

# Non-equilibrium QCD in Heavy-Ion Collisions

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*Reviews:*

*SS, Teaney, Ann.Rev.Nucl.Part.Sci. 69 (2019) 447-476*

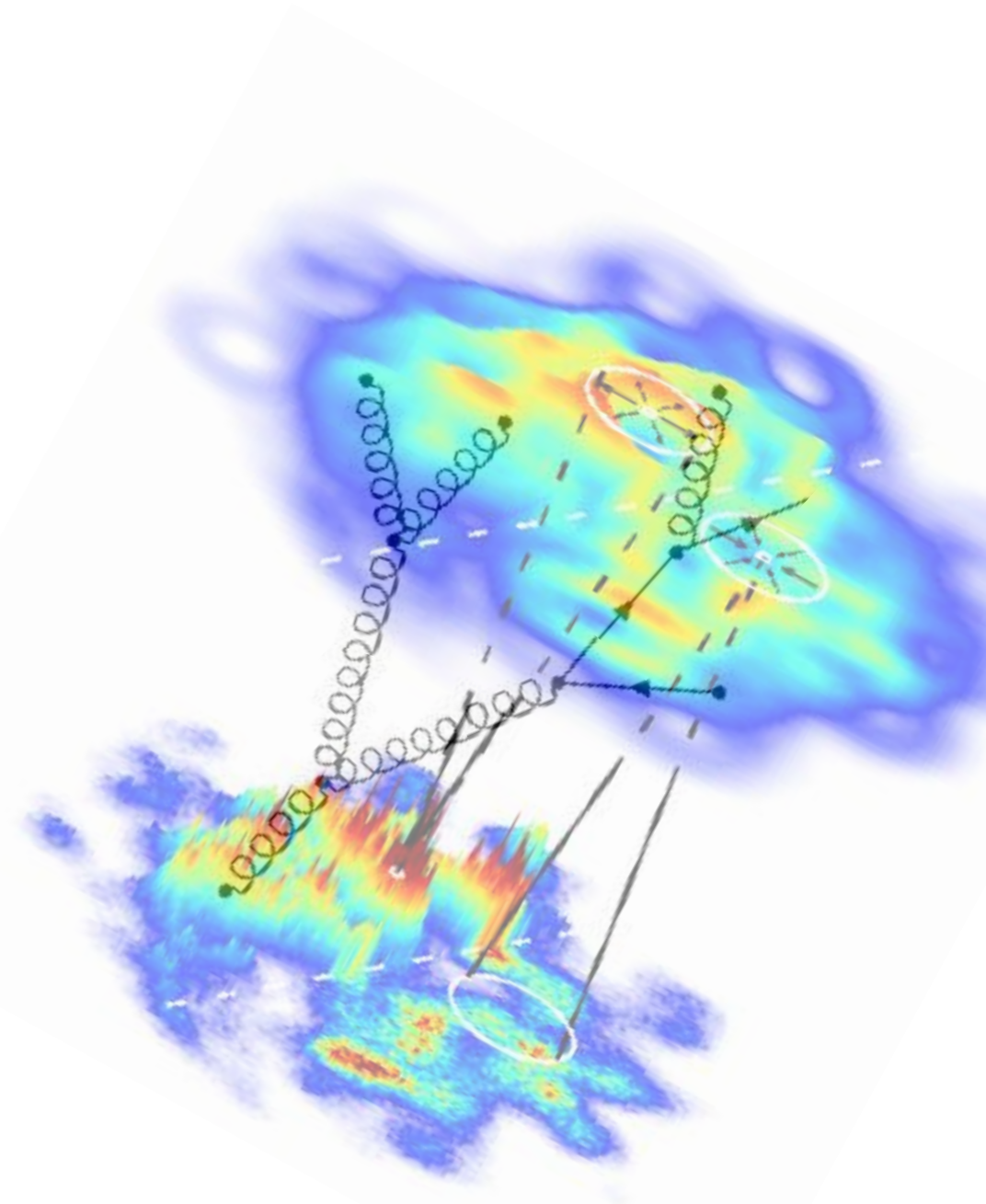
*Berges, Heller, Mazeliauskas, Venugopalan arXiv:2005.12299*

