On the phase structure of QCD

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Arizona State University, September 15th 2021

for the fQCD collaboration

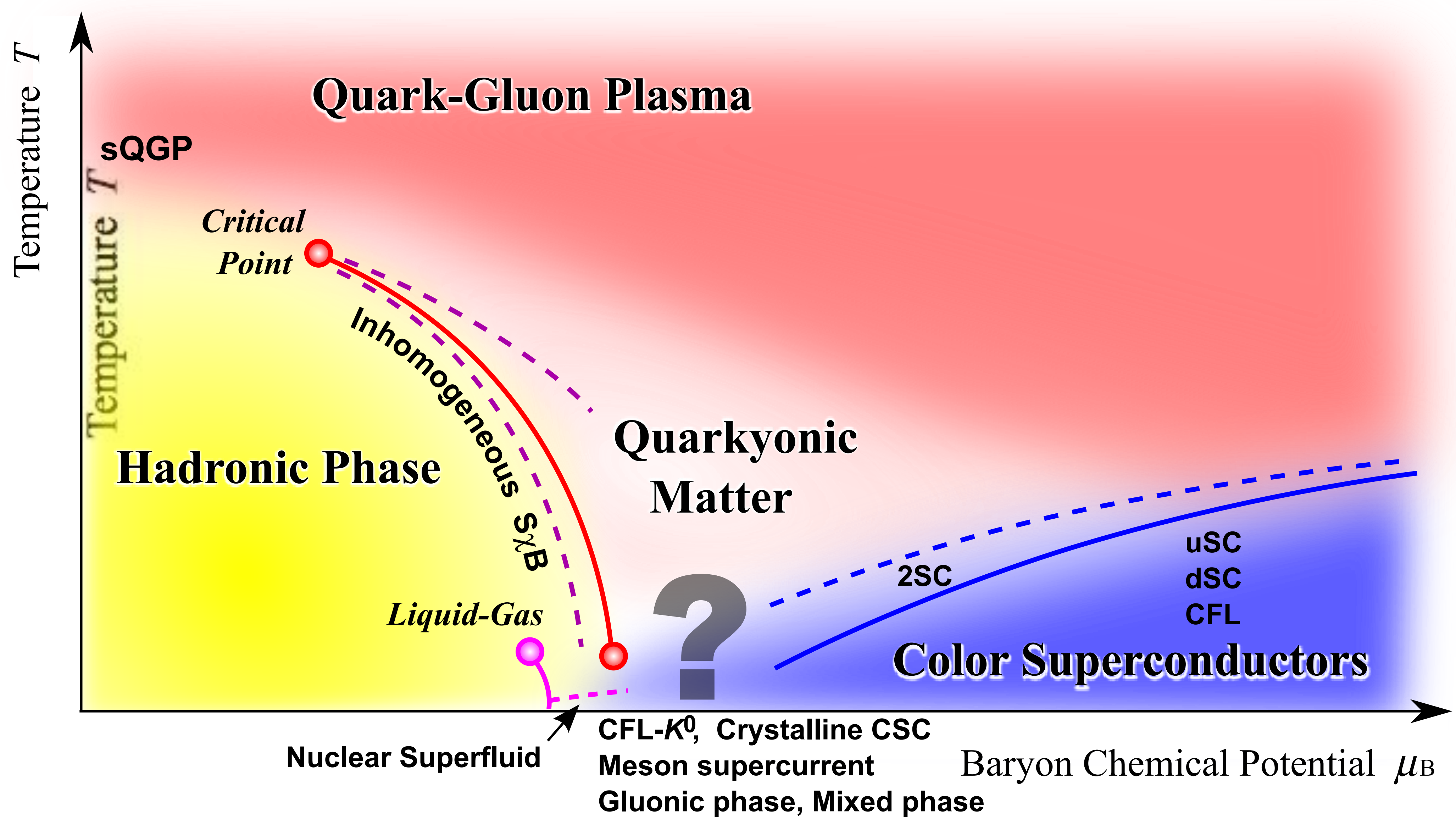
GEFÖRDERT VOM

Bundesministerium für Bildung und Forschung

STRUCTURES CLUSTER OF EXCELLENCE

isoQuant SFB1225

EMMI



fQCD collaboration

Braun, Chen, Fu, Ihssen, Geissel, Horak, Huang, JMP, Rennecke, Sattler, Schallmo, Schneider, Tan, Töpfel, Wen, Wessely, Wink, Yin

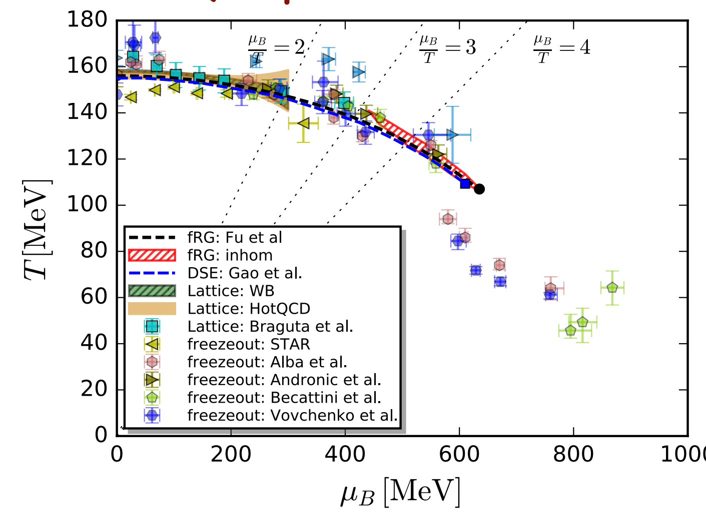
Dalian, Darmstadt, Heidelberg, Gießen

Outline

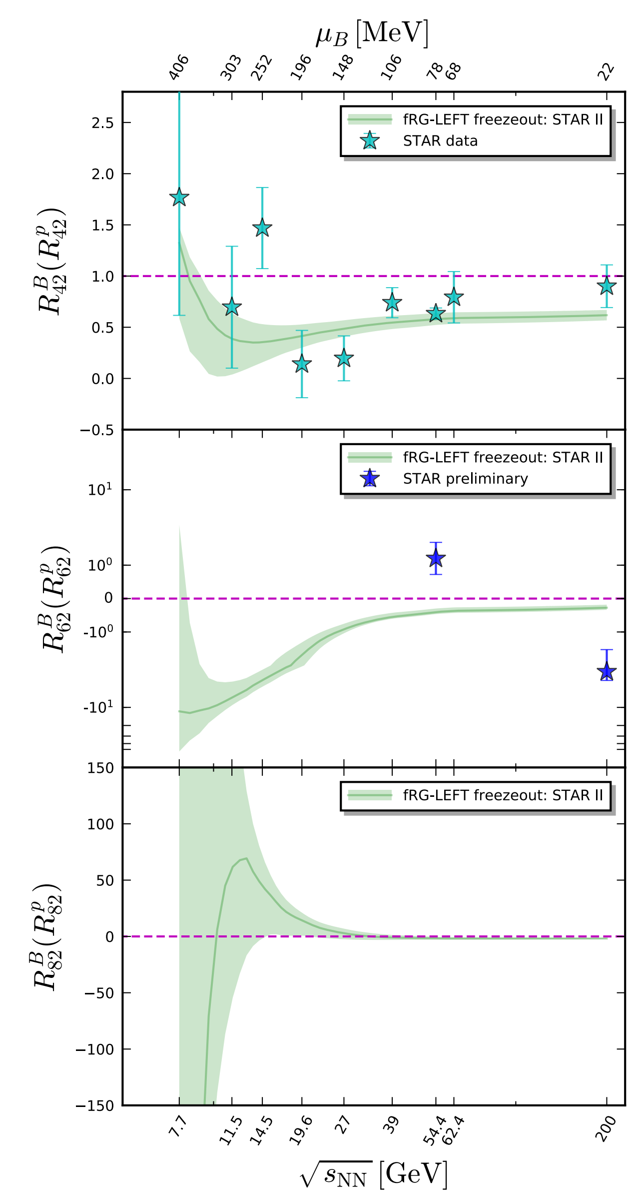
- **QCD from functional methods**

- **QCD phase structure**

QCD phase structure



Hyper-fluctuations



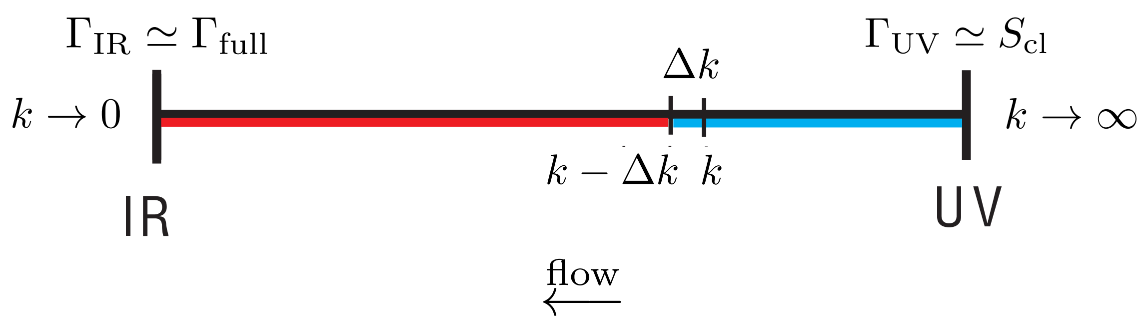
- **Fluctuations of conserved charges**

- **Summary & outlook**

Functional Methods for QCD

FRG: JMP, NPA 931 (2014) 113
Dupuis et al, Phys.Rept. 910 (2021) 1
DSE: Fischer, PPNP 105 (2019) 1

free energy at momentum scale k

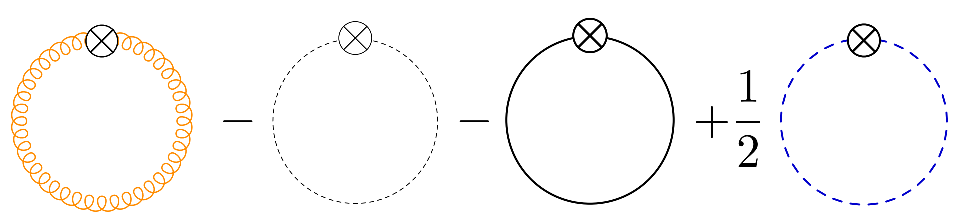


ab initio

glue quantum fluctuations

hadronic quantum fluctuations

functional RG: $\partial_t \Gamma_k[\phi] = \frac{1}{2}$



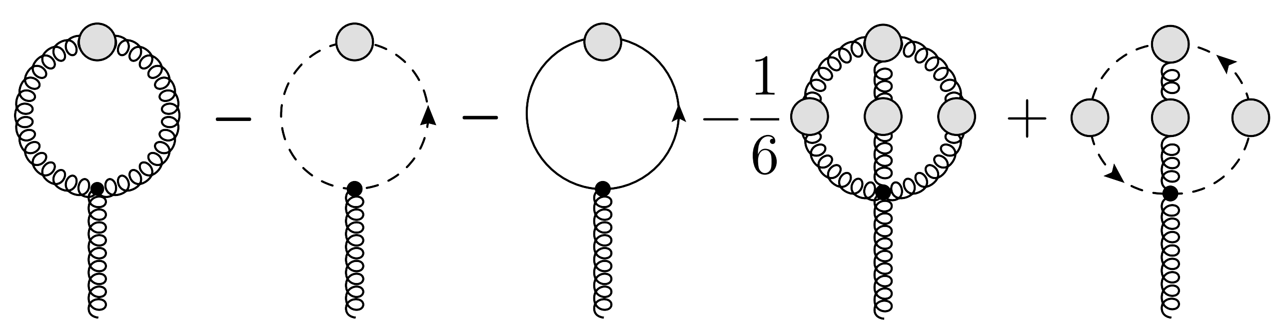
free energy/ grand potential

quark quantum fluctuations

$\boxed{\text{RG-scale k: } t = \ln k}$

closed form

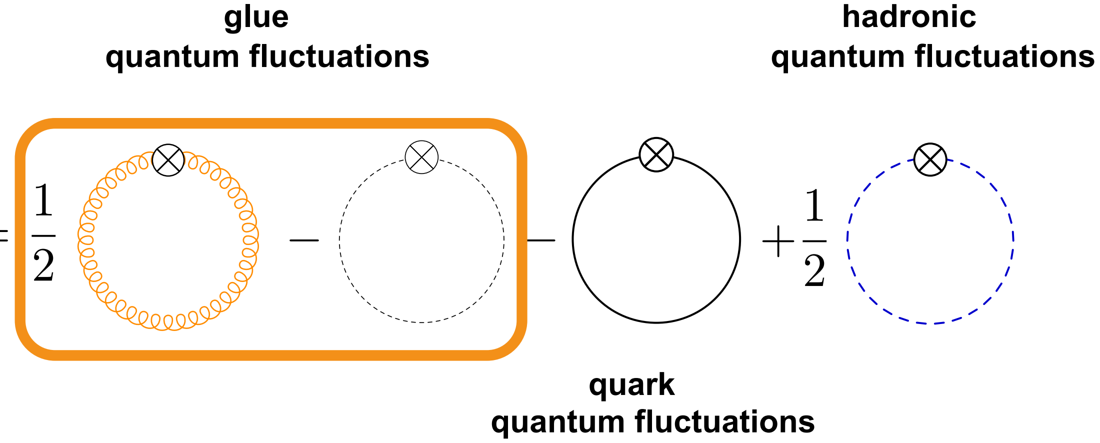
functional DSE : $\frac{\delta\,(\Gamma - S)}{\delta A_0} = \frac{1}{2}$



A_0 **: background field**

Functional Methods for QCD

functional RG: $\partial_t \Gamma_k[\phi] =$



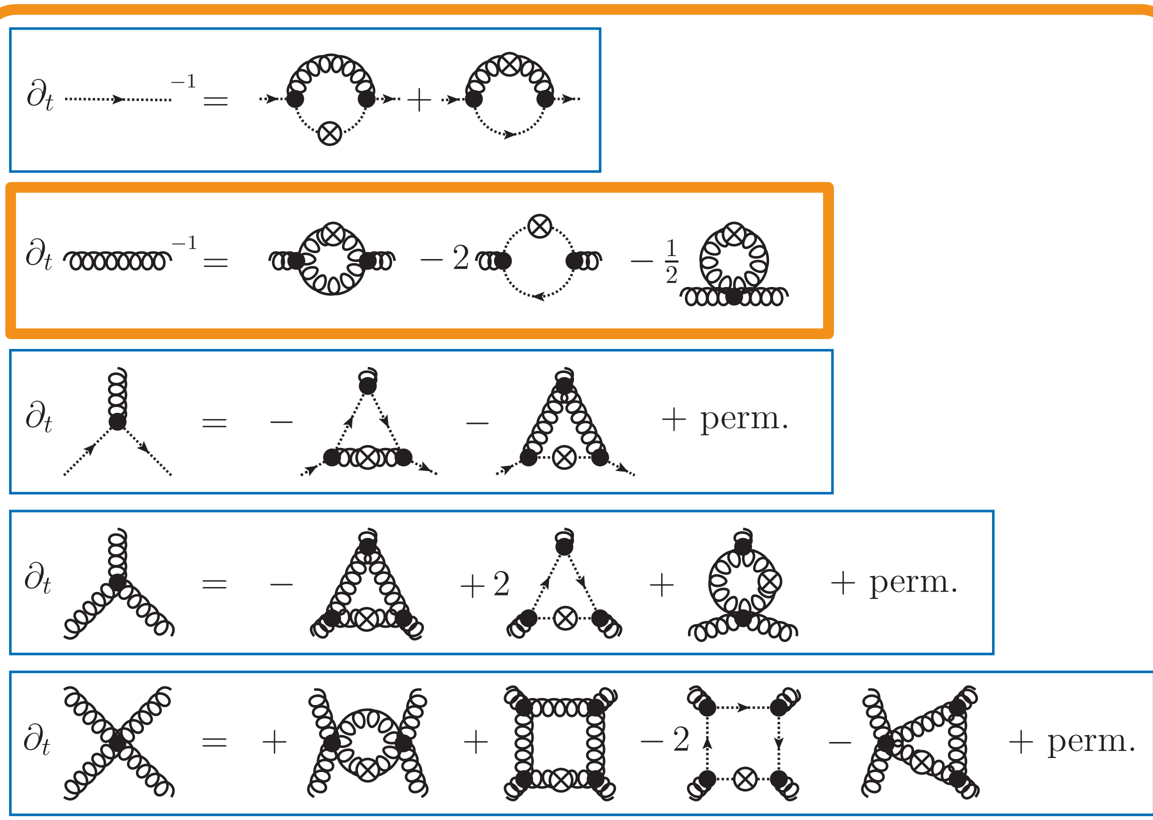
free energy/
grand potential

Correlation functions

gluon propagator

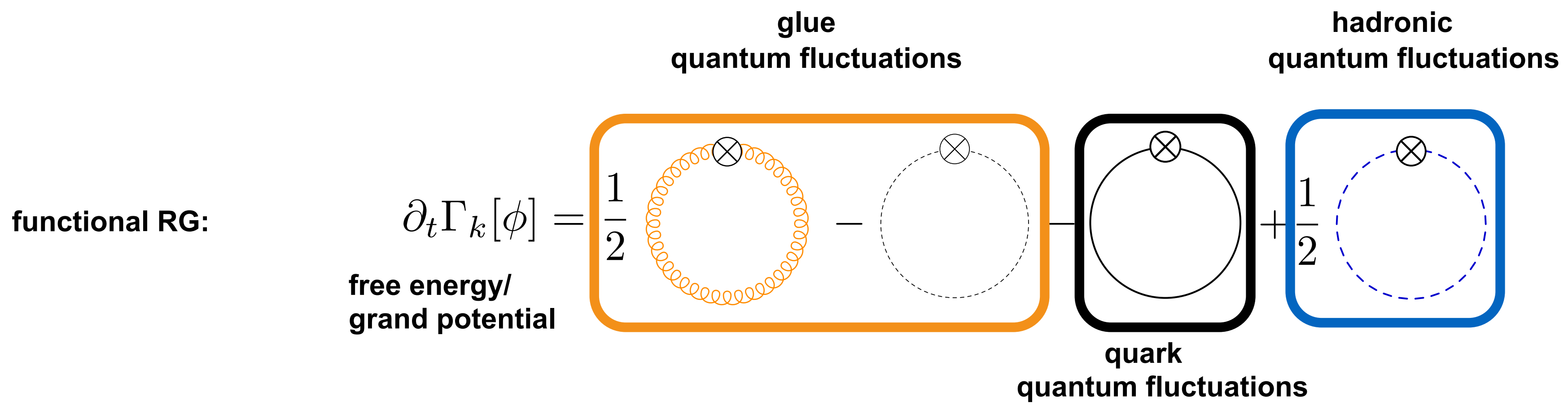
$$\langle A_\mu A_\nu \rangle (p)$$

Pure glue



+ matter loops

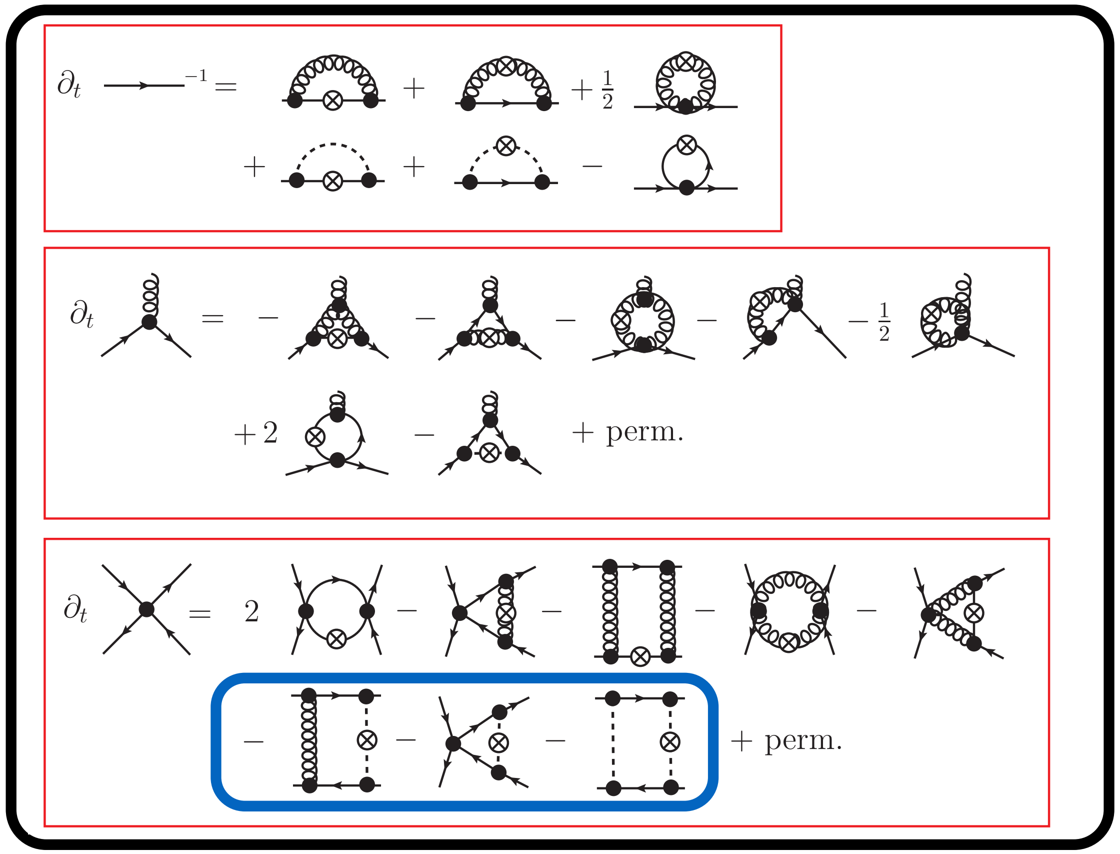
Functional Methods for QCD



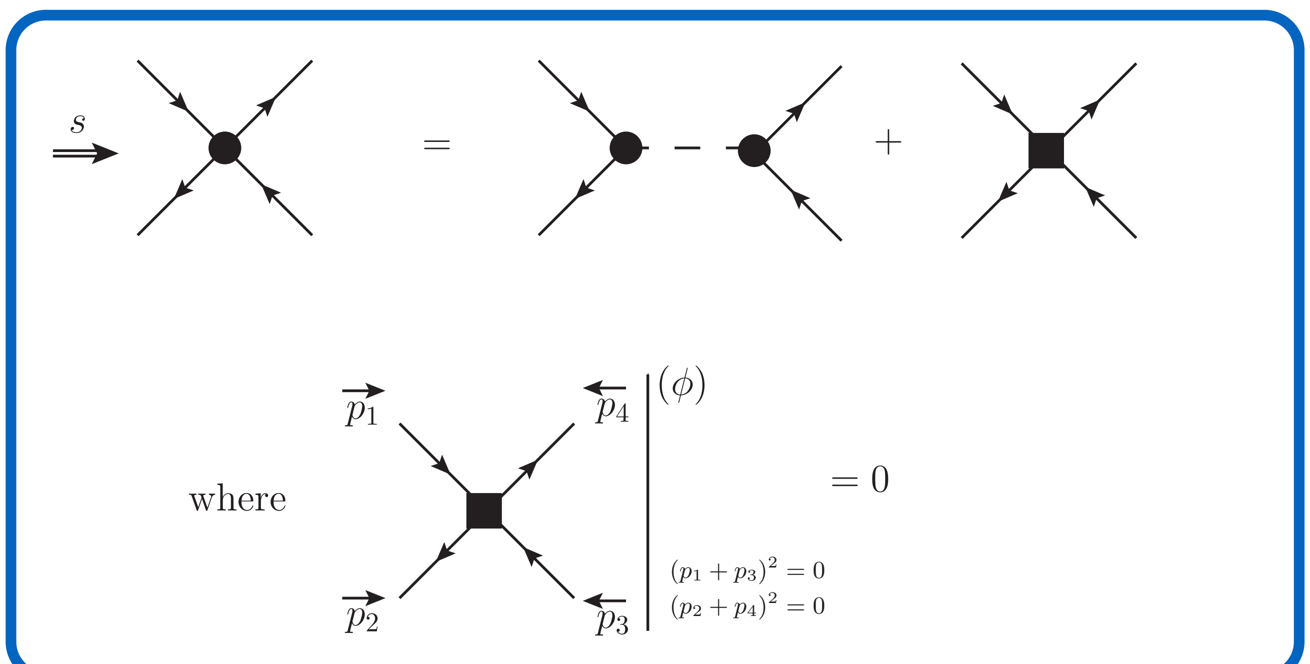
Correlation functions

gluon propagator	quark propagator	quark-gluon vertex	quark—anti-quark scatterings
$\langle A_\mu A_\nu \rangle(p)$	$\langle q\bar{q} \rangle(p)$	$\langle q\bar{q}A_\mu \rangle(p_1, p_2)$	$\langle q\bar{q}q\bar{q} \rangle(p_1, p_2, p_3)$

Eight tranverse tensor structures

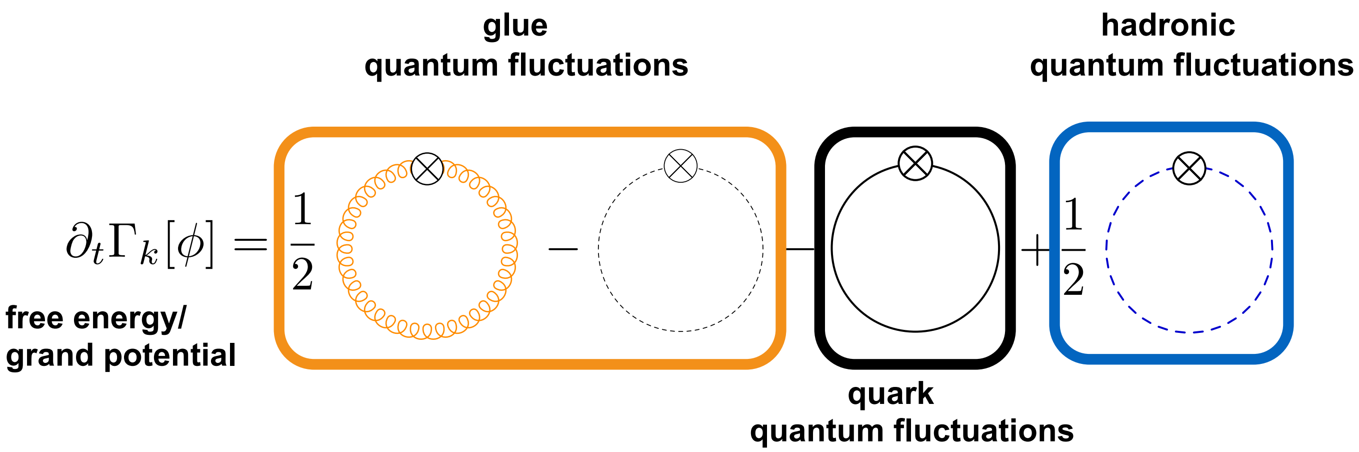


Dynamical hadronisation



Functional Methods for QCD

functional RG:



Correlation functions

gluon propagator

$$\langle A_\mu A_\nu \rangle (p)$$

quark propagator

$$\langle q\bar{q} \rangle (p)$$

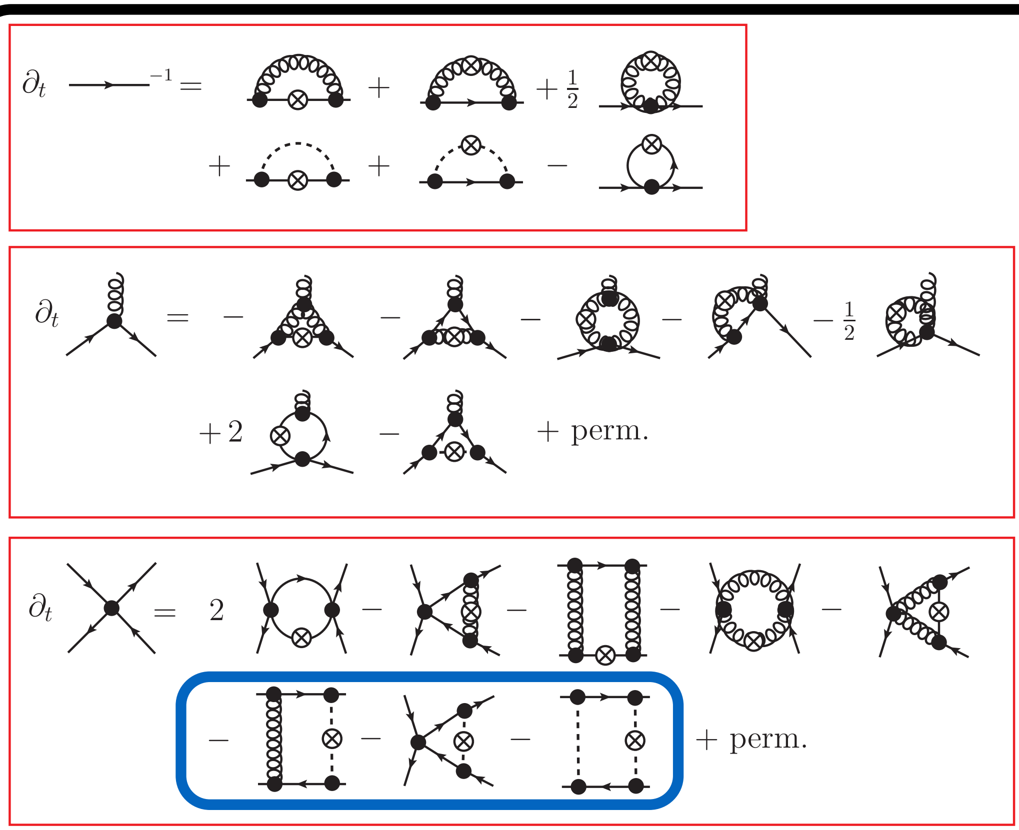
quark-gluon vertex

$$\langle q\bar{q} A_\mu \rangle (p_1, p_2)$$

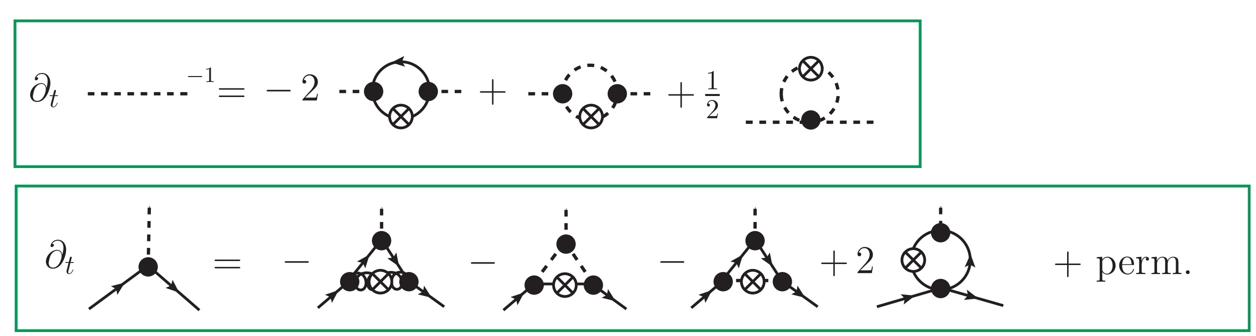
Eight tranverse tensor structures

quark—anti-quark scatterings

$$\langle q\bar{q} q\bar{q} \rangle (p_1, p_2, p_3)$$



Dynamical hadronisation



Hadronisation of mesons, diquarks, baryons

Fukushima, JMP, Strodthoff, 2103.01129

vacuum QCD with the fRG

1^{st} principles

Input: fundamental parameters of QCD at a large momentum scale: $\Lambda = 20\,\mathrm{GeV}$

2-flavour QCD

(i) $\alpha_{s,\Lambda}$

(ii) $m_{u,\Lambda} = m_{d,\Lambda} = m_{l,\Lambda}(m_\pi)$ $\qquad m_\pi = 140\,\mathrm{MeV}$

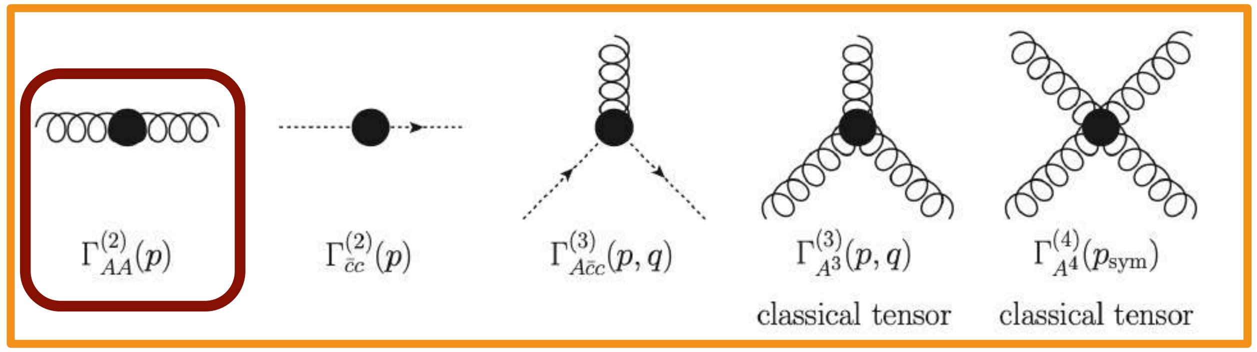
2+1-flavour QCD

(i) $\alpha_{s,\Lambda}$

(ii) $m_{u,\Lambda} = m_{d,\Lambda} = m_{l,\Lambda}(m_\pi)$ $\qquad m_\pi = 140\,\mathrm{MeV}$

