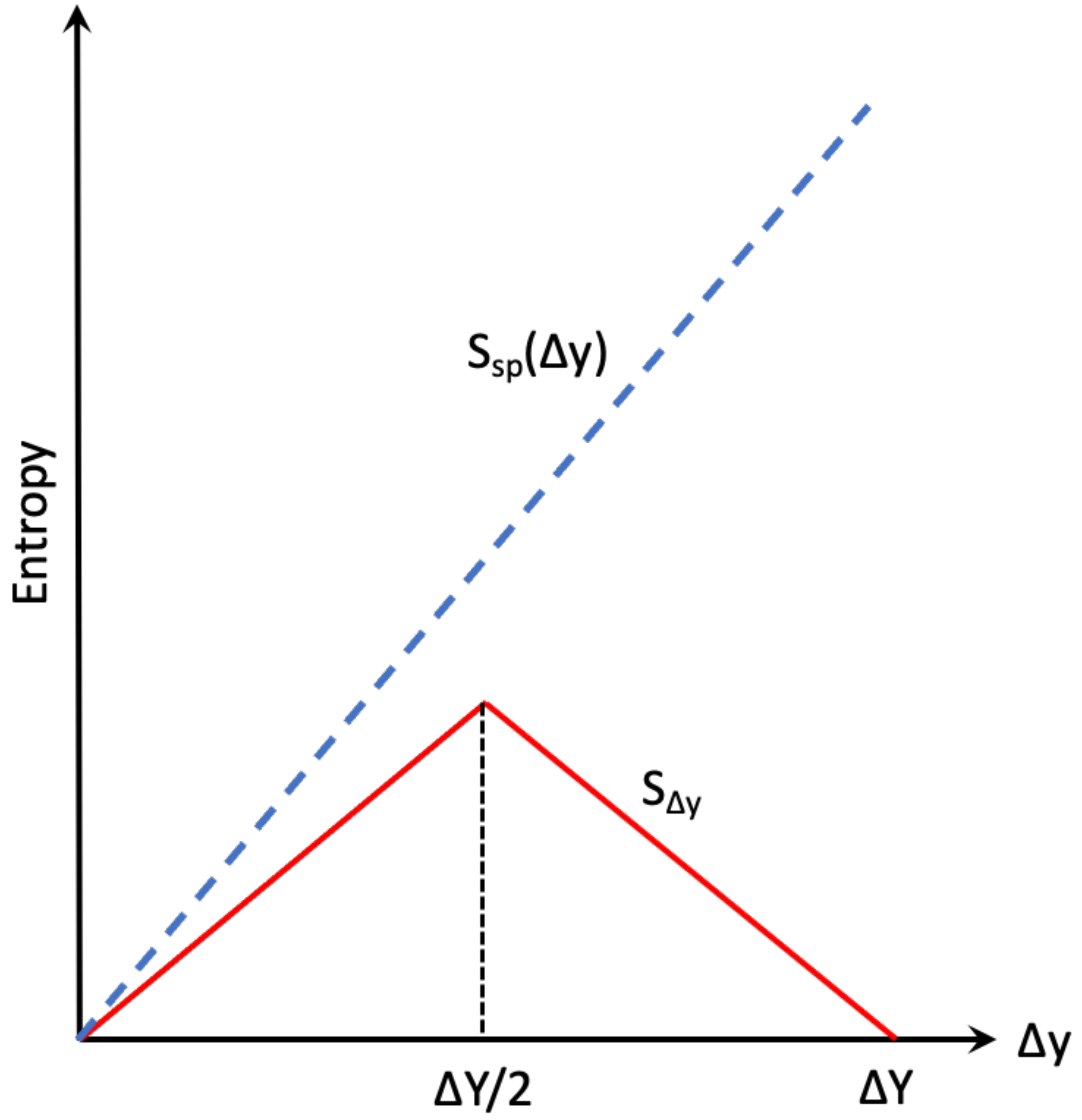
$S_{\Delta y} \propto \Delta y$ when $\Delta y \ll \Delta Y$,
but $S_{\Delta y} \to 0$ when $\Delta y \to \Delta Y$,
because overall the state is pure.




Usually, the final-state entropy is inferred from the single-particle spectrum: $S_{\rm sp}(\Delta y)$. This quantity ignores correlations among particles within Δy: $S_{\rm sp}(\Delta y) > S_{\Delta y}$.

$S_{\rm sp}(\Delta y)$ continues to grow as $\Delta y \to \Delta Y$.

$S_{\rm sp}(\Delta y)$ has been called "entropy of ignorance" [Duan 2001.01726]



# Holography

# The principle of holography

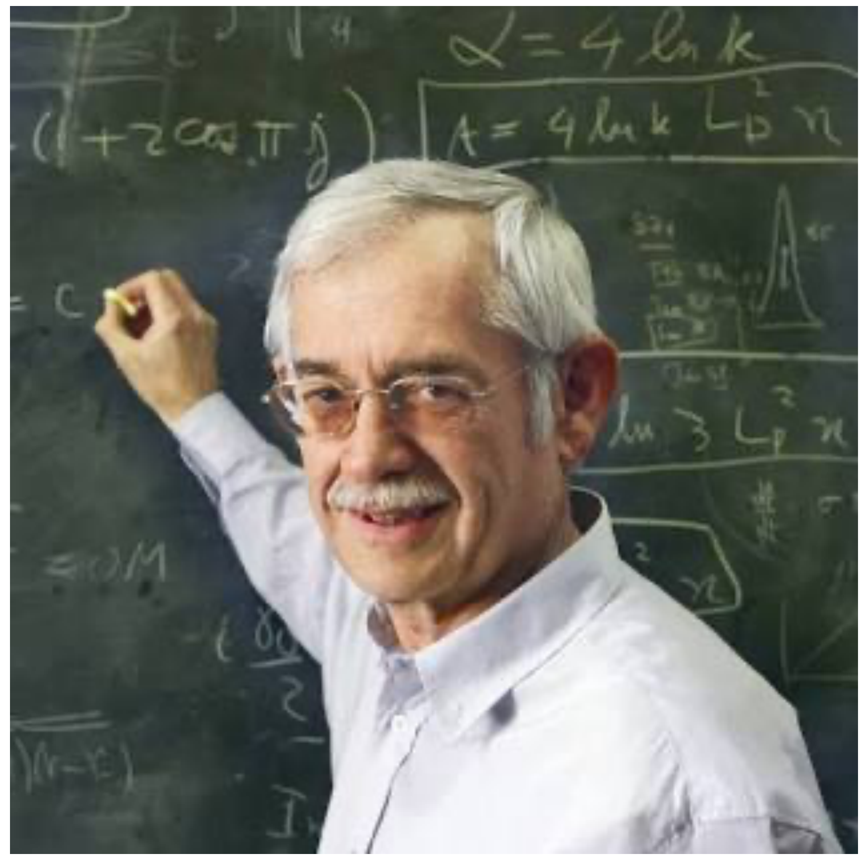
The entropy of a volume of space is maximized by the Bekenstein entropy of a black hole whose event horizon encloses this volume.

This entropy is proportional to the surface area of the event horizon measured in units of the Planck length scale, not the volume.

This suggests that at the Planck scale all degrees of freedom of 3-dimensional space are encoded in a 2-dimensional hologram (’t Hooft 1993, Susskind 1994).

What was missing was a concrete model.

This model was provided by Maldacena’s discovery in 1997 of the AdS/CFT duality (18,257 citations!).



# AdS/CFT duality - holography

- Maldacena proposed that a strongly coupled "cousin" of QCD − the SU($N_c$) gauge theory with $\mathcal{N}$=4 supersymmetries and many colors $N_c \gg 1$− is *dual* to a weakly coupled (and thus solvable!) string theory on the 10-dimensional space-time of topology $AdS_5$ x $S_5$.

AdS-metric: $$ds^2 = \frac{1}{z^2}(-dt^2 + dz^2 + d\vec{x}^2)$$

- In the limit $N_c \to \infty$ the string theory effectively becomes classical gravity, which can be solved using standard analytical and numerical techniques.

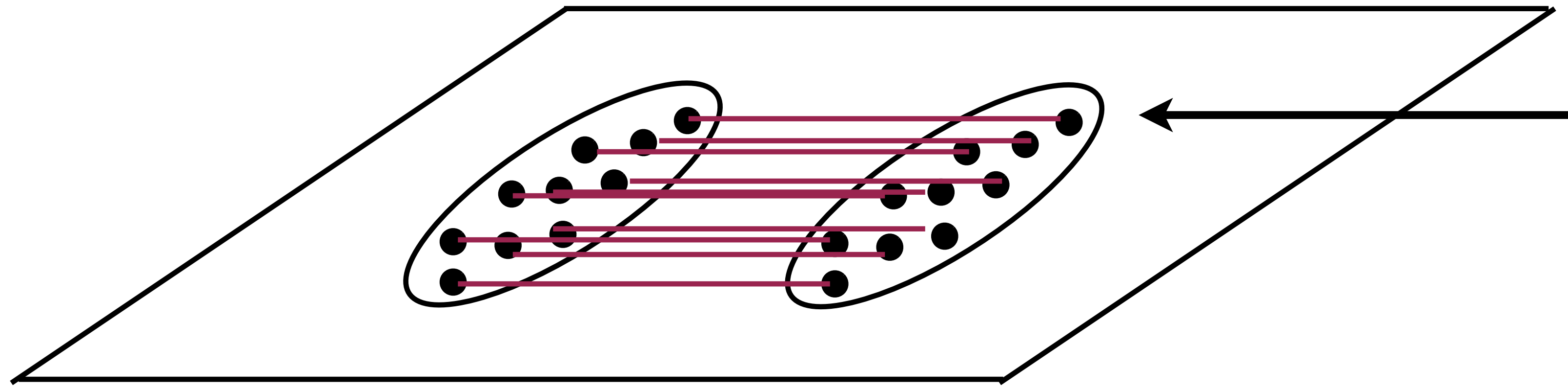
- The $\mathcal{N}$=4 SUSY gauge theory "lives" on the edge of $AdS_5$ space. The d*uality* means that every observable in the gauge theory can be mapped onto an object in the string theory.

- If $AdS_5$ space contains a black hole, the boundary gauge theory is thermally excited; the temperature is given by the horizon radius.

AdS-BH metric: $$ds^2 = \frac{1}{z^2}\left(-(1 - Mz^4)dt^2 + \frac{dz^2}{1 - Mz^4} + d\vec{x}^2\right)$$

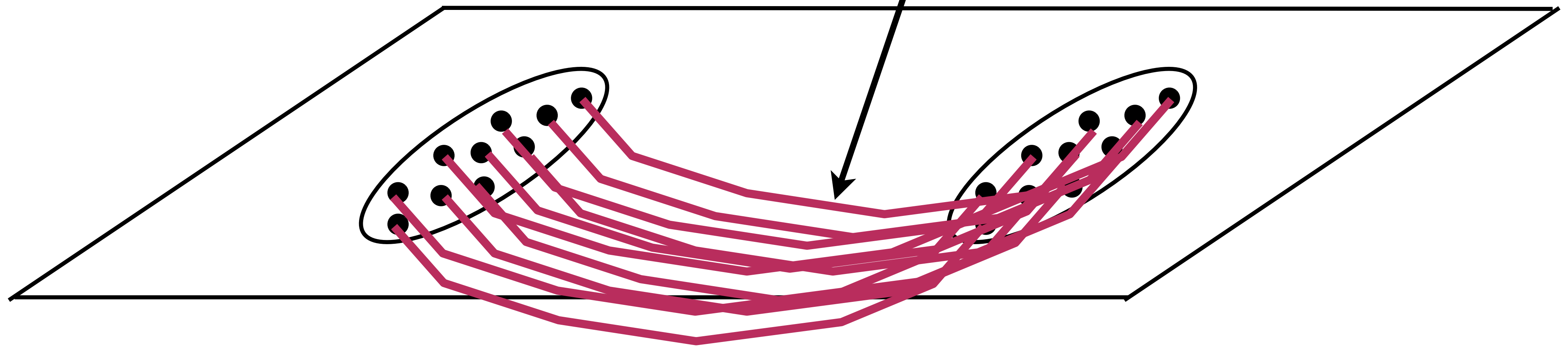
# Thermalization

# Idealized heavy ion collision

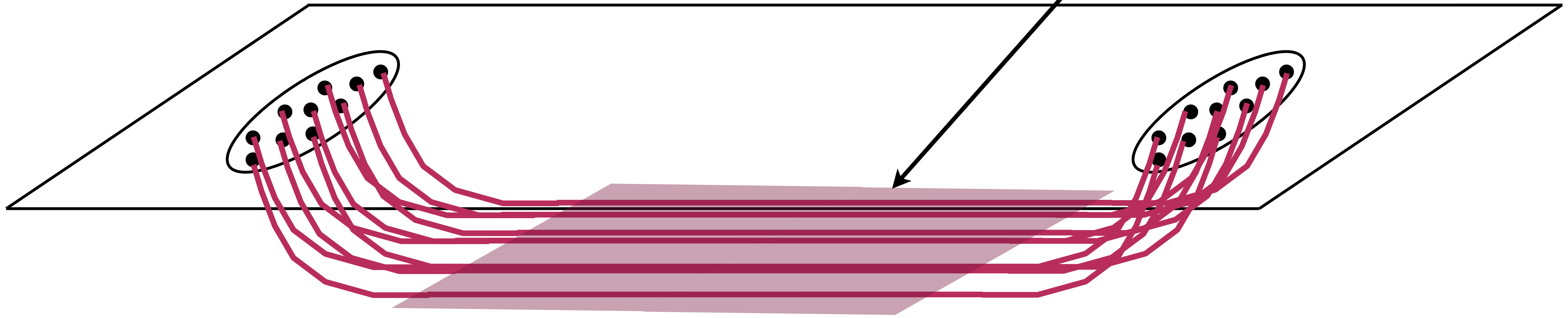


Picture the precursor of the QGP as collection of flux tubes connecting nuclear color charges moving apart at high speed.

