QCD at non-zero density and phenomenology

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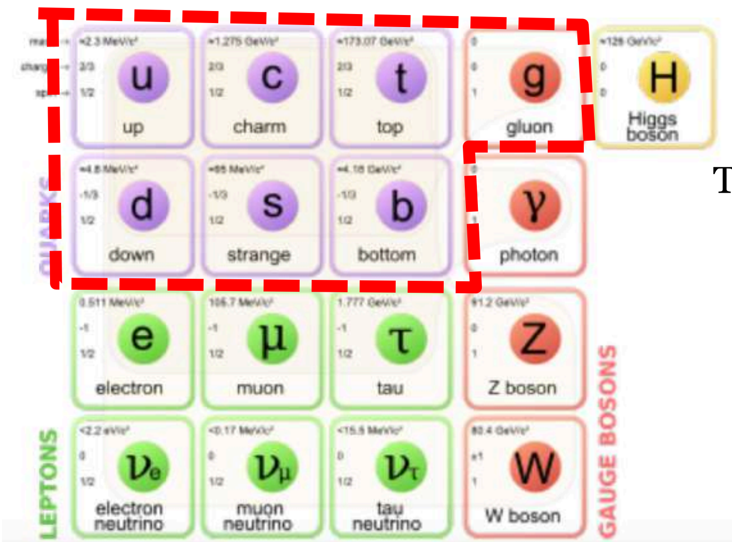
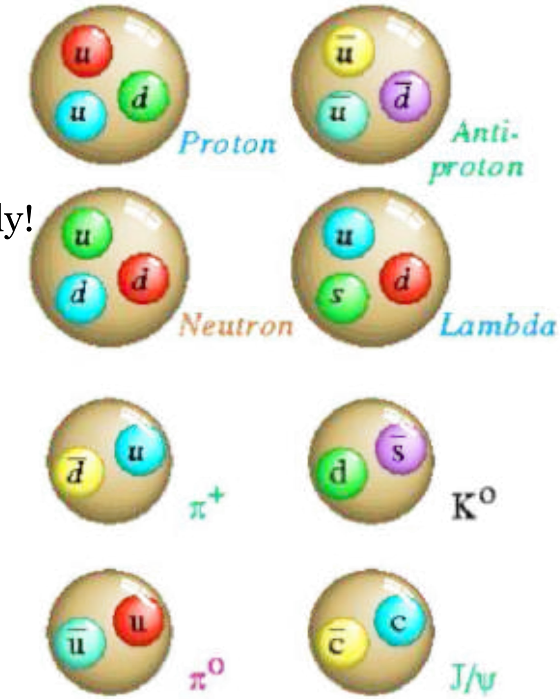
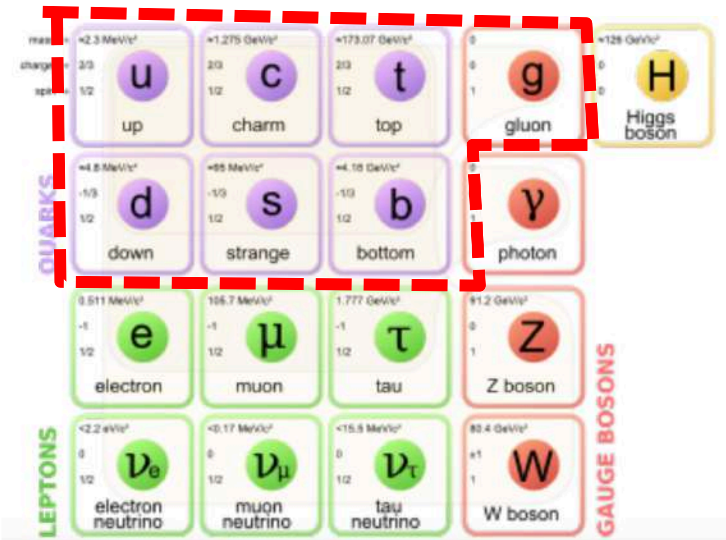
Matter in the Universe



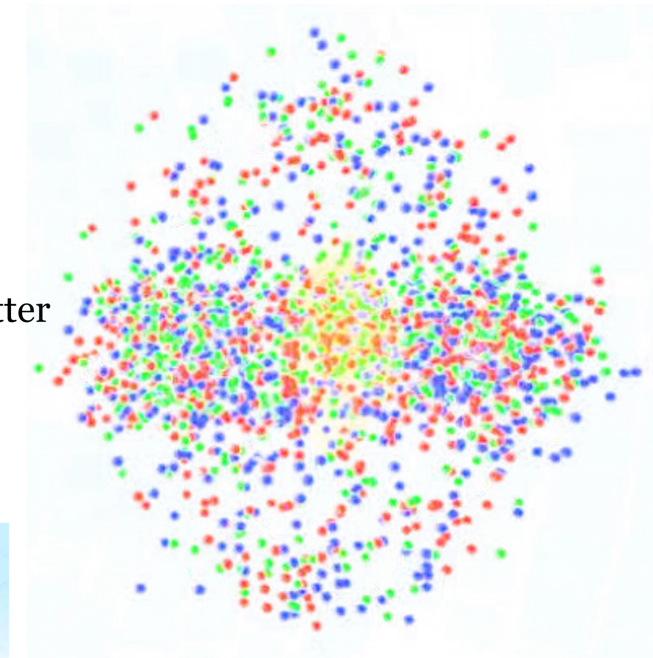
Two- and three-quark states only!



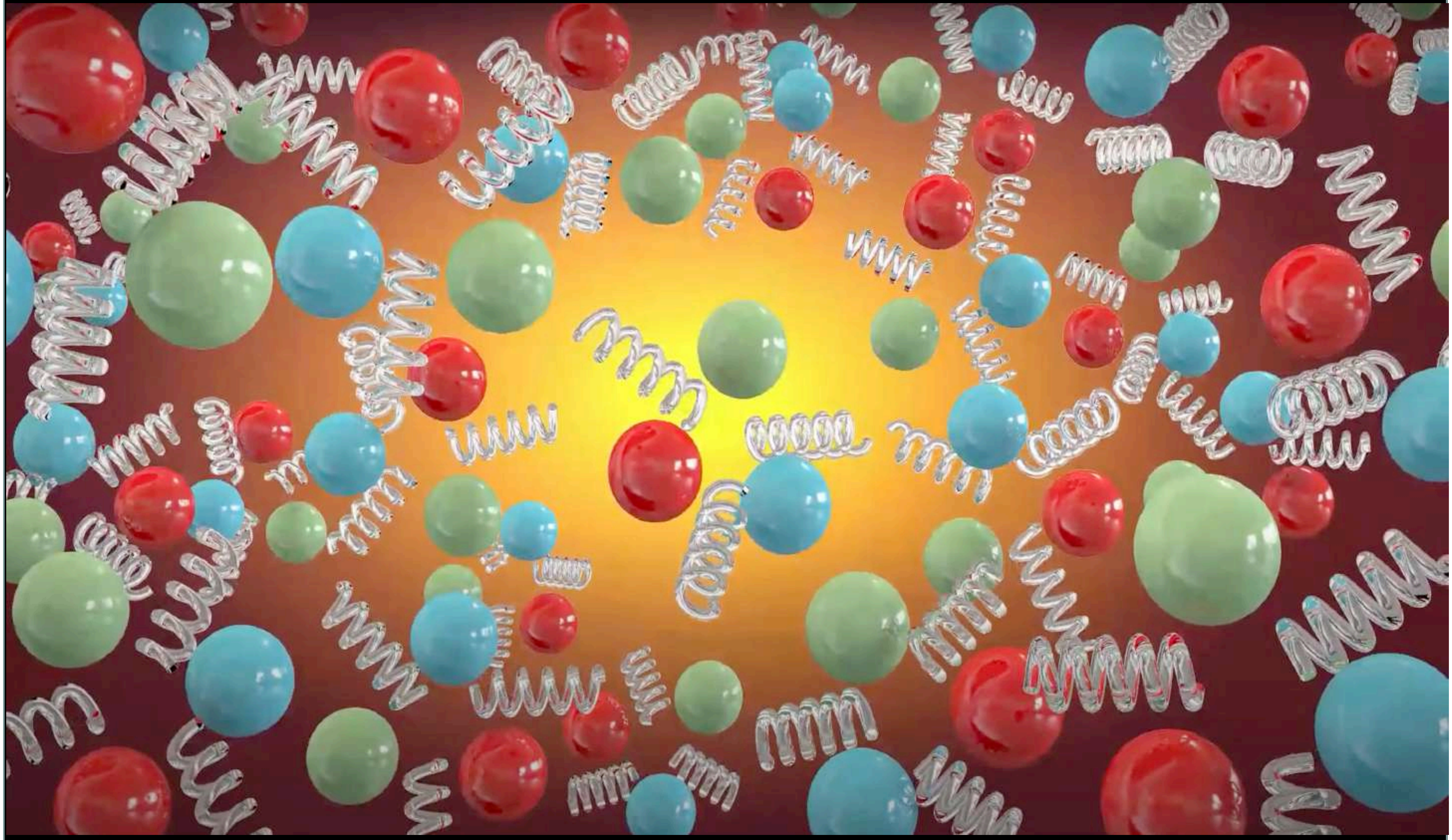
Matter in the Universe



Heat and compress matter



Quark-Gluon Plasma:
new phase of matter at very high temperatures (or densities)



Graphics credit to: ООО ИнтерГрафика

QCD matter under extreme conditions

Research Council of the National Academies: Eleven science questions for the new century

- How did the Universe begin?
- What are the new states of matter at exceedingly high density and temperature?
- What is Dark Matter?
- What is the nature of Dark Energy?
- What are the masses of the neutrinos, how have they shaped the evolution of the Universe?
- Did Enstein have the last word on Gravity?
- How do cosmic accelerators work and what are they accelerating?
- Are protons unstable?
- Are there additional space-time dimensions?
- How were the elements from Iron to Uranium made?
- Is a new theory of matter and light needed at the highest energies?

QCD matter under extreme conditions

Research Council of the National Academies: Eleven science questions for the new century

- How did the Universe begin?
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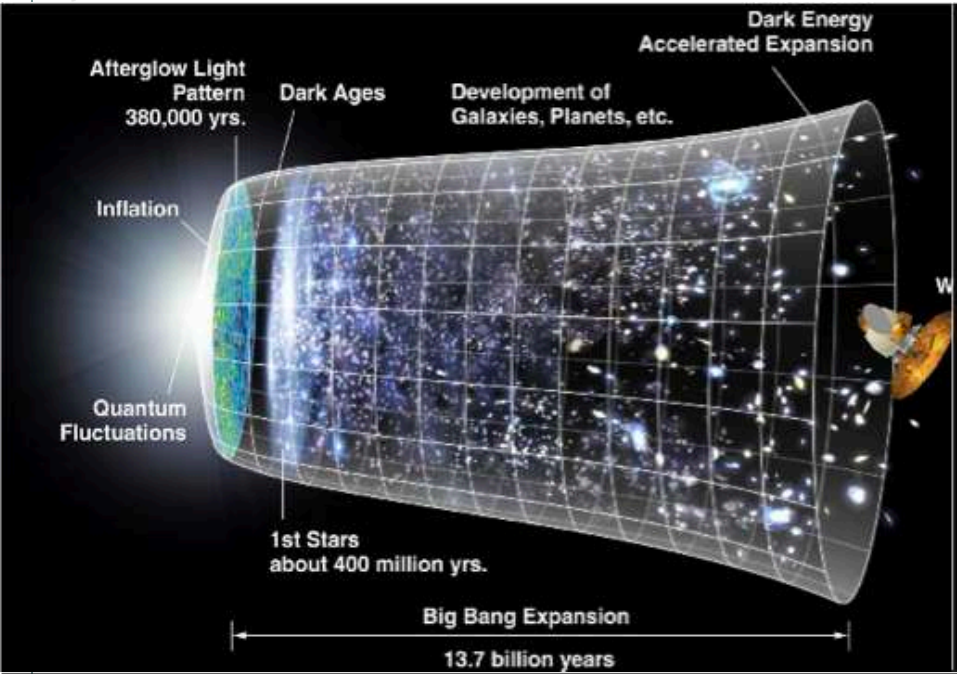
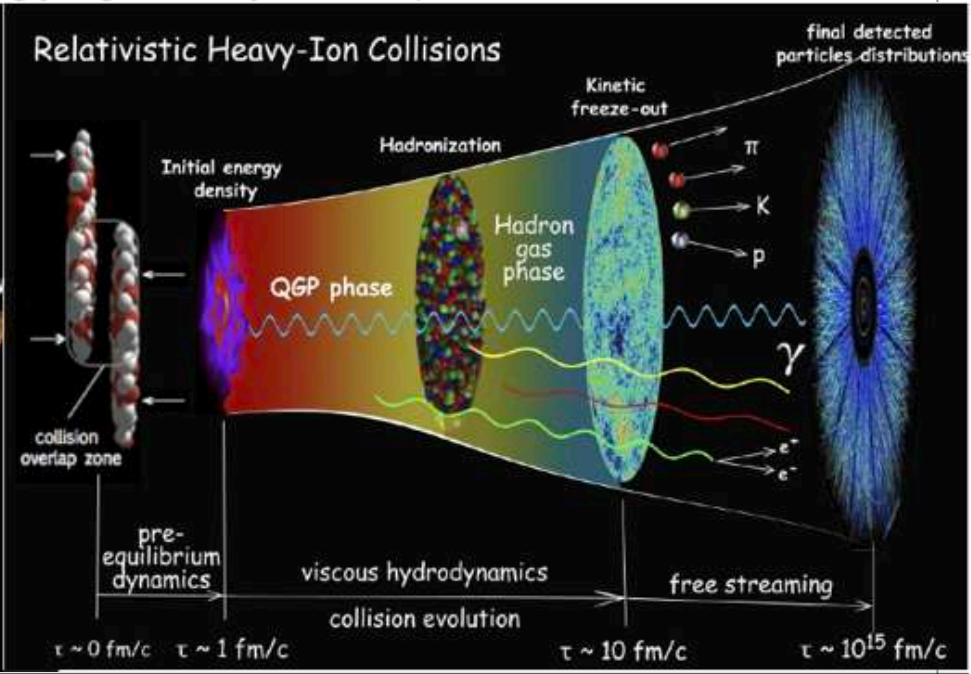




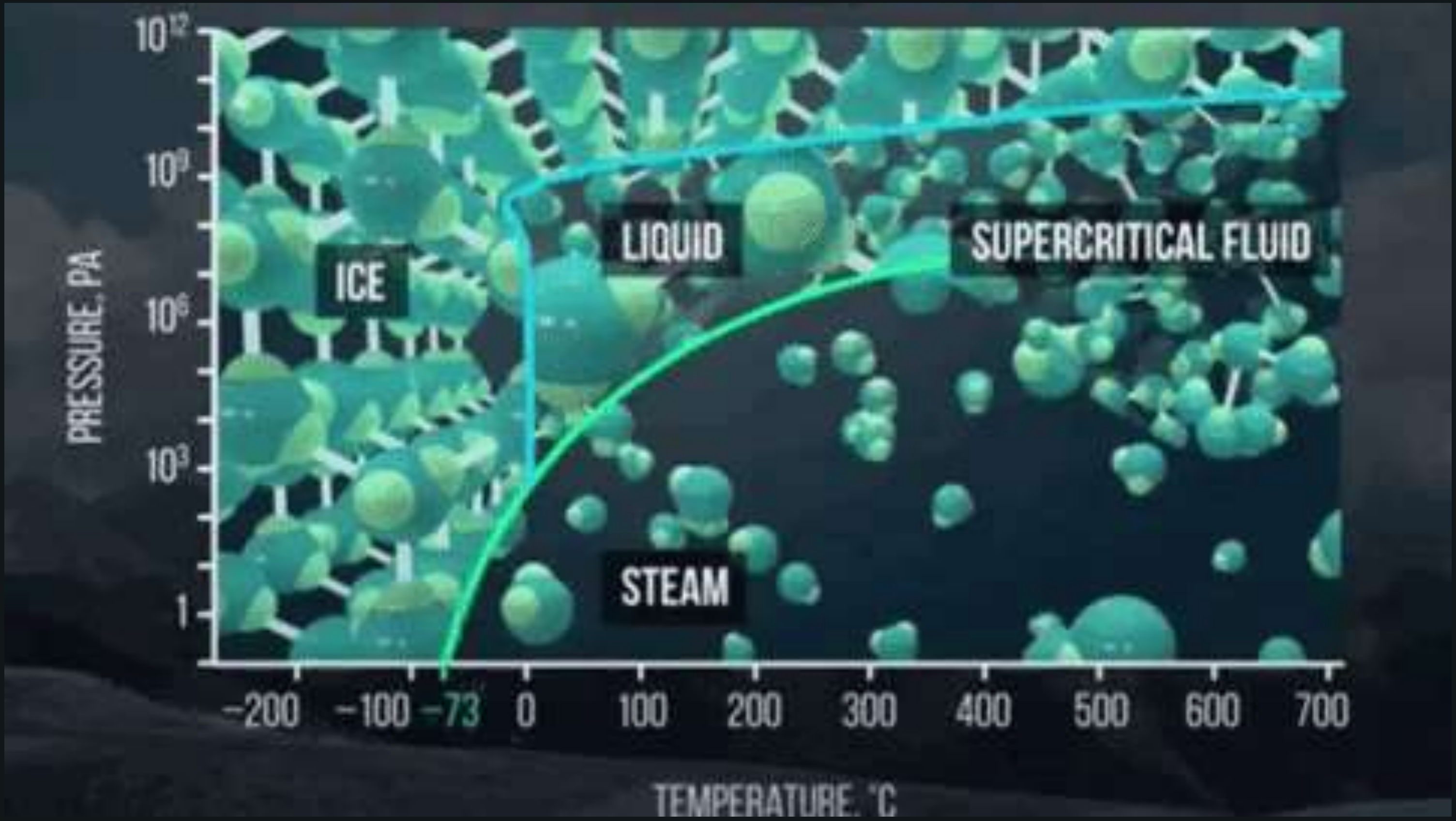
The two questions are related!

