Open-Quantum-Systems: Precision Thermometry at the Extremes

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Some selected references:

O. Ålund, Y. Akamatsu, F. Laurén T. Miura, J. Nordstöm, A.R. JCP **425** (2021) 109917

A.R. "Heavy Quarkonium in Extreme Conditions" Phys.Rept. **858** (2020)

T. Miura, Y. Akamatsu, M. Asakawa, A.R. PRD**101** (2020) 034011

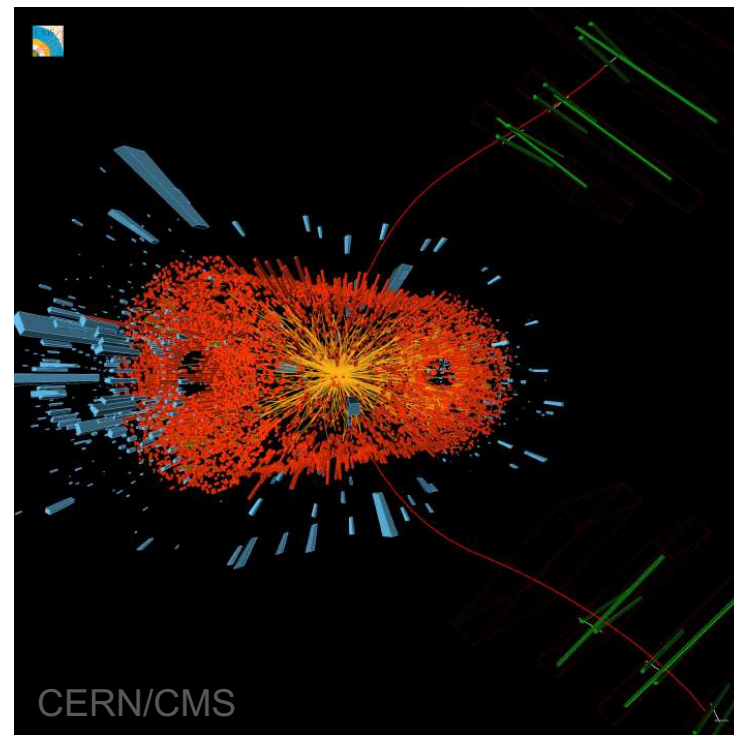M. Mehboudi et.al. PRL**122** (2019) 030403, Olf et.al. Nature Physics **11** (2015) 720

NPACT
Norwegian Particle, Astroparticle
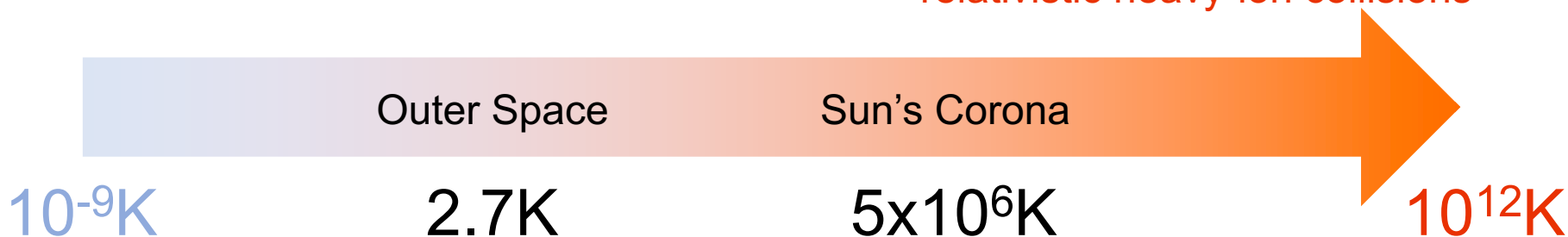& Cosmology Theory network

Temperatures at the extremes
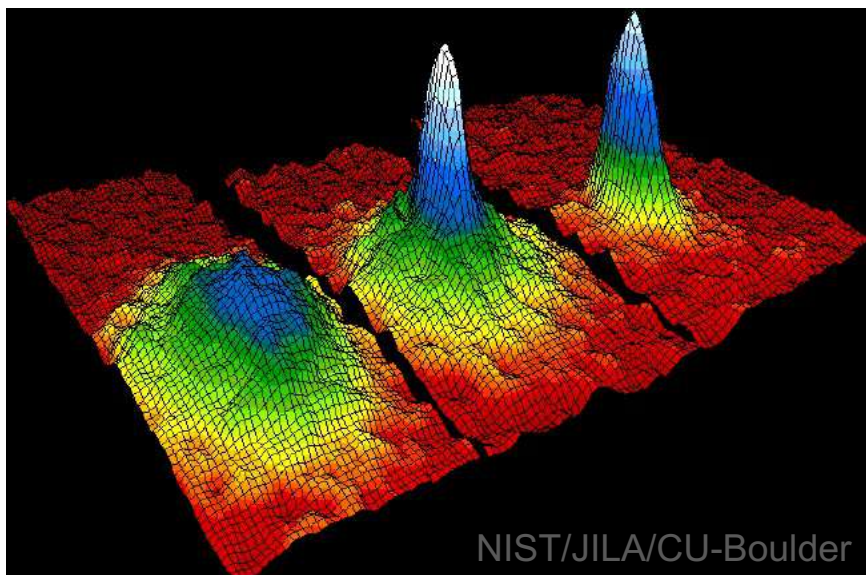
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Bose Einstein Condensate (BEC)
from trapped ultracold atoms

CERN/CMS

Quark-Gluon Plasma (QGP) from
relativistic heavy-ion collisions

	Outer Space	Sun's Corona	
10^{-9}K	2.7K	5×10^{6}K	10^{12}K

What is Temperature

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For a system with large enough number of degrees of freedom

$$\rho(T) = \sum_k \frac{1}{Z} e^{-E_k/k_B T} |E_k\rangle\langle E_k|$$

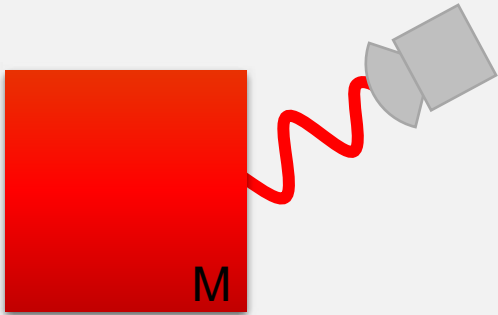**density matrix**   **probability for a state with E_k**   **stationary state of energy E_k**

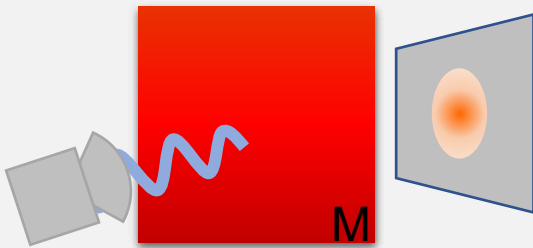T is a **parameter** governing the **statistical distribution** of **energy**

Thermometry – Measuring Temperature

Open quantum systems

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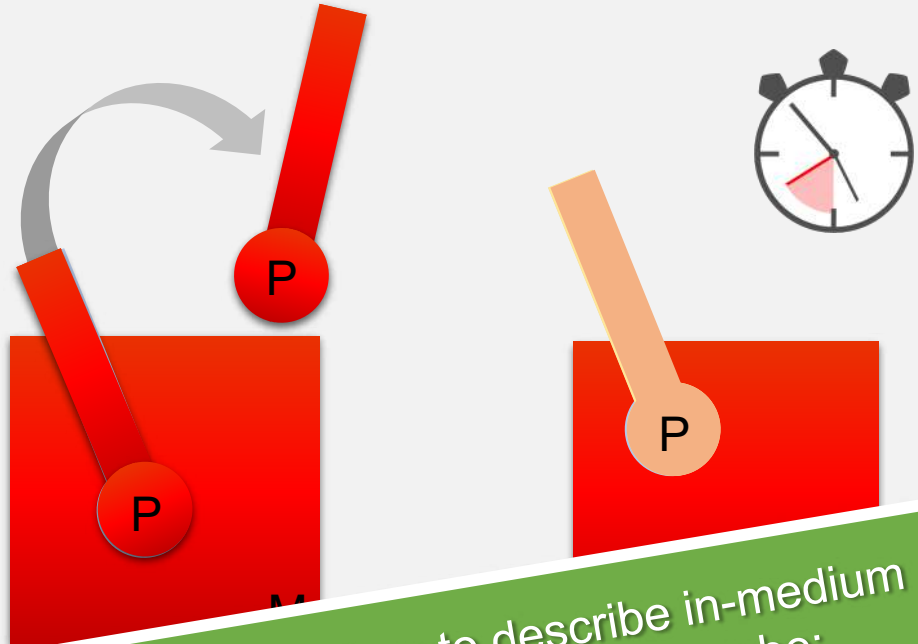
Need general approach to describe (small) probe coupled to a thermal medium

$$H_{\text{tot}} = H_{\text{probe}} \otimes I_{\text{M}} + I_{\text{probe}} \otimes H_{\text{M}} + H_{\text{int}} = H_{\text{tot}}^{\dagger}$$

$$\rho_{\text{tot}} = \sum_k \frac{1}{Z_{\text{tot}}} e^{-E_k / k_B T} |E_k\rangle \langle E_k|$$