Theoretical Physics Colloquium @ ASU

Phoenix, AZ, Sept 2020



**UNIVERSITÄT  
BIELEFELD**

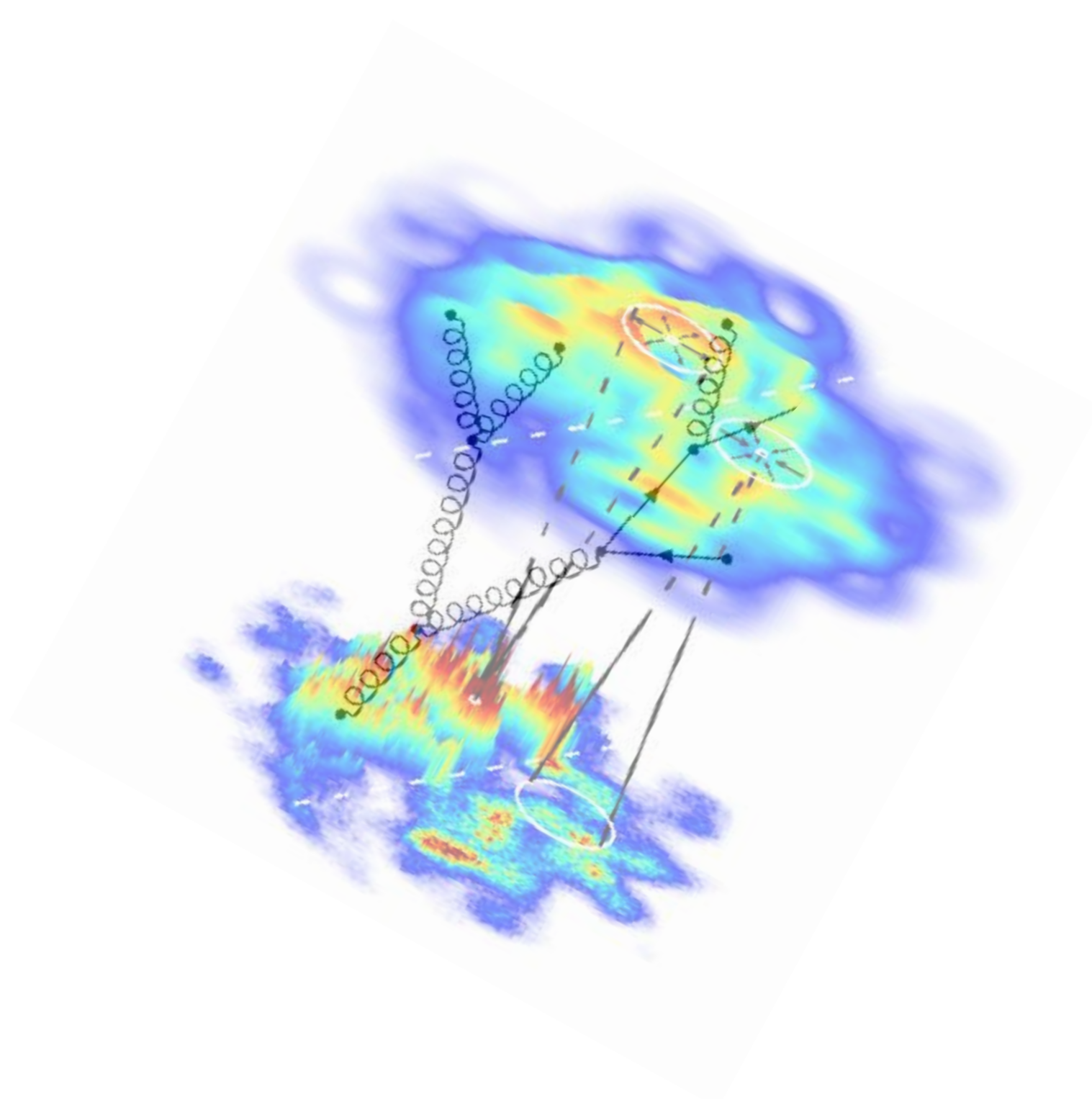


# Overview

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- ① Introduction & Motivation
- ② Basics of weak coupling thermalization in QCD plasmas
- ③ Early time dynamics & entropy production in high-energy Heavy-Ion Collisions
- ③ Conclusions & Outlook