Quark-Gluon Plasma (QGP) is at $T>10^{12}$K and $\rho \sim 10^{40}$ cm^{-3}

The Universe was in the QGP phase a few μs after Big Bang

Ultimate goals

Phase diagram of water



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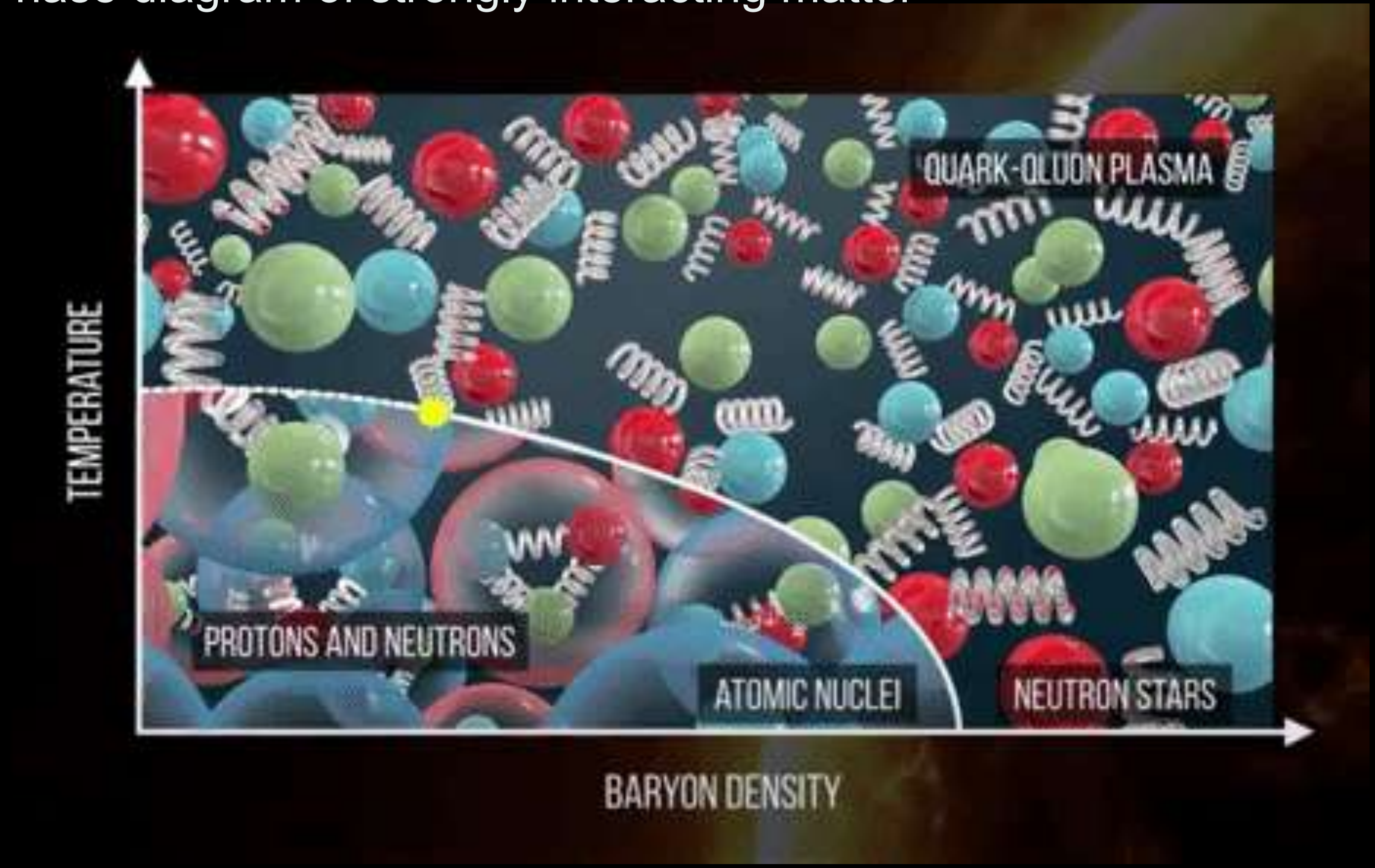
Ultimate goals

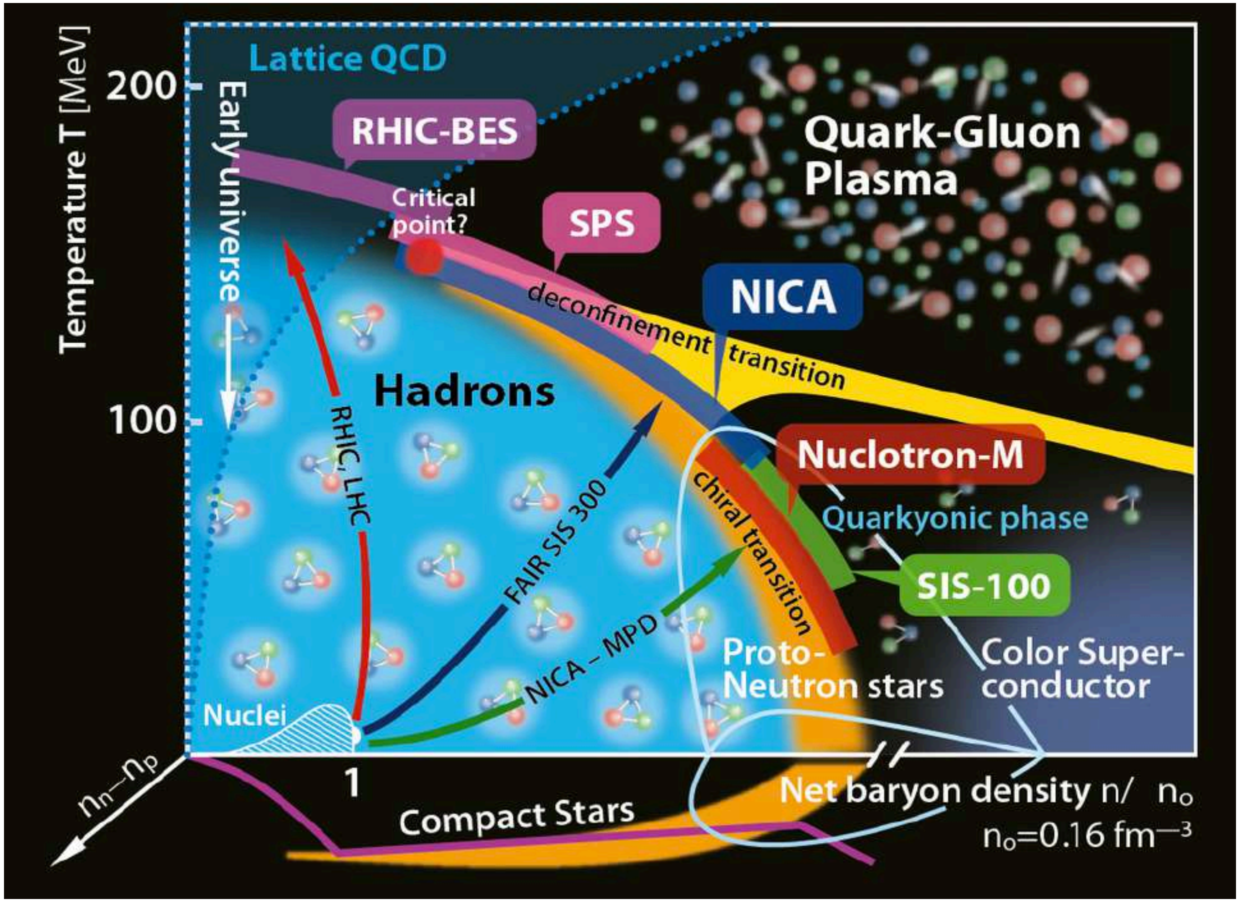
Phase diagram of strongly interacting matter



Graphics credit to: ООО ИнтерГрафика

Open Questions

- Is there a critical point in the QCD phase diagram?
- What are the degrees of freedom in the vicinity of the phase transition?
- Where is the transition line at high density?
- What are the phases of QCD at high density?
- Are we creating a thermal medium in experiments?



QCD matter under extreme conditions

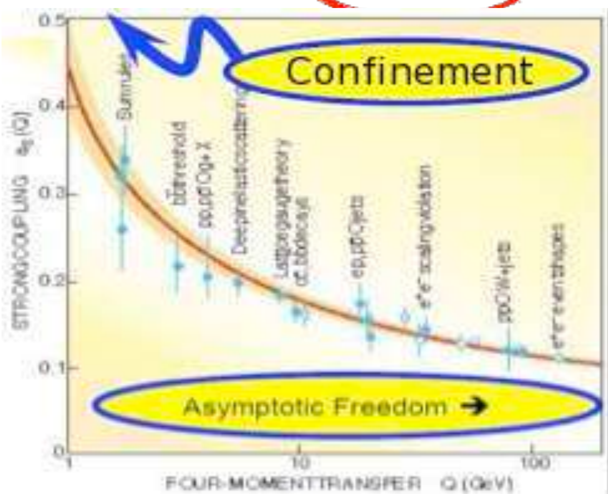
To address these questions, we need fundamental theory and experiment

Theory: Quantum Chromodynamics

- QCD is the fundamental theory of strong interactions
- It describes interactions among quarks and gluons

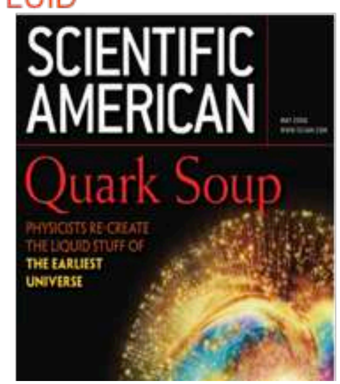
$$L_{QCD} = \sum_{i=1}^{n_f} \overline{\psi}_i \gamma_\mu \left(i\partial^\mu - g A_a^\mu \frac{\lambda_a}{2} \right) \psi_i - m_i \overline{\psi}_i \psi_i - \frac{1}{4} \sum_a F_a^{\mu\nu} F_a^{\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + i f_{abc} A_b^\mu A_c^\mu$$

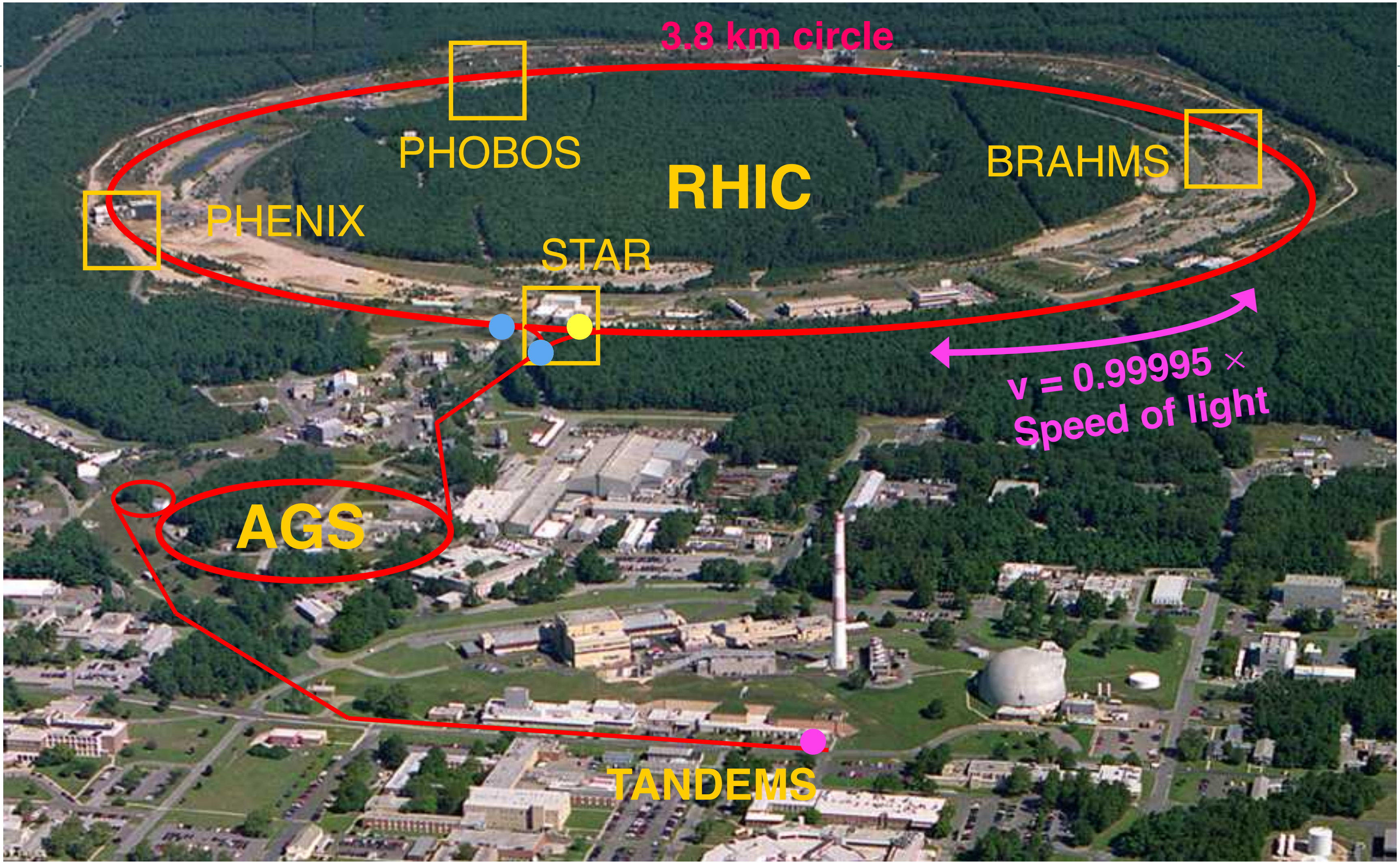


Experiment: heavy-ion collisions

- Quark-Gluon Plasma (QGP) discovery at RHIC and LHC:
- SURPRISE!!! QGP is a PERFECT FLUID
- Changes our idea of QGP (no weak coupling)
- Microscopic origin still unknown



Relativistic Heavy Ion Collider

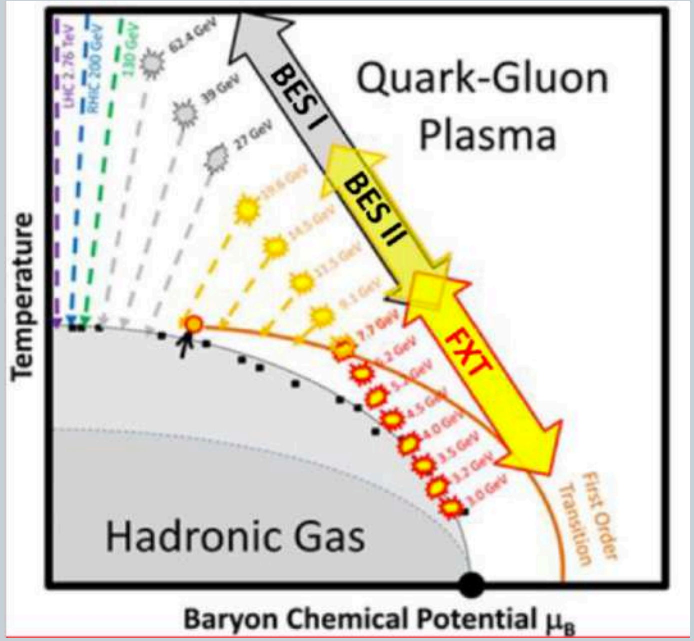


Gold nuclei, with 197 protons + neutrons each, are accelerated

The beams go through the experimental apparatus 100,000 times per second!

Second Beam Energy Scan (BESII) at RHIC

- Planned for 2019-2020
- 24 weeks of runs each year
- Beam Energies have been chosen to keep the μ_B step ~50 MeV
- Chemical potentials of interest: $\mu_B/T \sim 1.5 \ldots 4$



√s (GeV)	19.6	14.5	11.5	9.1	7.7	6.2	5.2	4.5	3.0
μ_B (MeV)	205	260	315	370	420	487	541	589	720
# Events	400M	300M	230M	160M	100M	100M	100M	100M	100M

Collider | Fixed Target

Comparison of the facilities

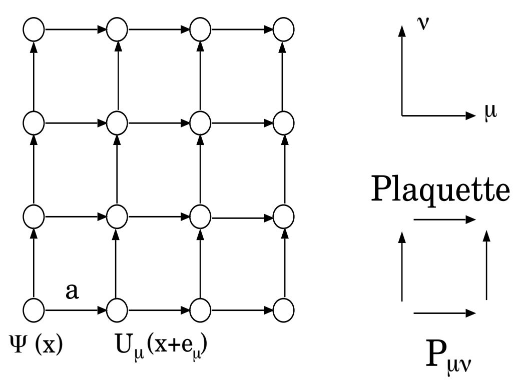
Compilation by D. Cebra

Facilty	RHIC BESII	SPS	NICA	SIS-100 SIS-300	J-PARC HI
Exp.:	STAR +FXT	NA61	MPD + BM@N	CBM	JHITS
Start:	2019-20 2018	2009	2020 2017	2022	2025
Energy: $\sqrt{s_{NN}}$ (GeV)	7.7– 19.6 2.5-7.7	4.9-17.3	2.7 - 11 2.0-3.5	2.7-8.2	2.0-6.2
Rate: At 8 GeV	100 HZ 2000 Hz	100 HZ	<10 kHz	<10 MHZ	100 MHZ
Physics:	CP&OD	CP&OD	OD&DHM	OD&DHM	OD&DHM
	Collider Fixed target	Fixed target Lighter ion collisions	Collider Fixed target	Fixed target	Fixed target

CP=Critical Point OD= Onset of Deconfinement DHM=Dense Hadronic Matter

The theory of strong interactions

- ✧ Quantum ChromoDynamics (QCD) Nobel prize 2004
- ✧ Analytic solutions of QCD are not possible in the non-perturbative regime
- ✧ Numerical approach to solve QCD
- ✧ Simulations are running on the most powerful supercomputers in the world



Fundamental fields

How can lattice QCD support the experiments?