As color charges separate, flux tubes sink into the bulk.

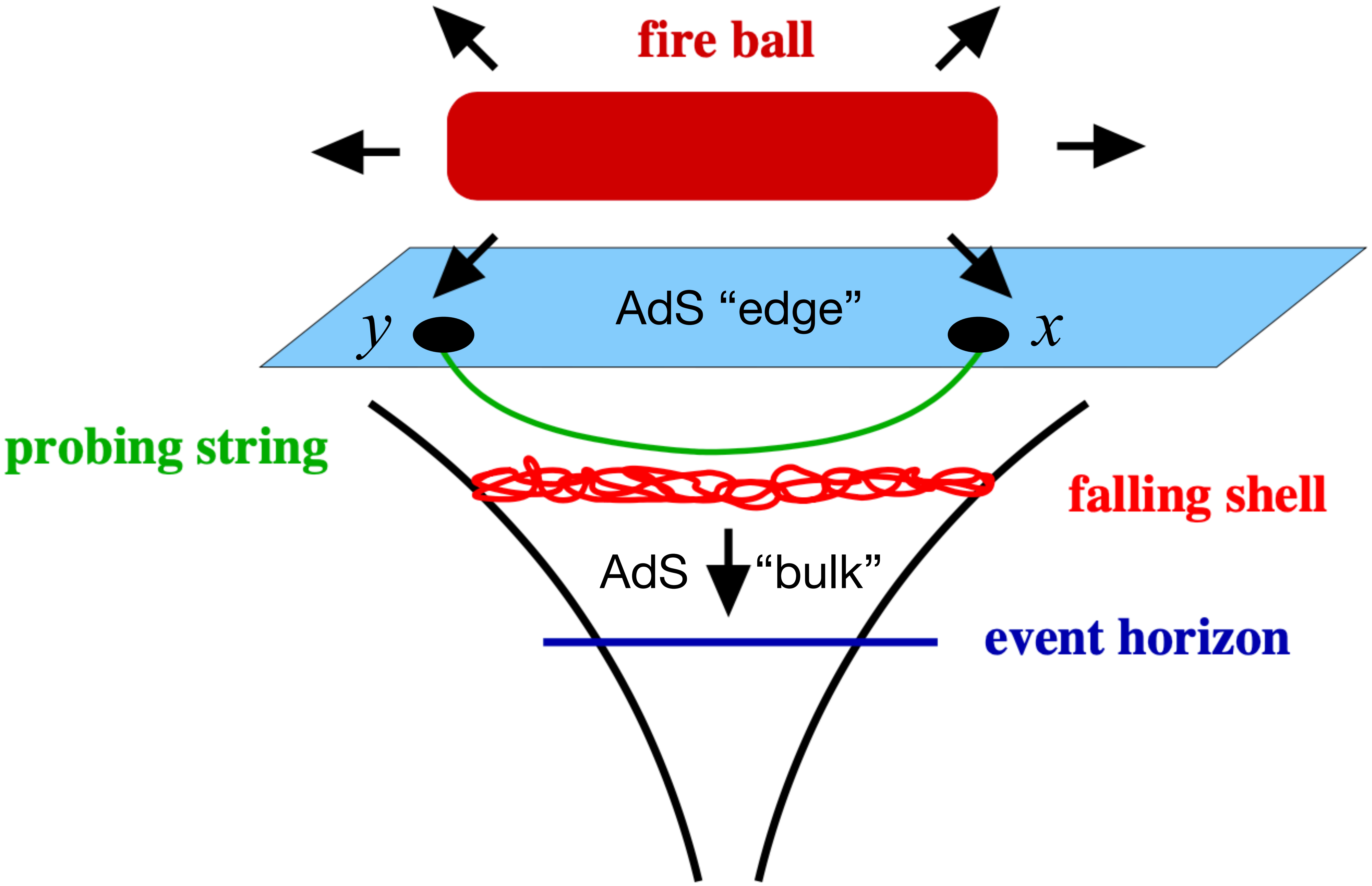


Collection of parallel flux tubes is idealized as thin mass shell moving deeper into the bulk.



When the shell sinks below its Schwarzschild radius, thermalization is reached.

# “Holographic” thermalization



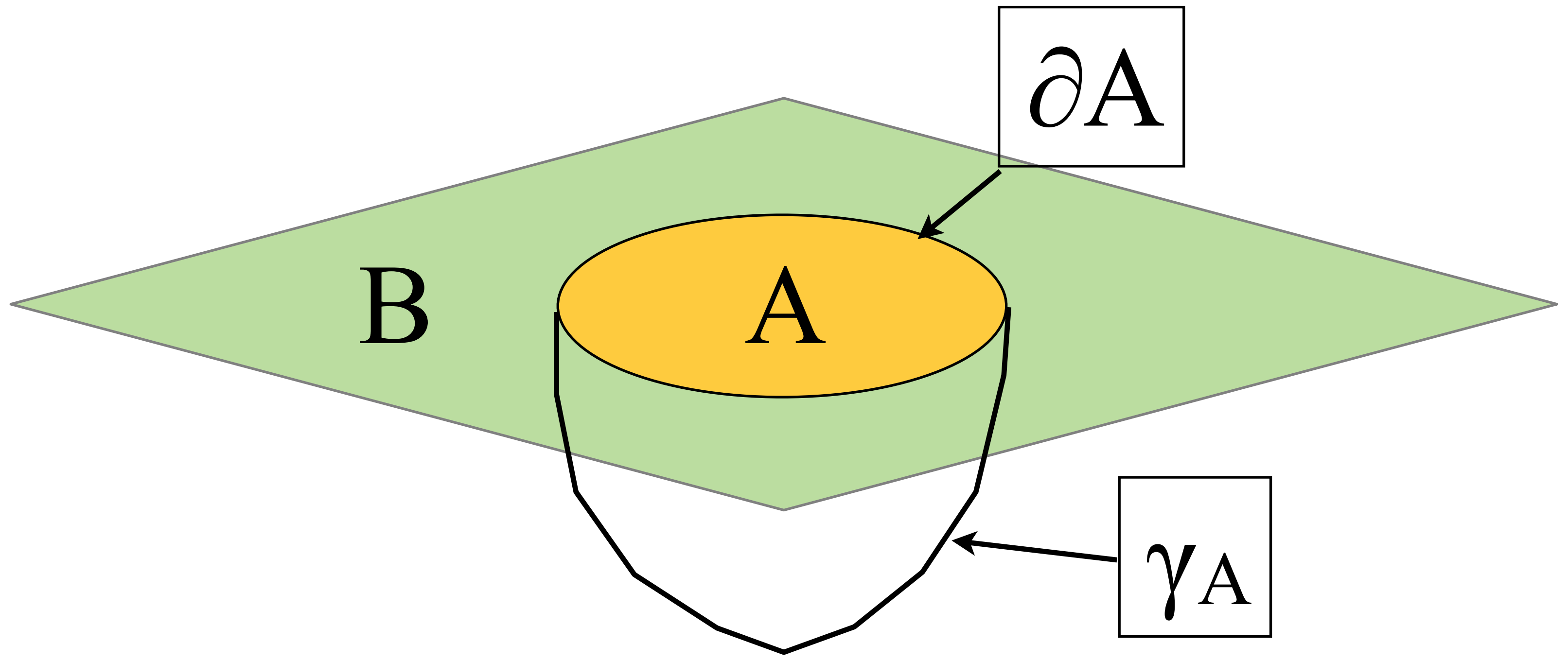

- Probe for thermalization with a QFT observable $\langle O(y)O(x)\rangle$ in the $AdS_5$ “edge”
- Such bi-local observables are needed, e.g., to find the momentum spectrum
- The dual in the $AdS_5$ “bulk” geometry is a string that hangs between $x$ and $y$
- Evaluate in the presence of the falling massive shell

*V. Balasubramanian, et al.*, PRL 106 (2011) 191601; PRD 84 (2011) 026010

# Entanglement entropy

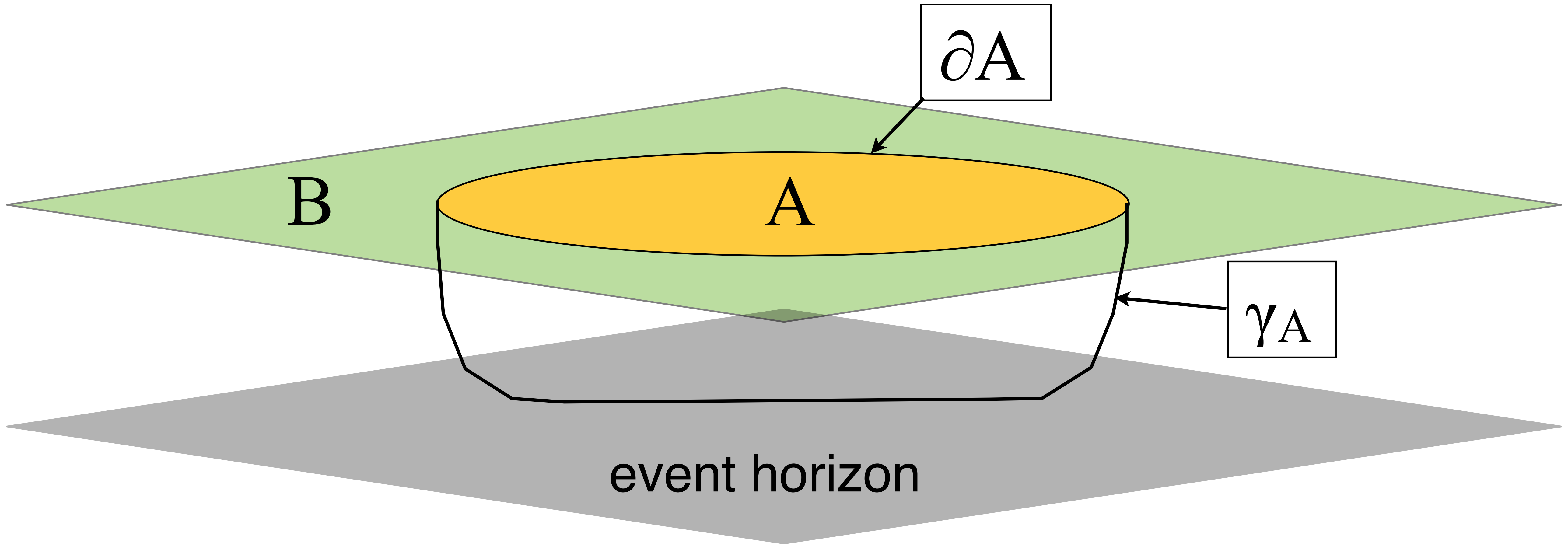
For us, the most important quantity is the entropy contained in a certain volume A on the boundary.