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# Introduction & Motivation

# High energy Heavy-Ion Collisions

Science goal is to explore the structure and dynamics of the fundamental constituents of matter

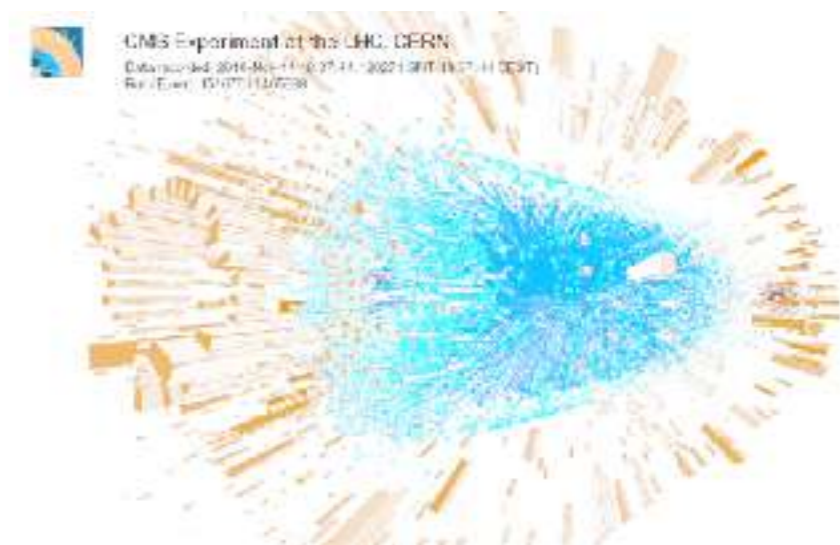
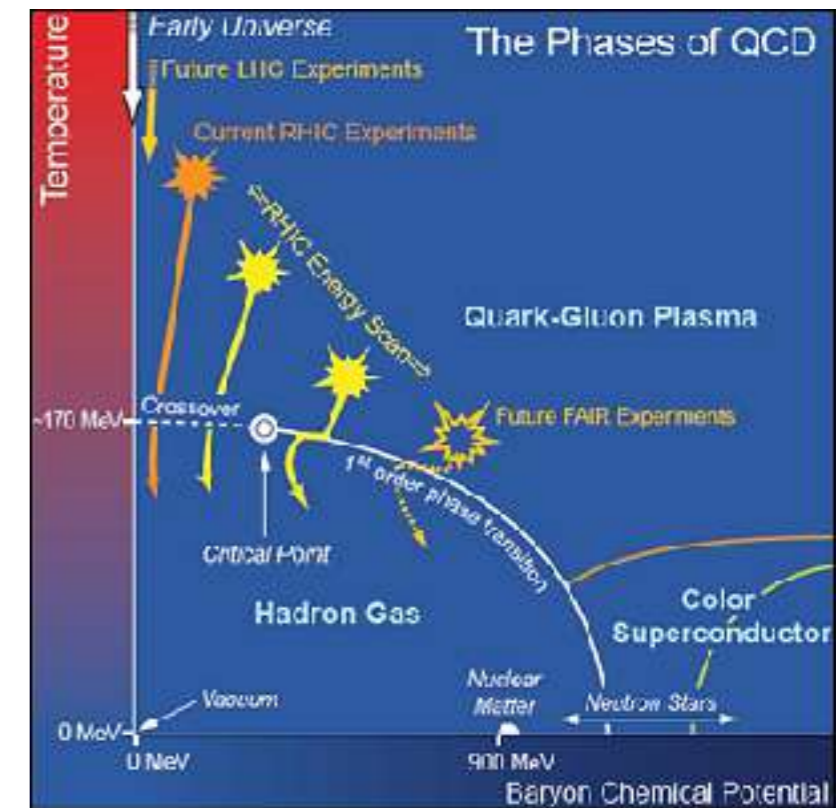
-> described by theory of strong interactions of quarks & gluons (QCD)