- Equation of state
  - Needed for hydrodynamic description of the QGP
- QCD phase diagram
  - Transition line at finite density
  - Constraints on the location of the critical point
- Fluctuations of conserved charges
  - Can be simulated on the lattice and measured in experiments
  - Can give information on the evolution of heavy-ion collisions
  - Can give information on the critical point

QCD Equation of State at finite density

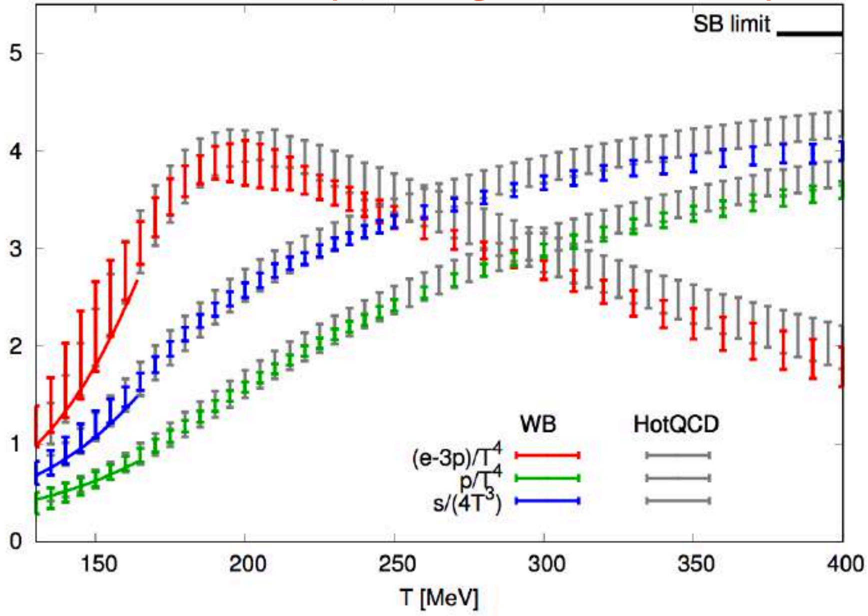
TAYLOR EXPANSION

ANALYTICAL CONTINUATION FROM IMAGINARY CHEMICAL POTENTIAL

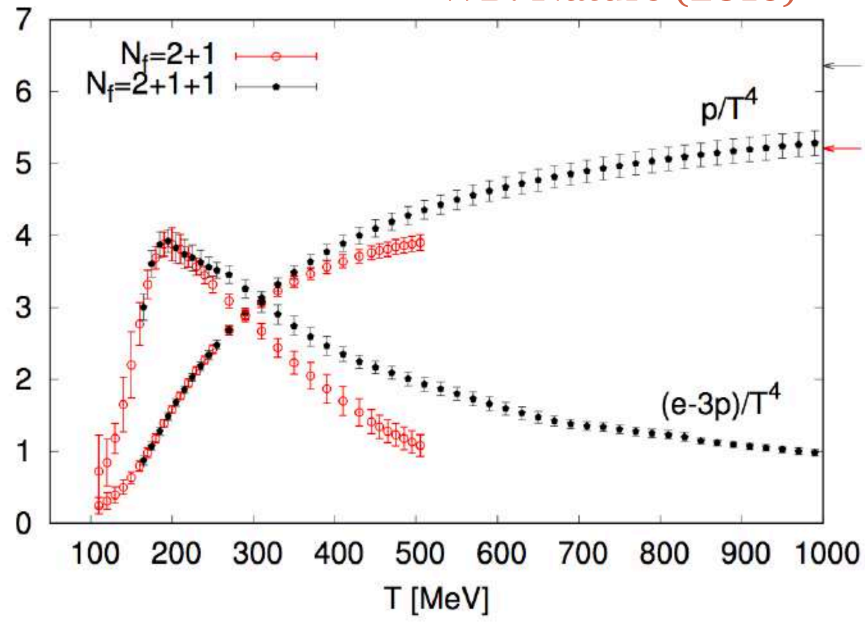
ALTERNATIVE EQUATION OF STATE AT LARGE DENSITIES

QCD EoS at μ_B=0

WB: PLB (2014); HotQCD: PRD (2014)



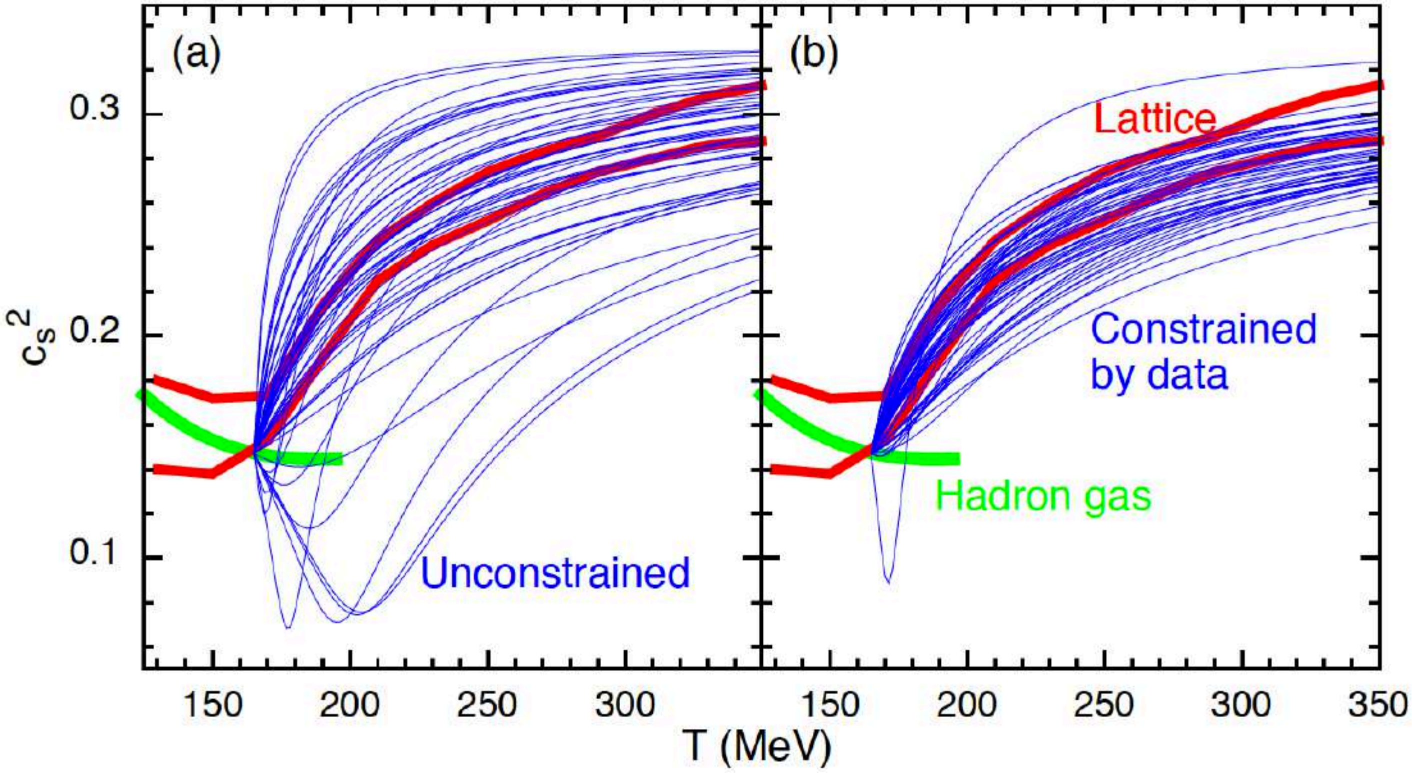
WB: Nature (2016)



- EoS for N_f=2+1 known in the continuum limit since 2013
- Good agreement with the HRG model at low temperature
- Charm quark relevant degree of freedom already at T~250 MeV

Constraints on the EoS from the experiments

S. Pratt et al., PRL (2015)



- Comparison of data from RHIC and LHC to theoretical models through Bayesian analysis
- The posterior distribution of EoS is consistent with the lattice QCD one

Taylor expansion of EoS

- Taylor expansion of the pressure:

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{\mathrm{d}^{2\mathrm{n}}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

- Two ways of extracting the Taylor expansion coefficients:
  - Direct simulation
  - Simulations at imaginary μ_B

- Two physics choices:
  - $\mu_B \neq 0$, $\mu_S = \mu_Q = 0$
  - μ_S and μ_Q are functions of T and μ_B to match the experimental constraints:

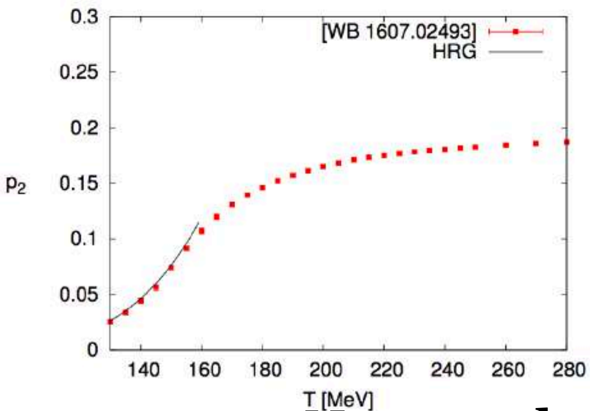
$\langle n_S \rangle = 0$ $\quad\quad$ $\langle n_Q \rangle = 0.4 \langle n_B \rangle$

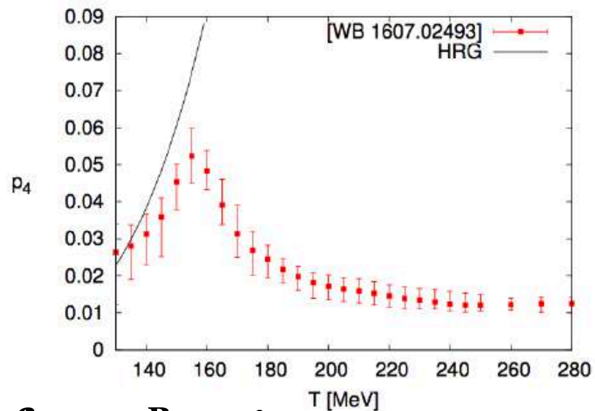
Pressure coefficients

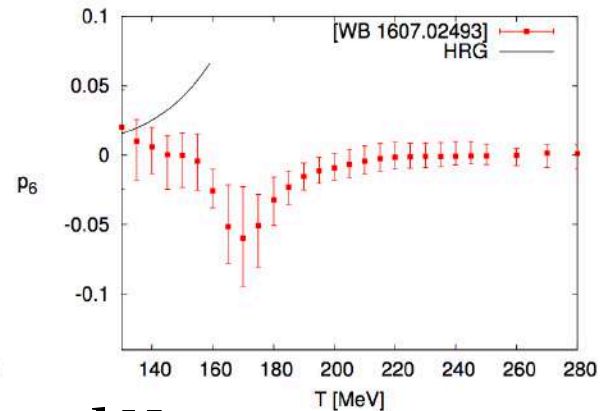
Simulations at imaginary μ_B:

Continuum, O(10[4]) configurations, errors include systematics (WB: NPA (2017))

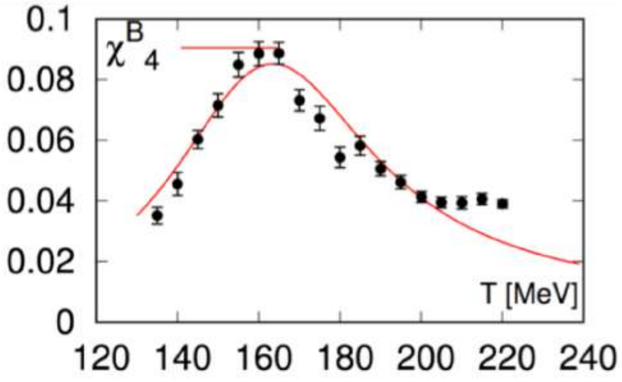
Strangeness neutrality

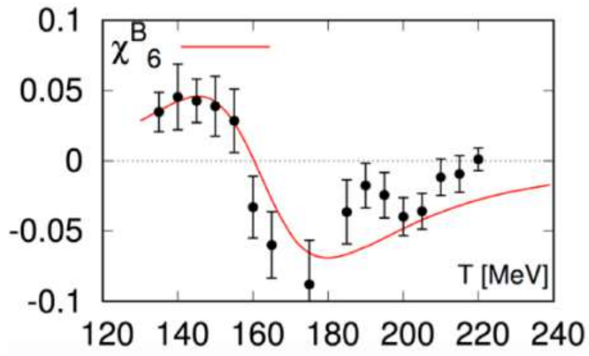


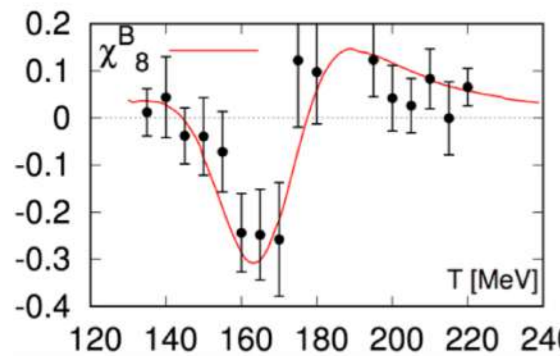




New results for $\chi_n^B = n! c_n$ at $\mu_S=\mu_Q=0$ and Nt=12





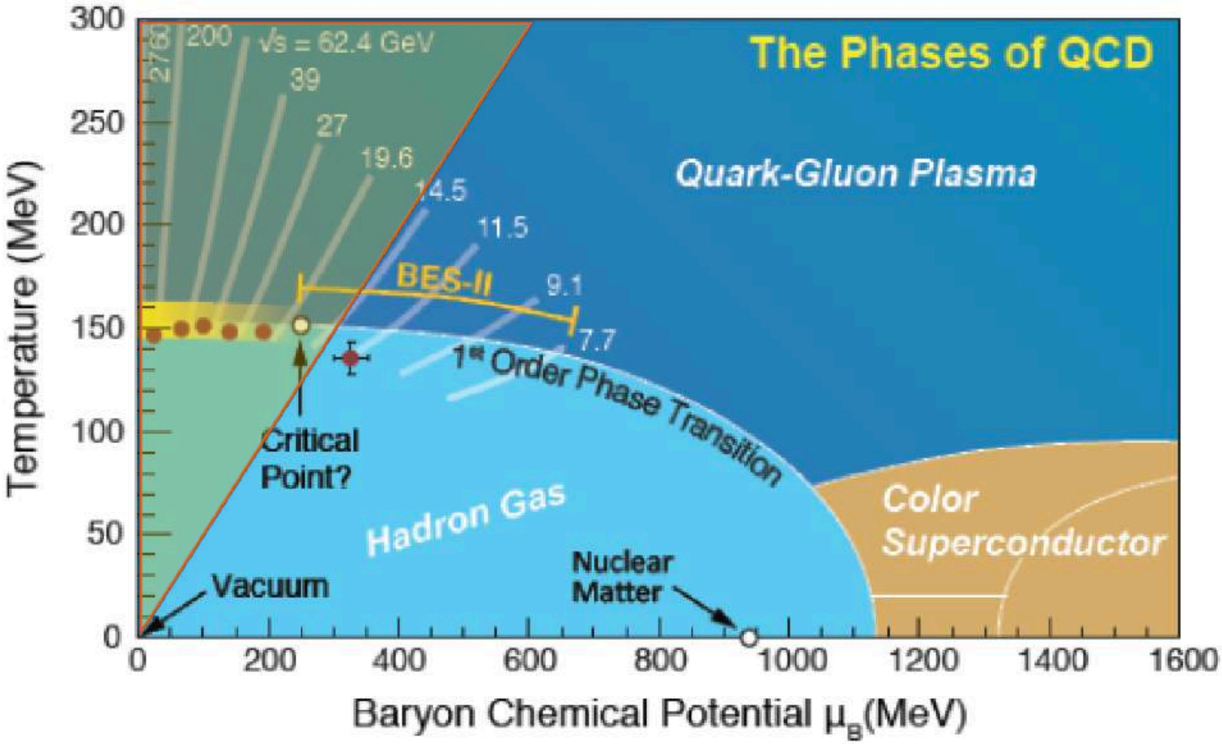


WB, JHEP (2018)

Range of validity of equation of state

- We now have the equation of state for $\mu_B/T \leq 2$ or in terms of the RHIC energy scan:

$$\sqrt{s} = 200,\ 62.4,\ 39,\ 27,\ 19.6,\ 14.5\text{GeV}$$



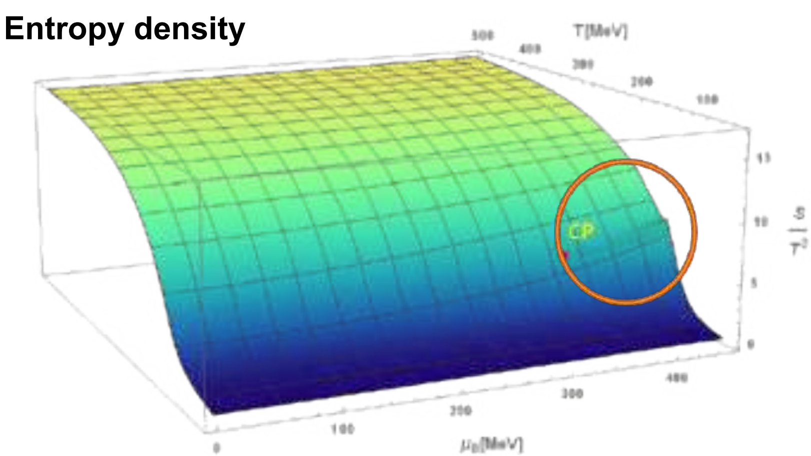
Alternative EoS at large densities

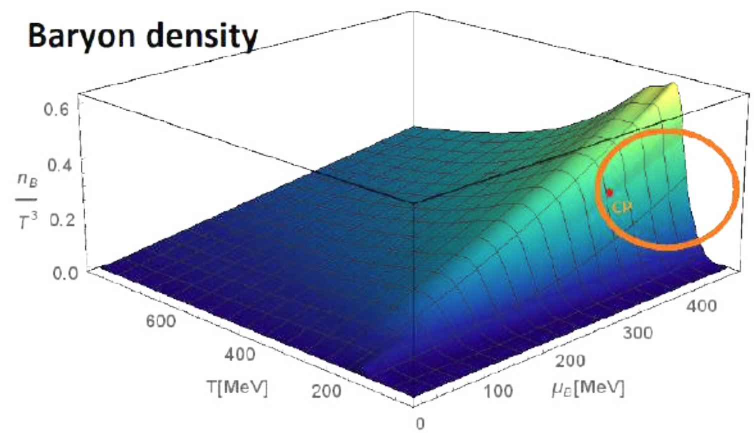
P. Parotto, C. R. et al., PRC (2020)

- EoS for QCD with a 3D-Ising critical point

$$T^4 c_n^{LAT}(T) = T^4 c_n^{Non\text{-}Ising}(T) + T_c^4 c_n^{Ising}(T)$$

- Implement scaling behavior of 3D-Ising model EoS
- Define map from 3D-Ising model to QCD
- Estimate contribution to Taylor coefficients from 3D-Ising model critical point
- Reconstruct full pressure





Open-source code at https://www.bnl.gov/physics/best/resources.php

- Entropy and baryon density discontinuous at $\mu_B > \mu_{Bc}$

QCD phase diagram

TRANSITION TEMPERATURE

TRANSITION LINE

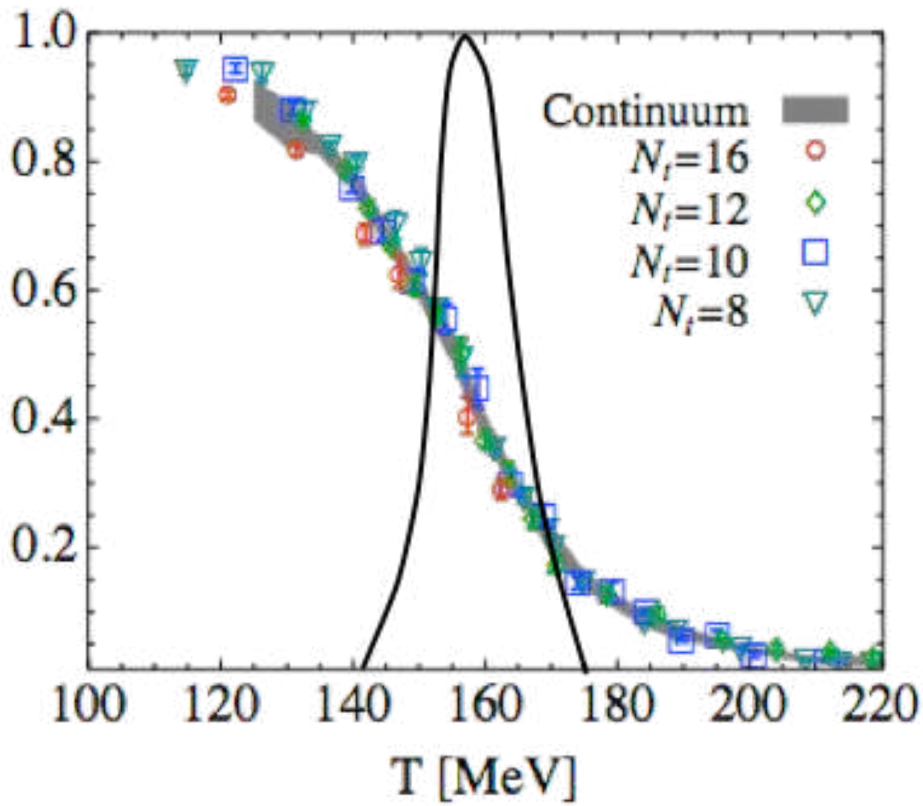
TRANSITION WIDTH

Phase Diagram from Lattice QCD

- The transition at μ_B=0 is a smooth crossover

Aoki et al., Nature (2006)

Borsanyi et al., JHEP (2010)
Bazavov et al., PRD (2012)



