Extracting the Diffusivity and Charge Susceptibilities of the QGP from Experiment

Scott Pratt
Department of Physics & Astronomy …

S.P., J. Kim & C.Plumberg PRC(2018)
S.P. & C.Plumberg PRC(2019)
S.P. PRC (2020)
S.P. & R.Steinhorst, PRC (2020)
S.P. & C.Plumberg, PRC (2020)

Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

Transport Coefficients

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

Charge balance functions are principal tool

Transport Coefficients

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

Charge balance functions also important for:

- CME background
- Background for fluctuations for phase transitions

I. Theory of Correlations and Balance Functions

Charge Correlations
(Equilibrated System)

$$C_{ab}(\vec{r}_1, \vec{r}_2) \equiv \langle \delta\rho_a(\vec{r}_1)\delta\rho_b(\vec{r}_1)\rangle = \chi_{ab}\delta(\vec{r}_1 - \vec{r}_2),$$

$$\chi_{ab} = \frac{1}{V}\langle \delta Q_a \delta Q_b \rangle = \frac{T^2}{Z}\frac{\partial^2}{\partial\mu_a\partial\mu_b}\ln Z,$$

3x3 matrix

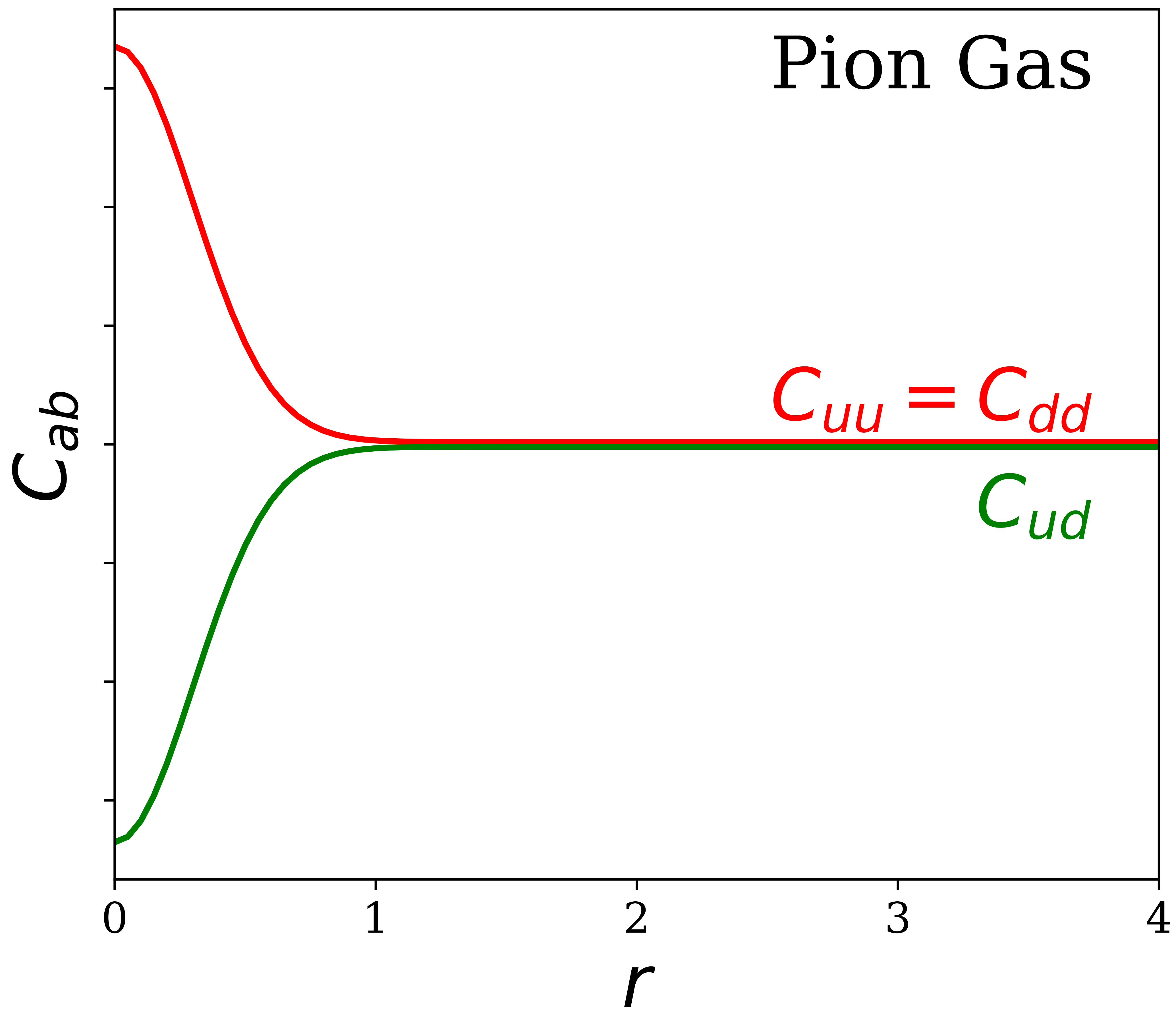
$$= \int d^3r\ C_{ab}(0, \vec{r}).$$

Quark Gas:

$$\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$$

Hadron Gas:

$$\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$$

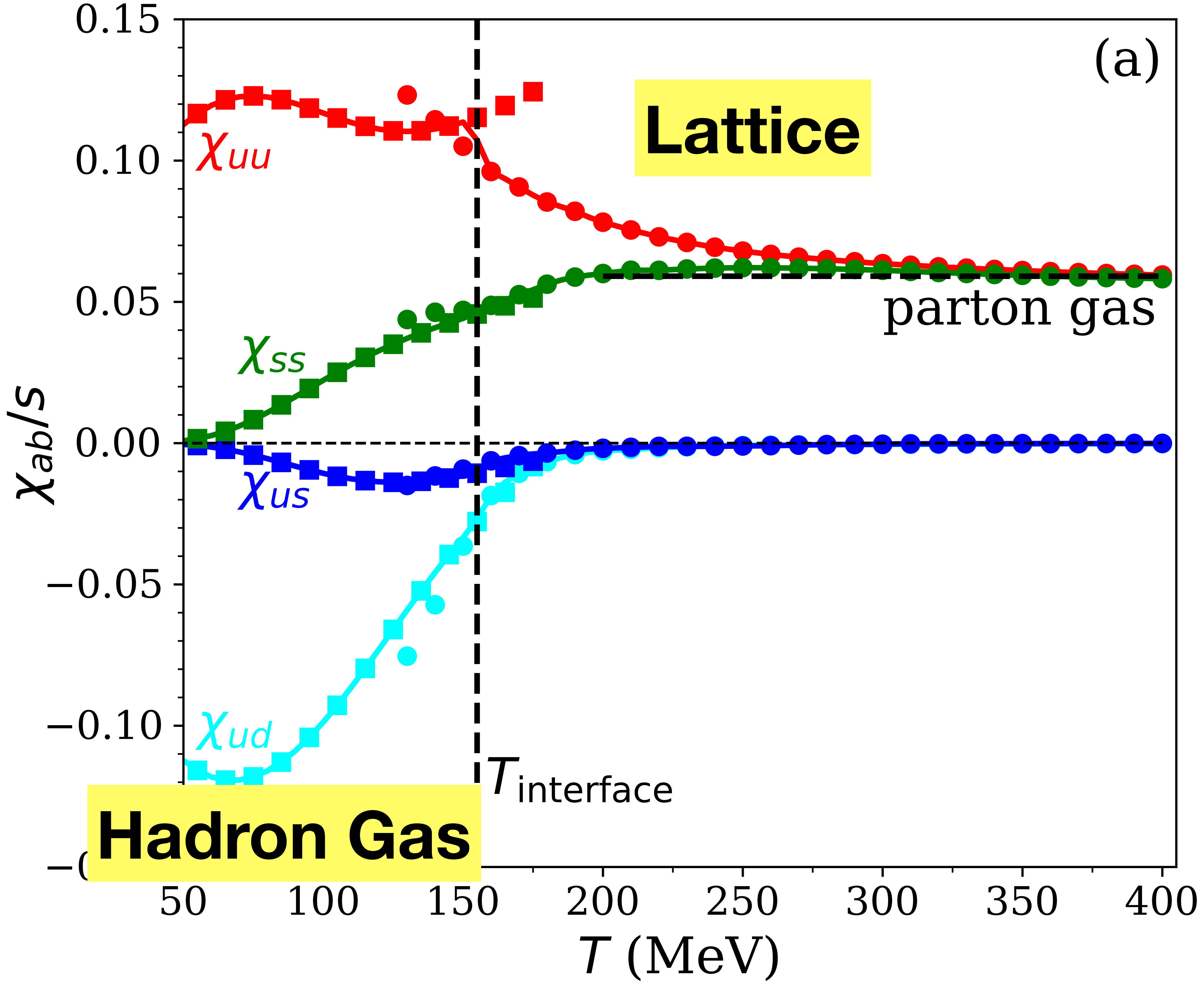


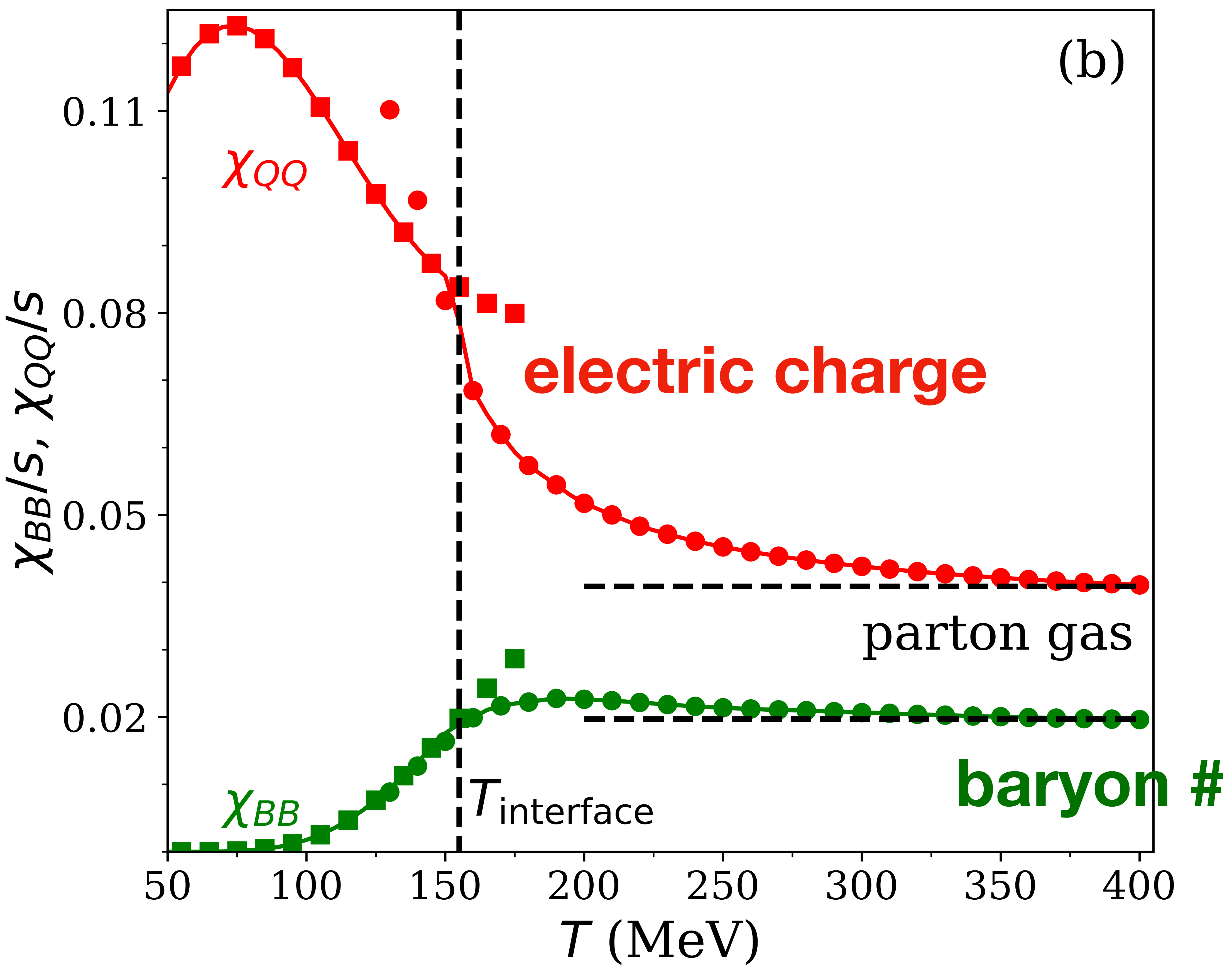
Susceptibility
(Lattice, BW-Claudia Ratti)

For hadron gas: $\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$

a=(u,d,s)

For parton gas: $\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$





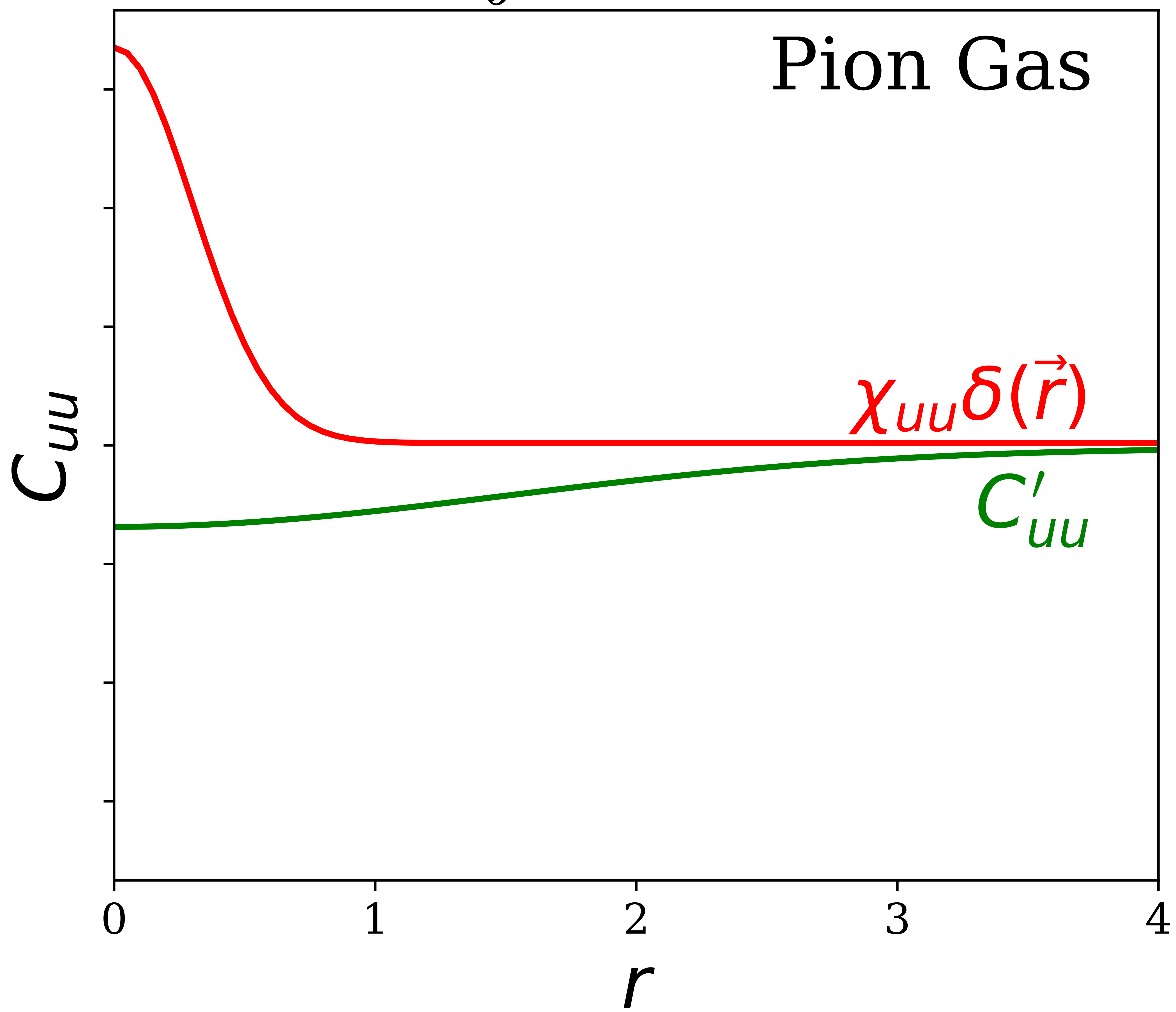
Charge Correlations

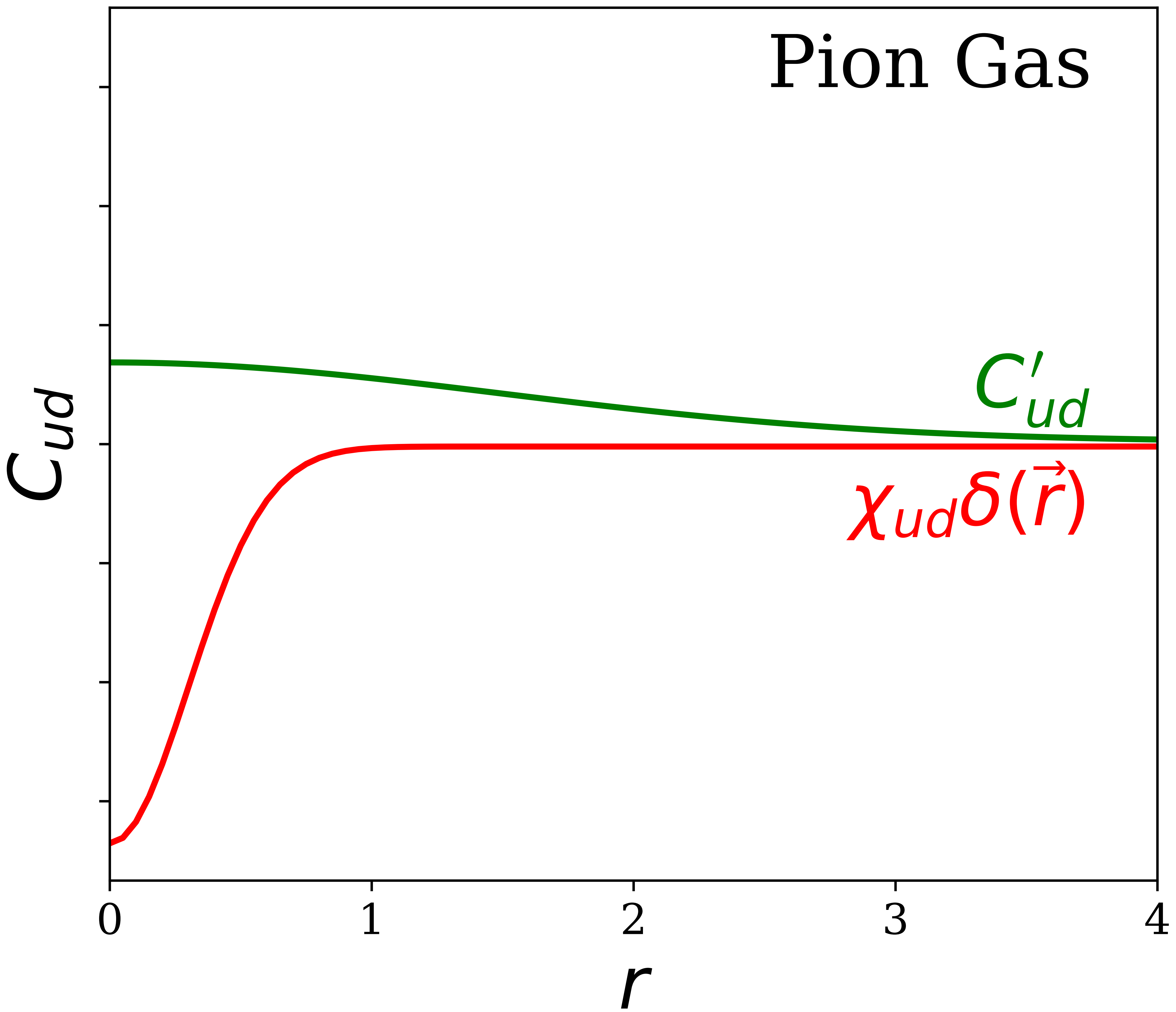
(Dynamic System)