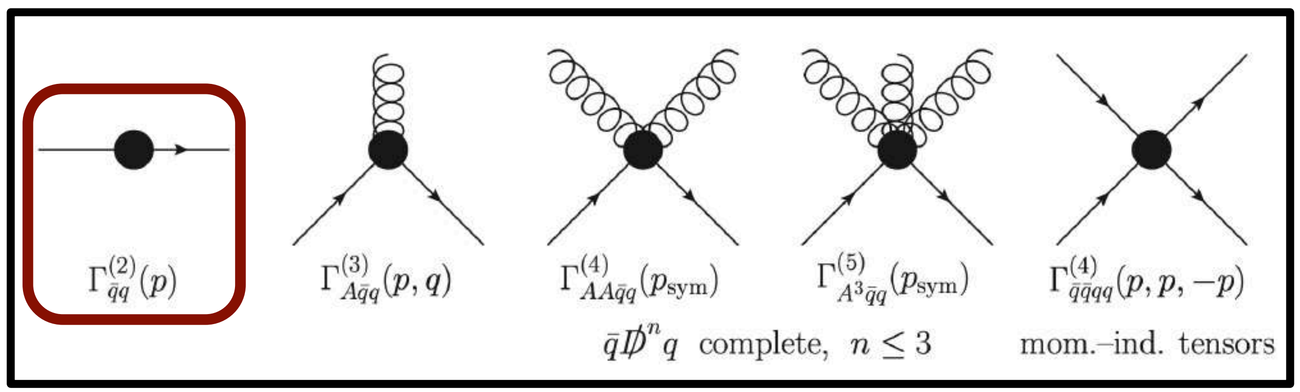
(iii) $\frac{m_{l,\Lambda}}{m_{s,\Lambda}} = 27$

vacuum fQCD: current set of correlation functions

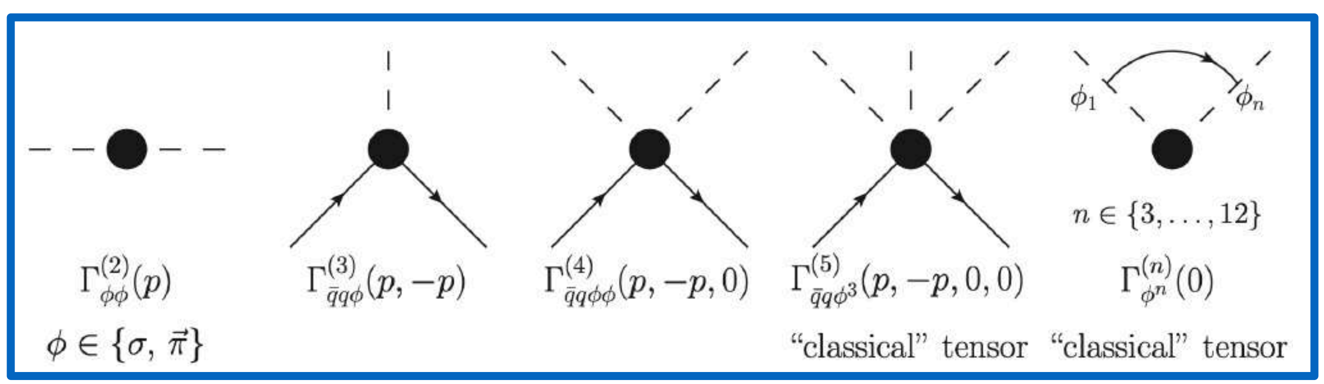


glue sector

FRG



quark-glue sector



quark-meson sector

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006,
PRD 97 (2018) 054015

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

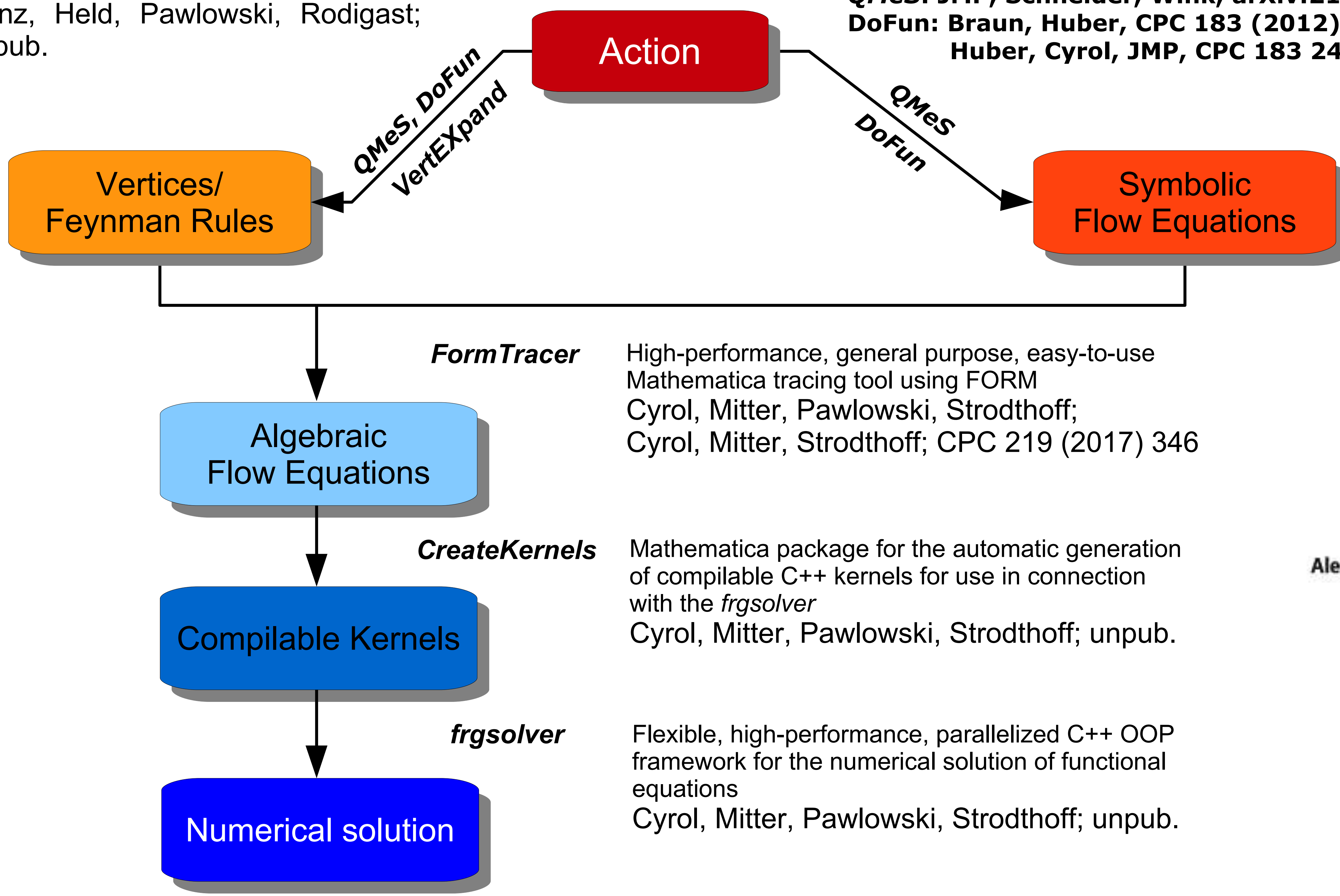
Extension, work in progress:

Fu, Huang, Ihssen, JMP, Sattler, Schneider, Tan, Wink

fQCD: workflow

VertEXpand
Mathematica package for the derivation of vertices from a given action using FORM
Denz, Held, Pawlowski, Rodigast; unpub.

QMeS, DoFun
Mathematica package for the derivation of functional equations
***QMeS*: JMP, Schneider, Wink, arXiv:2102.01410**
DoFun: Braun, Huber, CPC 183 (2012) 1290
Huber, Cyrol, JMP, CPC 183 248 (2020) 107058



FormTracer High-performance, general purpose, easy-to-use Mathematica tracing tool using FORM
Cyrol, Mitter, Pawlowski, Strodthoff;
Cyrol, Mitter, Strodthoff; CPC 219 (2017) 346

CreateKernels Mathematica package for the automatic generation of compilable C++ kernels for use in connection with the *frgsolver*
Cyrol, Mitter, Pawlowski, Strodthoff; unpub.

frgsolver Flexible, high-performance, parallelized C++ OOP framework for the numerical solution of functional equations
Cyrol, Mitter, Pawlowski, Strodthoff; unpub.

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

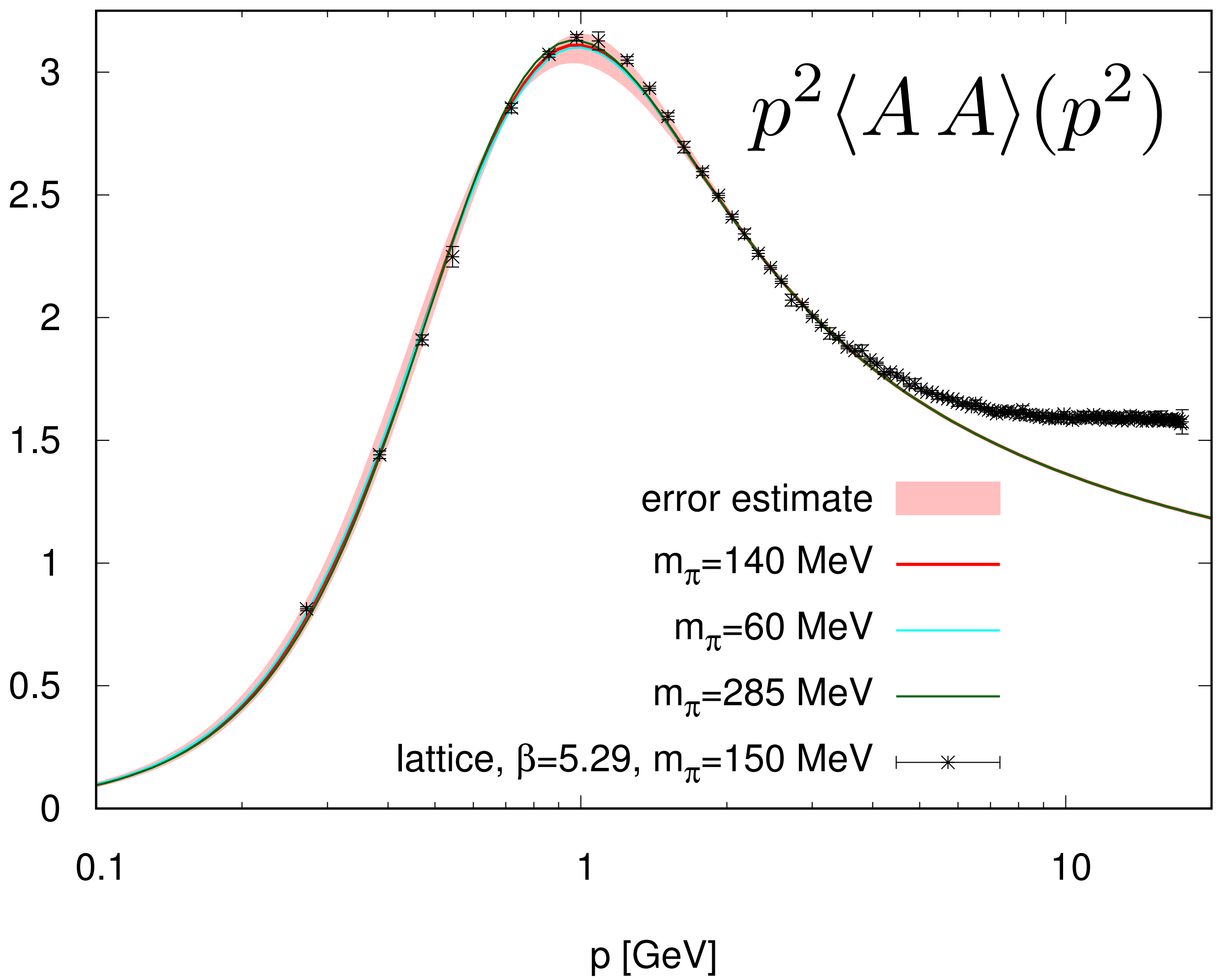
Alexander von Humboldt Stiftung/Foundation

FWF Der Wissenschaftsfonds.

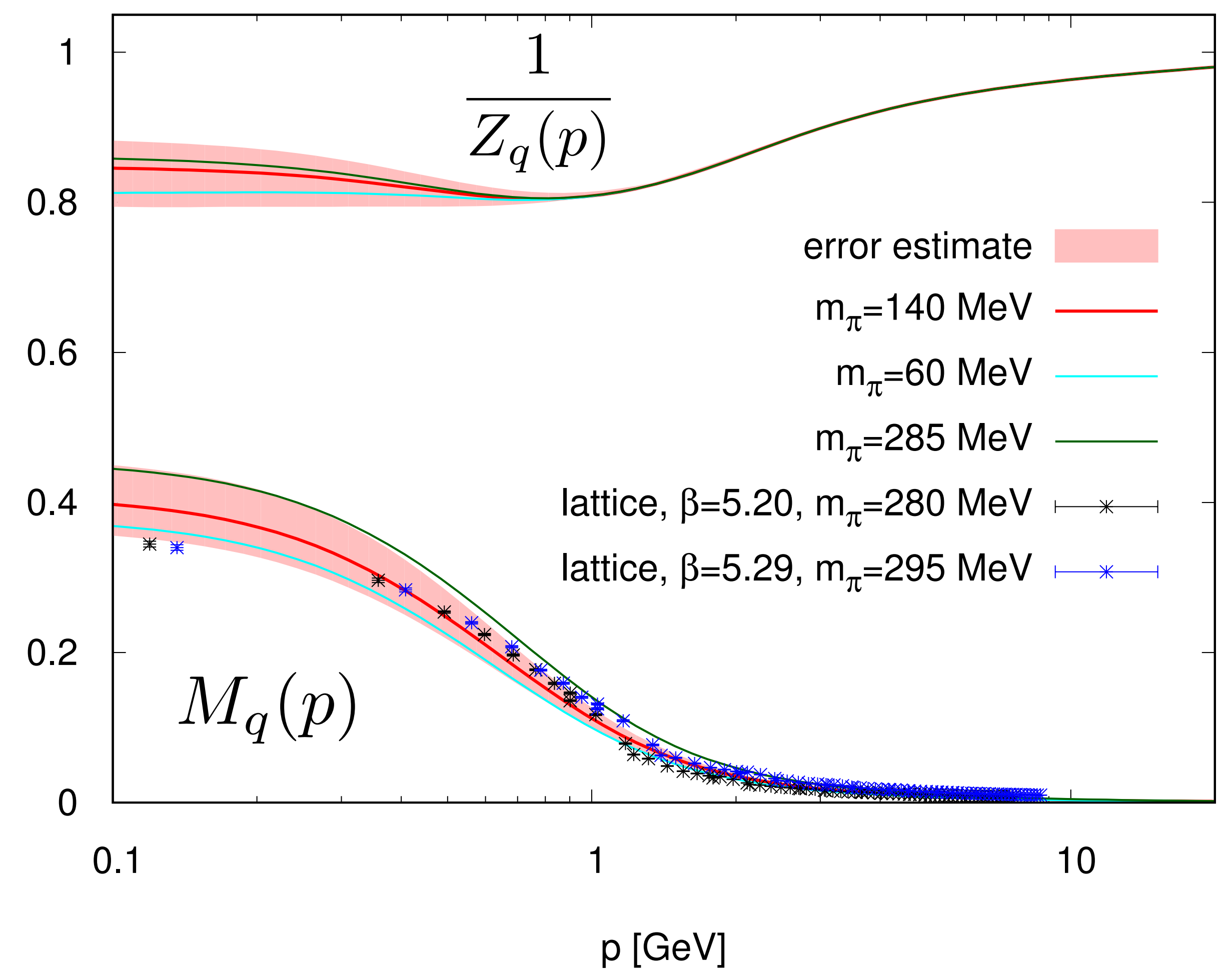
erc European Research Council
Established by the European Commission

vacuum QCD: Euclidean propagators

Two-flavour QCD



$$\frac{1}{Z_q(p)}\,\frac{1}{i\not{p}+M_q(p)}$$

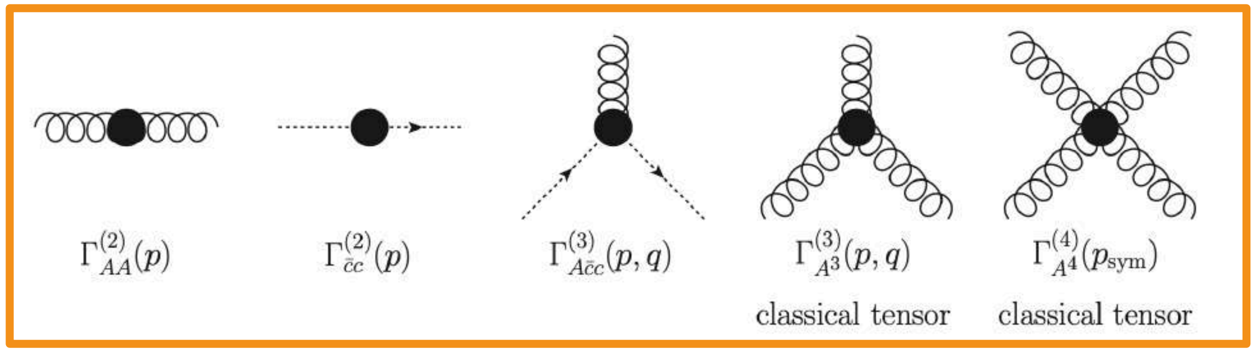


lattice, e.g.: Oliviera et al, Acta Phys.Polon.Supp. 9 (2016) 363
Sternbeck et al, PoS LATTICE2016 (2017)
A. Athenodorou et al, PLB 761 (2016) 444

simple correlations

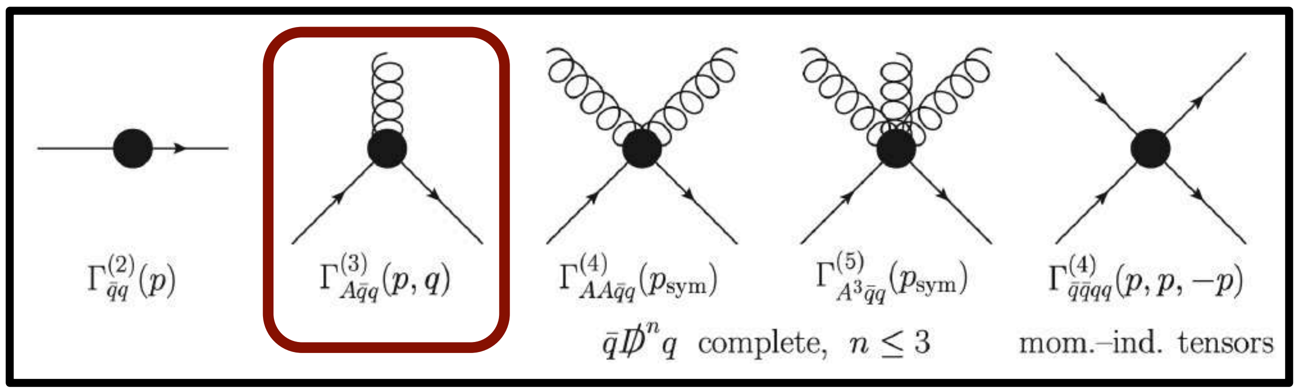
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

vacuum QCD: current set of correlation functions



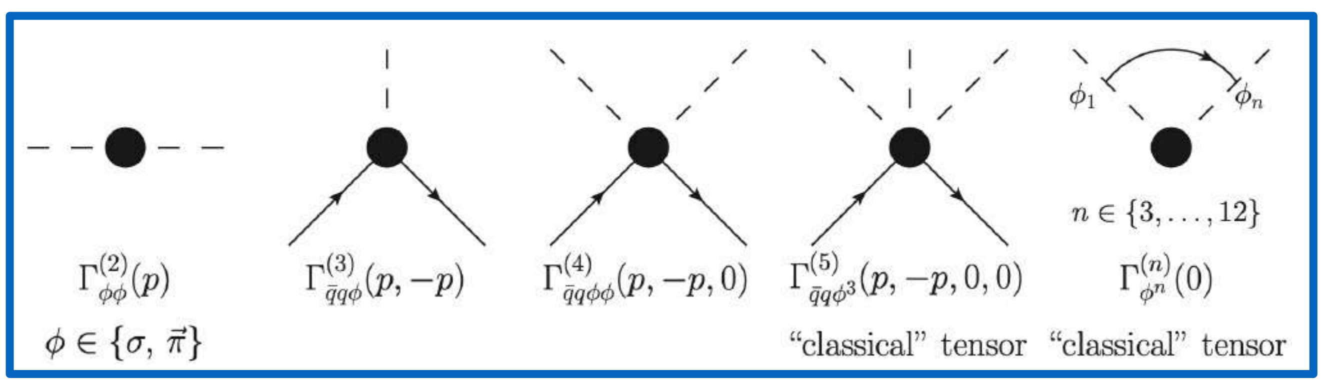
glue sector

FRG



Eight tranverse tensor structures

quark-glue sector



quark-meson sector

Aiming at apparent convergence

Extension, work in progress:

Fu, Huang, Ihssen, JMP, Schneider, Tan, Wink

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006,
PRD 97 (2018) 054015

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Quark-gluon vertex

$$\left[\Gamma^{(3)}_{\bar{q}qA}\right]^a_\mu (p,q) = 1^{\text{flav}}_{2\times 2}\, T^a \sum_{i=1}^{8} \lambda_i(p,q) \left[\mathcal{T}^{(i)}_{\bar{q}qA}\right]_\mu (p,q)$$

covariant expansion scheme

$\bar{q}\not{D}q :$ $\quad \left[\mathcal{T}^{(1)}_{\bar{q}qA}\right]_\mu (p,q) = -i\,\gamma_\mu$

$\bar{q}\not{D}^2 q :$ $\quad \left[\mathcal{T}^{(2)}_{\bar{q}qA}\right]_\mu (p,q) = (p-q)_\mu\, 1_{4\times 4}$

$$\left[\mathcal{T}^{(3)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}-\not{q})\gamma_\mu$$

$$\left[\mathcal{T}^{(4)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}+\not{q})\gamma_\mu$$

$\bar{q}\not{D}^3 q :$ $\quad \left[\mathcal{T}^{(5)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}+\not{q})(p-q)_\mu$

$$\left[\mathcal{T}^{(6)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}-\not{q})(p-q)_\mu$$

$$\left[\mathcal{T}^{(7)}_{\bar{q}qA}\right]_\mu (p,q) = \frac{i}{2}[\not{p},\not{q}]\gamma_\mu$$

Aiming at apparent convergence

quenched: **Mitter, JMP, Strodthoff, PRD 91 (2015) 054035**

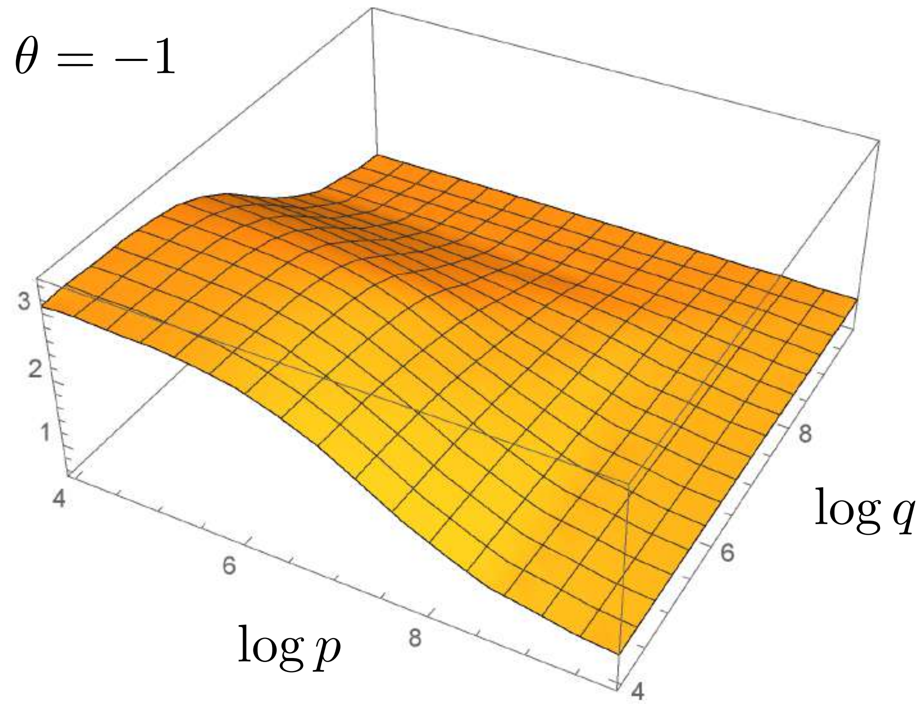
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

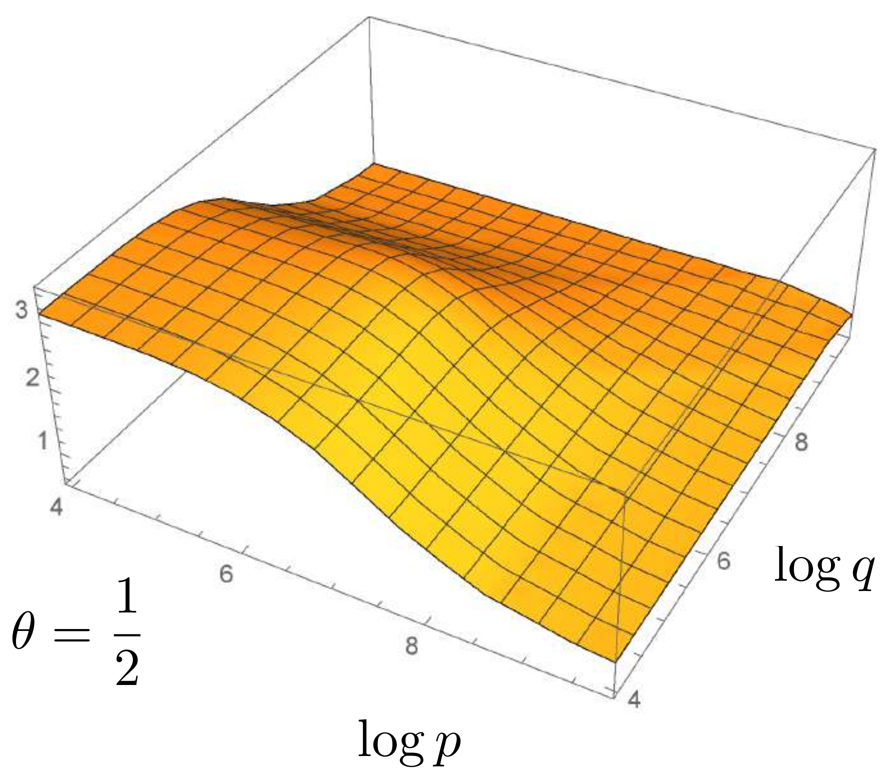
Quark-gluon vertex

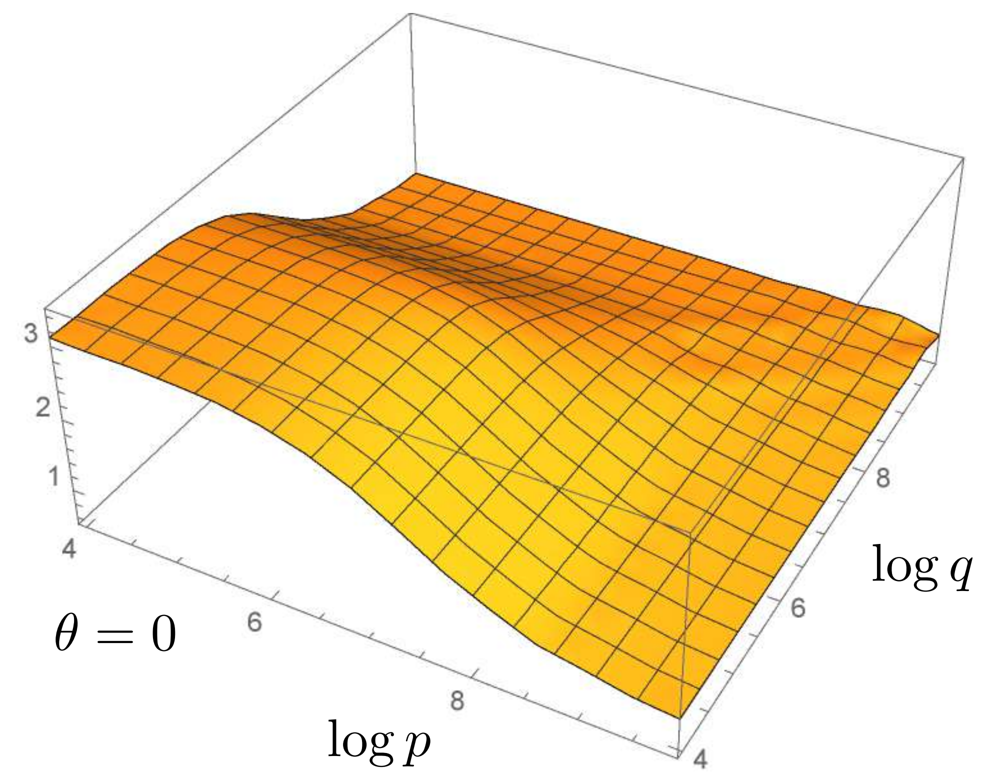
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV

$\lambda_1(p,q)$







Aiming at apparent convergence

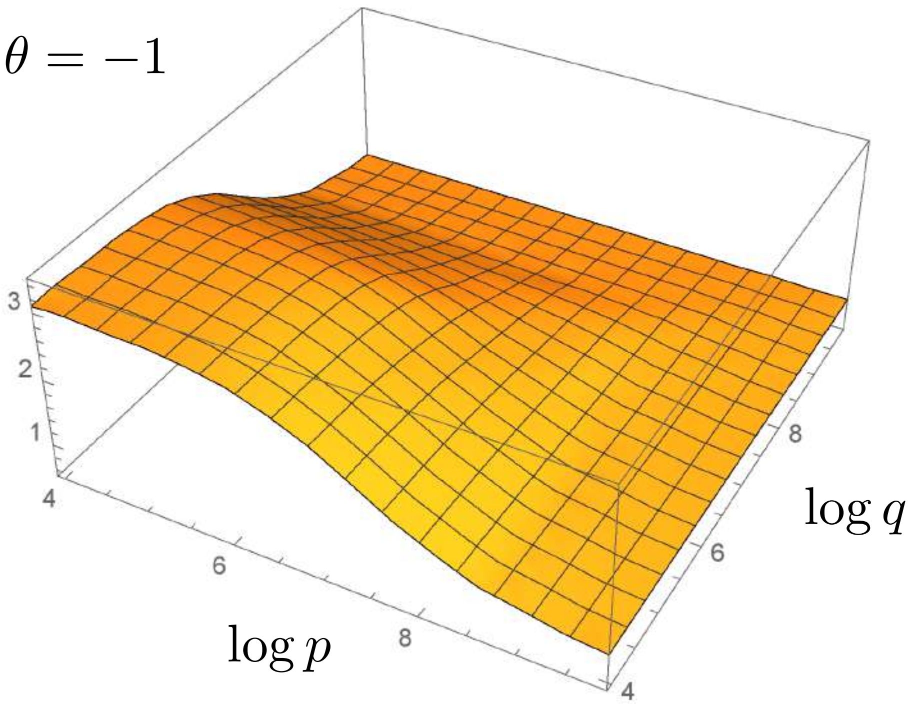
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Quark-gluon vertex

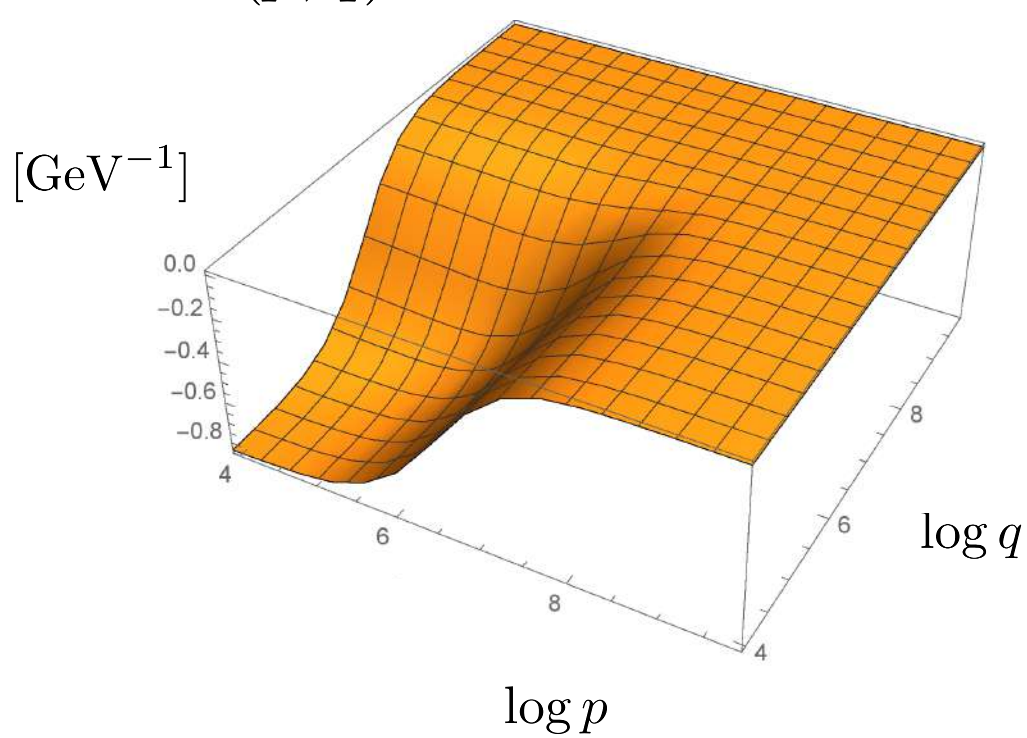
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV

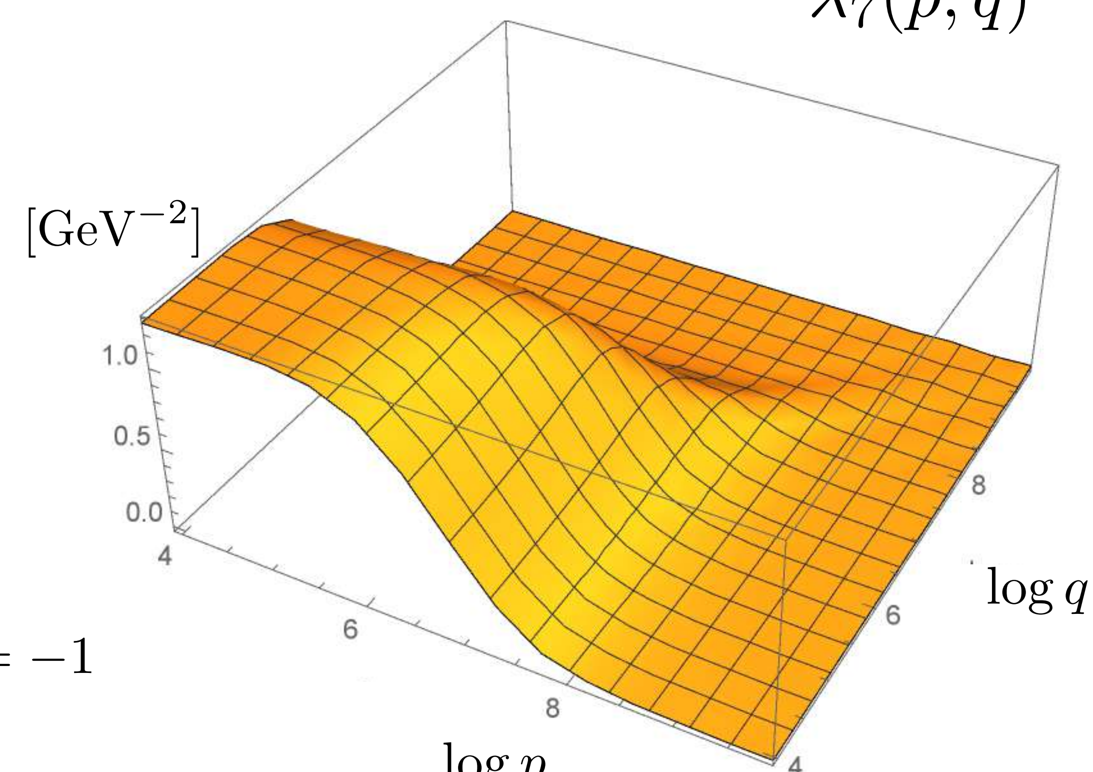
$\lambda_1(p,q)$



$\lambda_4(p,q)$



$\lambda_7(p,q)$



$\theta = -1$

Aiming at apparent convergence

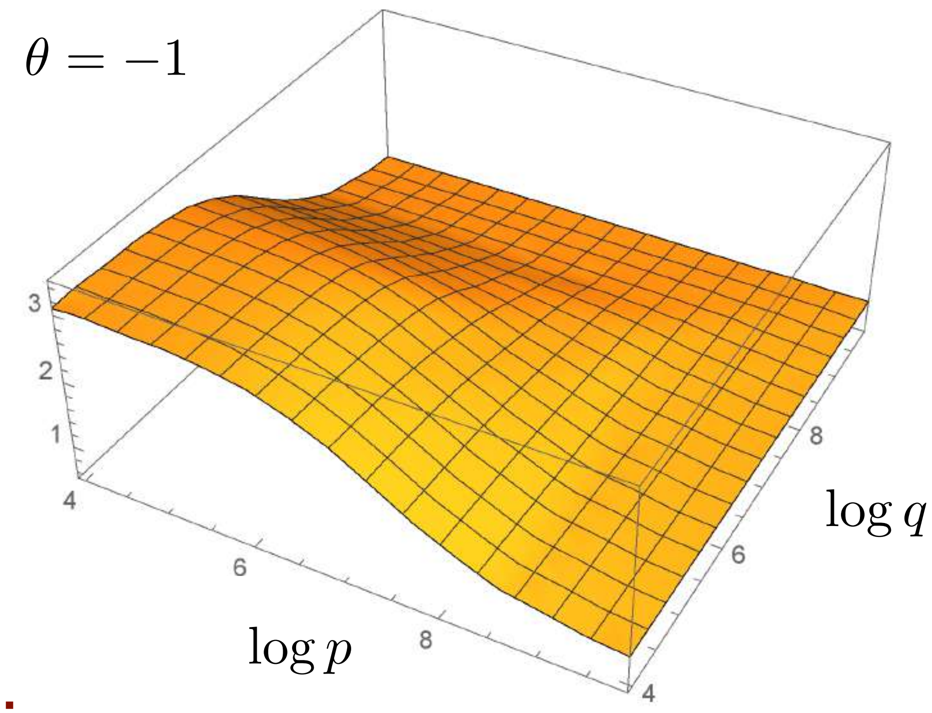
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

vacuum QCD: Quark-gluon vertex

Two-flavour QCD

$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV



$\lambda_1(p,q)$

All (eight) tensor structures!

simple correlations

up-to-date 1st principles works:

FunMethods:

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035
Gao, Papavassiliou, JMP, 2102.13053

Williams, EPJ A51 (2015) 57
Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, Fischer, Heupel, PRD 93 (2016) 034026
Contant, Huber, Fischer, Welzbacher, Williams, APP.Supp. 11 (2018) 483

Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027
Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 95 (2017) 031501
Aguilar, Cardona, Ferreira, Papavassiliou, PRD 96 (2017) 014029
PRD 98 (2018) 014002

Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012

Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

lattice, e.g.: Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363

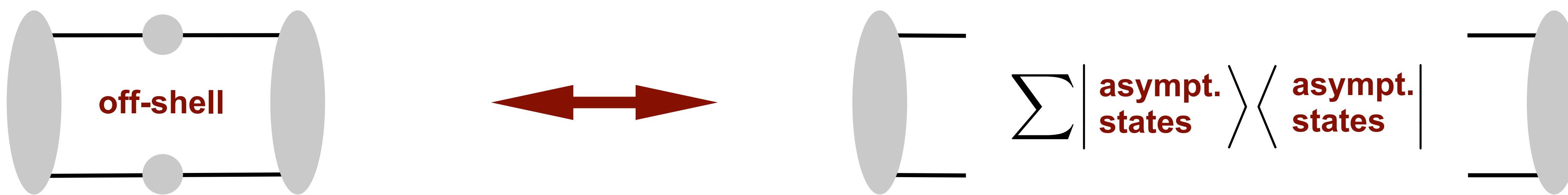
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Three remarks on Functional Methods for QCD

- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$

on-shell



pressure, trace anomaly,
fluctuations, volume flucs., ...

e.g. hadron resonances

- **gauge fixing = parameterisation**

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

Consequences

I: simple correlations

II: Difficult access to some observables

'No free lunch theorem'

- 'Your mean field is not my mean field'

$$\frac{\delta S_{\rm cl}[\phi]}{\delta\phi}\bigg|_{\phi=\bar\phi} = 0$$

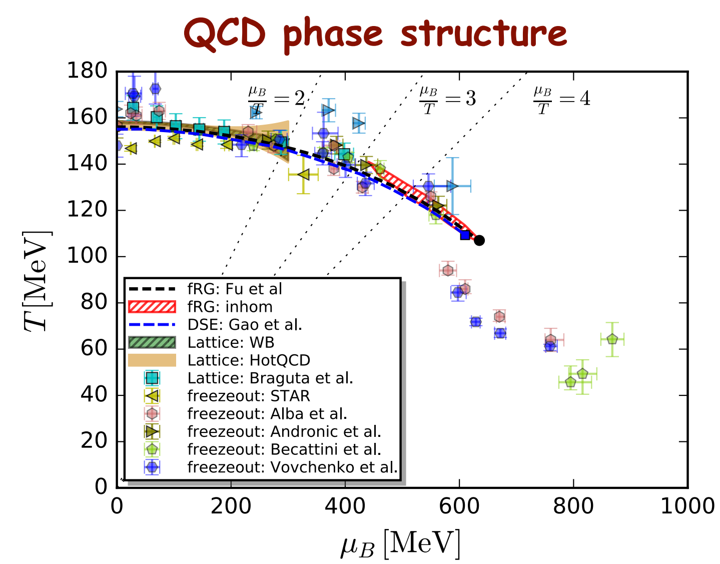
$$\frac{\delta\Gamma[\phi]}{\delta\phi}\bigg|_{\phi=\bar\phi_{\rm quant}} = 0$$

full quantum equation of motion

Outline

- **QCD from functional methods**

- **QCD phase structure**



- **Fluctuations of conserved charges**

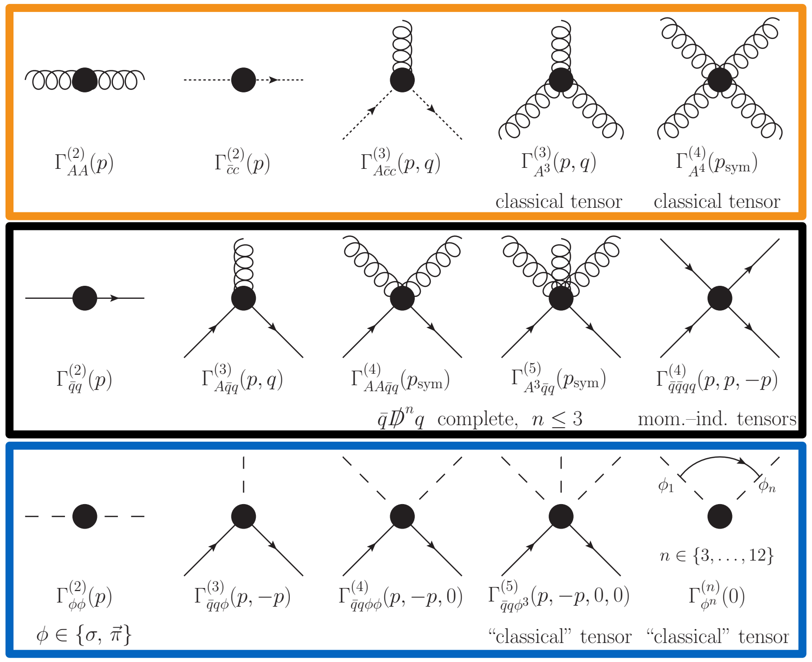
- **Summary & outlook**

QCD at finite density

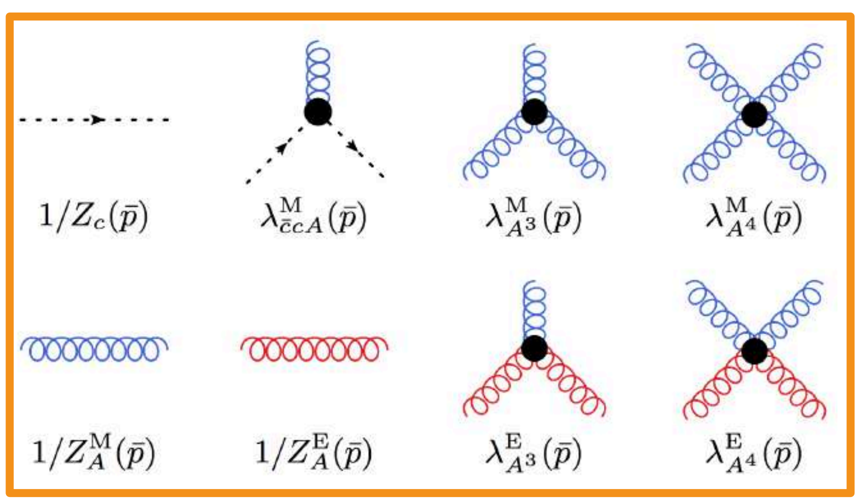
Approximation scheme

Input

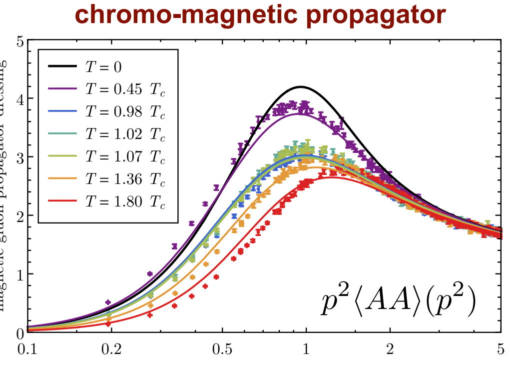
two flavour vacuum QCD



finite T Yang-Mills



chromo-magnetic propagator



Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

Output

$$\boxed{\partial_t \Delta\Gamma^{(n)}} = \left[\partial_t\Gamma^{(n)}\right]_{\text{Input}} + \Delta\text{Flow}^{(n)}\left[\left\{\left[\Gamma^{(m)}\right]_{\text{Input}}\right\}, \boxed{\left\{\Delta\Gamma^{(m)}\right\}}\right]$$

vacuum: Braun, Fister, Pawlowski, Rennecke, PRD 94, 034016 (2016)

$$\Gamma^{(n)} = \left[\Gamma^{(n)}\right]_{\text{Input}} + \boxed{\Delta\Gamma^{(n)}}$$

Fu, JMP, Rennecke, PRD 101, (2020) 054032

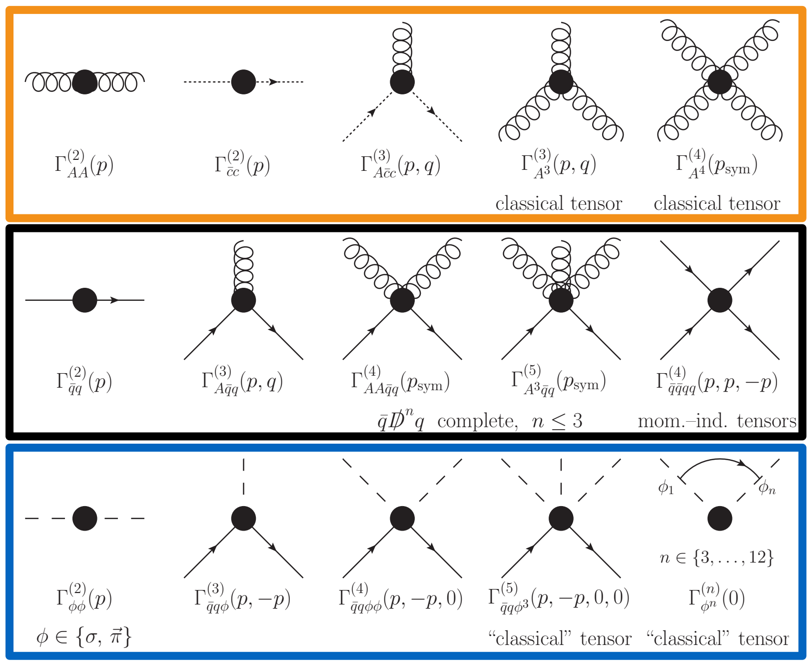
fRG

QCD at finite density

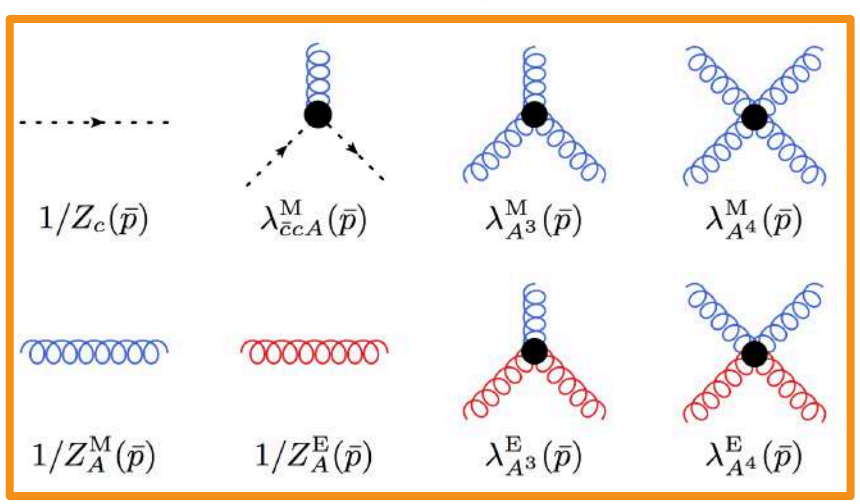
Approximation scheme

Input

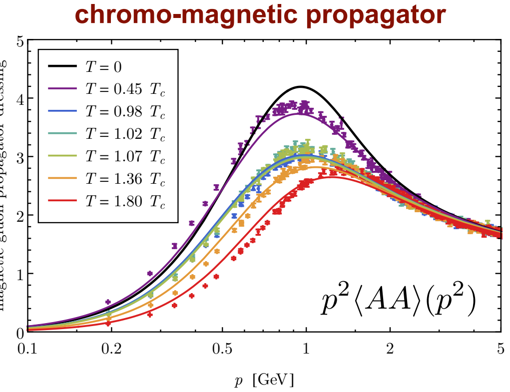
two flavour vacuum QCD



finite T Yang-Mills



chromo-magnetic propagator



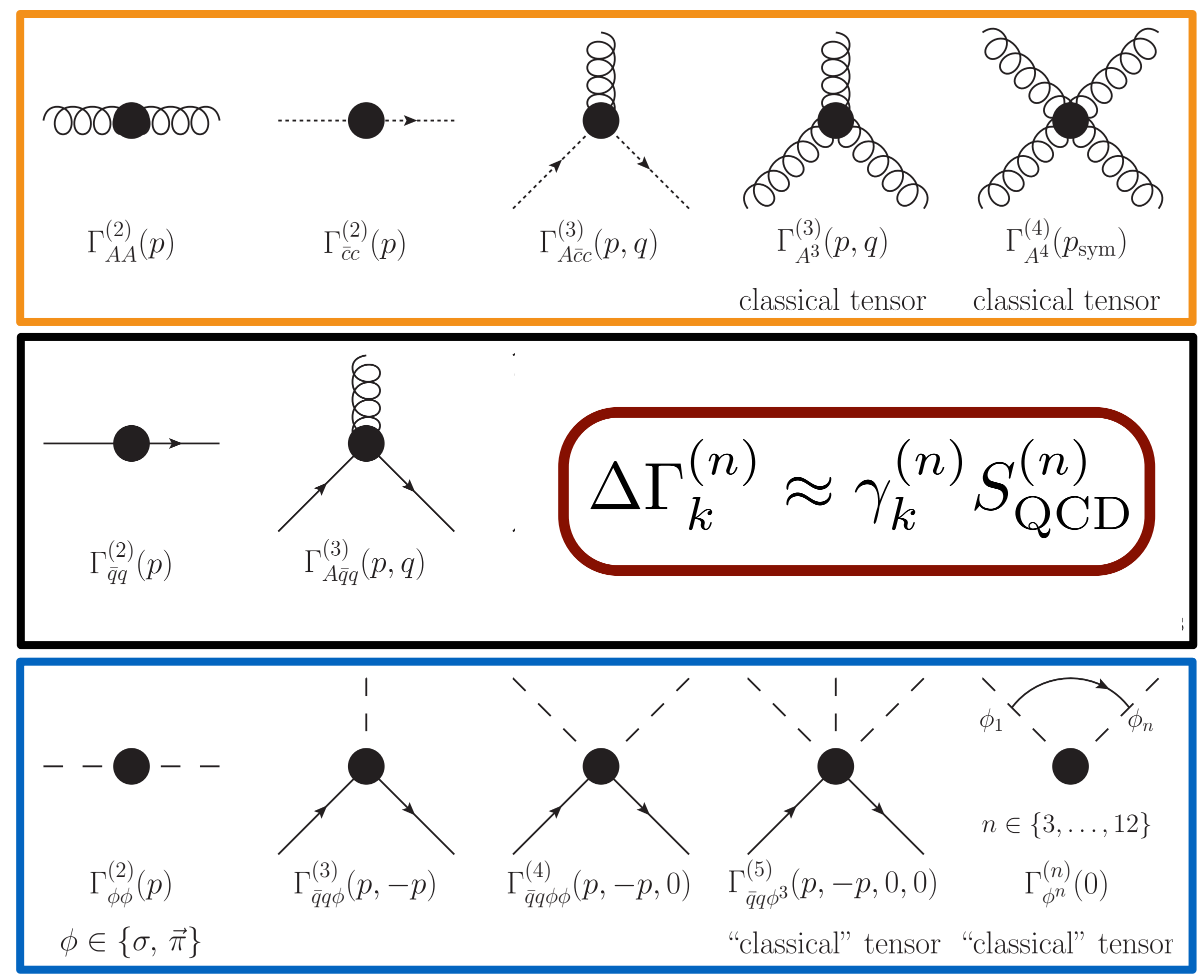
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

Output

$$\boxed{\partial_t \Delta\Gamma^{(n)}} = \Big[\partial_t \Gamma^{(n)}\Big]_{\rm Input} + \Delta \mathrm{Flow}^{(n)}\left[\left\{\Big[\Gamma^{(m)}\Big]_{\rm Input}\right\}, \boxed{\left\{\Delta\Gamma^{(m)}\right\}}\right]$$

vacuum: Braun, Fister, Pawlowski, Rennecke, PRD 94, 034016 (2016)

2+1 flavour QCD at finite T & mu



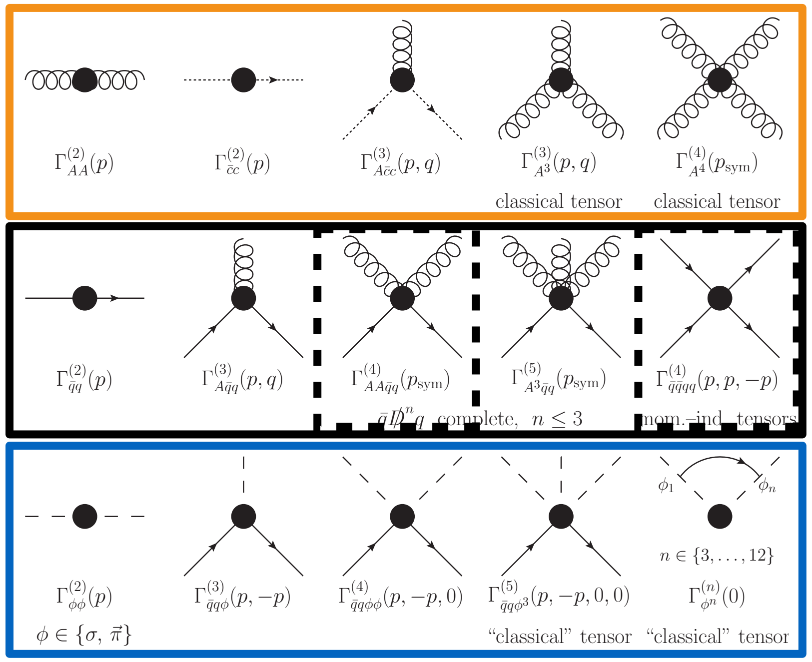
Fu, JMP, Rennecke, PRD 101, (2020) 054032

QCD at finite density

Approximation scheme

Input

two flavour vacuum QCD



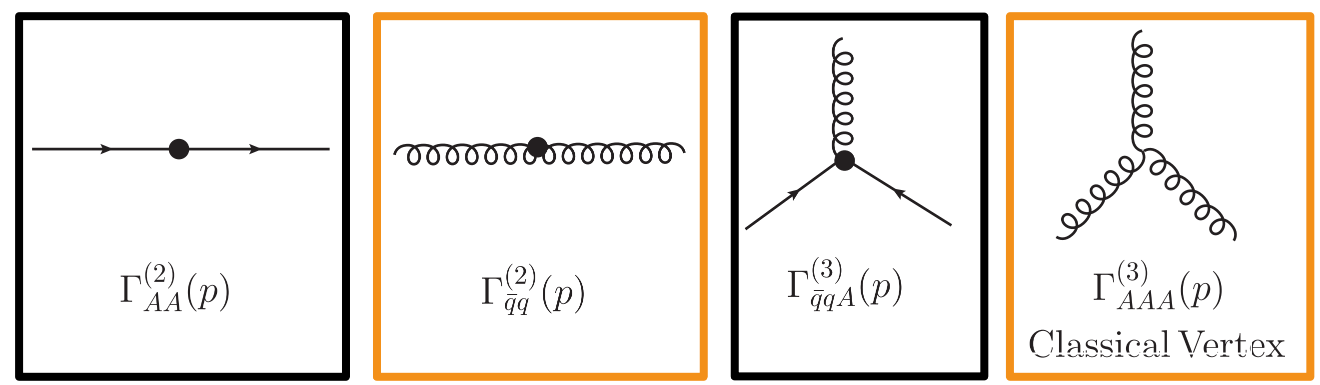
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Output

$$\Delta\Gamma^{(n)} = \left[\Gamma^{(n)}\right]_{\text{Input}} + \Delta\text{DSE}^{(n)}\left[\left\{\left[\Gamma^{(m)}\right]_{\text{Input}}\right\}, \left\{\Delta\Gamma^{(m)}\right\}\right]$$

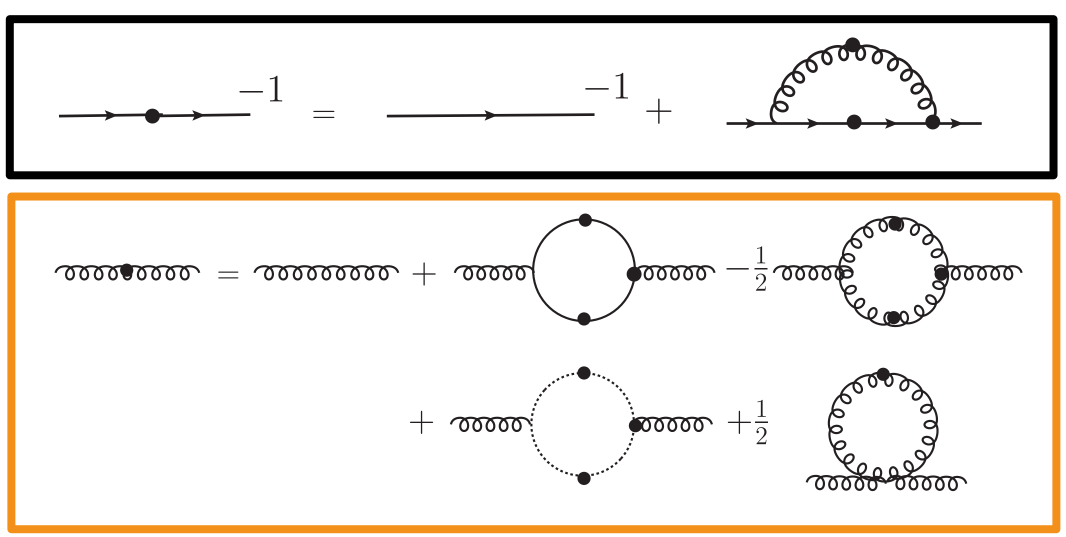
fRG-assisted DSE

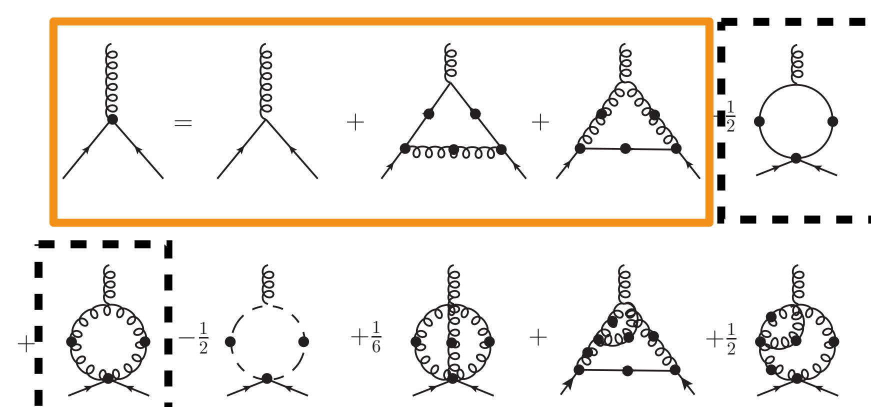
2+1 flavour QCD at finite T & mu



New: all eight tensor structures

System of DSEs



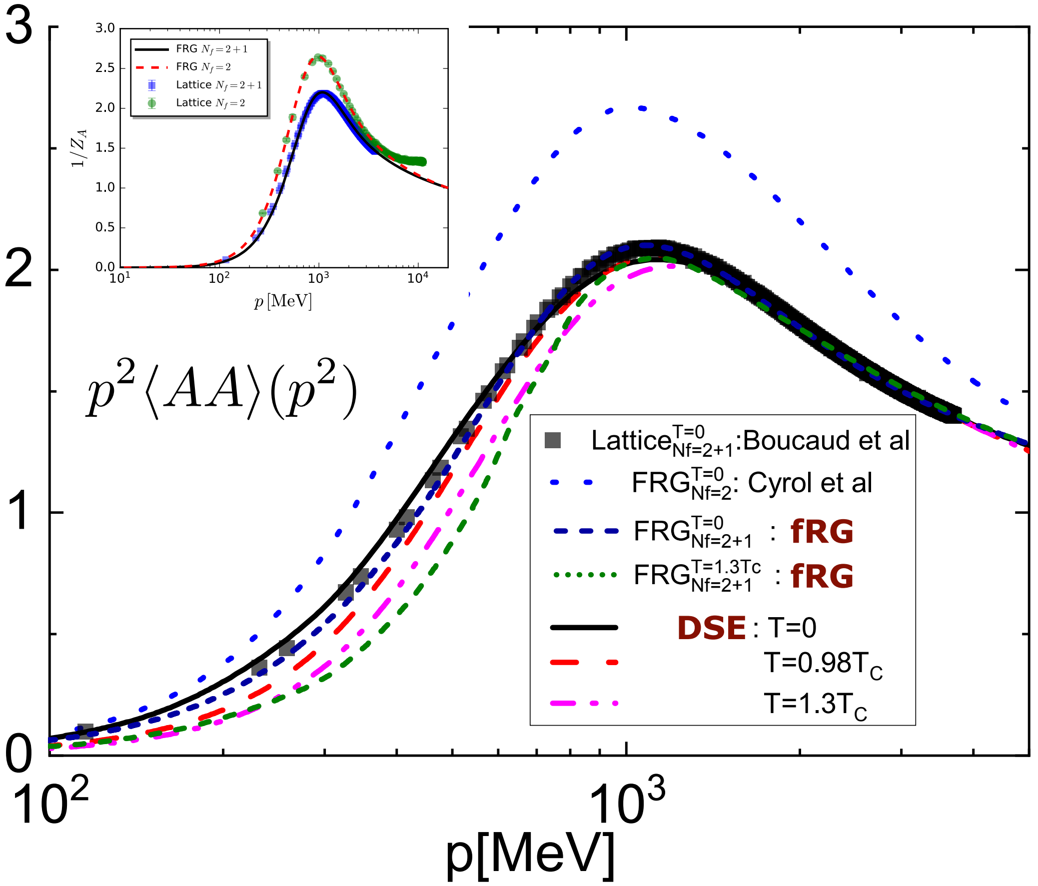


Next step

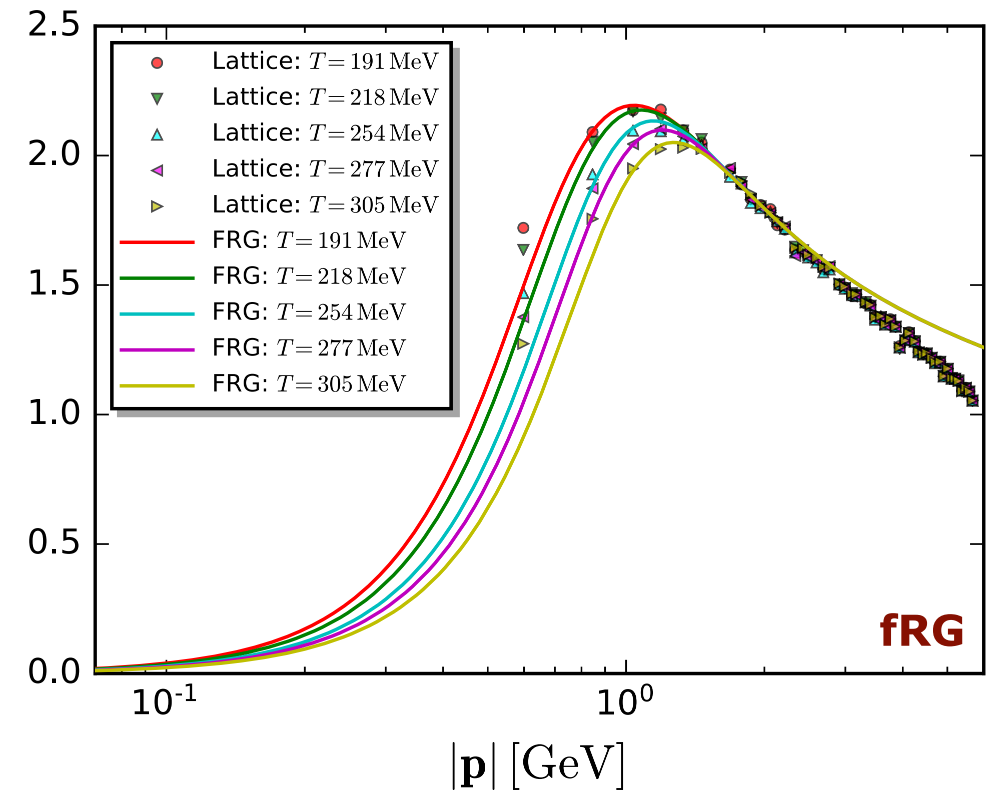
Gao, JMP, PRD 102, (2020) 034027
arXiv:2010.137005

QCD at finite density

Nf=2+1 Gluon and quark benchmark results in the vacuum and at finite T



$$\vec{p}^2\langle AA\rangle(\vec{p}^2)$$



fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PRD 102, (2020) 034027
PLB 820 (2021) 136584

DSE: vacuum & finite T

Fischer, Luecker, PLB 718 (2013) 1036
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