However under normal conditions, quarks & gluons confined in color neutral bound states ( $p, n, \pi, K, \dots$ )

-> need extreme conditions (high resolution, high temperature) to “see” fundamental constituents

Explored in high-energy Heavy-Ion collisions experiments at RHIC (BNL) and LHC (CERN) as well as GSI/FAIR and NICA (Dubna)

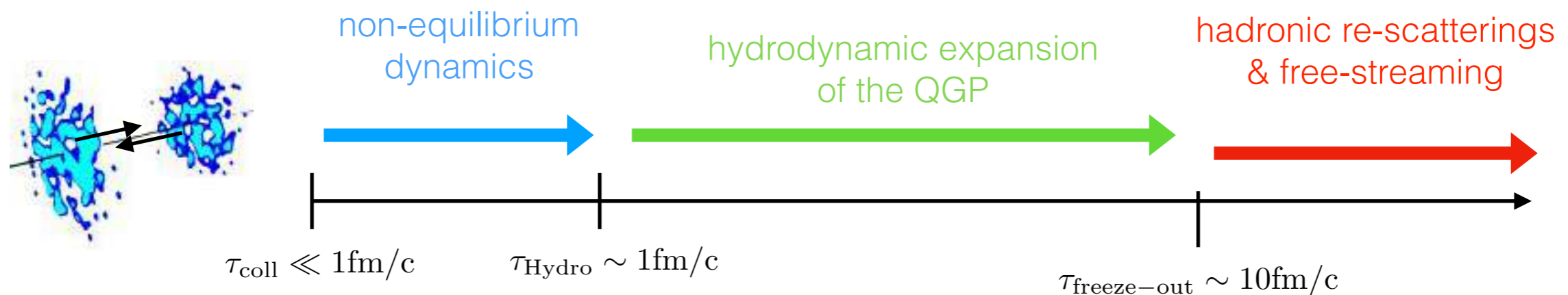
Extraction of QCD properties from heavy-ion experiments requires profound understanding of the space-time dynamics of the collision



# Dynamics of HICs

Dynamical description of heavy-ion collisions from underlying theory of QCD remains an outstanding challenge

Standard model of nucleus-nucleus (A+A) collisions based on macroscopic description of the space-time dynamics of the QGP



Space-time dynamics of HICs dominated by near-equilibrium hydrodynamic expansion

Extremely successful phenomenology based on hydrodynamic models with parametrized initial state  $\sim 1\text{ fm}/c$

Small effects of pre-equilibrium dynamics on typical observables ( $v_n, \langle p_T \rangle, \dots$ )