$$C_{ab}(\vec{r}_1, \vec{r}_2) = \chi_{ab}\delta(\vec{r}_1 - \vec{r}_2) + C'_{ab}(\vec{r}_1, \vec{r}_2)$$

$$\int d^3r C'_{ab}(\vec{r}) = -\chi_{ab}$$

***Balancing* correlation**

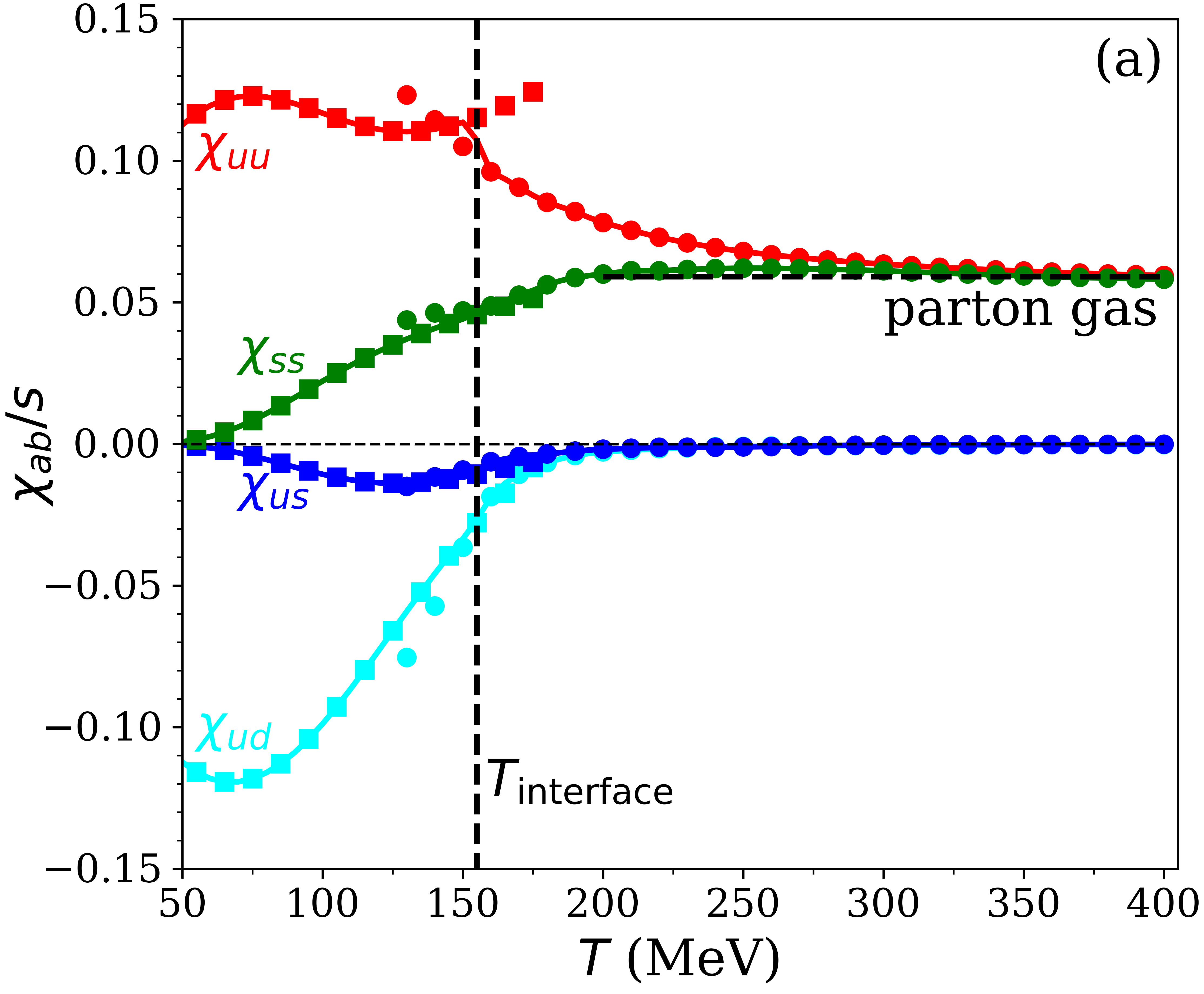


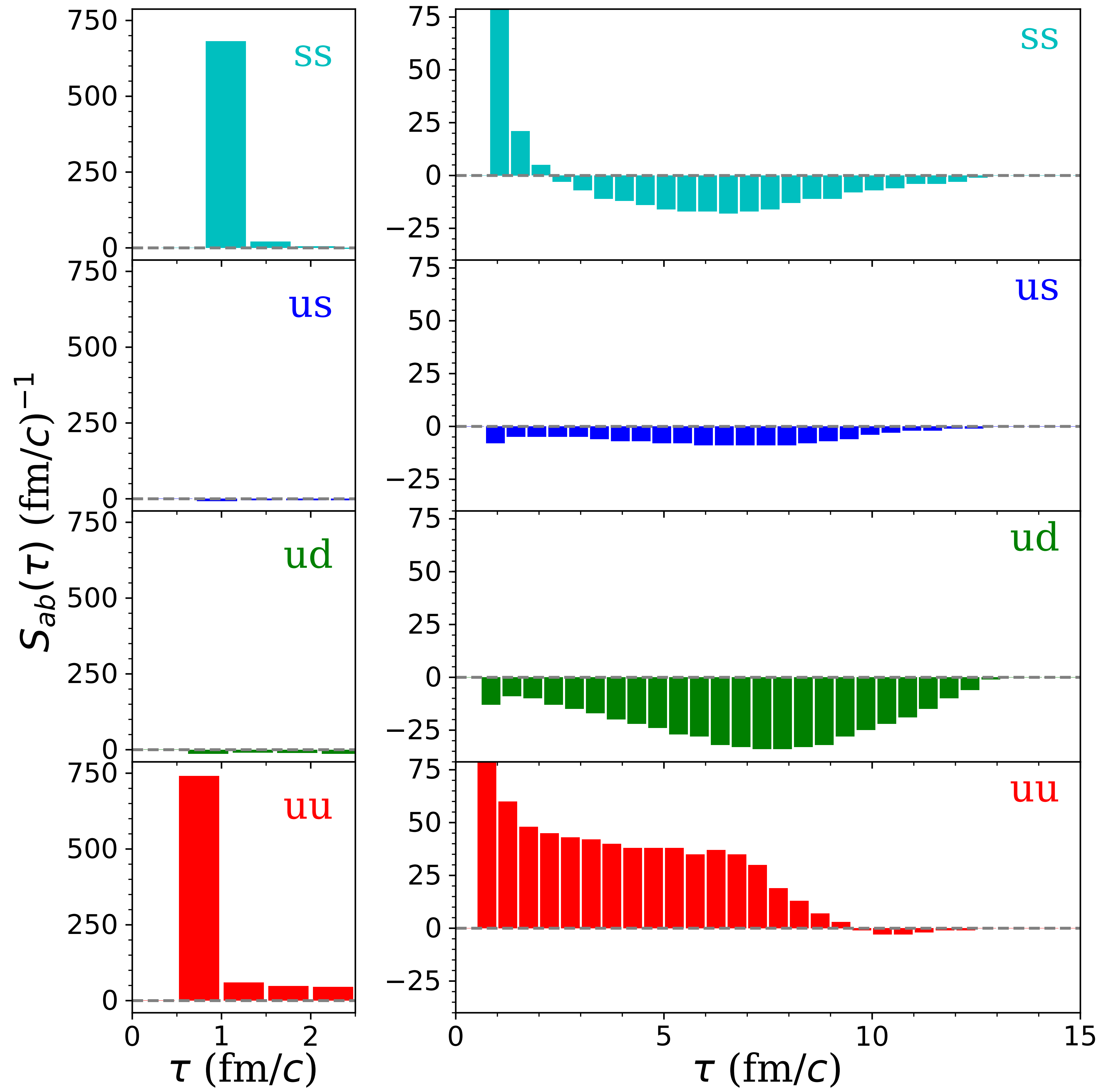


Eq.s of motion

$$\partial_t C'_{ab}(\vec{r}_1 - \vec{r}_2) - D_{ab}\nabla_1^2 C'_{ab}(\vec{r}_1, \vec{r}_2) - D_{ab}\nabla_2^2 C'_{ab}(\vec{r}_1, \vec{r}_2) = S_{ab}(\vec{r}_1, t)\delta(\vec{r}_1 - \vec{r}_2),$$

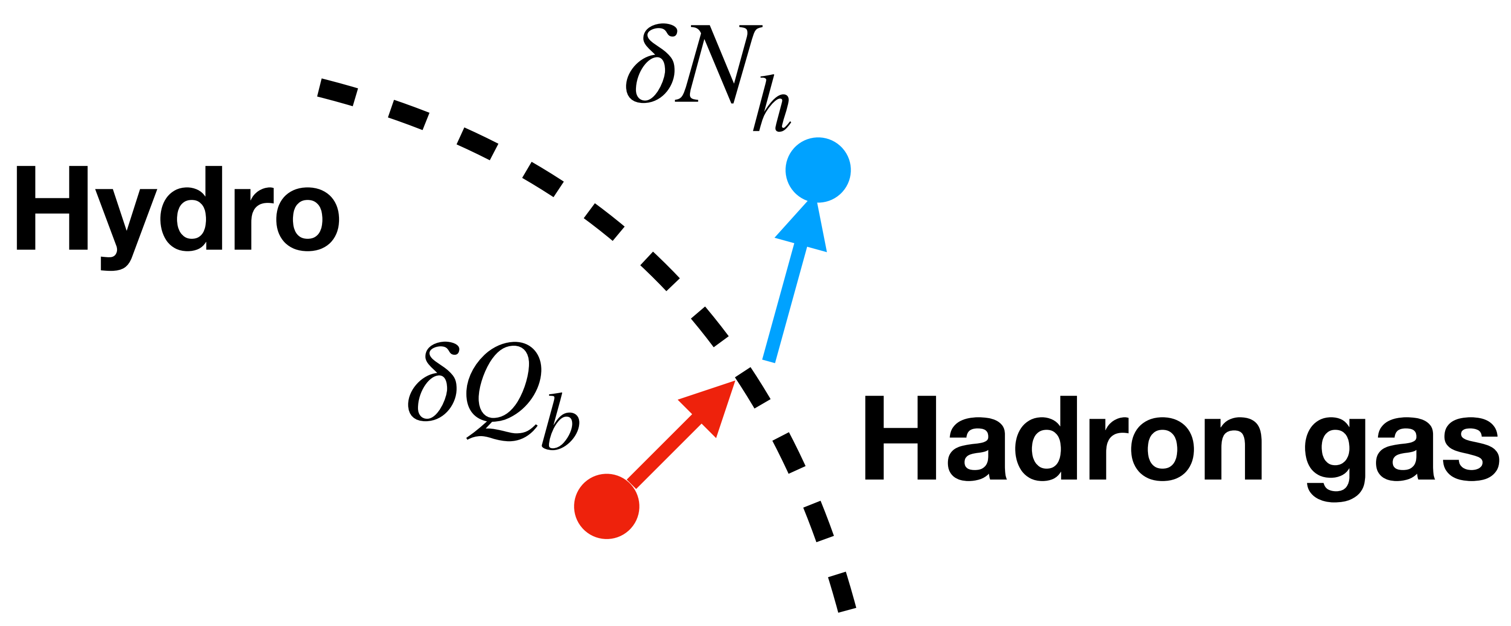
$$S_{ab}(\vec{r}, t) = \left[\partial_t + \vec{v}\cdot\vec{\nabla} + (\nabla\cdot\vec{v})\right]\chi_{ab}(\vec{r}, t) \approx s(\vec{r}, t)\left[\partial_t + \vec{v}\cdot\vec{\nabla}\right]\frac{\chi_{ab}(\vec{r}, t)}{s(\vec{r}, t)}$$





Translate C_{ab} into $C_{hh'}$

$$\delta N_h = \chi^{-1}_{ab}(T_{\rm interface}) q_{ha} n_h \delta Q_b$$



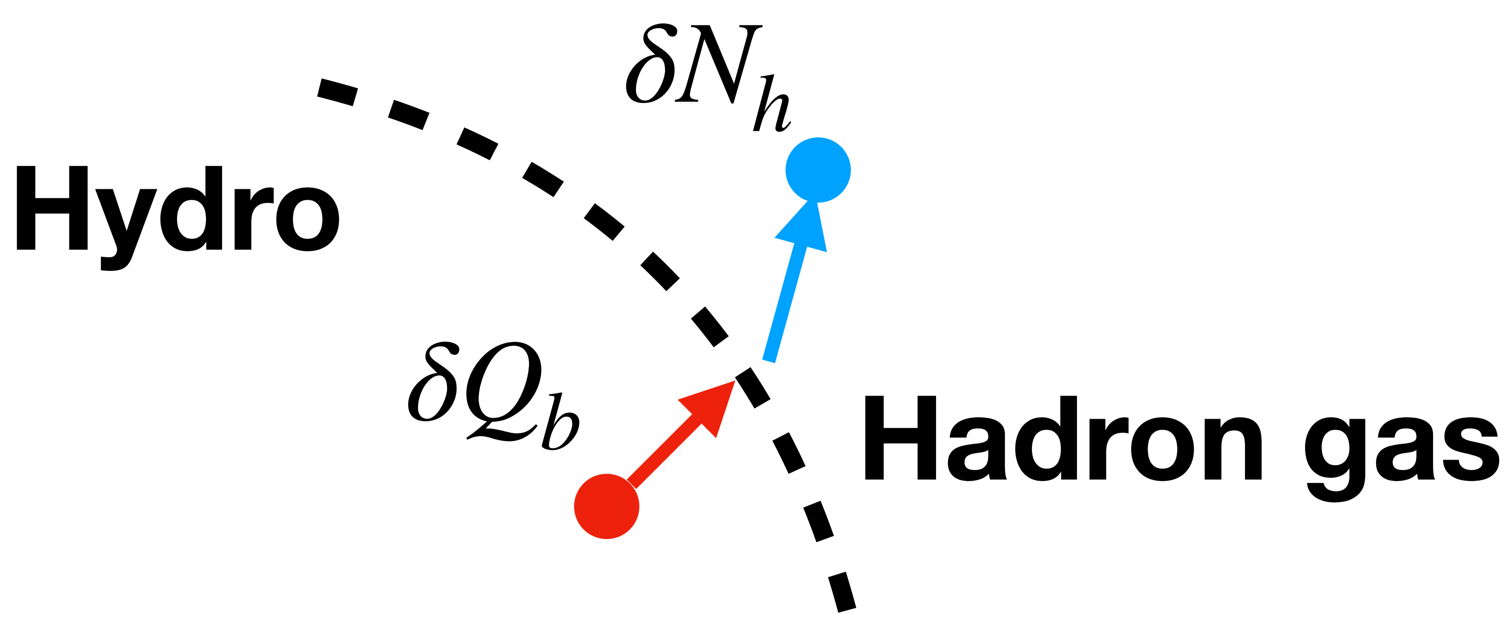
Charge Balance Function

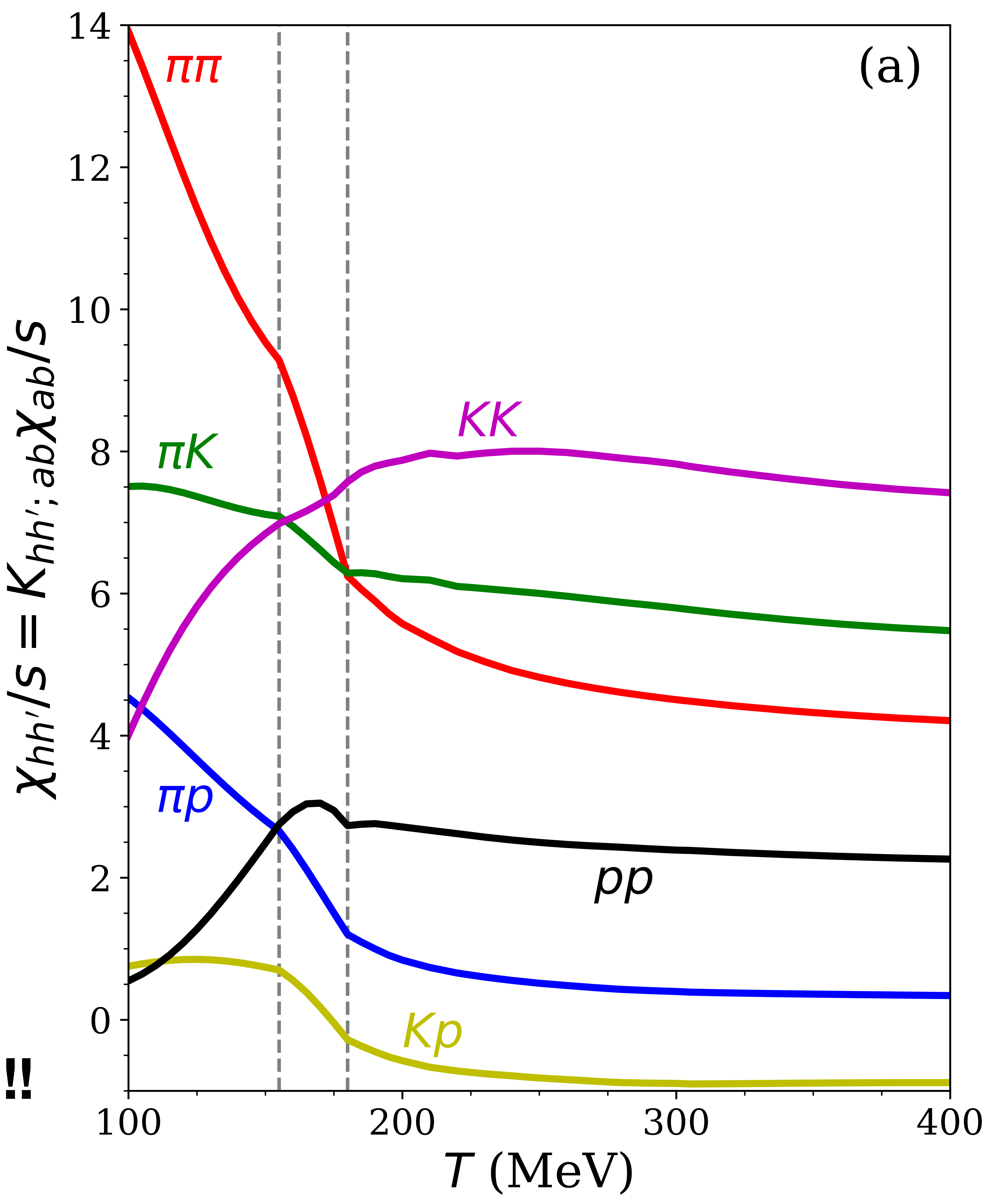
$$C_{hh'}(\vec{r}_1,\vec{r}_2) = \langle [\delta n_h(\vec{r}_1) - \delta n_{\bar{h}}(\vec{r}_1)][\delta n_{\bar{h}'}(\vec{r}_2) - \delta n_{h'}(\vec{r}_2)]\rangle,$$

$$B_{h|h'}(\vec{p}_1|\vec{p}_2) = \frac{1}{N_{h'}(\vec{p}_2) + N_{\bar{h}'}(\vec{p}_2)} \langle [\delta N_h(\vec{p}_1) - \delta N_{\bar{h}}(\vec{p}_1)][\delta N_{\bar{h}'}(\vec{p}_2) - \delta N_{h'}(\vec{p}_2)]\rangle$$