Unitary time evolution

$$\frac{d}{dt} \rho_{\text{tot}} = -i[H_{\text{tot}}, \rho_{\text{tot}}]$$

$$\rho_{\text{probe}} = \text{Tr}_{\text{M}}[\rho_{\text{tot}}]$$

reduced density matrix

Dissipative dynamics

$$\frac{d}{dt} \rho_{\text{probe}} = \ \textbf{?}$$

**see e.g. Breuer, Pettrucione
The theory of Open Quantum Systems**
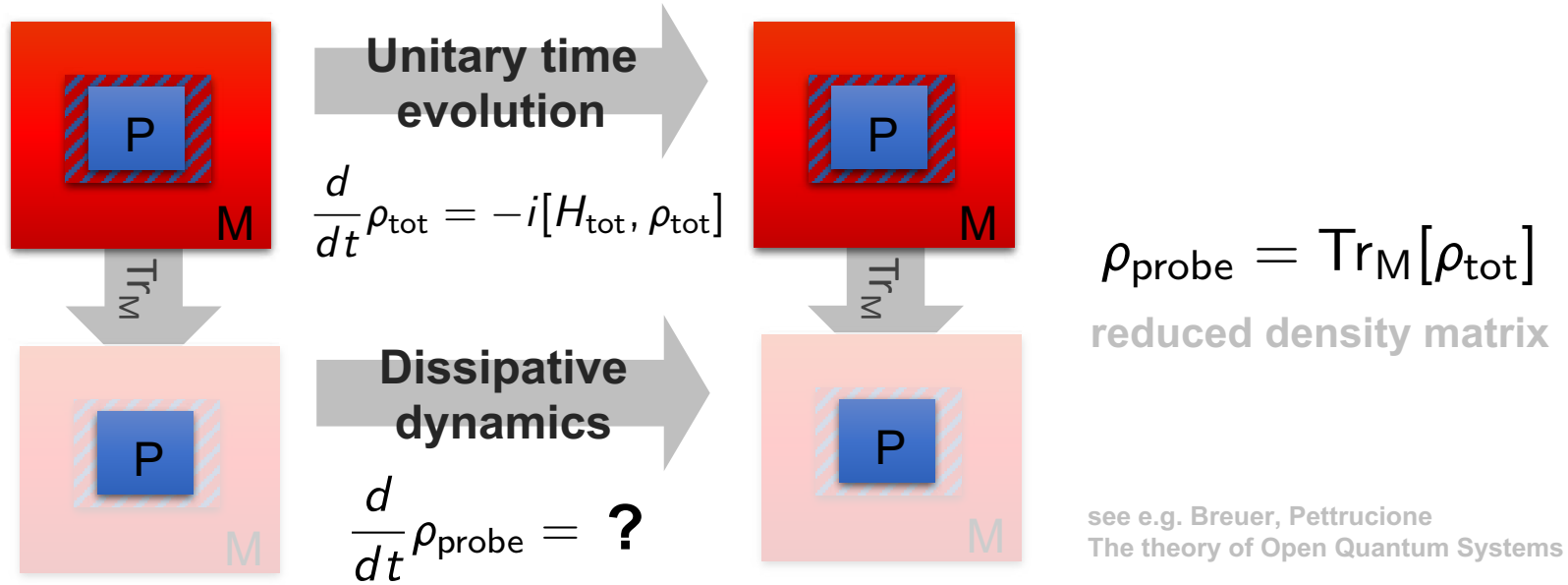
In general, e.o.m. is a dissipative **master equation** with memory of past:

BEYOND THE STANDARD SCHRÖDINGER EQUATION

Separation of time-scales determines the nature of the e.o.m. :

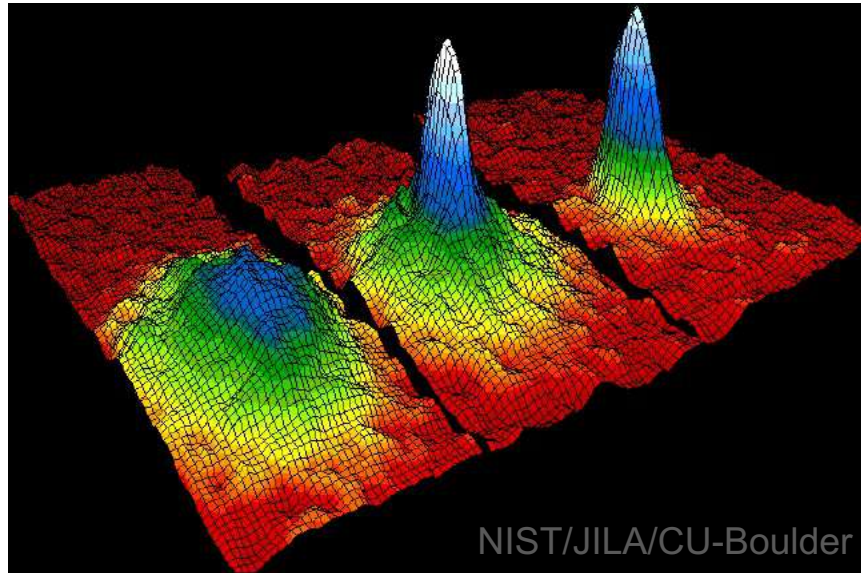Medium relaxation scale τ_{M} :   intrinsic probe scale τ_{P} :   probe relaxation scale τ_{rel} :

$$\langle \hat{O}_{\text{M}}(t)\hat{O}_{\text{M}}(0)\rangle \sim e^{-t/\tau_{\text{M}}}$$

$$\tau_{\text{P}} \sim 1/|\omega_i - \omega_j|$$

$$\langle p(t)\rangle \propto e^{-t/\tau_{\text{rel}}}$$

The coldest matter in the universe

NIST/JILA/CU-Boulder

Bose Einstein Condensate (BEC)
from trapped ultracold atoms

Bose-Einstein Condensation

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In 1924 Einstein & Bose predicted a **new phase** of bosonic matter at low T: **BEC**

A. Einstein König. Preuß. Akad. Wiss. (1924) 261
S. N. Bose *Zeitschrift für Physik*. **26 (1924) 178**

Below a **critical temperature**, significant
fraction of bosons **condense** into
ground state with **macroscopic** wavefunction.

boson gas momentum profile

condensed
atoms

regular atoms
Maxwell-Boltzmann

p
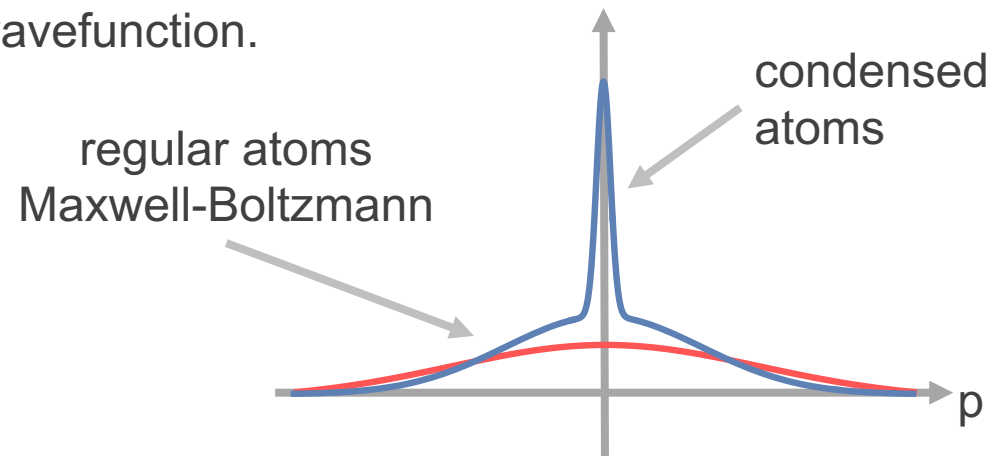
Laser light shone on monoatomic gas induces viscous
force, which reduces gas kinetic energy**:   T ~ 100nK**