For a quantum field theory with a holographic gravity dual, $S_A$ can be calculated in the dual theory from the area of the minimal surface $\gamma_A$ in the bulk that has the same boundary ∂A as $A$: $\partial(\gamma_A) = \partial A$ .




$$S_A = \frac{\text{Area of } \gamma_A}{4G}$$

[Hrubeny-Ryu-Takayanagi (HRT)]

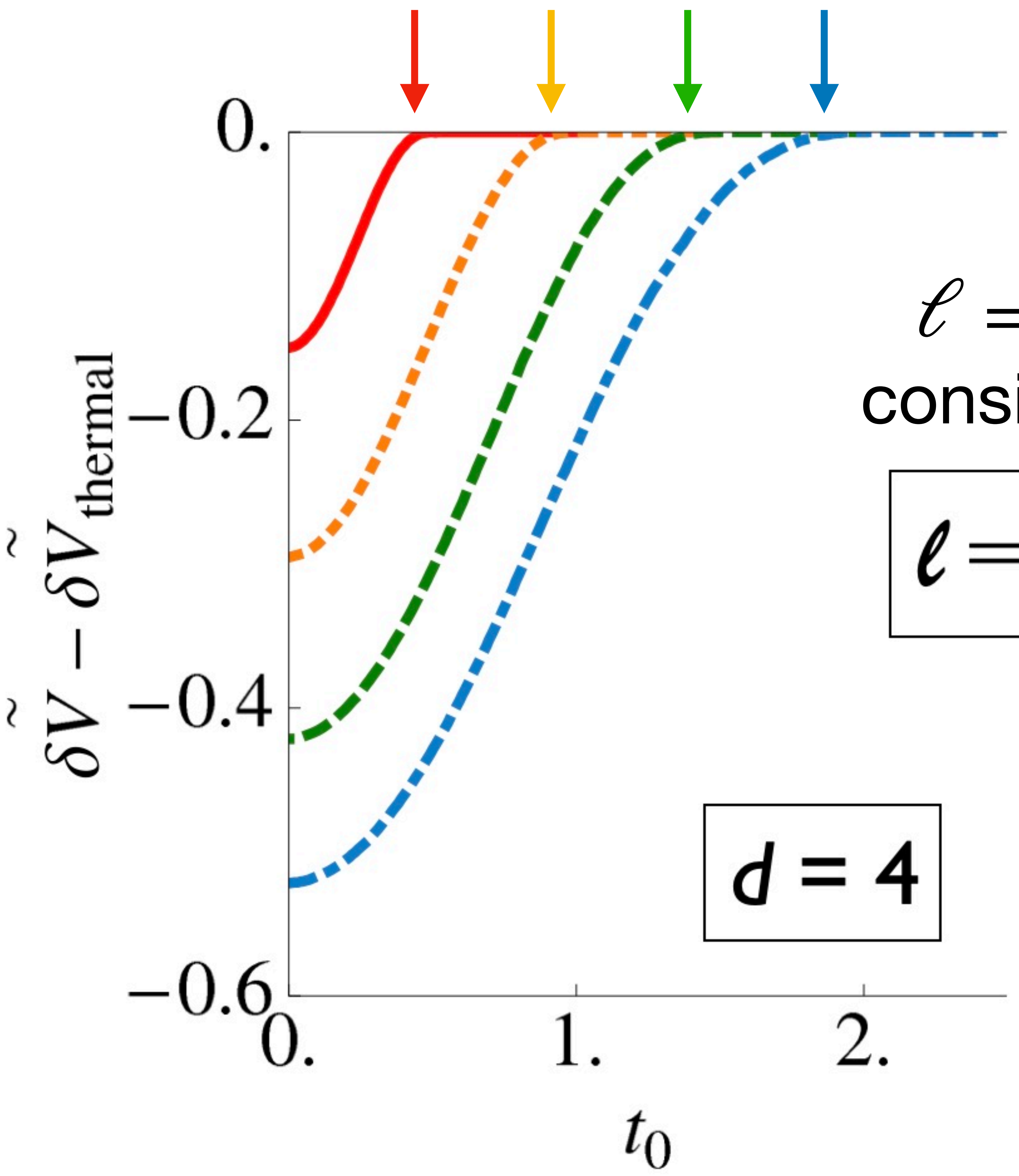



At finite temperature, a BH is present, and the surface $\gamma_A$ picks up a part of the event horizon, thus accounting for the thermal equilibrium entropy of $A$.

Review by Nishioka, Ryu, and Takayanagi, arXiv:0905.0932

# Entropy thermalization

Thermalization time




$\ell$ = diameter of considered volume

$\ell = 1, 2, 3, 4$

Thermalization time for the (entanglement) entropy is $\tau_{\rm th} = \ell/2$.

This is the time for information to escape from the center to the surface at the speed of light.

It is the fastest thermalization time compatible with causality.

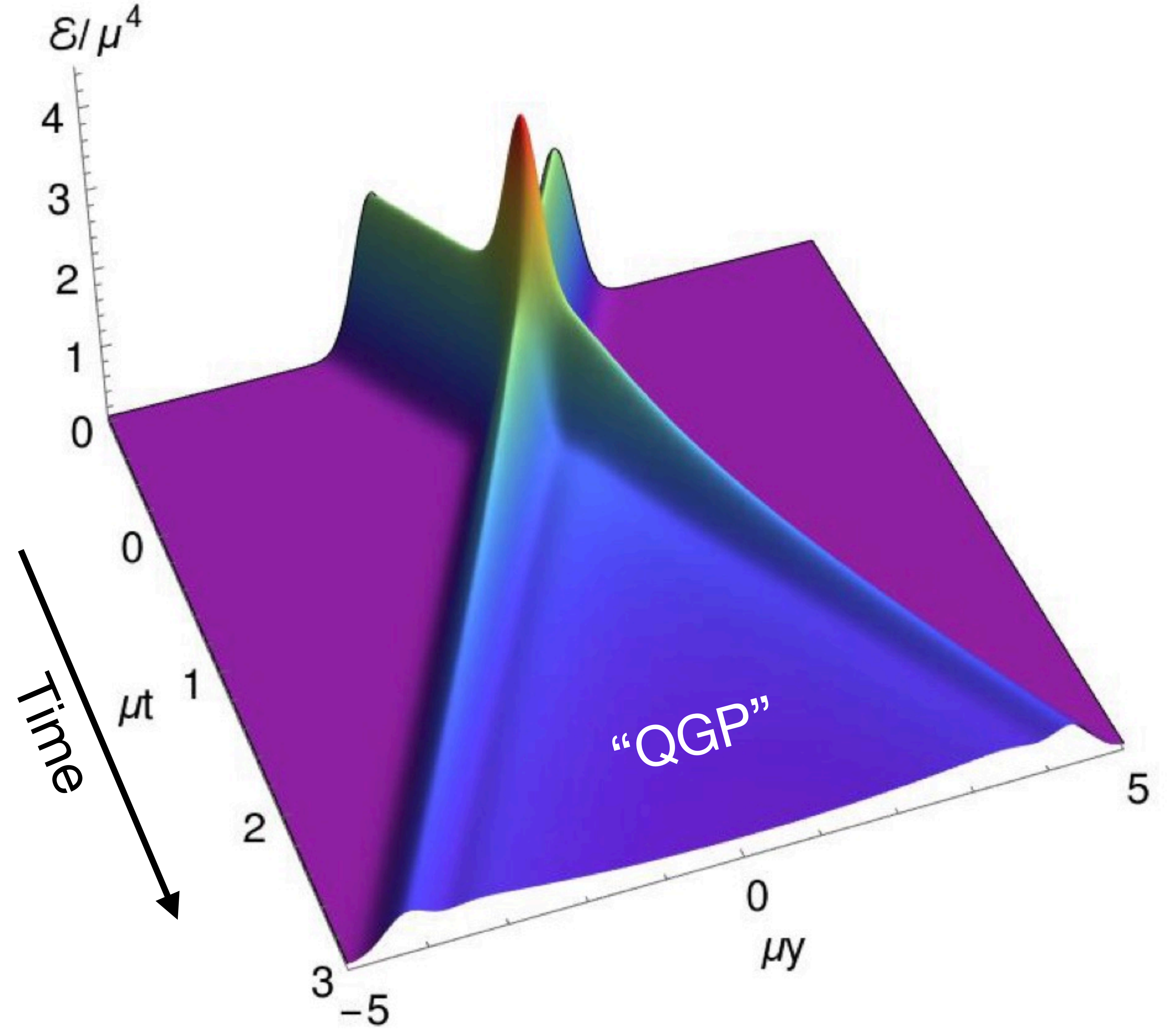
Rough estimate is $\ell \geq \hbar/T$ and thus:

$\tau_{\rm th} \geq 0.5\ \hbar/T \approx 0.3$ fm/c for $T$ = 300 MeV

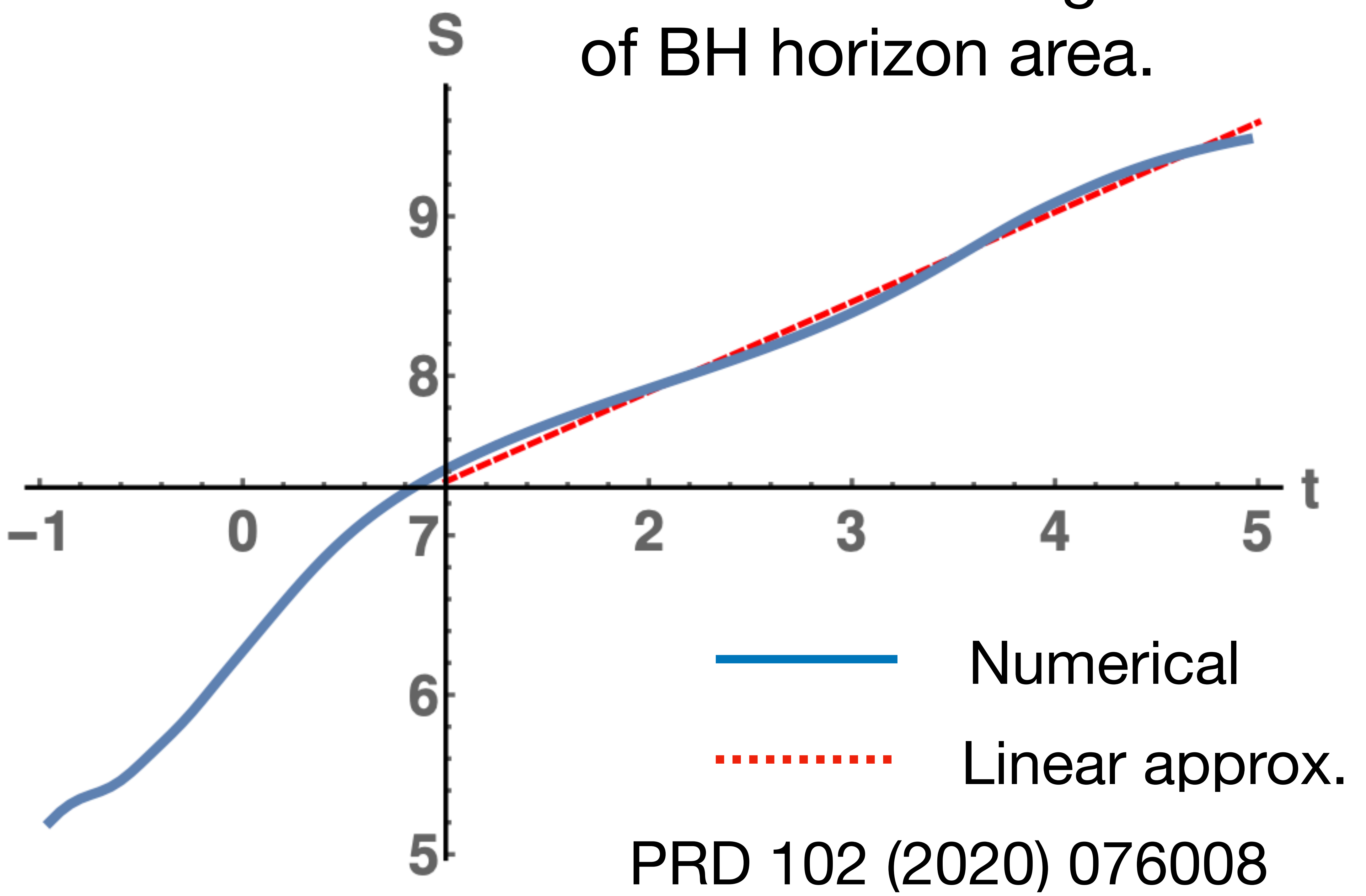
A heavy ion collision is much more complicated than a sudden quench where unstructured energy is injected into the quantum field. Several groups of theorists have performed increasingly sophisticated numerical calculations where two energetic shock waves collide and studied thermalization via the formation of a black hole horizon in $AdS_5$.

# Shock wave collisions

The collision of two energetic narrow shock waves is the analogue of a relativistic heavy ion collision. The shocks can be varied in amplitude and thickness.