Translate C_{ab} into $C_{hh'}$

$$\delta N_h = \chi^{-1}_{ab}(T_{\text{interface}}) q_{ha} n_h \delta Q_b$$





$B_{\pi\pi}$ **has contribution from hadronization stage**

B_{pp} **and** B_{KK} **are sourced at thermalization**

$B_{\pi\pi}(\Delta y)$ **should be narrower than** $B_{KK}(\Delta y)$ **or** $B_{pp}(\Delta y)$**!!!**

Diffusivity

$$\vec{j}_a = -D_{ab}\nabla\rho_b,$$
3x3 matrix (colors)
$$= -\sigma_{ab}\nabla(\mu_b/T),$$
$$\sigma = \chi D,$$
$$\chi_{ab} = \langle\delta Q_a \delta Q_b\rangle/V = \partial\rho_a/\partial(\mu_b/T)$$

susceptibility

Kubo Relation

$$\sigma_{ab} = \frac{1}{2T}\int d^4x \; \langle\{j_a(0), j_b(x)\}\rangle$$

difficult (not impossible) for lattice gauge theory

II. The Model

Diffusion = Random walk

Monte Carlo procedure:

A) Overlay with hydro evolution to create $S_{ab}(t, \vec{r})$

B) Generate partners (uu,dd,ss,ud,us,ss) proportional to $S_{ab}(t, \vec{r})$ with weights

C) Move particles in random directions punctuated by re-directioning according to $\tau_{\rm coll}$

D) Translate δQ_a to δN_h at hyper surface

E) Collide (fixed σ) and decay particles

F) Combine decay products with those from partner

G) Correlations created during hadronic phase: create uncorrelated hadrons, run through cascade, combine ALL particles to create BF

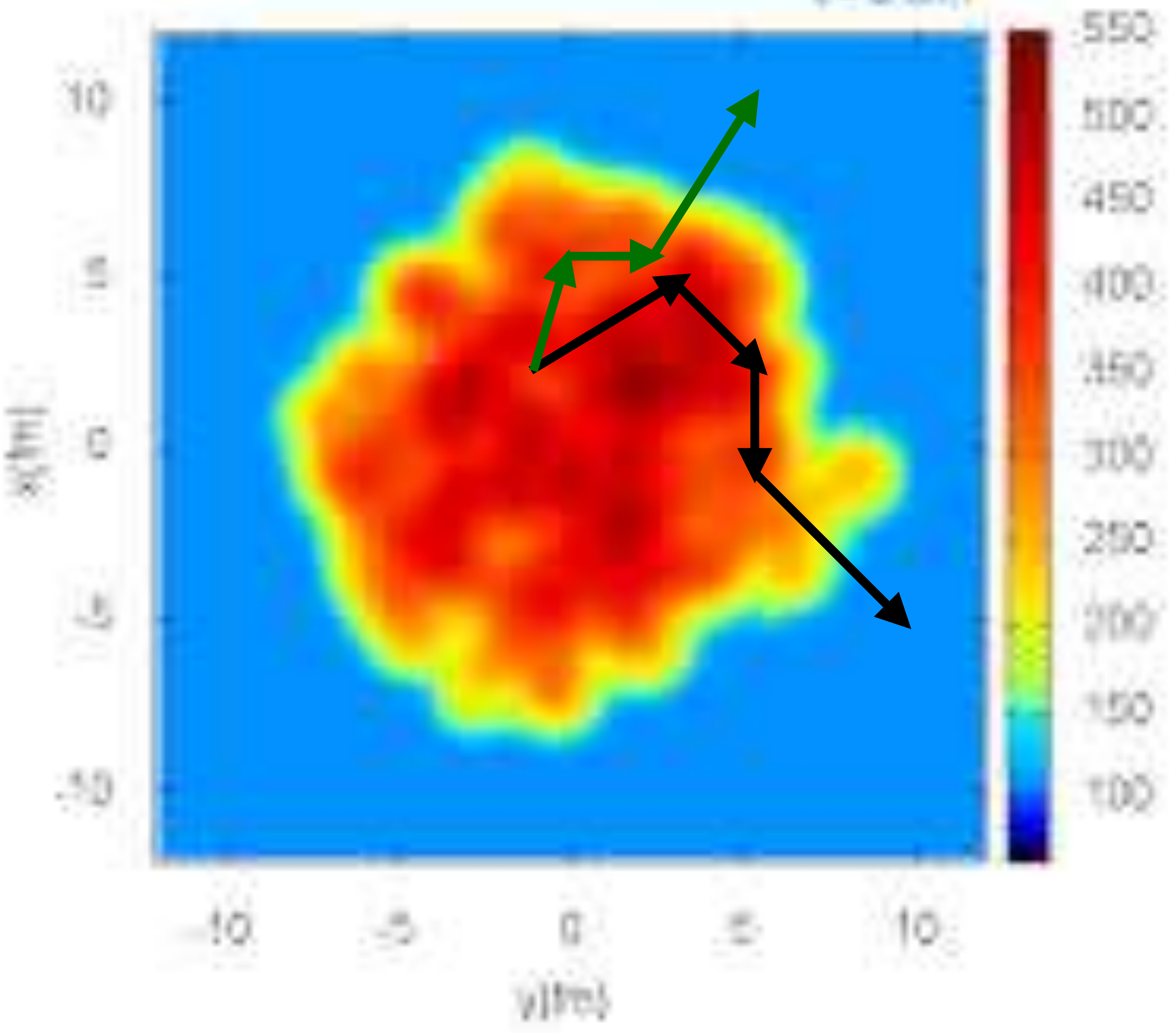
H) Add contributions from (E) and (F)

I) Fold with acceptance/efficiency

J) Test sum rules

ALGORITHM

$$\tau_{\rm coll} = 6D$$



ALGORITHM

Correlations from Hydro:

— Depends of D and σ_0

— Only a few hours of CPU

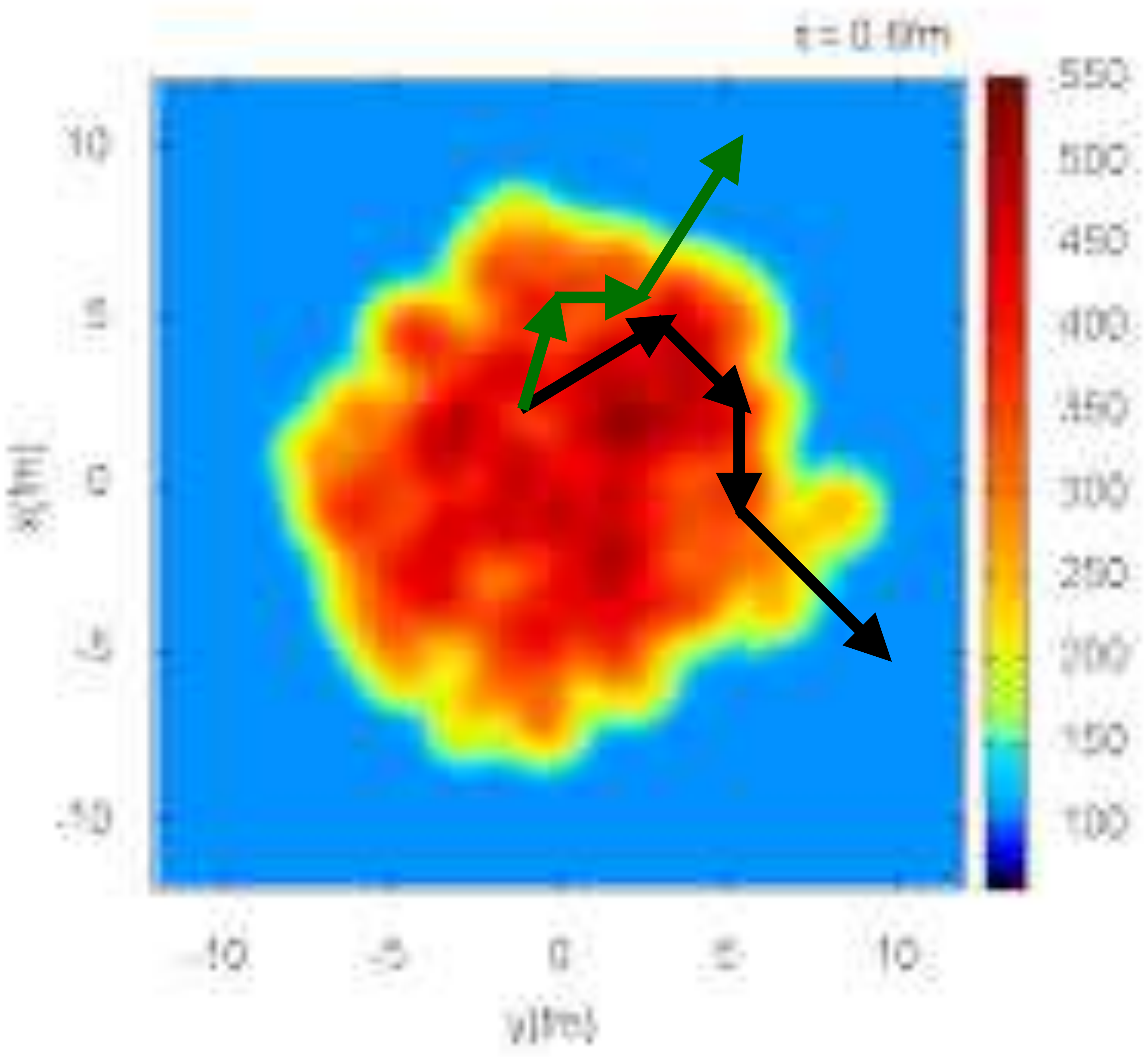
— track charges from same source point

Correlations from Cascade

— Weeks of CPU

— One hydro event (independent of D, σ_0)

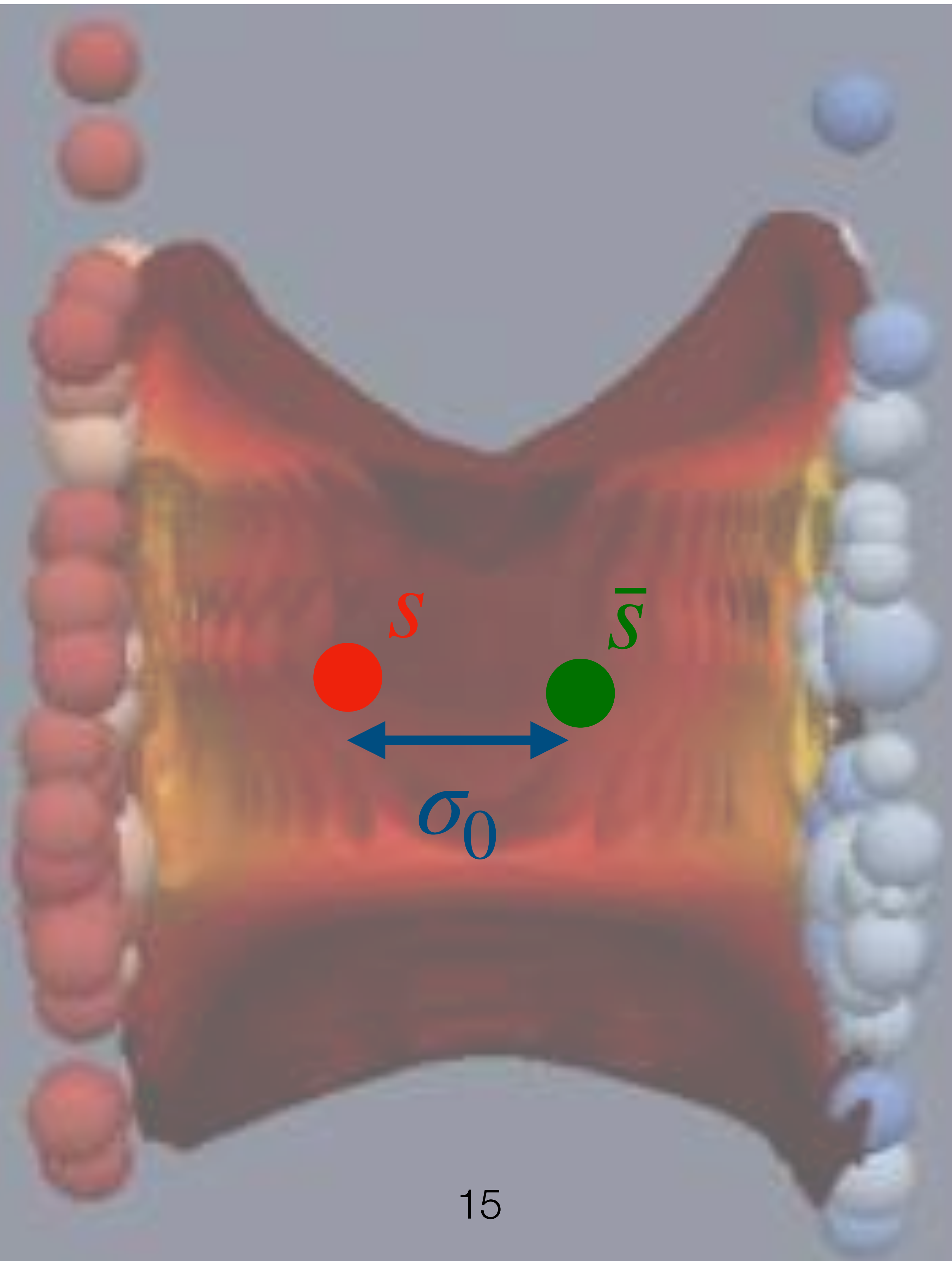
— Millions of cascade events



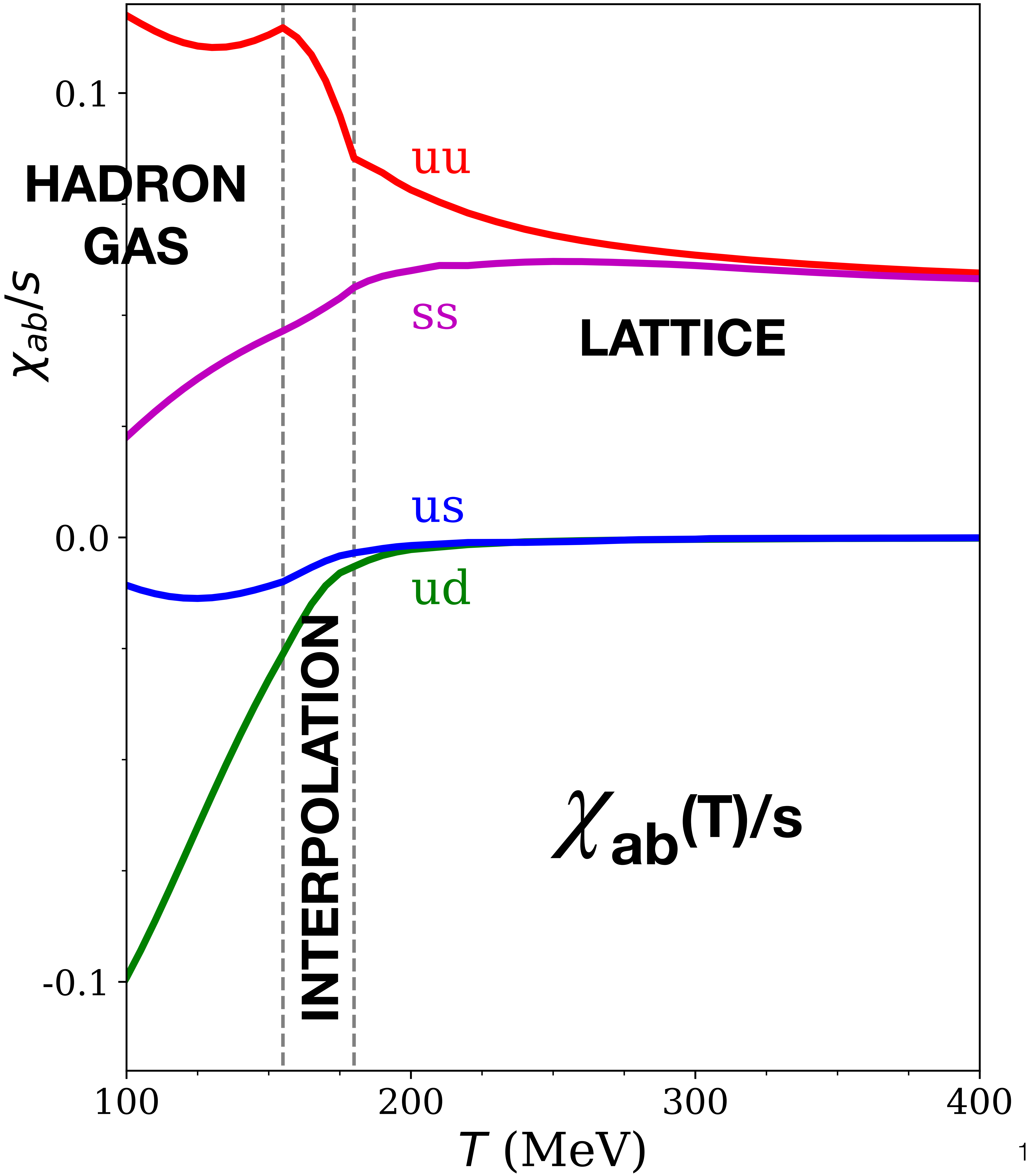
Adjustable Parameters

1. **Diffusion Constant $D(T)$ (multiples of lattice values)**
2. **T_h = 155 MeV**
3. **σ_0 = spread in spatial rapidity at τ_0 = 0.6 fm/c**

