



Fundamental Physics with Nuclei

ASU Theoretical Physics Colloquium

30 November 2022

Saori Pastore

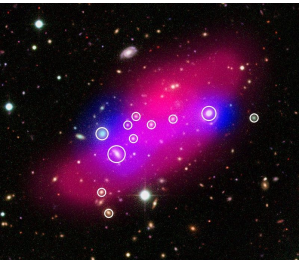
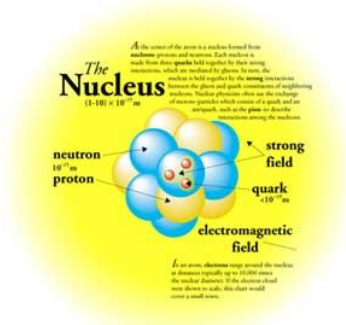
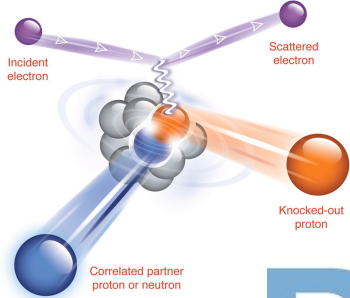
<https://physics.wustl.edu/quantum-monte-carlo-group>

Quantum Monte Carlo Group @ WashU

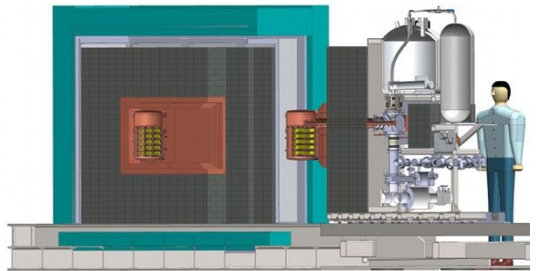
Lorenzo Andreoli (PD) Jason Bub (GS) Garrett King (GS) Maria Piarulli and Saori Pastore

Computational Resources awarded by the DOE ALCC and INCITE programs

Understand Nuclei to Understand the Cosmos



ESA, XMM-Newton, Gaspardello, CFHTL



The Standard Model

mass →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS					
	≈4.8 MeV/c ²	≈95 MeV/c ²	≈4.18 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	γ photon	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS					
	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
					GAUGE BOSONS

Neutrinos ν

Chargeless particles, come in 3 flavours, interact only via the weak interaction (10^{-4} EM and 10^{-9} Strong)

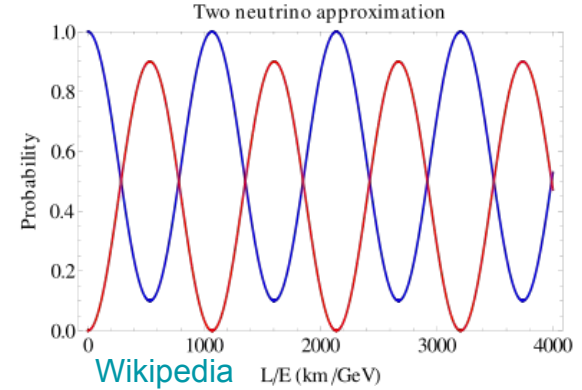
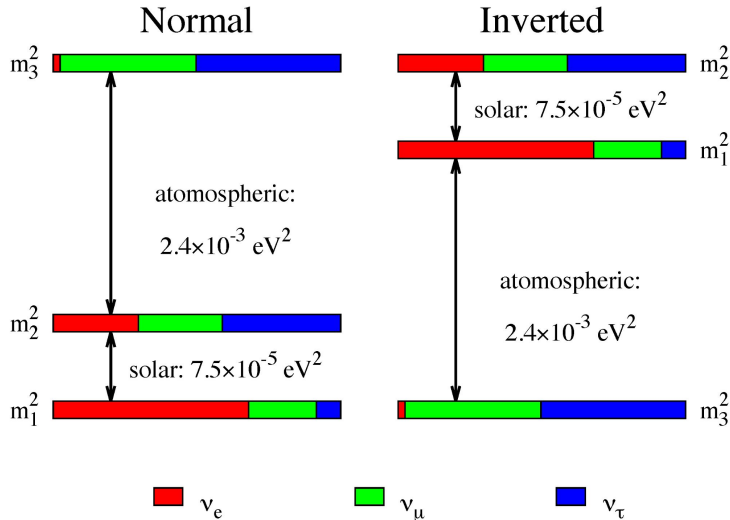
The Sun is a huge source of neutrinos on Earth, every sec $\sim 10^{11}$ solar neutrinos cross 1 cm^2

1 interaction per 100 years occur in our body (huge detectors are needed to see more events)

According to the Standard Model, neutrinos are massless...
To Be Continued...

Neutrino Oscillations

Neutrinos oscillate → they have a tiny mass
Beyond the Standard Model physics



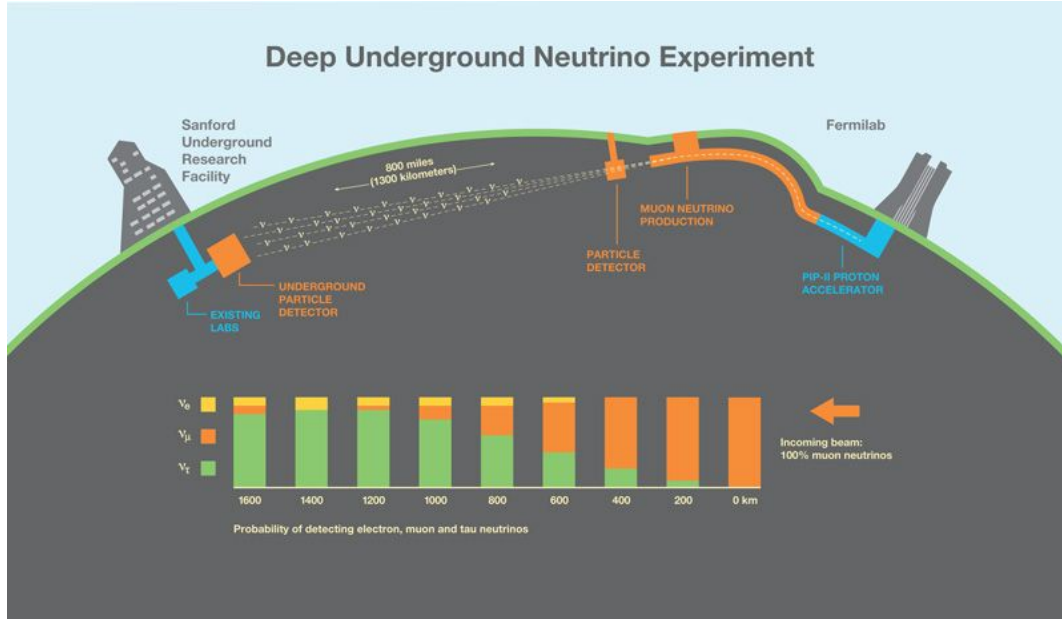
Simplified two-flavour picture

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

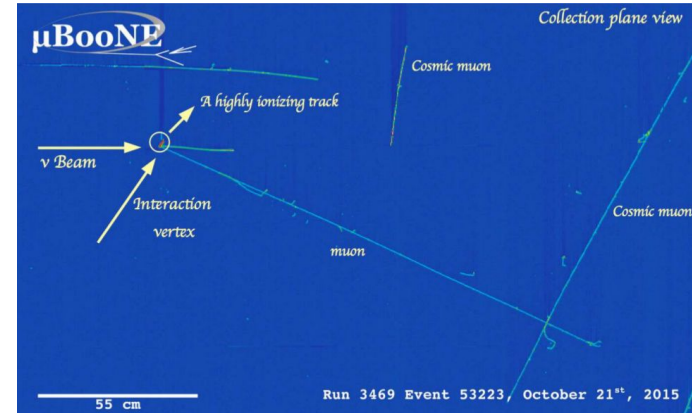
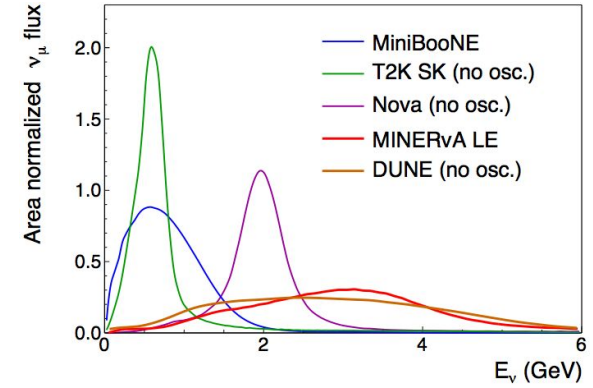
Probability of conversion

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(\frac{(m_2^2 - m_1^2) L}{2E_\nu} \right)$$

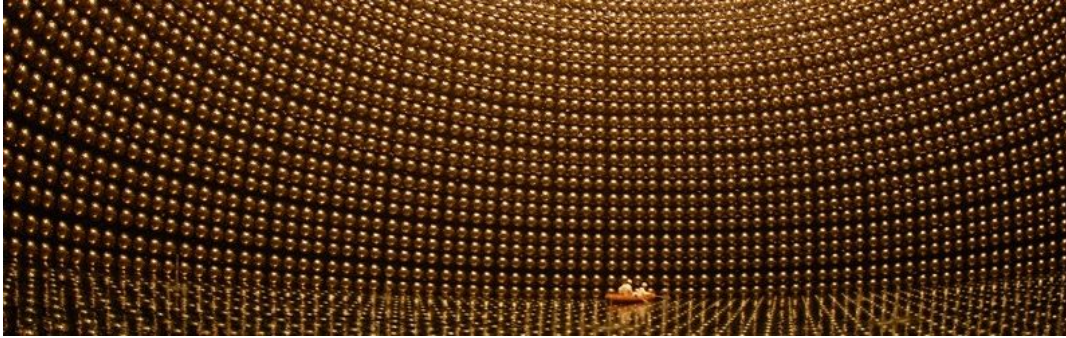
Accelerator Neutrinos' Experiments



DUNE - Fermilab

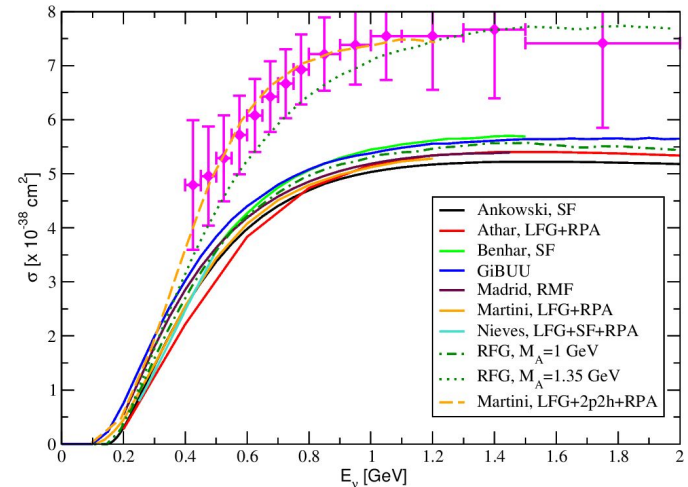


Nuclei for Neutrino Oscillations' Experiments

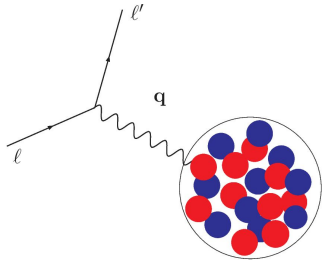


Neutrino- ^{12}C cross section

CCQE on ^{12}C



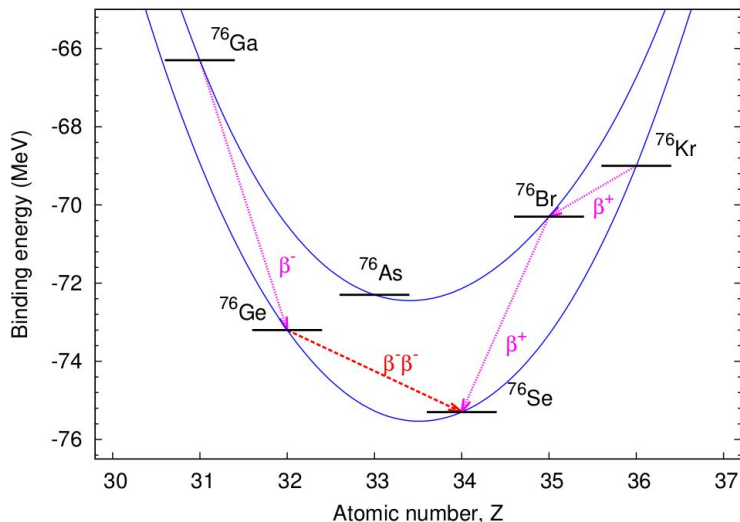
$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{2E_\nu} \right)$$



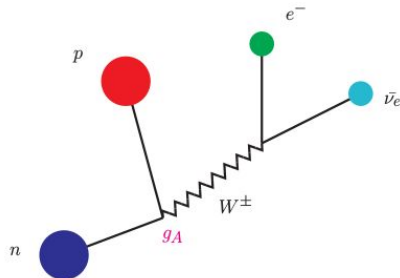
Nuclei are the active material in the detector. The energy of the incident neutrino is reconstructed from the observed final states using **neutrino event generators** that require **theoretical cross-sections**.