D. J. Wineland et.al. PRL40 (1978) 1639

By changing the trapping potential the Maxwell tail
of the gas can be **evaporated** off: **T < nK**
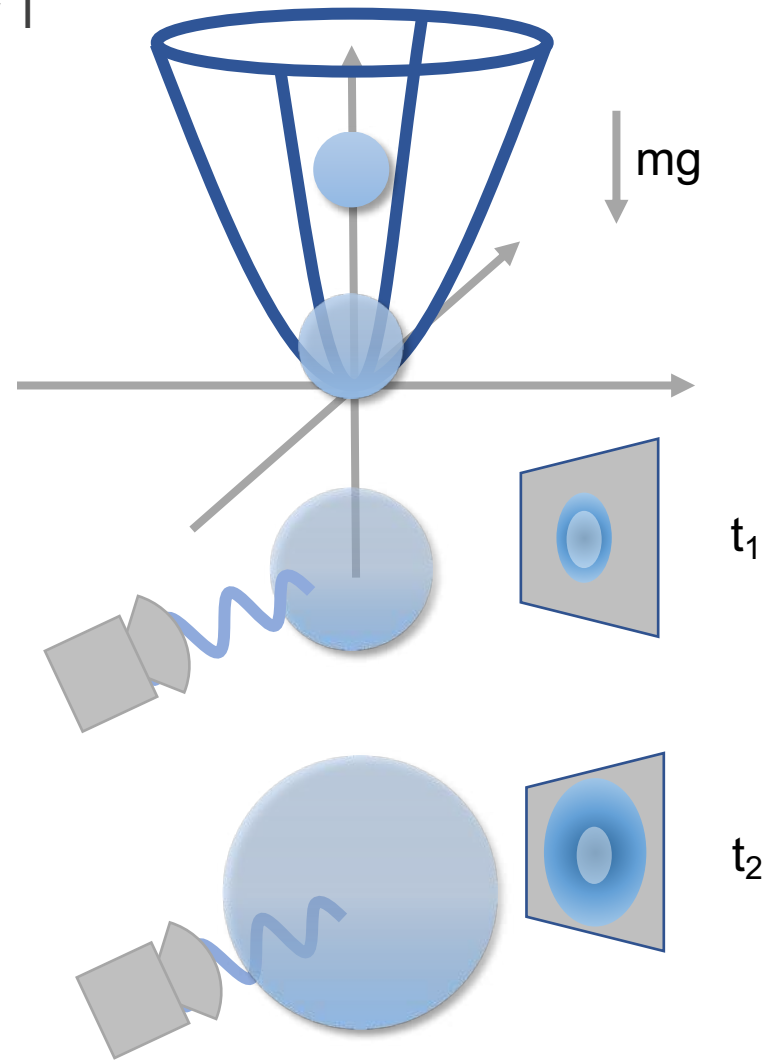
see e.g. Anderson et.al. Science 269 (1995) 198
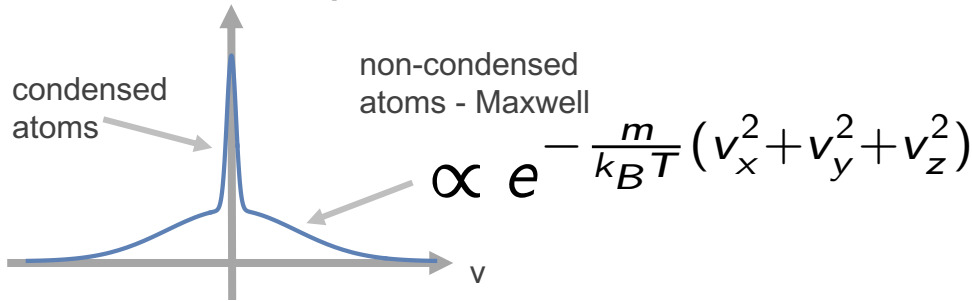
A common way to measure T

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Goal: measure statistical distribution of v at low T via active imaging: **Time-Of-Flight method**
see e.g. P. Lett, et.al. PRL61 (1988) 169

Velocity mixture in Maxwell-Boltzmann leads to a T-dep. expansion of free-falling gas cloud

BEC momentum profile

condensed atoms

non-condensed atoms - Maxwell

$$\propto e^{-\frac{m}{k_B T}\left(v_x^2 + v_y^2 + v_z^2\right)}$$

v

$$P_{\text{TOF}}(t) \propto exp\left[-\frac{g(t_0 - t)^2}{2\frac{k_B T}{m}} \right]$$

System of interest is **destroyed** during the process!

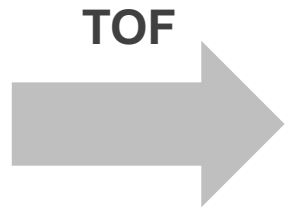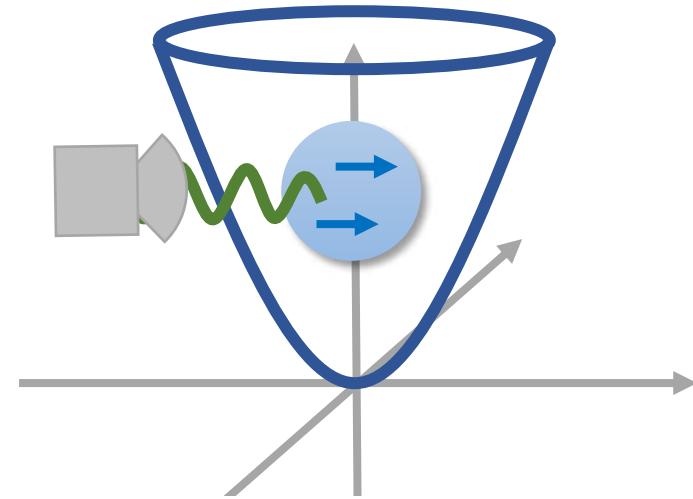Only non-condensed atoms relevant, signal may be **shadowed** by condensated peak

mg

t_1

t_2

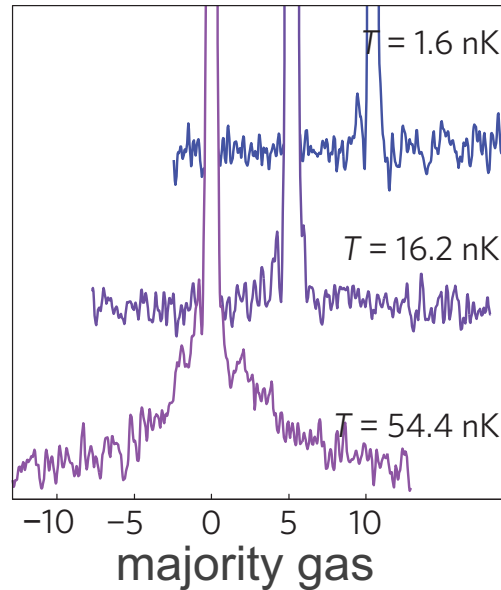# Probes as precision enhancer

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"**Too cold to touch**" – impurity probes must be embedded in the ensemble (thermal equilibrium)

majority gas |F=1, m$_F$=-1>, minority of states via RF pulse into **|F=1, m$_F$=0>** quasiparticle impurities **spinwaves**
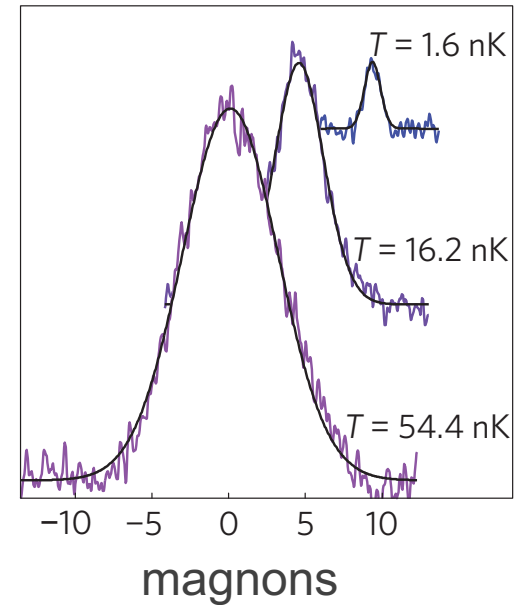
Quasiparticle magnons quickly thermalize but kept dilute enough not to condensate

TOF

m$_F$≠0 particles
weeded out
by microwave

T = 1.6 nK

T = 16.2 nK

T = 54.4 nK

−10   −5   0   5   10
majority gas

Olf et.al. Nature Physics 11 (2015) 720

T = 1.6 nK

T = 16.2 nK

T = 54.4 nK

−10   −5   0   5   10
magnons

Olf et.al. Nature Physics 11 (2015) 720

# Non-destructive precision thermometry

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"**Too cold to touch**" – impurities of different atom species (polarons) embedded in the ensemble (equilibrium)

Laser cool **majority gas ^{39}K** together with **minority gas ^{174}Yb**. (Quantum) Brownian motion!

see e.g. M. Mehboudi et.al. PRL122 (2019) 030403

What properties of the probe allow us to obtain most precise measurement (sensitivity)?