   — creation before τ_0 or tunneling (flux tubes)

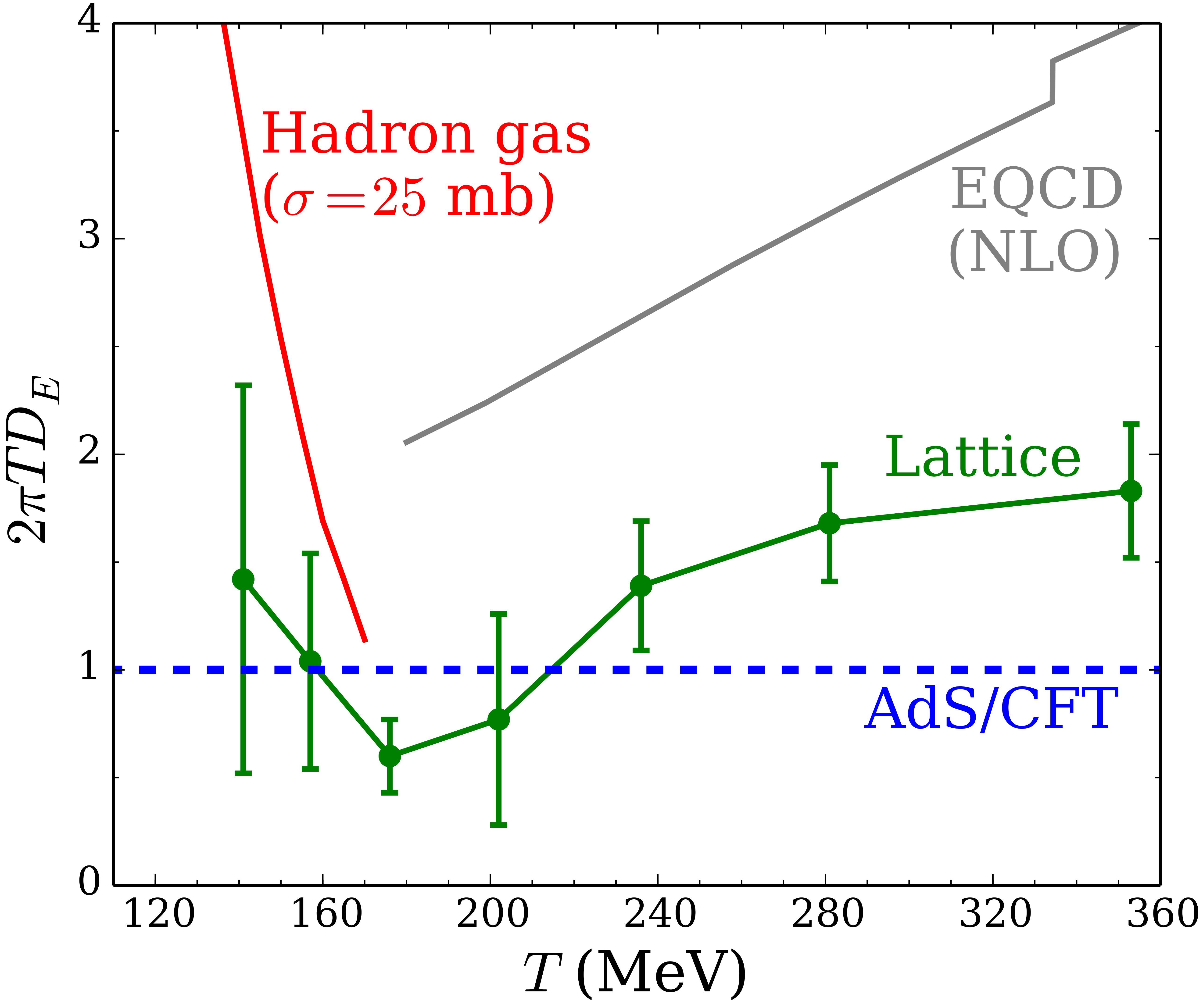


Model input Susceptibility



Claudia Ratti
BW Collaboration

$D(T)$ — No Clear Consensus

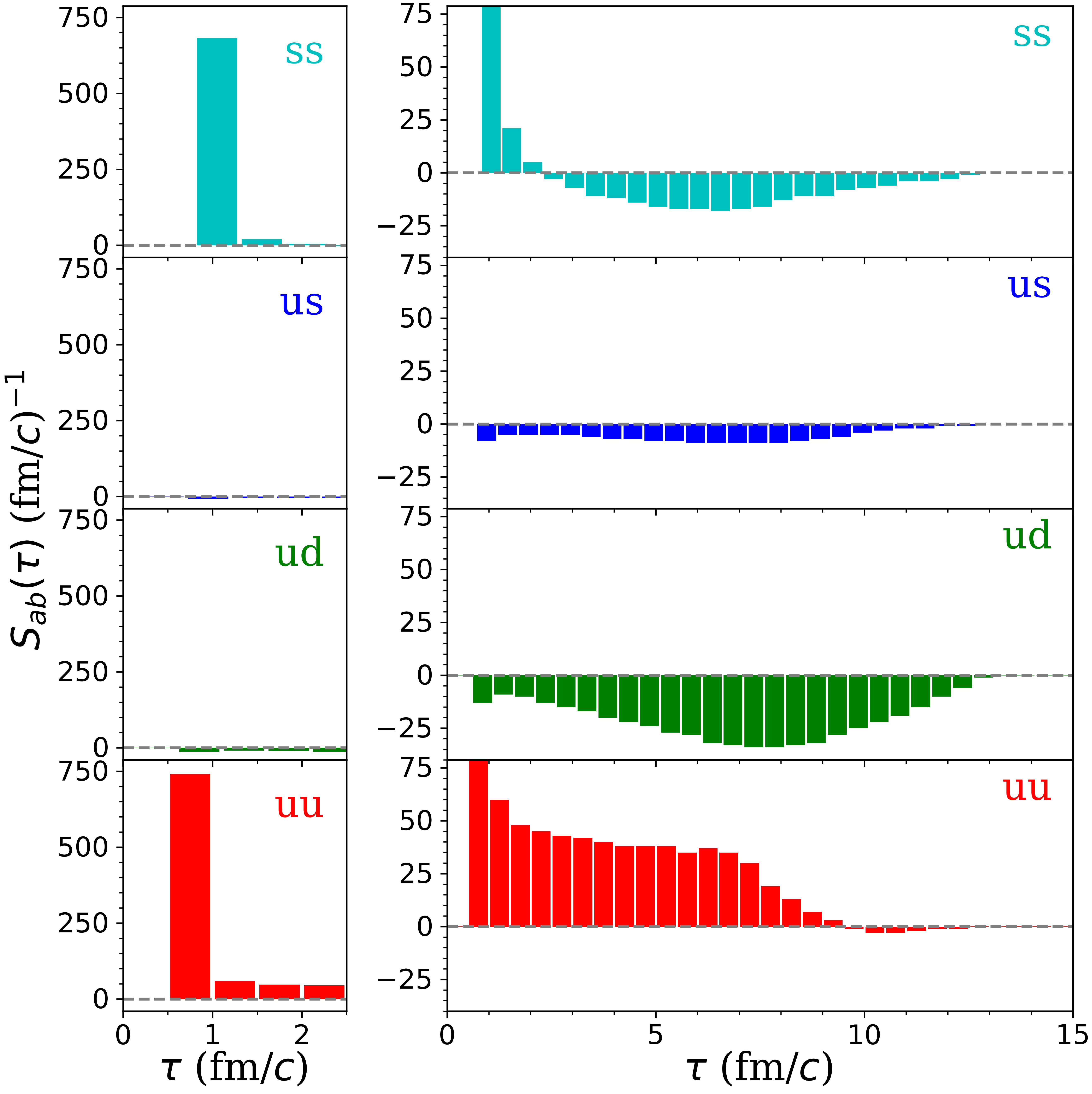


G.Aarts et al, JHEP (2015)
J.Ghiglieri et al, JHEP (2018)
G.Policastro et al, JHEP (2002)

Model input
Hydro history

Chris Plumberg

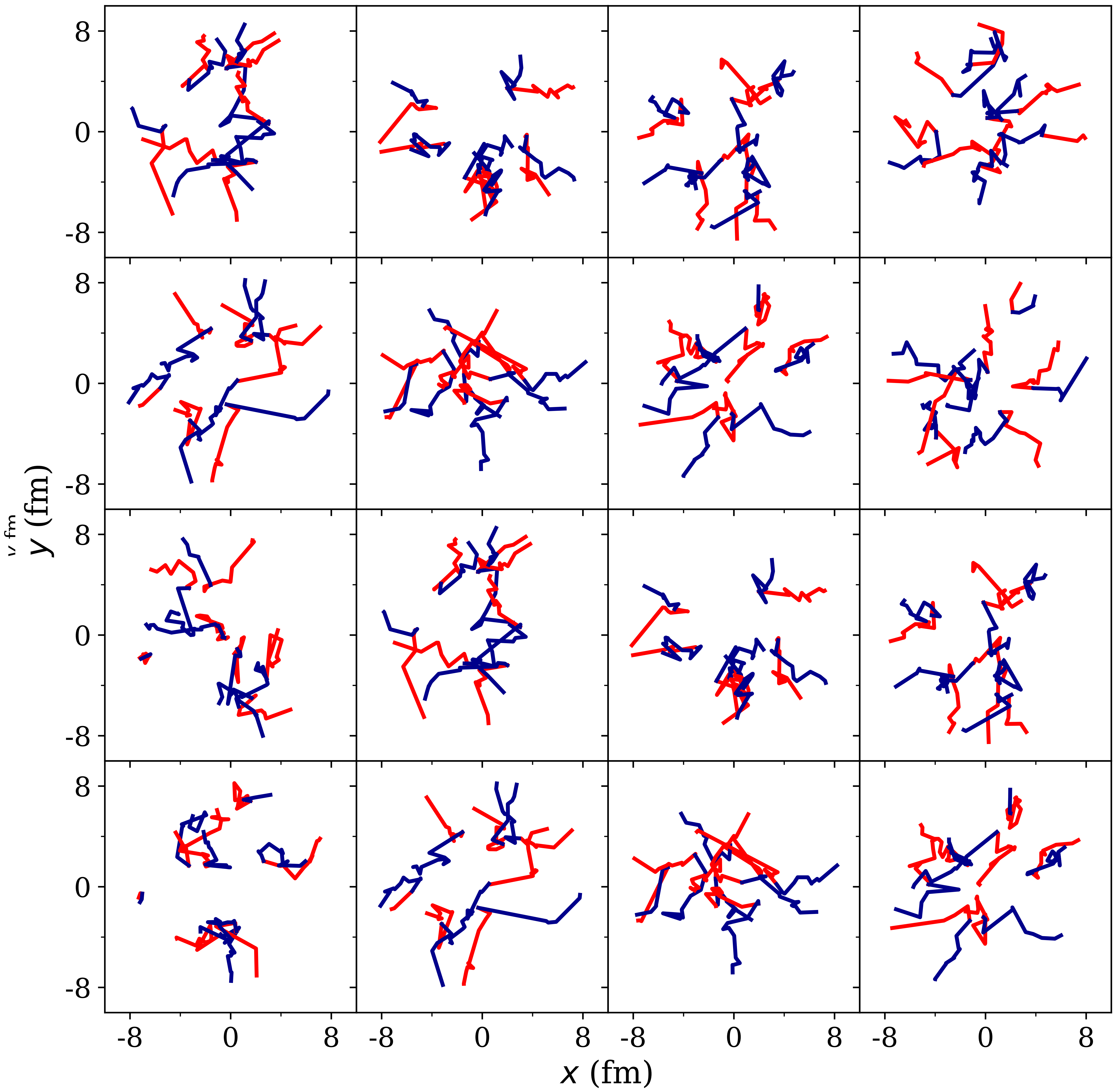
VISHNU Hydro, Au+Au (200A GeV)



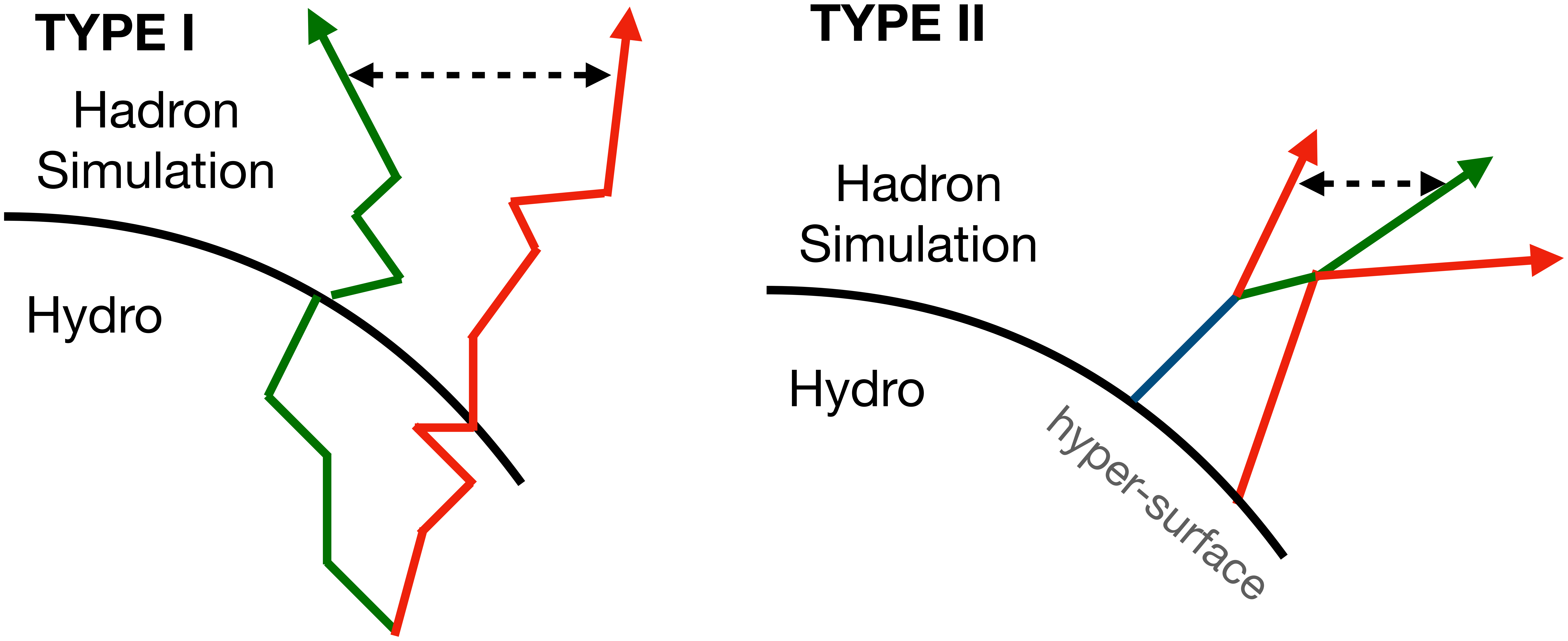
Source Function

- First surge when QGP is created
- uu,dd continuously created
- ss nearly steady
- ud,us,ds at hadronization