Alvarez-Ruso arXiv:1012.3871

Single and Double Beta Decays



J. Menéndez arXiv:1703.08921v1



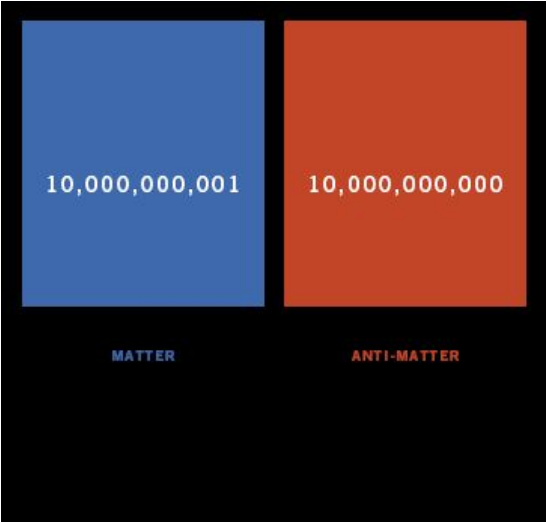
Maria Goeppert-Mayer

Single beta decay $(Z, N) \rightarrow (Z + 1, N - 1) + e + \bar{\nu}_e$

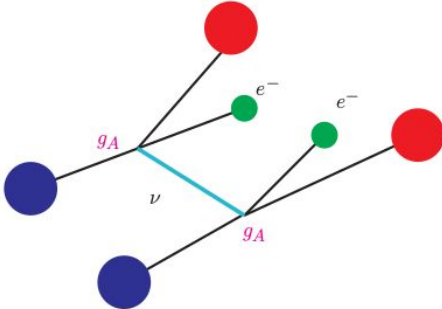
Double beta decay $(Z, N) \rightarrow (Z + 2, N - 2) + 2e + 2\bar{\nu}_e$

Here the lepton number is conserved

Neutrinoless double beta decay



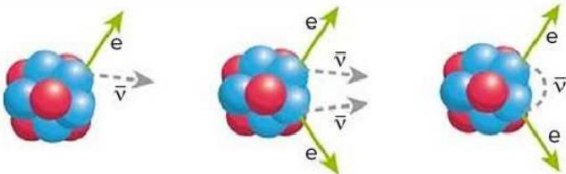
Hitoshi Murayama



Ettore Majorana

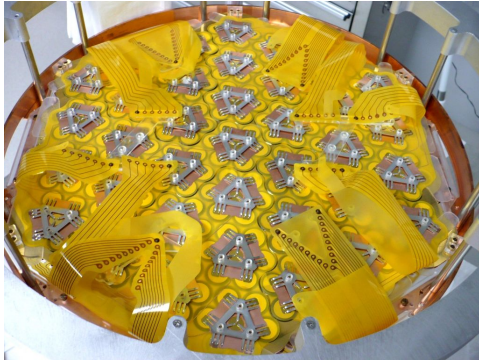
$$(Z, N) \rightarrow (Z + 2, N - 2) + 2e$$

Here the lepton number is not conserved

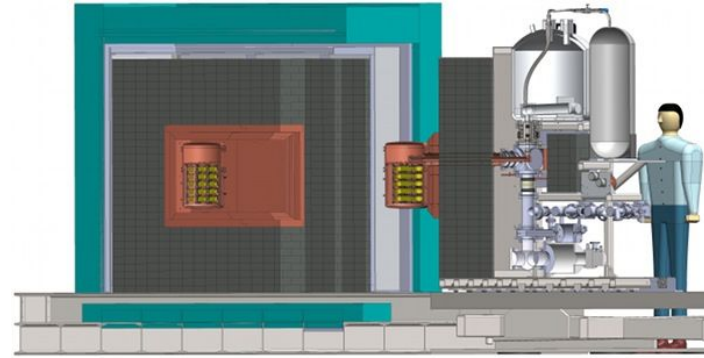


2015 Long Range Plan for Nuclear Physics

Nuclear Physics for Neutrinoless Double Beta Programs



EXO-200 Collaboration



Majorana Demonstrator

Neutrinoless double beta decay half-life $T_{1/2} \gtrsim 10^{25}$ years (age of the universe 1.4×10^{10} years)
1 ton of material is required to see few events per year

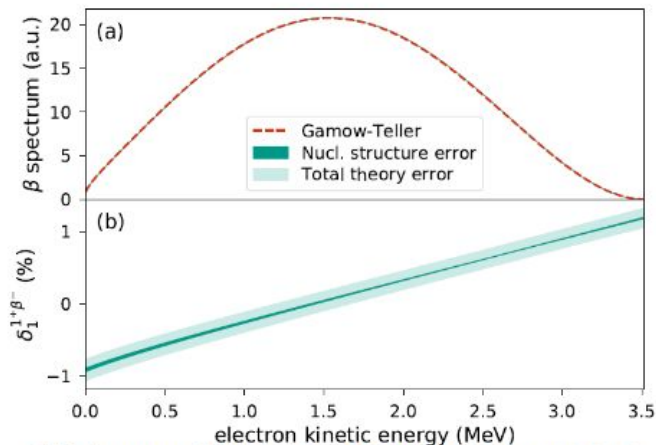
$$\text{Decay Rate} \propto (\text{nuclear matrix element})^2 \times (m_{\beta\beta})^2$$

Beta decay spectrum

${}^6\text{He}$ Beta decay spectrum for BSM searches with NCSL, He6-CRES, LPC-Caen



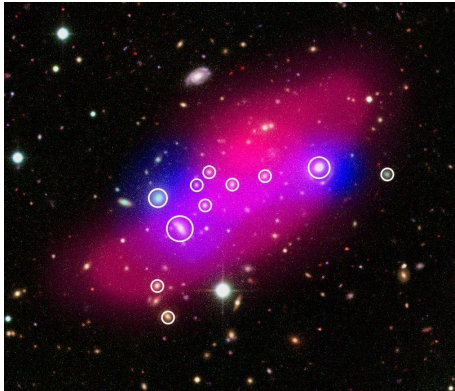
${}^6\text{He}$ beta-decay spectrum from NCSM



Glick-Magid et al. arXiv:2107.10212

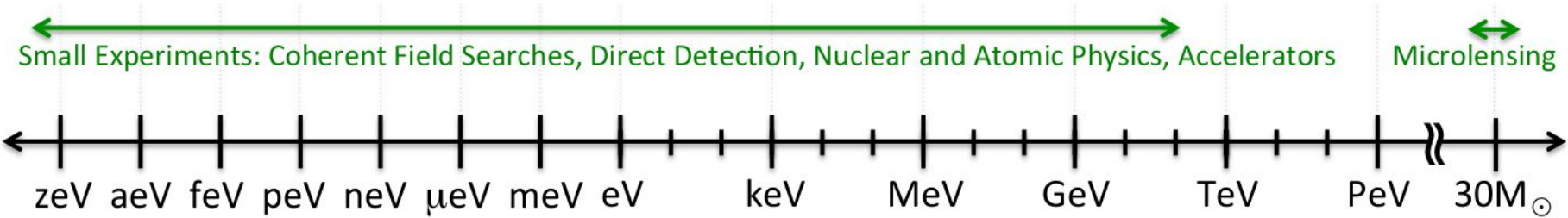
$$\frac{d\Gamma}{d\varepsilon} = \frac{d\Gamma_0}{d\varepsilon} \times (1 + \text{corrections})$$

Dark Matter

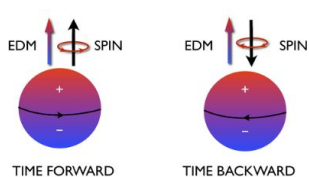


ESA, XMM-Newton, Gastaldello, CFHTL

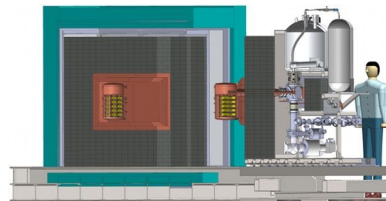
Candidates



Ground States'
Electroweak Moments,
Form Factors, Radii



Neutrinoless Double
Beta Decay,
Muon-Capture



Accelerator Neutrino
Experiments,
Lepton-Nucleus XSecs

$(\omega, q) \sim 0$ MeV

$\omega \sim \text{few MeVs}$
 $q \sim 0$ MeV

$\omega \sim \text{few MeVs}$
 $q \sim 10^2$ MeV

$\omega \sim \text{tens of MeVs}$

$\omega \sim 10^2$ MeV



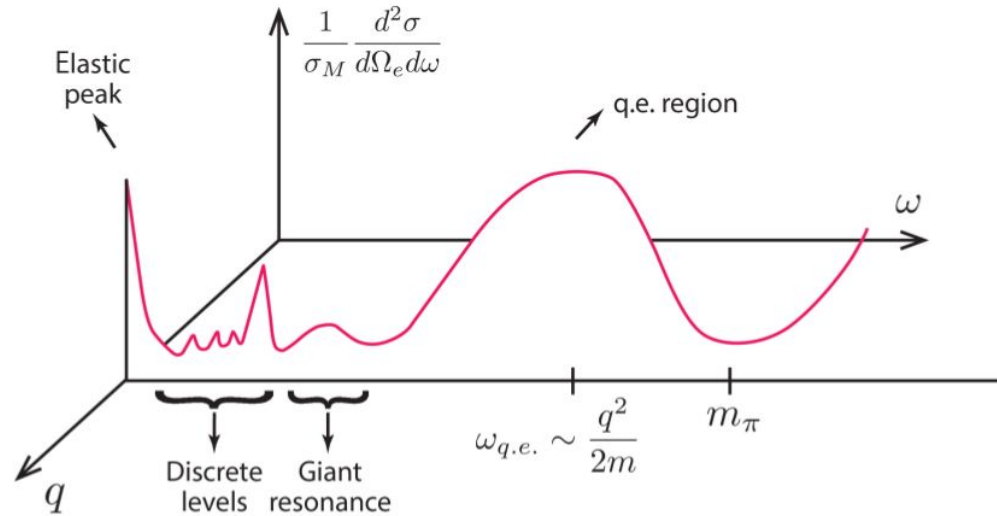
Electromagnetic
Decay, Beta Decay,
Double Beta Decay &
inverse processes



Nuclear Rates for
Astrophysics



Electron-Nucleus Scattering Cross Section



Energy and momentum transferred (ω, q)

Current and planned experimental programs rely on theoretical calculations at different kinematics

Strategy

Validate the Nuclear Model against available data for strong and electroweak observables

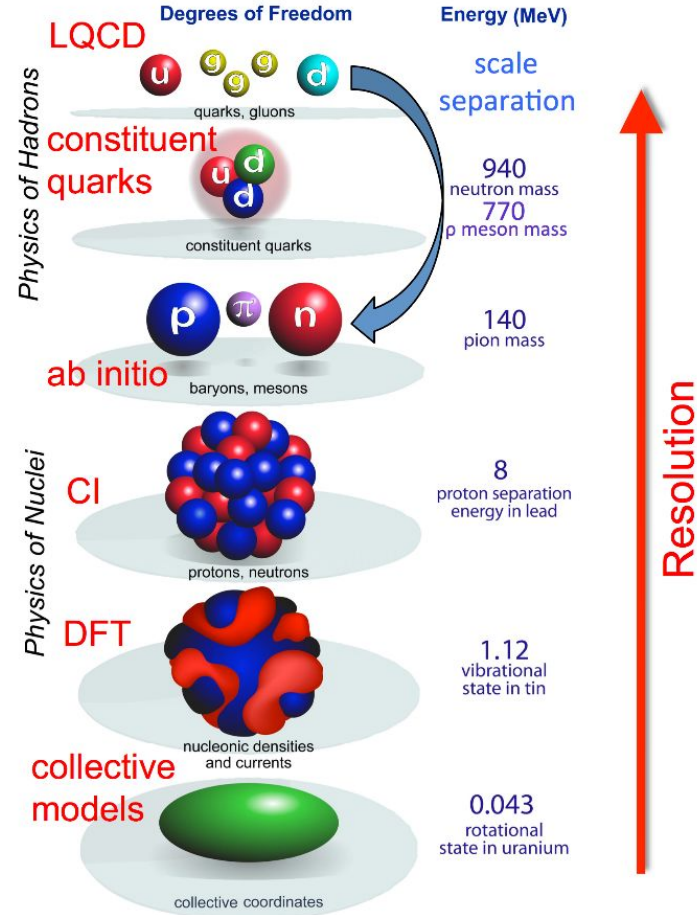
- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- Electron-Nucleus Scattering Cross Sections, ...