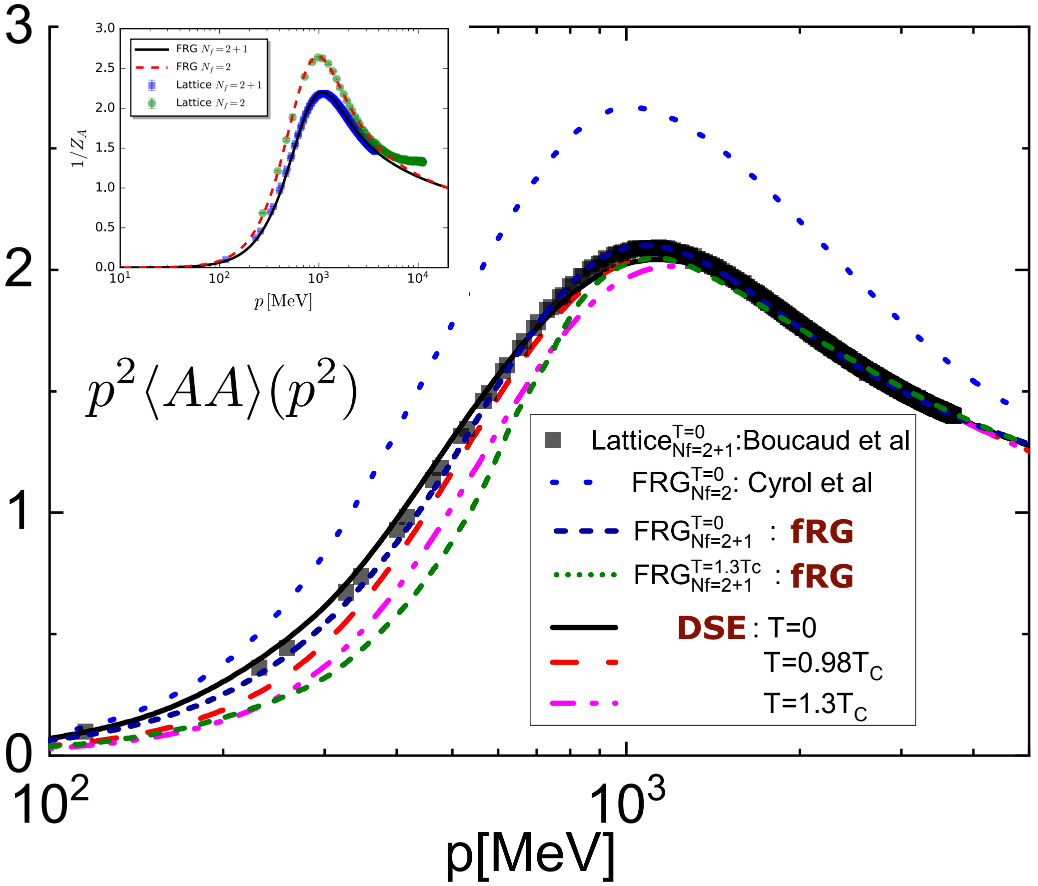
lattice: Nf=2: Sternbeck, Maltman, Müller-Preussker, von Smekal, PoS LATTICE2012, 243 (2012)

Nf=2+1: Aguilar, De Soto, Ferreira, Papavassiliou, Rodriguez-Quintero, Zafeiropoulos, EPJC 80 (2020) 2, 154,
Boucaud, De Soto, Raya, Rodriguez-Quintero, Zafeiropoulos, PRD 98, 114515 (2018)

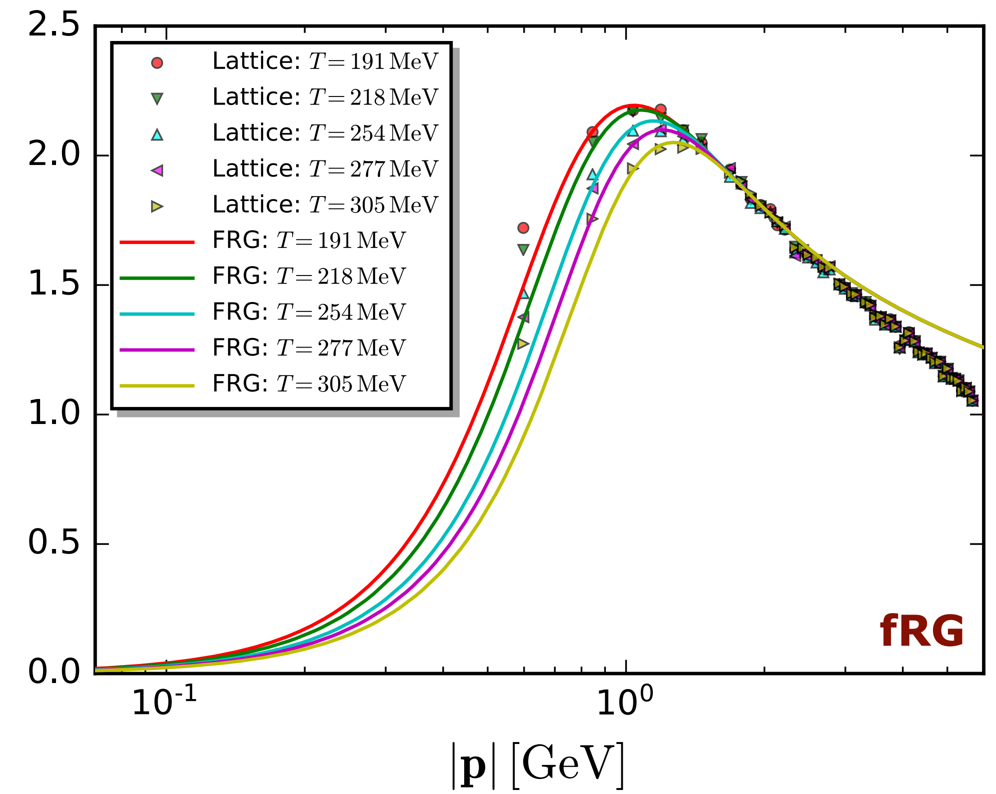
Finite T: Ilgenfritz, JMP, Rothkopf, Trunin, EPJ C78, 127 (201) (Nf=2+1+1)

QCD at finite density

Nf=2+1 Gluon and quark benchmark results in the vacuum and at finite T

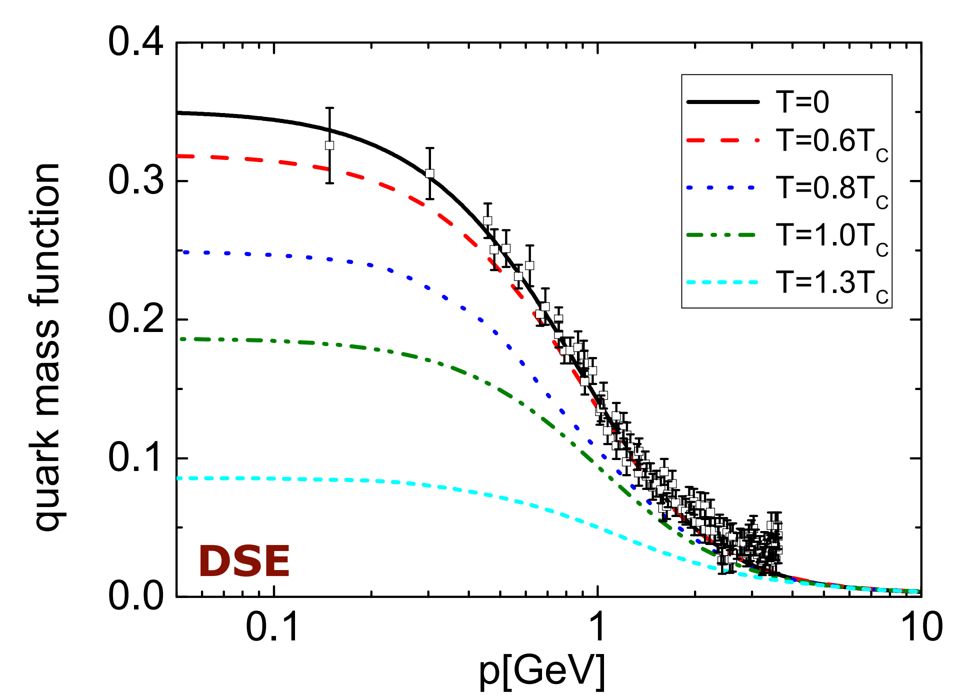


$\vec{p}^2\langle AA\rangle(\vec{p}^2)$



fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**

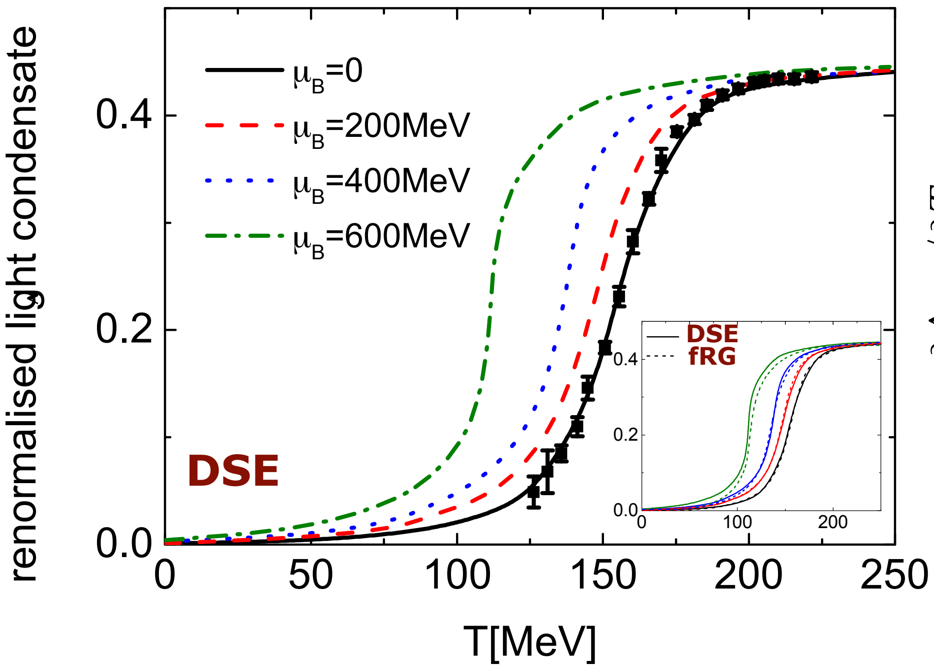
DSE: **Gao, JMP, PRD 102, (2020) 034027**
PLB 820 (2021) 136584

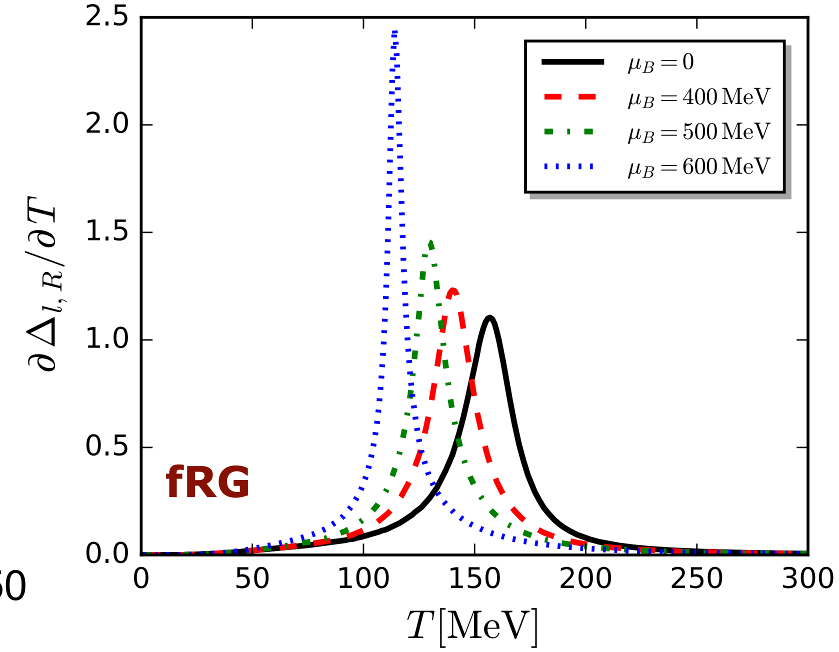


QCD at finite density

Chiral order parameter benchmark results at finite T

renormalised condensate





$$\Delta_{l,R}(T,\mu_B) \simeq \Delta_l(T,\mu_B) - \Delta_l(0,0)$$

$$\Delta_q(T,\mu_B) = \frac{T}{\mathcal{V}}\, m_q^0 \int_x \langle \bar{q}(x) q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

DSE: quark condensates

See also

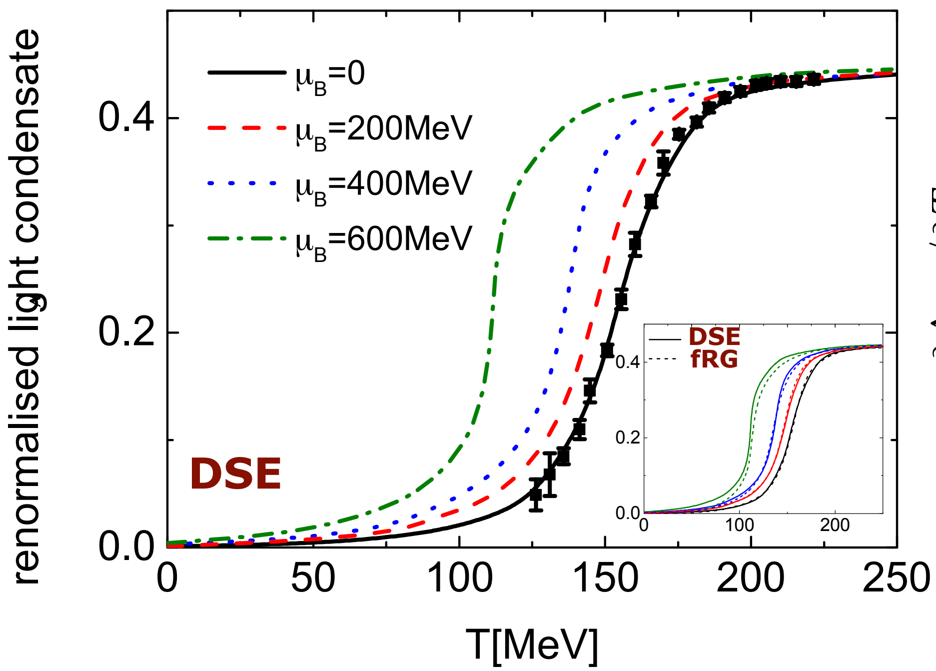
Fischer, Luecker, PLB 718 (2013) 1036
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

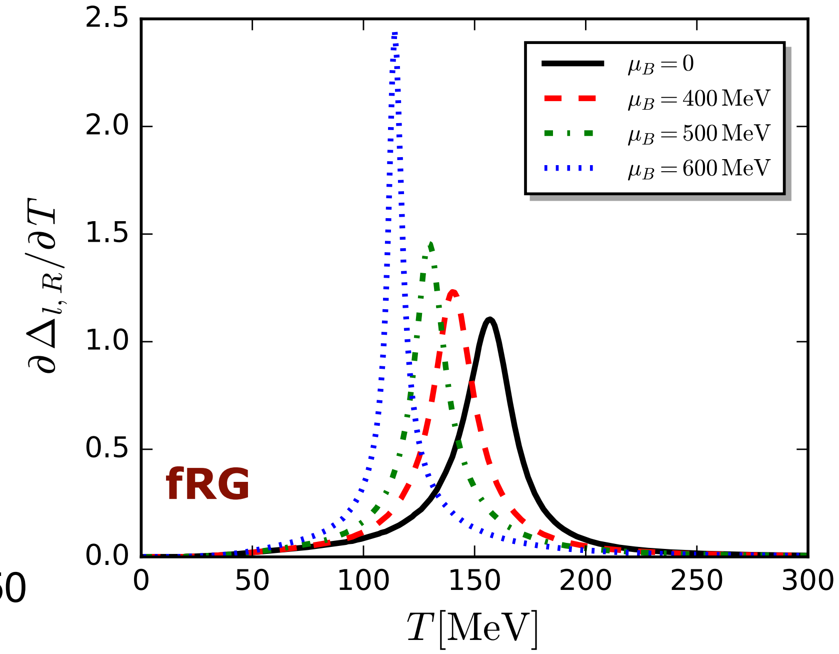
fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**
DSE: **Gao, JMP, PLB 820 (2021) 136584**

QCD at finite density

Chiral order parameter benchmark results at finite T

renormalised condensate



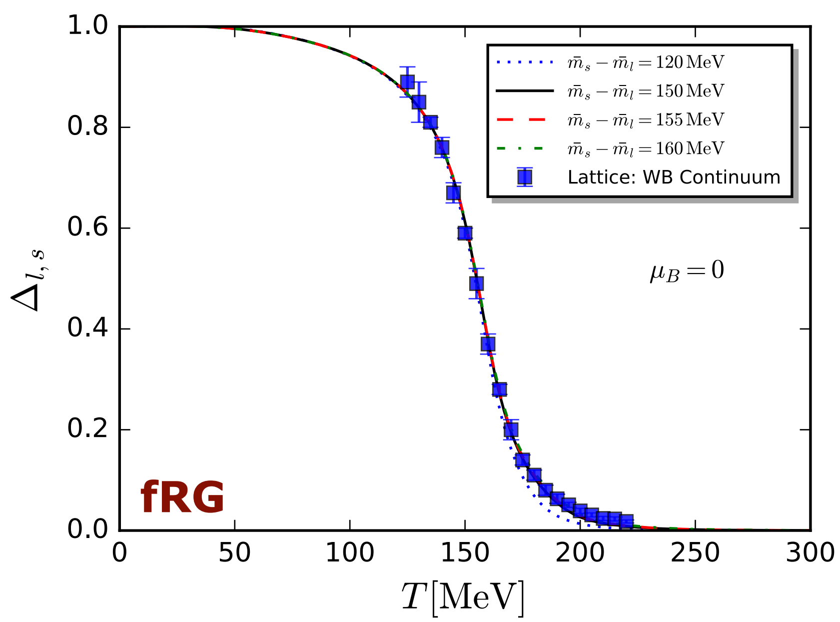


$$\Delta_{l,R}(T,\mu_B) \simeq \Delta_l(T,\mu_B) - \Delta_l(0,0)$$

$$\Delta_q(T,\mu_B) = \frac{T}{\mathcal{V}}\, m_q^0 \int_x \langle \bar{q}(x) q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

reduced condensate

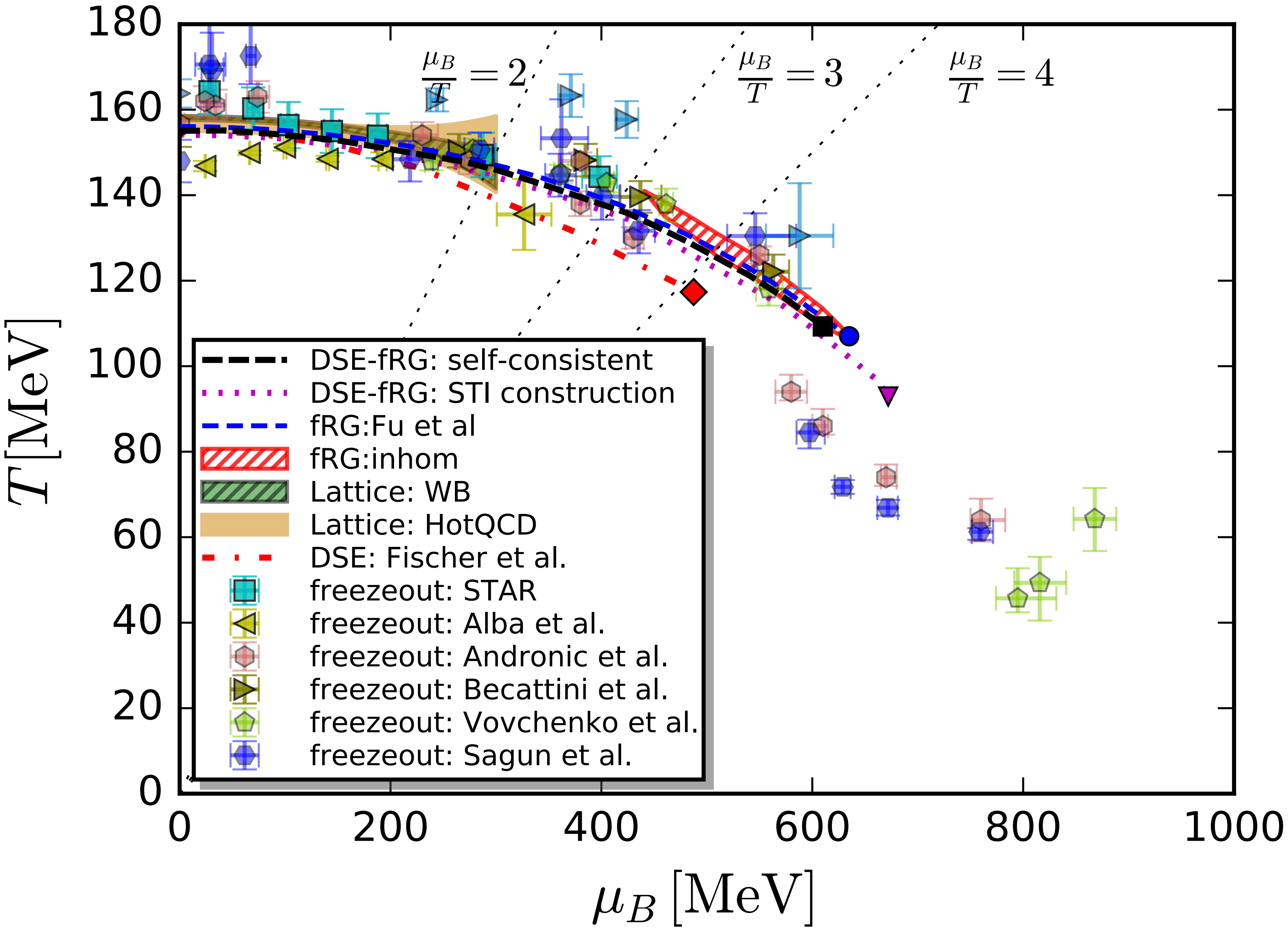


$$\Delta_{l,s}(T,\mu_B) = \frac{\Delta_l(T,\mu_B) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_B)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure



See also

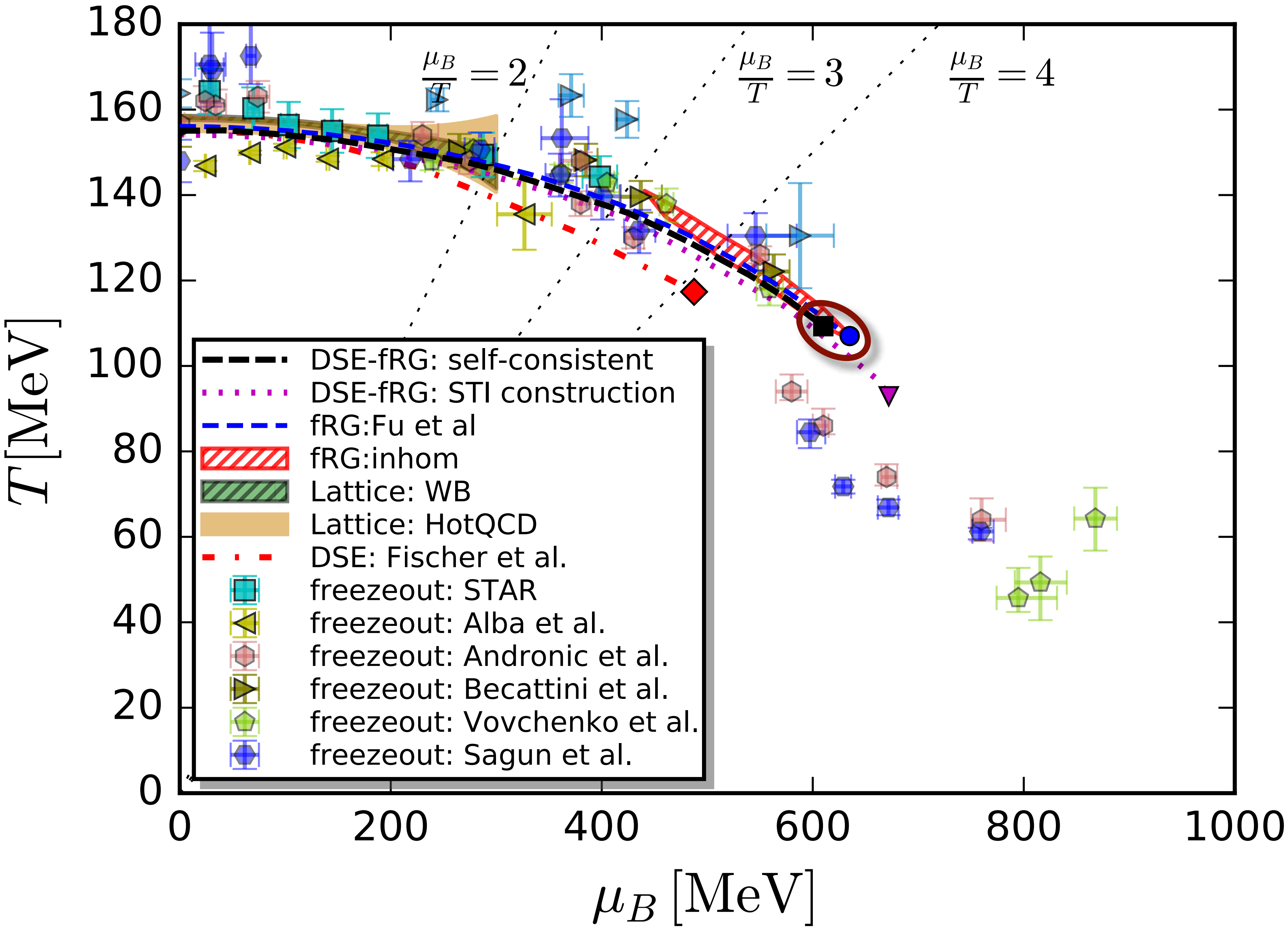
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022

Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure



CEP fRG-DSE

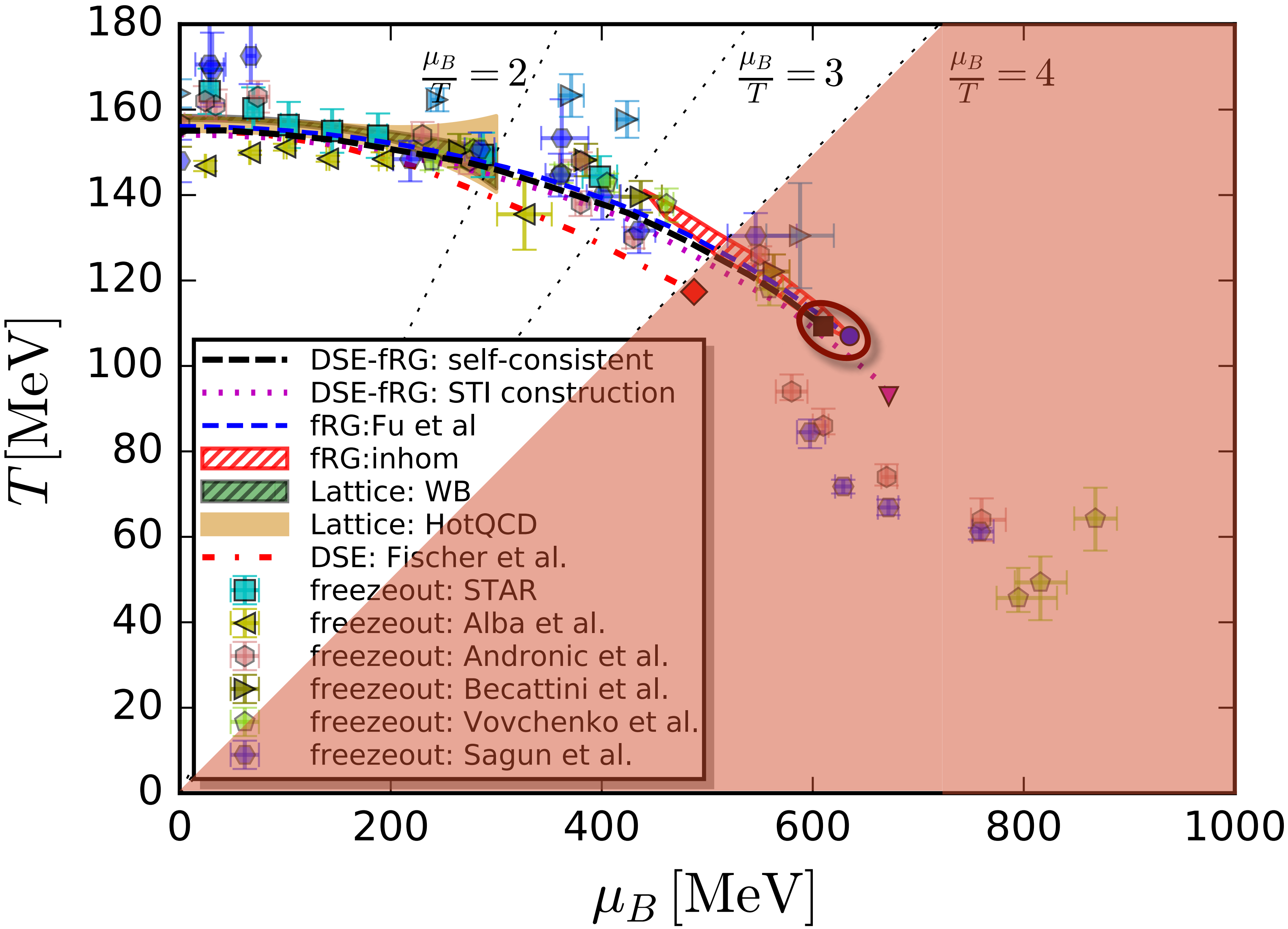
$$(T,\mu_B)_{\mathrm{CEP}} = (107, 635)\,\mathrm{MeV}$$

$$(T,\mu_B)_{\mathrm{CEP}} = (109, 610)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure



curvature fRG-DSE

$$\kappa_{\rm FRG} = 0.0142(2)$$

$$\kappa_{\rm DSE} = 0.0147(5)$$

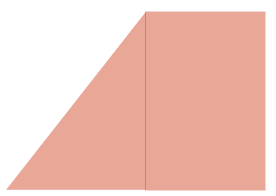
curvature lattice

$$\kappa_{\rm WB} = 0.0149(21)$$

WB, PLB 751 (2015) 559

$$\kappa_{\rm hotQCD} = 0.015(4)$$

hotQCD, PLB 795 (2019) 15



area beyond quantitative reliability bound

$$\mu_{\rm B}/{\rm T} \gtrsim 4$$

CEP fRG-DSE

$$(T,\mu_B)_{\rm CEP} = (107, 635)\,\mathrm{MeV}$$

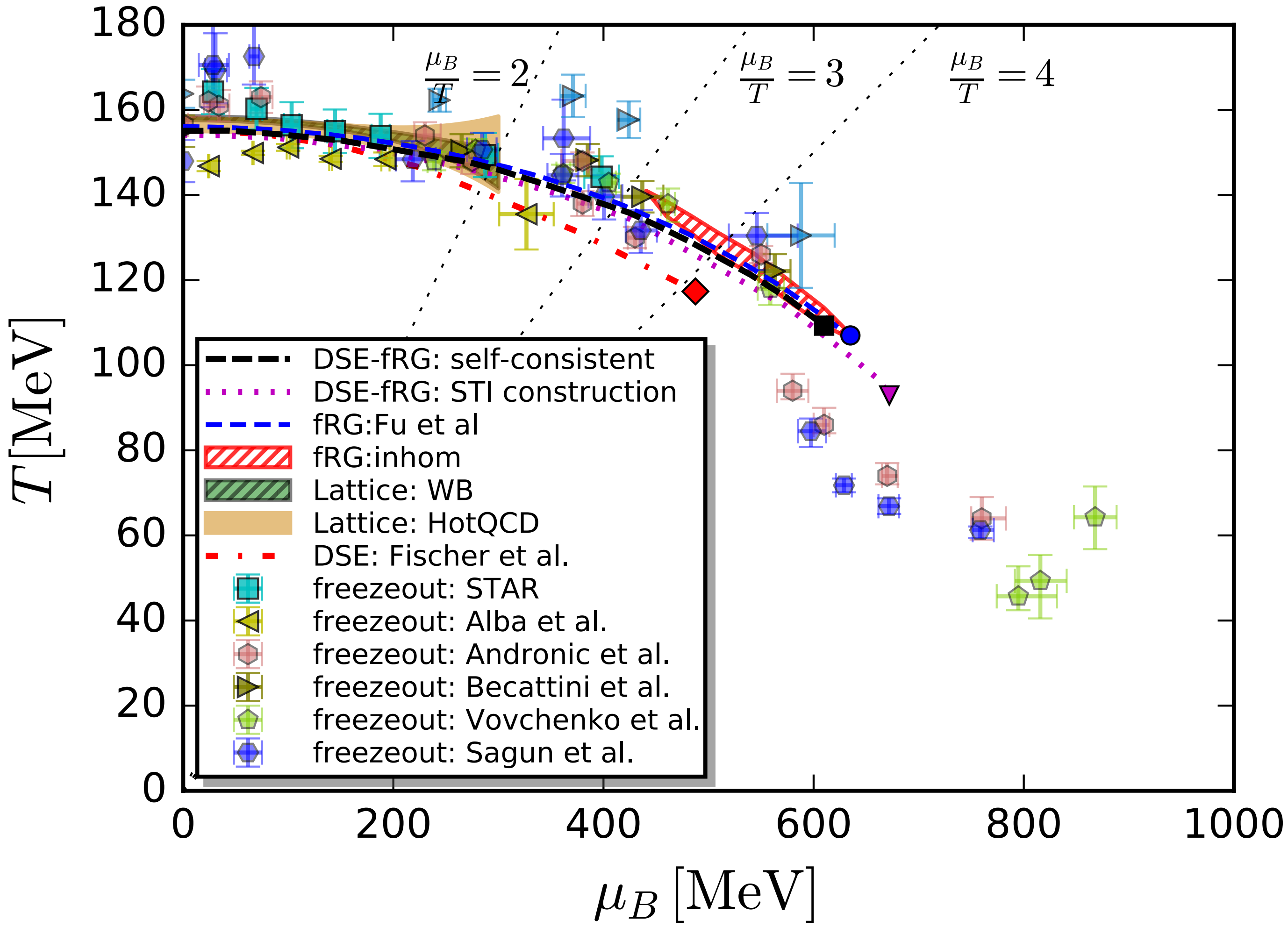
$$(T,\mu_B)_{\rm CEP} = (109, 610)\,\mathrm{MeV}$$

fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

QCD phase structure

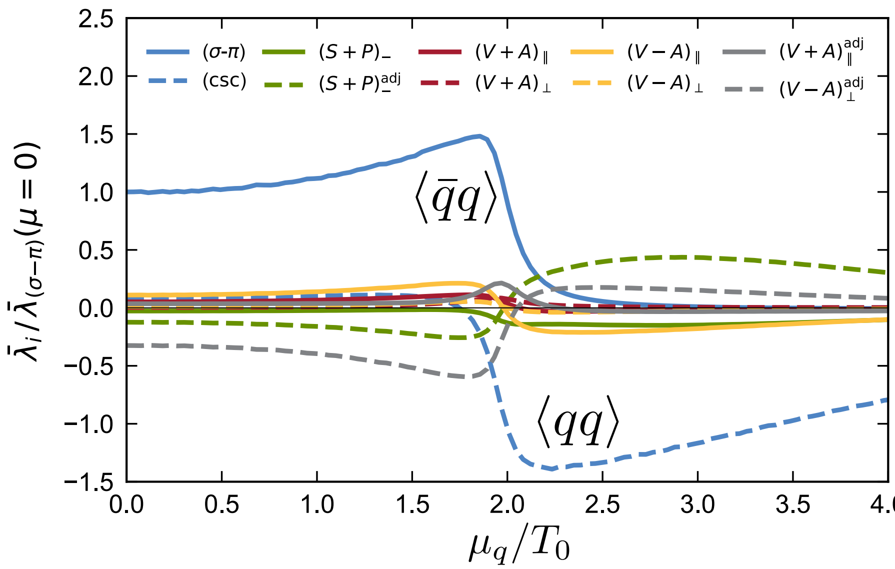
Reliability considerations



I+II → **Fierz-complete computation**

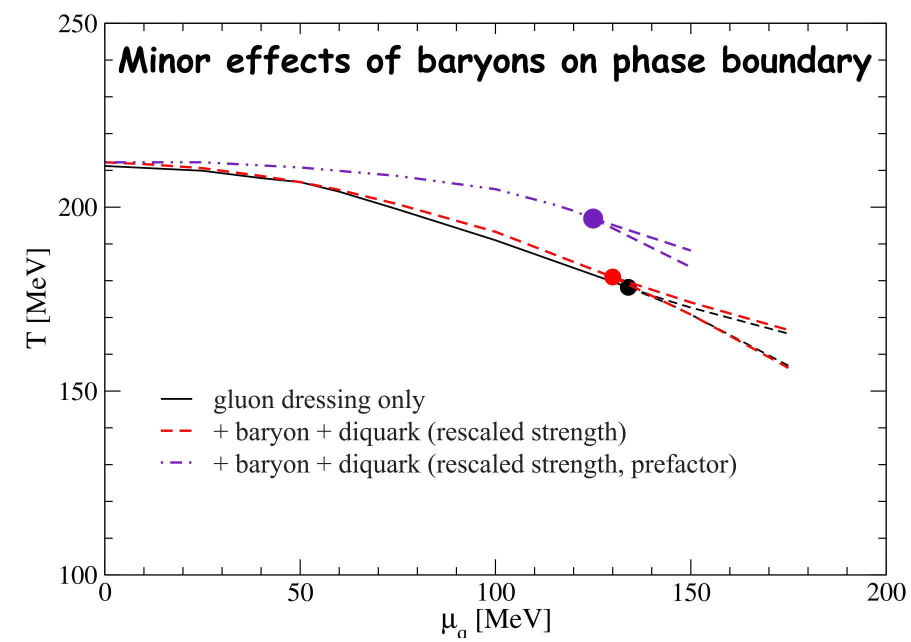
Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