see e.g. M. Mehboudi et.al. J. Phys. A52 (2019) 303001

^{39}K

^{174}Yb
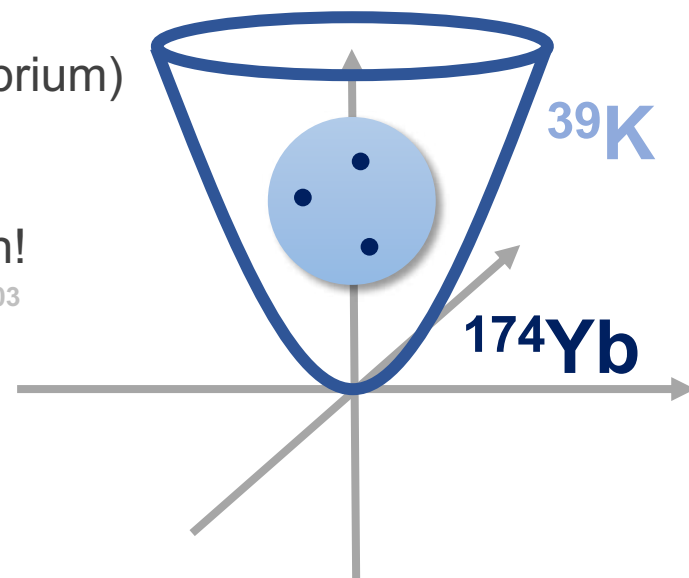
error in T measurement   spread in impurity property

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2}{\sqrt{N \chi_T^2[\hat{O}]}}$$

classically random var.

$$\sim \frac{1}{\sqrt{N}}$$

quantum Cramer-Rao bound

$$\geq \frac{1}{\sqrt{N \mathcal{F}[T]}}$$

\# of measurements   how sensitive is O to T

Fisher information

Optimum sensitivity $\chi_T[\hat{O}] = \partial_\xi \mathrm{Tr}[\rho_{\mathrm{probe}}(\xi)\hat{O}]_{\xi=T}$ reached via unique quantity $\hat{\Lambda}_T$:

$$\hat{\Lambda}_T \hat{\rho}_{\mathrm{probe}} + \hat{\rho}_{\mathrm{probe}} \hat{\Lambda}_T = 2 \partial_T \hat{\rho}_{\mathrm{probe}}$$

$$\hat{\Lambda}_T^{\mathrm{QBM}} = C_x(\hat{x}^2 - \langle \hat{x}^2 \rangle) + C_p(\hat{p}^2 - \langle \hat{p}^2 \rangle)$$

"symmetric logarithmic derivative"   optimal observable from ρ_{probe}

Quantum Brownian Motion

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Iconic model of an Open Quantum System: **heavy point-like impurity** in a medium of harmonic oscillators
see e.g. Breuer, Pettrucione The theory of Open Quantum Systems

wave packet undergoing stochastic evolution

$$H_{\text{tot}} = H_{\text{probe}} \otimes I_{\text{M}} + I_{\text{probe}} \otimes H_{\text{M}} + H_{\text{int}}$$

g_k: all details M-P interaction

$$\frac{\hat{p}^2}{2m_p} + \sum_k E_k \hat{b}_k^\dagger \hat{b}_k + \sum_k \hbar g_k \hat{x}(\hat{b}_k + \hat{b}_k^\dagger)$$

how to derive an evolution equation for the probe particle?

$$\frac{d}{dt}\rho_{\text{probe}}(t) = -i\text{Tr}_{\text{M}}\left[[H_{\text{tot}}, \rho_{\text{probe}} \otimes \rho_{\text{M}}]\right]$$
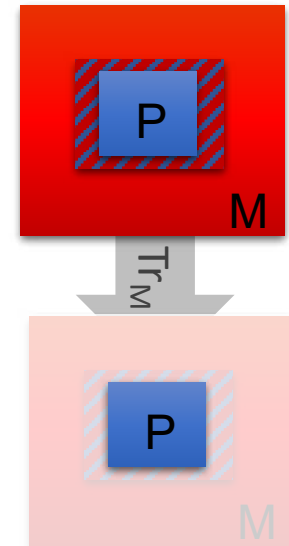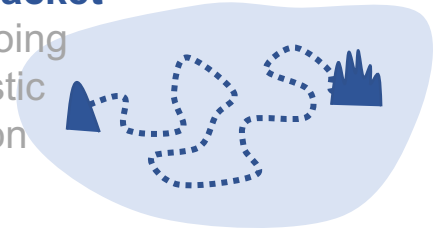
Born approximation

$$\tau_{\text{rel}} > \tau_{\text{M}}$$

Markovian approximation

$$\tau_{\text{P}} > \tau_{\text{M}}$$

Brownian motion limit

P

M

Tr_M

P

M

yields the **Caldeira-Leggett** master equation "damping rate" γ(g_k) explicitly computable

$$\frac{d}{dt}\hat{\rho}_{\text{probe}} = -\frac{i}{\hbar}[\hat{H}_{\text{probe}}, \hat{\rho}_{\text{probe}}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\text{probe}}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\text{probe}}\}] - \frac{\gamma}{8m_p k_B T}[\hat{p}, [\hat{p}, \hat{\rho}_{\text{probe}}]]$$

coherent dynamics   fluctuations   dissipation

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587 & Annals of Phys. 149 (1983) 374

The Lindblad equation

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A **general mathematical structure** is hiding in the Quantum-Brownian motion e.o.m.

Summarize medium effects on probe by **one operator** and **one damping rate**

$$\hat{\mathcal{L}} = \sqrt{\frac{4 m_p k_B T}{\hbar^2}}\,\hat{x} + i\sqrt{\frac{1}{4 m_p k_B T}}\,\hat{p}$$

Lindblad operator for QBM

$$\frac{d}{dt}\hat{\rho}_{\text{probe}} = -\frac{i}{\hbar}[\hat{H}_{\text{probe}}, \hat{\rho}_{\text{probe}}] + \sum_k \gamma_k \left(\hat{\mathcal{L}}_k \hat{\rho}_{\text{probe}} \hat{\mathcal{L}}_k^\dagger - \frac{1}{2}[\hat{\mathcal{L}}_k^\dagger \hat{\mathcal{L}}_k, \hat{\rho}_{\text{probe}}] \right)$$

Linblad equation for **Markovian** open quantum systems

G. Lindblad Commun. Math. Phys. 48 (1976) 119,
V. Gorini, et.al. J. Math. Phys. 17 (1976) 821

proven

$\langle$ ...

sanity

$\frac{d}{dt}\langle\hat{x}\rangle$

$\frac{d}{dt}\langle\hat{p}\rangle$

$\infty \frac{k_B T}{m_p \gamma} t$

to classical Brownian motion

All ingredients are ready to compute predictions for precision low T thermometry using Quantum Brownian probe particles

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2}{\sqrt{N \chi_T^2[\hat{O}]}} \qquad \hat{\Lambda}_T^{\text{QBM}} = C_x(\hat{x}^2 - \langle \hat{x}^2 \rangle) + C_p(\hat{p}^2 - \langle \hat{p}^2 \rangle)$$
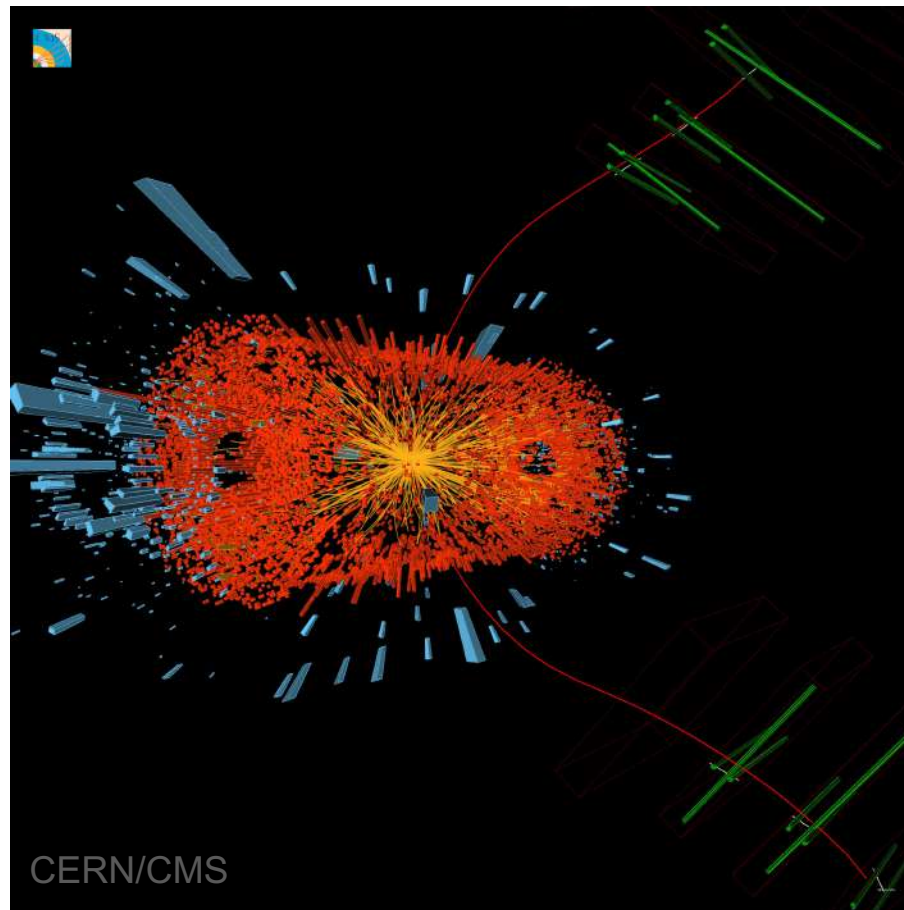
caveat: neglected possible memory effects etc. - simplest case study