Use attained information to make (accurate) predictions for BSM searches and precision tests

- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- Neutrino-Nucleus Scattering Cross Sections, ...
- ...

From Quarks to Nuclei

- Nuclei are complex systems made of interacting **protons** and **neutrons**, which in turns are composite objects made of interacting constituent quarks.
- All fundamental forces are at play in nuclei.
- **EFTs** low-energy approximations of QCD whose d.o.f. are bound states of QCD (e.g., protons, neutrons, pions, ...)
- **EFTs** are used to construct many-nucleon interactions and currents

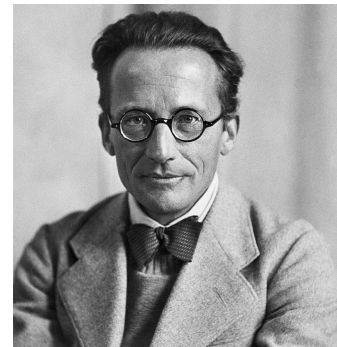


Microscopic (or *ab initio*) Description of Nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

- Accurate understanding of the interactions/correlations between nucleons in **pairs, triplets, ... (two- and three-nucleon forces)**
- Accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated nucleon-pairs, ... (**one- and two-body electroweak currents**)
- **Computational methods** to solve the many-body nuclear problem of strongly interacting particles



Erwin Schrödinger

$$H\Psi = E\Psi$$

Many-body Nuclear Problem

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A, s_1, s_2, \dots, s_A, t_1, t_2, \dots, t_A)$$

Ψ are **spin-isospin** vectors in **3A** dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components

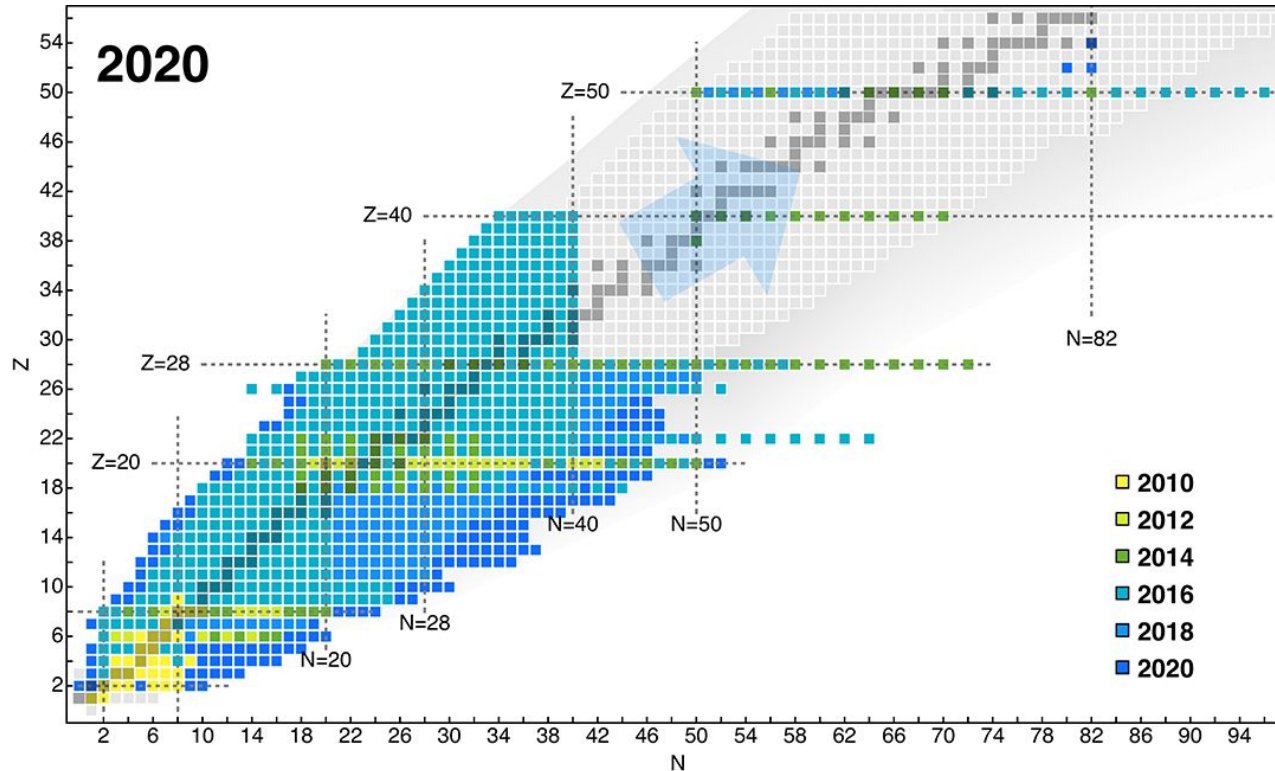
Develop Computational Methods to solve (numerically) exactly or within approximations that are under control the many-body nuclear problem

${}^4\text{He}$: 96
 ${}^6\text{Li}$: 1280
 ${}^8\text{Li}$: 14336
 ${}^{12}\text{C}$: 540572



<http://exascale.org/np/>

Current Status



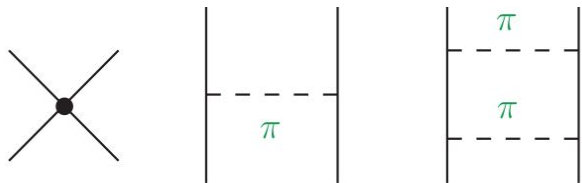
H. Hergert
Front. Phys.
07 October 2020

Many-body Nuclear Interactions

Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

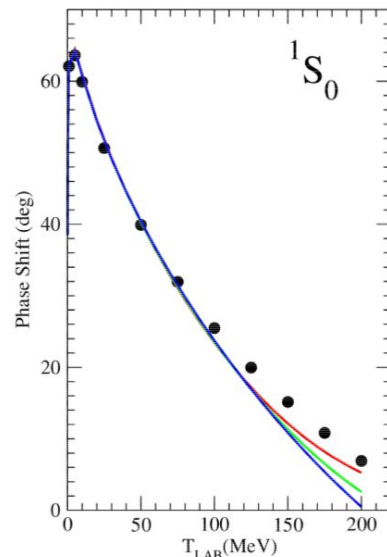
v_{ij} and V_{ijk} are **two-** and **three-**nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



Contact term: short-range

Two-pion range: intermediate-range $r \propto (2m_\pi)^{-1}$

One-pion range: long-range $r \propto m_\pi^{-1}$



SP et al. PRC80(2009)034004



Hideki Yukawa

AV18+UIX; **AV18+IL7**

Wiringa, Schiavilla, Pieper
et al.

chiral $\pi N\Delta$

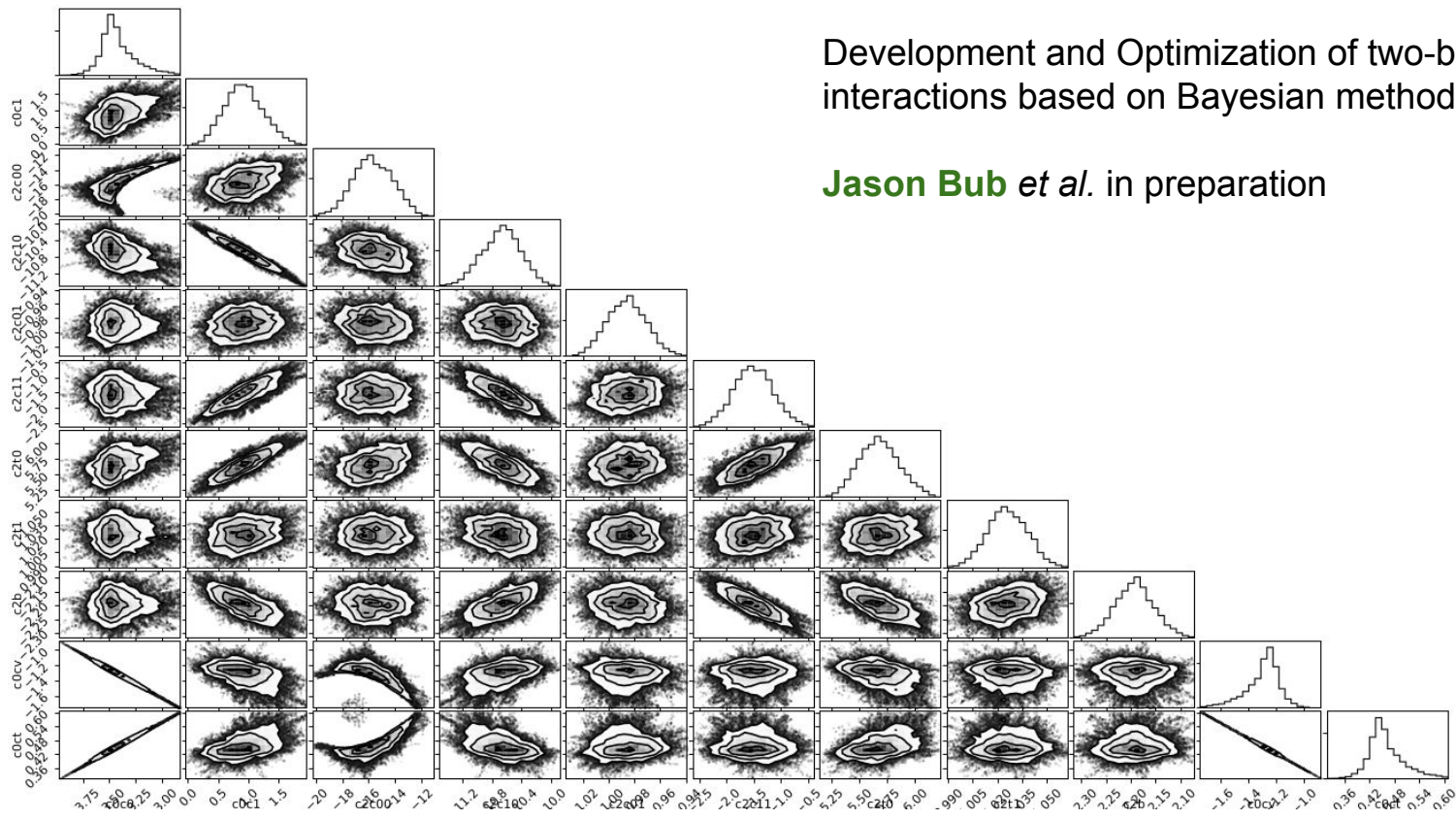
N3LO+N2LO Piarulli *et al.*

al. **Norfolk Models**

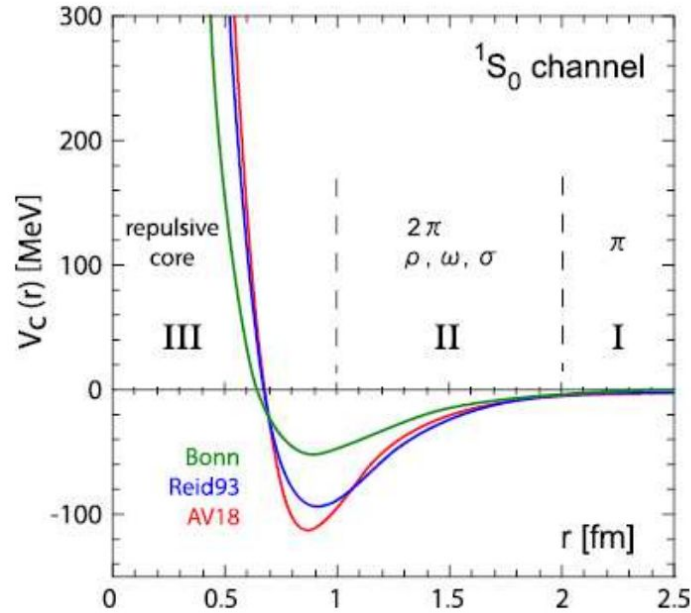
Optimization of Nuclear Two-body Interactions