Dominant channels II (DSE)

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



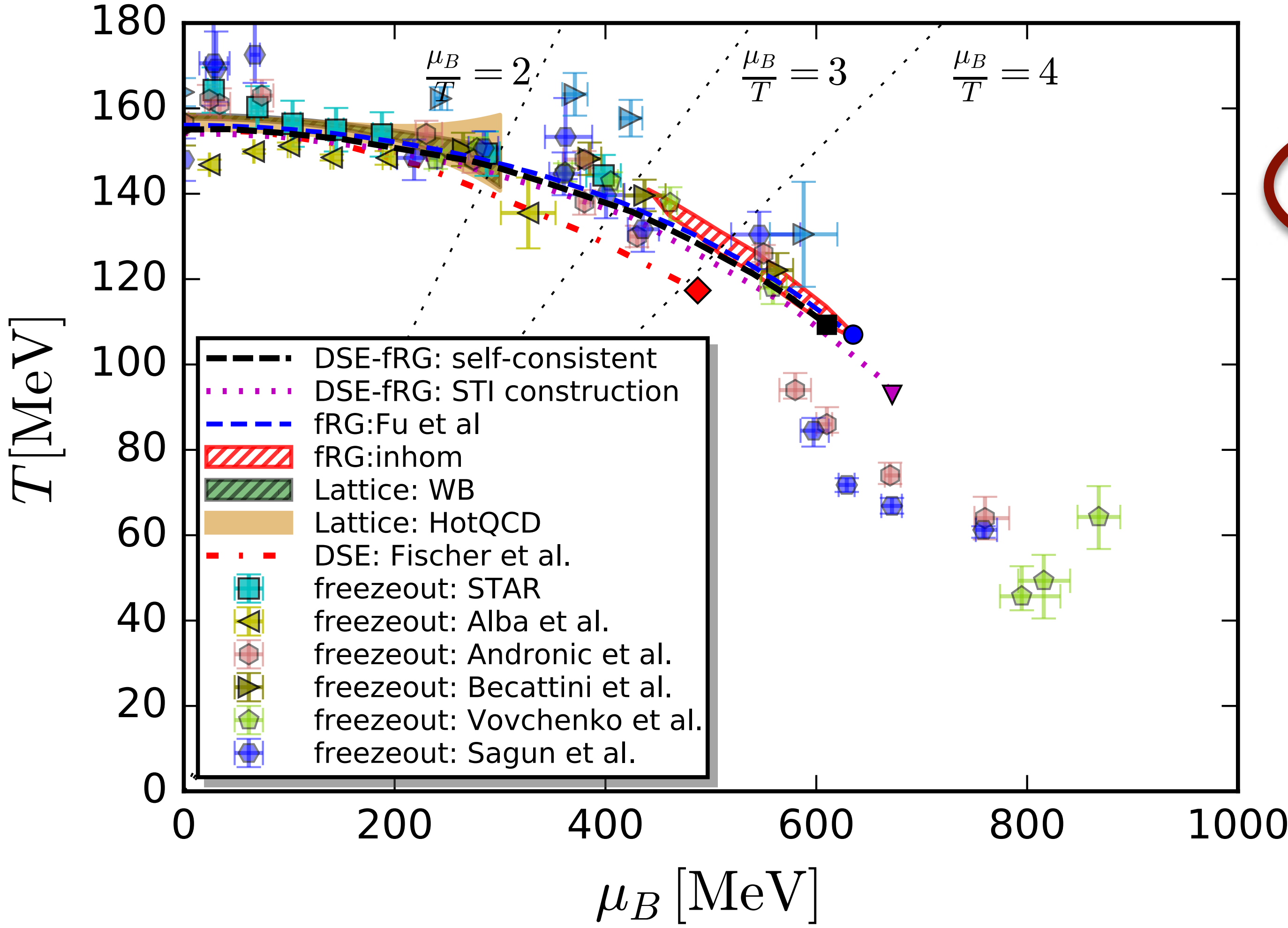
Minor effects of mesons: Gunkel, Fischer, 2106.08356

fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

QCD phase structure

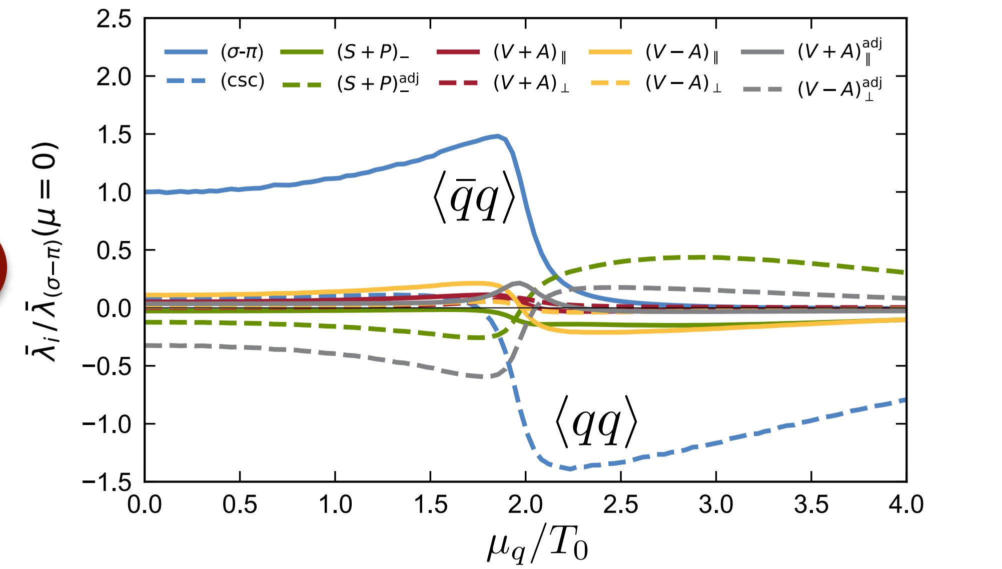
Reliability considerations



$\mu_\mathrm{B}/\mathrm{T} \lesssim 4$

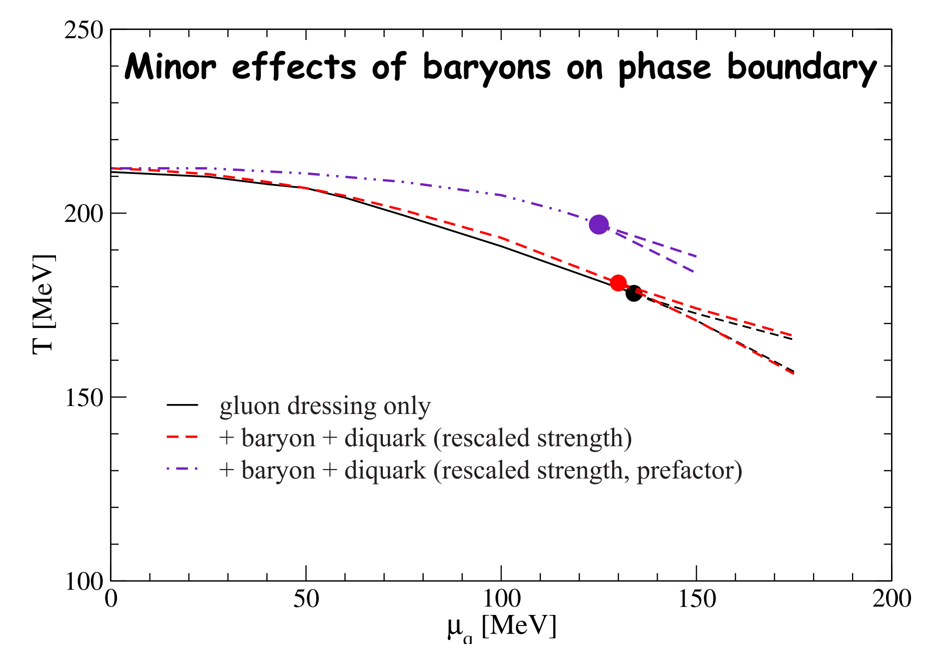
Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



Dominant channels II (DSE)

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



Minor effects of mesons: **Gunkel, Fischer, 2106.08356**

'Inhomogeneous' phase

$\mu_\mathrm{B}/\mathrm{T} \lesssim 4$

: Pion dispersion has minimum at non-vanishing spatial momentum

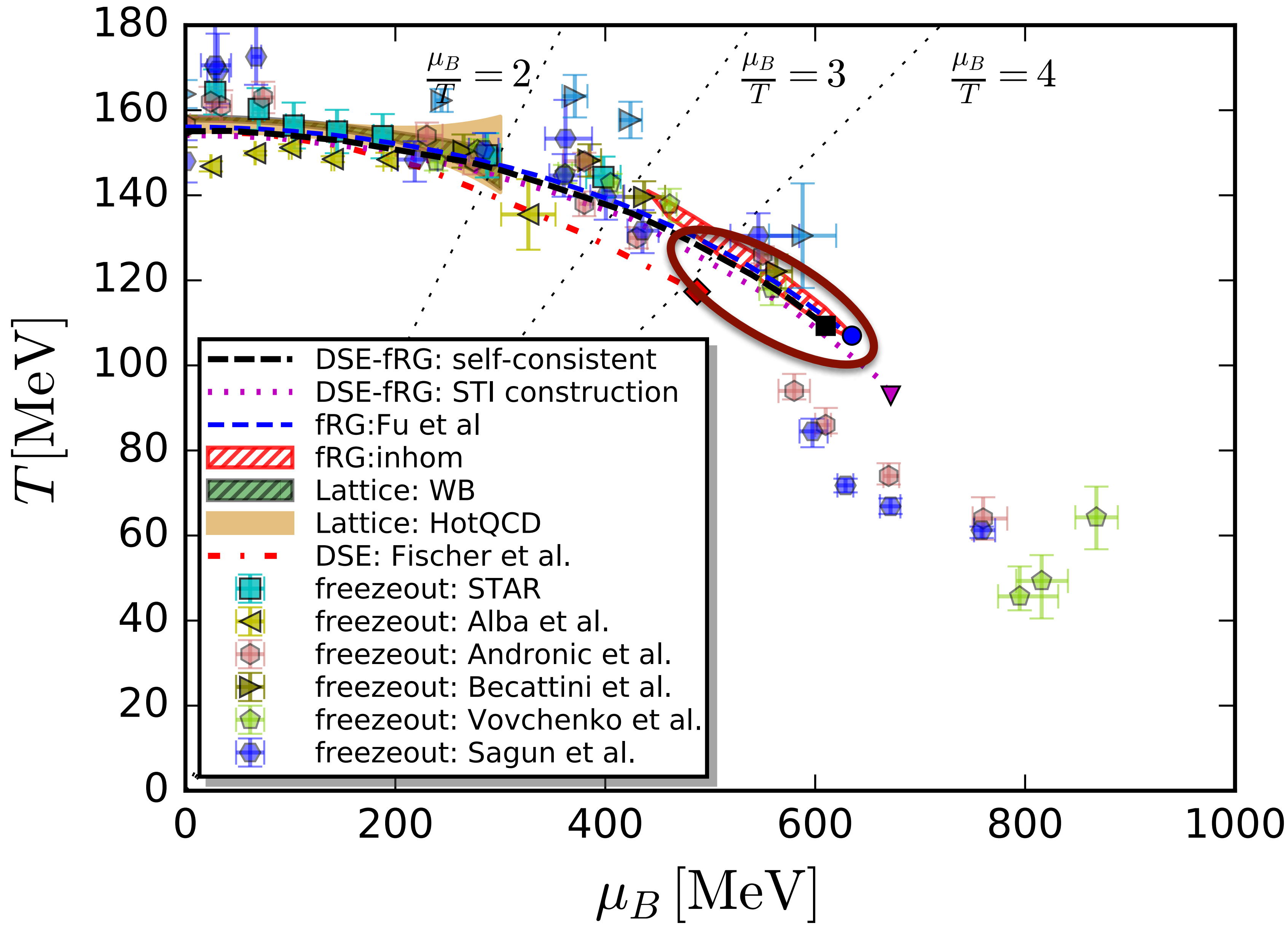
→ Non-trivial background

fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

QCD phase structure

Estimate for CEP



Stay tuned!

CEP-estimate fRG-DSE

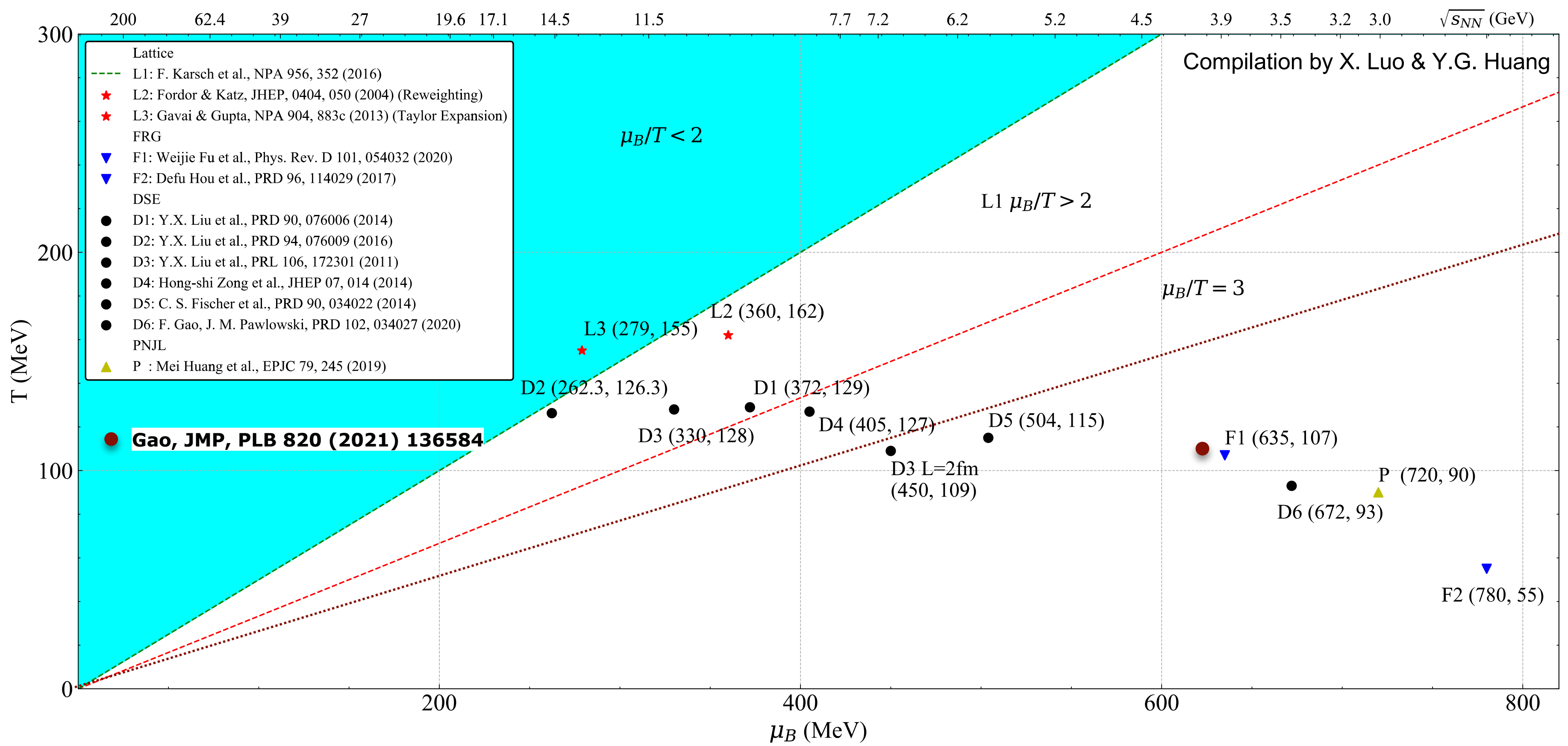
$$(135\,,\,450\,)\,\mathrm{MeV} \lesssim (T_{\mathrm{CEP}}\,,\,\mu_{B_{\mathrm{CEP}}}) \lesssim (100\,,\,650)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

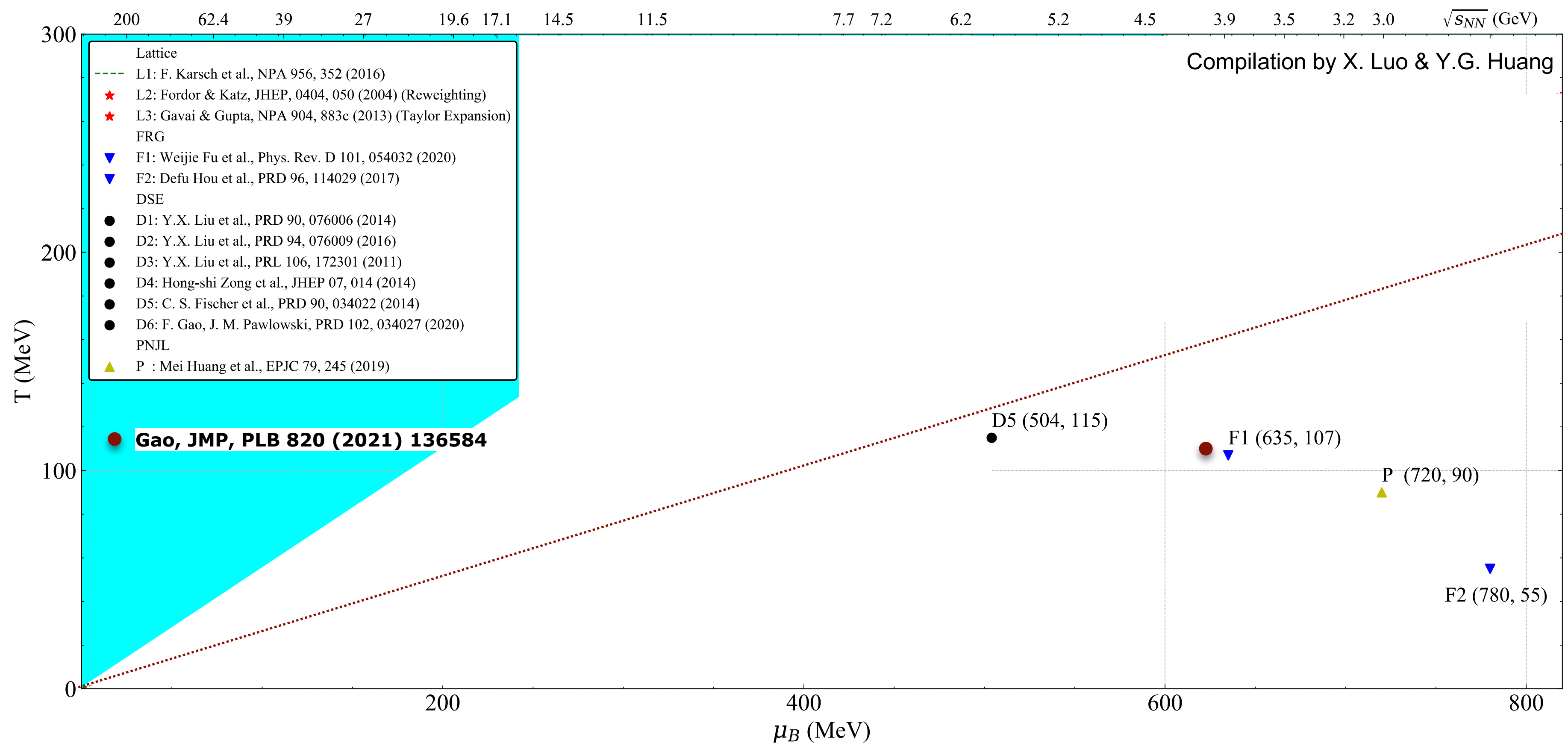
Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

Remove for CEP-predictions:

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

Remove for CEP-predictions:

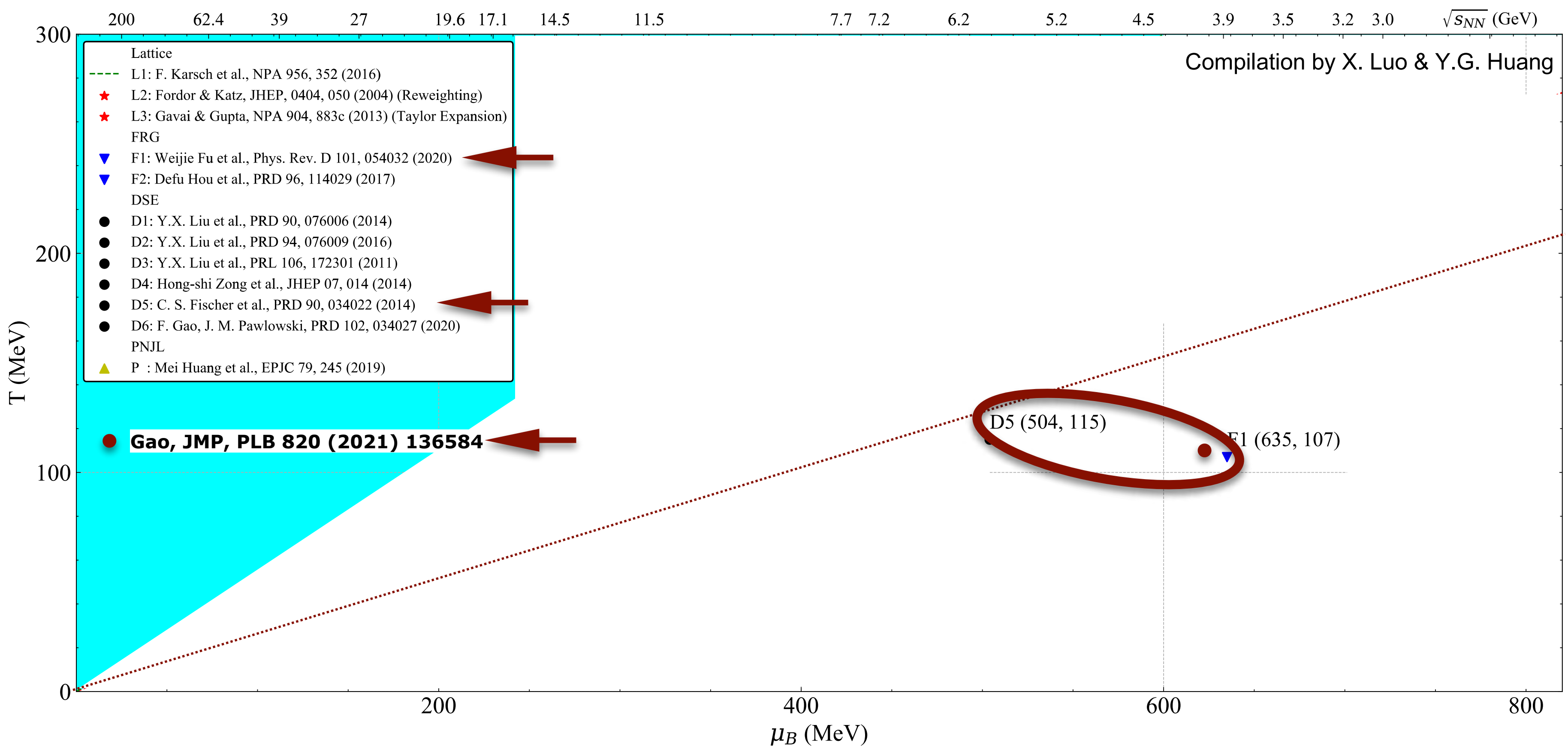
(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B=0

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

Still uncertainties for the estimation of CP location.

Remove for CEP-predictions:

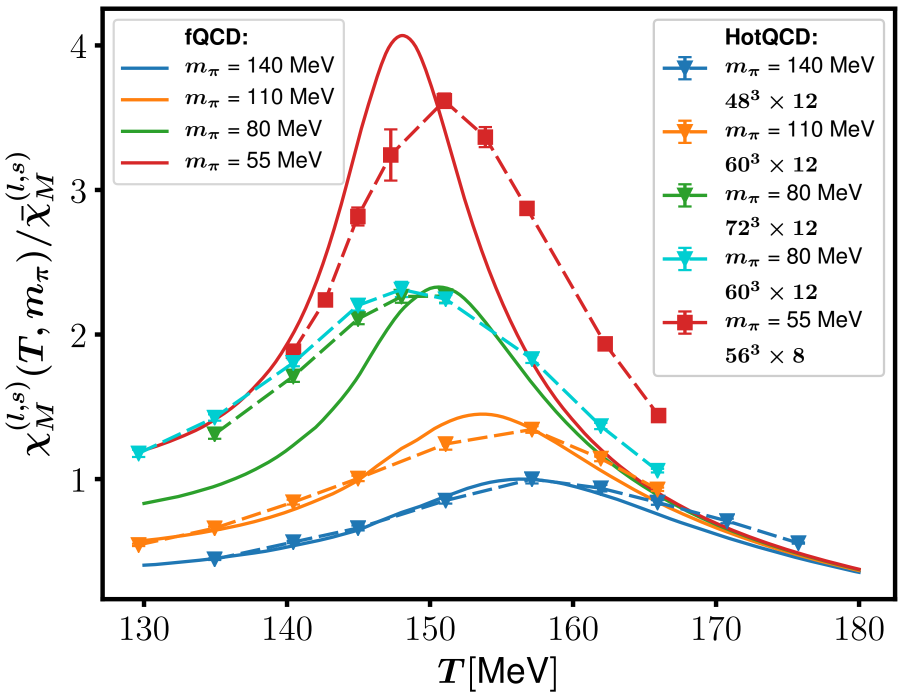
(i) ‘old’ CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B=0

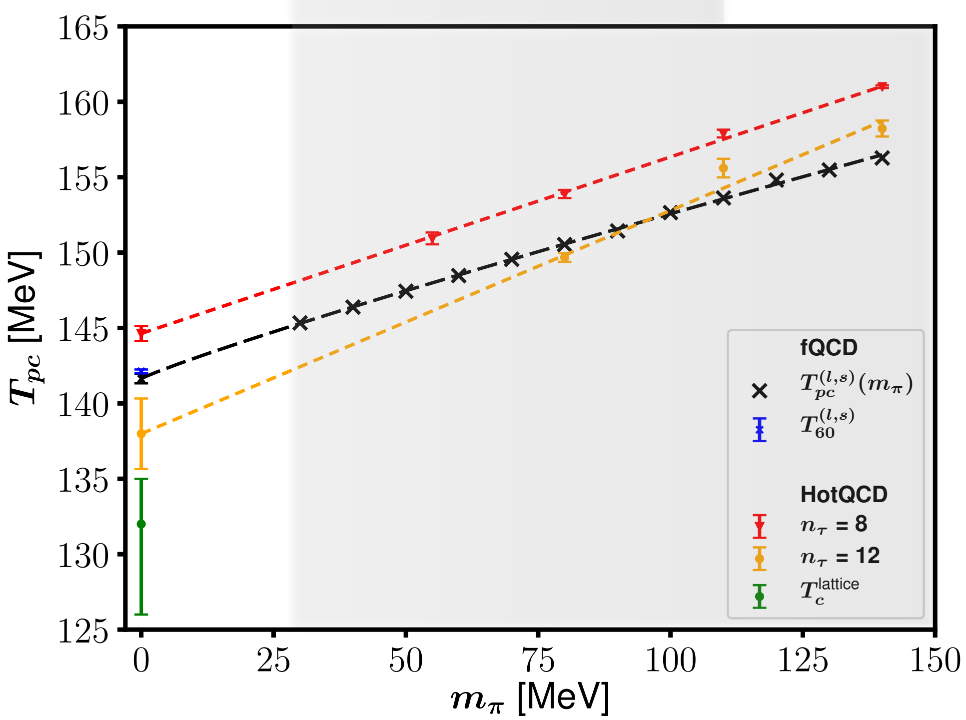
(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Some applications (fQCD)

Magnetic EoS

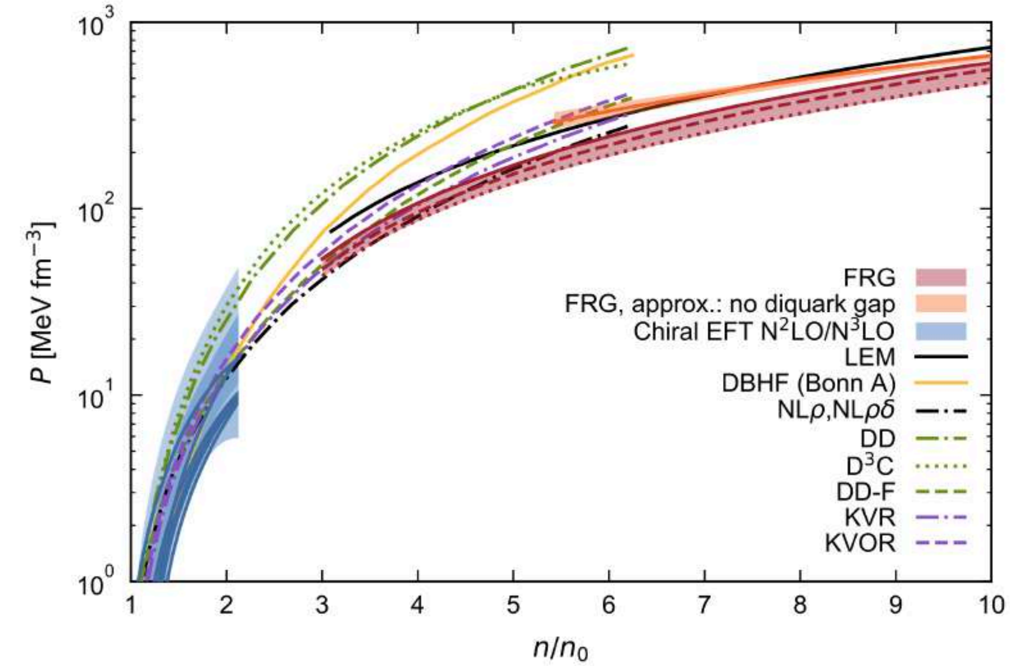


No critical scaling



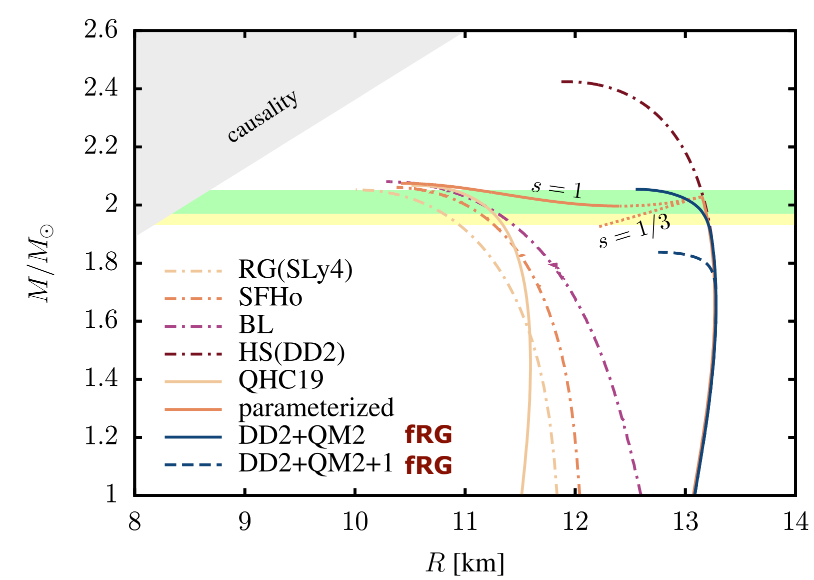
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