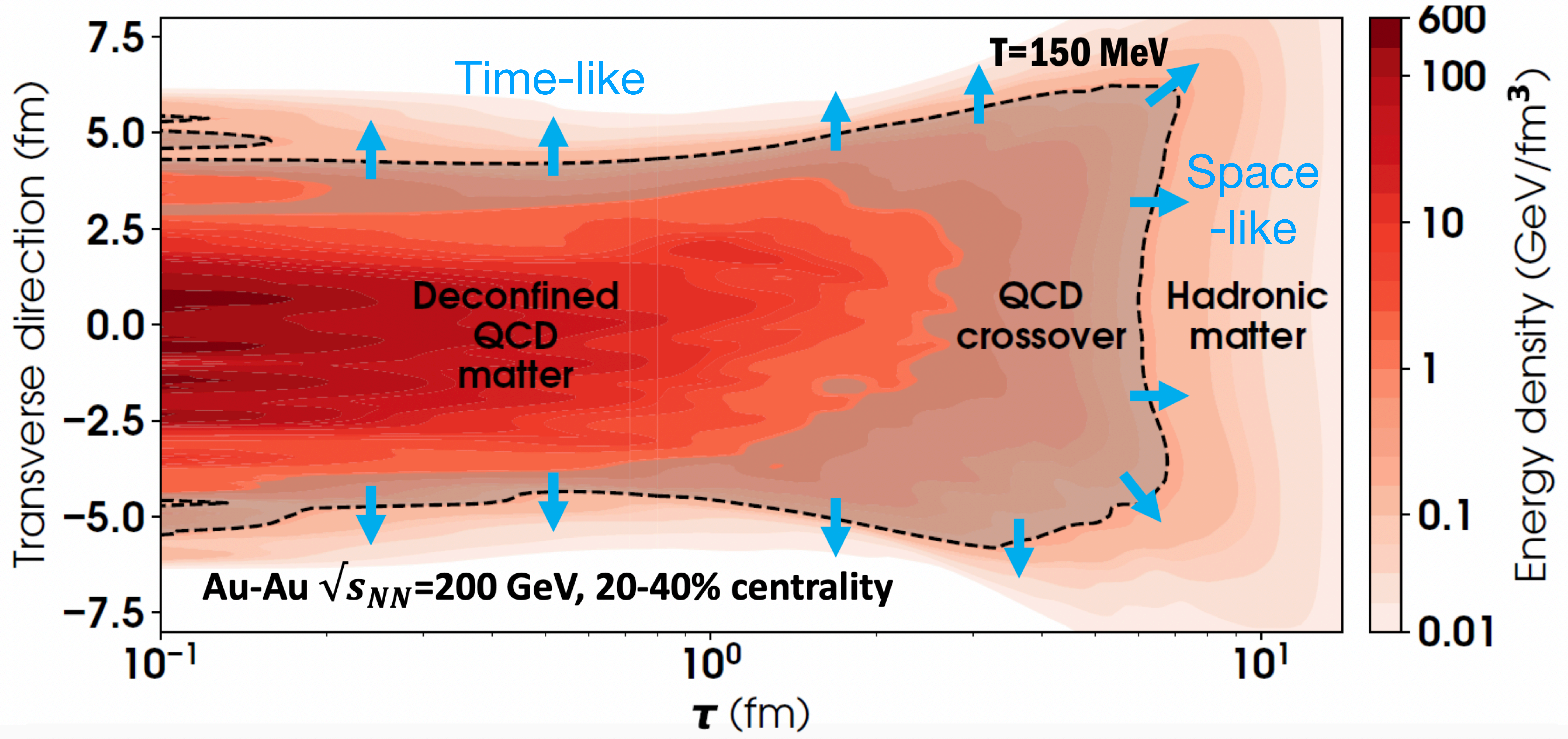

Entropy growth can be calculated from growth of BH horizon area.




PRD 102 (2020) 076008

# Hadronization

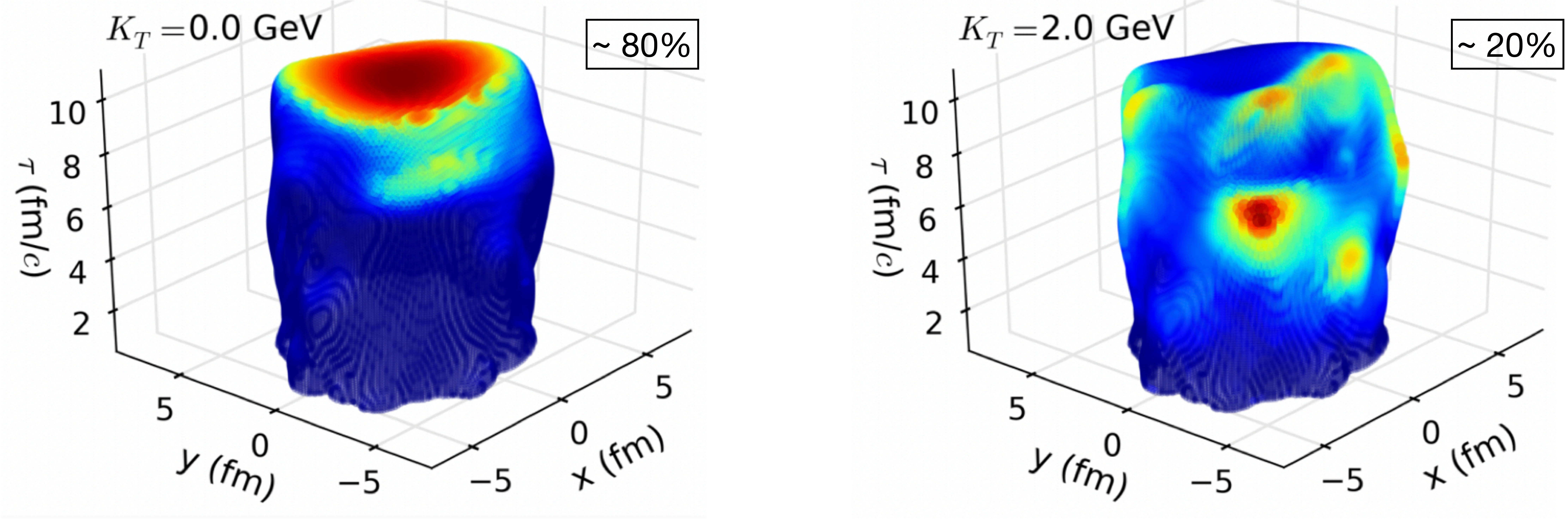
# Hadronization surface




J-F Paquet

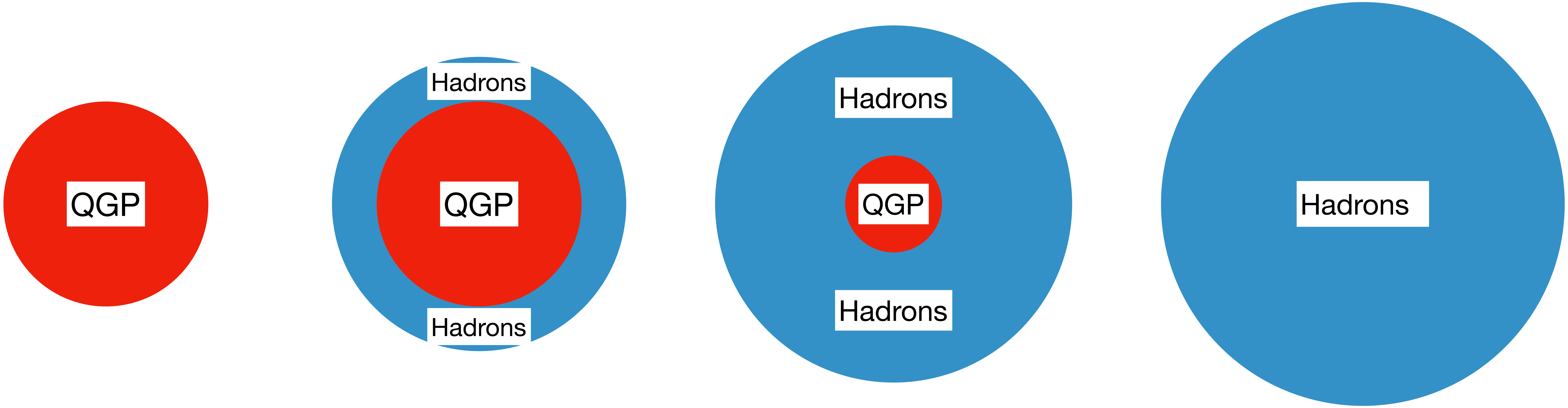
# Two hadronization components

C. Plumberg & U. Heinz, PRC 91 (2015) 054905




Freeze-out hyper surface monitored by two-pion correlations

# QGP Hadronization




Entangled QGP state

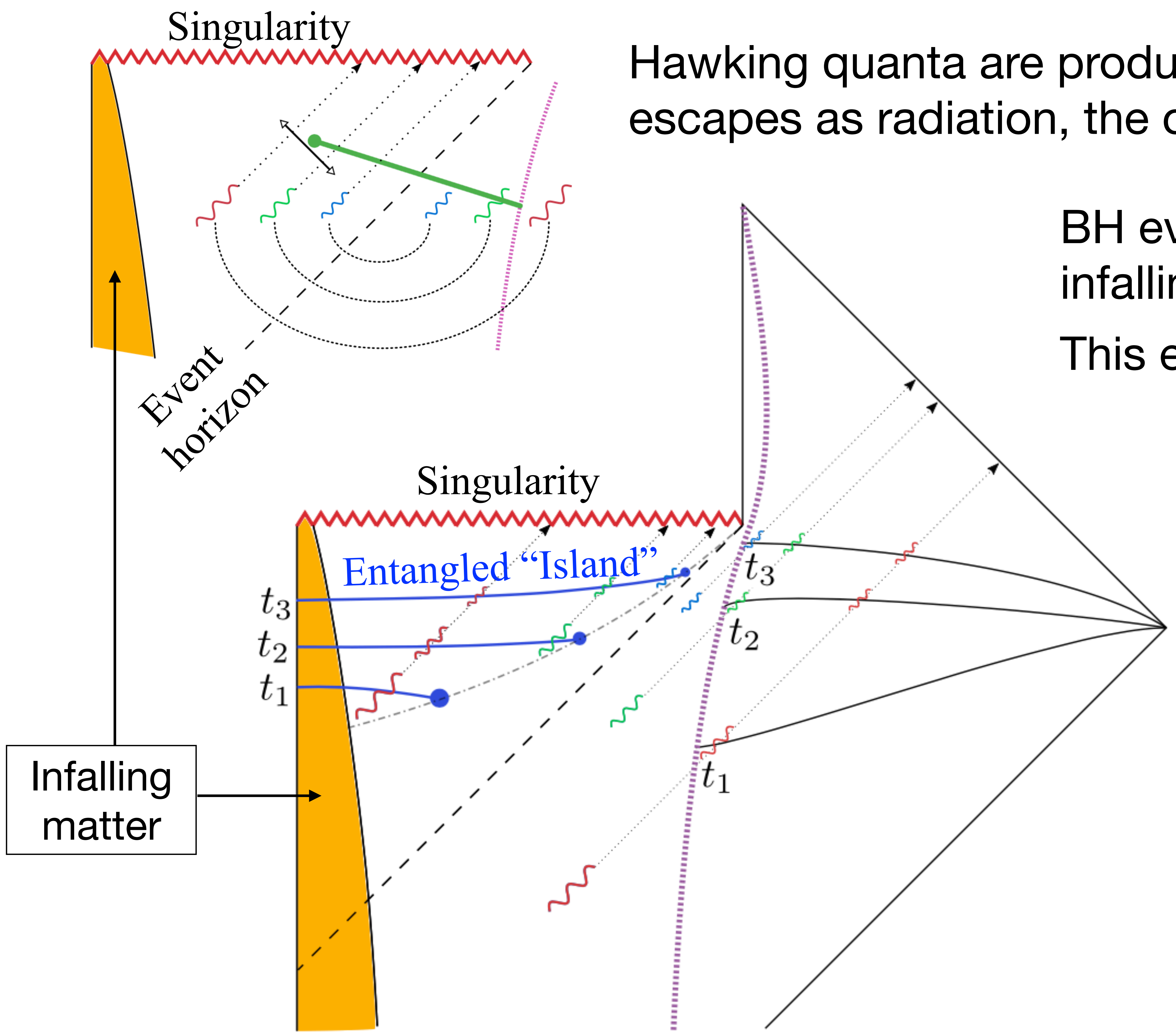
Emitted hadrons mostly entangled with QGP