Development and Optimization of two-body interactions based on Bayesian methods

Jason Bub *et al.* in preparation



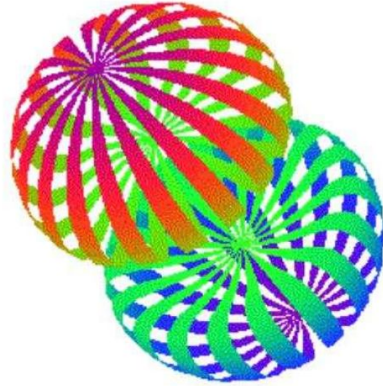
Nucleon-Nucleon Potential



Aoki *et al.* Comput.Sci.Disc.1(2008)015009

The Deuteron

$M = \pm 1$



$M = 0$



Constant density surfaces for a polarized deuteron in the $M = \pm 1$ (left) and $M = 0$ (right) states

Carlson and Schiavilla Rev.Mod.Phys.70(1998)743

Quantum Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: $H = T + V_{ij} + V_{ijk}$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

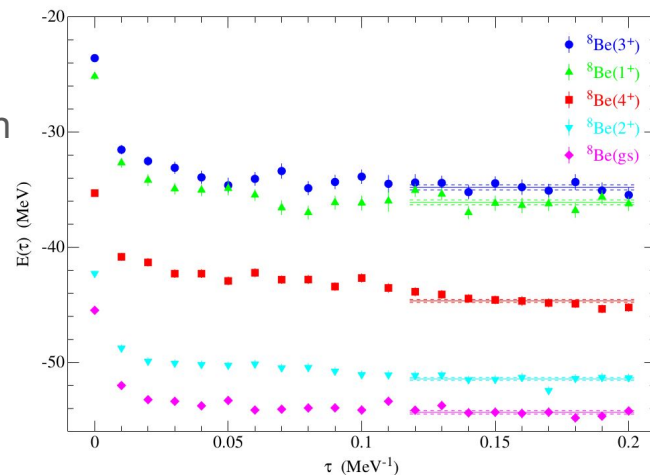
using the trial wave function:

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

Further improve the trial wave function by eliminating spurious contaminations via a Green's Function Monte Carlo propagation in imaginary time

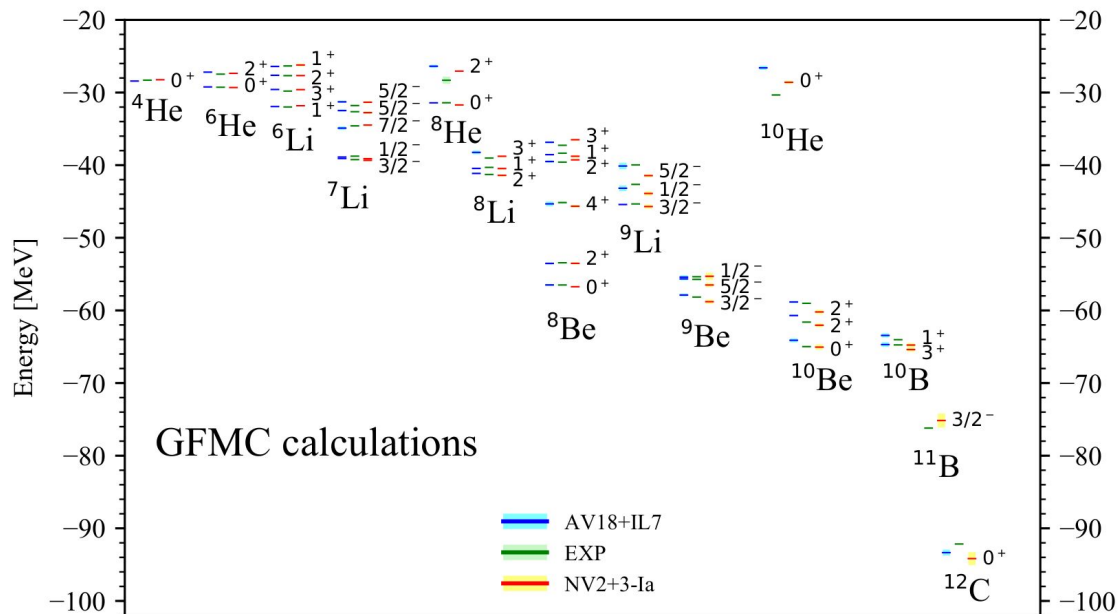
$$\Psi(\tau) = \exp[-(H - E_0)\tau] \Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \rightarrow \infty) = a_0 \psi_0$$



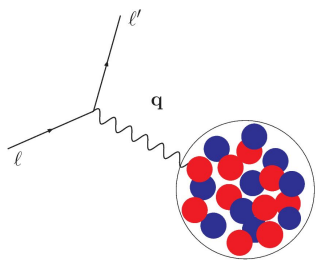
Carlson, Wiringa, Pieper *et al.*

Energies

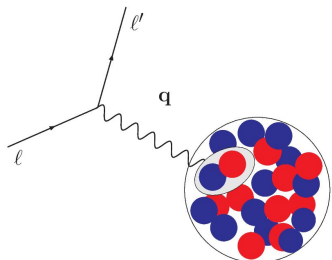


Piarulli *et al.* PRL120(2018)052503

Many-body Nuclear Electroweak Currents



one-body



two-body

- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

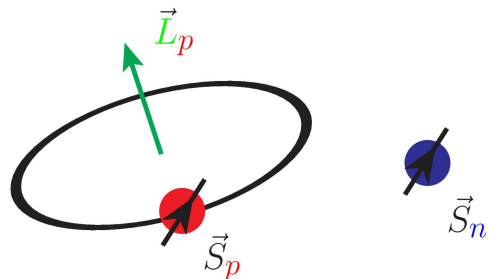
$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

Nuclear Charge Operator

$$\rho = \sum_{i=1}^A \rho_i + \sum_{i<j} \rho_{ij} + \dots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$



Magnetic Moment: Single Particle Picture

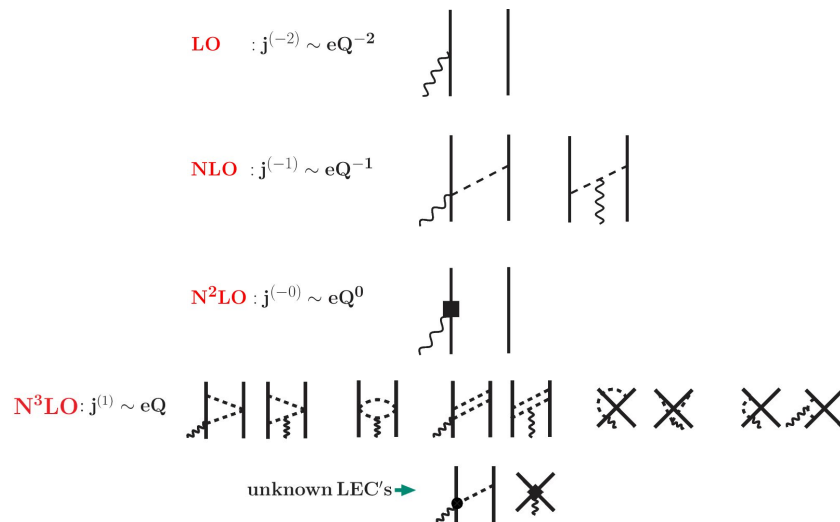
Many-body Currents

- **Meson Exchange Currents (MEC)**

Constrain the MEC current operators by imposing that the current **conservation relation is satisfied with the given two-body potential**

- **Chiral Effective Field Theory Currents**

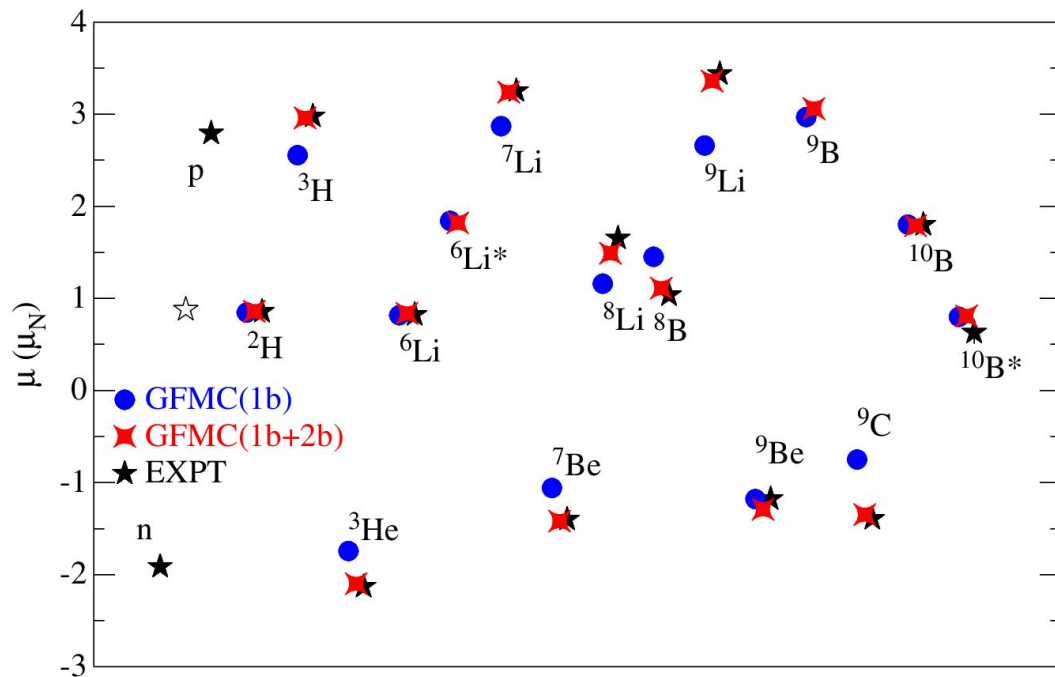
Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (**LECs**), need to be **determined by either fits to experimental data or by Lattice QCD calculations**



Electromagnetic Current Operator

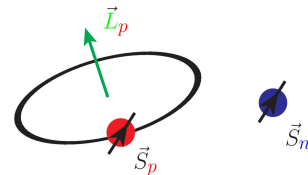
SP *et al.* PRC78(2008)064002, PRC80(2009)034004,
 PRC84(2011)024001, PRC87(2013)014006
 Park *et al.* NPA596(1996)515, Phillips (2005)
 Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

Magnetic Moments of Light Nuclei



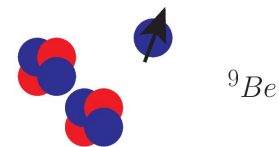
SP *et al.* PRC87(2013)035503

Single particle picture



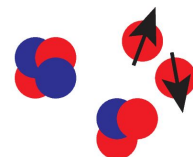
$$\mu_N(1b) = \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$$

