





Impact of dark matter on compact stars

properties Mass and Radi

Tidal deformability ar waveform

NS cooling and heating

Fermionic DN

Bosonic DN

simulations of DM admixed NS binaries

Conclusions

Violetta Sagun

University of Coimbra, Portugal











Theoretical Physics



Accumulation

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DM candidates

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DM accumulation regimes

During

During the star formation stage the initial mixture of DM and BM contracting to form the progenitor star. Trapped DM undergoes scattering processes with baryons leading to its kinetic energy loss and thermalisation.

■ Main sequence (MS) star

From this stage of star evolution accretion rate increases due to big gravitational potential of the star. In the most central Galaxy region $M_{acc} \approx 10^{-5} M_{\odot} - 10^{-9} M_{\odot}$.

■ Supernova explosion & formation of a proto-NS

The newly-born NS should be surrounded by the dense cloud of DM particles with the temperature and radius that corresponds to the last stage of MS star evolution, i.e. a star with a silicone core.

Kouvaris & Tinyakov 2010

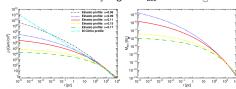
In addition, a significant amount of DM can be produced during the supernova explosion and mostly remain trapped inside the star.

■ Equilibrated NS

Progenitor

$$M_{acc} \approx 10^{-14} \left(\frac{\rho_{\chi}}{0.3 \frac{\text{GeV}}{\text{m}^3}} \right) \left(\frac{\sigma_{\chi n}}{10^{-45} \text{cm}^2} \right) \left(\frac{t}{\text{Gyr}} \right) M_{\odot},$$
 (1)

In the most central Galaxy region $M_{acc} \approx 10^{-5} M_{\odot} - 10^{-8} M_{\odot}$.



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DM and NS structure

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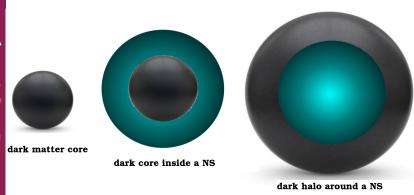
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Dark matter and baryon components do not expel each other but overlap due to absence of non-gravitational interaction

Effect of DM on Mass and Radius

Accumulation of DM in star

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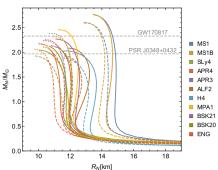
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- DM core ⇒ decrease of the maximum mass and observed stellar radius
- DM halo ⇒ increase of the maximum mass and the outermost radius

Ciarcelluti & Sandin 2011; Nelson+ 2019; Deliyergiyev+ 2019; Ivanytskyi+2020; Das+ 2020; Del Popolo+ 2020; Karkevandi+ 2022



DM core contributing to 5% of the total NS mass $\sqrt{\sigma_{\rm D}}/m_{\rm D}^3 = 0.05\,{\rm GeV}^{-2}$

Ellis+ 2018

TOV equations - two fluid system

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2 TOV equations:

$$\begin{split} \frac{dp_B}{dr} &= -\frac{(\epsilon_B + p_B)(M + 4\pi r^3 p)}{r^2 \left(1 - 2M/r\right)} \\ \frac{dp_D}{dr} &= -\frac{(\epsilon_D + p_D)(M + 4\pi r^3 p)}{r^2 \left(1 - 2M/r\right)} \end{split}$$

BM and DM are coupled only through gravity, and their energy-momentum tensors are conserved separately

total pressure
$$p(r) = p_B(r) + p_D(r)$$
 gravitational mass $M(r) = M_B(r) + M_D(r)$, where $M_j(r) = 4\pi \int\limits_0^r \epsilon_j(r')r'^2 dr'$ (j=B,D)

$$M_T = M_B(R_B) + M_D(R_D)$$
 - total gravitational mass

Fraction of DM inside the star:

$$f_{\chi} = \frac{M_D(R_D)}{M_T}$$

Tidal deformabilities of DM-admixed NS

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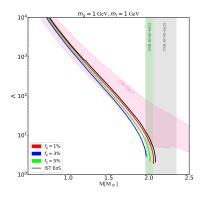
Tidal

heating

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Numerical simulations of DM admixed

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Tidal deformability parameter

$$\Lambda = rac{2}{3} k_2 \left(rac{R_{
m outermost}}{M_{
m tot}}
ight)^5$$

k2 - Love's number.