Diffusive Trajectories



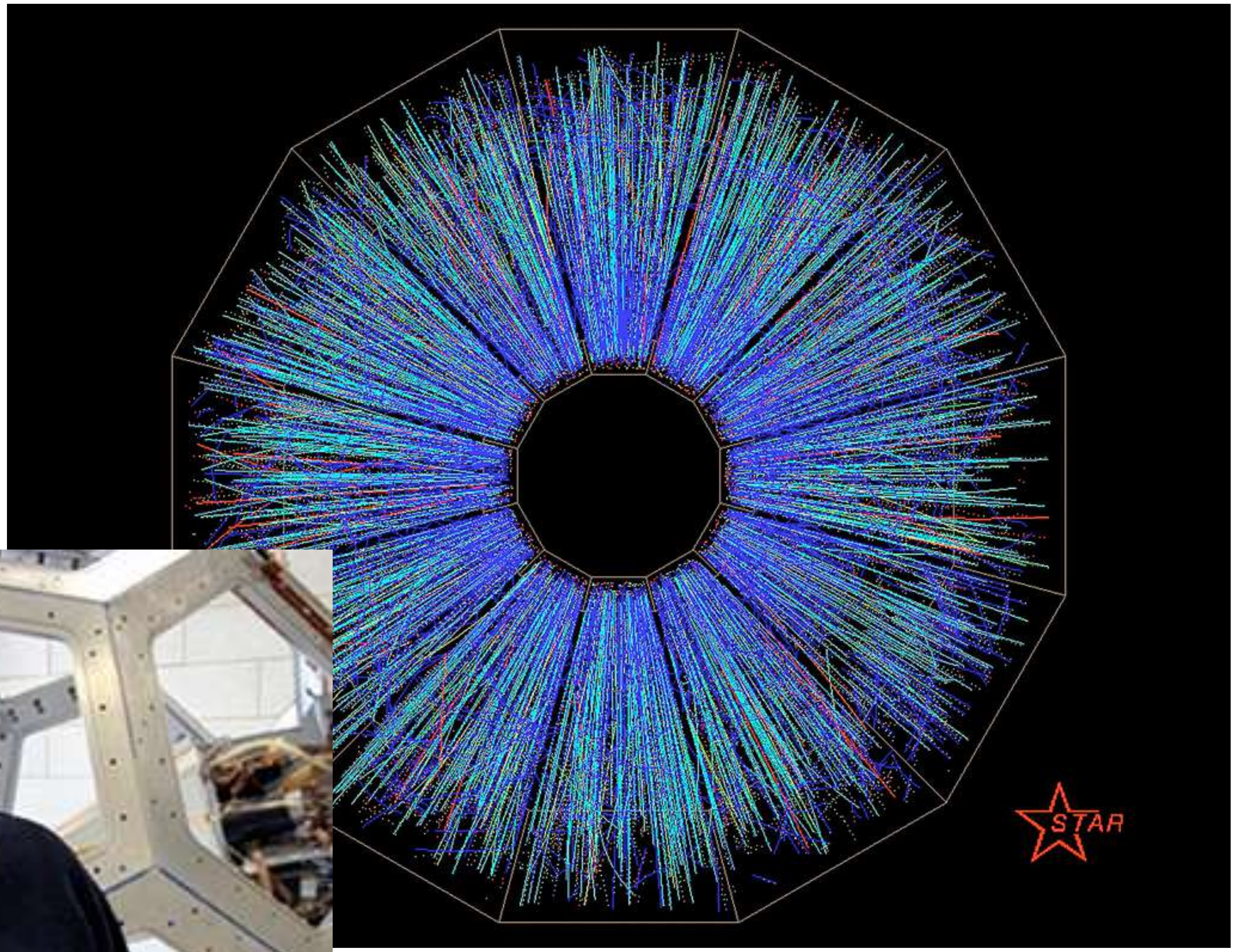
ALGORITHM

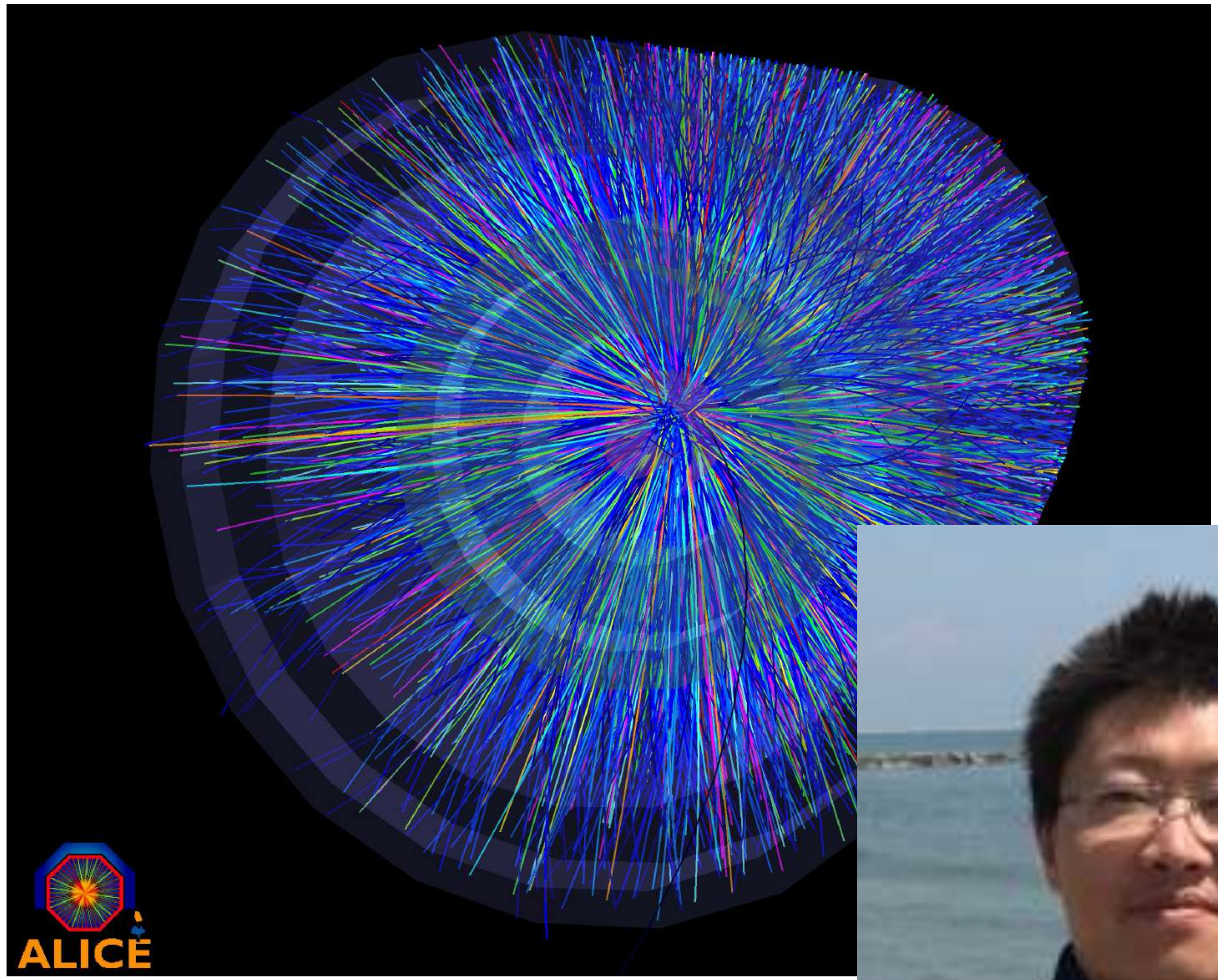


TYPE I
Hadron Simulation
Hydro
TYPE II
Hadron Simulation
Hydro
hyper-surface

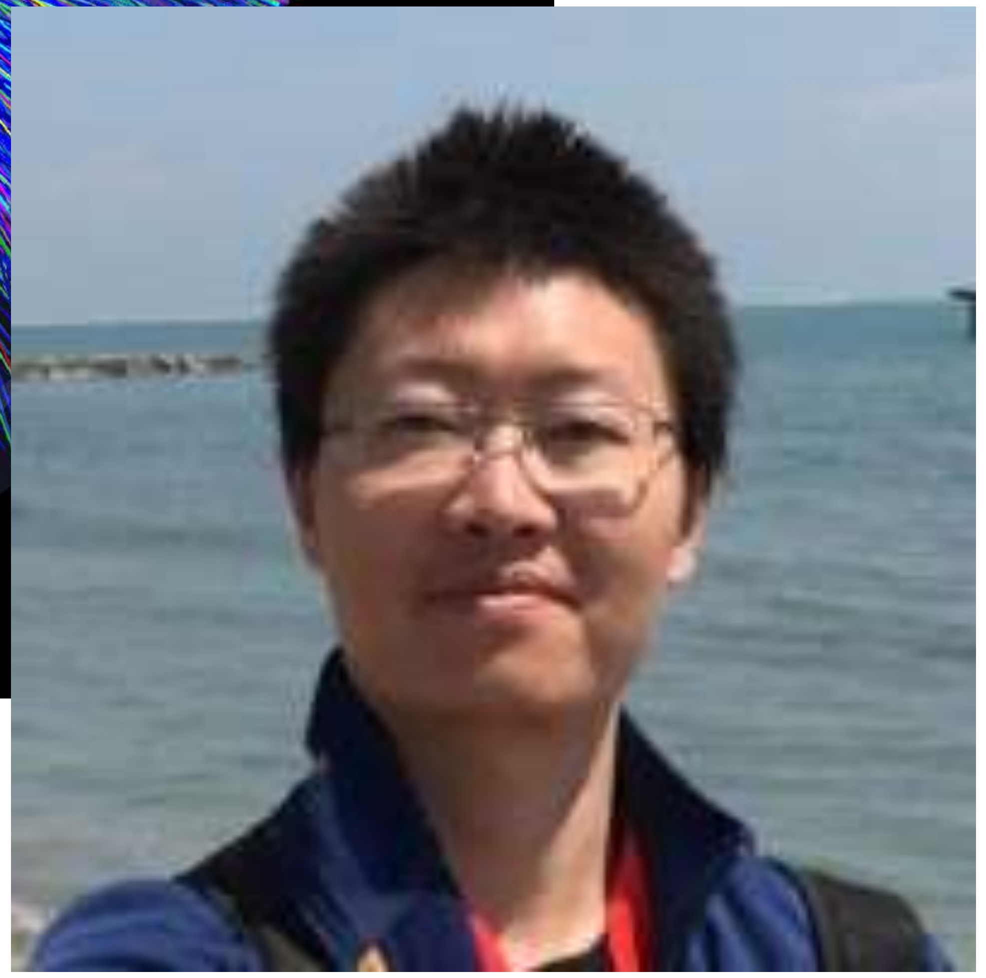
III. Results vs. Data

Experimental Acceptance/Efficiency

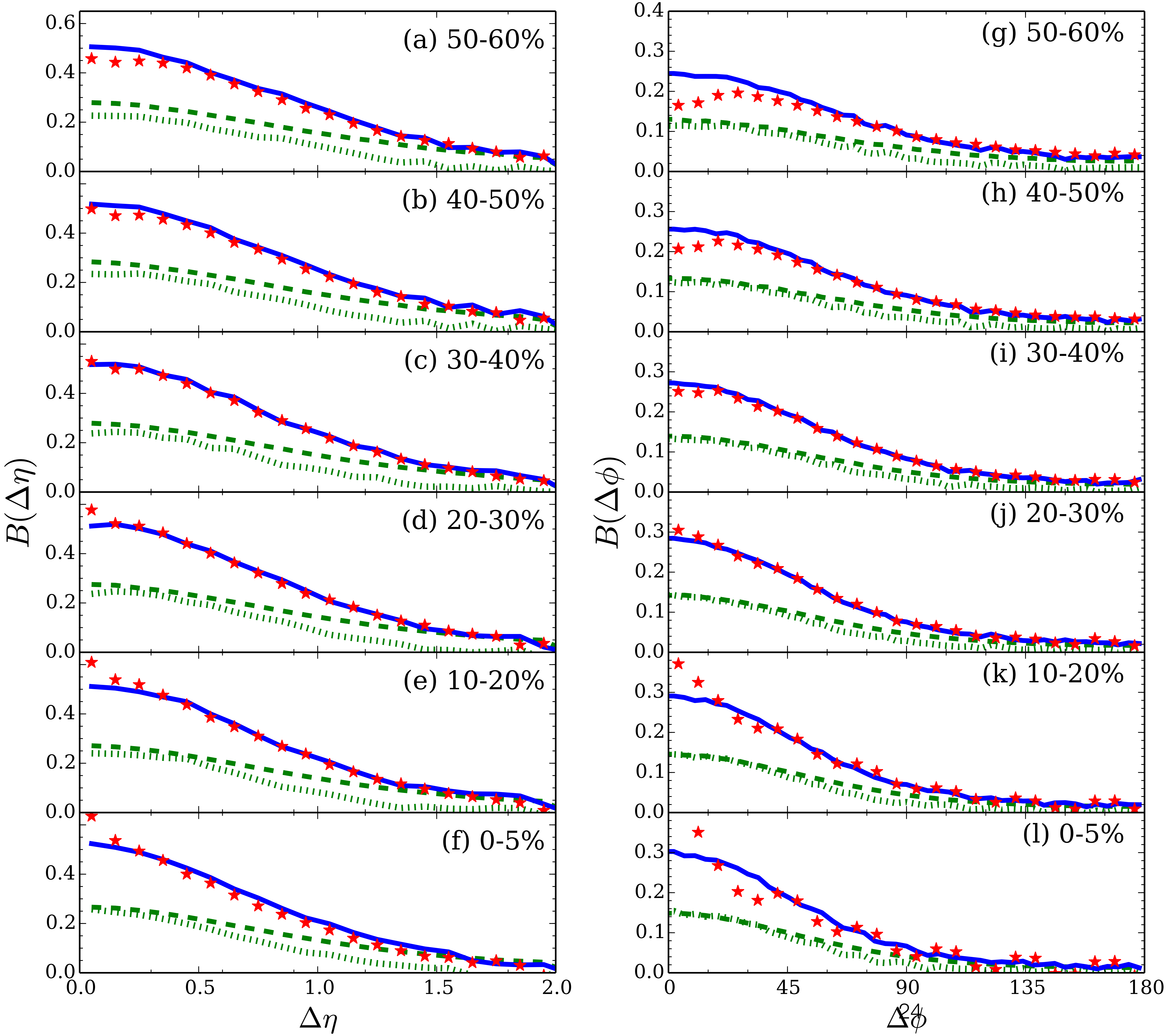




Gary Westfall
MSU



Jinjin Pan
Wayne State



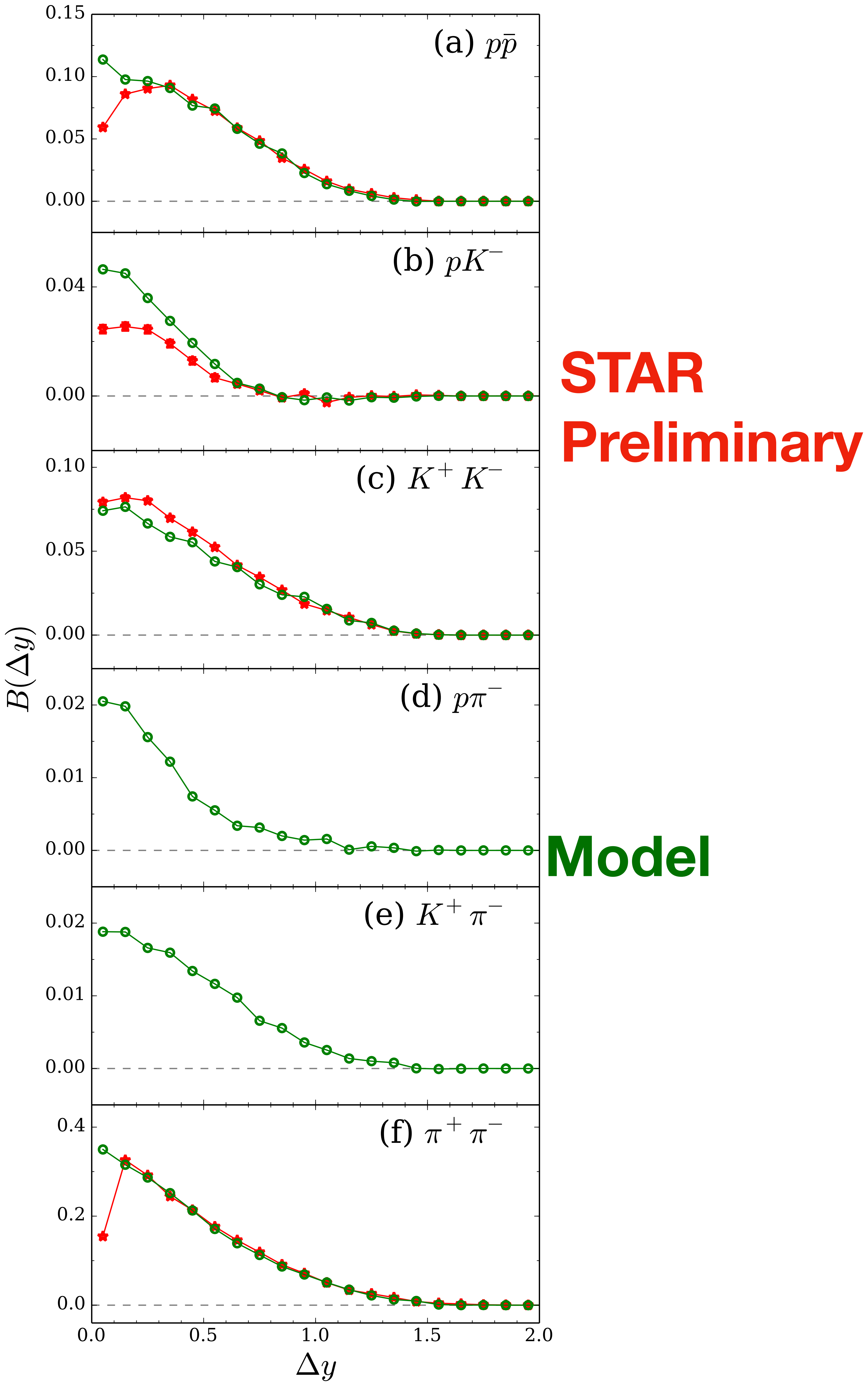
Model vs. STAR

Unidentified Particles

★ STAR Preliminary

Model, Type 1 + Type 2
Type 1 (dashes, hydro)
Type 2 (dots, cascade)

Model vs. STAR

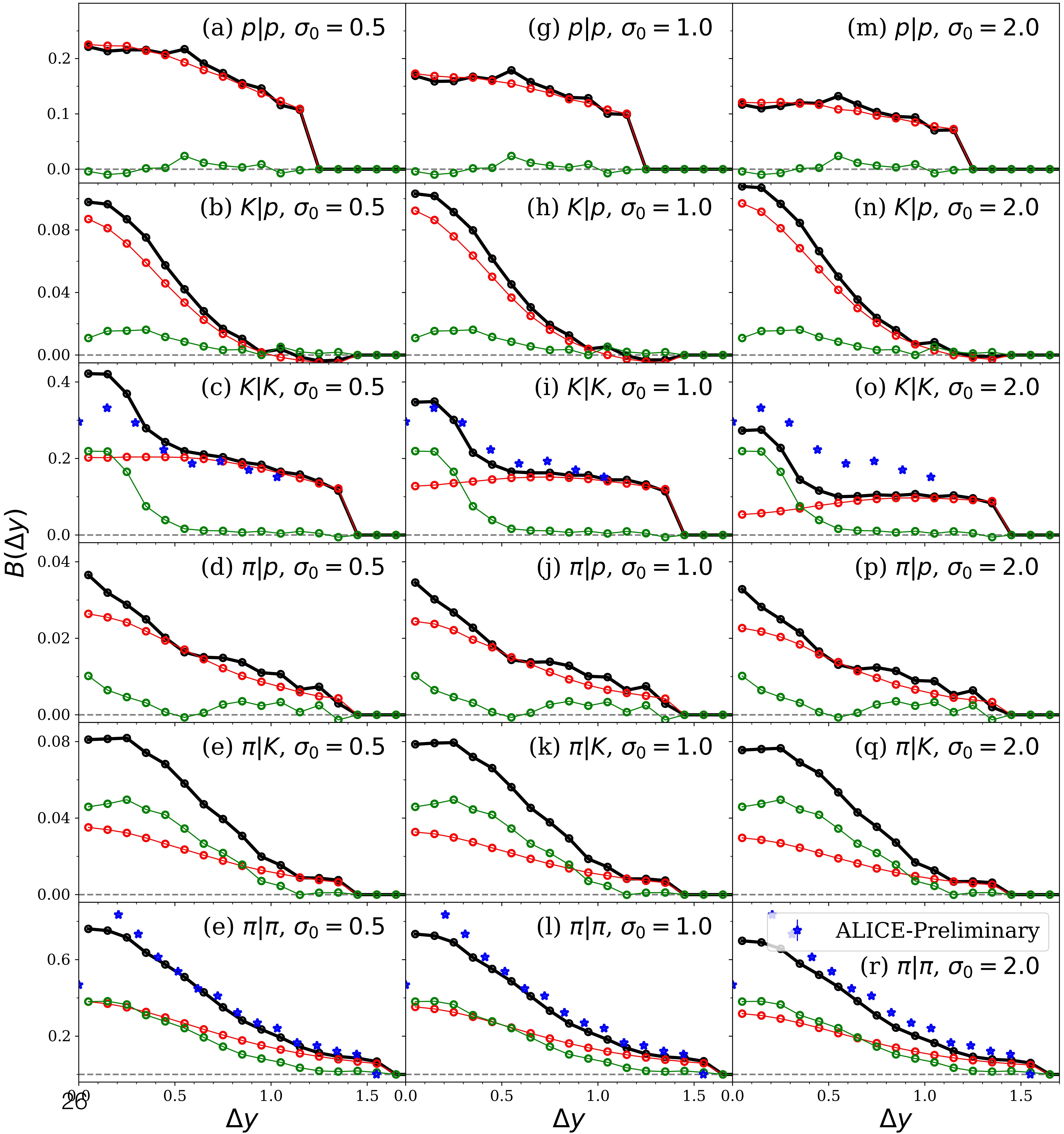


- Identified particles (vs. Δy)
- pK is off
- pp is off (annihilation missing)