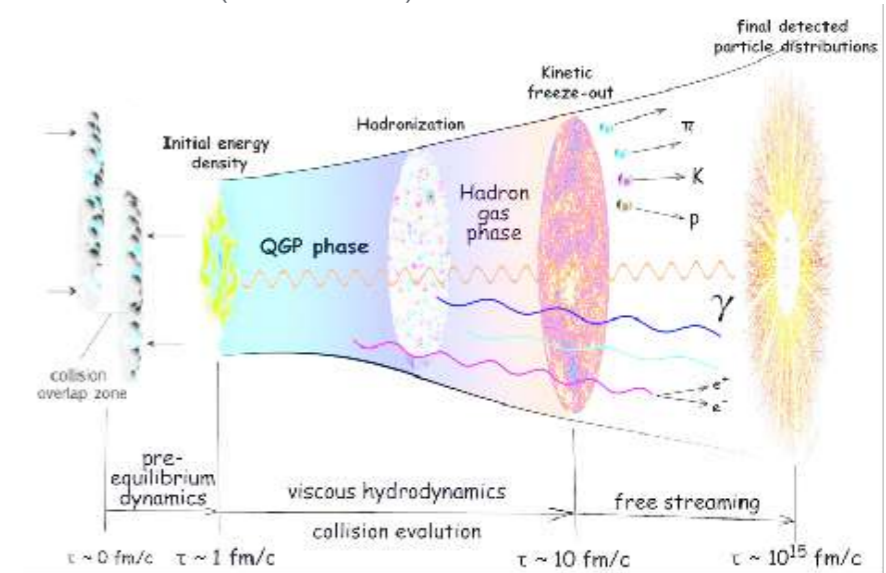
Theoretical description of pre-equilibrium dynamics highly desirable

# Equilibration of QCD plasmas

## High-Energy Heavy-Ion Collisions:

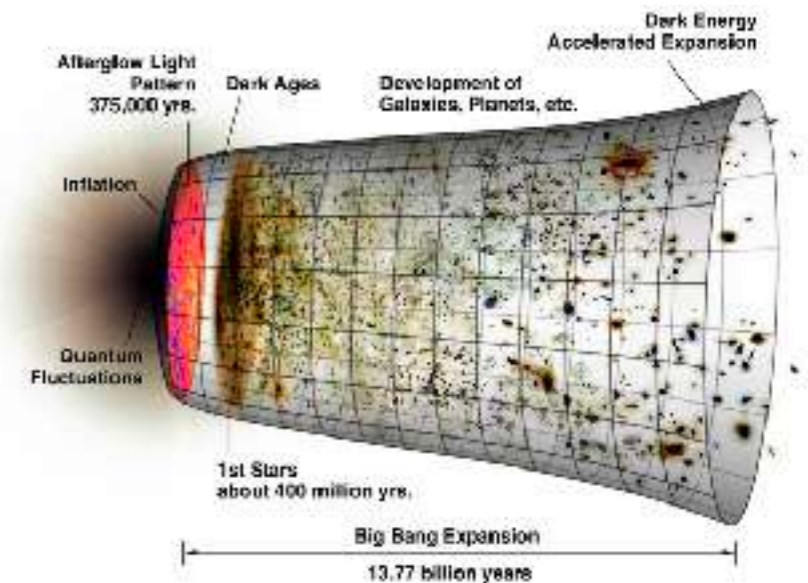
How does macroscopic description as viscous fluid (macroscopic variables  $e, u^\mu$ ) emerge from non-equilibrium dynamics of “primordial” far-from equilibrium plasma created in the collision?