- $R_{outermost} = R_B > R_D$ DM core
- \blacksquare $R_{outermost} = R_D > R_B$ DM halo

Speed of sound should be calculated for two-fluid system Das+ 2020

Ellis+ 2018; Bezares+ 2019, Sagun+ 2022; Karkevandi+2022; Miao+2022; Leung+2022

Effect of DM on GW waveform

Accumulation of DM in sta

Effect of DM on NS properties

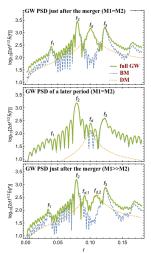
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Giudice+ 2016: Ellis+ 2018: Bezares+ 2019

The DM cores may produce a supplementary peak in the characteristic GW spectrum of NS mergers, which can be clearly distinguished from the features induced by the baryon component

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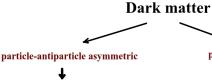
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accumulated inside a star



- DM particles are fermions -> the Pauli blocking may prevent them from collapsing into a black hole
- DM particles are bosons -> at zero temperature could form Bose-Einstein condensate leading to gravitational collapse of the bosonic DM leading to the formation of a black hole

Models of asymmetric DM should allow old NSs to exist

Kouvaris 2013

particle-antiparticle symmetric



DM particles can annihilate



- possibility of its detection via X-ray, γ-ray or neutrino telescopes

 Konvaris 2008
- late-time heating -> higher surface temperature of old NSs

de Lavallaz & Fairbairn 2010

Hamaguchi+ 2019

Equation for thermal balance

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The time evolution of the red-shifted temperature is determined by

$$C\frac{dT^{\infty}}{dt} = -L_{\nu}^{\infty} - L_{\gamma}^{\infty} + L_{H}^{\infty}$$

C - total heat capacity of the NS

L[∞] - red-shifted luminosity of the neutrino

 L_{γ}^{∞} - red-shifted luminosity of the photon emissions

 L_H^{∞} - red-shifted heating power

The photon emission luminosity is given by $L_{\gamma} = 4\pi R^2 \sigma_B T_S^4$, where σ_B is the Stefan-Boltzmann constant and R is the NS radius.

NS cooling

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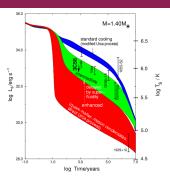
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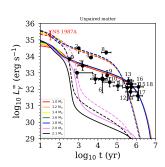
heating

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Numerical simulations of DM admixed

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Credits: Fridolin Weber

Light DM particles, such as axions, could contribute as an additional cooling channel in compact stars and their mergers

Creation mechanisms:

- nucleon bremsstrahlung
- Cooper pair breaking and formation processes

Buschmann+ 2022; Dietrich & Clough 2019

Cooling of NS with DM

Accumulation of DM in star

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Tidal deformability :

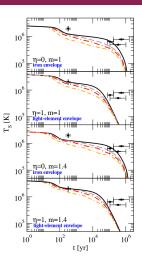
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The emission of axions alters the observable surface temperature

Sedrakian 2016; 2019

Heating of NS with DM

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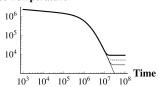
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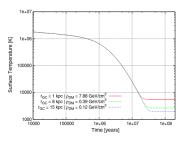
DM particles annihilation can cause heating of old NS

For a typical WIMP, its annihilation and capture rates equilibrate in old NSs.

Surface Temperature



Kouvaris 2008; Kouvaris & Tinyakov 2010; Hamaguchi+ 2019



Evolution of the surface temperatures of a 1.44 M_{\odot} neutron star situated at various galactic radii. In the present case, $m_{\chi}=10$ GeV, $\sigma_{0}=1.5\times 10^{-41}$ cm² and $(r_{-2},\alpha)=(16$ kpc, 0.19).

Lavallaz & Fairbairn 2010

DM admixed NSs

2 NSs with mass above $2M_{\odot}$

- PSR J0348-0432: $M = 2.01^{+0.04}_{-0.04} M_{\odot}$ (Antoniadis+ 2013)
- PSR J0740+6620: $M = 2.14^{+0.20}_{-0.18} M_{\odot}$ (Cromartie+ 2019)

Dark matter EoS

 Asymmetric dark matter relativistic Fermi gas of noninteracting particles with the spin 1/2

Nelson+ 2019

Baryon matter EoS

■ EoS with induced surface tension (IST EoS)

consistent with:

nuclear matter ground state properties,
proton flow data,
heavy-ion collisions data,
astrophysical observations,
tidal deformability constraint from the NS-NS merger (GW170817)