Model vs ALICE

Thesis of Jin-Jin Pan

Binned by Δy



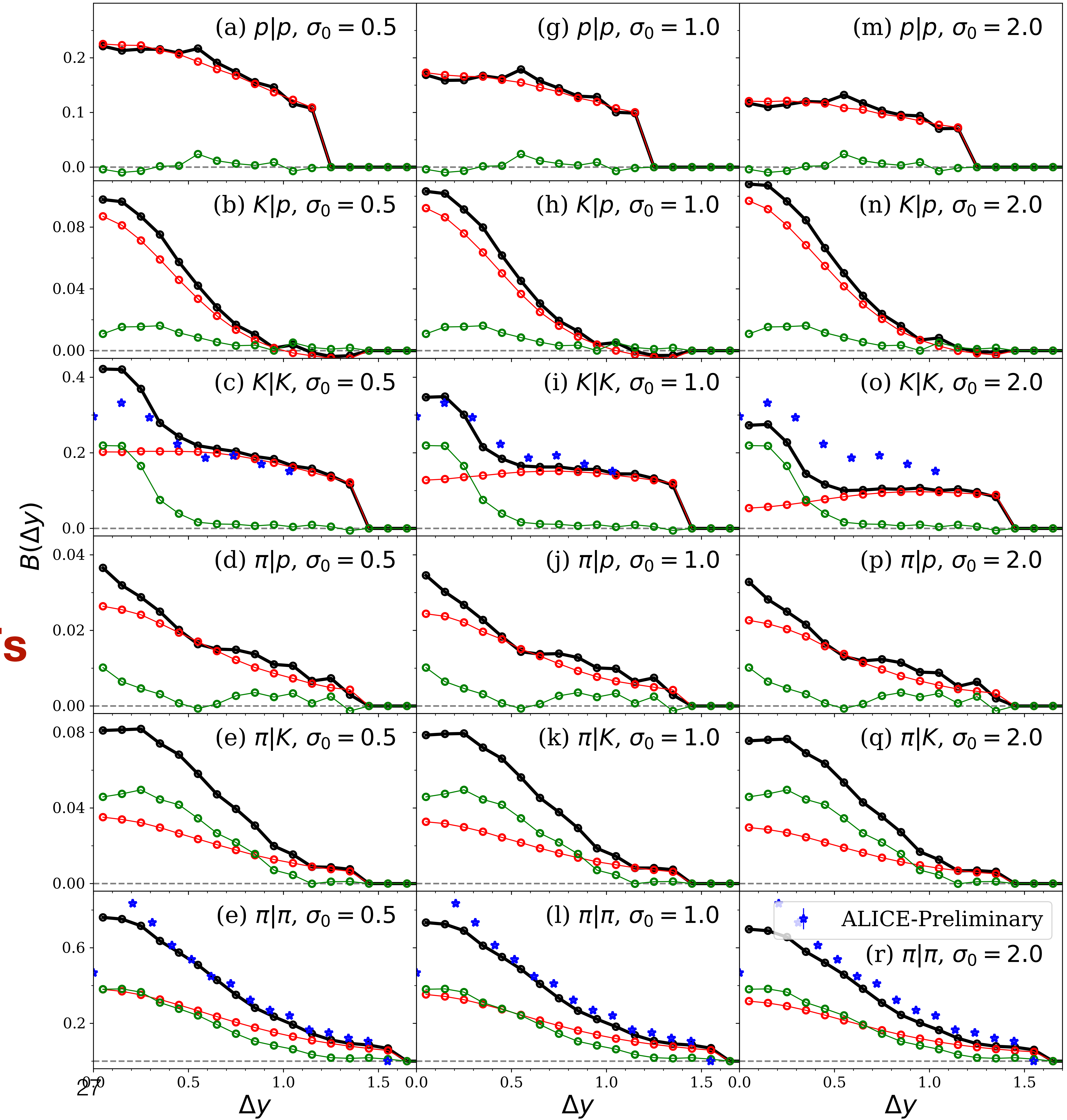
Type 1 + Type 2

Type 1 (hydro)

Type 2 (cascade)

Evidence of early chemical equilibrium

- $p\bar{p}$, K^+K^- **BFs broader than** $\pi^+\pi^-$ **BFs**
- $\sigma_0 > 0$



First Conclusion:

KK **and** pp **BFs are wider than** $\pi\pi$ **BFs**

- **Strong evidence of early charge creation!**

$\sigma_0 \gtrsim 0$

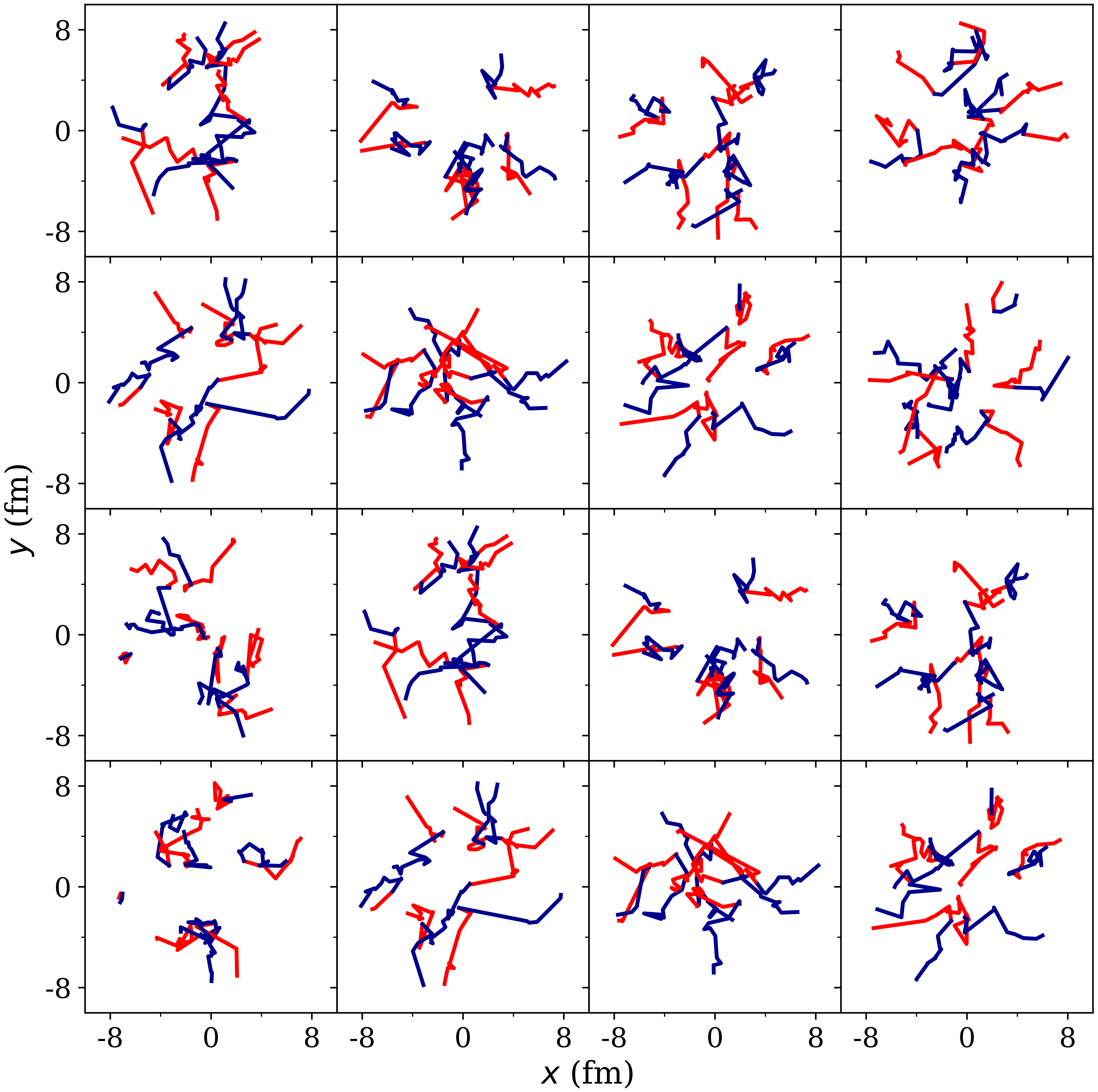
- **Charge (quarks) must have been created by** $\tau_0 = 0.6$ **fm/c!**

BUT if diffusion underestimated:

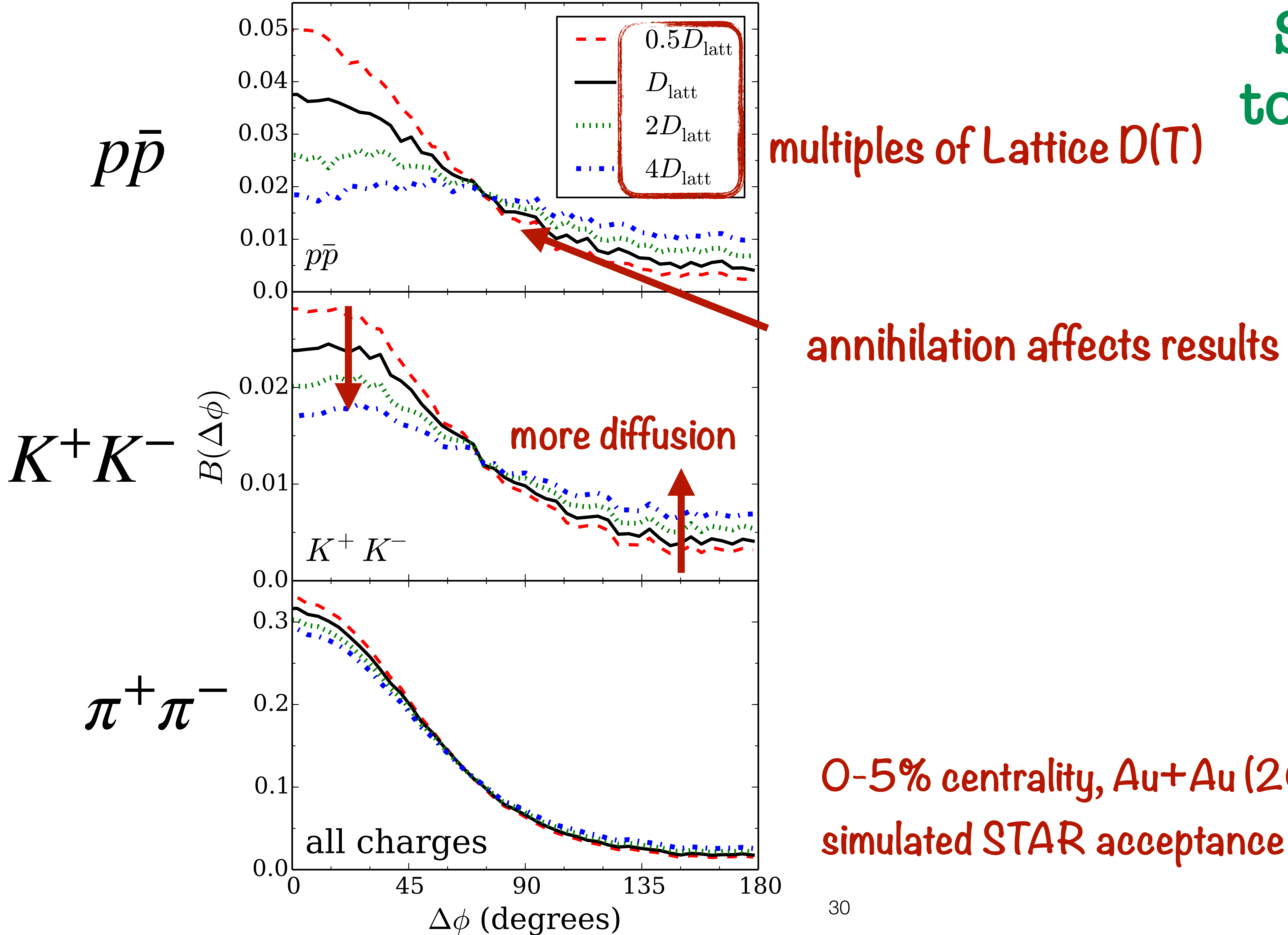
Charge production could have been later

Look at $B(\Delta\phi)$

- **Insensitive to pre-thermal separation**
- **Eliminate sensitivity to late-production**
  - pp **or** KK **BFs**
  - **Only consider** $\Delta y \gtrsim 1.0$



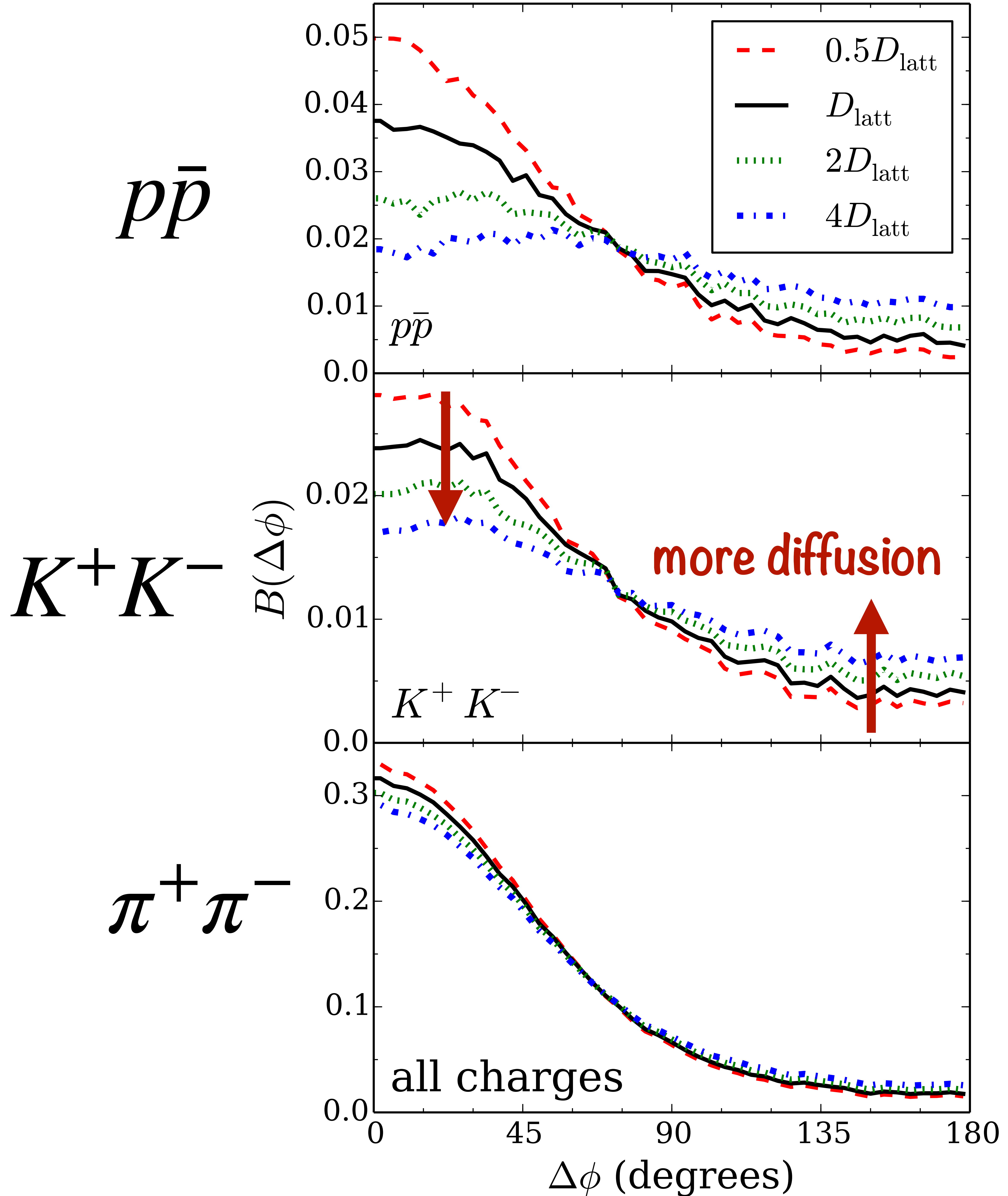
Sensitivity to Diffusivity



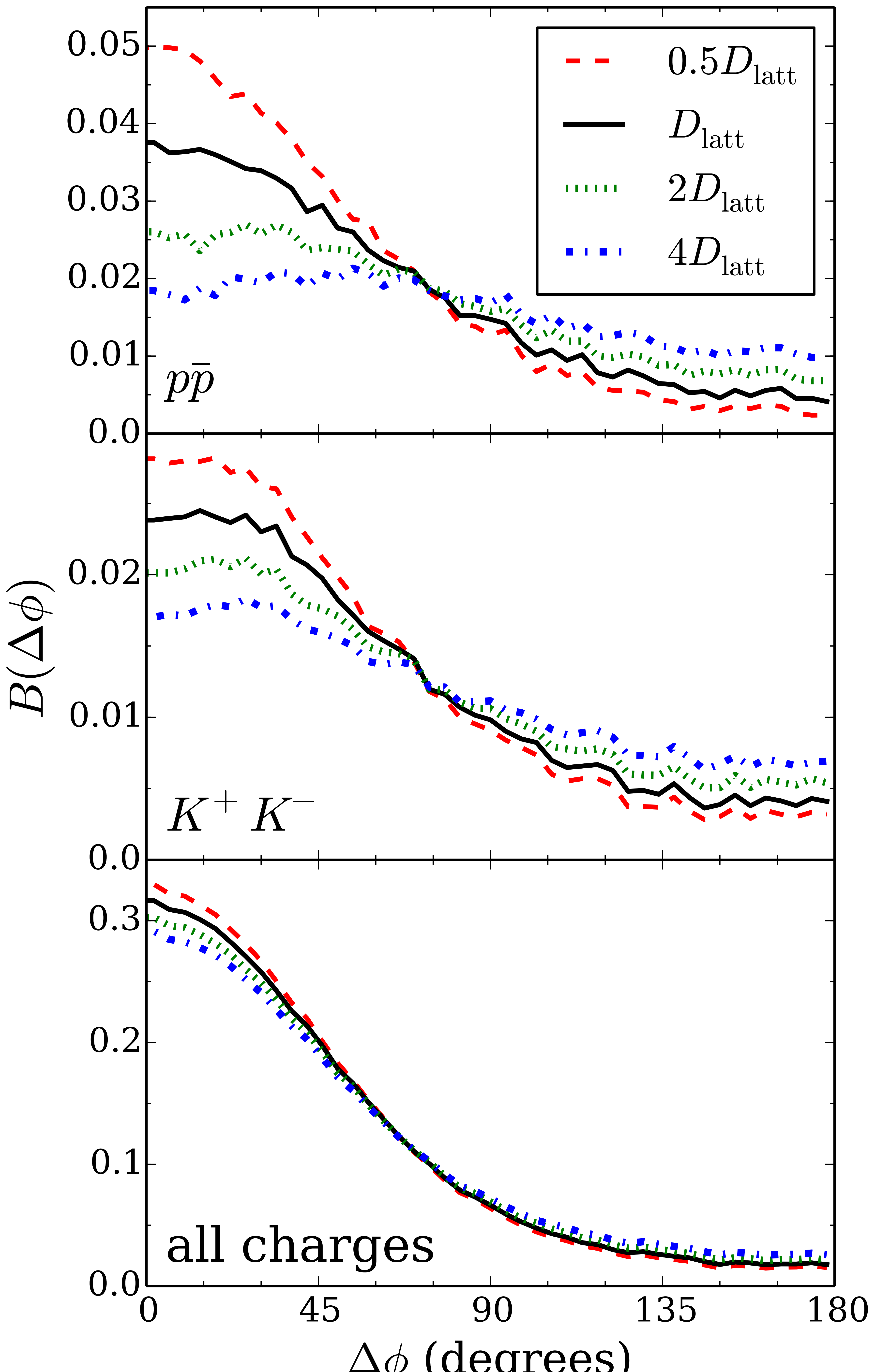
0-5% centrality, Au+Au (200A GeV)

simulated STAR acceptance

Sensitivity to Diffusivity



- $\Delta\phi$ binning reduces dependence on σ_0
- kaons or protons best suited:
- χ_{ss}/s roughly constant
  $\approx$ only phi contributes from final state
- χ_{BB}/s roughly constant
  annihilation an issue



Sensitivity to Diffusivity

Extract D ~ ±50 % ?