Emitted hadrons mostly entangled among themselves

When the QGP has evaporated, the hadrons are in a highly entangled pure quantum state that looks “thermal” to all practically feasible experiments

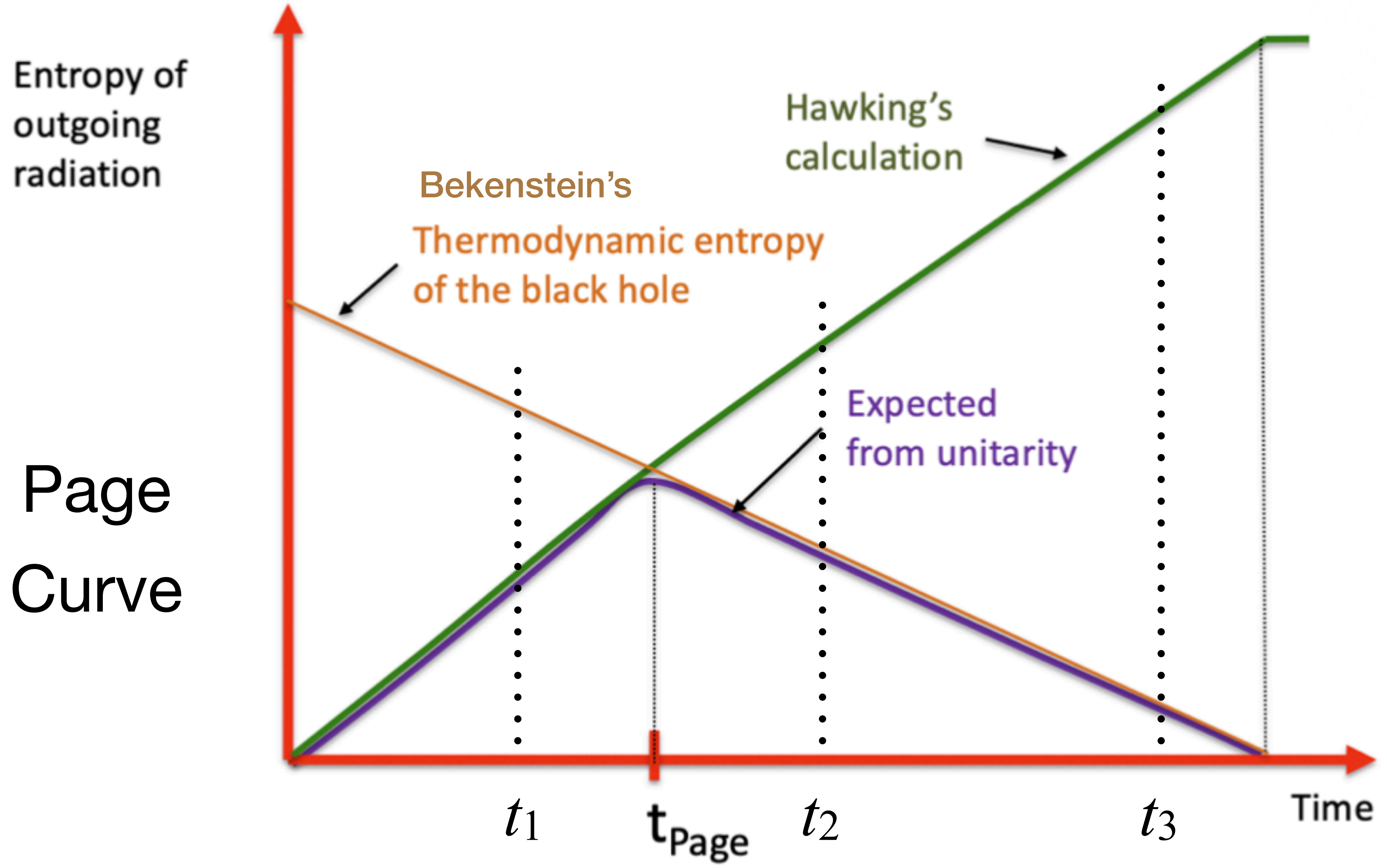
# Black hole evaporation

Hawking quanta are produced near the BH event horizon. One quantum escapes as radiation, the other falls into the BH, creating entanglement with BH.

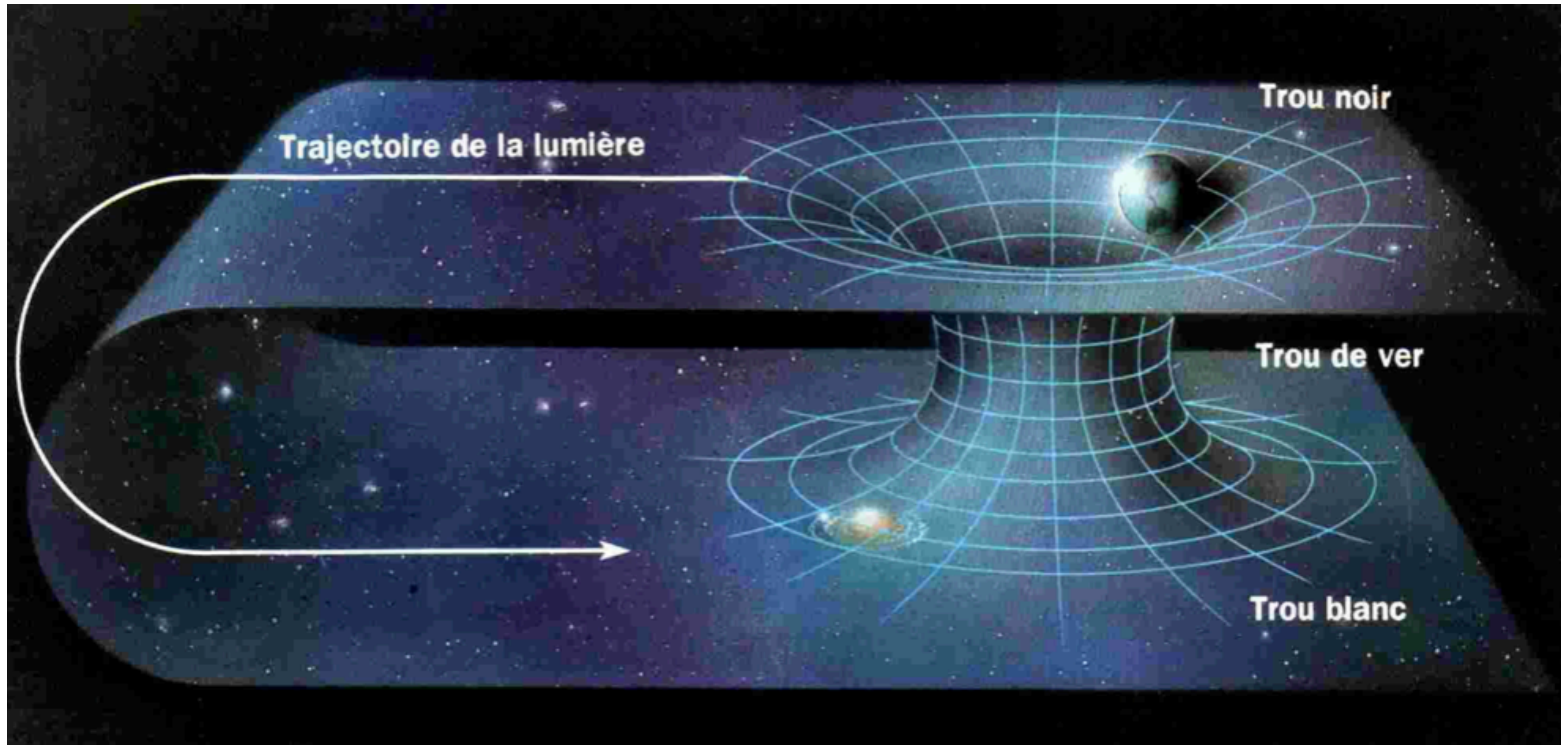



BH evaporation picture benefits from the fact that the infalling quanta propagate ballistically in the geometry.

This enables a simple geometric description.