C. Shen (PhD Thesis)

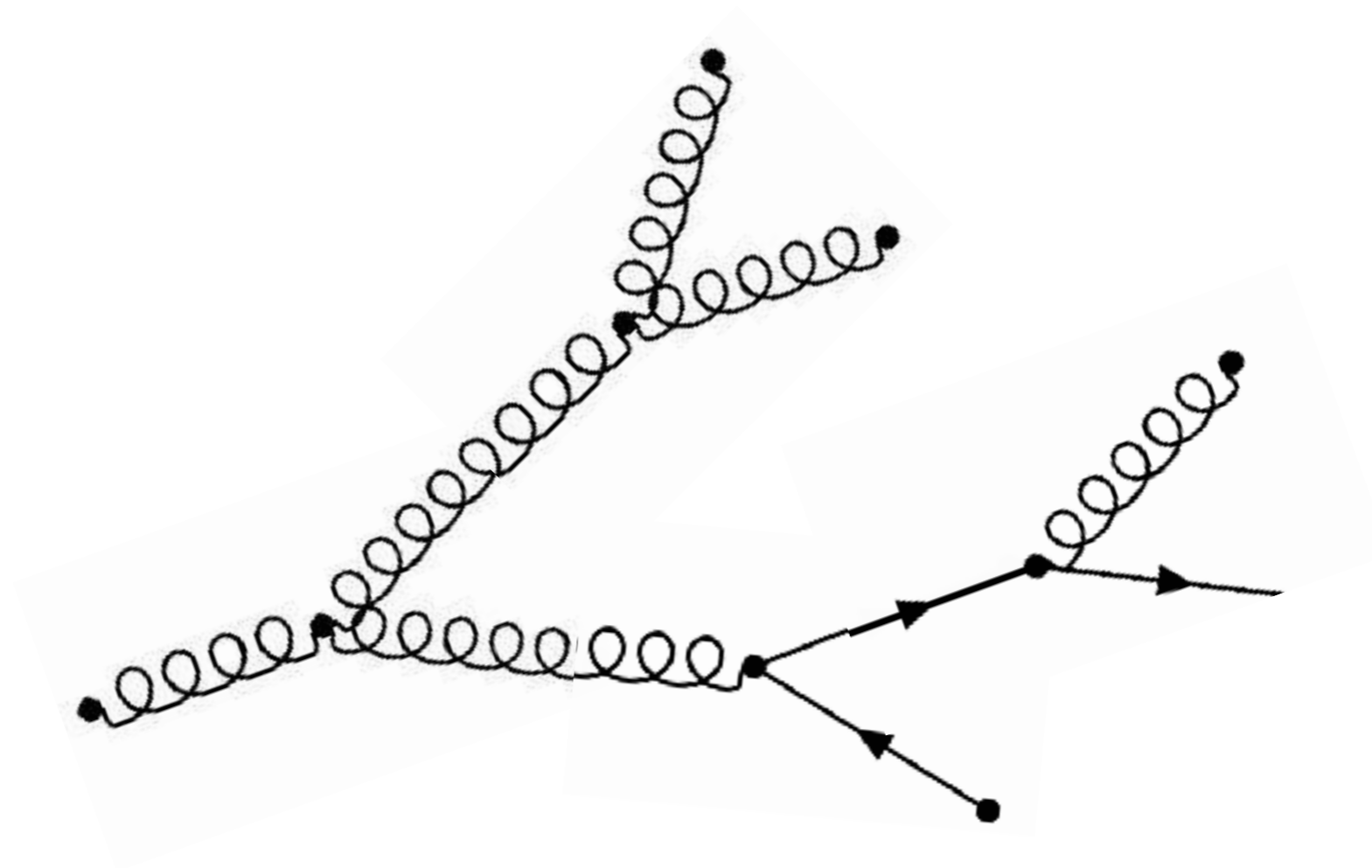


## Early Universe:

How is Standard Model Matter produced and equilibrated between end of Inflation and BBN?



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Basics of weak coupling  
thermalization in QCD plasmas

# Non-equilibrium QCD

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Generally there is no exact way to study non-equilibrium dynamics in an interacting quantum field theory

In high-energy QCD first principles studies are feasible in two limits

Weak coupling limit of QCD

Description in terms of the fundamental degrees and freedom (quarks & gluons) based on kinetic theory/classical-statistical simulations

Strong coupling limit ( $g^2 N_c \gg 1$ ) of related theories

Description based on holographic methods for strongly coupled gauge theories (N=4 SYM)

Will use weak-coupling description based on (LO) QCD kinetic theory ( $\alpha_s \ll 1$ ) to describe dynamics of non-equilibrium QCD plasmas

-> Extrapolate findings to realistic coupling strength ( $\alpha_s \sim 0.3$ ) when addressing HICs



# Non-equilibrium QCD

## Effective kinetic theory of QCD at LO\*

Arnold, Moore, Yaffe JHEP 0301 (2003) 030

$$p^\mu \partial_\mu f(x, p) = \mathcal{C}_{2 \leftrightarrow 2}[f] + \mathcal{C}_{1 \leftrightarrow 2}[f]$$