Small two-body current effects



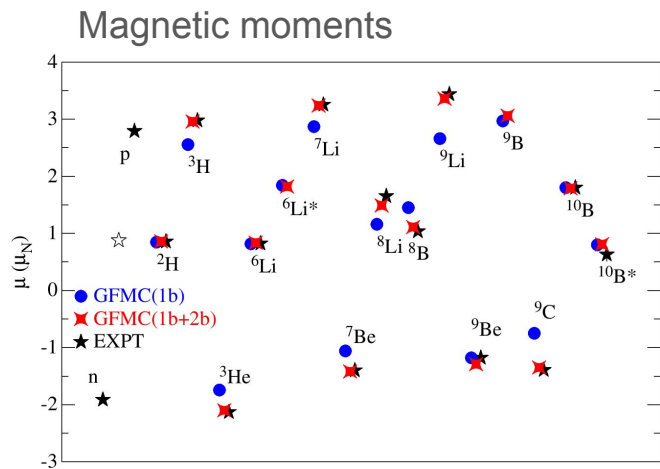
${}^9\text{Be}$

Large two-body current effects



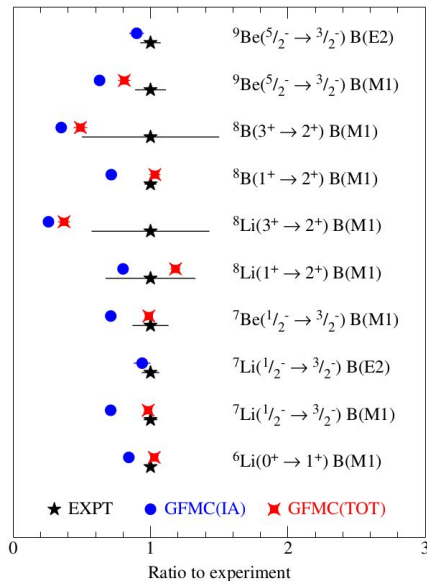
${}^9\text{C}$

Electromagnetic Observables

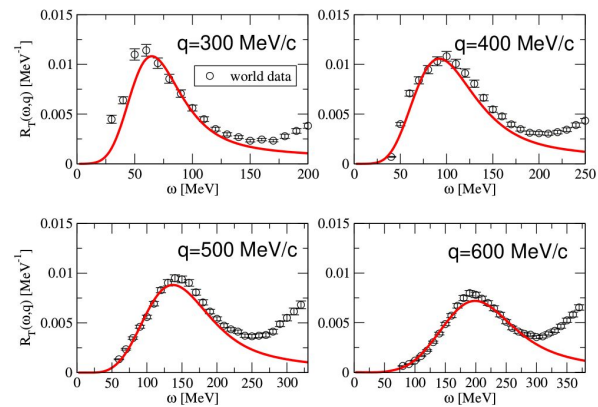


SP *et al.* PRC87(2013)035503,
 PRC101(2020)044612

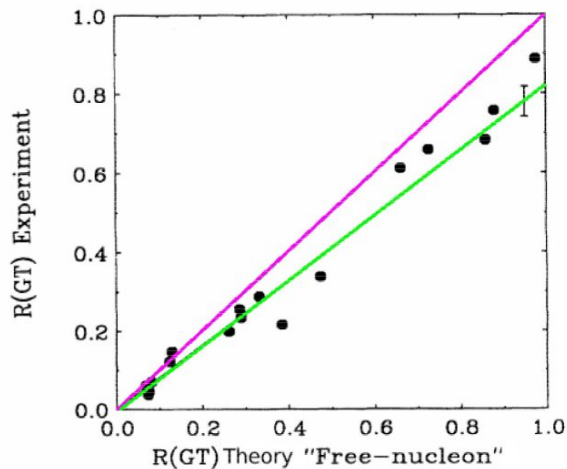
EM decay



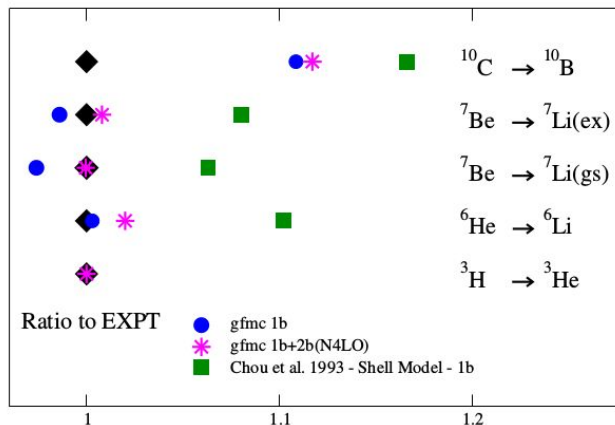
e - ${}^4\text{He}$ particle scattering



Beta decay

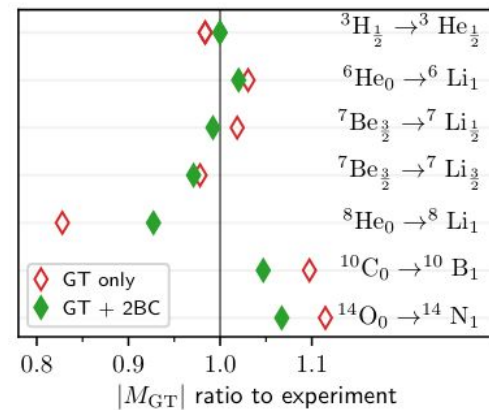


Chou et al. PRC47(1993)163



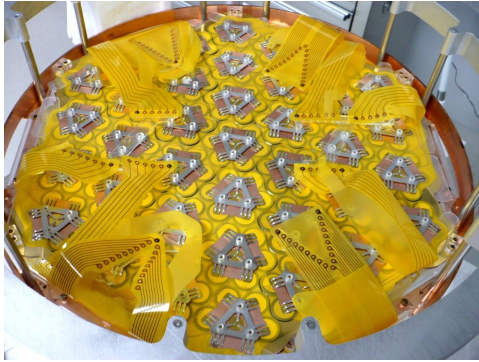
gfm1b (blue) and gfm1b+2b (pink); shell model (green)

SP et al. PRC97(2018)022501

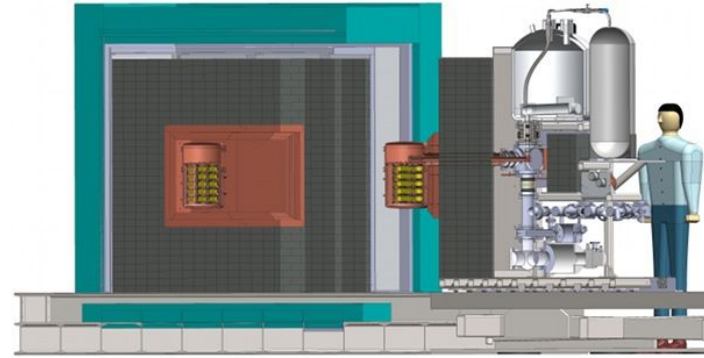


P. Gysbers *Nature Phys.* 15 (2019)

Nuclear Physics for Neutrinoless Double Beta Programs



EXO-200 Collaboration

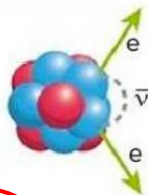


Majorana Demonstrator

Neutrinoless double beta decay half-life $T_{1/2} \gtrsim 10^{25}$ years (age of the universe 1.4×10^{10} years)
1 ton of material is required to see few events per year

Decay Rate \propto (nuclear matrix element)² \times $(m_{\beta\beta})^2$

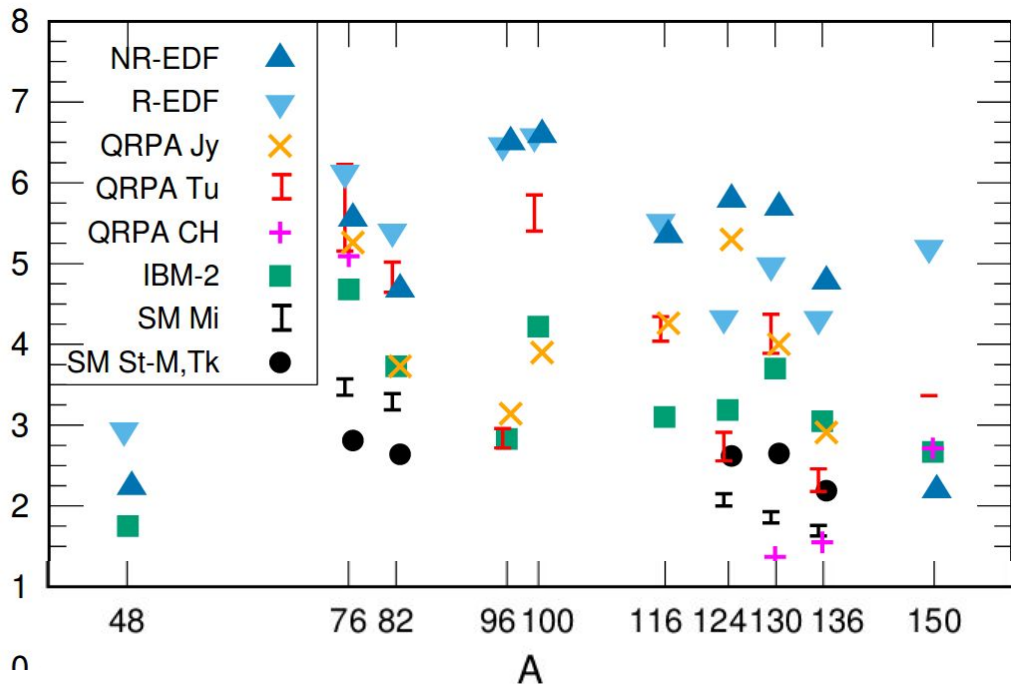
Neutrinoless Double Beta Decay



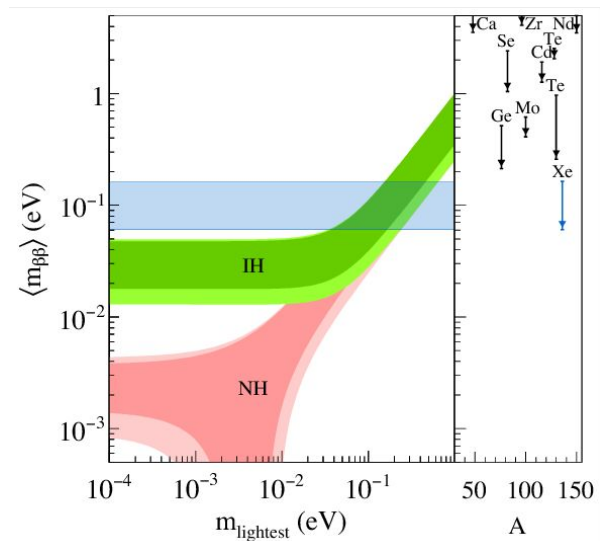
$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q, Z) |M_{0\nu}|^2 m_{\beta\beta}^2$$



$M_{0\nu}$



Engel & Menendez Rep.Progr.Phys80(2017)046301



Partial muon capture rates: VMC calculations

$$\Gamma_{\text{VMC}}(\text{avg.}) = 1495 \text{ s}^{-1} \pm 19 \text{ s}^{-1}$$

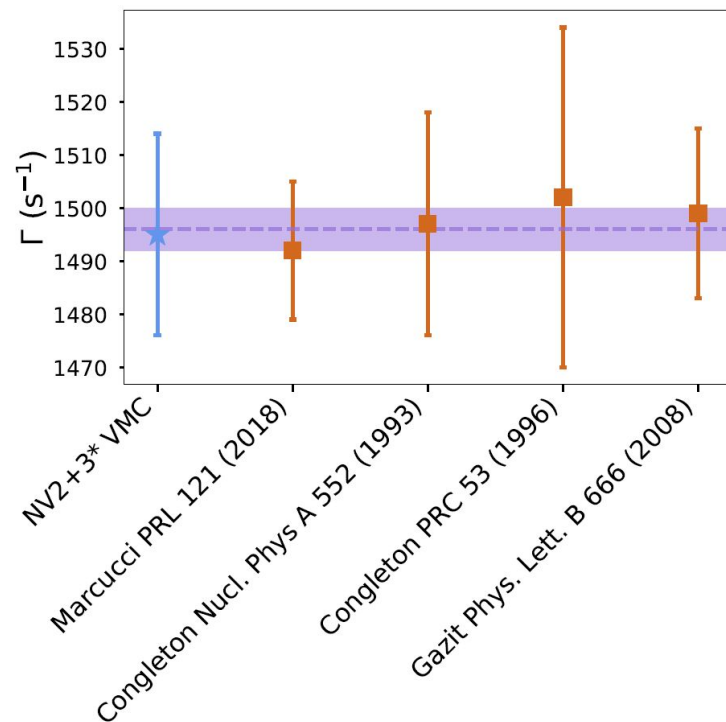
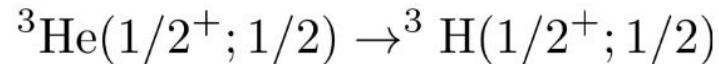
$$\Gamma_{\text{expt}} = 1496.0 \text{ s}^{-1} \pm 4.0 \text{ s}^{-1}$$