VS+ 2019; VS+ 2014

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Mass-Radius diagram of the DM admixed NSs

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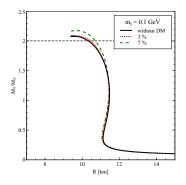
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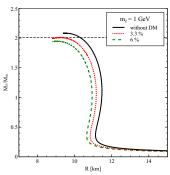
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 $M_{max} > 2 M_{\odot}$ for any f_{χ}



for $f_\chi=3.3$ % M_{max} equals to 2 M_\odot further increase of the DM fraction leads to $M_{max}<2$ M_\odot

Ivanytskyi+ 2020

Internal structure of the stars

Accumulation of DM in star

Effect of DM on NS properties

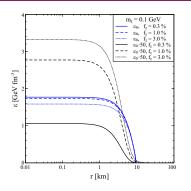
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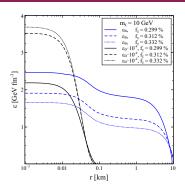
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$$R_D = 9.4 \text{ km for } f_\chi = 0.3\%$$

 $R_D = 21.2 \text{ km for } f_\chi = 1.0 \%$
 $R_D = 135.2 \text{ km for } f_\chi = 3.0 \%$

Large values of R_D relate to the existence of dilute and extended halos of DM around a baryon core of NS

DM admixed NSs

Mass-Radius diagram





Mass and Radius
Tidal
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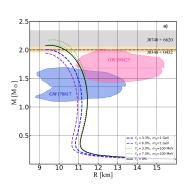
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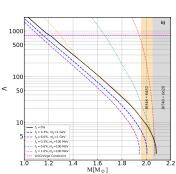
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$$\Lambda = \frac{2}{3}k_2 \left(\frac{R_{\rm outermost}}{M_{\rm tot}}\right)^5 \longrightarrow \Lambda(1.4{\rm M}_{\odot}) < 800; \tag{2}$$

Abbott+ 2018

Maximal mass of NS as a function of the DM fraction

Accumulation of DM in stars

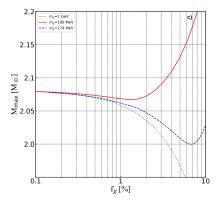
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for m_χ = 0.174 GeV $M_{\it max}$ is 2 M_\odot

DM particles with $m_\chi \leq 0.174$ GeV are consistent with the 2 M_\odot constraint for any f_χ For heavier DM particles the NS mass can reach 2 M_\odot only if f_χ is limited from above

DM constraint in the Galaxy center

Accumulation of DM in star

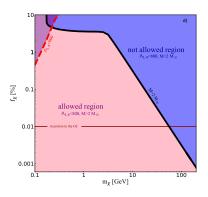
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- $2M_{\odot}$ NS in the GC $\Rightarrow m_{\chi} < 60$ GeV
- high DM fractions are not supported by GW170817

Measurements of M and R of compact stars at the Galaxy center will put more tight constraints on m_{χ} and f_{χ} .

What is the nature of the GW190814 secondary component?

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The compact binary merger event GW190814 had primary mass component, a black hole, with $M=23.2~M_{\odot}$ and the second component with $M=2.5-2.67~M_{\odot}$. The nature of the secondary component raised a lot of questions.

Possible explanations:

- NS with exotic degrees of freedom, e.g. hyperons and/or quarks

 [Tan+ 2020: Desheimer+ 2021]
- highly spinning NS [Zhang & Li 2020]
- NS matter with extra stiffening of the EoS at high densities [Fattoyev+ 2020]
- BH from the 'mass gap' [Tews+ 2021; Essick & Landry 2020]

An alternative explanation, the secondary component of GW190814 is a DM-admixed NS

GW190814 secondary component as a dark matter admixed neutron star

Accumulation

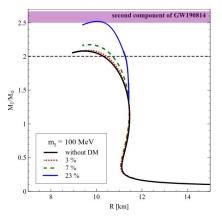
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Secondary component of GW190814 could be explained by the DM extended halo formation around a NS with the DM fraction $f_\chi=23\%$ for $m_\chi=100$ MeV.