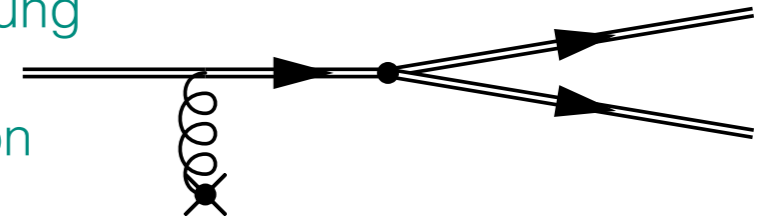
### Characteristic features:

- ultra-relativistic massless quasi-particles (g, u, u-bar, d, d-bar, s, s-bar)
- elastic (2 $\leftrightarrow$ 2) & in-elastic (1 $\leftrightarrow$ 2) processes at the same order

elast. 2 $\leftrightarrow$ 2 scattering  
screened by Debye mass



collinear 1 $\leftrightarrow$ 2 Bremsstrahlung  
incl. LPM effect  
via eff. vertex re-summation



Solve numerically as integro-differential equation, with in-medium matrix elements for 2 $\leftrightarrow$ 2 and 1 $\leftrightarrow$ 2 processes self-consistently determined

\*Note that expansion is in  $g$  rather than  $\alpha_s = g^2/4\pi$

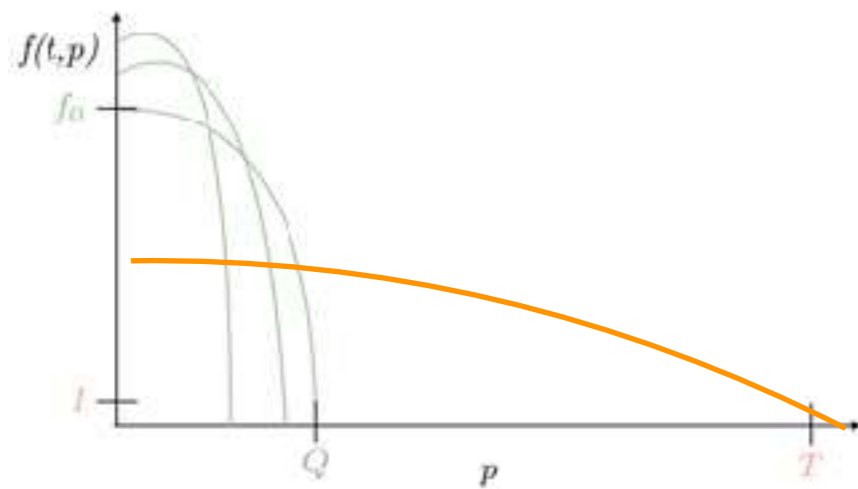
# Basics of weak-coupling thermalization

Before addressing the complex problem of thermalization in HICs it is interesting to start with thermalization of homogenous & isotropic QCD plasmas

Kurkela, Moore *JHEP* 12 (2011) 044

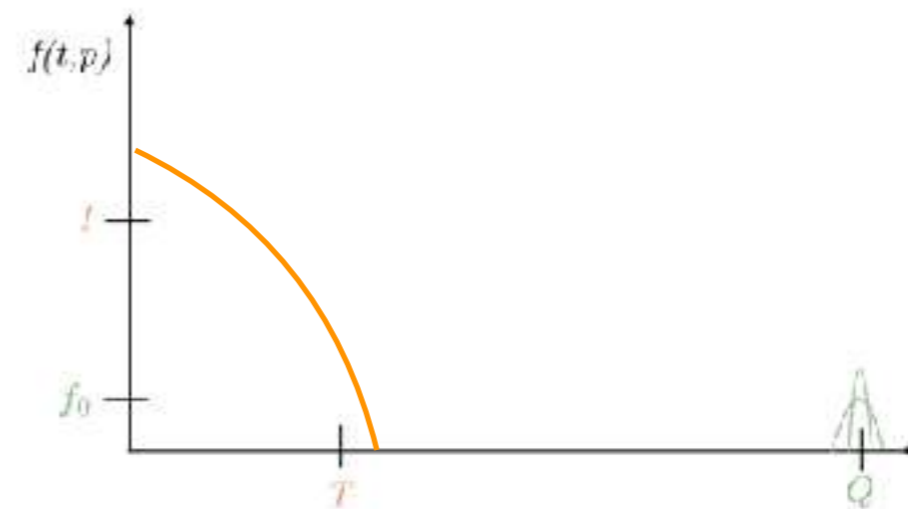
If we focus on initial states from from-equilibrium can distinguish two qualitatively different cases