EoS of symmetric nuclear matter



Leonhardt, Pospiech, Schallmo, Braun, Drischler, Hebeler, Schwenk, PRL 125 (2020) 142502

Recent fRG work on EoS in cold and dense matter



Otto, Oertel, Schaefer, PRD 101 (2020) 10, 103021; 2007.07394

Outline

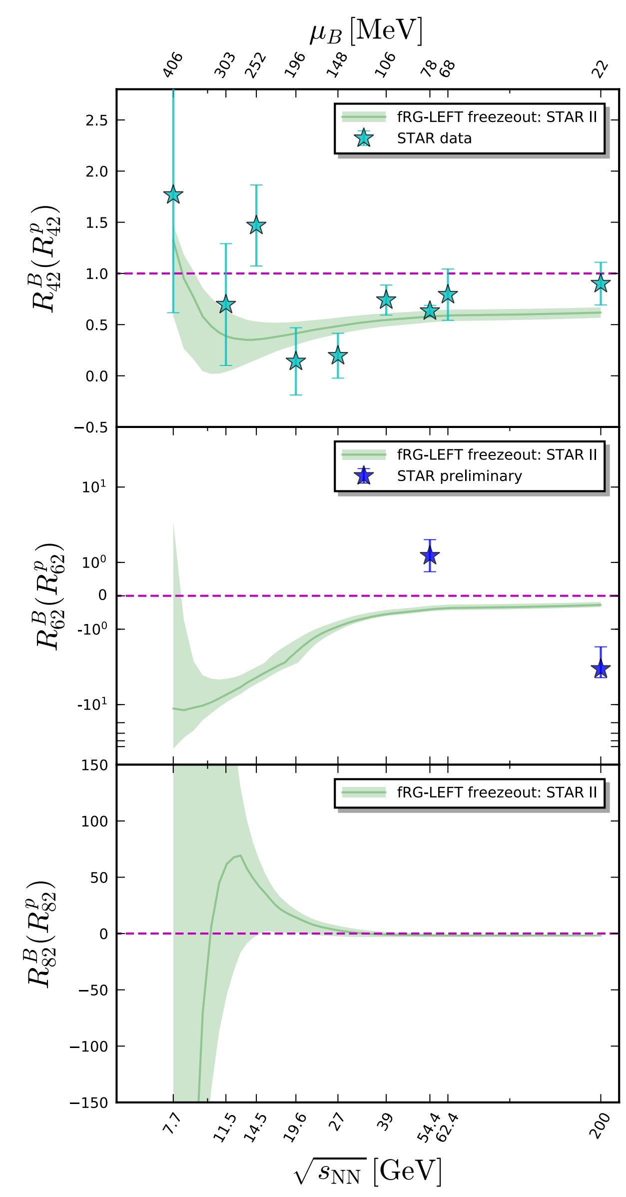
- QCD from functional methods

- QCD phase structure

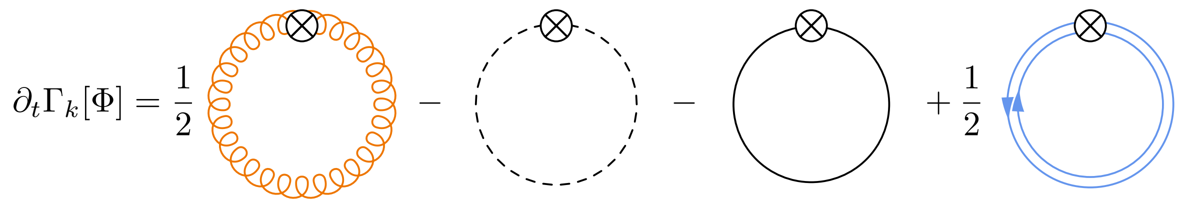
- **Fluctuations of conserved charges**

- Summary & outlook

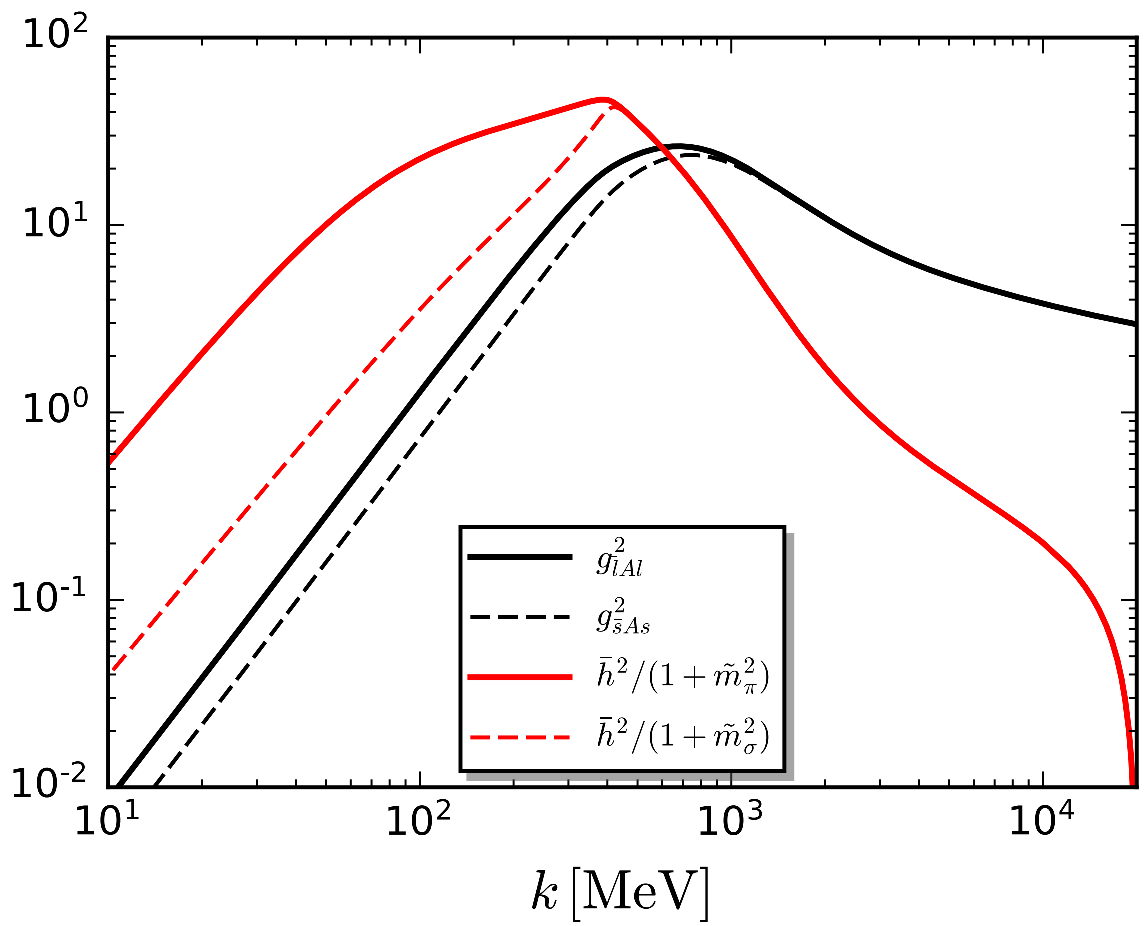
Hyper-fluctuations



On the unreasonable effectiveness of low energy effective theories



Sequential decoupling of gluon, quark, sigma, pion fluctuations



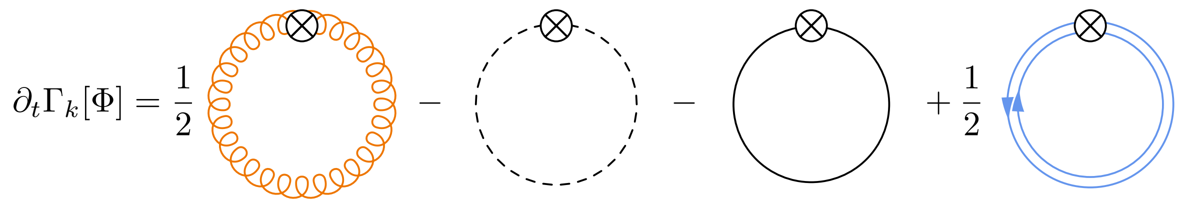
Fu, JMP, Rennecke, PRD 101, (2020) 054032

Based on:

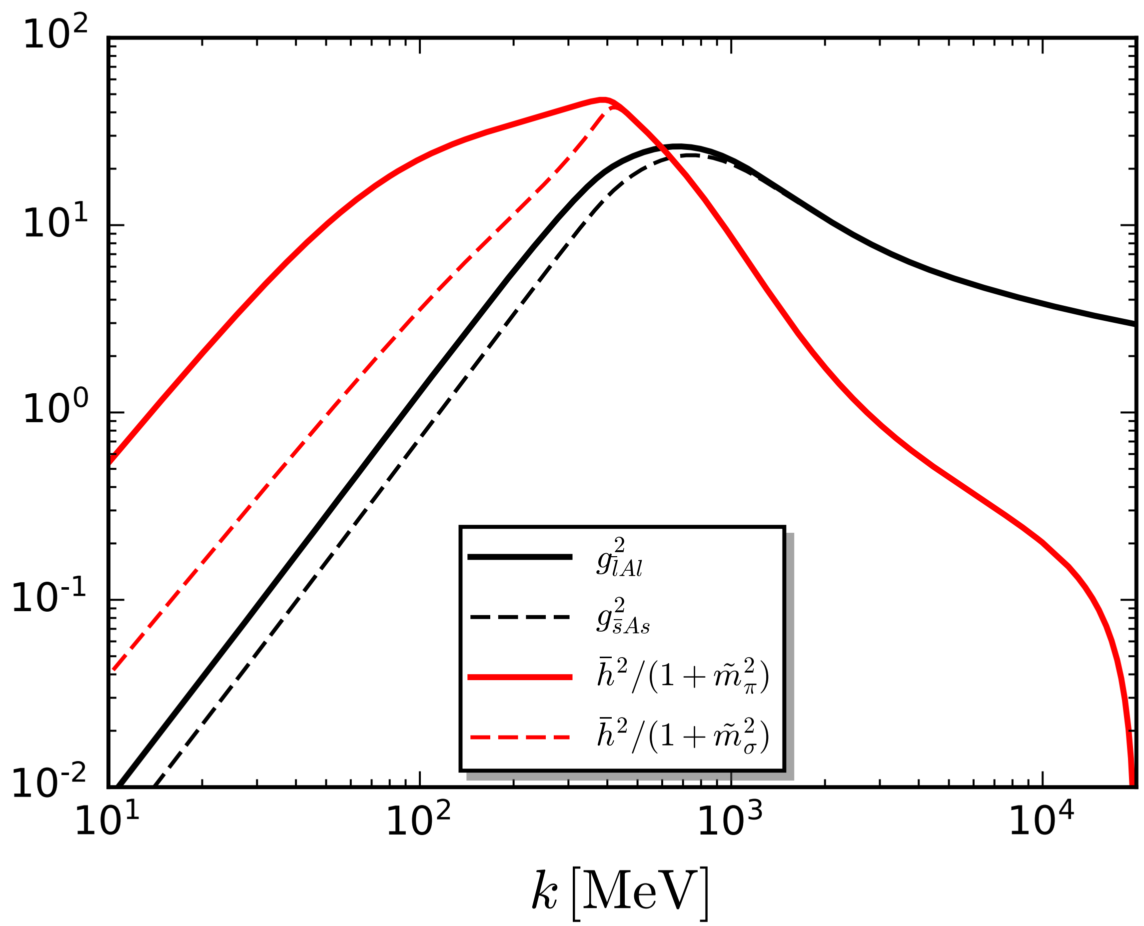
Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Rennecke, PRD 92 (2015) 076012

On the unreasonable effectiveness of low energy effective theories

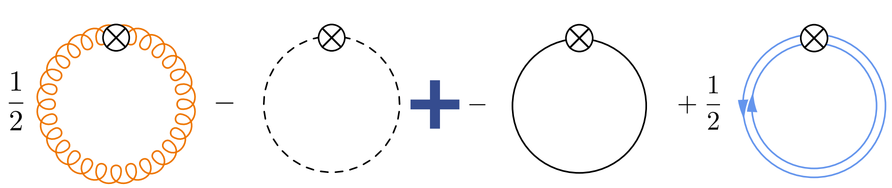


Sequential decoupling of gluon, quark, sigma, pion fluctuations

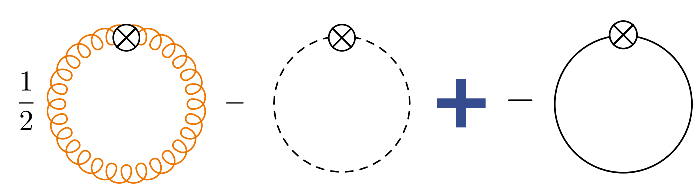


Fu, JMP, Rennecke, PRD 101, (2020) 054032

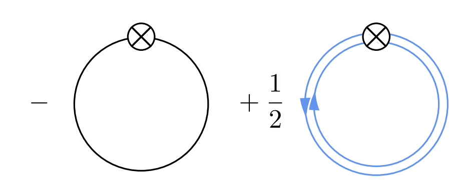
PQM-model



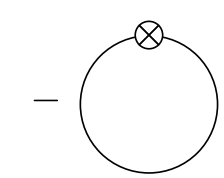
PNJL-model



QM-model



NJL-model

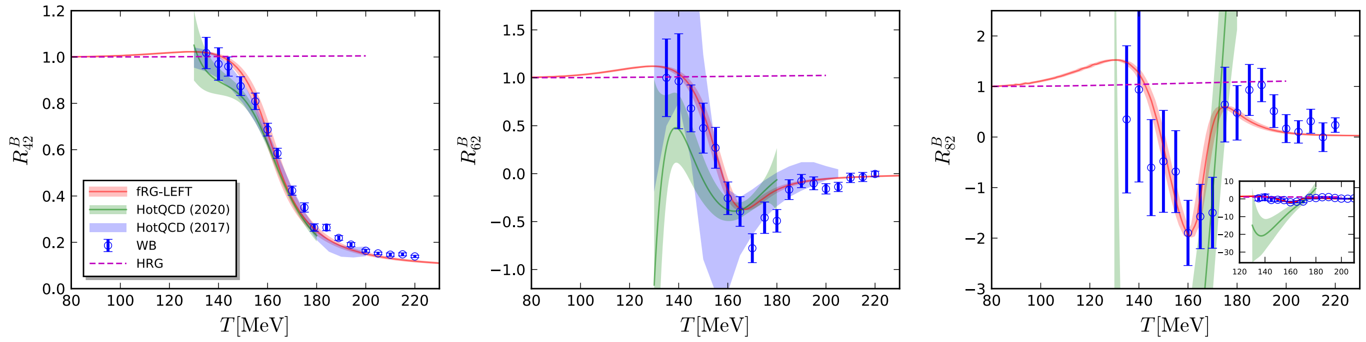


QCD-assisted low energy effective theories

Fluctuations of conserved charges

QCD-assisted LEFT

Benchmark at vanishing density



Builds on

Fu, JMP, PRD 93 (2016) 091501
Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020
Fu, JMP, Rennecke, PRD 101, (2020) 054032

Strangeness

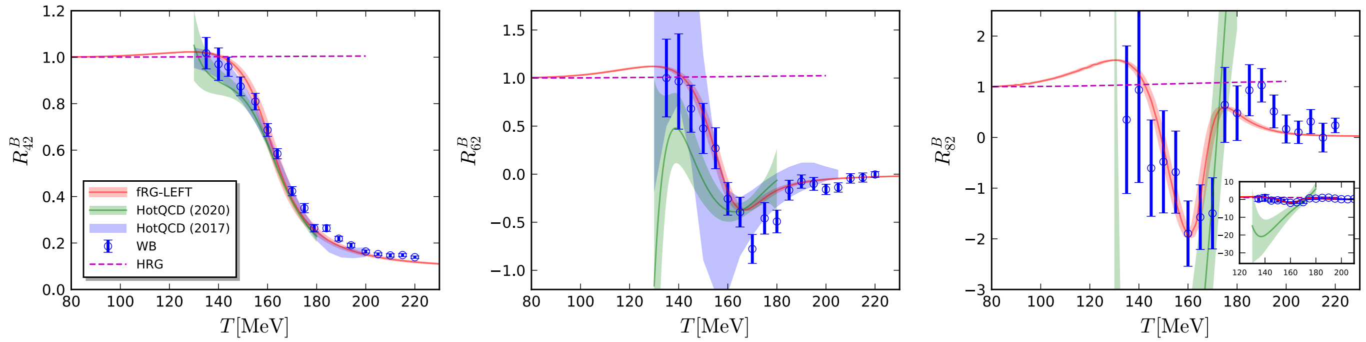
Fu, JMP, Rennecke, SciPost Phys. Core 2, 002 (2020)
PRD 100 (2019) 11, 111501
Wen, Huang, Fu, PRD 99 (2019) 094019

Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

Fluctuations of conserved charges

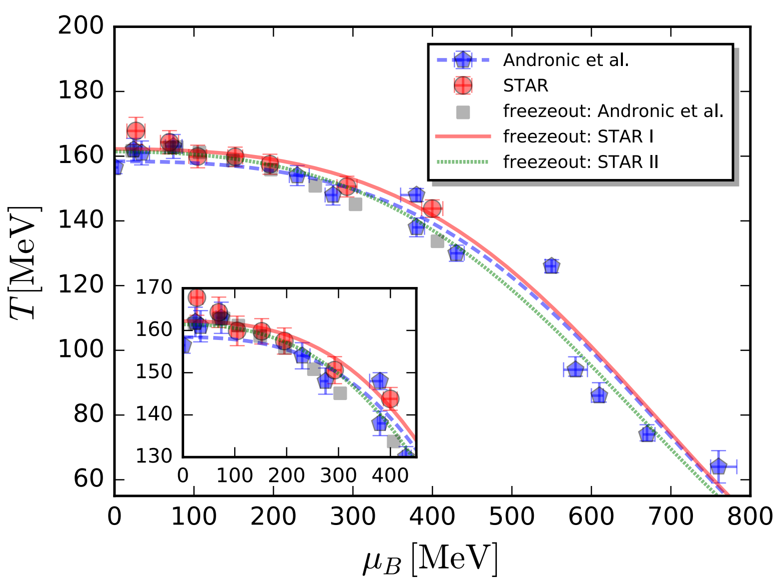
QCD-assisted LEFT

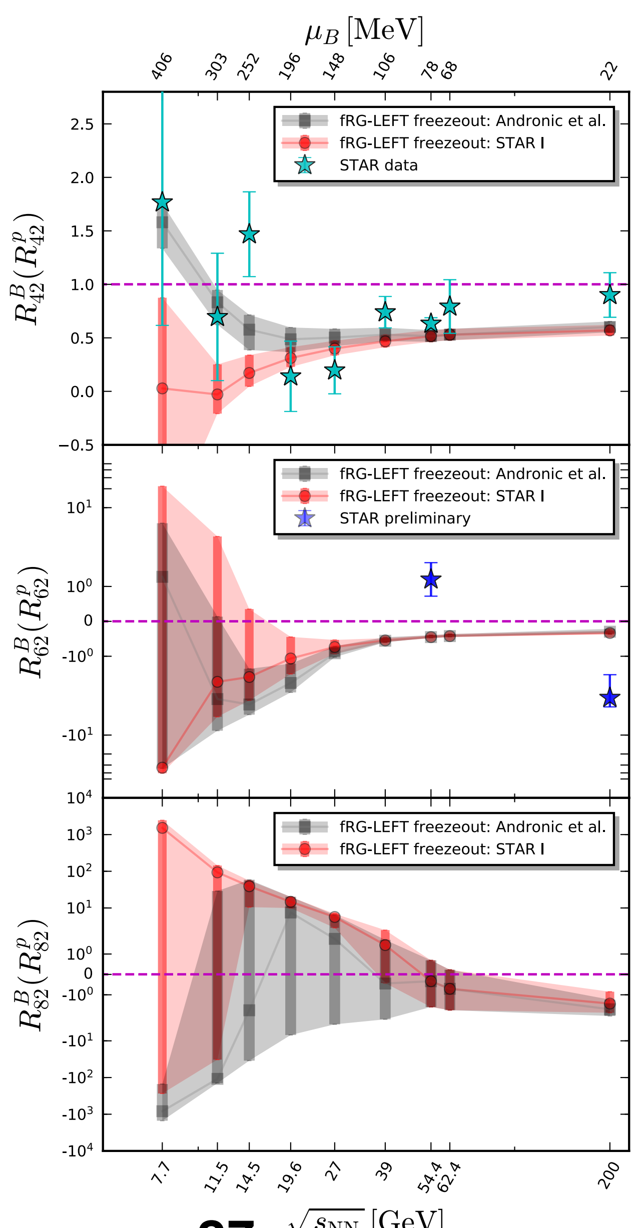
Benchmark at vanishing density

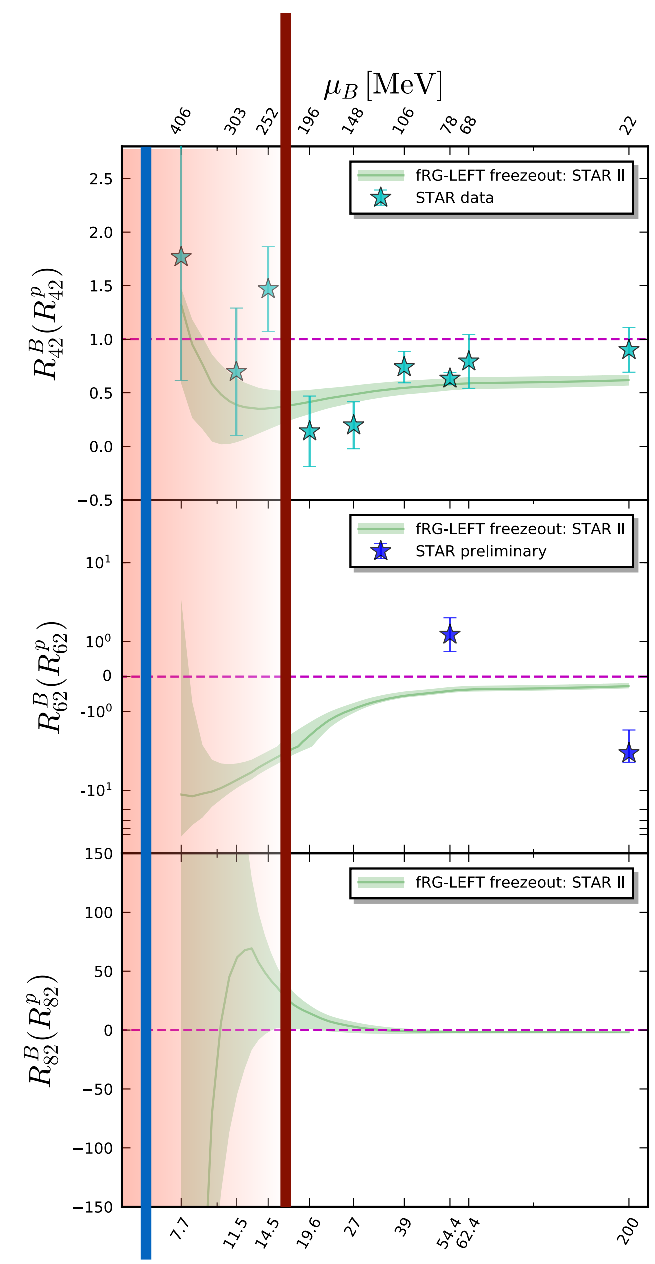


QCD-assisted LEFT

Freezeout curve







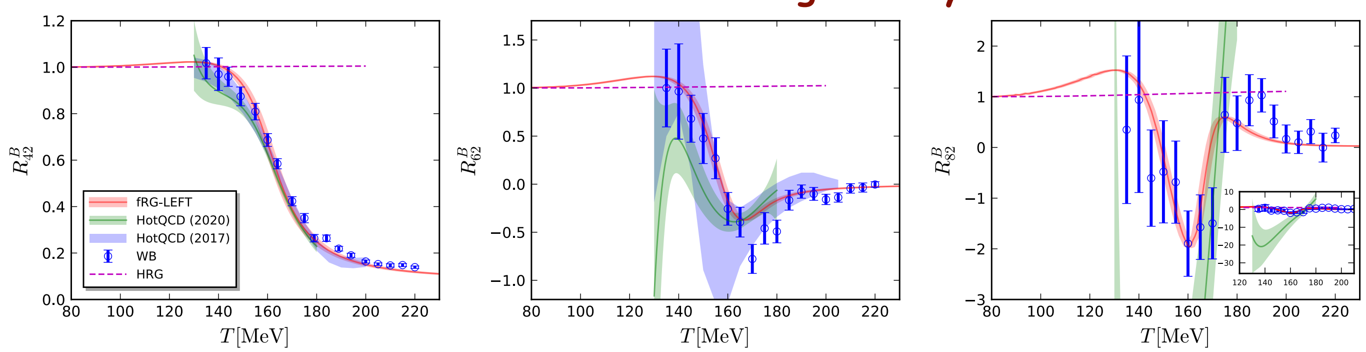
Fun QCD, in progress

Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

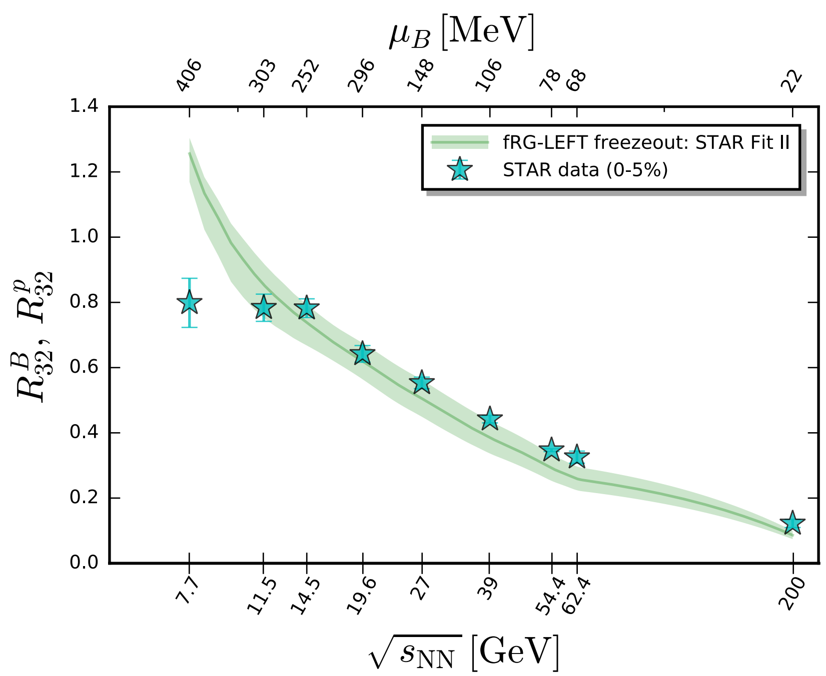
Fluctuations of conserved charges

Benchmark at vanishing density

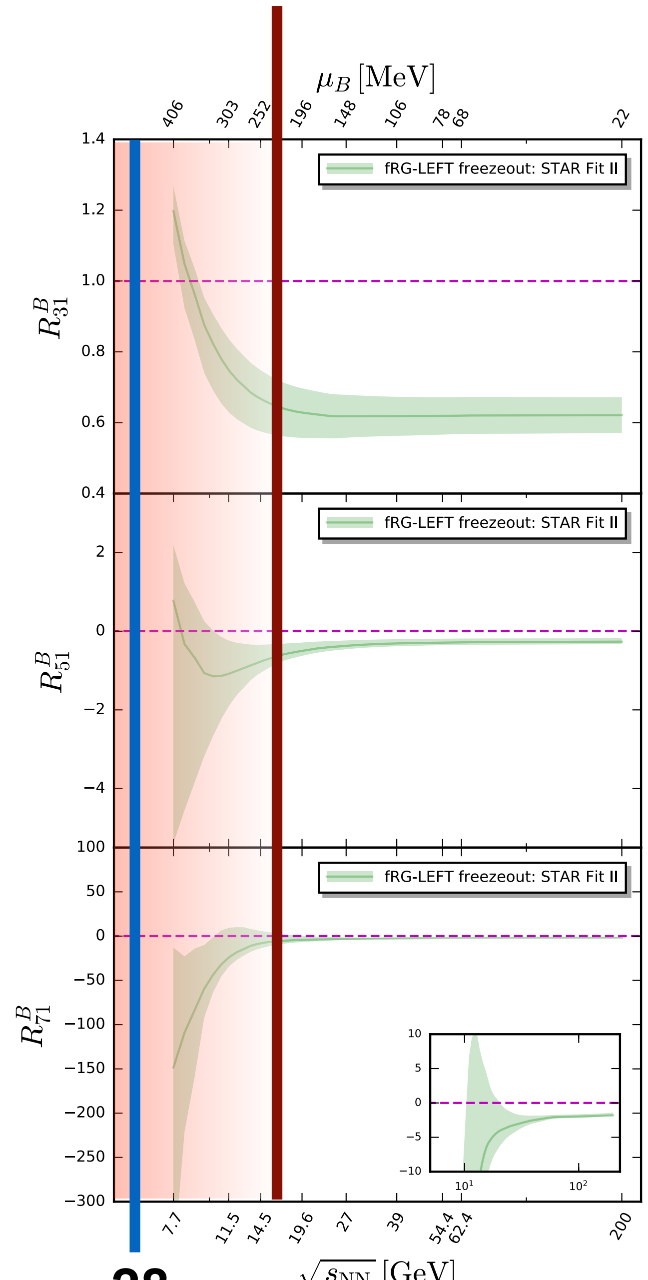
QCD-assisted LEFT

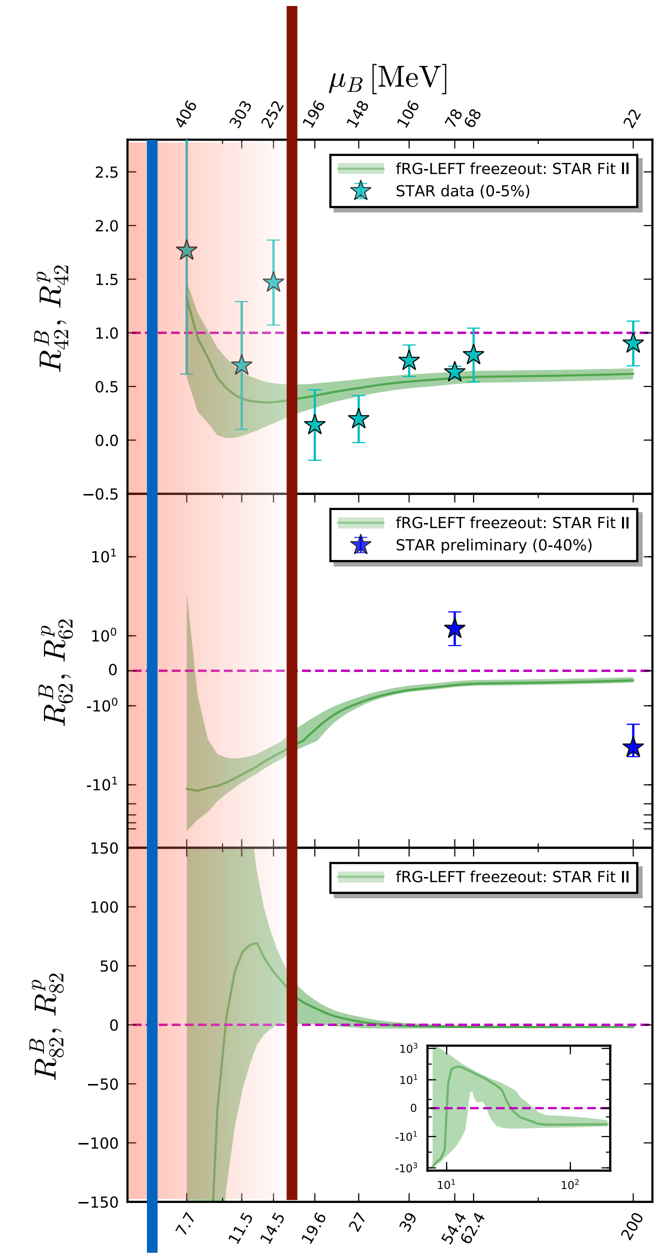


Baryon number conservation?