The hottest matter in the universe

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CERN/CMS

Quark-Gluon Plasma (QGP) from
relativistic heavy-ion collisions

Quark-Gluon Plasma

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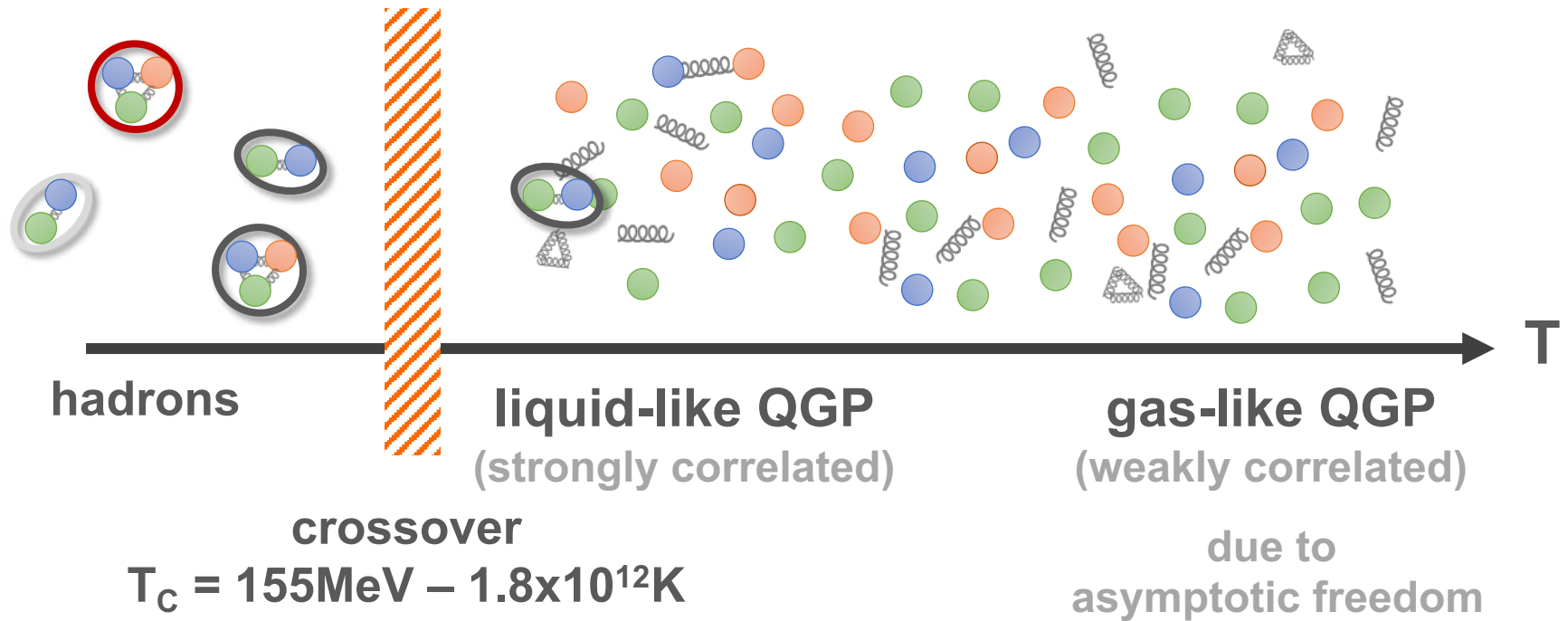
In 1965 Hagedorn hinted at existence of a **new phase** of nuclear matter at high T: **QGP**
R. Hagedorn Nuovo Cimento, Suppl. 3, no. CERN-TH-520 (1965): 147

nuclear matter at T=276K: strongly-interacting quarks and gluons **confined** into hadrons
for a review see T. Hatsuda and K. Fukushima Rept.Prog.Phys. 74 (2011) 014001

hadrons   **liquid-like QGP**   **gas-like QGP**
                     (strongly correlated)   (weakly correlated)

crossover   due to
T_C = 155MeV – 1.8x10^{12}K   asymptotic freedom

QGP filled the universe shortly after the **Big Bang** – possibly exists inside **neutron stars**
Hatsuda et. al. "QGP: From Big Bang to little Bangs" Cambridge   Annala et.al. Nature Physics 16 (2020) 907

Theory basis: **Quantum ChromoDynamics (QCD)**, the field theory of quarks & gluons
endowed with a triple valued **charge called color** (r,g,b)

Relativistic heavy-ion collisions


p+p collision (reference)   A+A heavy-ion collision
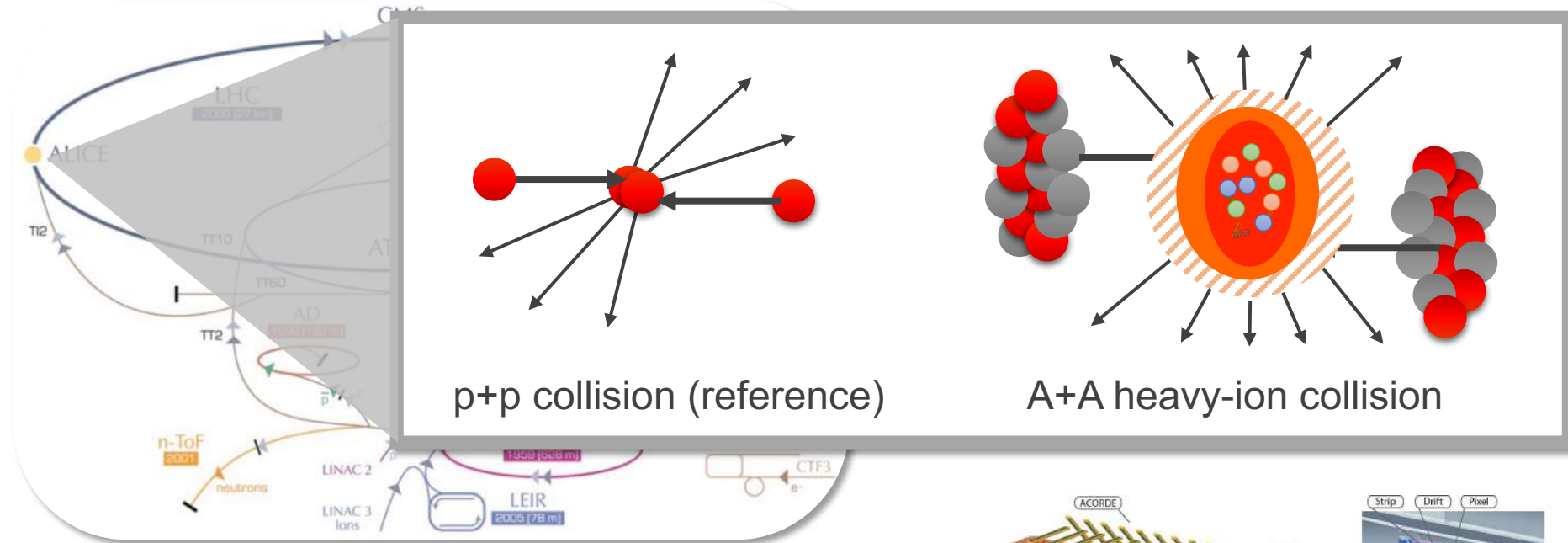
- Vast amount of kinetic energy transformed into new quarks and gluons

- In RHICs: high enough number density of quarks to form a locally thermal QGP

- Compare p+p result vs. A+A results to infer presence and properties of medium

Radiation Thermometry

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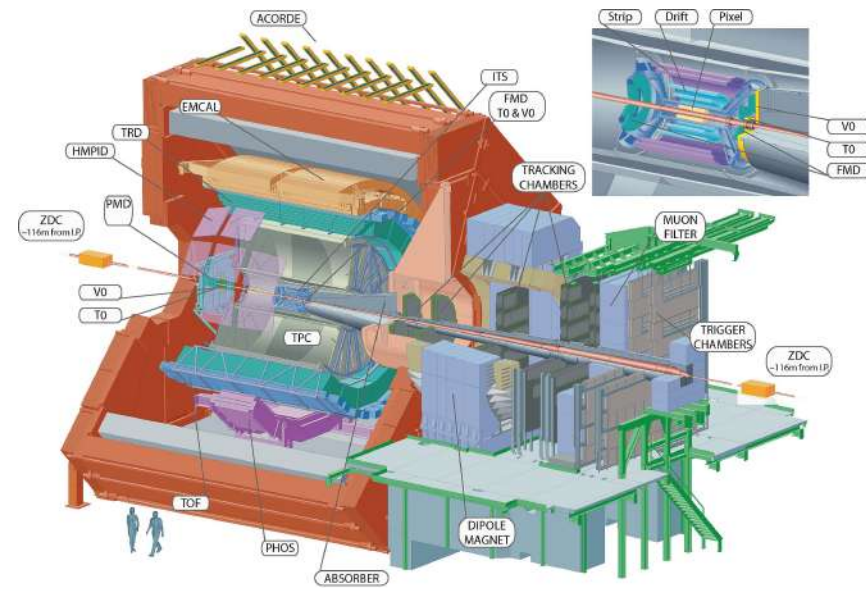
Photon spectrum emitted by a RHIC