Ackerbauer *et al.* PLB417, 224(1998)

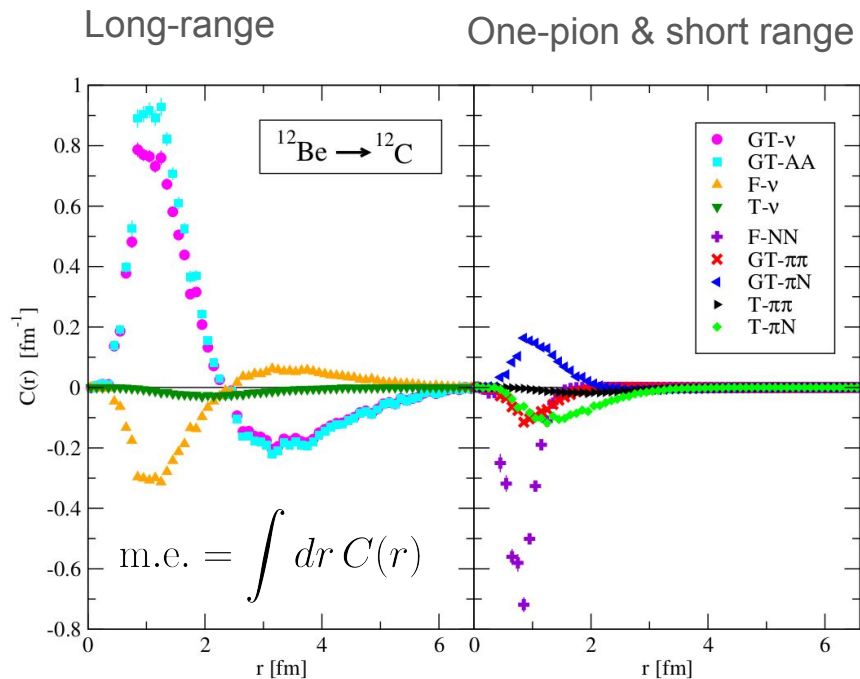
Momentum transfer $q \sim 100 \text{ MeV}$

Two-body correction is $\sim 8\%$ of total rate on average for $A=3$

Garrett King *et al.* PRC2022



Neutrinoless Double Beta Decay Matrix Elements



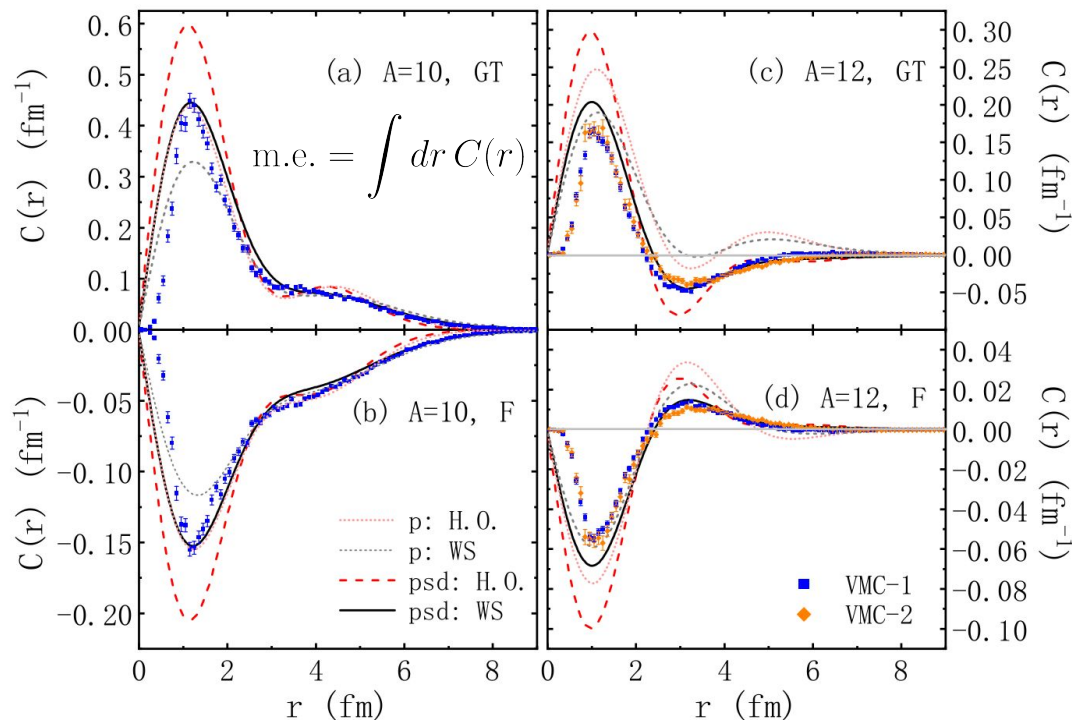
SP *et al.* PRC97(2018)014606



Cirigliano Dekens DeVries Graesser Mereghetti *et al.*
 PLB769(2017)460, JHEP12(2017)082, PRC97(2018)065501

- Leading operators in neutrinoless double beta decay are two-body operators
- These observables are particularly sensitive to short-range and two-body physics
- Transition densities calculated in momentum space indicate that the momentum transfer in this process is of the order of $\mathbf{q} \sim 200 \text{ MeV}$

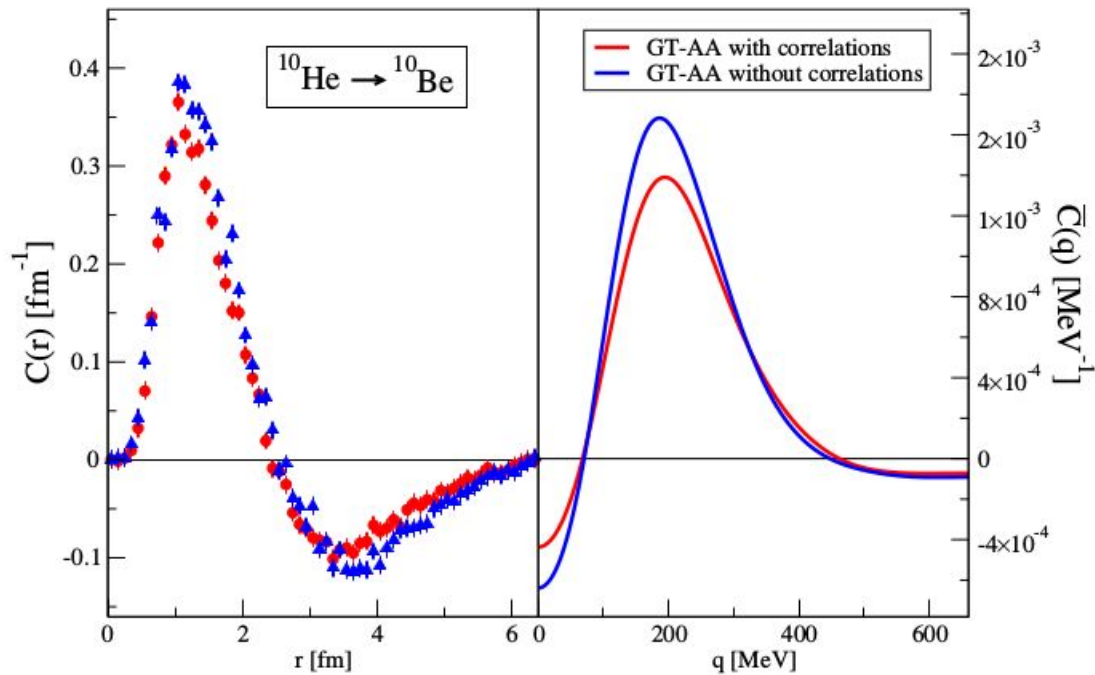
Comparison with Shell-Model Calculations



Closer agreement between Shell-Model calculations with Variational Monte Carlo results is reached by

- Increasing the size of the model space
- Wood-Saxon single particle wave functions are superior in describing the tails of the densities wrt harmonic oscillator wave functions
- Phenomenological Short-Range-Correlations functions further improve the agreement

Correlations in neutrinoless double beta decay ME

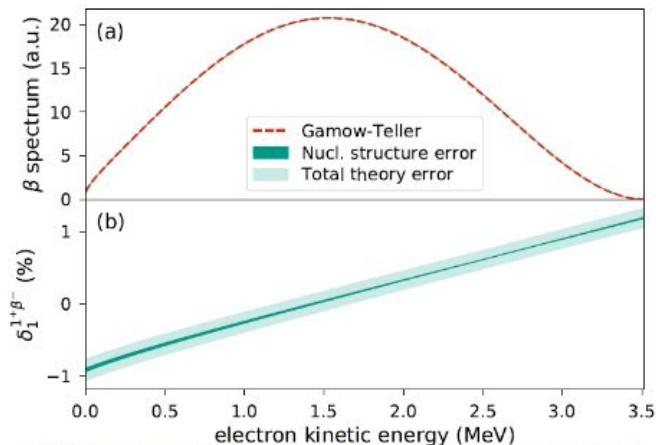


Beta decay spectrum

${}^6\text{He}$ Beta decay spectrum for BSM searches with NCSL, He6-CRES, LPC-Caen



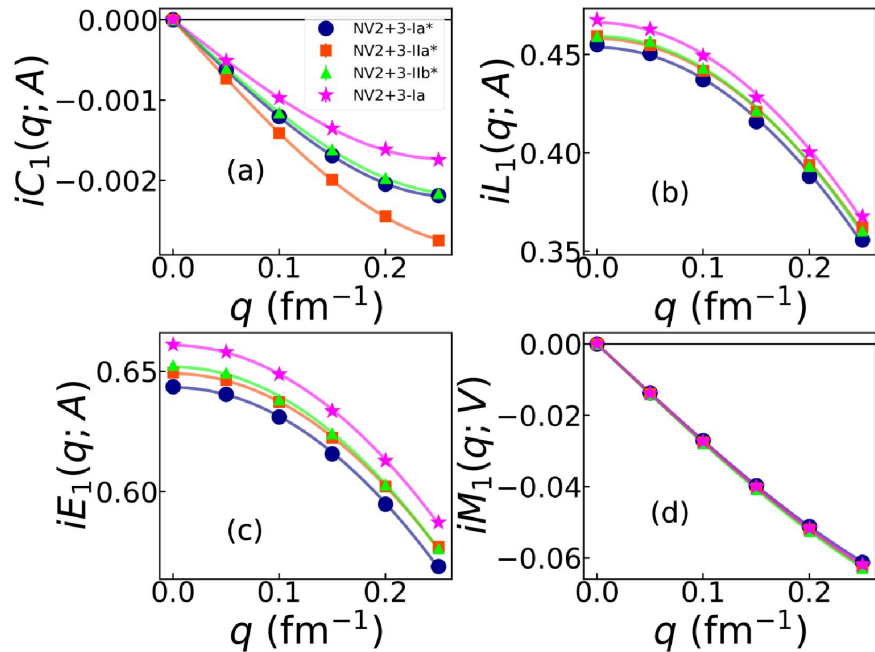
${}^6\text{He}$ beta-decay spectrum from NCSM