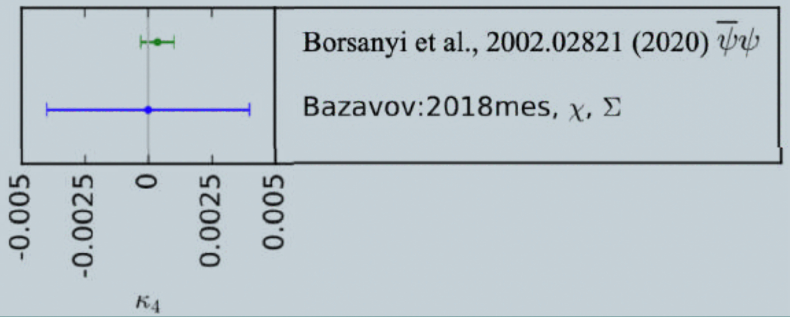
QCD transition temperature and curvature

Borsanyi, C. R. et al. PRL (2020)

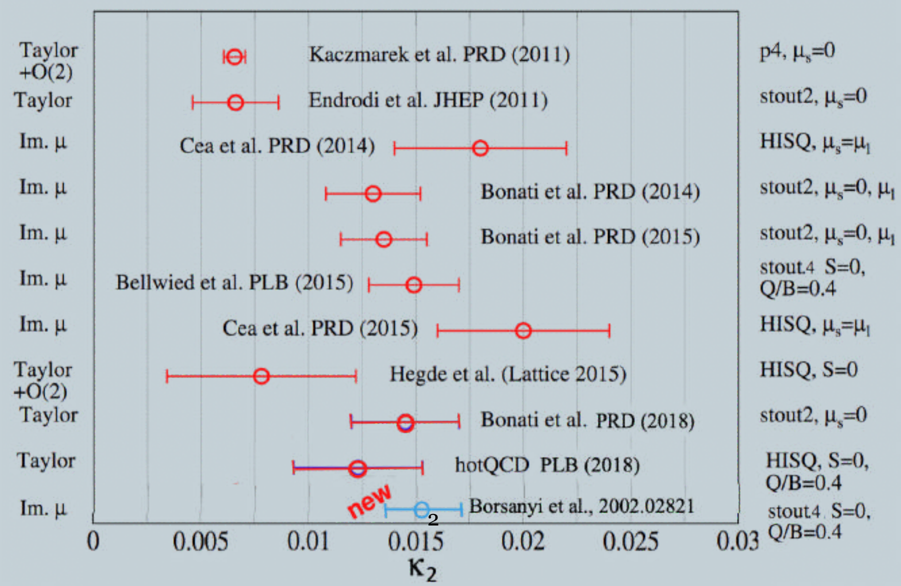
$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + O(\mu_B^6)$$

- QCD transition at μ_B=0 is a crossover
  Aoki et al., Nature (2006)
- Latest results on T_0 from WB collaboration based on subtracted chiral condensate and chiral susceptibility

T_0=158.0±0.6 MeV



Compilation by F. Negro



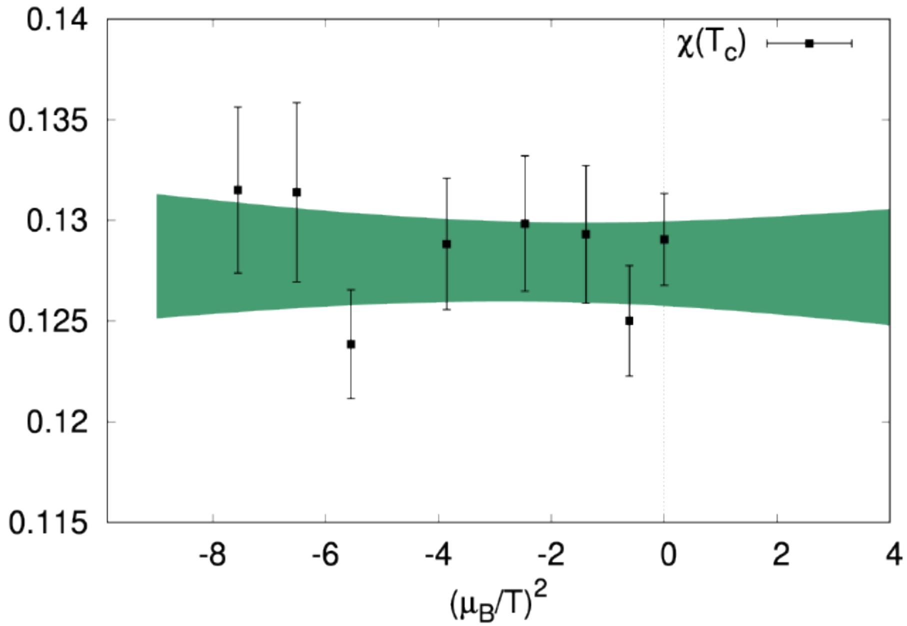
$$\kappa_2 = 0.0153 \pm 0.0018\ ,$$
$$\kappa_4 = 0.00032 \pm 0.00067$$

Limit on the location of the critical point

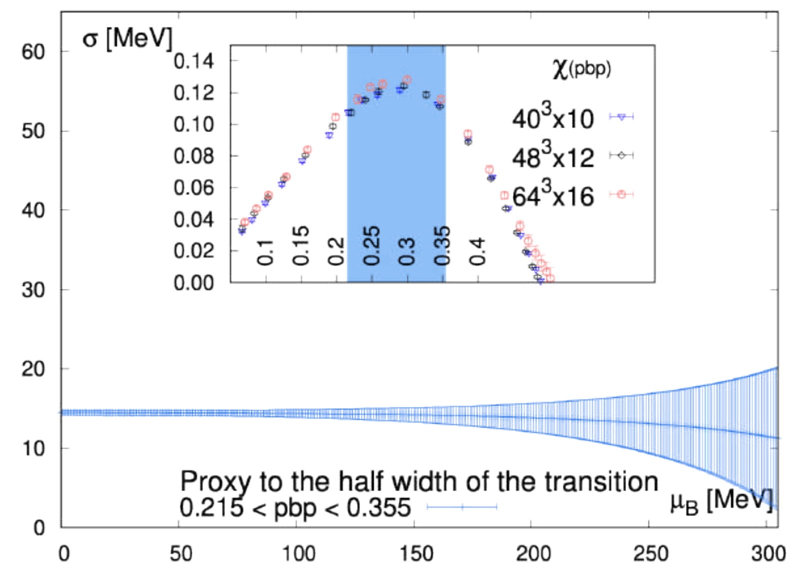
Borsanyi, C. R. et al. PRL (2020)

- For a genuine phase transition, the height of the peak of the chiral susceptibility diverges and the width shrinks to zero

Height of chiral susceptibility peak



Width of chiral susceptibility peak



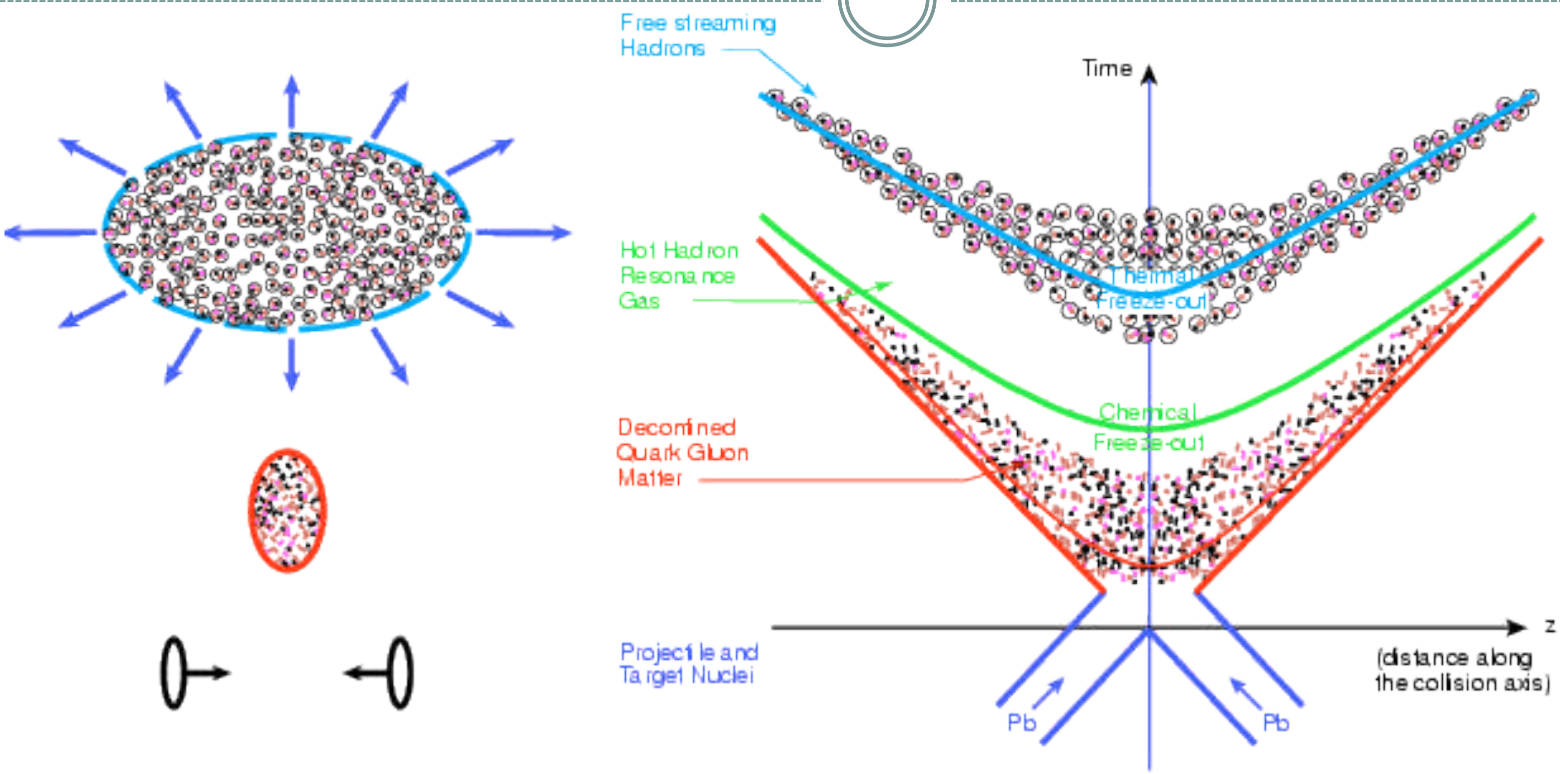
- No sign of criticality for μ_B<300 MeV

Fluctuations of conserved charges

COMPARISON TO EXPERIMENT: CHEMICAL FREEZE-OUT PARAMETERS

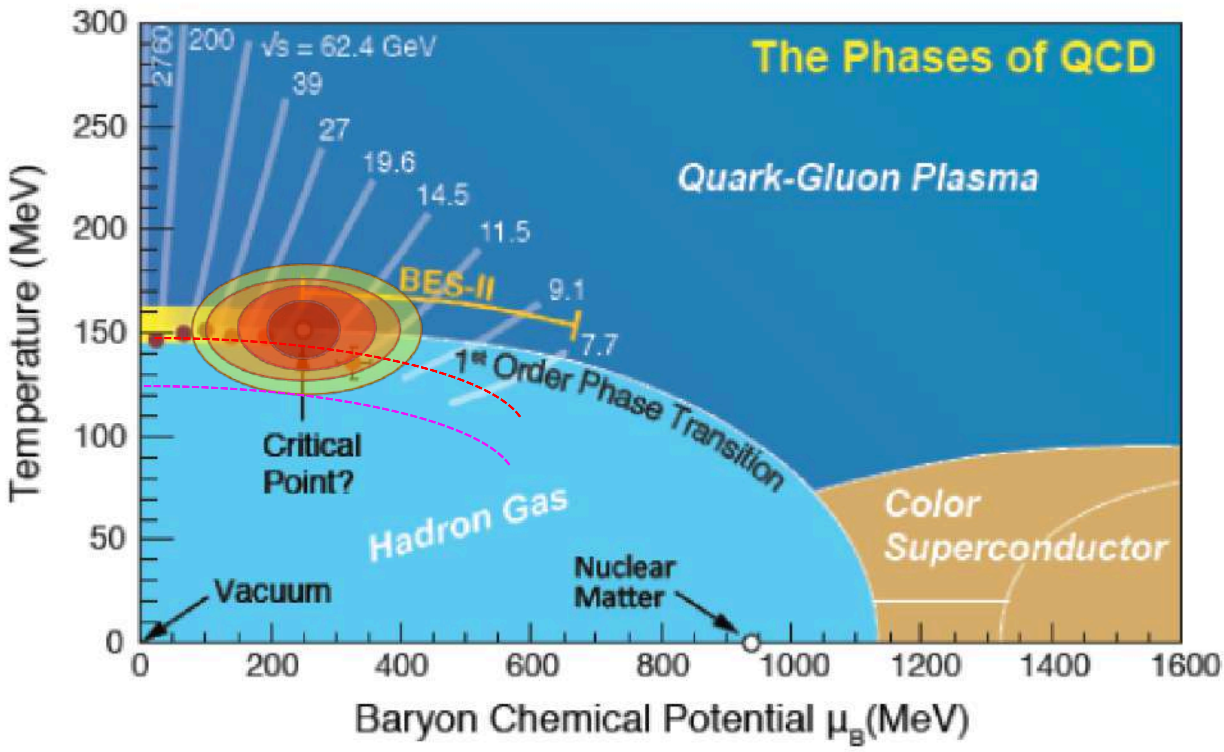
OFF-DIAGONAL CORRELATORS

Evolution of a heavy-ion collision



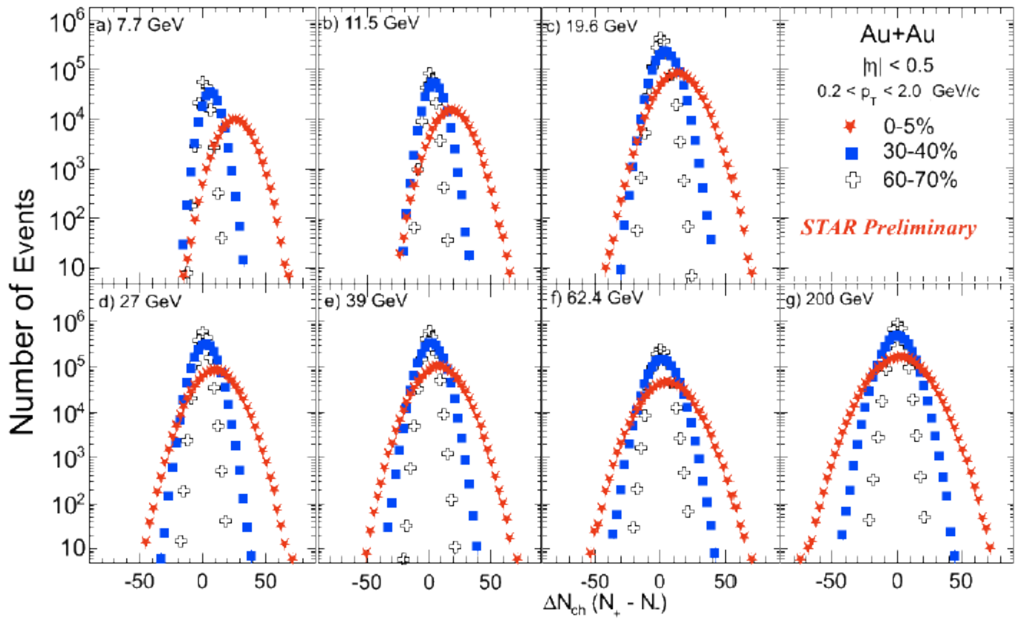
- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Freeze-out vs phase transition



Distribution of conserved charges

- Consider the number of electrically charged particles N_Q
- Its average value over the whole ensemble of events is $\langle N_Q \rangle$
- In experiments it is possible to measure its event-by-event distribution



STAR Collab.: PRL (2014)

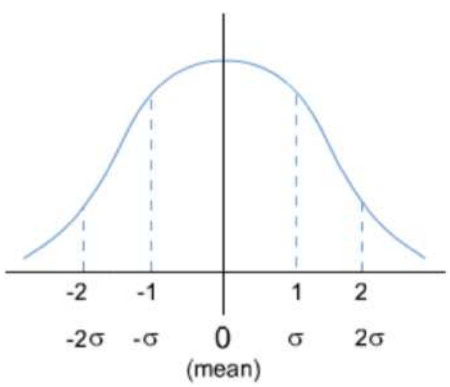
Cumulants of multiplicity distribution

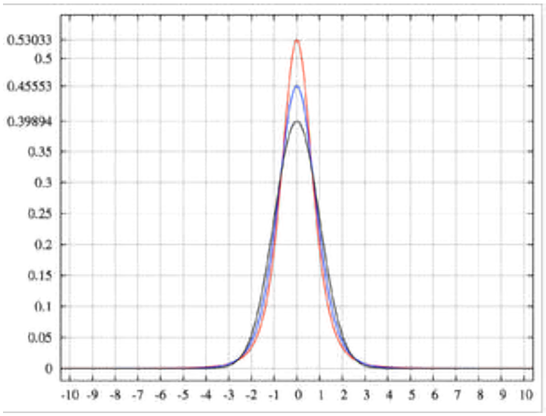
- Deviation of N_Q from its mean in a single event: $\delta N_Q = N_Q - \langle N_Q \rangle$
- The cumulants of the event-by-event distribution of N_Q are:

$$\chi_2 = \langle (\delta N_Q)^2 \rangle \qquad \chi_3 = \langle (\delta N_Q)^3 \rangle \qquad \chi_4 = \langle (\delta N_Q)^4 \rangle - 3\langle (\delta N_Q)^2 \rangle^2$$

- The cumulants are related to the central moments of the distribution by:

variance: $\sigma^2 = \chi_2$ *Skewness:* $S = \chi_3/(\chi_2)^{3/2}$ *Kurtosis:* $\kappa = \chi_4/(\chi_2)^2$





Fluctuations on the lattice

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution
- Definition: $$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$
- They can be calculated on the lattice and compared to experiment
- variance: $\sigma^2=\chi_2$ Skewness: $S=\chi_3/(\chi_2)^{3/2}$ Kurtosis: $\kappa=\chi_4/(\chi_2)^2$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