[ALICE collaboration] Phys.Lett.B 754 (2016) 235

VS
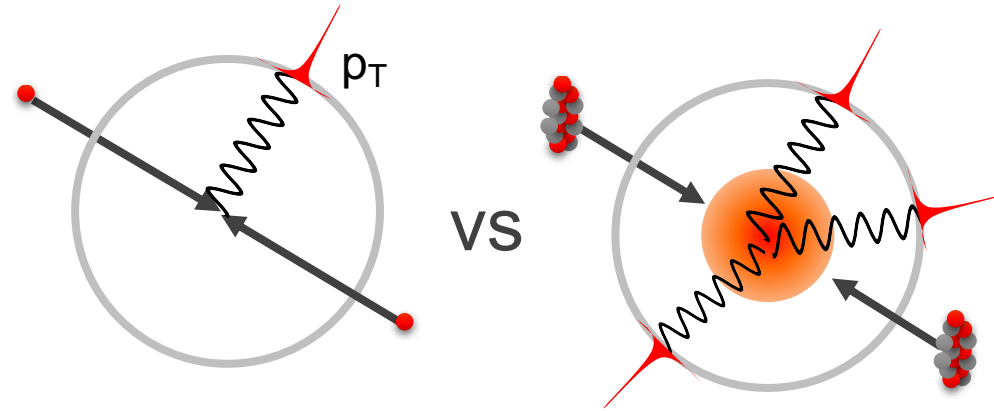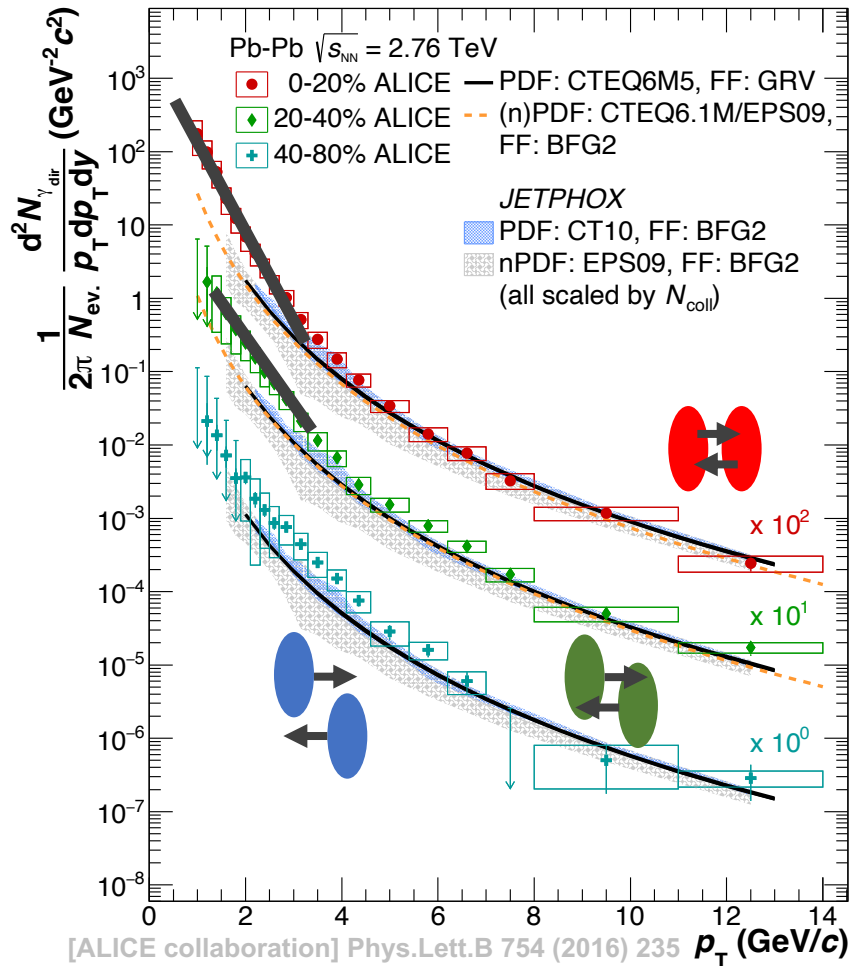
passive imaging

see e.g. K. Reygers CERN Courier 55 (2015) 22

- **At high p_T**: photons from highly energetic quark collisions before formation of QGP. similar to multiple p+p collisions added.

- **At low p_T**, expect photon emission from the QGP to dominate:
  excess over rescaled p+p

- Fitting excess with Maxwell-Boltzmann:
  $300 < T < 400$ MeV $\gg T_C$

- Challenge: photon source not obvious – really from the QGP?

Heavy Quarkonium

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QQ̄ pairs either color singlet (attractive) or octet (repulsive)

"**Too fast to watch**" – need to use collision remnants as probe particles.

Bound states of heavy quarks **cc̄** (charmonium) **bb̄** (bottomonium)

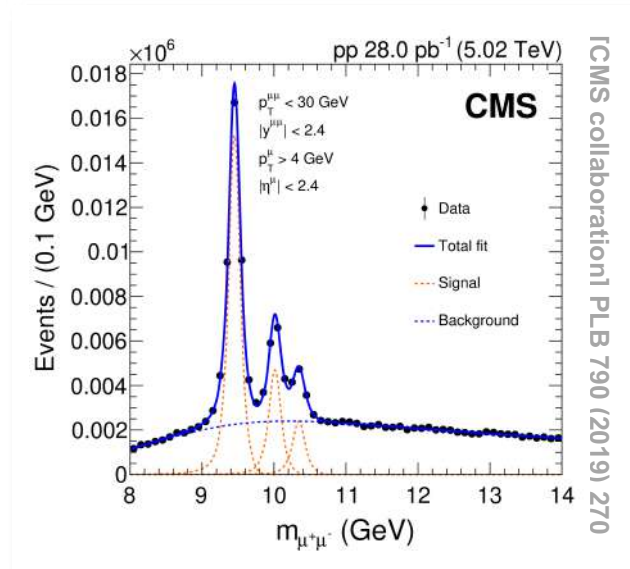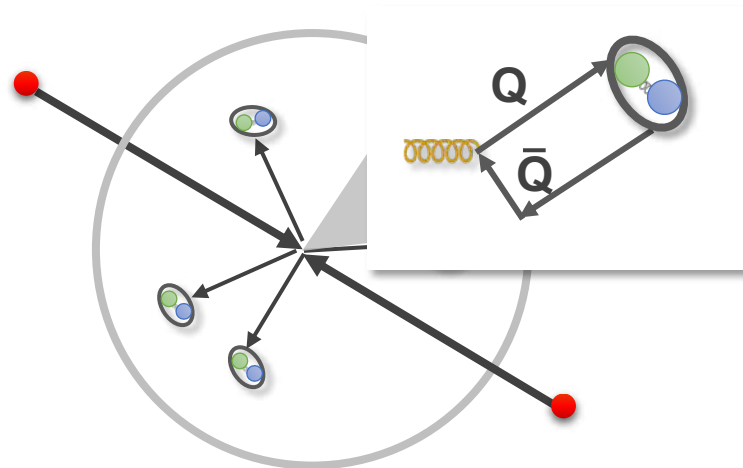for a review see Brambilla et.al.
Eur.Phys.J.C 71 (2011) 1534

- decay into hadrons forbidden (OZI rule): keV widths

- significant decay into dileptons: clean experimental signals

- non-relativistic since $\Lambda_{QCD}/m_Q \ll 1$ & described by a simple 2-body potential (known from QCD)

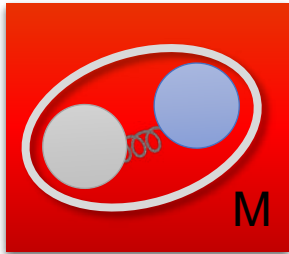Cornell potential

$\sim$ const.