Glick-Magid et al. arXiv:2107.10212

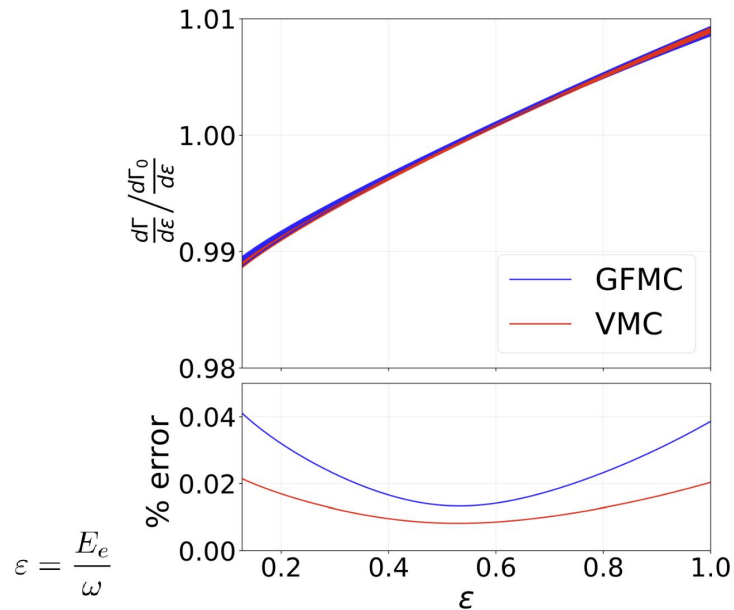
$$\frac{d\Gamma}{d\varepsilon} = \frac{d\Gamma_0}{d\varepsilon} \times (1 + \text{corrections})$$

Beta Decay Spectrum



Dominant terms $L_1^{(0)}$ and $E_1^{(0)}$ have model dependence of $\sim 1\%$ to $\sim 2\%$

Standard Model spectrum for ${}^6\text{He}$



$$\varepsilon = \frac{E_e}{\omega}$$

$$\tau_{\text{GFMC}} = 808 \pm 24 \text{ ms}$$

$$\tau_{\text{Expt.}} = 807.25 \pm 0.16 \pm 0.11 \text{ ms}$$

Garrett King et al. [arXiv:2207.11179](https://arxiv.org/abs/2207.11179)

Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

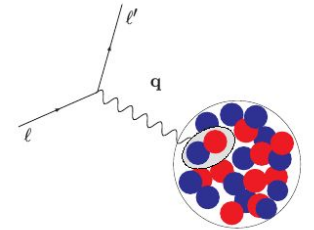
$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$

Transverse response induced by the current operator $O_T = \mathbf{j}$

5 Responses in neutrino-nucleus scattering

$$\frac{d^2 \sigma}{d\omega d\Omega} = \sigma_M [v_L R_L(\mathbf{q}, \omega) + v_T R_T(\mathbf{q}, \omega)]$$



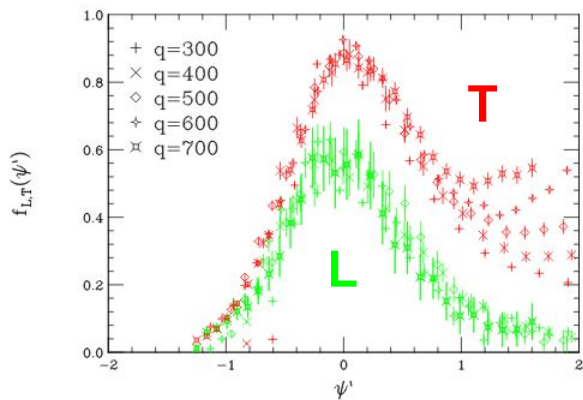
For a recent review on QMC, SF methods see

[Rocco Front. In Phys.8 \(2020\)116](#)

Lepton-Nucleus scattering: Data

Transverse Sum Rule

$$S_T(q) \propto \langle 0 | \mathbf{j}^\dagger \mathbf{j} | 0 \rangle \propto \langle 0 | \mathbf{j}_{1b}^\dagger \mathbf{j}_{1b} | 0 \rangle + \langle 0 | \mathbf{j}_{1b}^\dagger \mathbf{j}_{2b} | 0 \rangle + \dots$$

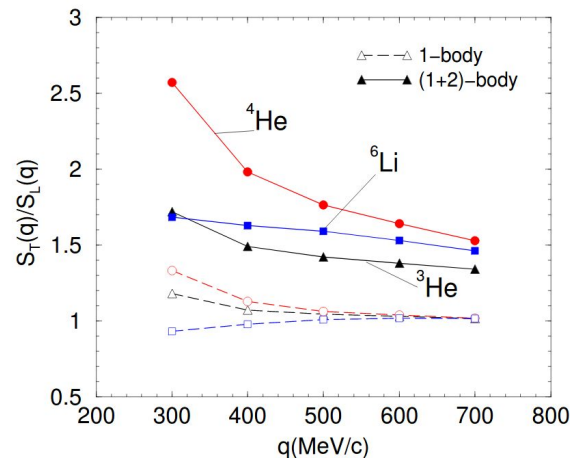


⁴He Electromagnetic Data
Carlson *et al.* PRC65(2002)024002

Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term

$\langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{1b} \rangle > 0$
 Leading one-body term

$\langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{2b} v_\pi \rangle \propto \langle v_\pi^2 \rangle > 0$
 Interference term

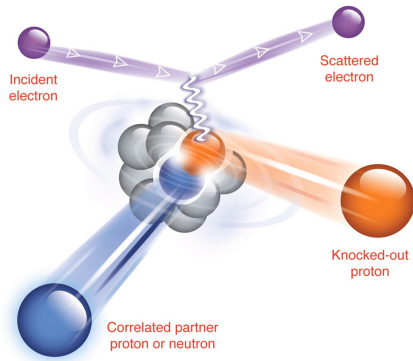


Transverse/Longitudinal Sum Rule
Carlson *et al.* PRC65(2002)024002

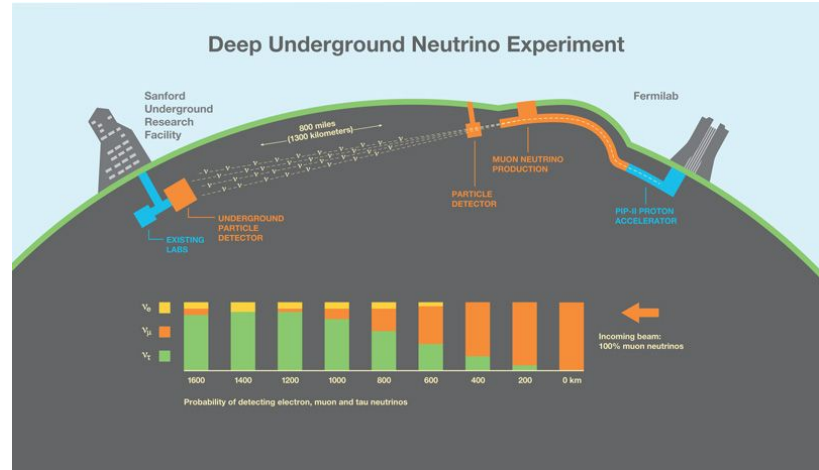
Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from $A > 12$ without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



Subedi et al. Science320(2008)1475

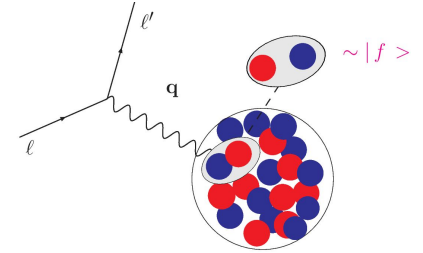


[Stanford Lab article](#)

[e4u collaboration](#)



Short-Time-Approximation



Short-Time-Approximation:

- Based on Factorization
- **Retains two-body physics**
- Response functions are given by the **scattering from pairs of fully interacting nucleons** that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides “more” exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions \propto Cross Sections

$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle|^2$$

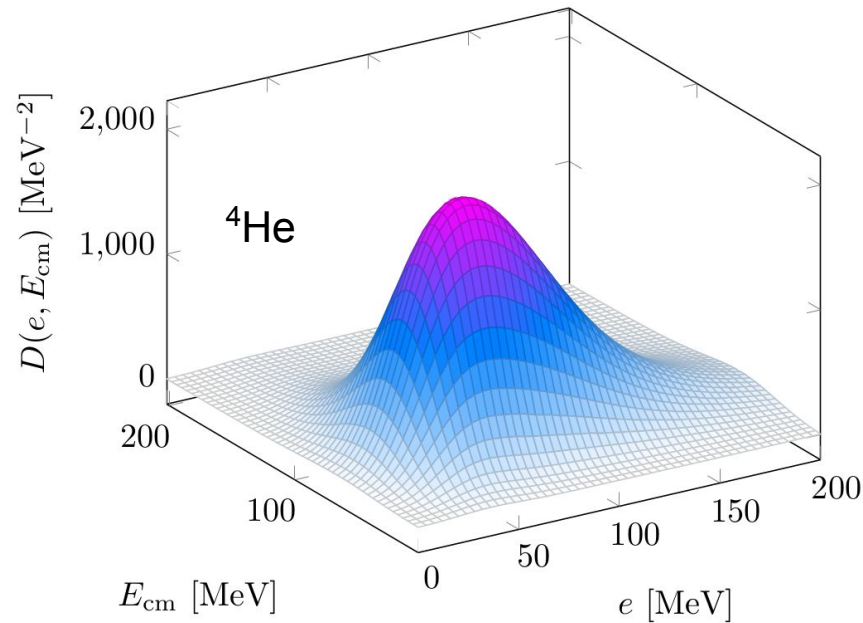
Response **Densities**

$$R(q, \omega) \sim \int \delta(\omega + E_0 - E_f) dP' dp' \mathcal{D}(p', P'; q)$$

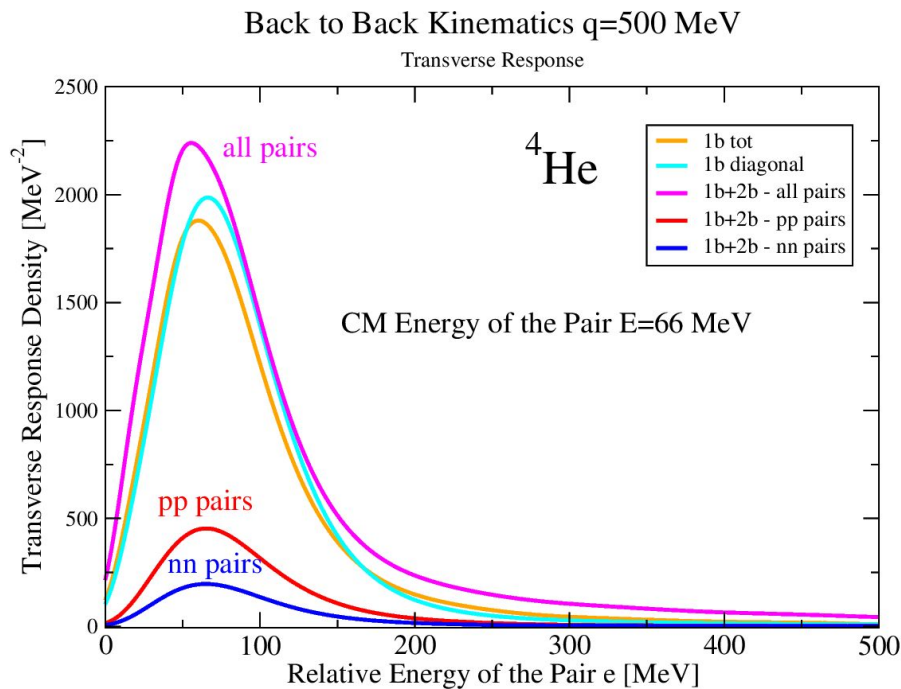
P' and p' are the CM and relative momenta of the struck nucleon pair