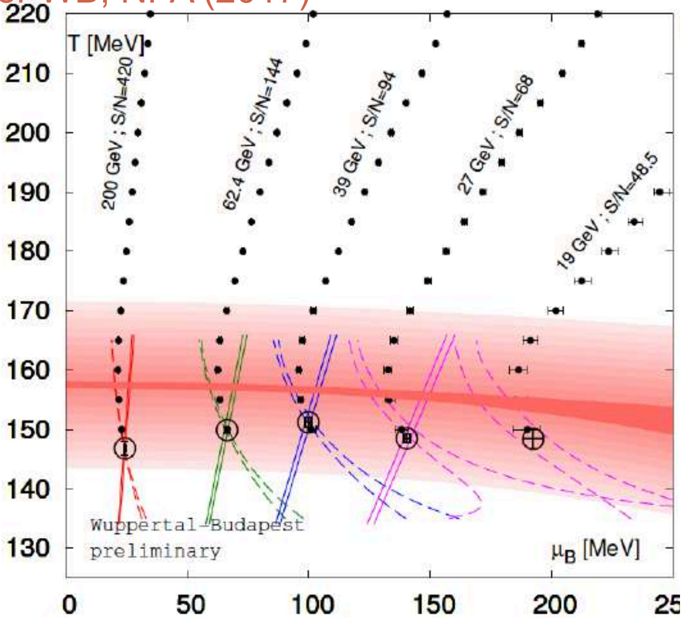
Freeze-out line from first principles

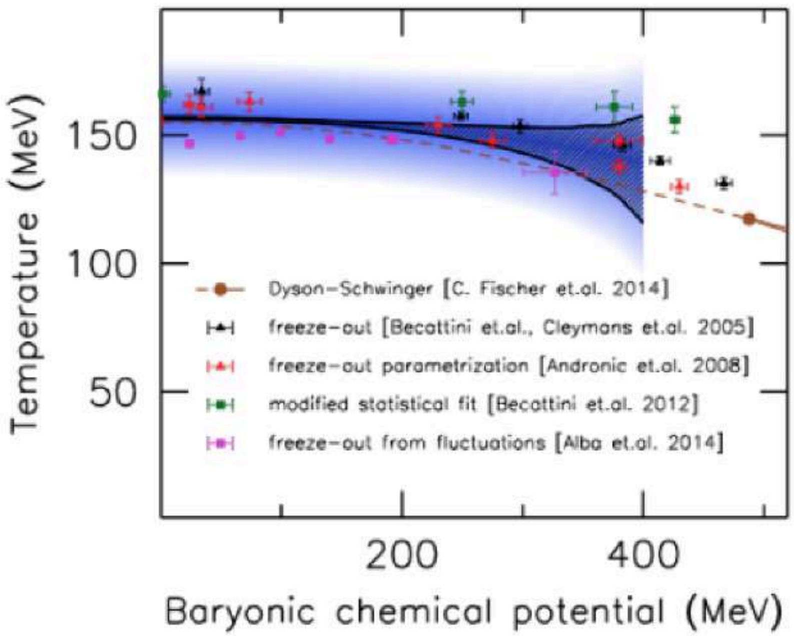
- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

C. Ratti for WB, NPA (2017)



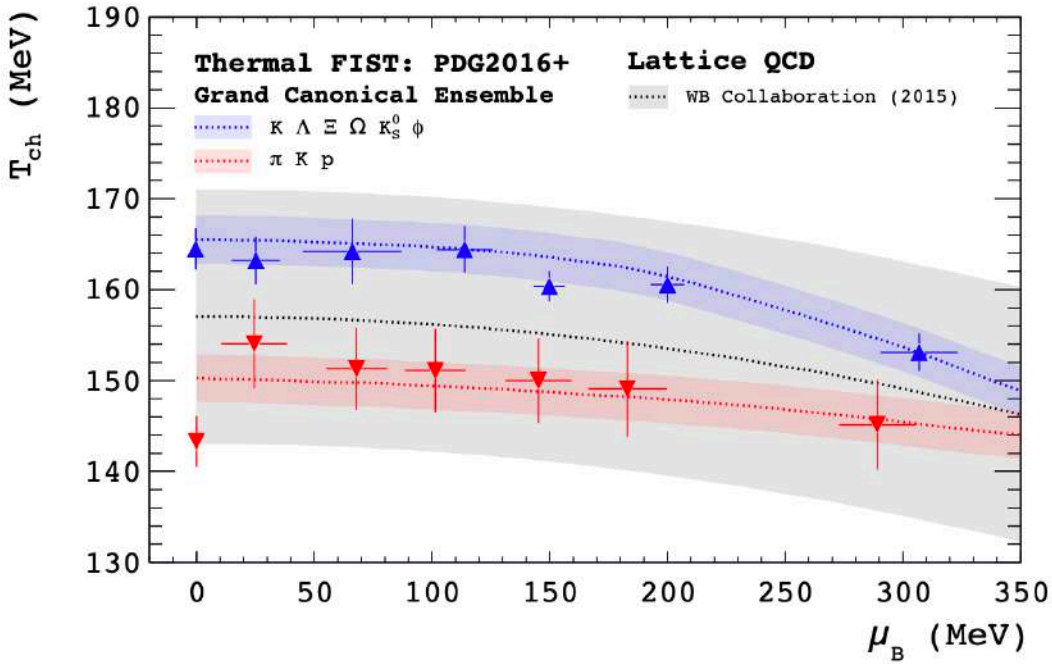


What about strangeness?

- Data for net-kaon fluctuations seem to prefer a higher freeze-out temperature. R. Bellwied, C. R. et al., Phys. Rev. C (2019)

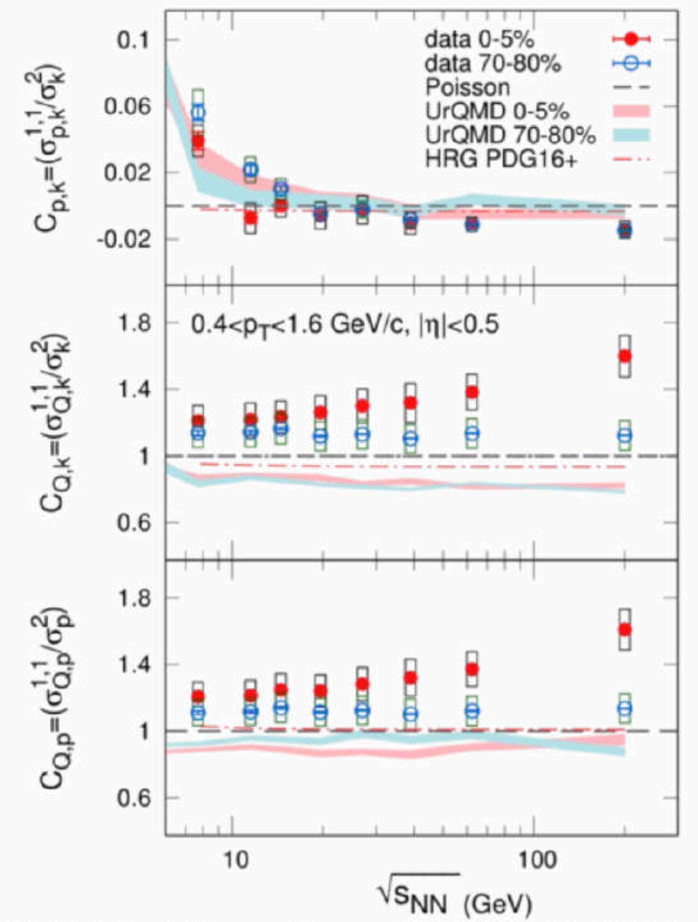
- Separate analysis of particle yields gives a similar result

P. Alba, C. R. et al., Phys. Rev. C (2020)



F. Flor et al., 2009.14781 (2020)

Off-diagonal fluctuations of conserved charges



STAR: Phys.Rev.C 100 014902 (2019)

- The measurable species in HIC are only a handful. How much do they tell us about the correlation between conserved charges?
- Historically, the proxies for B, Q and S have been p, p,π,K and K themselves → what about off-diagonal correlators?
- We want to find:
  - The main contributions to off-diagonal correlators
  - A way to compare lattice to experiment

Off-diagonal correlators

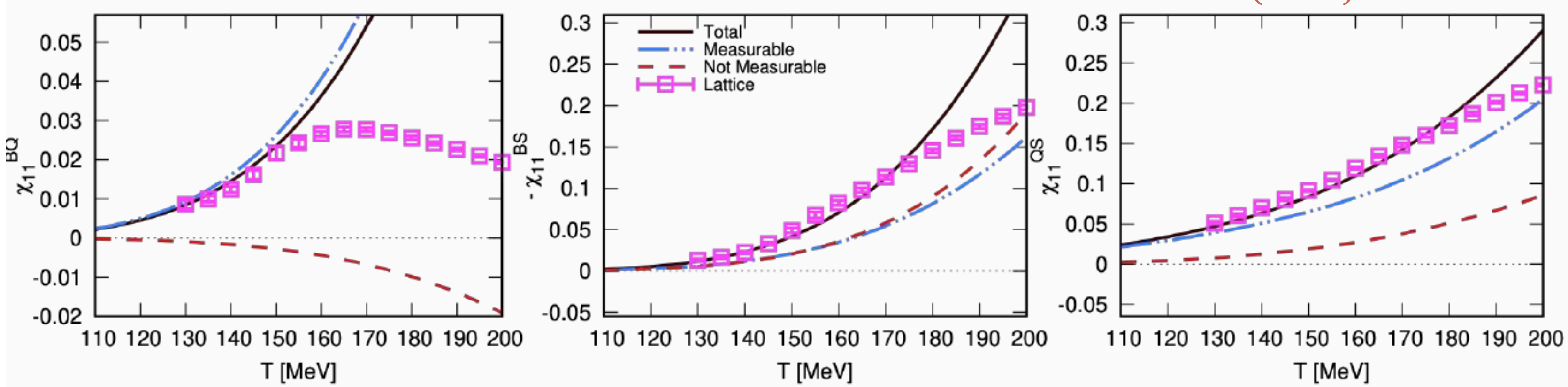
- The species that are stable under strong interactions, AND are **measurable**

$$\pi^{\pm},\ K^{\pm},\ p\,(\overline{p})\,,\ \Lambda(\overline{\Lambda}),\ \Xi^{-}(\overline{\Xi}^{+}),\ \Omega^{-}(\overline{\Omega}^{+})$$

$\longrightarrow$ we inevitably lose a good chunk of conserved charges!

- Thanks to the **separation between observable and non-observables species**, one can pinpoint what can be measured and what cannot of χ_{ijk}^{BQS}

R. Bellwied, C. R. et al., PRD (2020)

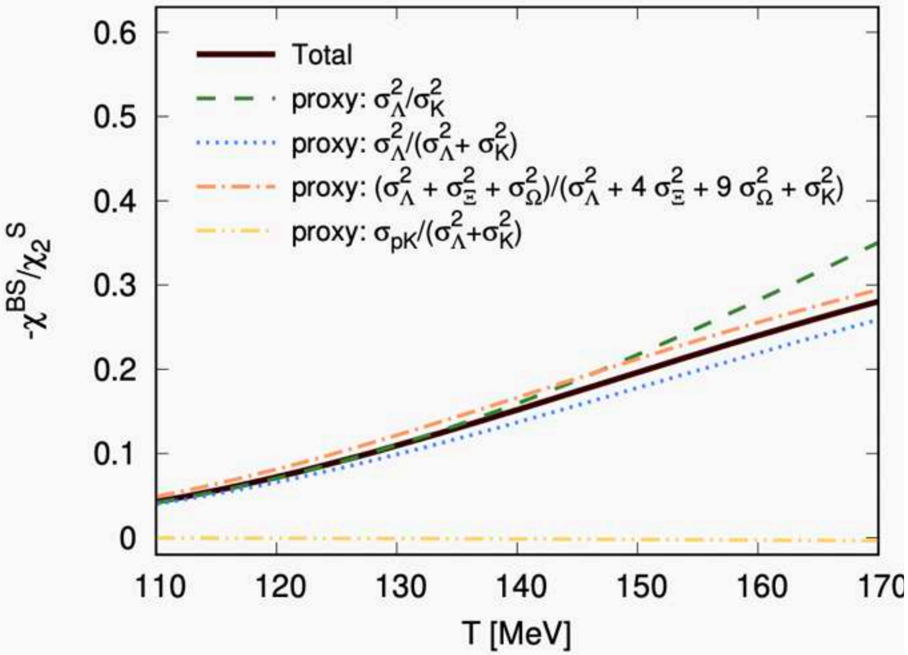


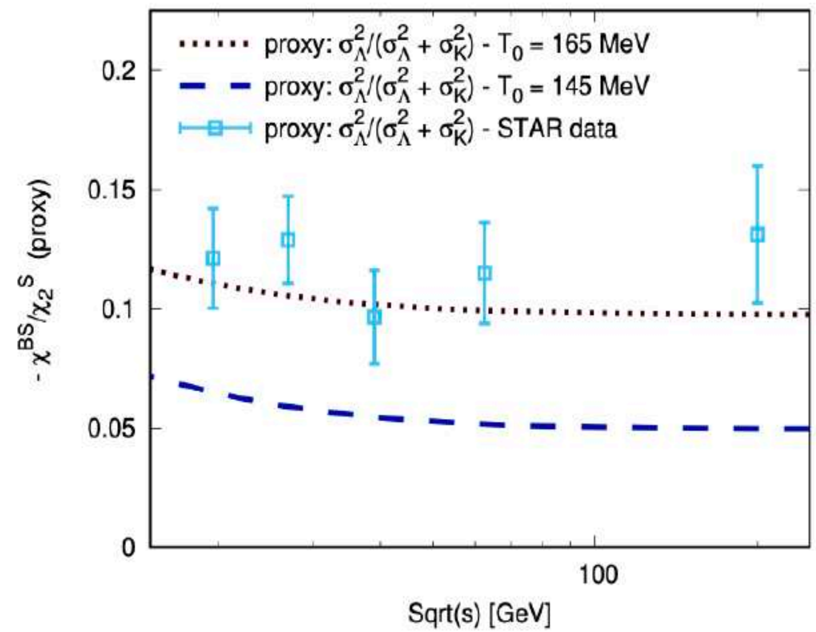
- For the **proton- and kaon-dominated** χ_{BQ} **and** χ_{QS}, a large part of the full correlator is carried by measurable particles
- χ_{BS} is less transparent, and requires careful analysis of its contributions

Hadronic proxies

R. Bellwied, C. R. et al., PRD (2020)

Constructing a proxy not a trivial task: consider main contributions to numerator and denominator

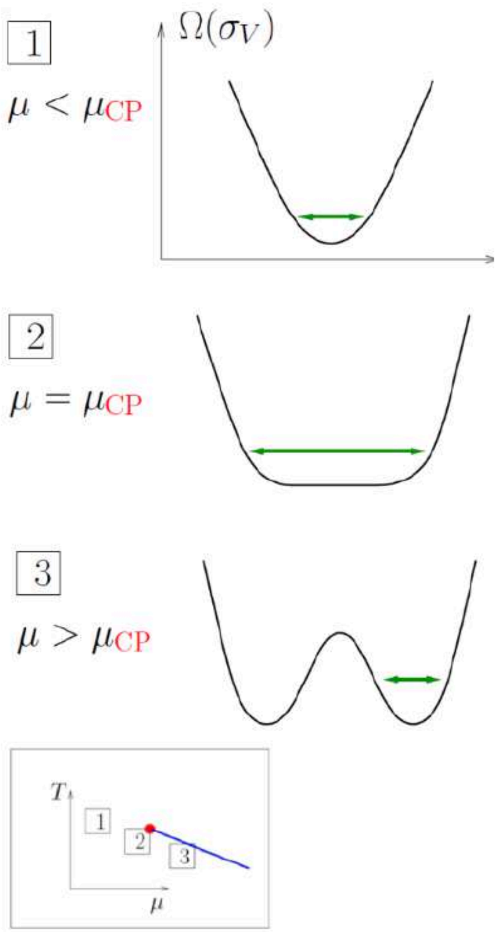




- Good proxy for χ_{11}^{BS}/χ_2^S: $\widetilde{C}_{BS,SS}^{\Lambda,\Lambda K} = \sigma_\Lambda^2/(\sigma_K^2+\sigma_\Lambda^2)$

Fluctuations at the critical point

M. Stephanov, PRL (2009).



The probability distribution for the order parameter

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma_2 + \frac{\lambda_3}{3}\sigma^3 + \cdots\right]$$

The correlation length ($\xi = m_\sigma^{-1}$)

$$\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0$$

$$\chi_2 = VT\xi^2$$

$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$

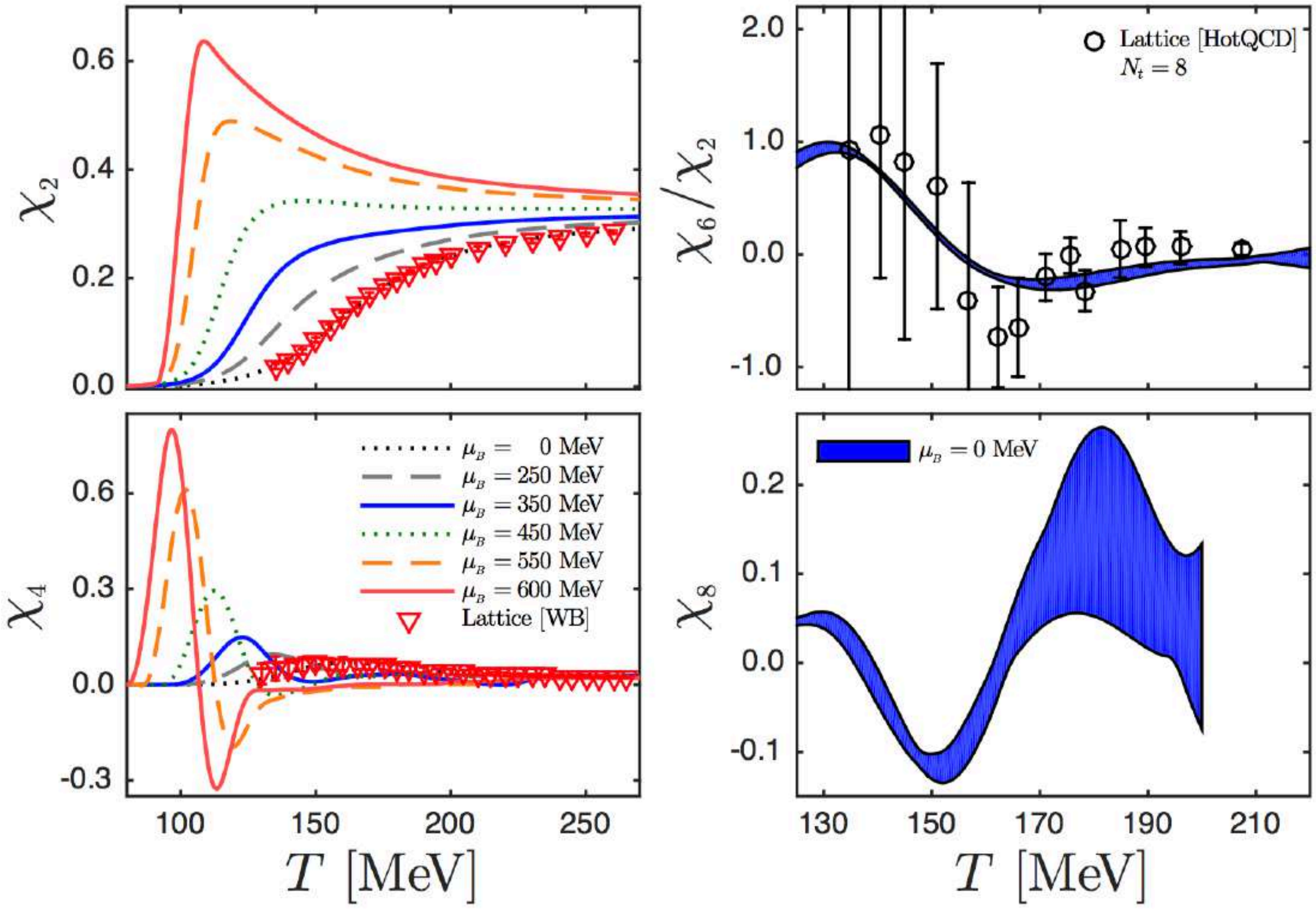
$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

A different approach at large densities

- Use AdS/CFT correspondence
- Fix the parameters to reproduce everything we know from the lattice
- Calculate observables at finite density
- Fluctuations of conserved charges: they are sensitive to the critical point