$\sim \sigma r$

$\sim -\dfrac{\alpha_S}{r}$

$\Upsilon(^3 3S_1)$

$\Upsilon(^3 2S_1)$

$\Upsilon(^3 1S_1)$

×10^6 pp 28.0 pb^{-1} (5.02 TeV)

CMS

$p_T^{\mu\mu} < 30$ GeV
$|y^{\mu\mu}| < 2.4$
$p_T^{\mu} > 4$ GeV
$|\eta^{\mu}| < 2.4$

Data
Total fit
Signal
Background

Events / (0.1 GeV)

$m_{\mu^+\mu^-}$ (GeV)

CMS collaboration1 PLB 790 (2019) 270

Quarkonium in heavy-ion collisions

"**Too fast to watch**" – need to use collision remnants as probe particles.

Bound states of heavy quarks **c$\bar{c}$** (charmonium) **b$\bar{b}$** (bottomonium)

- still non-relativistic since $T_C/m_Q \ll 1$
  Quantum Brownian Motion !

- presence of QGP modifies the interaction potential

see e.g. Y. Burnier, O. Kaczmarek, A.R. JHEP 12 (2015) 101

$$\sim -\alpha_S \frac{\exp[-r/r_S]}{r}$$

Υ($^3 2S_1$)

Υ($^3 1S_1$)
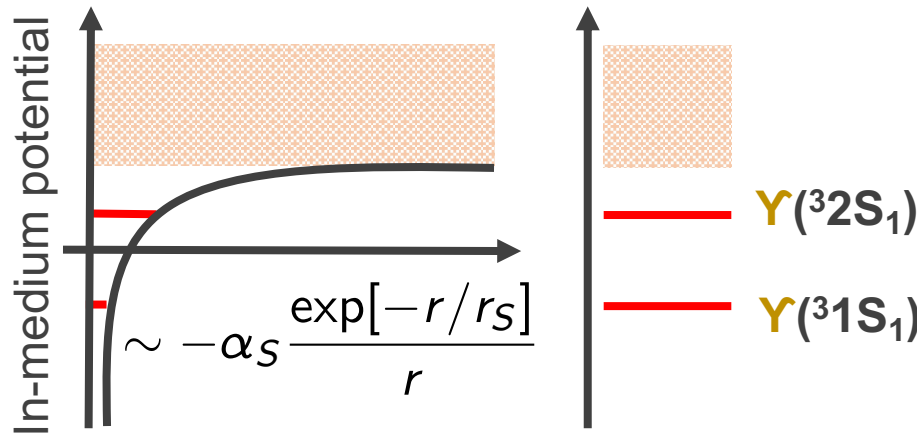
Early realization: screening by medium will weaken quarkonium and modifies production yields

Matsui & Satz
Phys.Lett.B 178 (1986) 416
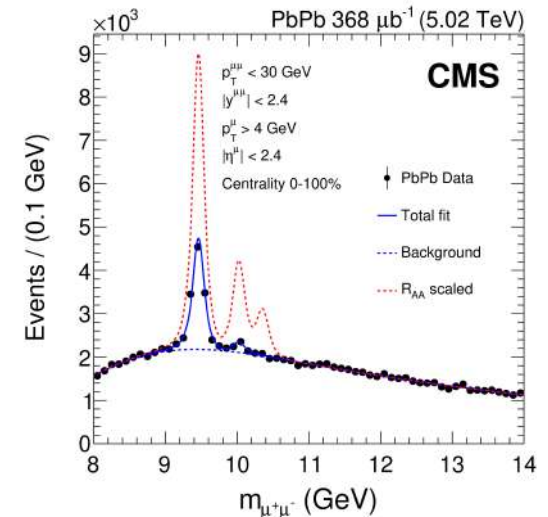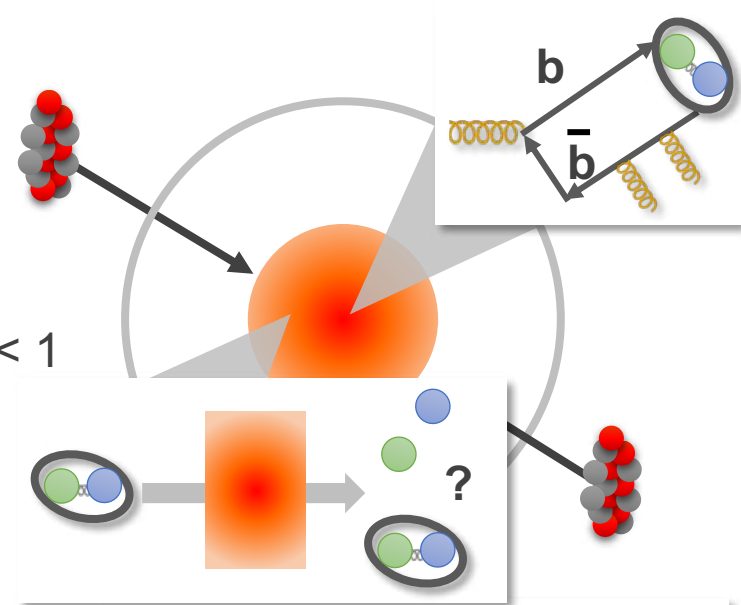
[CMS collaboration] PLB 790 (2019) 270

A Lindblad equation for quarkonium

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Caldeira Leggett not applicable as probe has finite extent and carries color charge.
Progress: QBM Lindblad equation from QCD at high T (**weakly coupled** regime)

$$\frac{d}{dt}\hat{\rho}_{Q\bar{Q}} = -\frac{i}{\hbar}[\hat{H}_{Q\bar{Q}}, \hat{\rho}_{Q\bar{Q}}] + \sum_k \left(\hat{\mathcal{L}}_k \hat{\rho}_{Q\bar{Q}} \hat{\mathcal{L}}_k^\dagger - \frac{1}{2}[\hat{\mathcal{L}}_k^\dagger \hat{\mathcal{L}}_k, \hat{\rho}_{Q\bar{Q}}] \right)$$

In-medium Hamiltonian exhibits a **screened potential:**

$$H_{Q\bar{Q}} = \frac{\mathbf{p}_{\text{rel}}^2}{m_Q} + V_{Q\bar{Q}}(\mathbf{r}) \qquad V_{Q\bar{Q}}(\mathbf{r}) = -\alpha_S \frac{\exp[-r/r_S(T)]}{r}$$

$r_S(T)$ explicitly known from QCD

Behind the scenes: In QCD, quarkonium interacts with medium via gluon exchange which induces **momentum transfer** & **color rotation**
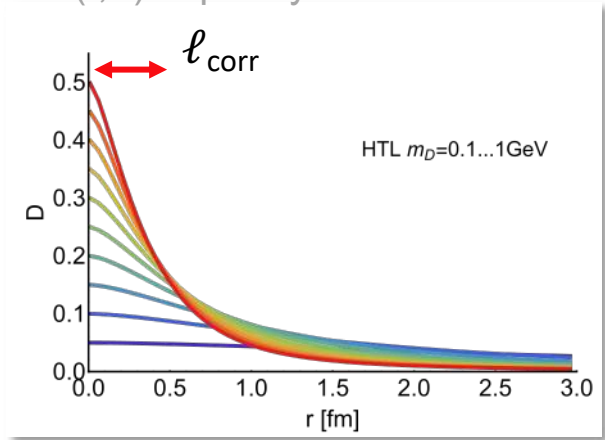
D(r,T) explicitly known from QCD

fluctuations   **dissipation**

$$\mathcal{L}_{\mathbf{k},a} = \sqrt{\frac{D(\mathbf{k})}{2}} \left[1 - \frac{\mathbf{k}}{4m_Q T} \cdot \left(\frac{1}{2}\mathbf{P}_{\text{CM}} + \mathbf{p}_{\text{rel}} \right) \right] e^{i\mathbf{k}\cdot\mathbf{r}/2} (T^a \otimes 1)$$

medium acting on the quark

T^a 3x3 matrix in color space

$$- \sqrt{\frac{D(\mathbf{k})}{2}} \left[1 - \frac{\mathbf{k}}{4m_Q T} \cdot \left(\frac{1}{2}\mathbf{P}_{\text{CM}} + \mathbf{p}_{\text{rel}} \right) \right] e^{i\mathbf{k}\cdot\mathbf{r}/2} (1 \otimes T^a)$$