VS+ 2022 (In prep)

Asymmetric Bosonic Dark Matter

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The minimal Lagrangian includes the complex scalar χ and real vector ω^μ fields, which are coupled through the covariant derivative $D^\mu = \partial^\mu - ig\omega^\mu$ with g being the corresponding coupling constant

$$\mathcal{L} = (D_{\mu}\chi)^* D^{\mu}\chi - m_{\chi}^2 \chi^* \chi - \frac{\Omega_{\mu\nu}\Omega^{\mu\nu}}{4} + \frac{m_{\omega}^2 \omega_{\mu}\omega^{\mu}}{2}$$
(3)

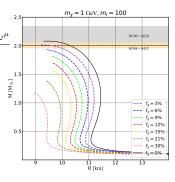
where $\Omega^{\mu\nu}=\partial^{\mu}\omega^{\nu}-\partial^{\nu}\omega^{\mu}$ and m_{ω} is the vector field mass.

Using a mean field approximation for ω , we get

$$p_{\chi} = \frac{m_l^2}{4} \left(m_{\chi}^2 - \mu_{\chi} \sqrt{2m_{\chi}^2 - \mu_{\chi}^2} \right),$$

$$\varepsilon_{\chi} = \frac{m_I^2}{4} \left(\frac{\mu_{\chi}^3}{\sqrt{2m_{\chi}^2 - \mu_{\chi}^2}} - m_{\chi}^2 \right),$$

(4)



Giangrandi+ 2022 (In prep.)

Chemical potential is limited

$$\mu_{\chi} \in [m_{\chi}, \sqrt{2}m_{\chi}], \quad m_{\chi}$$
 - boson mass $m_{I} = \frac{m_{\omega}}{\sigma}$ - interaction scale

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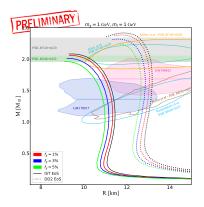
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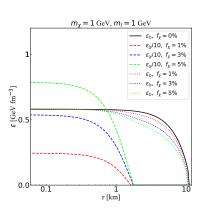
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Numerical Simulations of DM Admixed NS Binaries

Two-fluid 3D simulations of coalescencing binary NS systems admixed with DM

DM component: Mirror DM (mirrors the BM to a parallel hidden sector, the same particle physics as the observable world and couples to the latter through gravity)

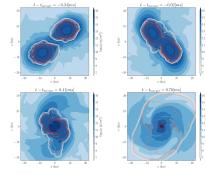
Berezhiani 2004; Ciancarella+ 2021

BM component: SLy EoS

Initial configurations

	$M_{A,B}(M_{\odot})$	Mirror dark matter %		$\rho_c^{dm} [\rho_{mc}]$	R _{A,B} [km]
SLy_M14_0	1.4	0%	3.866	0	11.45
SLy_M14_5	1.4	5%	4.360	2.234	11.00
SLy_M14_10	1.4	10%	4.713	2.854	10.60
SLy_M13_0	1.3	0%	3.624	0	11.46
SLy_M13_5	1.3	5%	4.058	2.087	11.04
SLy_M13_10	1.3	10%	4.366	2.679	10.63
SLy_M12_0	1.2	0%	3.398	0	11.46
SLy_M12_5	1.2	5%	3.791	1.960	11.04
SLv M12 10	1.2	10%	4.056	2.499	10.65

■ higher DM fraction ⇒ a longer inspiral likely due to a lower deformability of dark matter admixed neutron stars.



Emma+ 2022

Conclusion

Numerical simulations of DM admixed

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Gravitational waveform and frequency

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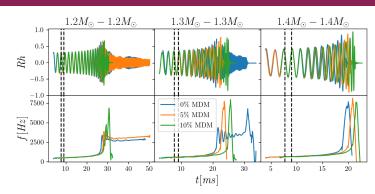
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- decrease of the disk mass ⇒ increasing DM fraction
- higher DM fraction ⇒ faster formation of the BH after the merger and harder to eject material from the bulk of the stars prior to the BH formation
- lack of DM ejecta and debris disks ⇒ is related to its concentration in the NS core

	M_{ej} sphere (M_{\odot})	M_{ej} integral (M_{\odot})	$M_{disk} (M_{\odot})$	fmerger[Hz]
SLy_M14_0	-	-	0.001	1770
SLy_M14_5	-	-	0.0008	2030
SLy_M14_10	-	-	0.0014	2058
SLy_M13_0	0.0168	$4.8 \cdot 10^{-3}$	0.062	1817
SLy_M13_5	0	$0.7 \cdot 10^{-3}$	0.001	1910
SLy_M13_10	0	$0.8 \cdot 10^{-3}$	0.0006	2221
SLy_M12_0	0	$0.3 \cdot 10^{-3}$	0.19*	1746
SLy_M12_5	0.0016	$2.6 \cdot 10^{-3}$	0.16*	1818
SLy_M12_10	0.0027	$3.3 \cdot 10^{-3}$	0.017	2198

Emma+ 2022

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- DM can be accumulated in the core of a NS ⇒ significant decrease of the maximum mass and radius of a star.
- DM halo ⇒ increase of the maximum mass and the outermost radius.
- The secondary component of the GW190814 binary merger might be a DM admixed NS.