Black Hole Susceptibilities

R. Critelli, C. R. et al., PRD (2017)

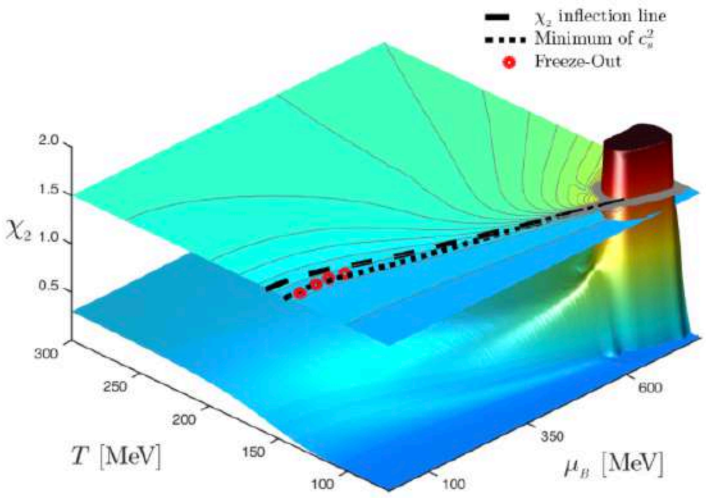


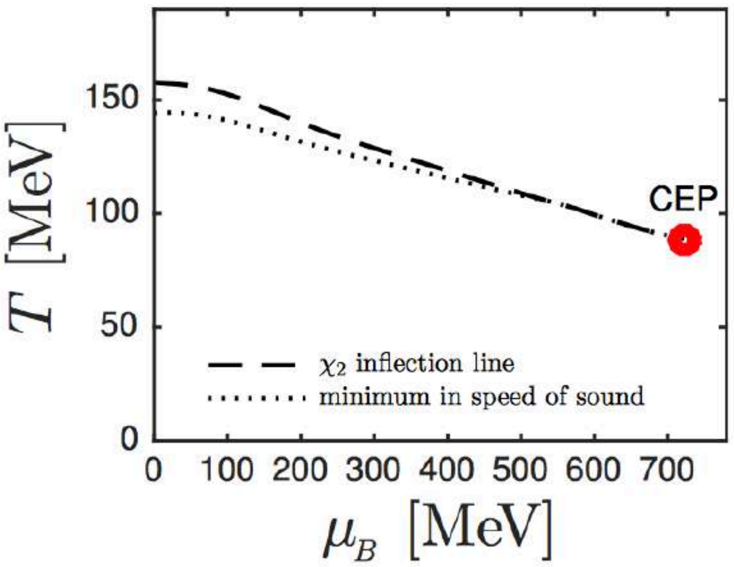
Black hole critical point

R. Critelli, C. R. et al., PRD (2017)

The black hole model contains a critical end point at

- $\mu_B = 723 \pm 36$ MeV
- $T = 89 \pm 11$ MeV





Conclusions

- Need for quantitative results at finite-density to support the experimental programs
  - Equation of state
  - Phase transition line
  - Fluctuations of conserved charges
- Current lattice results for thermodynamics up to $\mu_B/T \leq 2$
- Extensions to higher densities by means of lattice-based models
- No indication of Critical Point from lattice QCD in the explored μ_B range

Backup slides

Hadron Resonance Gas model

Dashen, Ma, Bernstein; Prakash, Venugopalan; Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3}\sum_{i\in\ mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3}\sum_{i\in\ baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{Vd_i}{2\pi^2}\int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

with energies $\varepsilon_i = \sqrt{k^2+m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right) \quad .$$

X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Fugacity expansion for $\mu_S=\mu_Q=0$: $\frac{p_B}{T^4} = \sum_{i\in B}\frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{N=1}^{\infty}(-1)^{N+1}N^{-2}K_2(N\frac{m_i}{T})\cosh\left[N\frac{\mu_B}{T}\right]$

Boltzmann approximation: N=1

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527

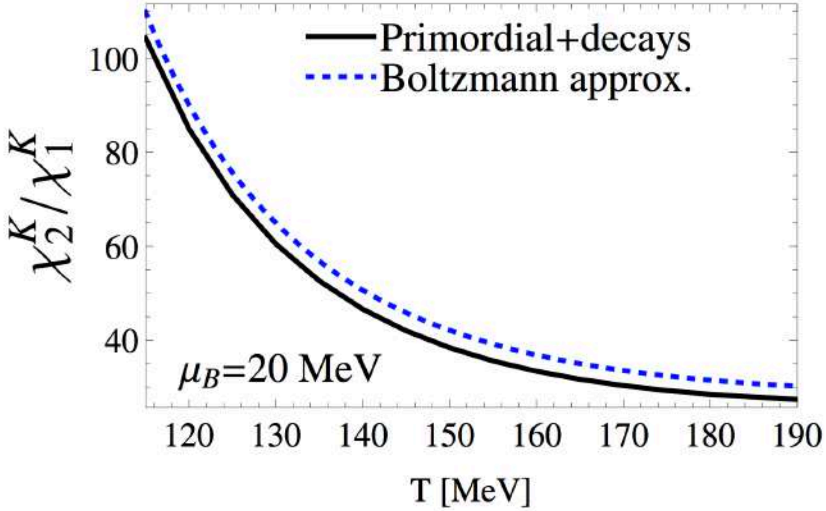
- Lattice QCD works in terms of conserved charges
- Challenge: isolate the fluctuations of a given particle species
- Assuming an HRG model in the Boltzmann approximation, it is possible to write the pressure as:

$$P(\hat{\mu}_B, \hat{\mu}_S) = P_{00}^{BS} + P_{10}^{BS}\cosh(\hat{\mu}_B) + P_{01}^{BS}\cosh(\hat{\mu}_S)$$
$$+ P_{11}^{BS}\cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS}\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P_{13}^{BS}\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

- Kaons in heavy ion collisions: primordial + decays
- Idea: calculate χ_2^K/χ_1^K in the HRG model for the two cases: only primordial kaons in the Boltzmann approximation vs primordial + resonance decay kaons

Kaon fluctuations on the lattice

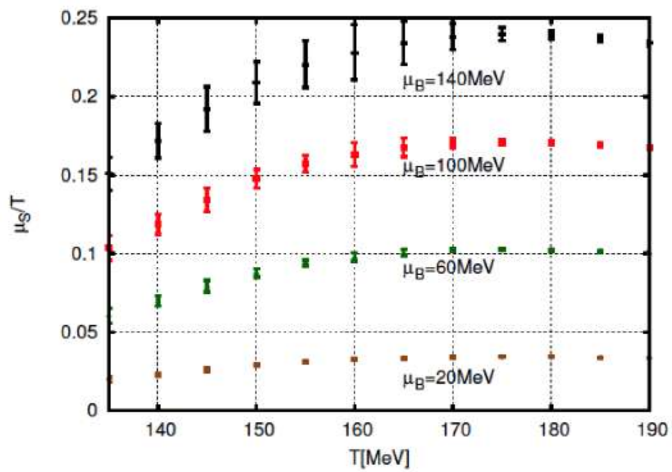
J. Noronha-Hostler, C.R. et al., forthcoming

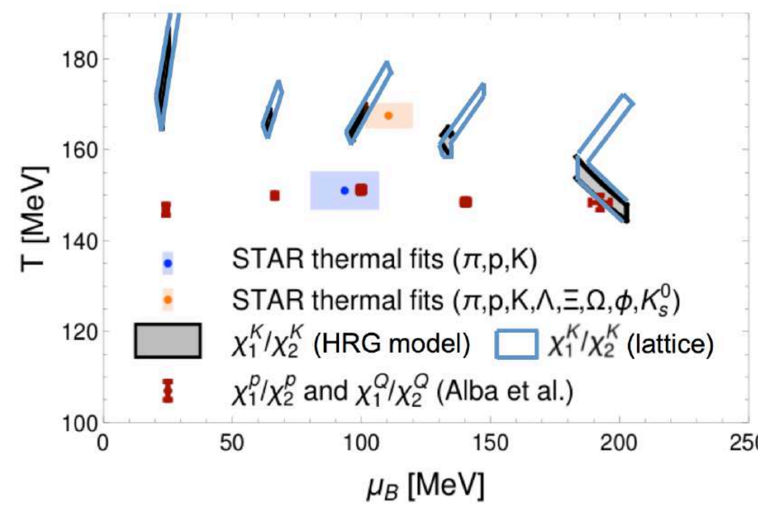


- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K/χ_1^K from primordial kaons + decays is very close to the Boltzmann approximation
- μ_S and μ_Q are functions of T and μ_B to match the experimental constraints:





Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

  V. Skokov et al., PRC (2013), P. Braun-Munzinger et al., NPA (2017), V. Begun and M. Mackowiak-Pawlowska (2017)
- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution

  A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance

  V. Koch, S. Jeon, PRL (2000)
- Baryon number conservation
  - Experimental data need to be corrected for this effect

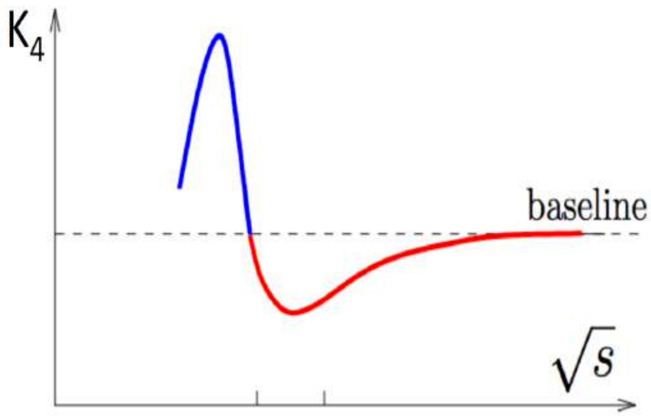
  P. Braun-Munzinger et al., NPA (2017)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

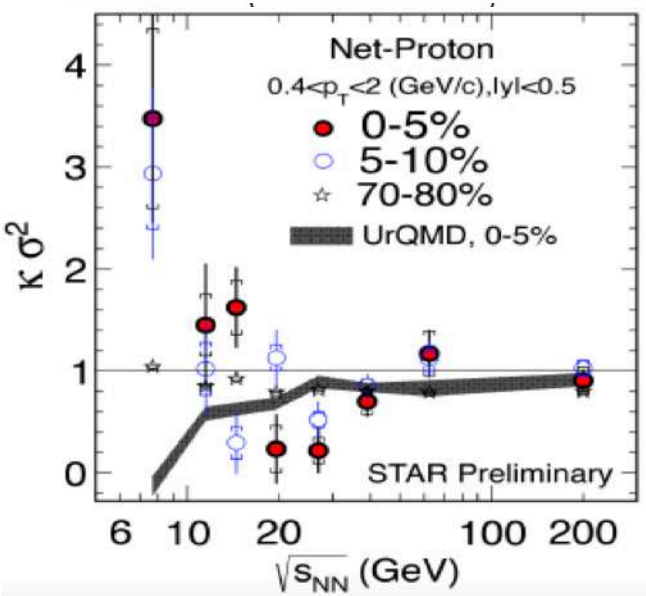
  M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

  J.Steinheimer et al., PRL (2013)

Fluctuations at the critical point

M. Stephanov, PRL (2009).





- Correlation length near the critical point
  $\xi \sim |T - T_c|^{-\nu}$ where $\nu > 0$

$$\chi_2 = VT\xi^2$$
$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$
$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

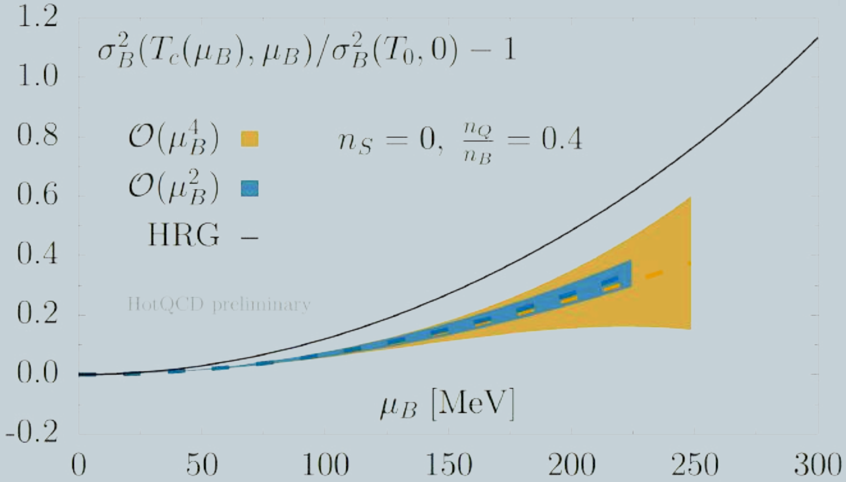
- Fluctuations are expected to diverge at the critical point
- Fourth-order fluctuations should have a non-monotonic behavior
- Preliminary STAR data seem to confirm this
- Can we describe this trend with lattice QCD?

Fluctuations along the QCD crossover

P. Steinbrecher for HotQCD, 1807.05607

Net-baryon variance

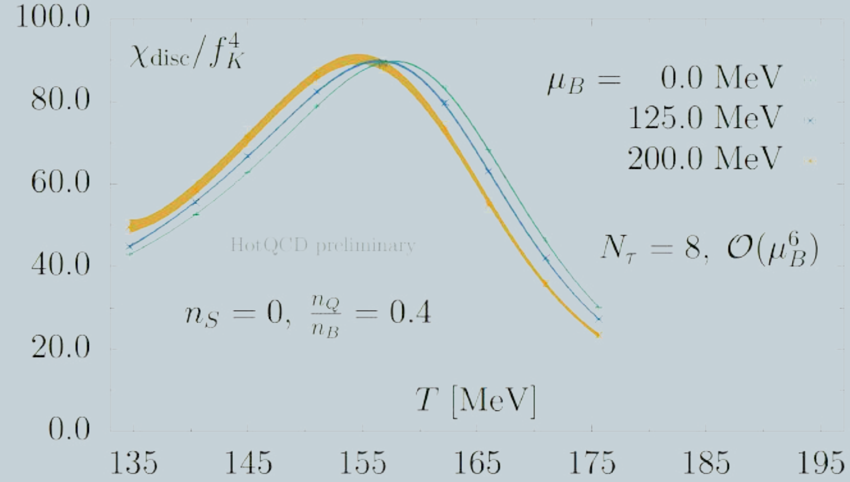
$$\frac{\sigma_B^2(T_c(\mu_B),\mu_B)-\sigma_B^2(T_0,0)}{\sigma_B^2(T_0,0)} = \lambda_2\left(\frac{\mu_B}{T_0}\right)^2 + \lambda_4\left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$



- Expected to be larger than HRG model result near the CP
- No sign of criticality

Disconnected chiral susceptibility

$$\chi_{\rm sub} \equiv \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Big[m_s(\Sigma_u + \Sigma_d) - (m_u + m_d)\Sigma_s\Big]$$

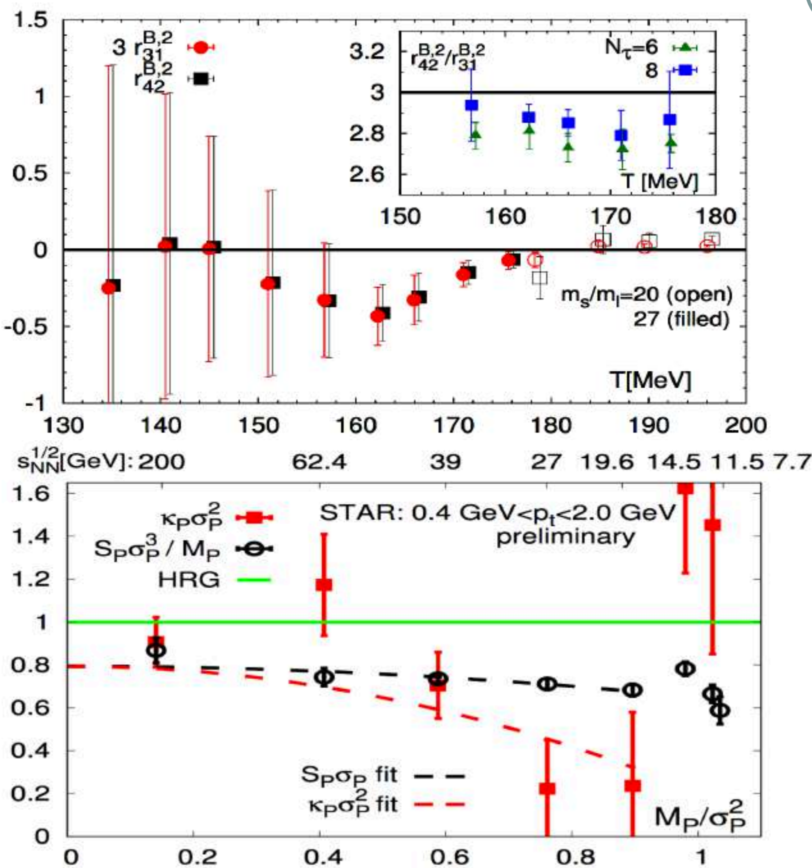


- Peak height expected to increase near the CP
- No sign of criticality

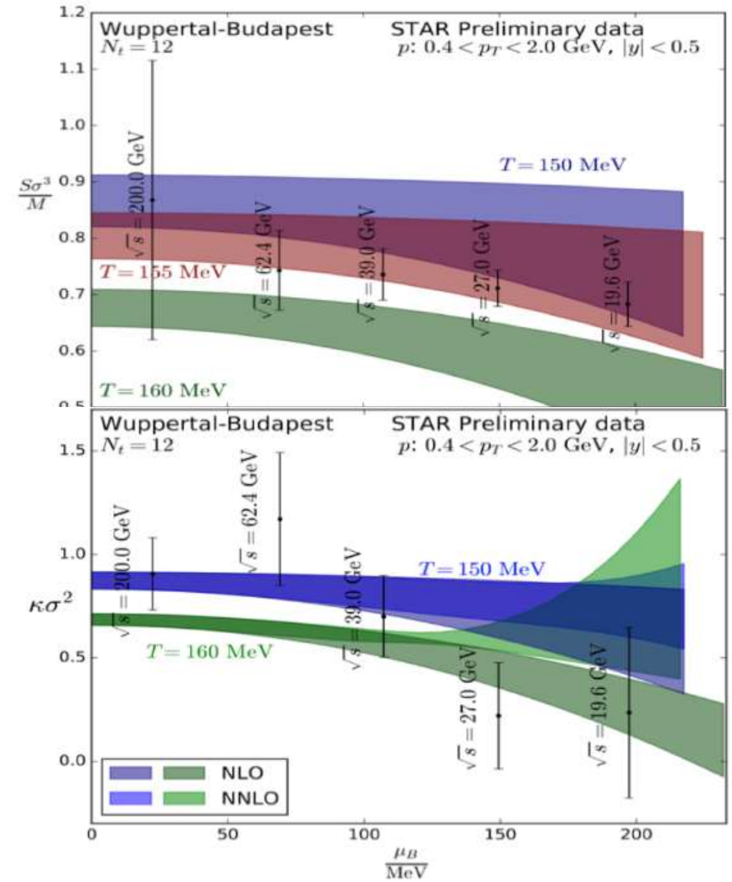
See talk by Patrick Steinbrecher on Wednesday

Higher order fluctuations

HotQCD, PRD (2017)