medium acting on the anti-quark

see T. Miura, Y. Akamatsu, M. Asakawa, A.R. PRD101 (2020) 034011

ℓ_{corr}

HTL m_D=0.1...1GeV

D

r [fm]
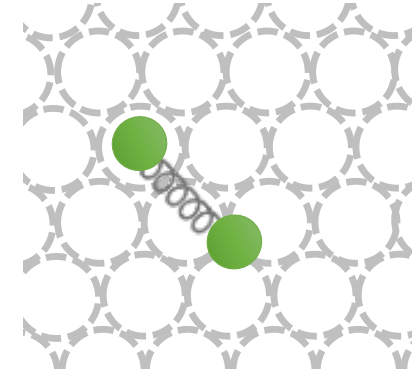
Screening and Decoherence
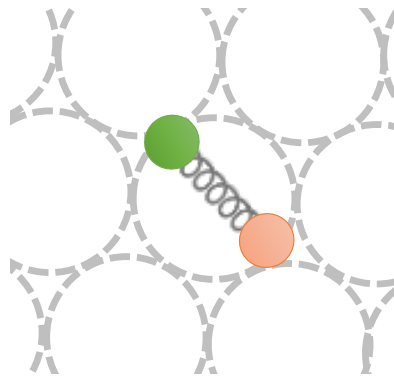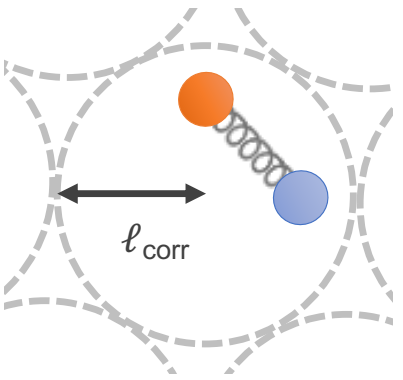
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screening
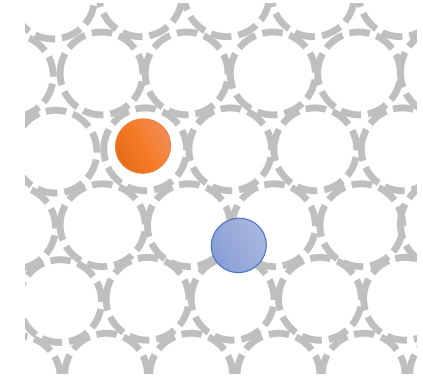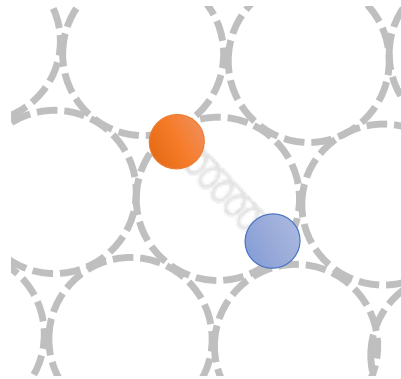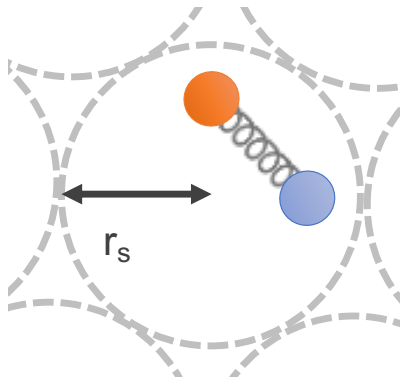
binding force acts efficiently among Q and $\bar{Q}$

medium impedes gluons mediating force

increasing temperature

decoherence

color rotation acts coherently on Q and $\bar{Q}$

color rotation acts individually on Q and $\bar{Q}$

see discussion in S. Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018), 014003

Solving the 1d Lindblad equation

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of Stavanger

■ For the first time: possible to **thermalize** quarkonium in a **fully quantum** fashion

N₀(0)=1, N₀(Mt)
N₁(Mt)
N₁(0)=1, N₀(Mt)
N₁(Mt)

T. Miura, Y. Akamatsu, M. Asakawa, A.R. PRD101 (2020) 034011
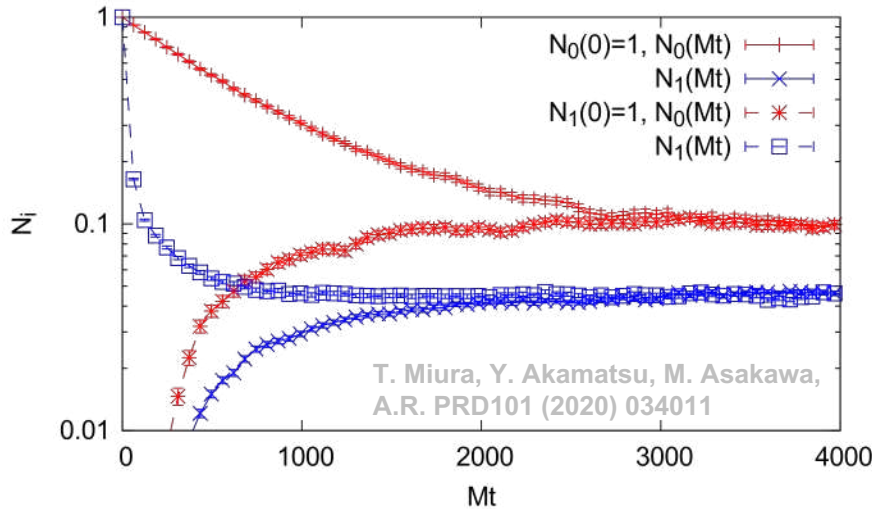
■ Start either with single ground state or single excited state and monitor survival probability

■ Late-time results independent of initial conditions: steady-state
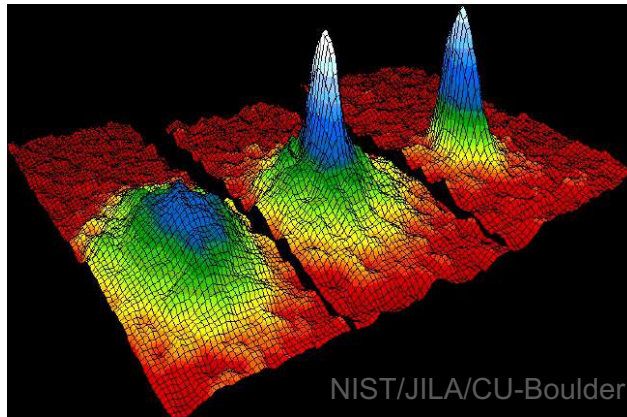

N₀(0)=1, T/M=0.1
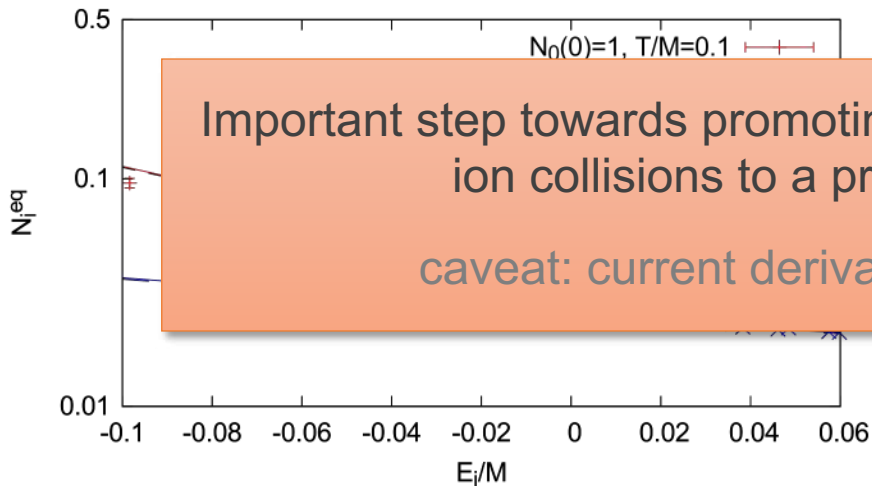
Important step towards promoting the study of quarkonium in heavy-ion collisions to a precision thermometry tool

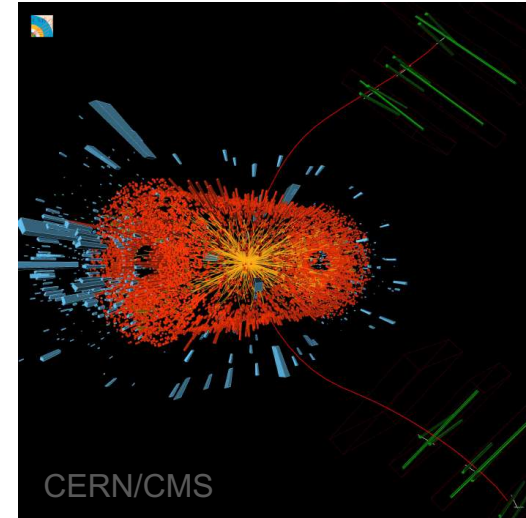caveat: current derivation only for weak coupling

Summary

University of Stavanger

- **Open Quantum Systems**: **impurities as quantum probes** play an essential role in **precision thermometry** at vastly different energy scales

Bose Einstein Condensate (BEC)
from trapped ultracold atoms

NIST/JILA/CU-Boulder

CERN/CMS

Quark-Gluon Plasma (QGP) from
relativistic heavy-ion collisions

- **Theory framework** developed to describe Open Quantum Systems (e.g. Lindblad equation) provides a **common language** among formerly disparate research fields

- Exciting prospect for **collaboration** between ultracold atoms and heavy-ion collision community on **impurity physics** (quarkonium, jets, hadronization,....)