Changing the position of the NS in the Galaxy the accretion rate of DM varies, which in turn leads to different amount of DM



different modifications of M, R, A, surface temperature, etc

Smoking gun of the presence of DM in NSs

■ by measuring mass, radius, and moment of inertia of NSs with few-%-accuracy.

To see this effect we need high precision measurement of M and R of compact stars as well as NS searches in the central part of the Galaxy with

radio telescopes: MeerKAT, SKA, ngVLA plan to increase radio pulsar timing and discover Galactic center pulsars.

space telescopes: NICER, ATHENA, eXTP, STROBE-X are expected to measure M and R of NSs with high accuracy.

DM core \Rightarrow mass and radius reduction of NSs toward the Galaxy center DM halo \Rightarrow mass increase of NSs toward the Galaxy center or variation of mass and radius in different parts of the Galaxy

by performing binary numerical-relativity simulations and kilonova ejecta for DM-admixed compacts stars for different DM candidates, their particle mass, interaction strength and fractions with the further comparison to GW and electromagnetic signals.

Large statistics on NS-NS, NS-BH mergers by LIGO/Virgo/KAGRA would be very helpful The smoking gun of the presence of DM could be:

supplementary peak in the characteristic GW spectrum of NS mergers; exotic waveforms; modification of the kilonova ejection;

post-merger regimes: the next generation of GW detectors, i.e., the Cosmic Explorer and Einstein Telescope.

by detecting objects that go in contradiction with our understanding.

As a potential candidate for a DM-admixed NS could be the secondary component of GW190814.

■ High/low surface temperature of NSs towards the Galaxy center

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Thanks for your attention!

Effect of DM

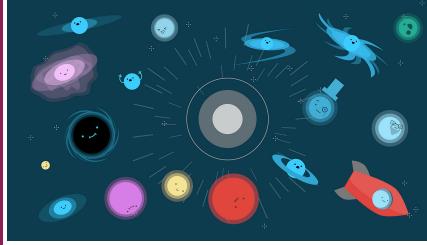
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Bosonic DM

Numerical simulations o DM admixed NS binaries



Asymmetric Bosonic Dark Matter

Accumulation of DM in star

on NS
properties

Mass and Radius
Tidal
deformability and
waveform
NS cooling and

Fermionic DM

Bosonic DN

Numerical simulations of DM admixed NS binaries

Conclusions

The minimal Lagrangian includes the complex scalar χ and real vector ω^μ fields, which are coupled through the covariant derivative $D^\mu = \partial^\mu - ig\omega^\mu$ with g being the corresponding coupling constant

$$\mathcal{L} = (D_{\mu}\chi)^* D^{\mu}\chi - m_{\chi}^2 \chi^* \chi - \frac{\Omega_{\mu\nu}\Omega^{\mu\nu}}{4} + \frac{m_{\omega}^2 \omega_{\mu}\omega^{\mu}}{2}$$
(5)

where $\Omega^{\mu\nu}=\partial^{\mu}\omega^{\nu}-\partial^{\nu}\omega^{\mu}$ and m_{ω} is the vector field mass.

Using a mean field approximation for $\boldsymbol{\omega},$ we get

$$p_{\chi} = \frac{m_I^2}{4} \left(m_{\chi}^2 - \mu_{\chi} \sqrt{2m_{\chi}^2 - \mu_{\chi}^2} \right),$$

$$\varepsilon_{\chi} = \frac{m_I^2}{4} \left(\frac{\mu_{\chi}^3}{\sqrt{2m_{\chi}^2 - \mu_{\chi}^2}} - m_{\chi}^2 \right),$$

(6)

Giangrandi+ 2022 (In prep.)

Chemical potential is limited

$$\mu_\chi \in [\frac{m_\chi}{\sigma}, \sqrt{2}m_\chi], \quad m_\chi$$
 - boson mass $m_l = \frac{m_\omega}{\sigma}$ - interaction scale