WB, 1805.04445 (2018)



$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{\chi_4^B + s_1\chi_{31}^{BS} + q_1\chi_{31}^{BQ}}{\chi_2^B + s_1\chi_{11}^{BS} + q_1\chi_{11}^{BQ}} + \mathcal{O}(\mu_B^2) \equiv r_{31}^{B,0} + r_{31}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^2) \equiv r_{42}^{B,0} + r_{42}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)\,,$$

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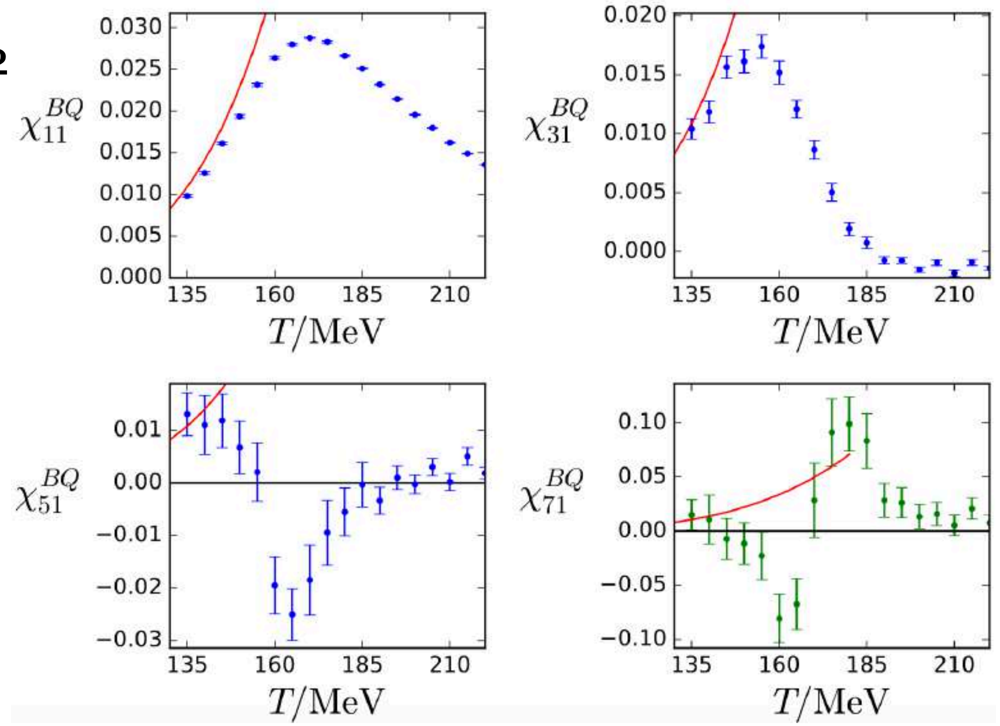
Alternative explanation: canonical suppression

Off-diagonal correlators

WB, 1805.04445 (2018)

- Simulation of the lower order correlators at imaginary μ_B
- Fit to extract higher order terms
- Results exist also for BS, QS and BQS correlators

Nt=12



$$\chi_{11}^{BS}(\hat{\mu}_B) = \chi_{11}^{BS} + \frac{1}{2!}\chi_{31}^{BS}\hat{\mu}_B^2 + \frac{1}{4!}\chi_{51}^{BS}\hat{\mu}_B^4 + \frac{1}{6!}\chi_{71}^{BS}\hat{\mu}_B^6 + \frac{1}{8!}\chi_{91}^{BS}\hat{\mu}_B^8$$

$$\chi_{21}^{BS}(\hat{\mu}_B) = \chi_{31}^{BS}\hat{\mu}_B + \frac{1}{3!}\chi_{51}^{BS}\hat{\mu}_B^3 + \frac{1}{5!}\chi_{71}^{BS}\hat{\mu}_B^5 + \frac{1}{7!}\chi_{91}^{BS}\hat{\mu}_B^7$$

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Forthcoming experimental data at RHIC

See talk by Jana Guenther on Wednesday

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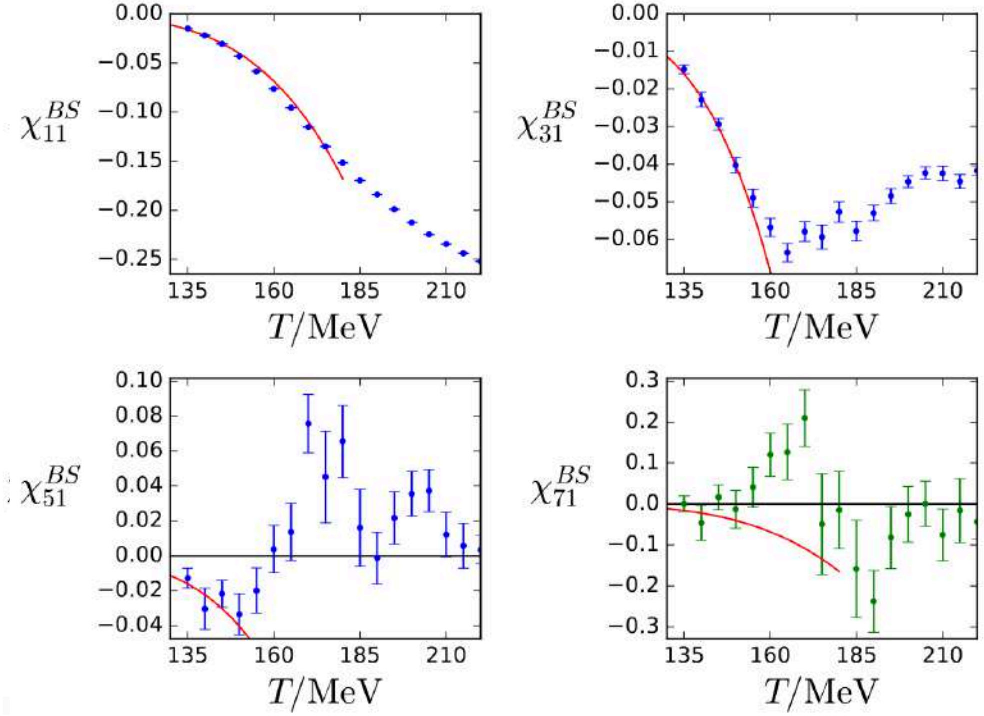

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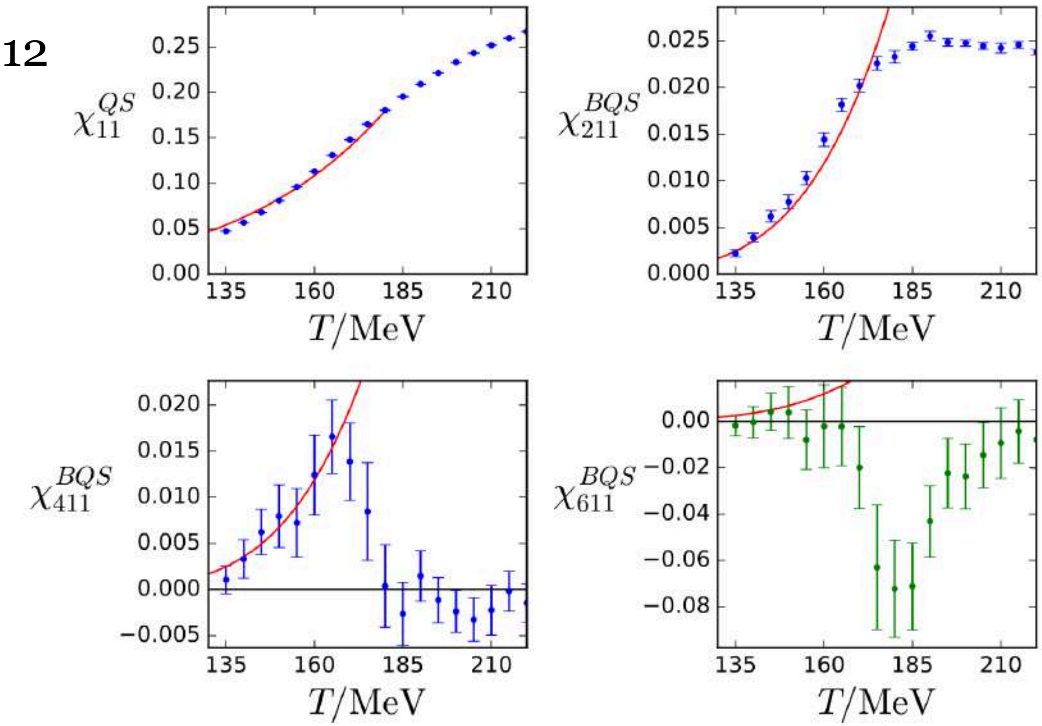

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Forthcoming experimental data at RHIC

See talk by Jana Guenther on Wednesday

Other approaches I did not have time to address

- Reweighting techniques (Fodor & Katz)
- Canonical ensemble (Alexandru et al., Kratochvila, de Forcrand, Ejiri, Bornyakov, Goy, Lombardo, Nakamura)
- Density of state methods (Fodor, Katz & Schmidt, Alexandru et al.)
- Two-color QCD (ITEP Moscow lattice group, Kogut et al., S. Hands et al., von Smekal et al.)
- Scalar field theories with complex actions (See talk by M. Ogilvie on Tuesday)
- Complex Langevin (see talks by D. Sinclair, S. Tsutsui, F. Attanasio, Y. Ito, A. Joseph on Monday)
- Lefshetz Thimble (see talks by K. Zambello, S. Lawrence, N. Warrington, H. Lamm on Monday)
- Phase unwrapping (see talks by G. Kanwar and M. Wagman on Friday)

Conclusions

- Need for quantitative results at finite-density to support the experimental programs
  - Equation of state
  - Phase transition line
  - Fluctuations of conserved charges
- Current lattice results for thermodynamics up to $\mu_B/T \leq 2$
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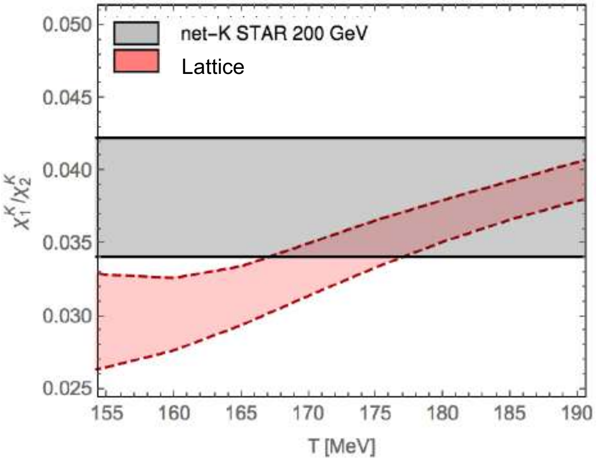
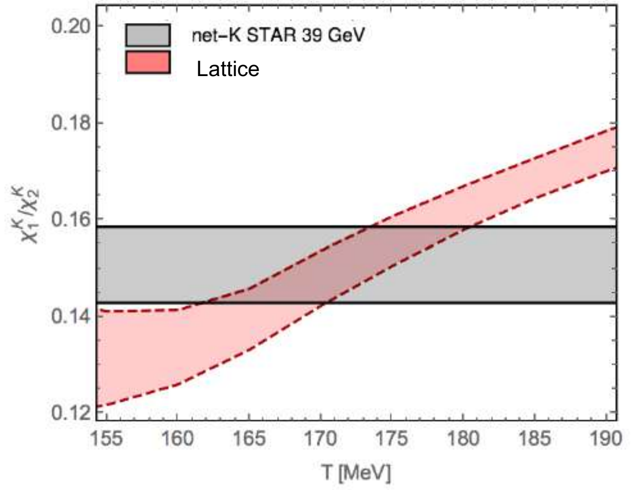
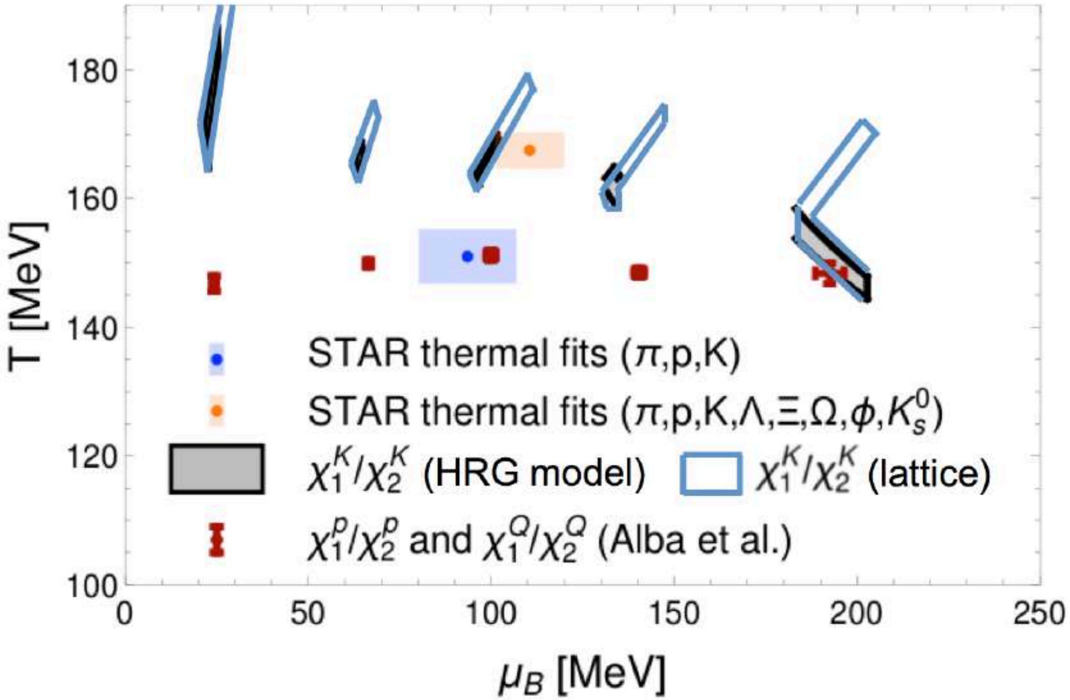
Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al. forthcoming





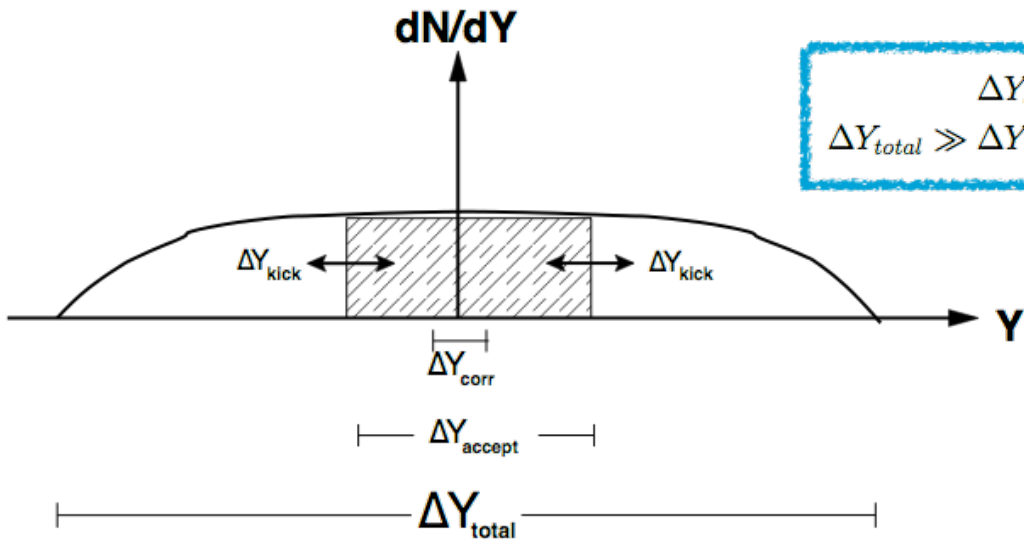


- Lattice QCD temperatures have a large uncertainty but they are above the light flavor ones

Fluctuations of conserved charges?

* If we look at the entire system, none of the conserved charges will fluctuate

* By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful



$$\Delta Y_{accept} \gg \Delta Y_{corr}$$
$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

- ❑ ΔYtotal: range for total charge multiplicity distribution
- ❑ ΔYaccept: interval for the accepted charged particles
- ❑ ΔYkick: rapidity shift that charges receive during and after hadronization

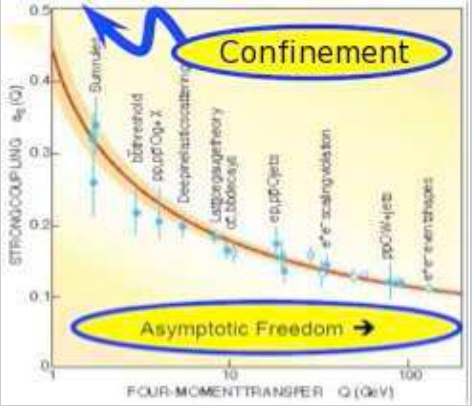
QCD matter under extreme conditions

To address these questions we need fundamental theory and experiment

Theory: Quantum Chromodynamics

- QCD is the fundamental theory of strong interactions
- It describes interactions among quarks and gluons

$$L_{QCD} = \sum_{i=1}^{n_f} \bar{\Psi}_i \gamma_\mu \left(i\partial^\mu - g A_a^\mu \frac{\lambda_a}{2} \right) \Psi_i - m_i \bar{\Psi}_i \Psi_i - \frac{1}{4} \sum_a F_a^{\mu\nu} F_a^{\mu\nu}$$



Experiment: heavy-ion collisions

- Quark-gluon plasma (QGP) discovery at RHIC and the LHC
- QGP is a strongly interacting (almost) perfect fluid

Cumulants of multiplicity distribution

- Deviation of N_Q from its mean in a single event: $\delta N_Q = N_Q - \langle N_Q \rangle$
- The cumulants of the event-by-event distribution of NQ are:

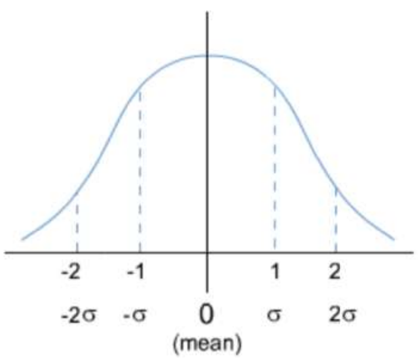
$$\chi_2 = \langle (\delta NQ)^2 \rangle \qquad \chi_3 = \langle (\delta NQ)^3 \rangle \qquad \chi_4 = \langle (\delta NQ)^4 \rangle - 3\langle (\delta NQ)^2 \rangle^2$$

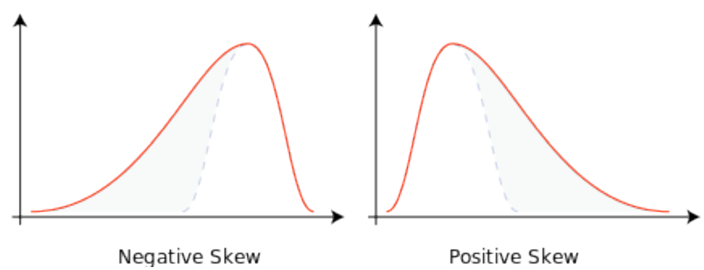
- The cumulants are related to the central moments of the distribution by:

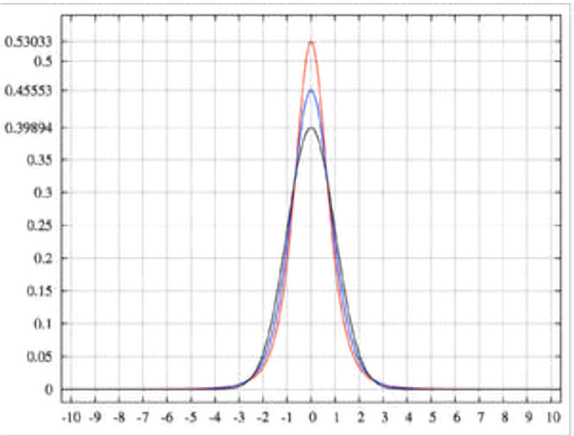
variance: $\sigma^2 = \chi_2$

Skewness: $S = \chi_3 / (\chi_2)^{3/2}$

Kurtosis: $\kappa = \chi_4 / (\chi_2)^2$





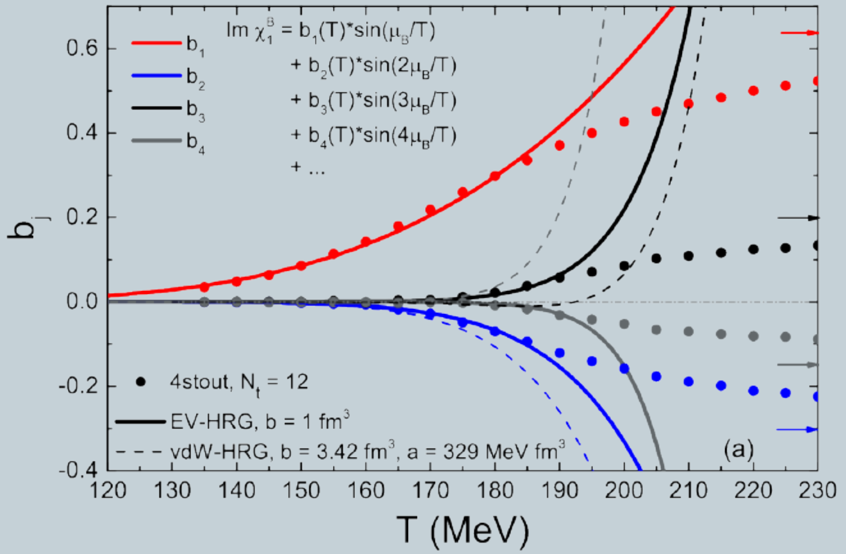


Fluctuations and hadrochemistry

$$\chi_1^B(T,\mu_B) = \frac{\rho_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T)\sinh(k\,\mu_B/T)$$

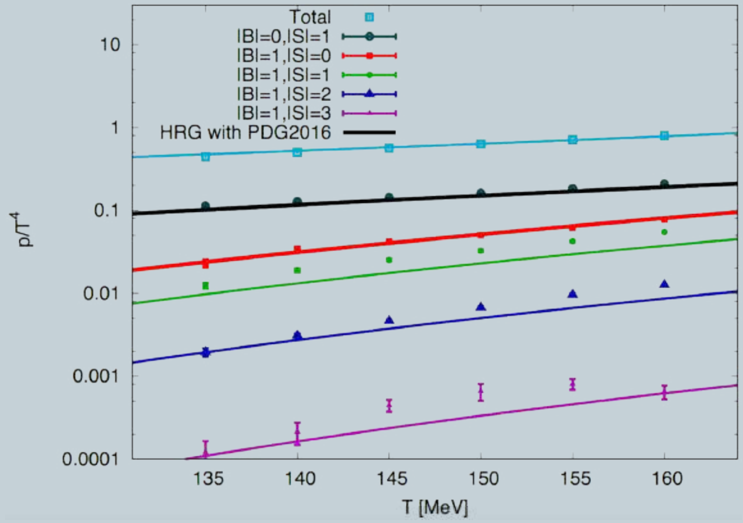
$$P(\hat\mu_B,\hat\mu_S) = P_{00}^{BS} + P_{10}^{BS}\cosh(\hat\mu_B) + P_{01}^{BS}\cosh(\hat\mu_S) + P_{11}^{BS}\cosh(\hat\mu_B-\hat\mu_S) + P_{12}^{BS}\cosh(\hat\mu_B-2\hat\mu_S) + P_{13}^{BS}\cosh(\hat\mu_B-3\hat\mu_S)$$

V. Vovchenko et al., PLB (2017)



- Consistent with HRG at low temperatures
- Consistent with approach to ideal gas limit
- b_2 departs from zero at T~160 MeV
- Deviation from ideal HRG

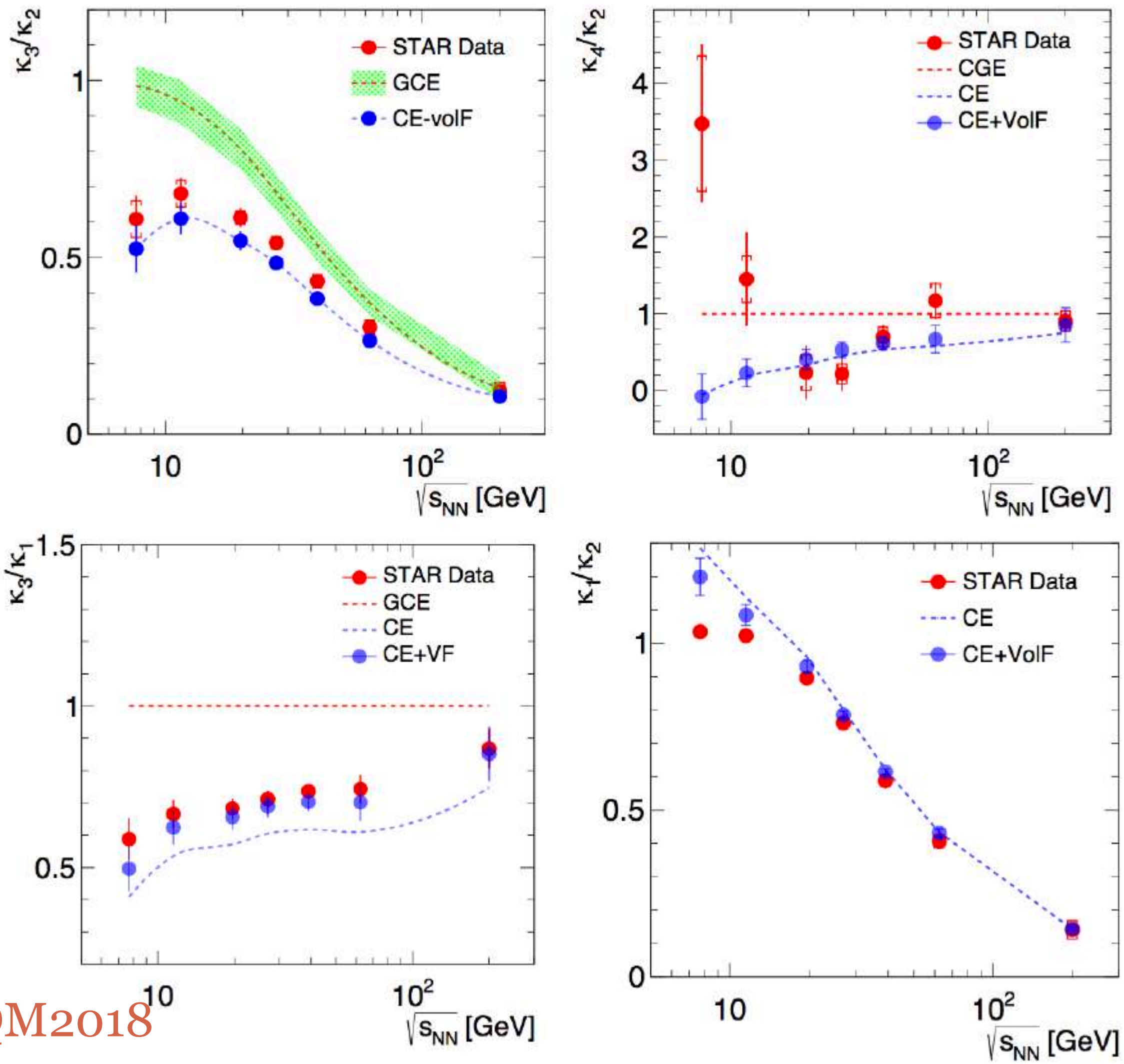
P. Alba et al., PRD (2017)



- Need of additional strange hadrons, predicted by the Quark Model but not yet detected
- First pointed out in Bazavov et al., PRL(2014)

(see talk by J. Glesaaen on Friday)

Canonical suppression



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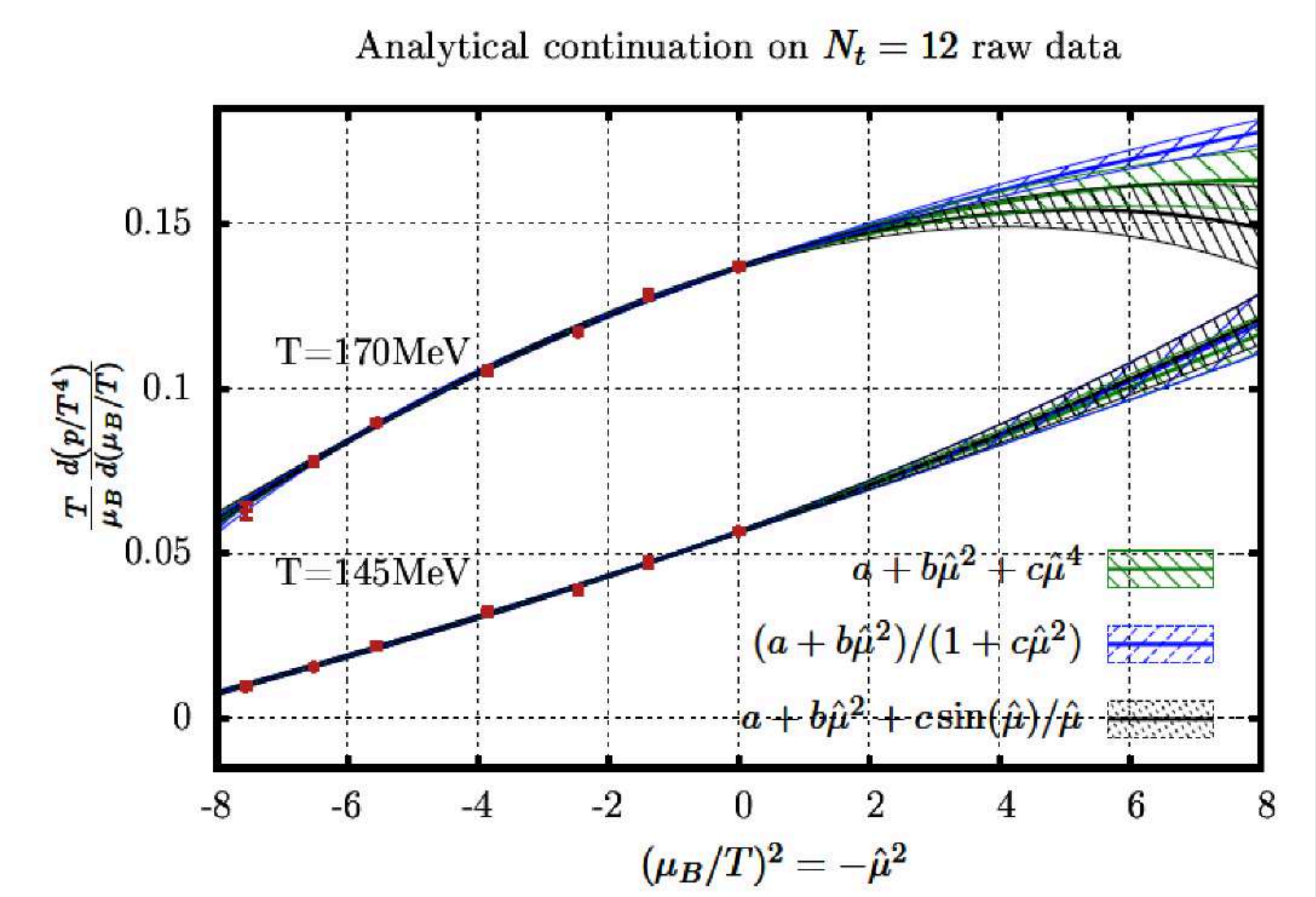
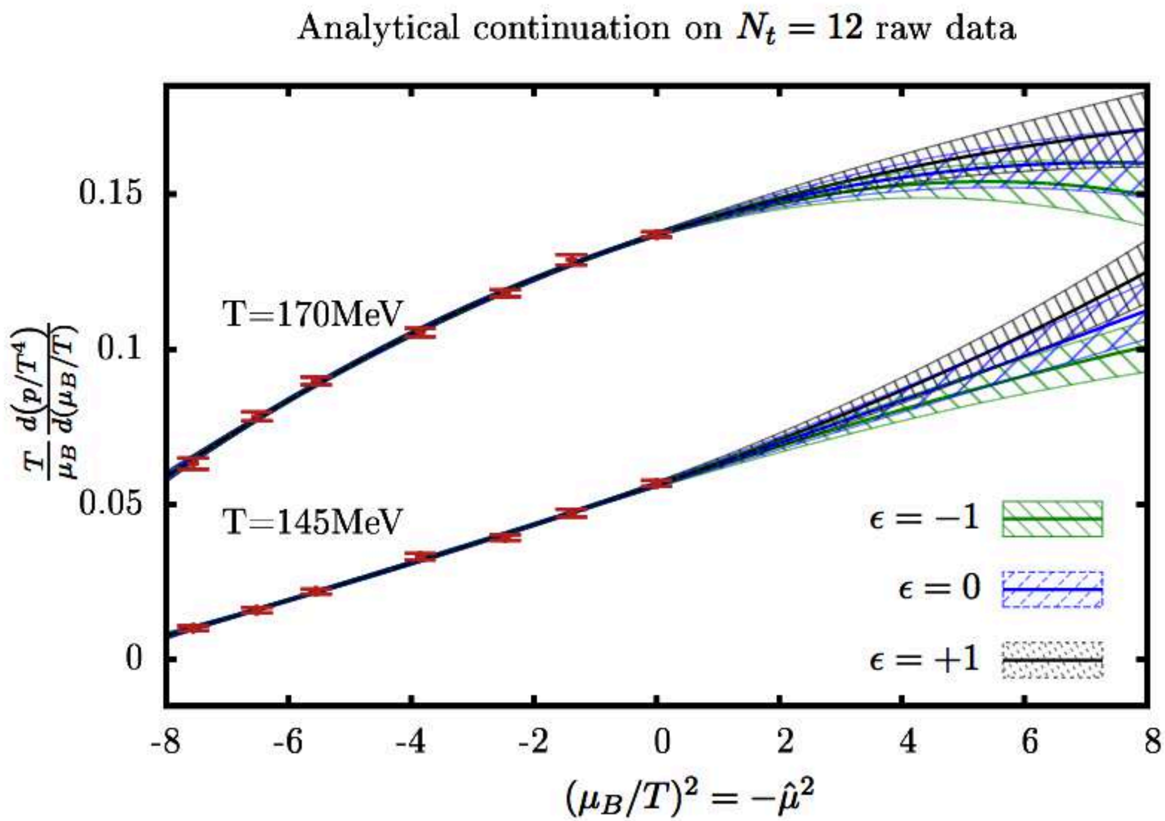
above 11.5 GeV CE suppression accounts for measured deviations from GCE

Analytical continuation – illustration of systematics



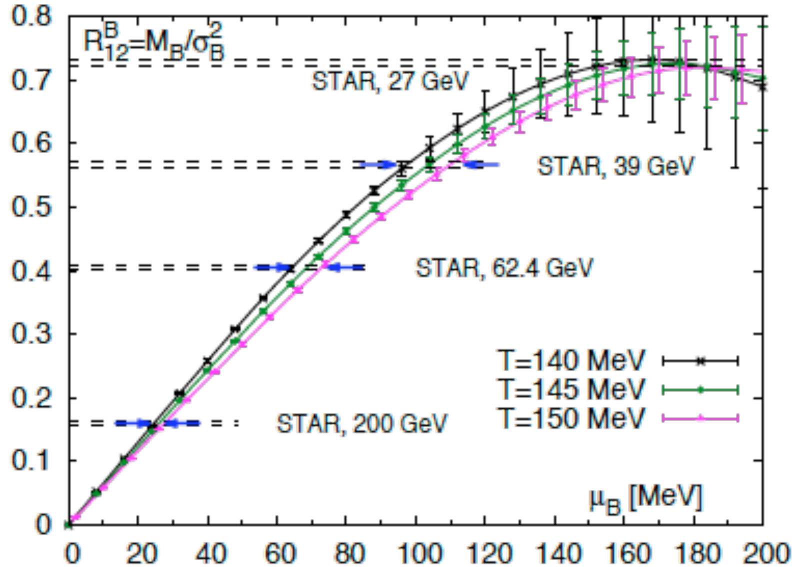
Analytical continuation – illustration of systematics

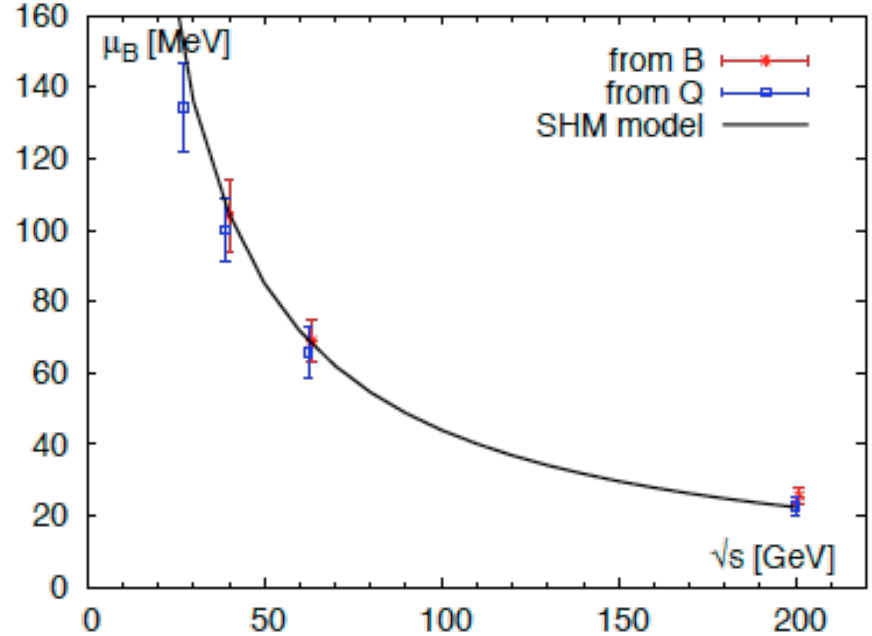
Condition: $\chi_8 \lesssim \chi_4 \longrightarrow f(\hat{\mu}_B) = a + b\hat{\mu}_B^2 + c\hat{\mu}_B^4 + \frac{b\epsilon}{840}\hat{\mu}_B^6$



Consistency between freeze-out of B and Q

- Independent fit of of R_{12}^Q and R_{12}^B: consistency between freeze-out chemical potentials





$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

WB: PRL (2014)
STAR collaboration, PRL (2014)


Geneva with the
Large Hadron Collider
Speed: 0.999995 x speed of light
26.2 km circle