QCD-assisted LEFT

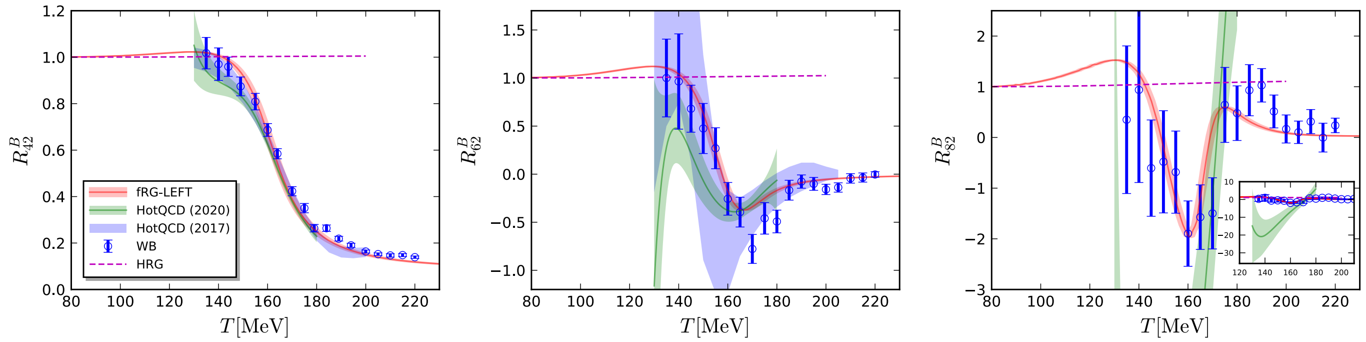




Fun QCD, in progress

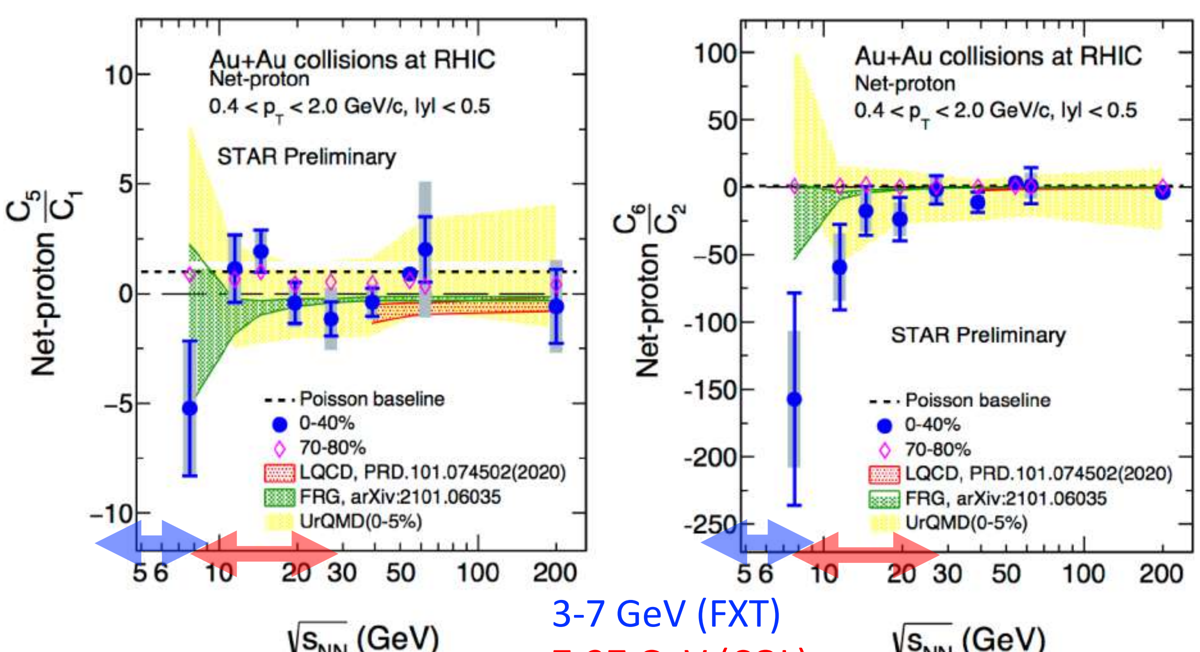
Fluctuations of conserved charges

Benchmark at vanishing density



QCD-assisted LEFT

STAR measurements



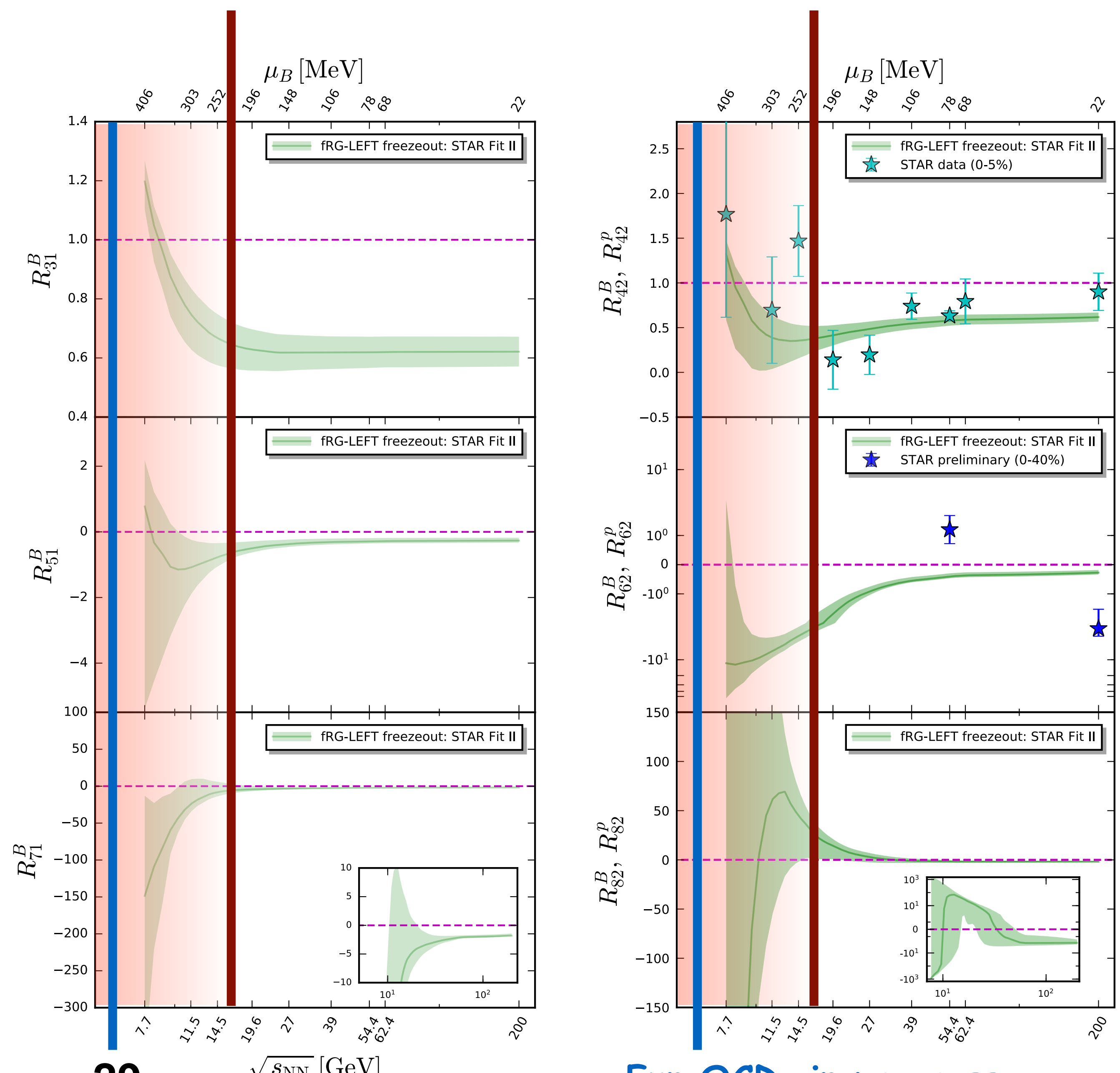
3-7 GeV (FXT)

7-27 GeV (COL)

S Results from RHIC-STAR, CPOD2021, 15-19/Mar, Online

S. Esumi, CPOD

QCD-assisted LEFT

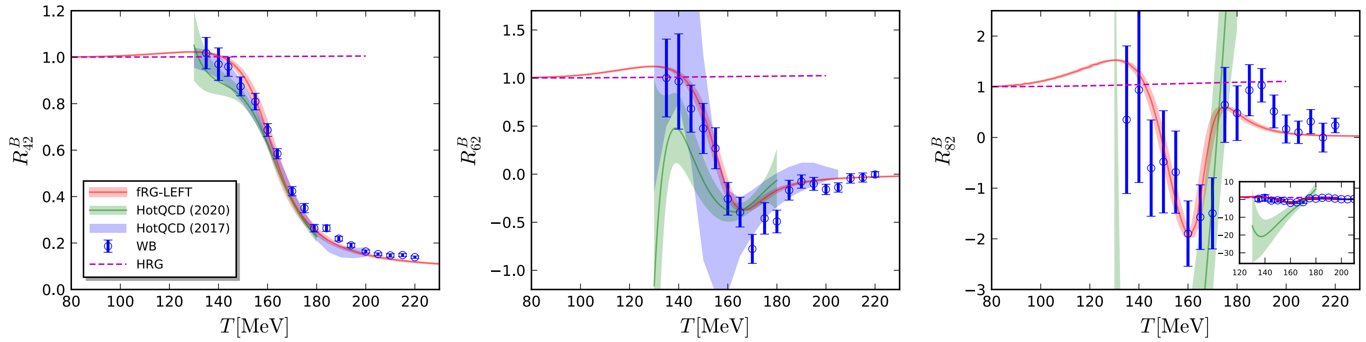


Fun QCD, in progress

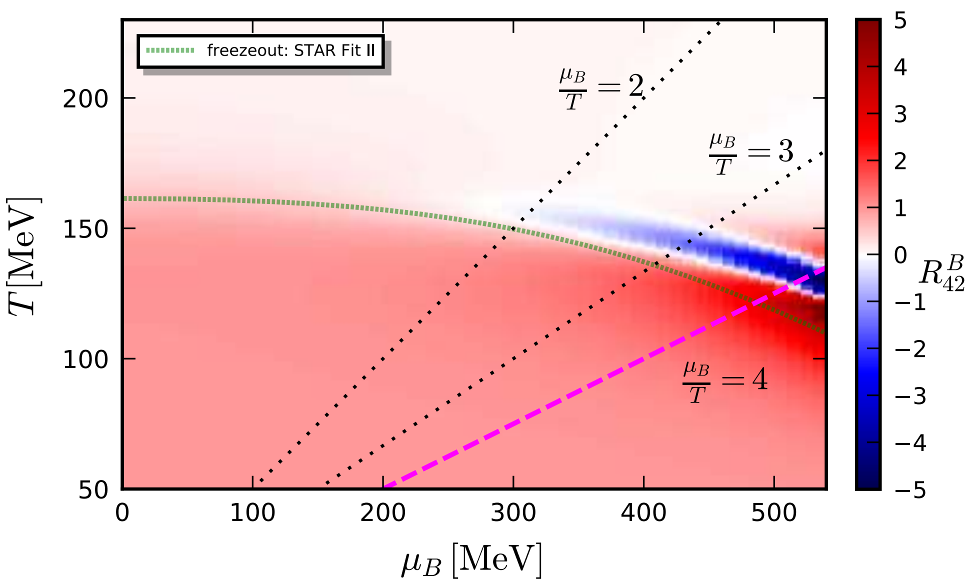
Fluctuations of conserved charges

QCD-assisted LEFT

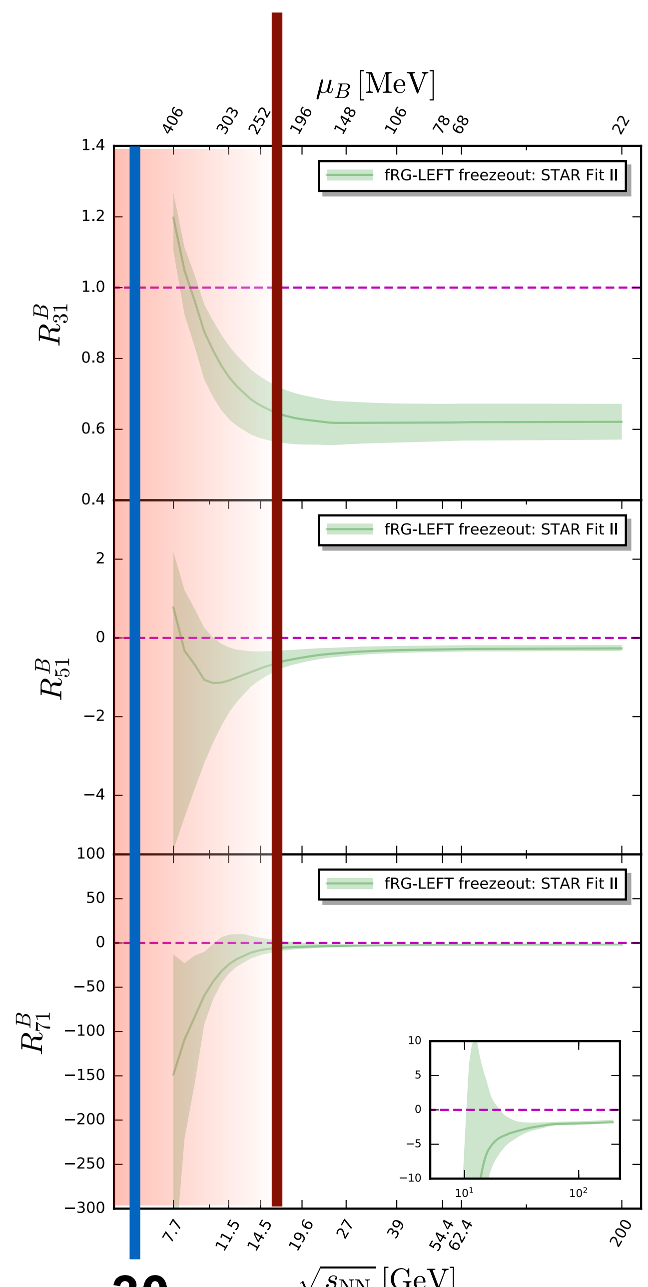
Benchmark at vanishing density

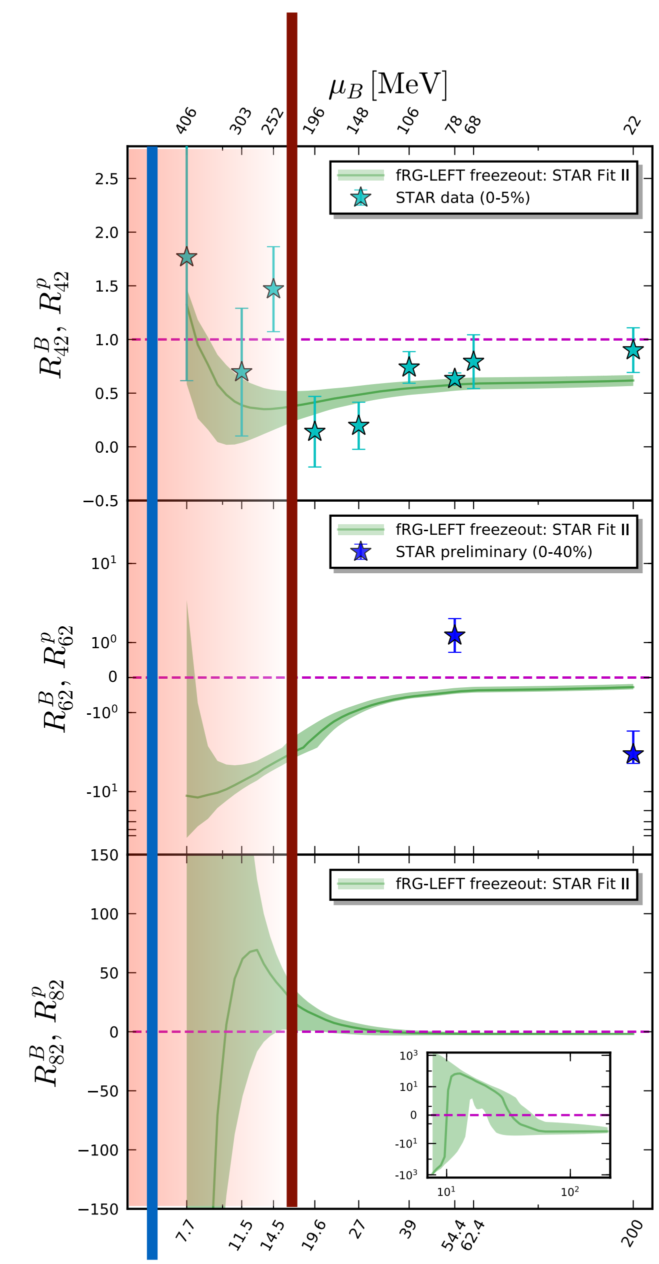


Freezeout curve



QCD-assisted LEFT



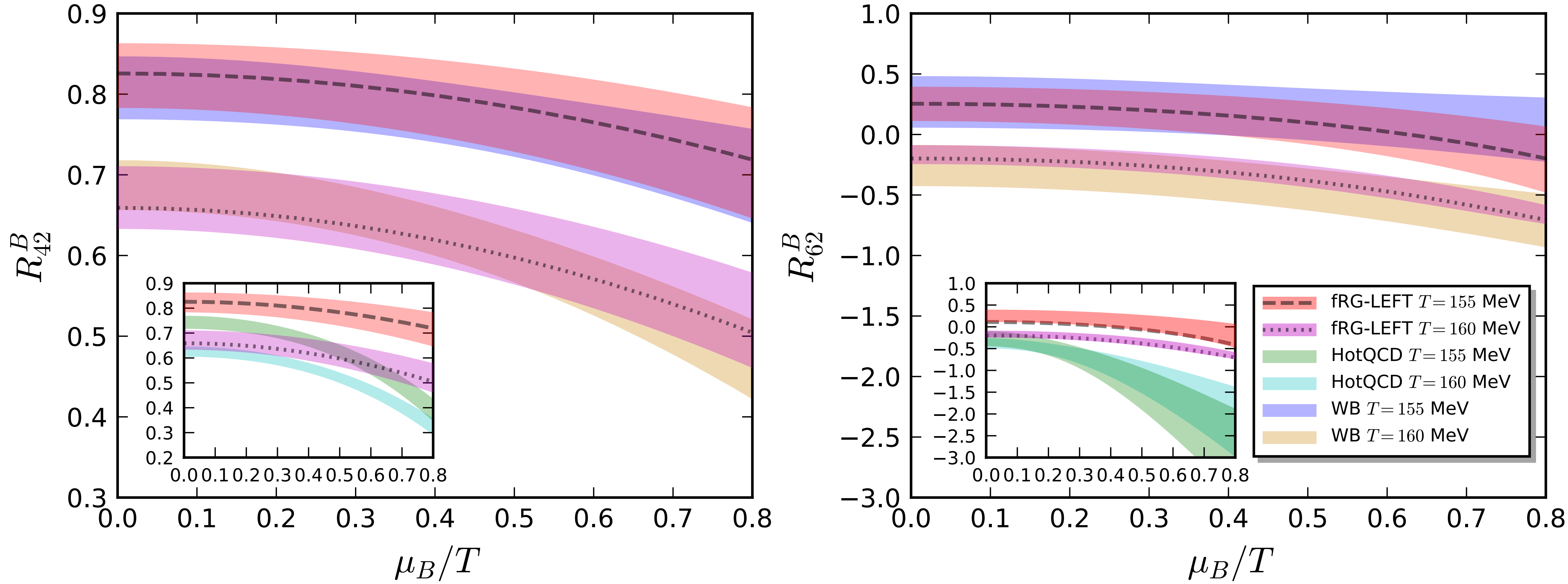


Fun QCD, in progress

Fluctuations of conserved charges

Fluctuation of conserved charges

QCD-assisted LEFT vs lattice results (Taylor expansion) at small chemical potential

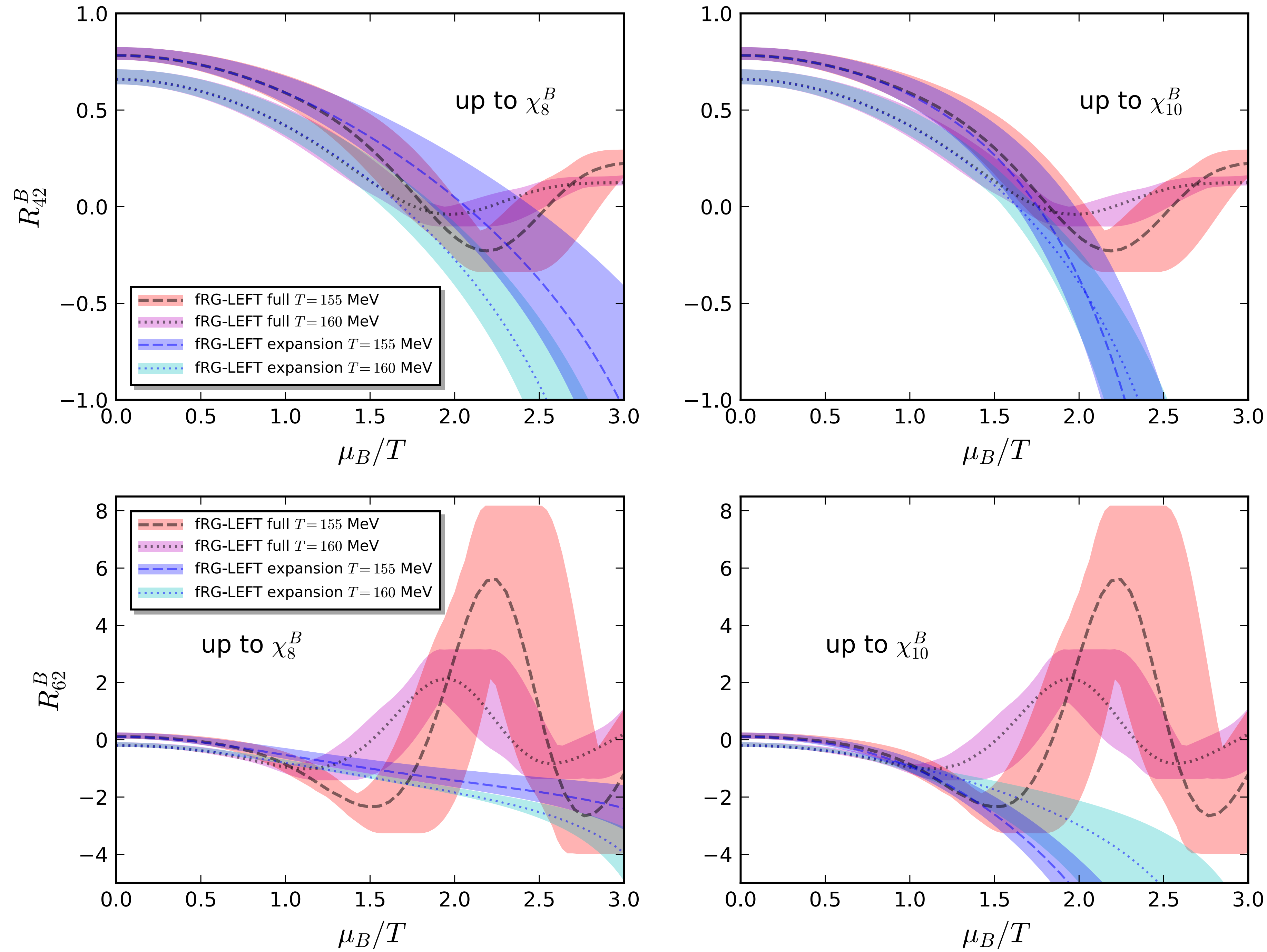


Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

Fluctuations of conserved charges

Fluctuation of conserved charges

QCD-assisted LEFT: Taylor expansion vs full results



Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

Outline

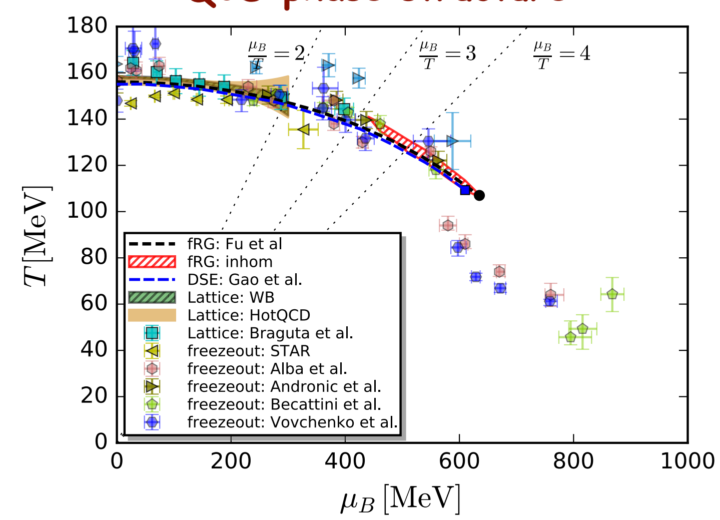
● QCD from functional methods

● QCD phase structure

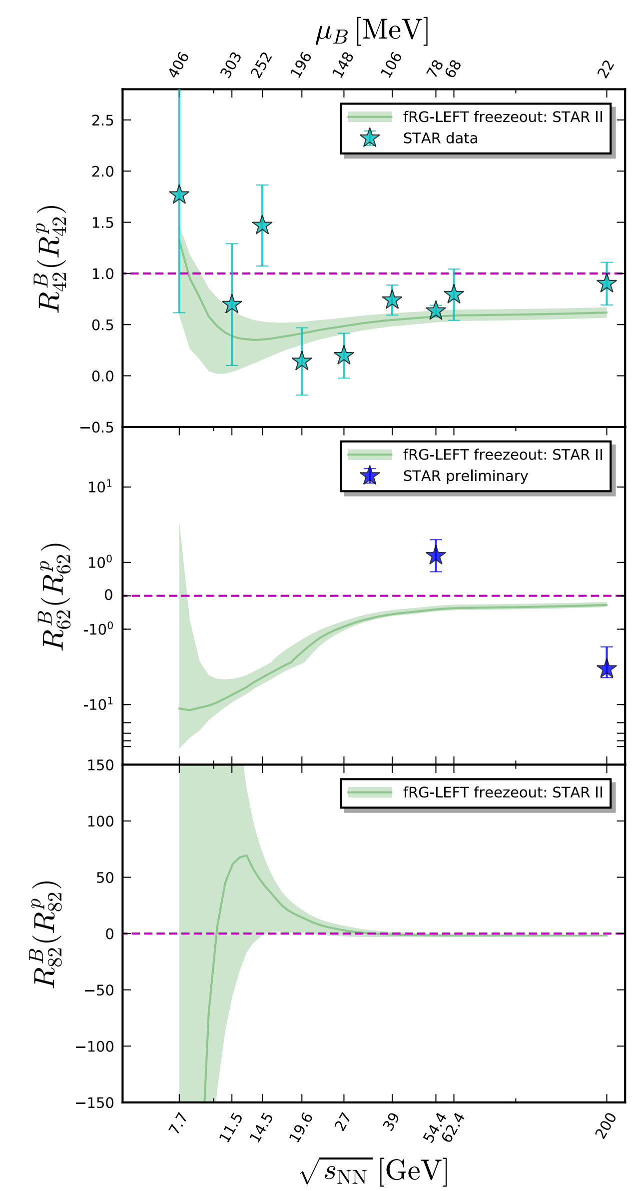
● Fluctuations of conserved charges

● Summary & outlook

QCD phase structure



Hyper-fluctuations

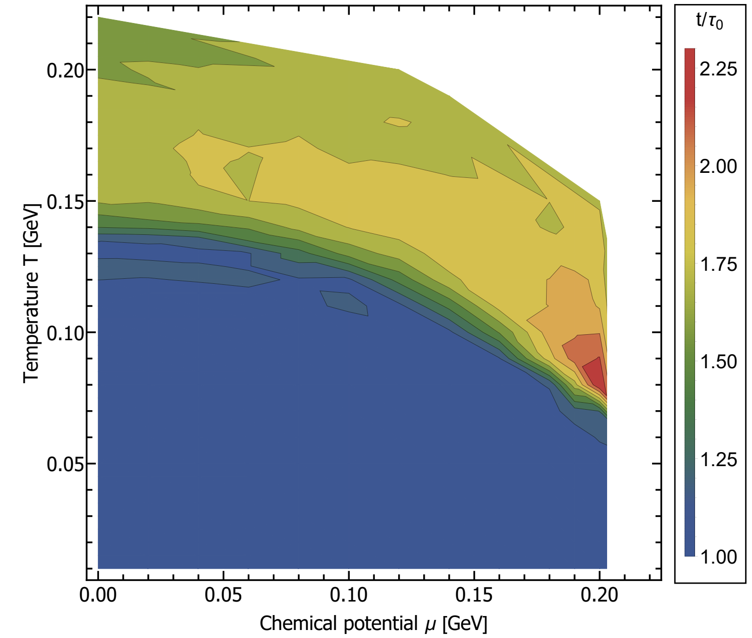


Summary & Outlook

- Towards apparent convergence in functional approaches to QCD
- Results & predictive power for the phase structure of QCD
  - Observables: quark condensates, fluctuations of conserved charges
- Towards quantitative precision at high densities
  - Systematic improvements under way for $\mu_\mathbf{B}/\mathbf{T} \gtrsim 4$

Summary & Outlook

- **Towards apparent convergence in functional approaches to QCD**
- **Results & predictive power for the phase structure of QCD**
  - **Observables: quark condensates, fluctuations of conserved charges**
- **Towards quantitative precision at high densities**
  - **Systematic improvements under way for** $\mu_\mathbf{B}/\mathbf{T} \gtrsim 4$
- **Transport, hydro, and critical region**
  - **Real-time correlation functions**
  - **Transport at finite mu & T**
  - **Transport coefficients**



Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

Transport approach to QCD

Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink

Time evolution of the critical (scalar) σ -mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion **noise field**

Extension of mean-field version

Nahrgang, Leupold, Herold, Bleicher PRC84 (2011)

see also

Stephanov, Rajagopal, Shuryak PRL 81 (1998) 4816

Mukherjee, Venugopalan, Yin PRC 92 (2015) 034912

Herold, Nahrgang, Yan, Kobdaj PRC 93 93 (2016) 021902

Nahrgang, Bluhm, Schäfer, Bass PRD 99 (2019) 116015

Transport approach to QCD

Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink

Time evolution of the critical (scalar) σ -mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion noise field

Input from equilibrium low energy effective action of QCD

$$\mathrm{Re}\,\Gamma_\sigma^{(2)}(\omega,\vec{p}) \qquad \mathrm{Im}\,\Gamma_\sigma^{(2)}(\omega,\vec{p}) \qquad U(\sigma)$$

kinetic term diffusion term $\eta\,\partial_t\sigma$ effective potential

Transport approach to QCD

Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink

Time evolution of the critical (scalar) σ -mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion noise field

Input from equilibrium low energy effective action of QCD

$$\mathrm{Re}\,\Gamma^{(2)}_\sigma(\omega,\vec{p}) \qquad \mathrm{Im}\,\Gamma^{(2)}_\sigma(\omega,\vec{p}) \qquad U(\sigma)$$

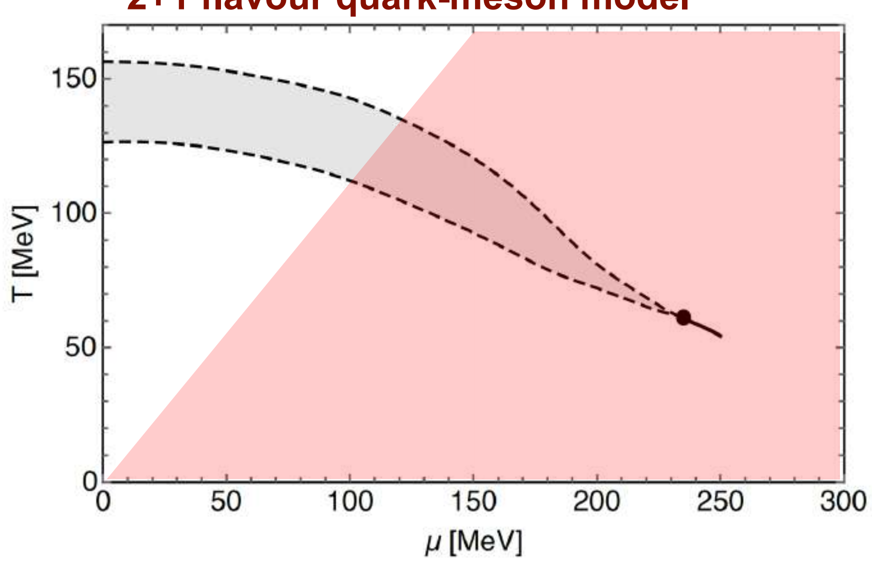
kinetic term

diffusion term $\eta\,\partial_t\sigma$

effective potential

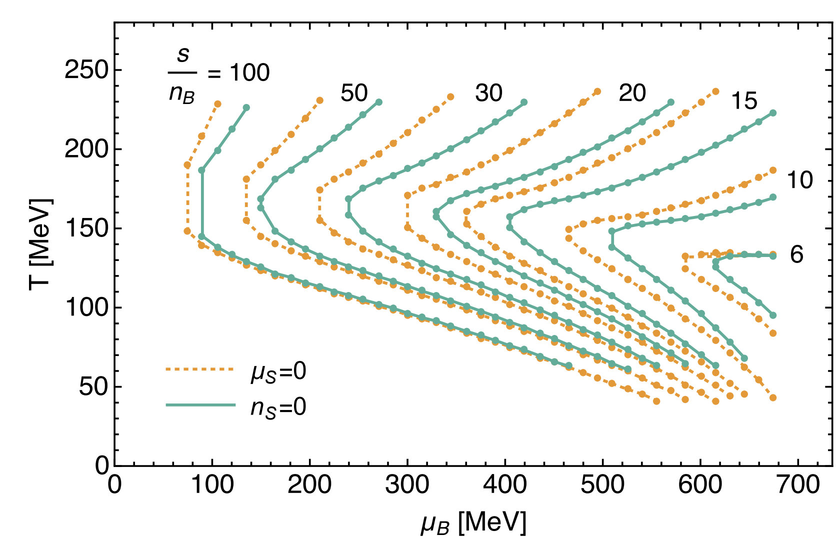
Phase structure of low energy QCD

2+1 flavour quark-meson model



Schaefer, Rennecke, PRD 96 (2017) 016009

strangeness neutrality & strangeness fluctuations



Fu, JMP, Rennecke, SciPost Core 002 (2020), PRD 100 (2019) 111501

$\mathrm{N_f} = 2$: **Nakano, Schaefer, Stokic, Friman, Redlich, PLB 682 (2010) 401**

Transport approach to QCD

Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink

Time evolution of the critical (scalar) σ -mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion noise field

Input from equilibrium low energy effective action of QCD

$$\mathrm{Re}\,\Gamma^{(2)}_{\sigma}(\omega,\vec{p}) \qquad \mathrm{Im}\,\Gamma^{(2)}_{\sigma}(\omega,\vec{p}) \qquad U(\sigma)$$