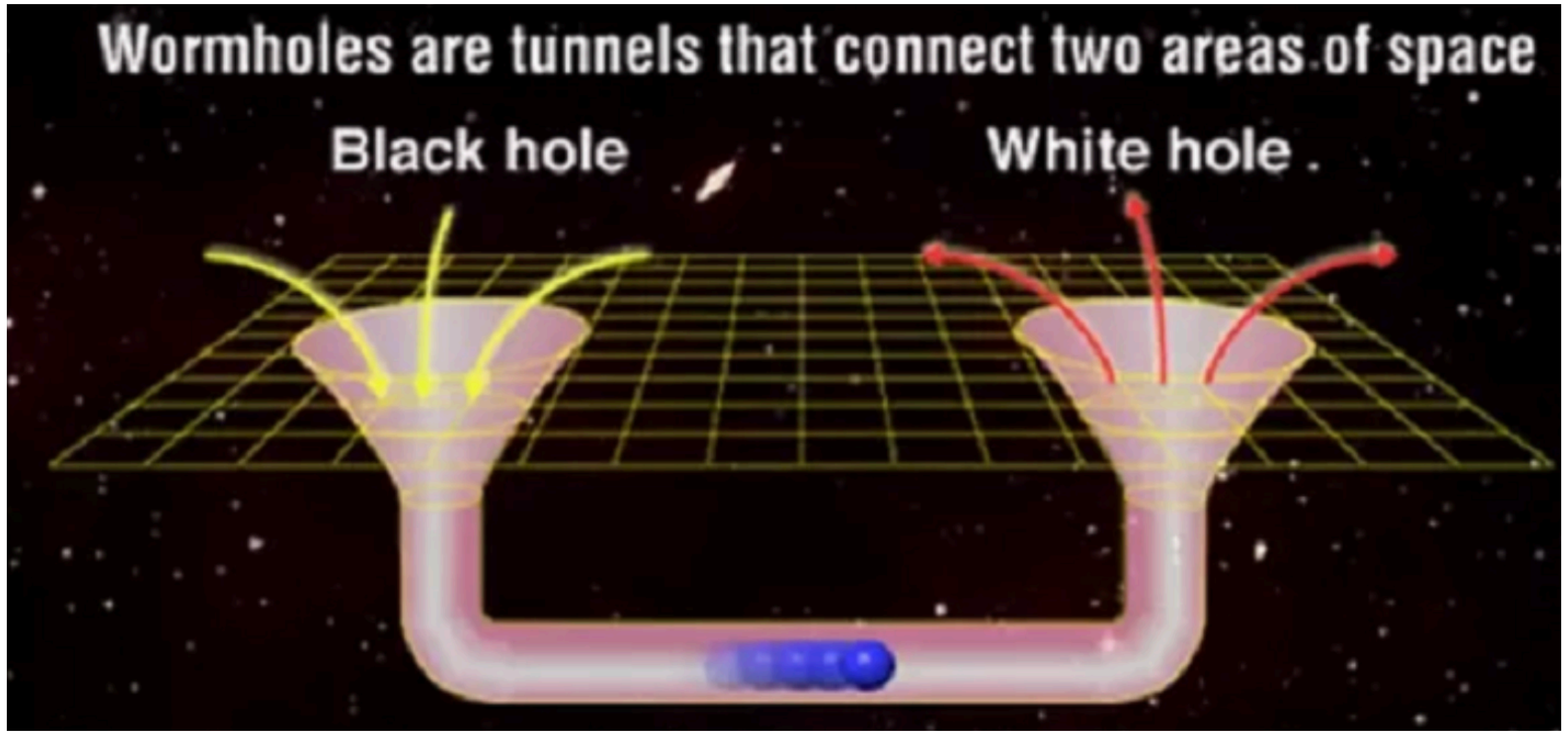


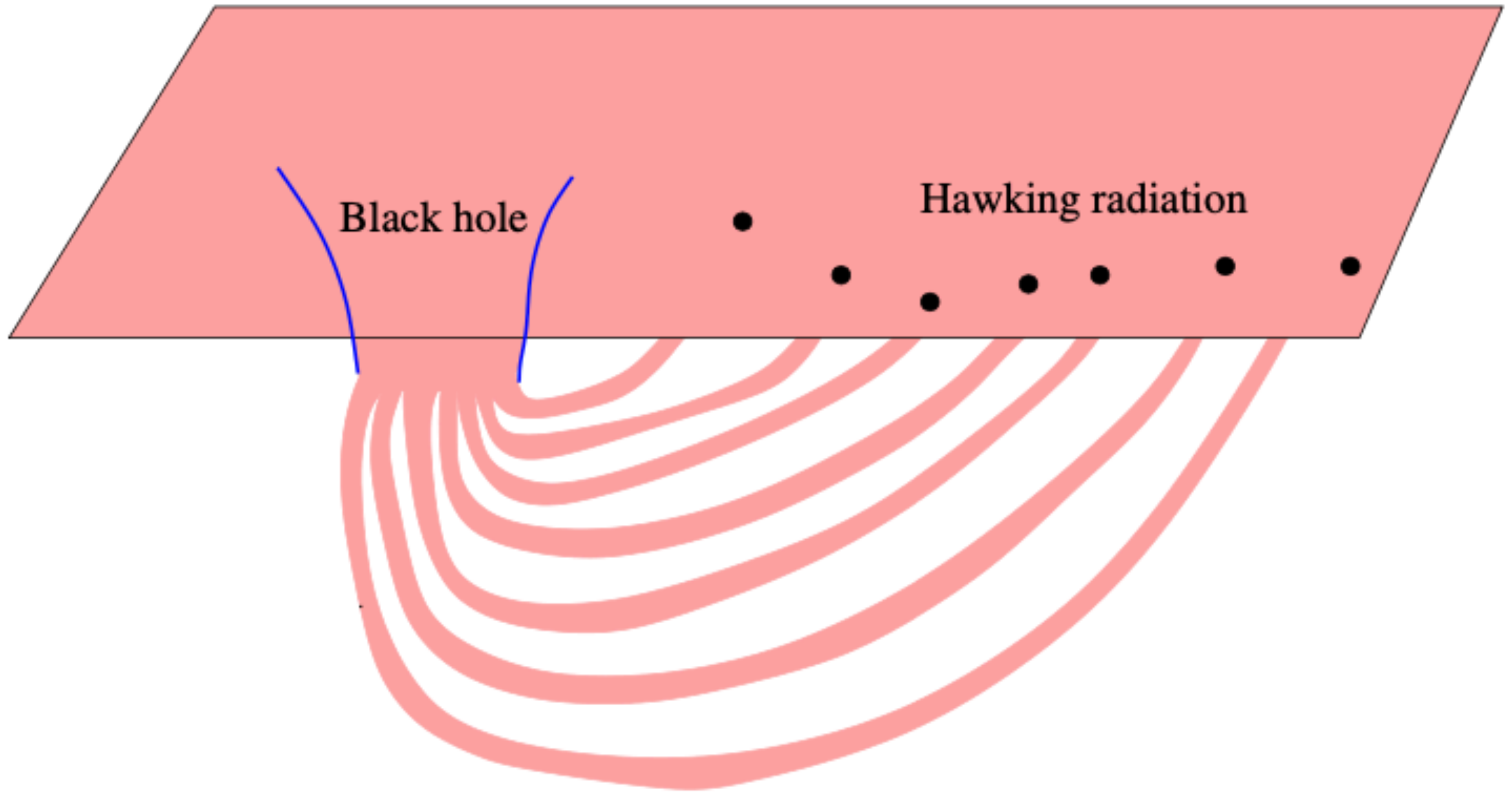
# Einstein-Rosen bridge = "wormhole"



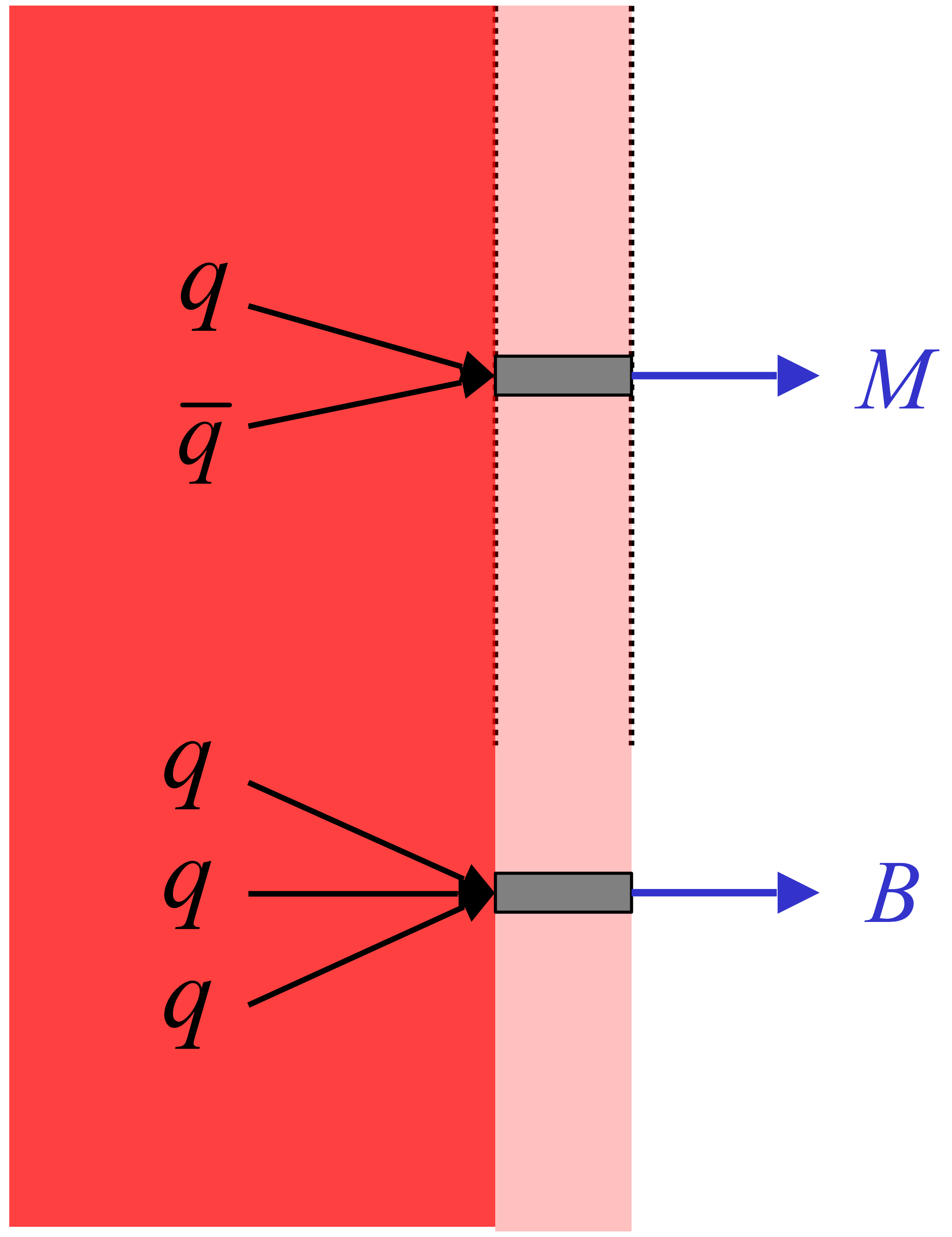

Einstein-Rosen bridges in AdS space encode quantum entanglement of different domains on the QFT "edge" (Maldacena & Susskind, 1306.0533).

In black hole evaporation, E-R bridges in the AdS bulk encode the entanglement between the BH and the Hawking radiation



# QGP Evaporation = Hadronization



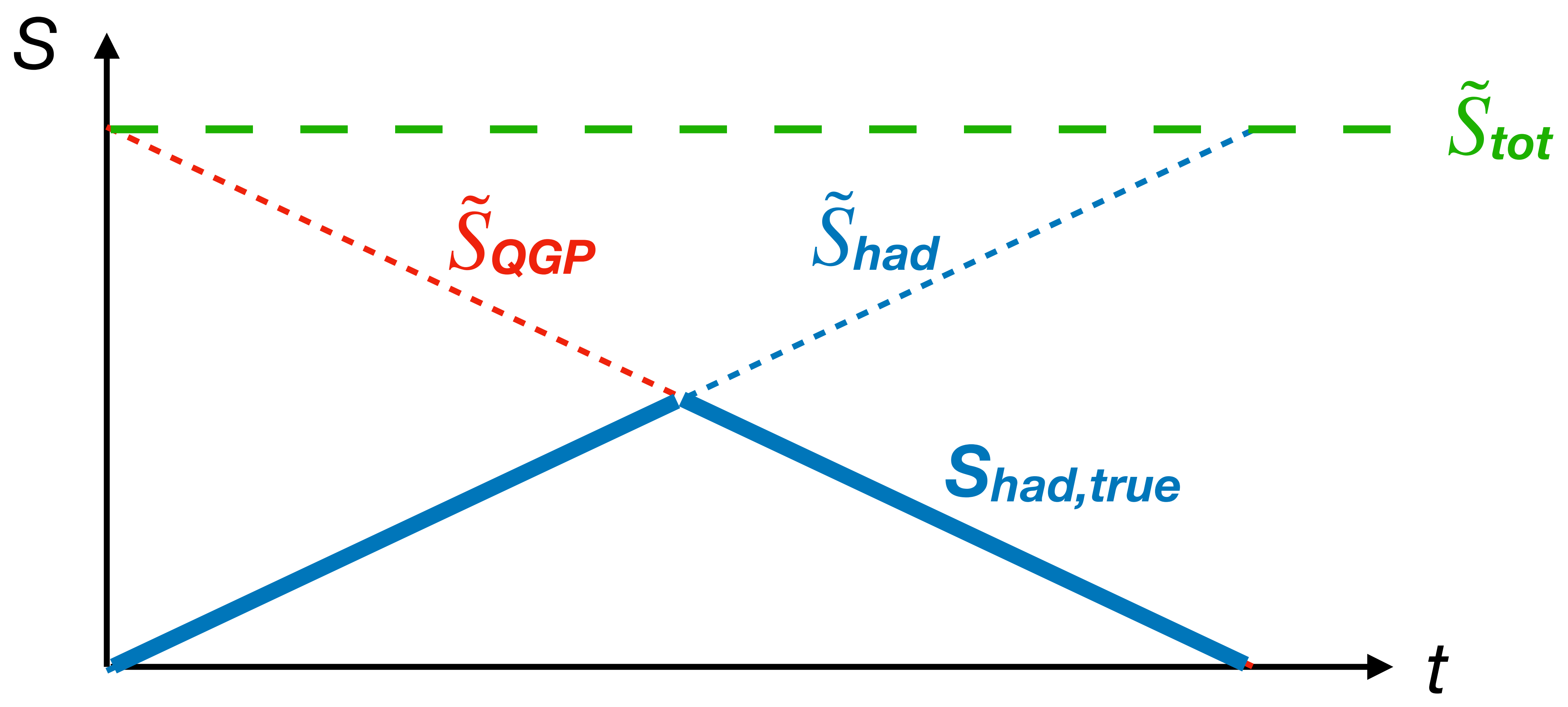

Emission of a meson (baryon) leaves behind a $q\bar{q}$ $(qqq)$ "hole" in the QGP.

There are no such quasi-hole excitations that propagate ballistically and can be traced geometrically.

Requires a direct calculation of the propagation of entanglement entropy.