But work needed:

- φ contribution to kaon B.F.
  — BF binned by Q_{inv}
- absorption of strangeness into baryons
  — look at pK, KΛ BFs
- strangeness annihilation
  — multiplicities and BF vs Δy

Model vs. ALICE

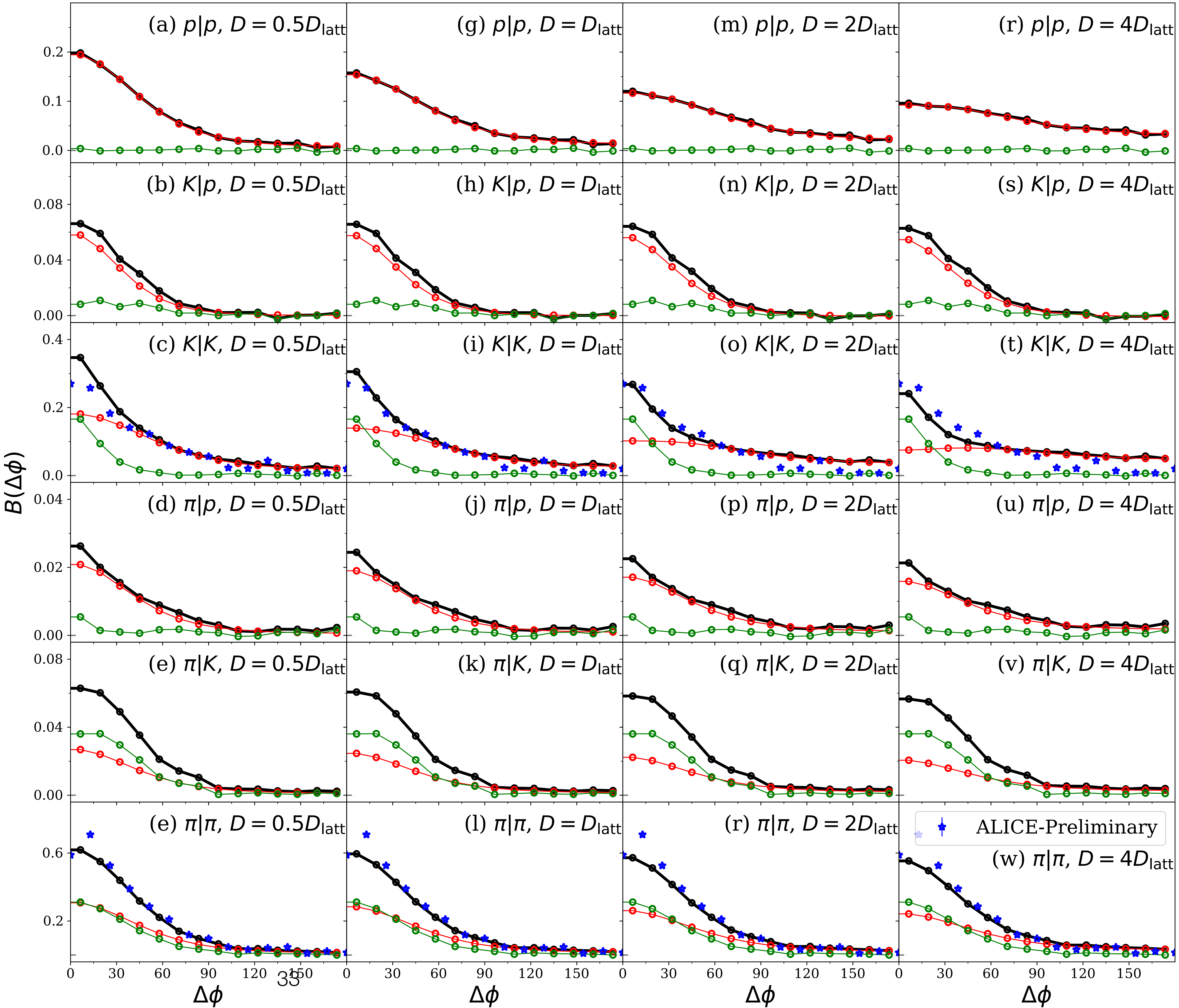
Binned by $\Delta\phi$

Lattice diffusion looks OK

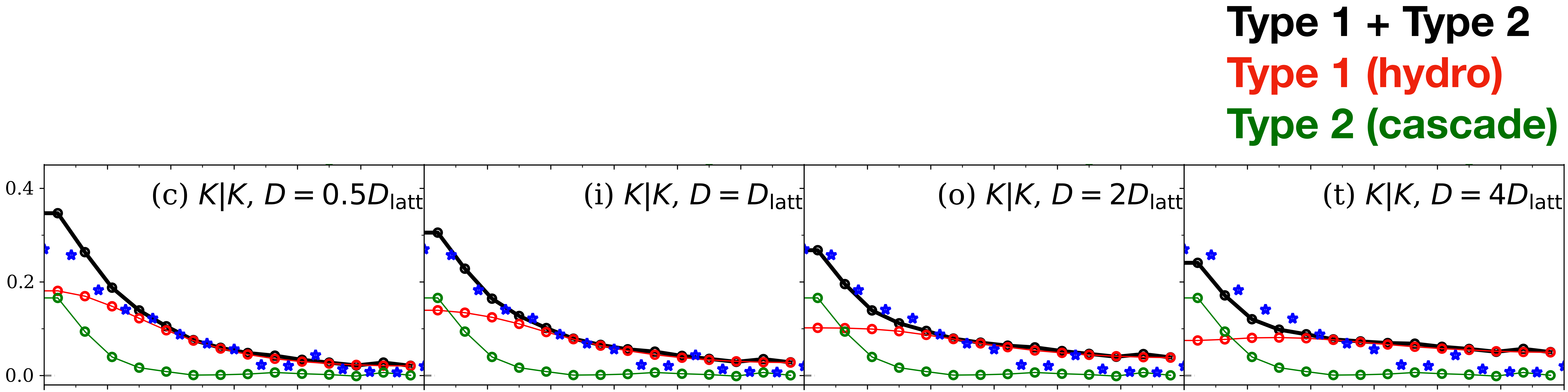
Type 1 + Type 2

Type 1 (hydro)

Type 2 (cascade)



Model vs. ALICE



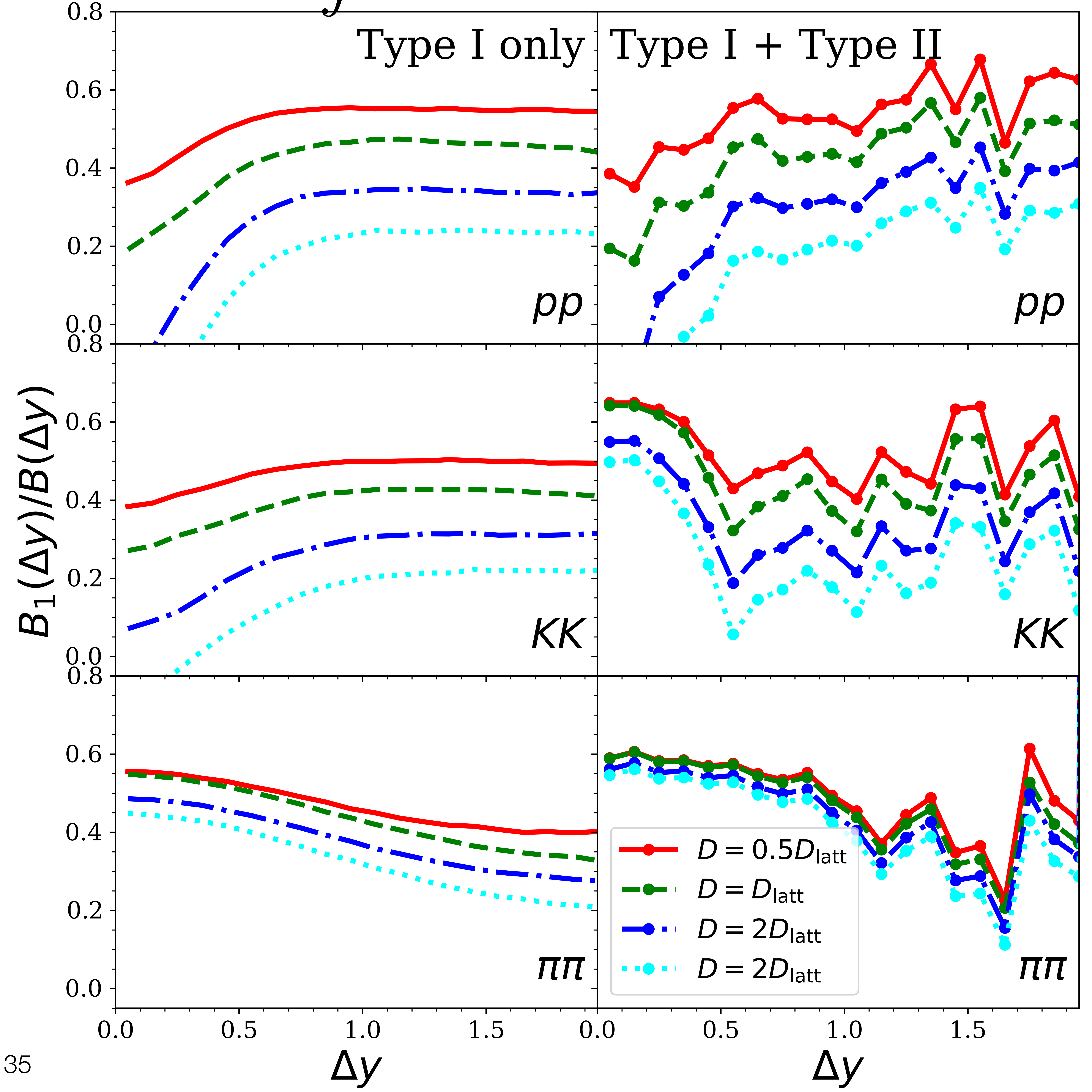
Lower diffusivities look better

Better Focus on Diffusivity
Analyze $B(\Delta\phi)$, Cutting on large Δy

Eliminate Effects from:

- HBT
- Resonant Decays
- Annihilation
- Experimental 2-track resolution
- $\Delta y \gtrsim 0.75$ should be good enough

$$B_1(\Delta y) \equiv \int d\Delta\phi B(\Delta y, \Delta\phi)\cos(\Delta\phi)$$



Analyze $B(\Delta\phi)$

Cutting on large Δy

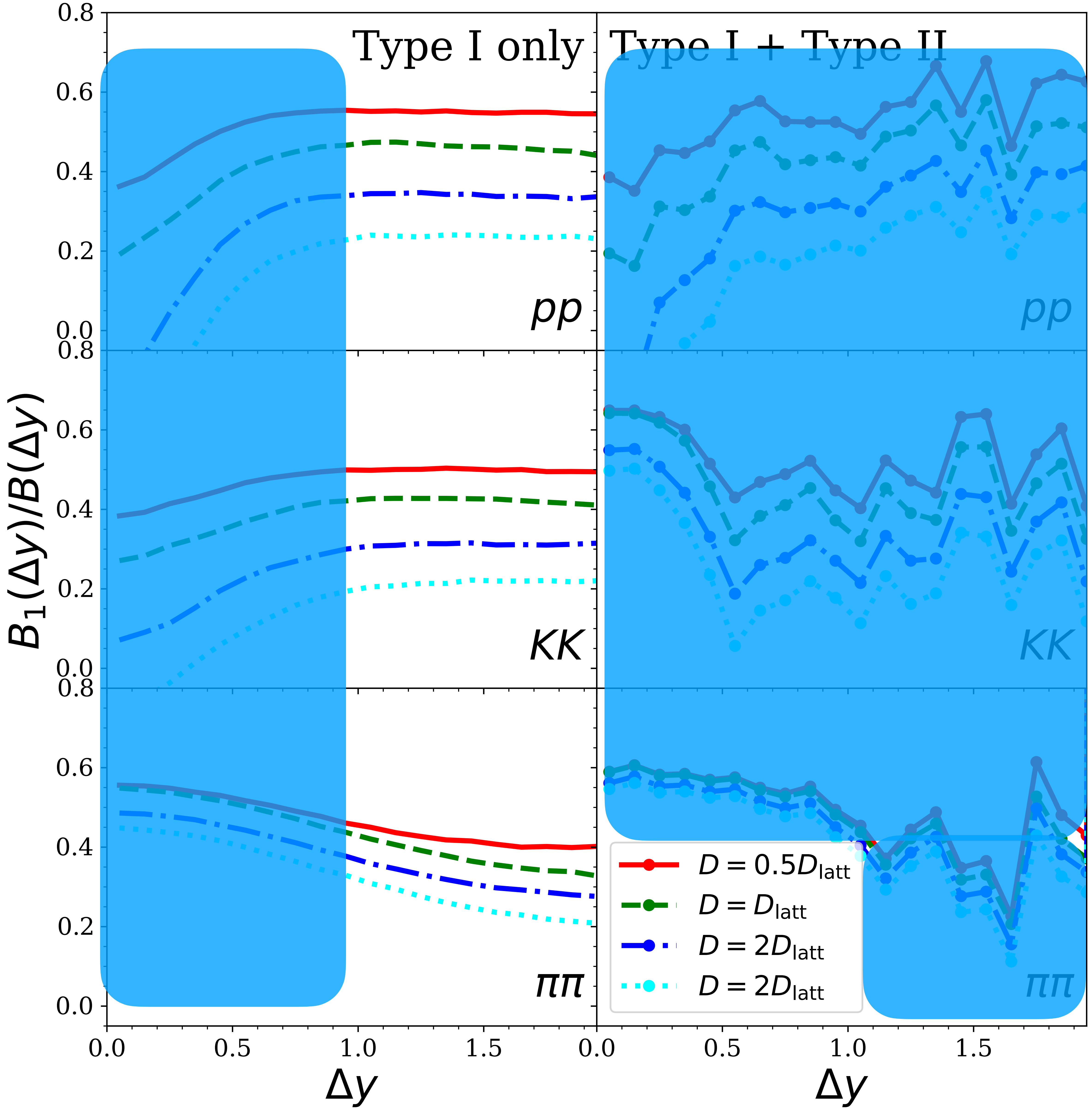
$$B_1(\Delta y) \equiv \int d\Delta\phi B(\Delta y, \Delta\phi)\cos(\Delta\phi),$$

$$B(\Delta y) = \int d\Delta\phi B(\Delta y, \Delta\phi)$$

Type II only provides noise for $\Delta y \gtrsim 1$

Robust extraction of diffusivity

for this window



Summary

- Charge correlations (order Q^2) calculated in "standard model"
- STAR/ALICE BFs vs Δy suggest early chemical equilibration
  $K^+K^-, p\bar{p}, \pi^+\pi^-$ systematics reproduced
  (STAR pK normalization off)
- Diffusivity can be extracted from BFs binned by $\Delta\phi$ cut on large Δy
  High statistics STAR & ALICE data coming
- Many opportunities for progress
  Both theoretical and experimental
  Both for diffusivity and for chemistry
  Similar to femtoscopy

U.S. DEPARTMENT OF ENERGY

Office of Science

Bonus Slides

- CME background
- Skewness/kurtosis background
- Theory for higher-order charge fluctuations

Model vs. STAR

Effect of Elliptic Flow

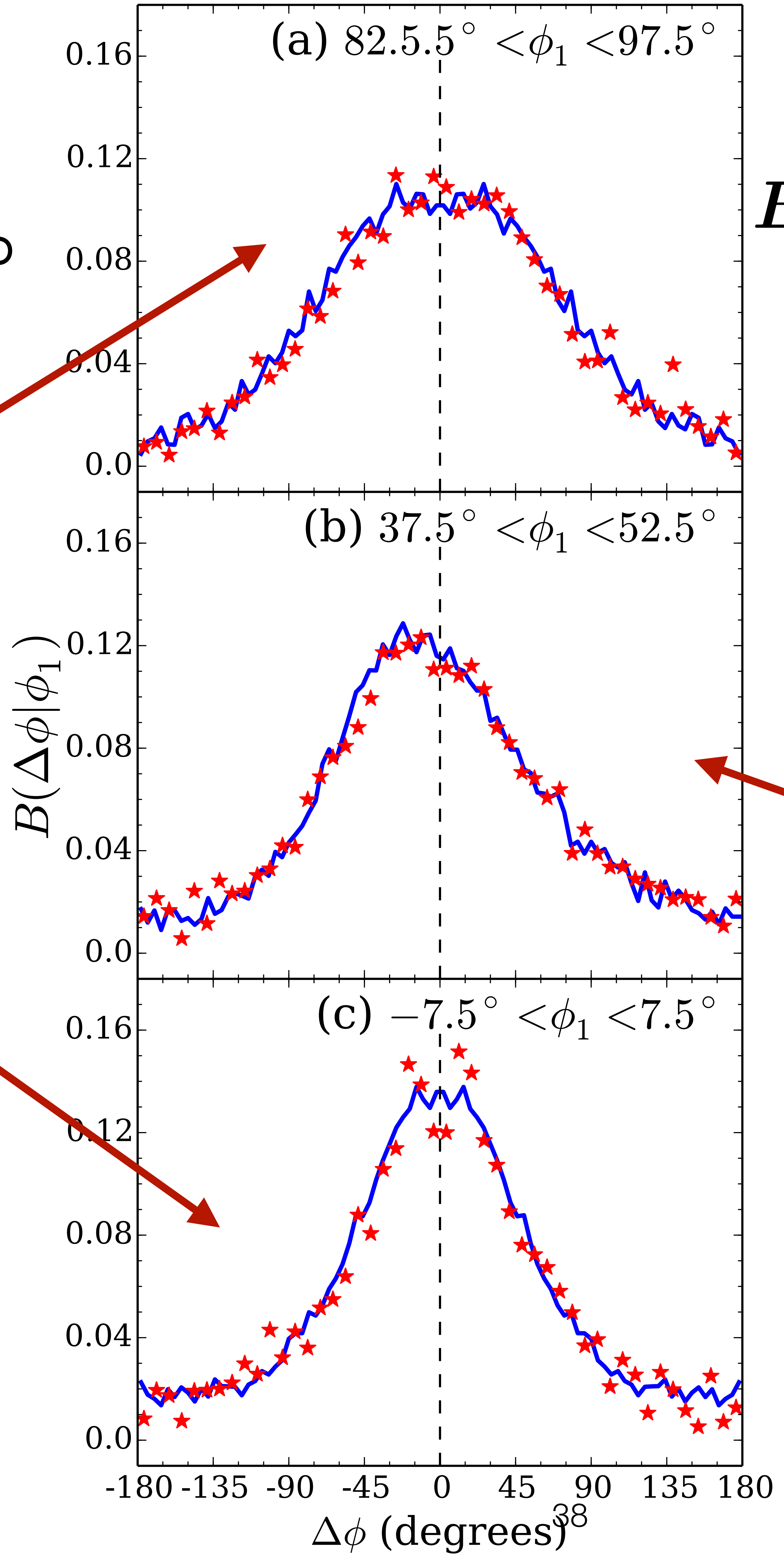
$$B(\Delta\phi|\phi_1) = \int d\phi_2 \, B(\phi_1,\phi_2)\delta(\Delta\phi - \phi_1 + \phi_2)$$