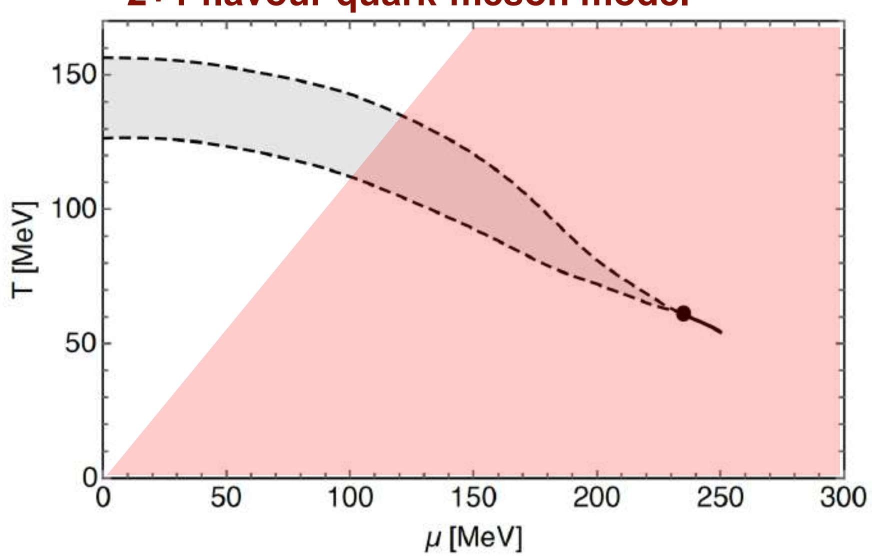
kinetic term

diffusion term $\eta\,\partial_t\sigma$

effective potential

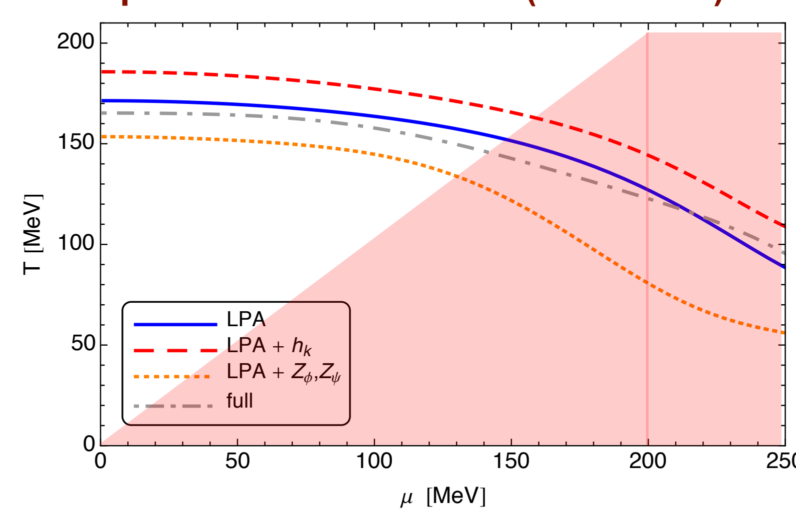
Phase structure of low energy QCD

2+1 flavour quark-meson model



Schaefer, Rennecke, PRD 96 (2017) 016009

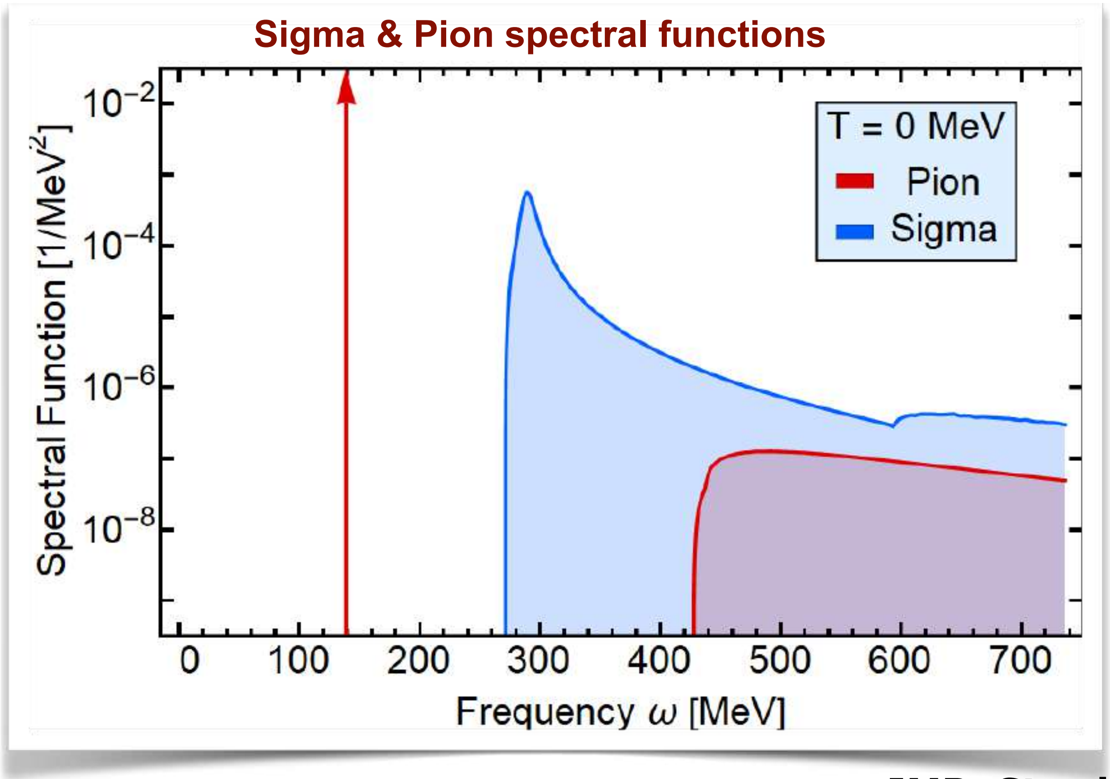
Comparison of truncations (2 flavours)



JMP, Rennecke, PRD 90 (2014) 076002

Pion & sigma spectral functions

Show case in linear sigma model



JMP, Strodthoff, Wink, PRD 98 (2018) 074008

Real-time FRG computations, e.g.

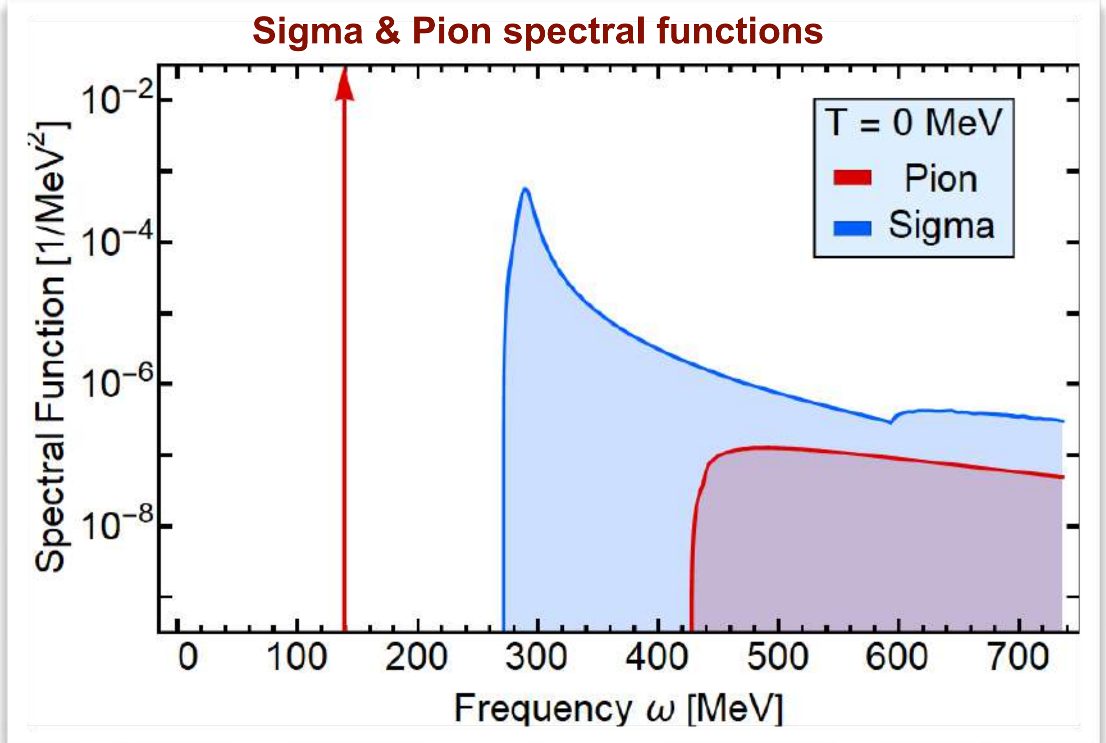
Flörchinger JHEP 1205 (2012) 021

Kamikado, Strodthoff, von Smekal, Wambach, EPJC 74 (2014) 2806

JMP, Strodthoff, PRD 92 (2015) 094009

Pion & sigma spectral functions

Show case in linear sigma model



JMP, Strodthoff, Wink, PRD 98 (2018) 074008

Real-time FRG computations, e.g.

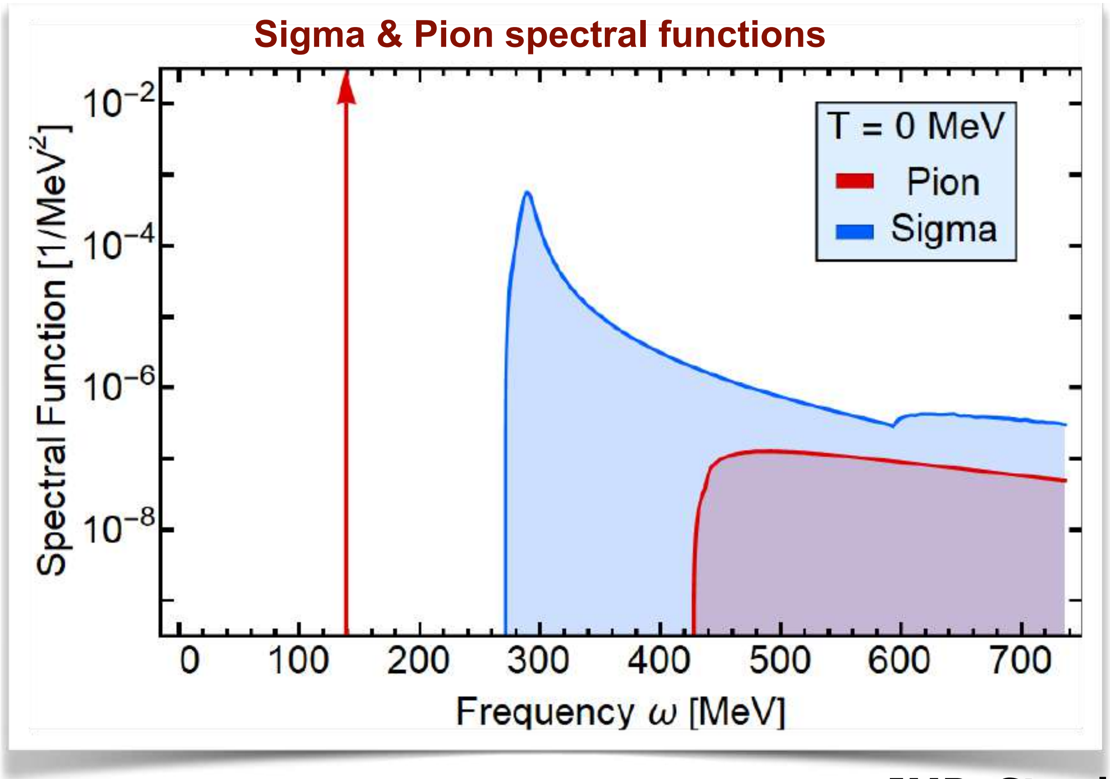
Flörchinger JHEP 1205 (2012) 021

Kamikado, Strodthoff, von Smekal, Wambach, EPJC 74 (2014) 2806

JMP, Strodthoff, PRD 92 (2015) 094009

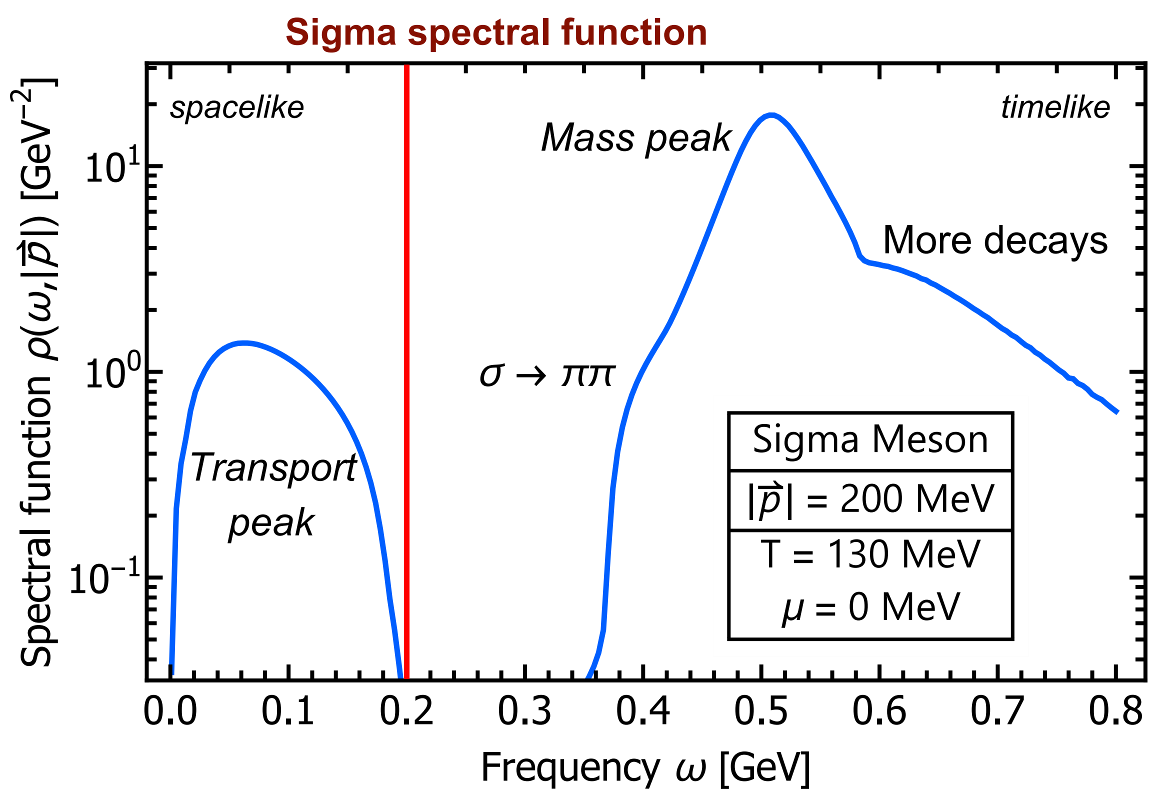
Pion & sigma spectral functions

Show case in linear sigma model



JMP, Strodthoff, Wink, PRD 98 (2018) 074008

2+1 flavour quark-meson model sigma spectral function

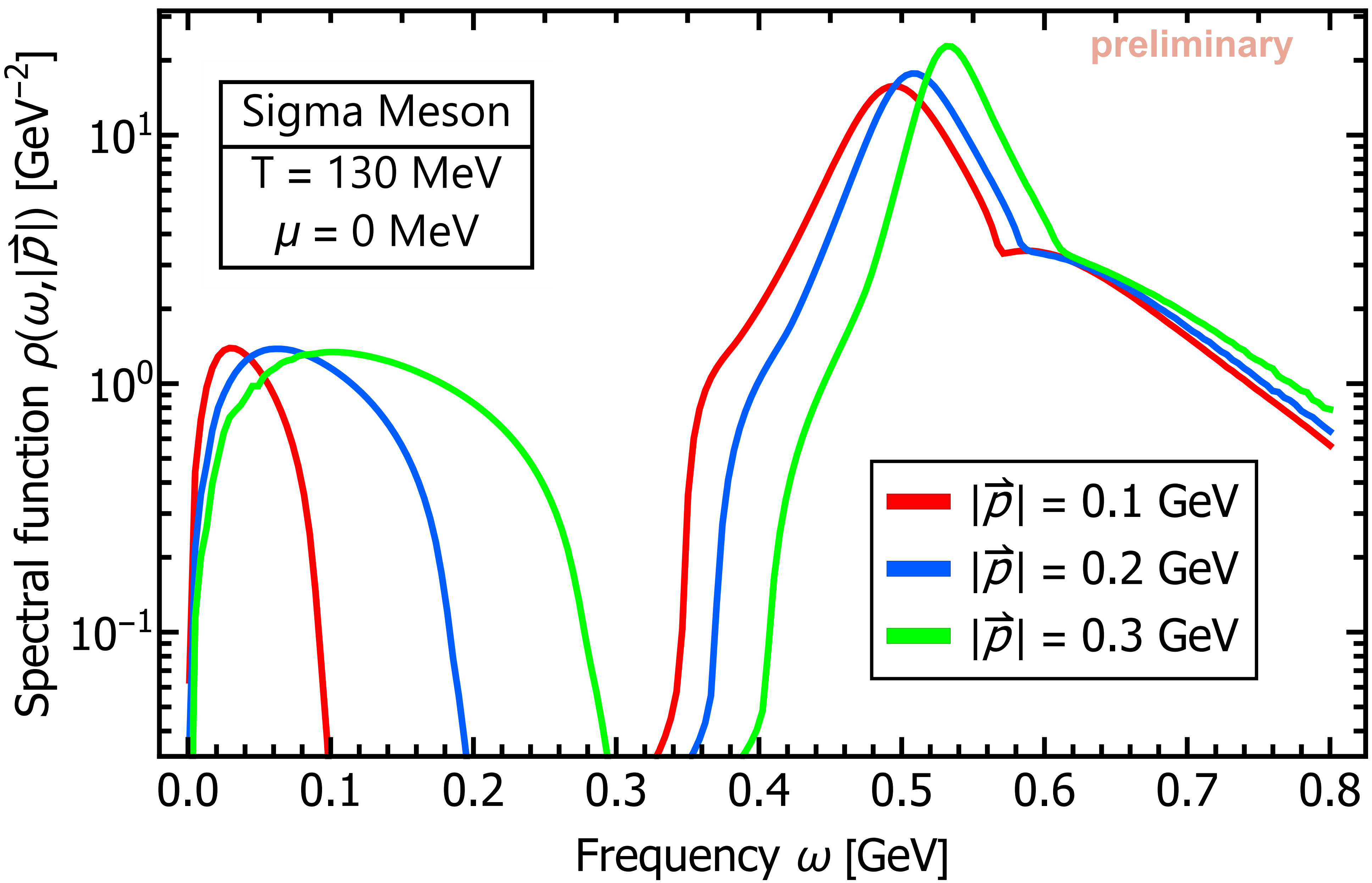


preliminary

JMP, Rennecke, Wink, in prep

Pion & sigma spectral functions

2+1 flavour quark-meson model sigma spectral function

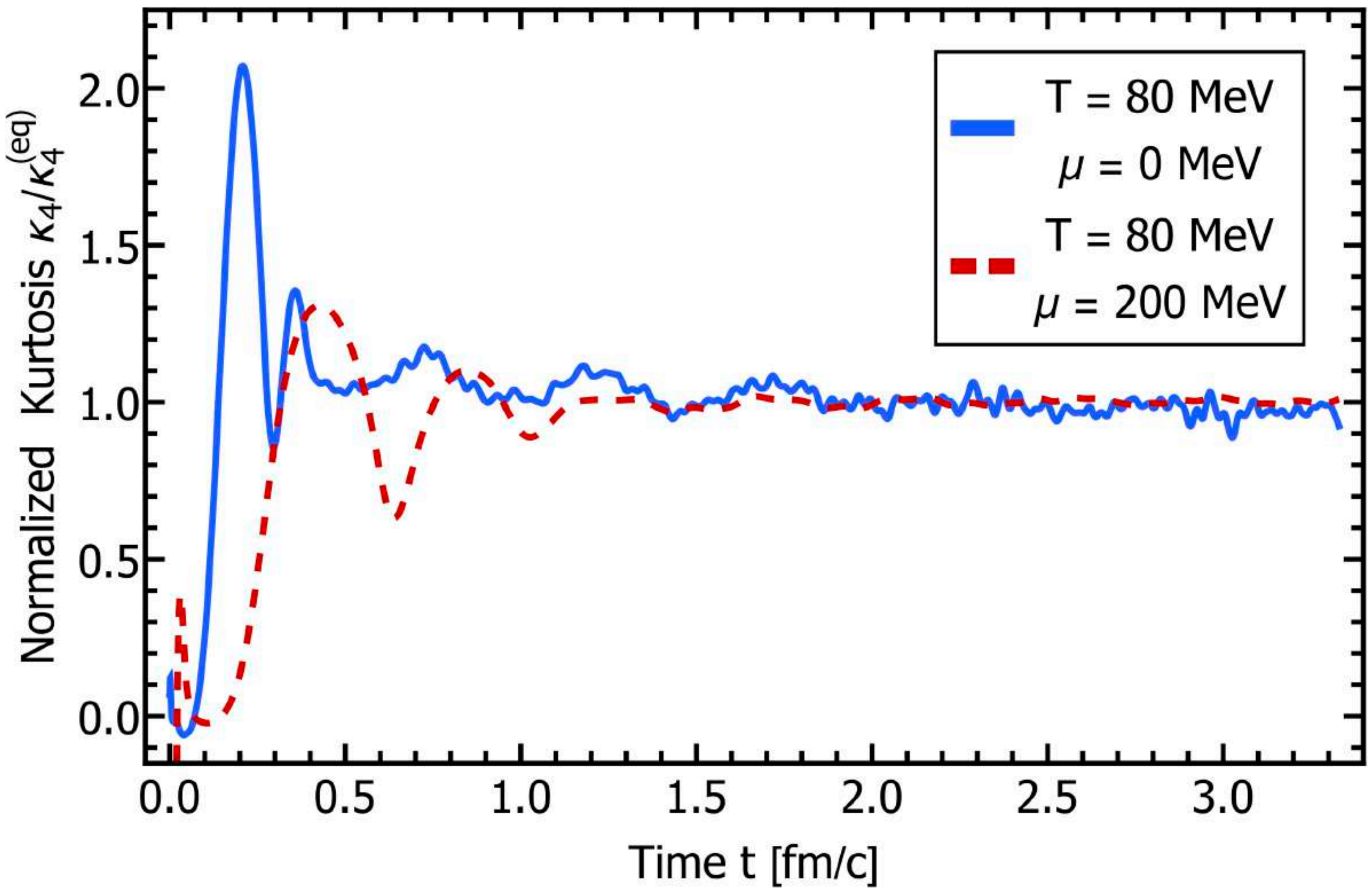


JMP, Rennecke, Wink, in prep

Time evolution of cumulants

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

Time evolution of sigma-kurtosis



Time evolution of the critical (scalar) σ - mode

noise field

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion

nth central moment of the sigma field: χ_n

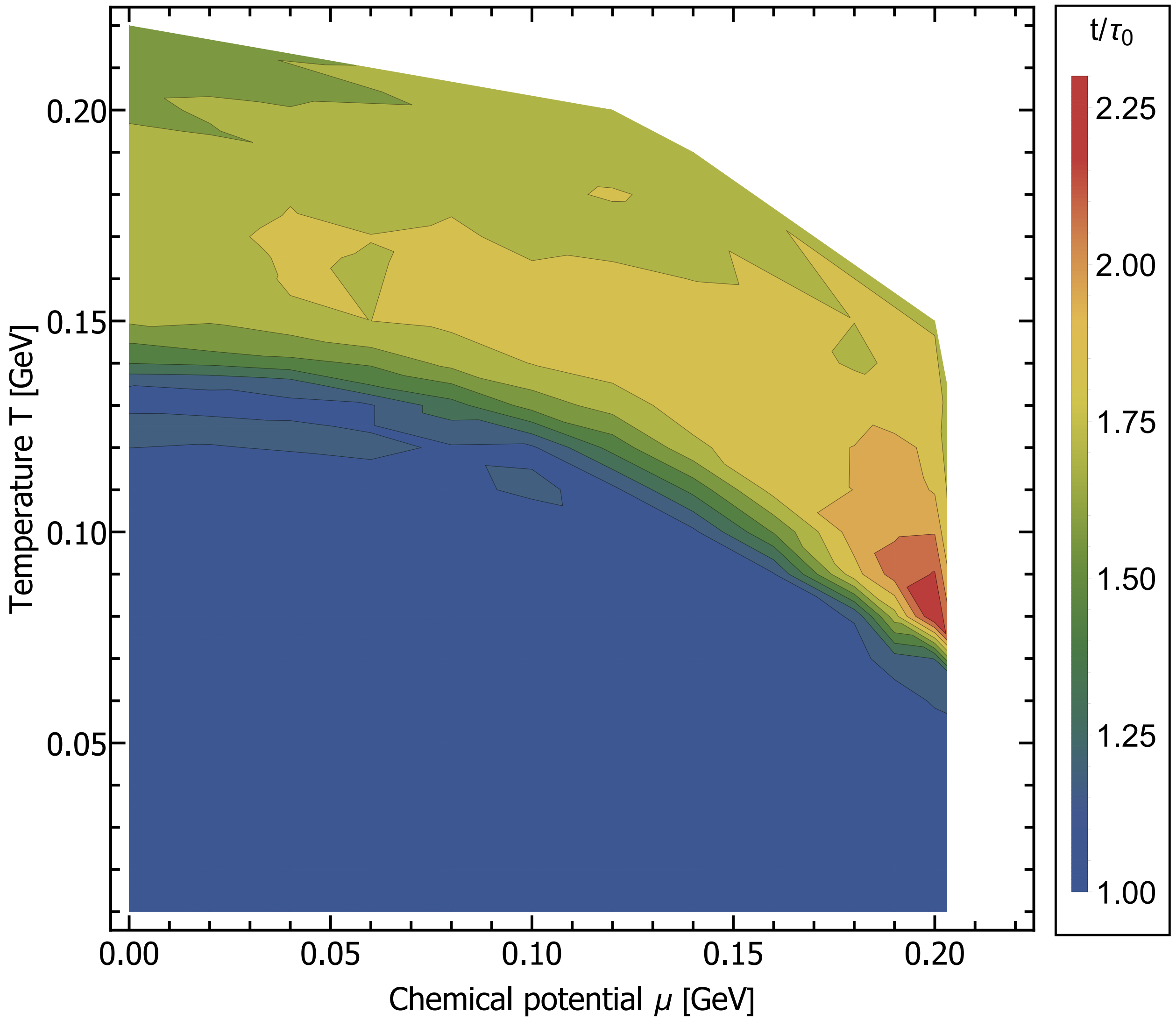
$$\chi_2 = \langle(\sigma - \langle\sigma\rangle)^2\rangle$$

kurtosis: $\kappa = \frac{\chi_4}{\chi_2^2} - 3$

Equilibration time phase structure

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

Equilibration time of sigma-kurtosis



nth central moment of the sigma field: χ_n

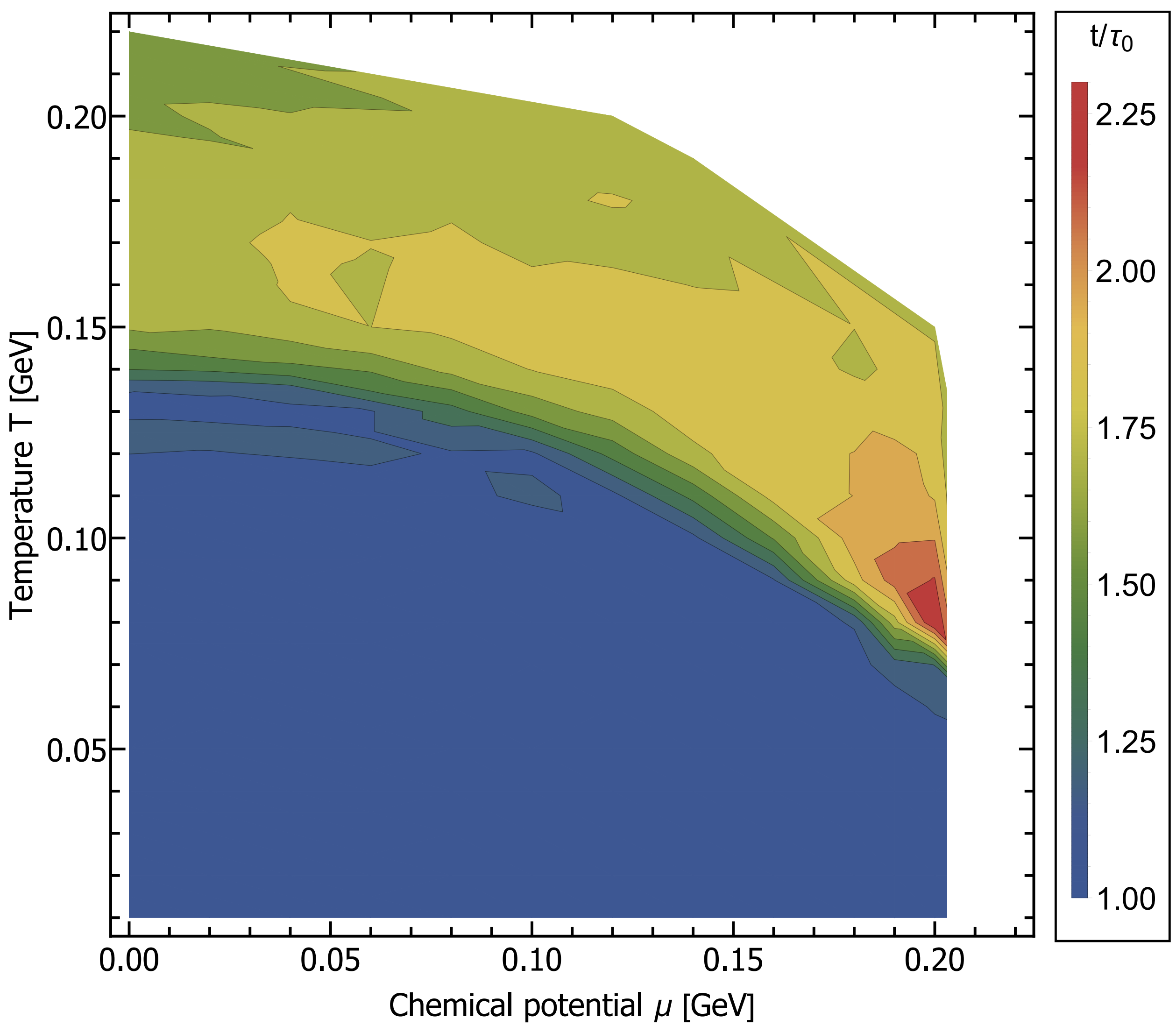
variance: $\chi_2 = \langle(\sigma - \langle\sigma\rangle)^2\rangle$

kurtosis: $\kappa = \frac{\chi_4}{\chi_2^2} - 3$

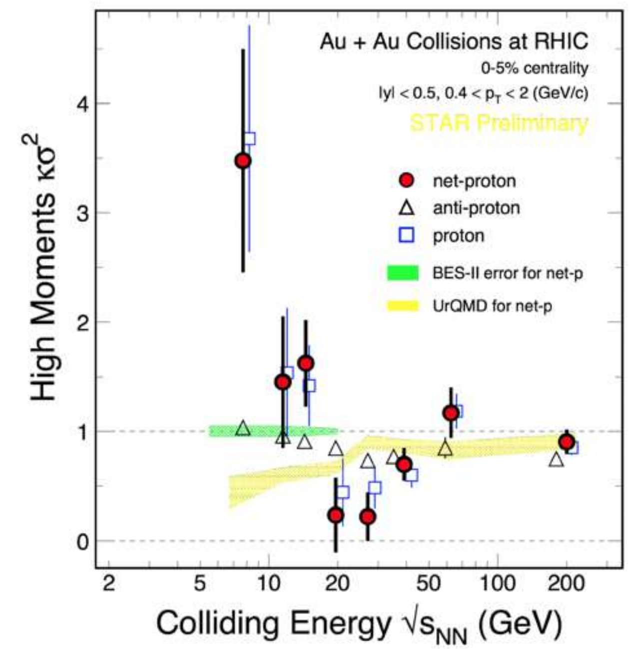
Equilibration time phase structure

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

Equilibration time of sigma-kurtosis

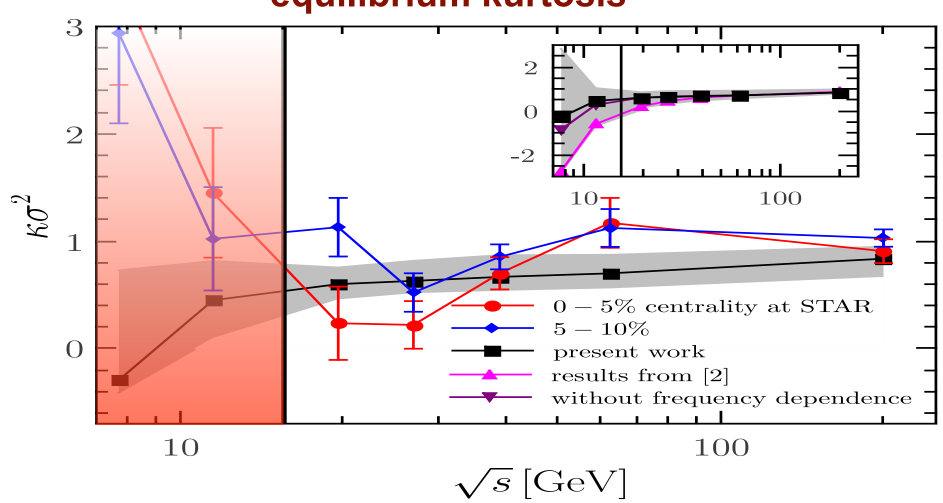


kurtosis of baryon number fluctuations



Luo, Cu, NST 28 (2017)

equilibrium kurtosis



nth central moment of the sigma field: χ_n

Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020

variance: $\chi_2 = \langle(\sigma - \langle\sigma\rangle)^2\rangle$

kurtosis: $\kappa = \frac{\chi_4}{\chi_2^2} - 3$