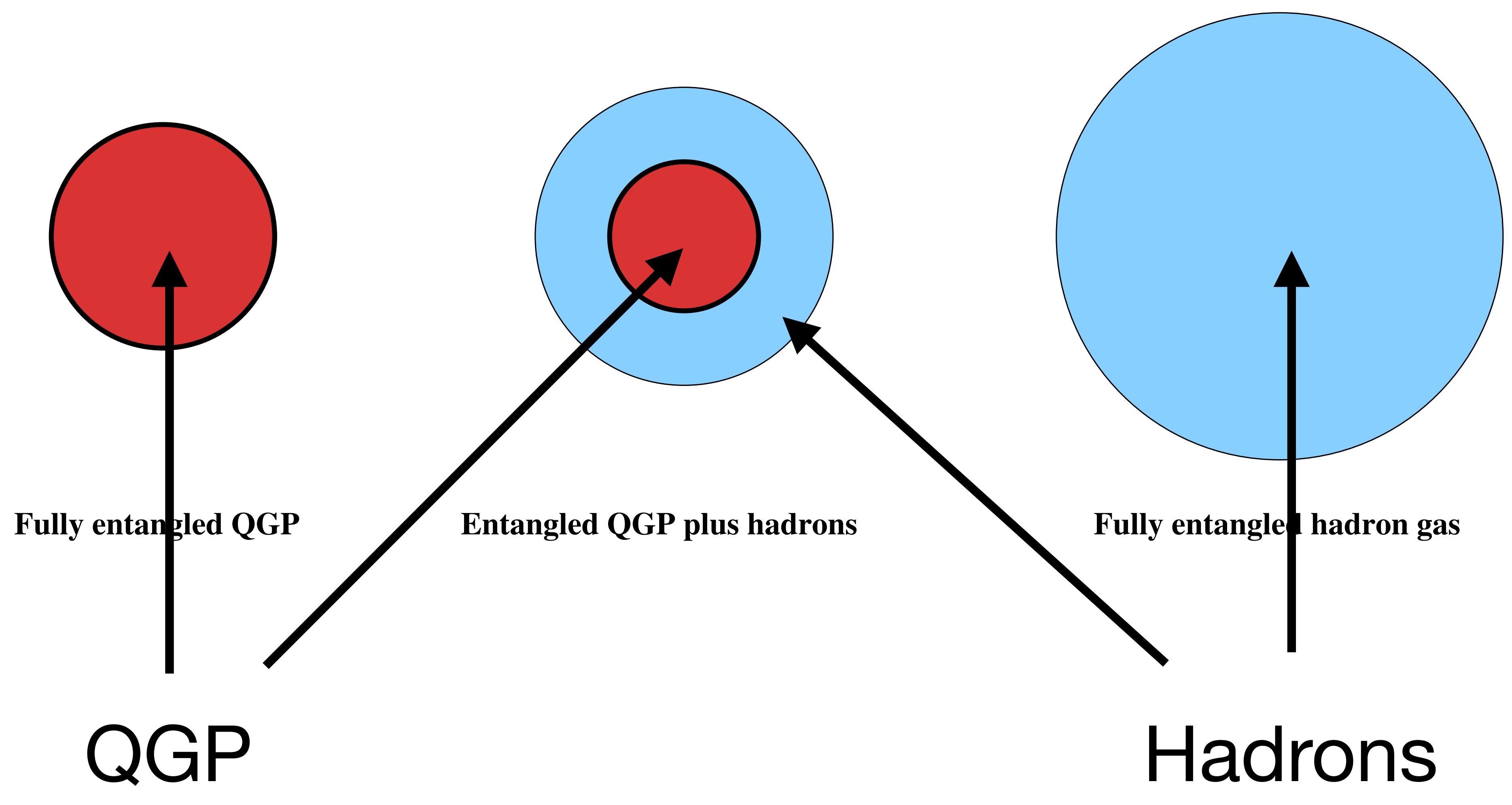
Advantage of the holographic dual, where entanglement entropy can be calculated geometrically via a minimal (HRT) surface in the bulk.

QGP "Page Curve"



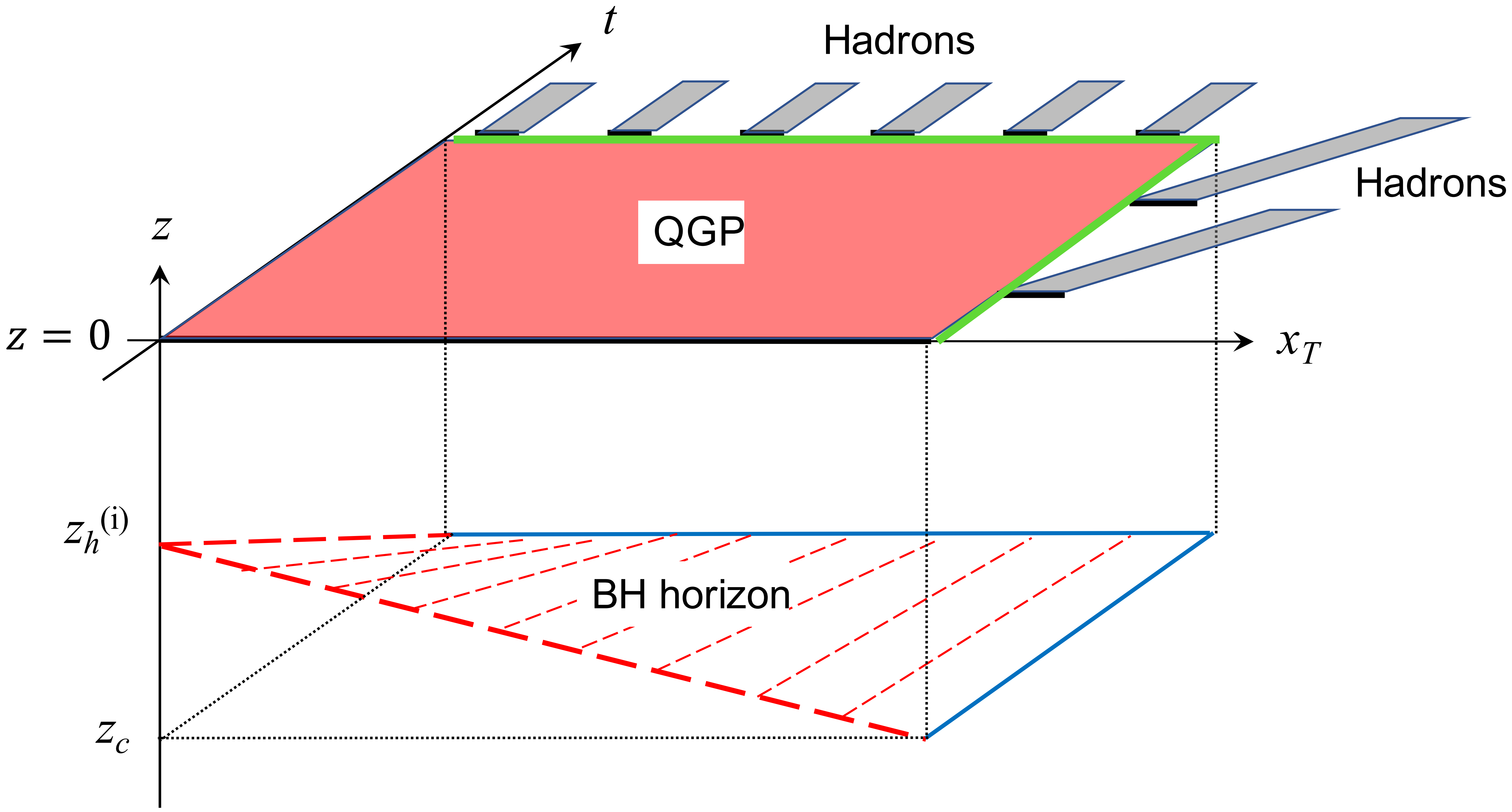
# Hadronization process

In the real (3-D) world




QGP evaporation process is similar to Black Hole evaporation (Hawking process)

Holographic dual



# Hawking-Page transition

What is the QGP - hadron (phase) transition in the holographic picture?

The temperature $T$ of the dual black hole is related to its "radius" in the AdS coordinate $z$: $$T \sim r_{\rm eq} \sim \frac{1}{z_{\rm BH}}$$

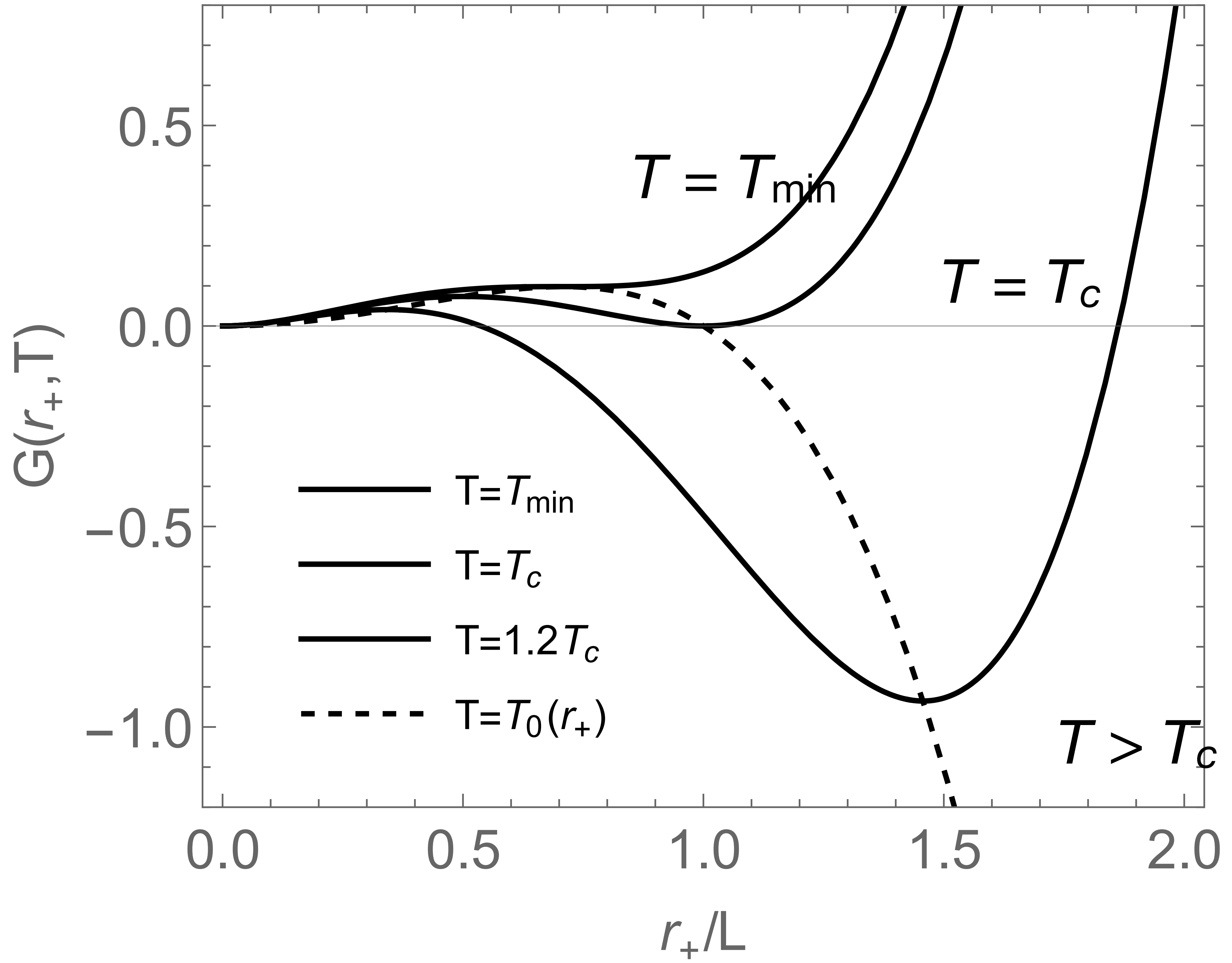
At a given $T$, only a black hole with a certain radius $r_{eq}$ is in equilibrium. This corresponds to the minimum of the free energy $G(r,T)$

When $T < T_c$, the $G(r_{eq},T) > 0$, and the black hole is no longer preferred.

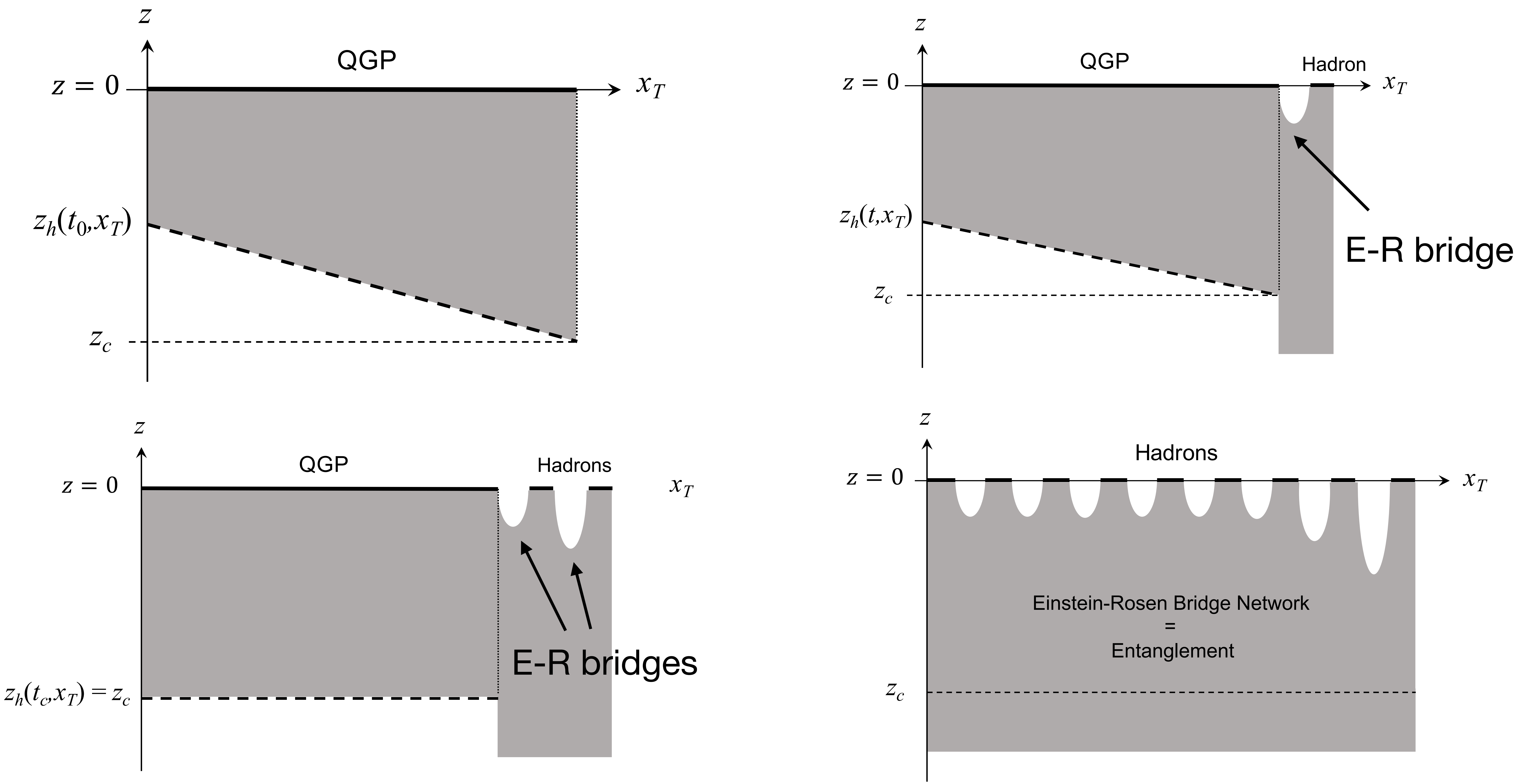
This is characteristic of a phase transition (Hawking-Page transition).