Transverse Response Density: e - ${}^4\text{He}$ scattering

Transverse Density $q = 500 \text{ MeV}/c$

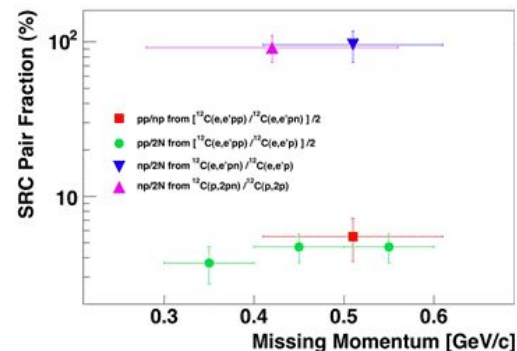


$e^{-4}\text{He}$ scattering in the back-to-back kinematic



SP *et al.* PRC101(2020)044612

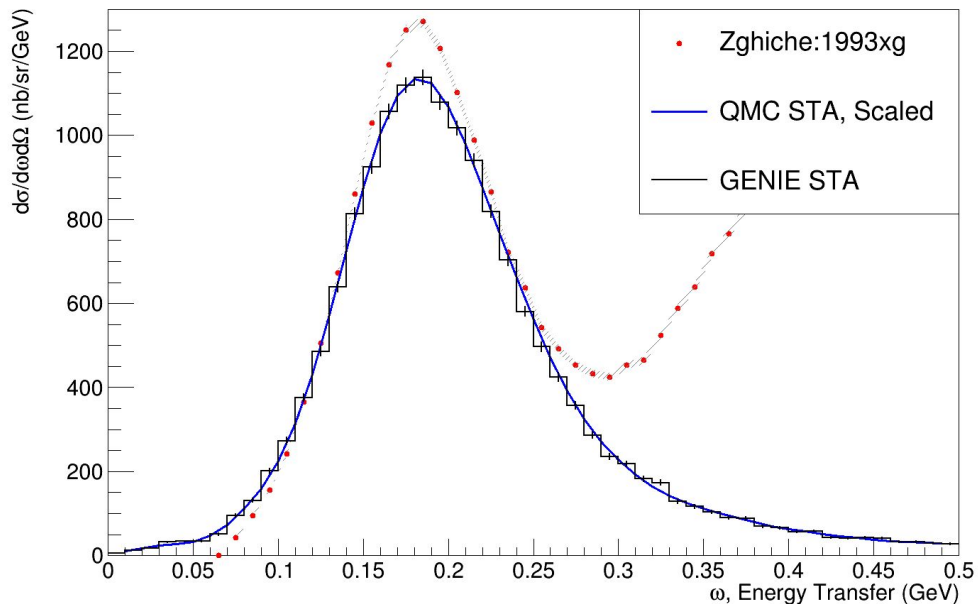
- pp pairs
- nn pairs
- all pairs 1body
- all pairs tot



Subedi *et al.* Science320(2008)1475

GENIE validation using e-scattering

Z = 2, A = 4, Beam Energy = 0.64 GeV, Angle = $60^\circ \pm 0.25^\circ$

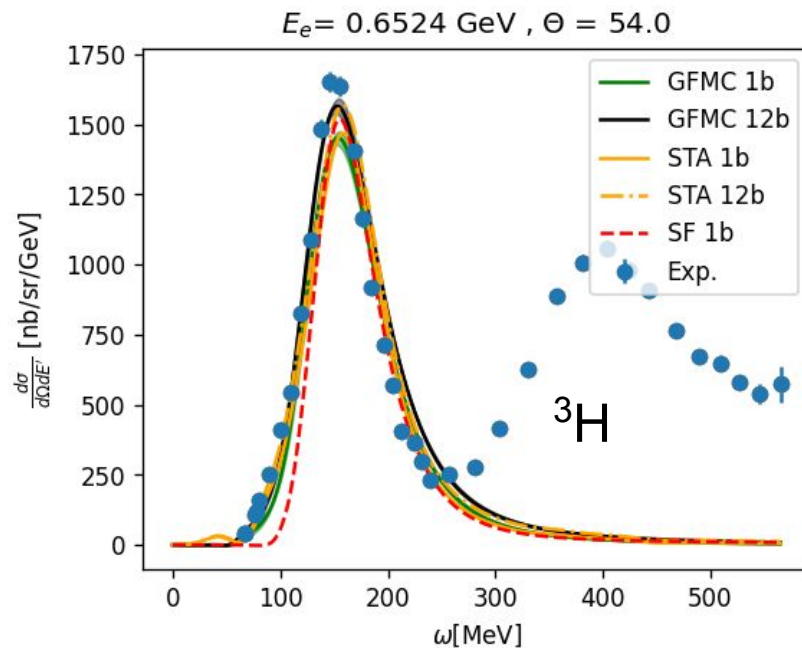
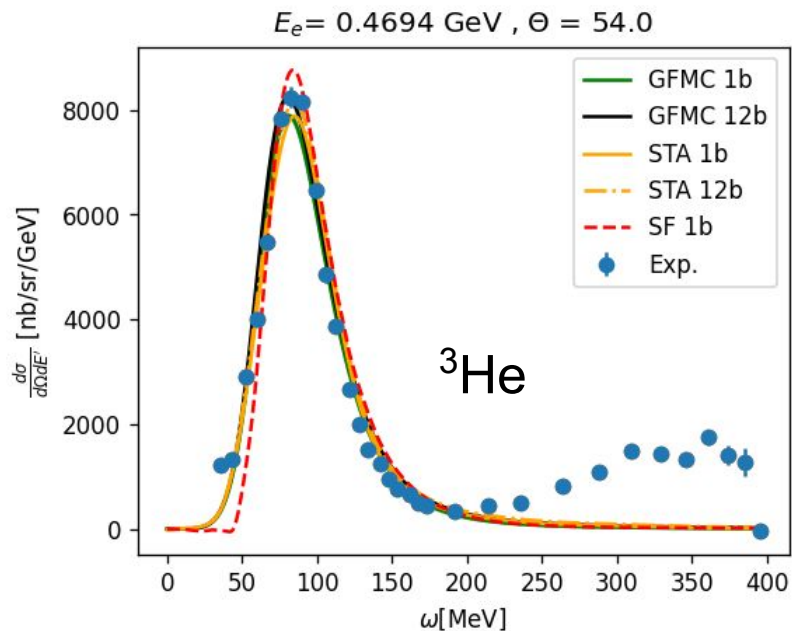


- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE (a Monte Carlo neutrino event generator)
- Here, we use electromagnetic processes (for which data are available) to validate the generator

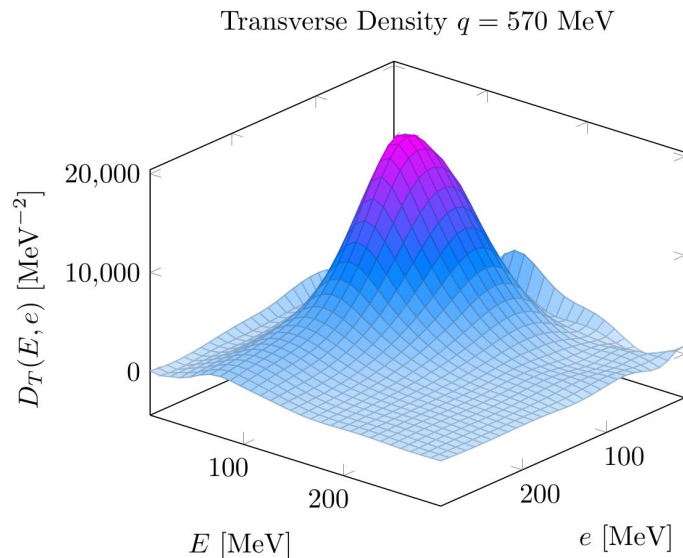
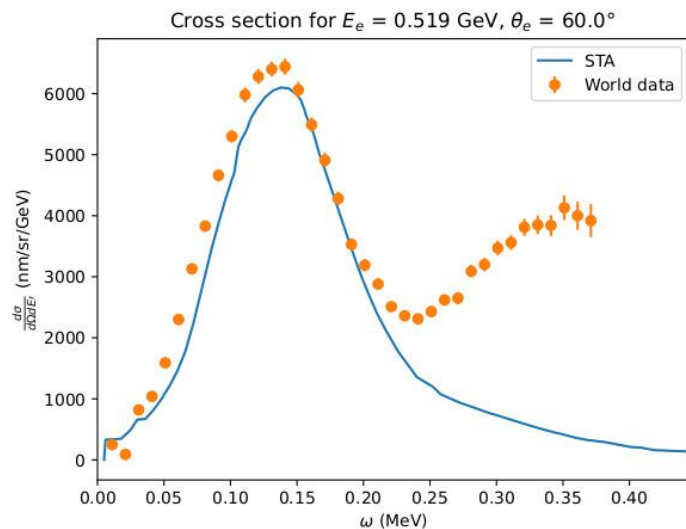
$$\frac{d^2 \sigma}{d\omega d\Omega} = \sigma_M [v_L R_L(\mathbf{q}, \omega) + v_T R_T(\mathbf{q}, \omega)]$$

Barrow, Gardiner, SP *et al.* PRD 103 (2021) 5, 052001

GFMC SF STA: Benchmark & error estimate



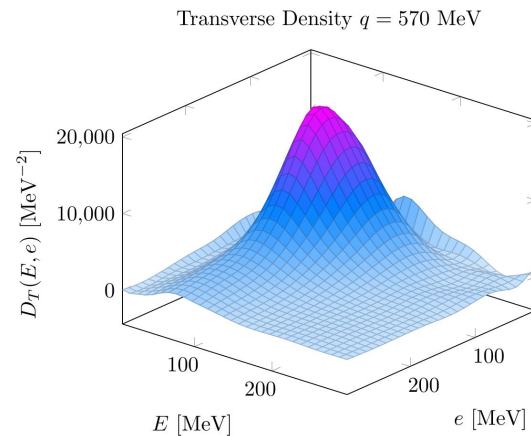
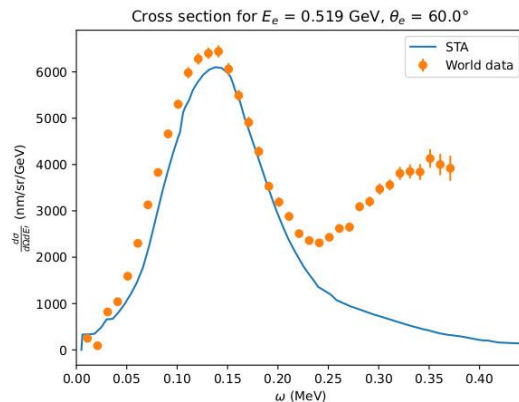
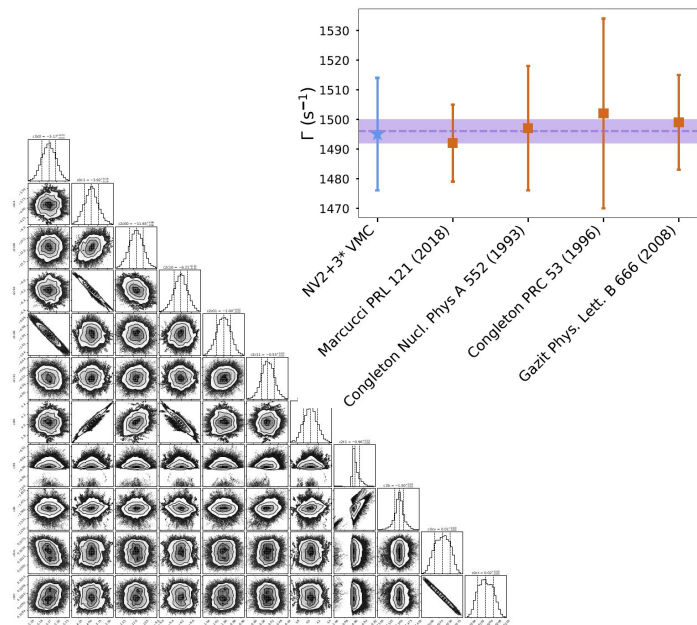
STA for Carbon 12: Preliminary results



Lorenzo Andreoli *et al.* in preparation

Summary

Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where **many-body effects play an essential role to explain available data.**



Close **c**ollaborations between **NP, LQCD, Pheno, Hep, Comp, Expt, ...** are required to progress e.g., NP is represented in the Snowmass process

It's a very exciting time!

Collaborators

WashU: **Andreoli Bub King Piarulli**

LANL: Baroni Carlson Cirigliano Gandolfi Hayes Mereghetti

JLab+ODU: Schiavilla

ANL: Lovato Rocco Wiringa

UCSD/UW: Dekens

Pisa U/INFN: Kievsky Marcucci Viviani

Salento U: Girlanda

Huzhou U: Dong Wang

Fermilab: Gardiner Betancourt

MIT: Barrow



Theory Alliance
FACILITY FOR RARE ISOTOPE BEAMS

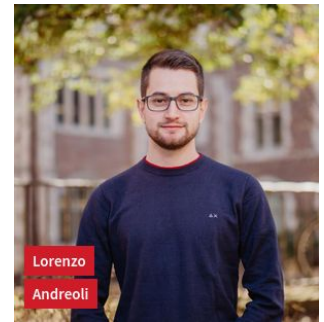


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Quantum Monte Carlo group



webpage: [Quantum Monte Carlo group](#)