$\phi_1 \sim 90^\circ$

$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$

correlation tighter in-plane due to elliptic flow



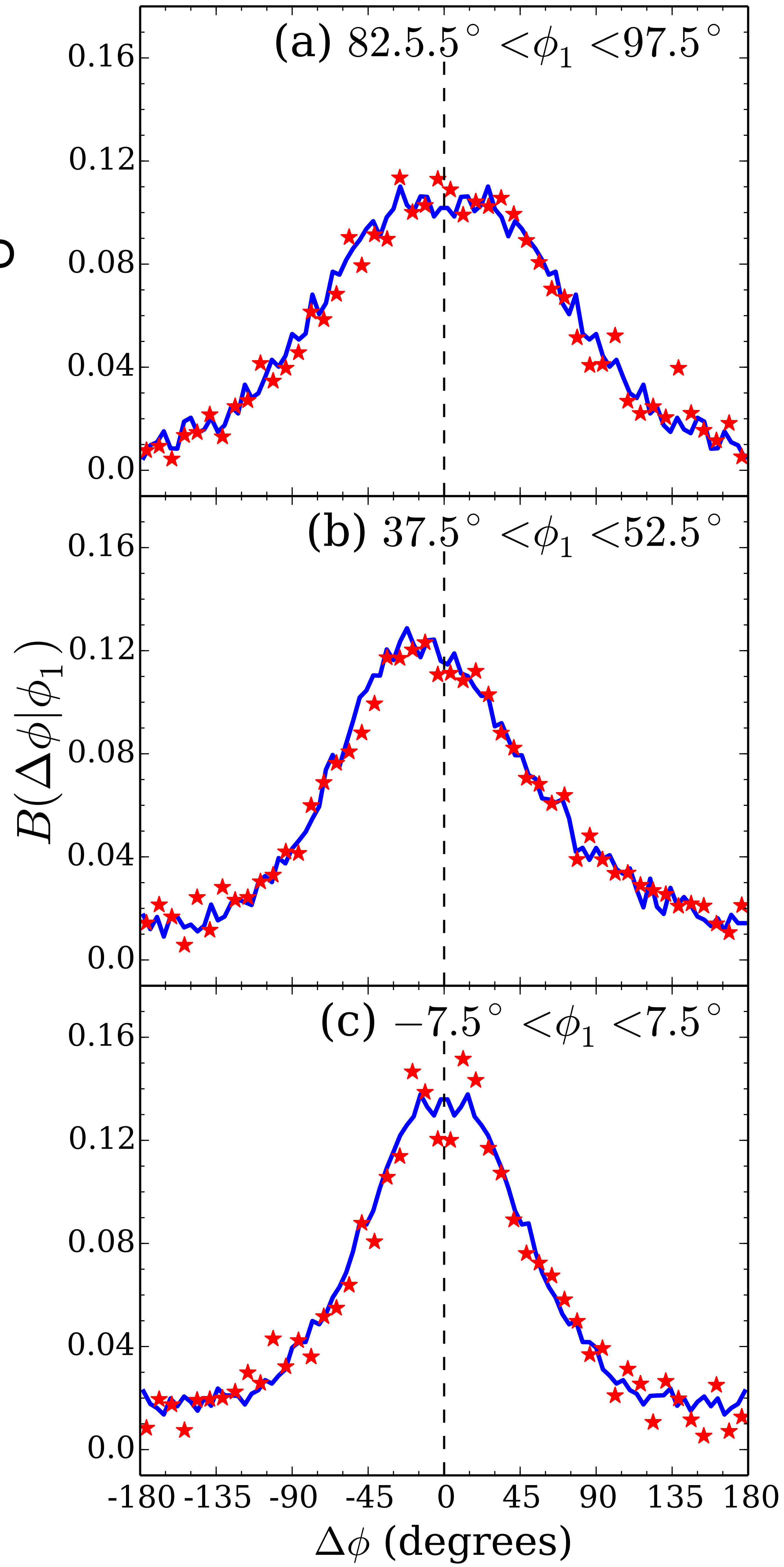
balancing charge more likely to be in-plane

ASIDE: CME correlator

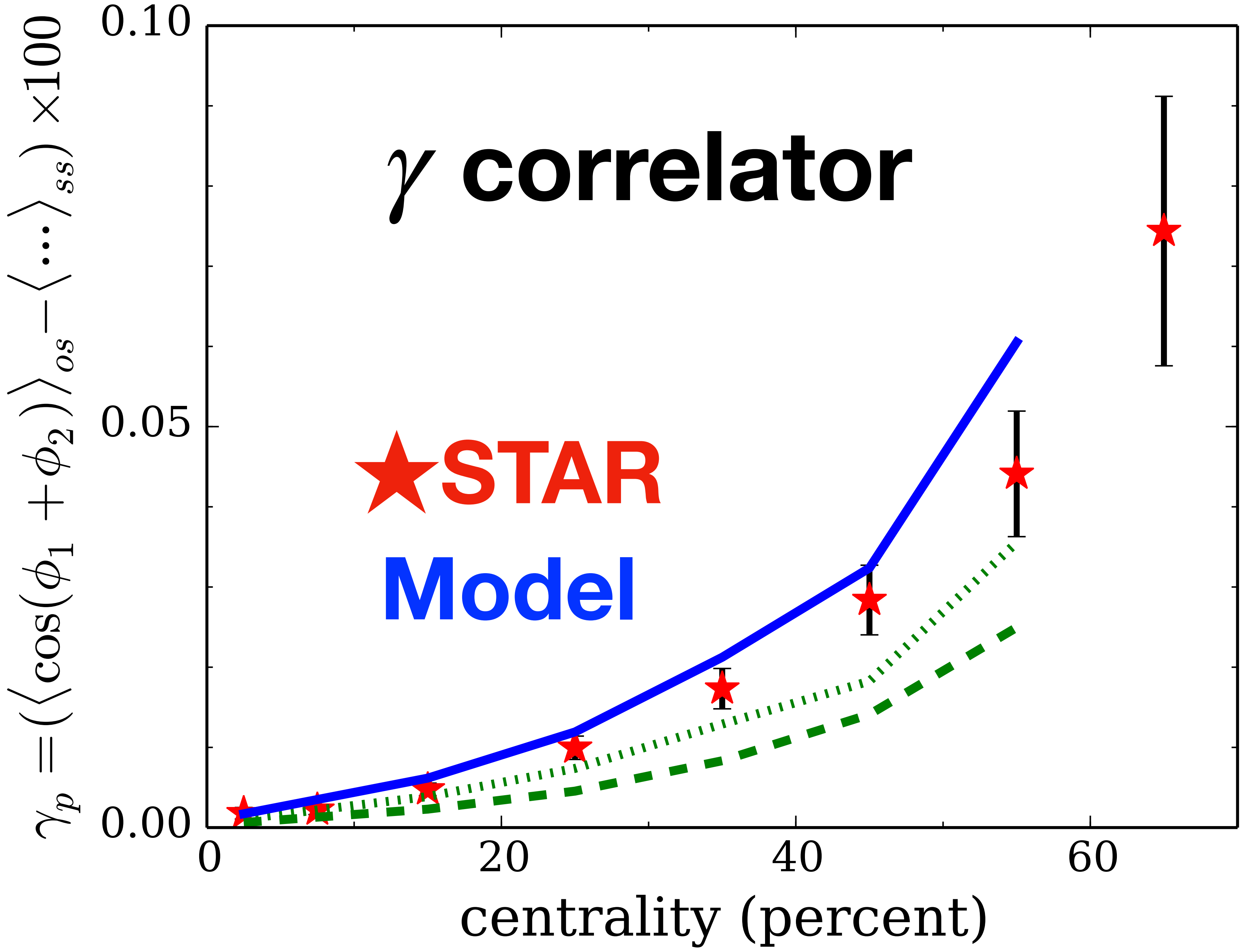
$\phi_1 \sim 90^\circ$

$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$



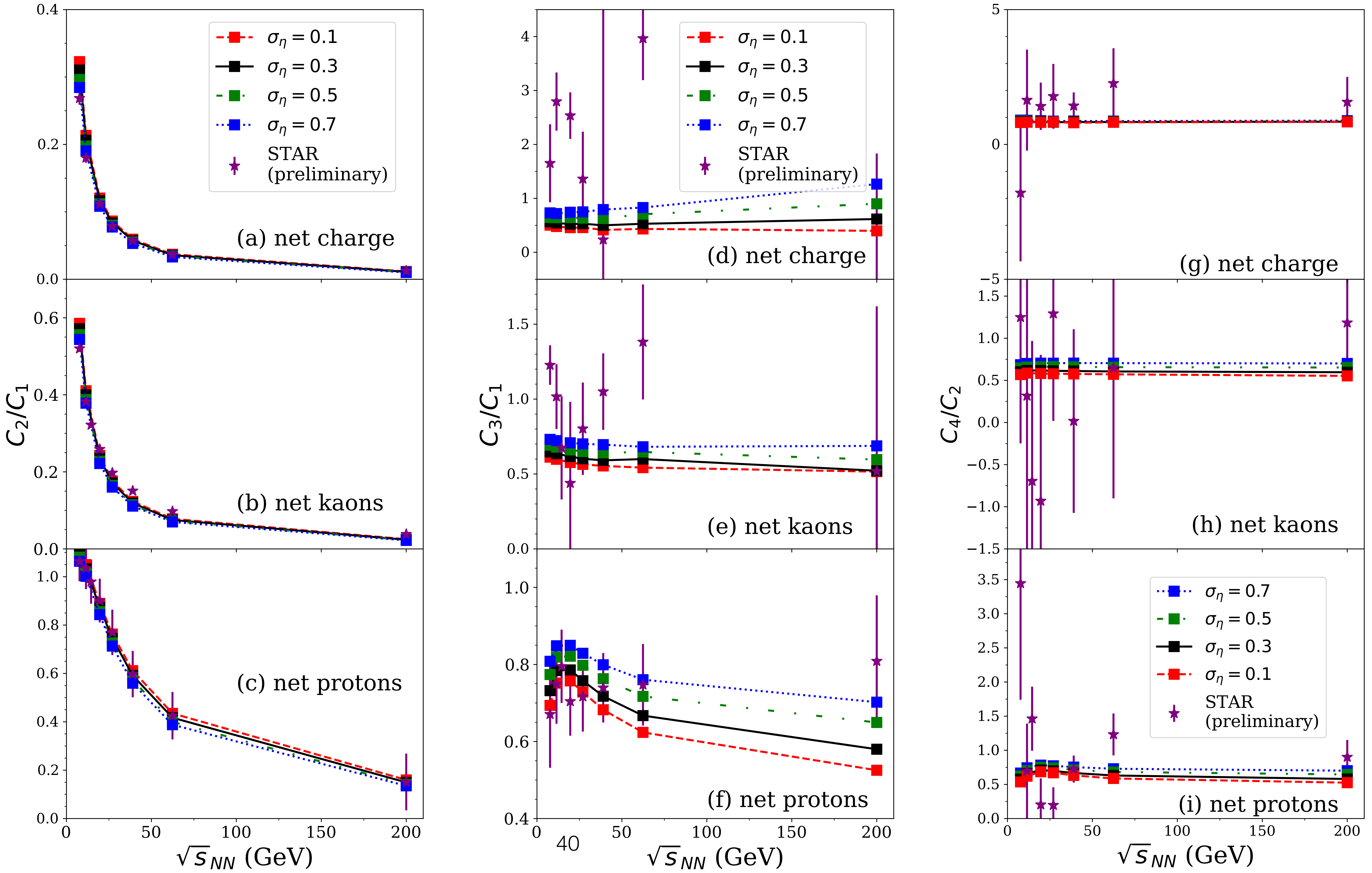
$$\gamma = \frac{1}{2}\{\langle \cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\rm opp.sign}\}$$
$$- \frac{1}{2}\{\langle \cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\rm same.sign}\}$$
$$= \frac{1}{M^2}\int d\phi_1 d\Delta\phi\ \frac{dM}{d\phi_1} B(\Delta\phi|\phi_1)\cos(2\phi_1 + \Delta\phi)$$



Model predicts ~ 115% of signal

Charge Conservation and Q^3, Q^4 correlations

b) Perform canonical ensemble on sub-volumes & superimpose on blast wave (crude)

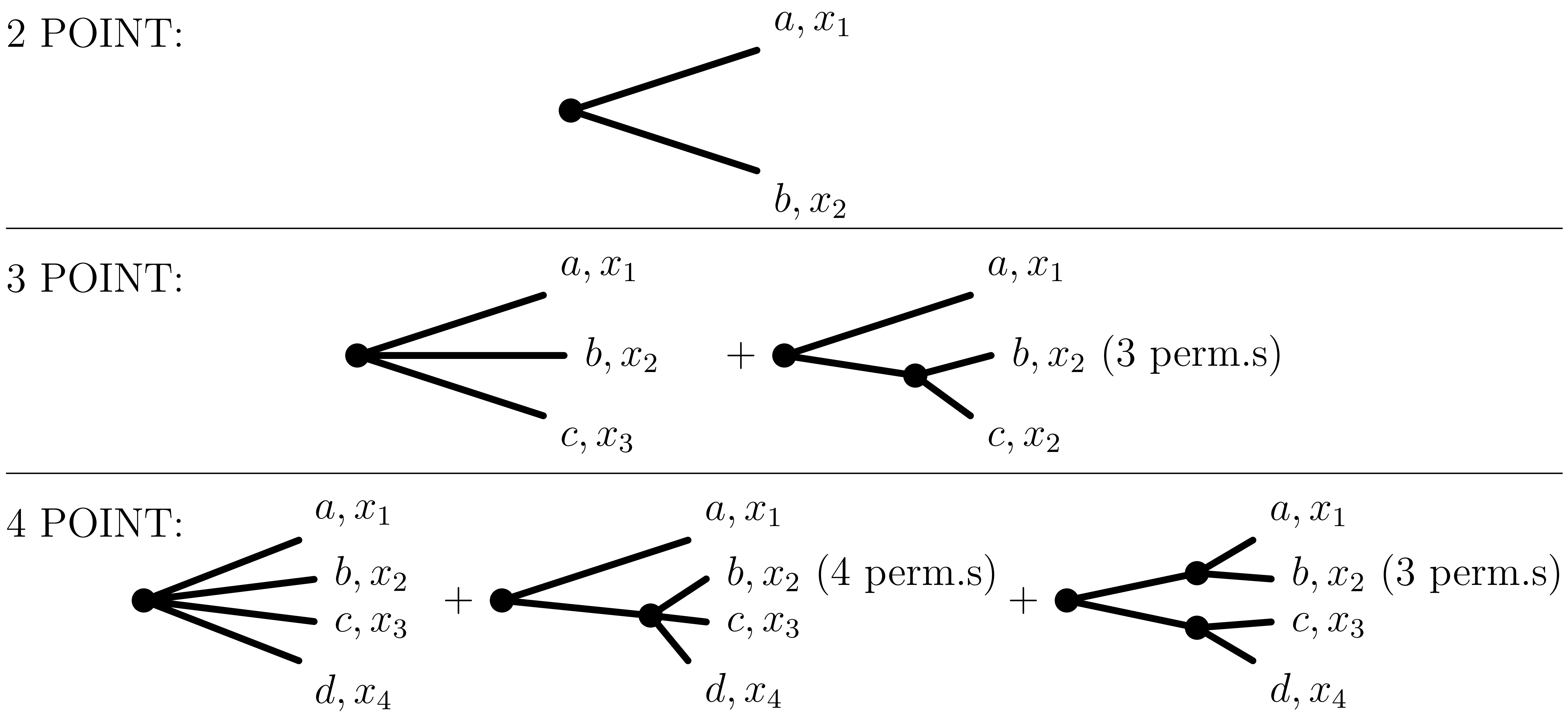


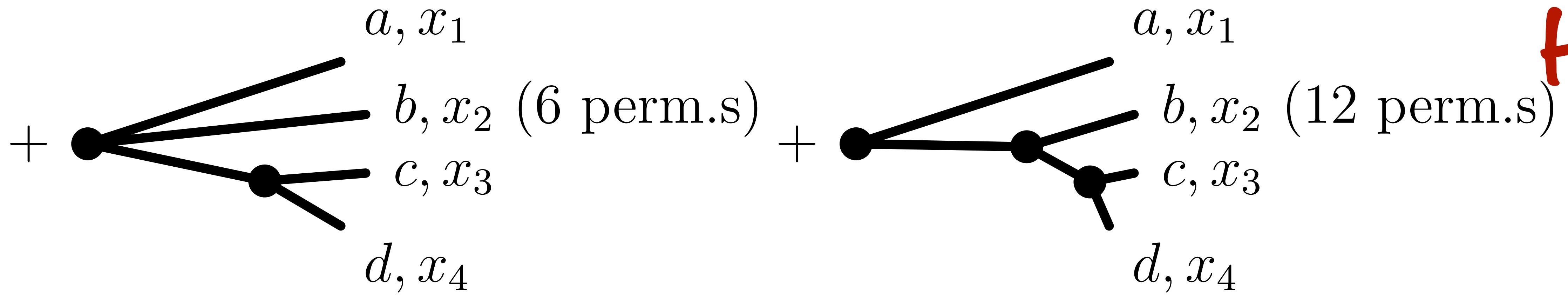
S.P. and R.Steinhorst
PRC (2020)

BONUS: Charge conservation and Q^3, Q^4 correlations (formalism)

a) Integrate n-point correlations to obtain skewness & kurtosis

S.P., PRC (2020)





Handles full 3x3 flavor dynamics

Tractable, but DIFFICULT