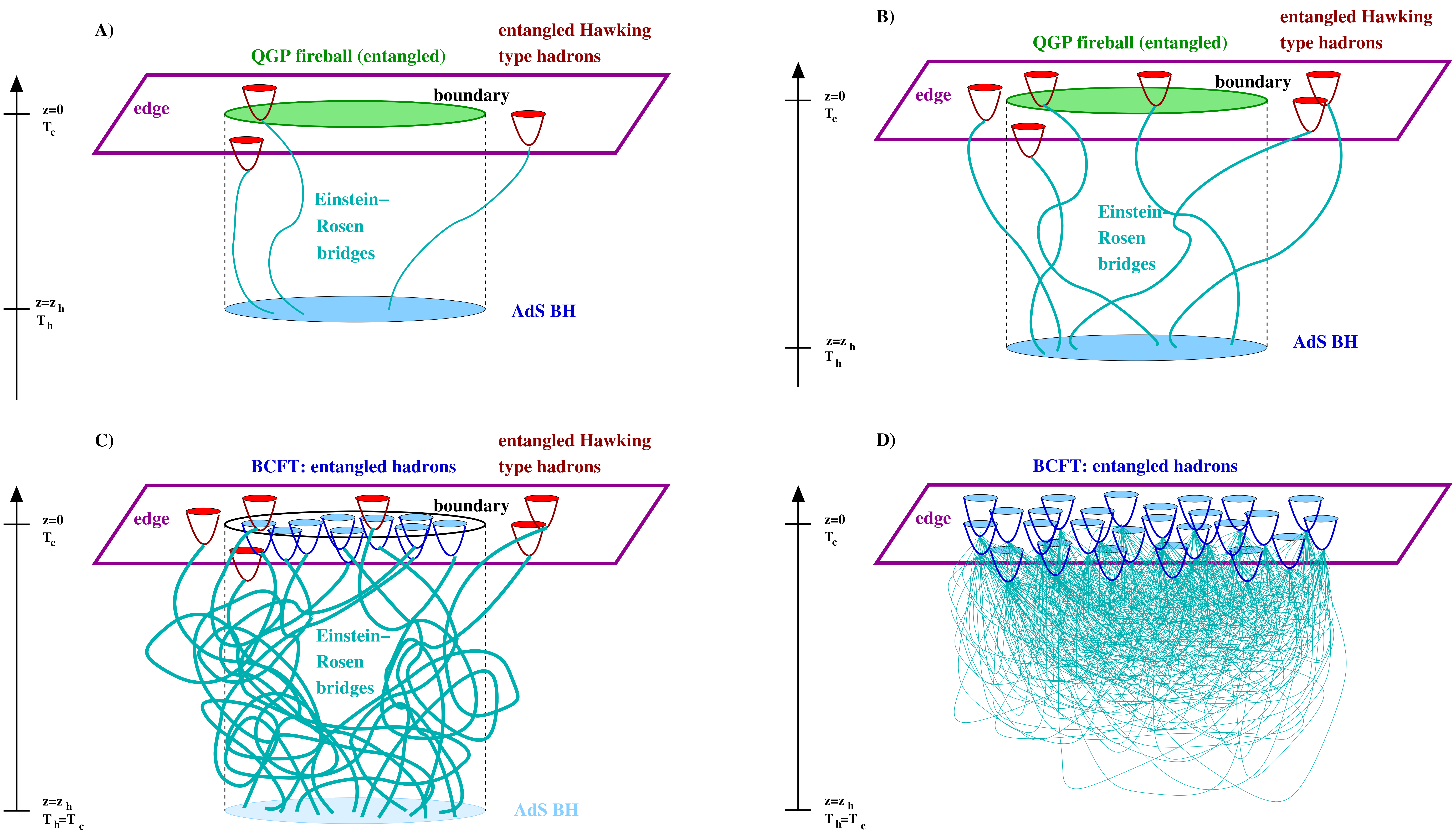
For $T < T_{\min}$, the minimum disappears completely.

Quantitative details can be modeled using dilaton gravity with dilation potential $U(\phi)$.



# Hadronization: Dual picture

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entangled Hawking
type hadrons
QGP fireball (entangled)
edge
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BCFT: entangled hadrons
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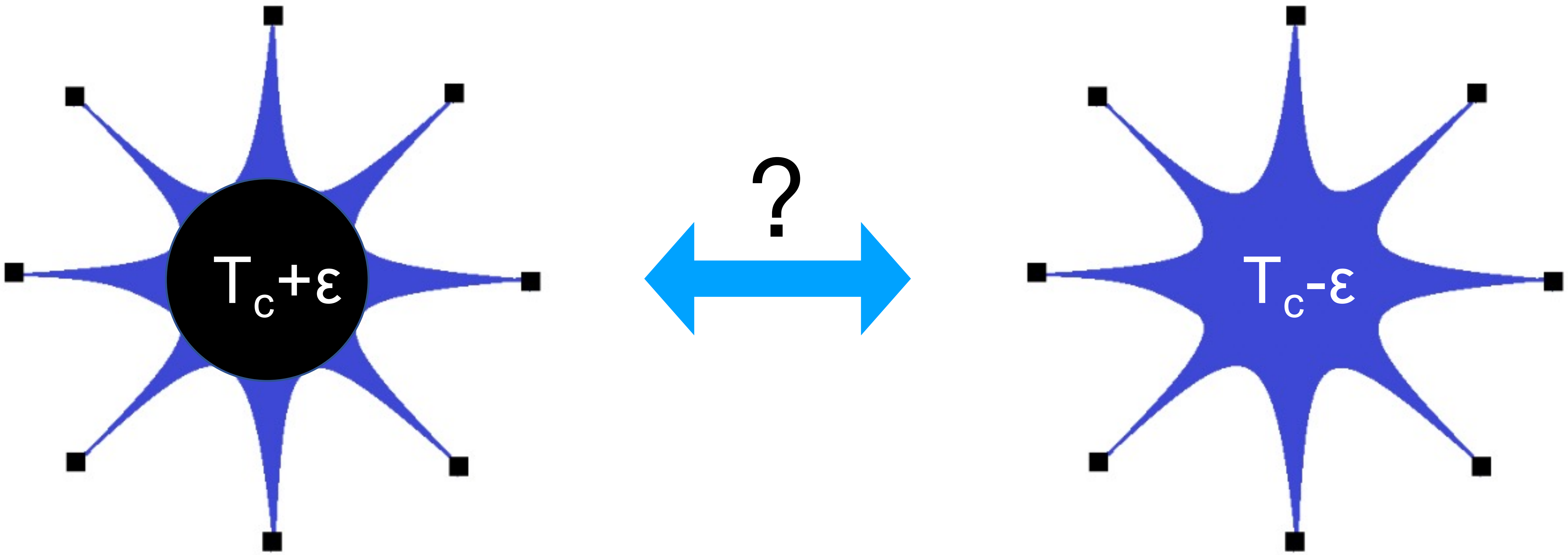
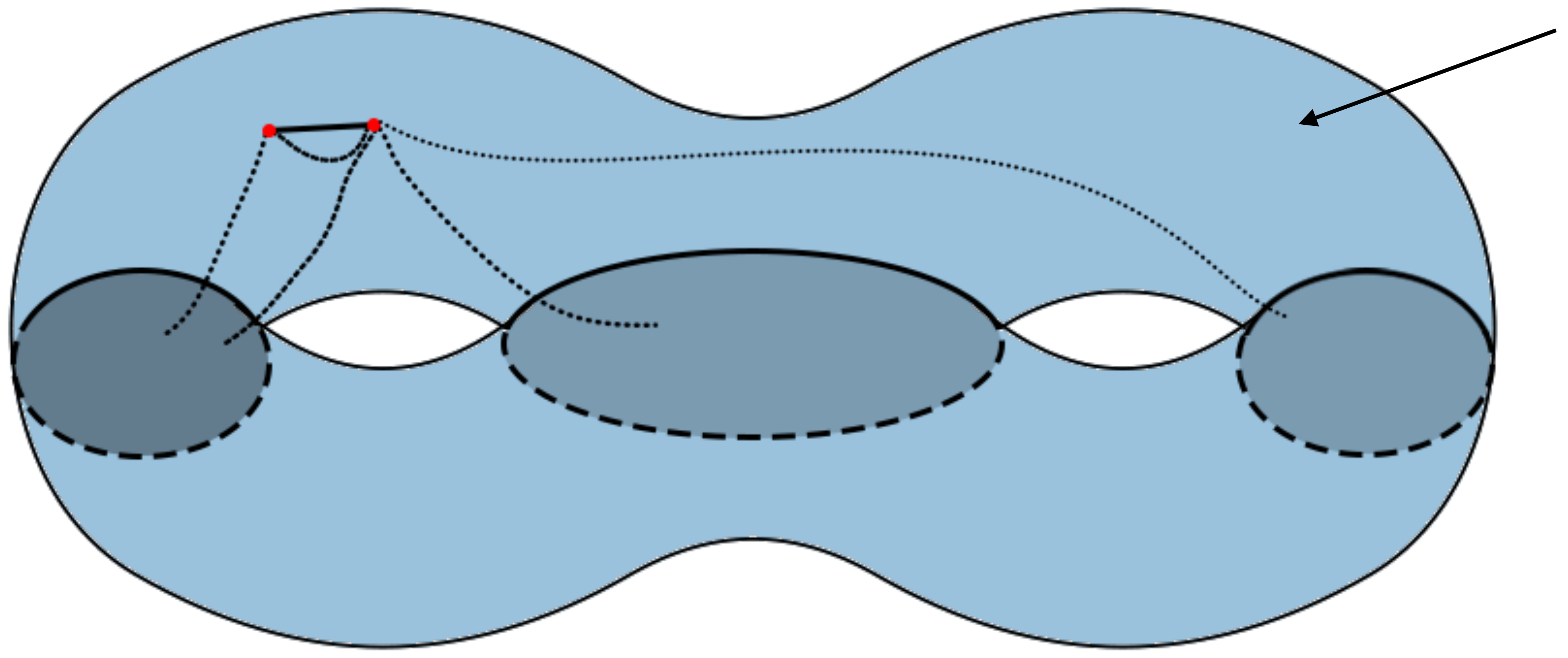
# A smooth transition?




Entanglement may provide for a rigorous equivalence of the edge QFT state across $T_c$ that is not apparent in the statistical treatment

# Open questions & outlook

- Does QCD exhibit ETH behavior - what are the time scales?
- Holographically modeling the QGP - hadron transition appears feasible.
- Entanglement among hadrons is captured by E-R bridges in AdS space
- Start with low-dim. model (1+1 dimension) - Joseph Lap



Holographic coordinate *z* conformally mapped to interior of a multiply connected torus

Entanglement entropy gets contributions from multiple geodesics (“wormholes”)

Splitting quench (Shimaji et al. 1812.01176)

Generalize to multiple splittings

| For details and more discussion of open questions see BM & A. Schafer, arXiv: 211.16265 |
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