Over-occupied QCD plasma



Energy carried by large number of low energy dof's  $\langle p \rangle \ll T$  (e.g. due to instabilities)

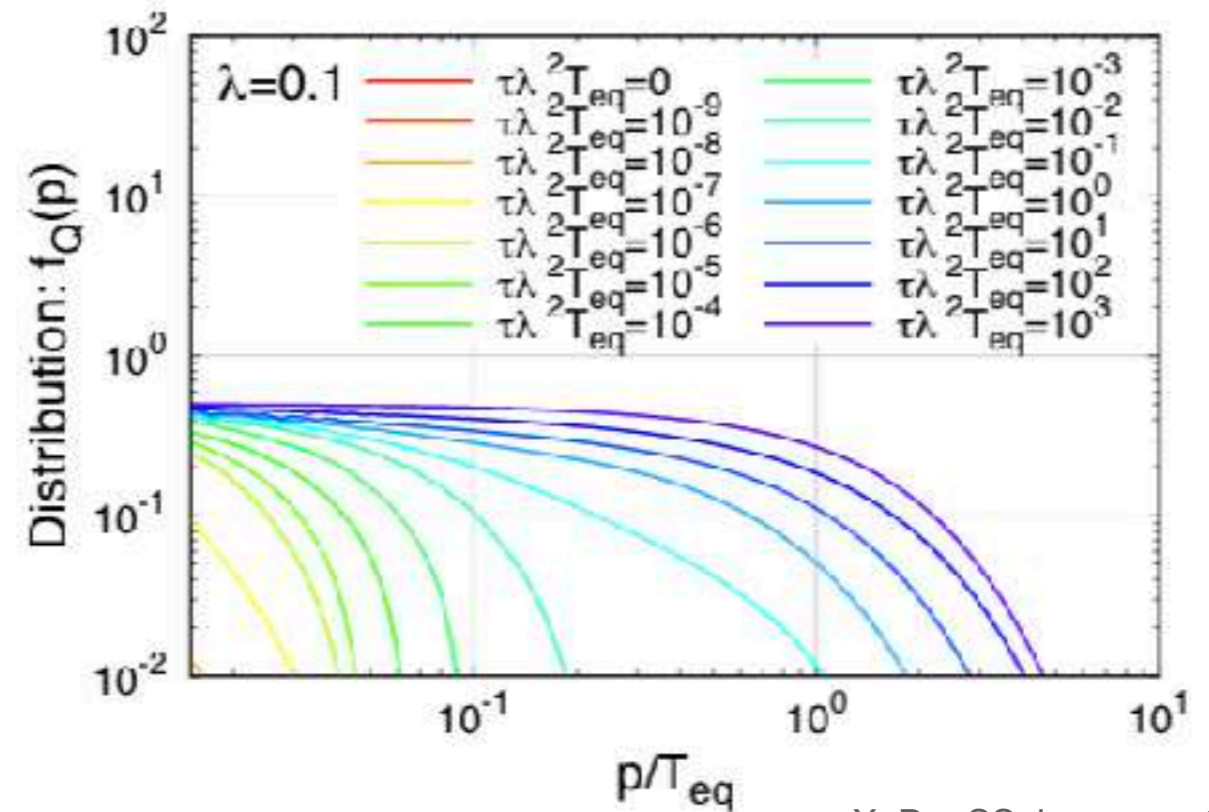
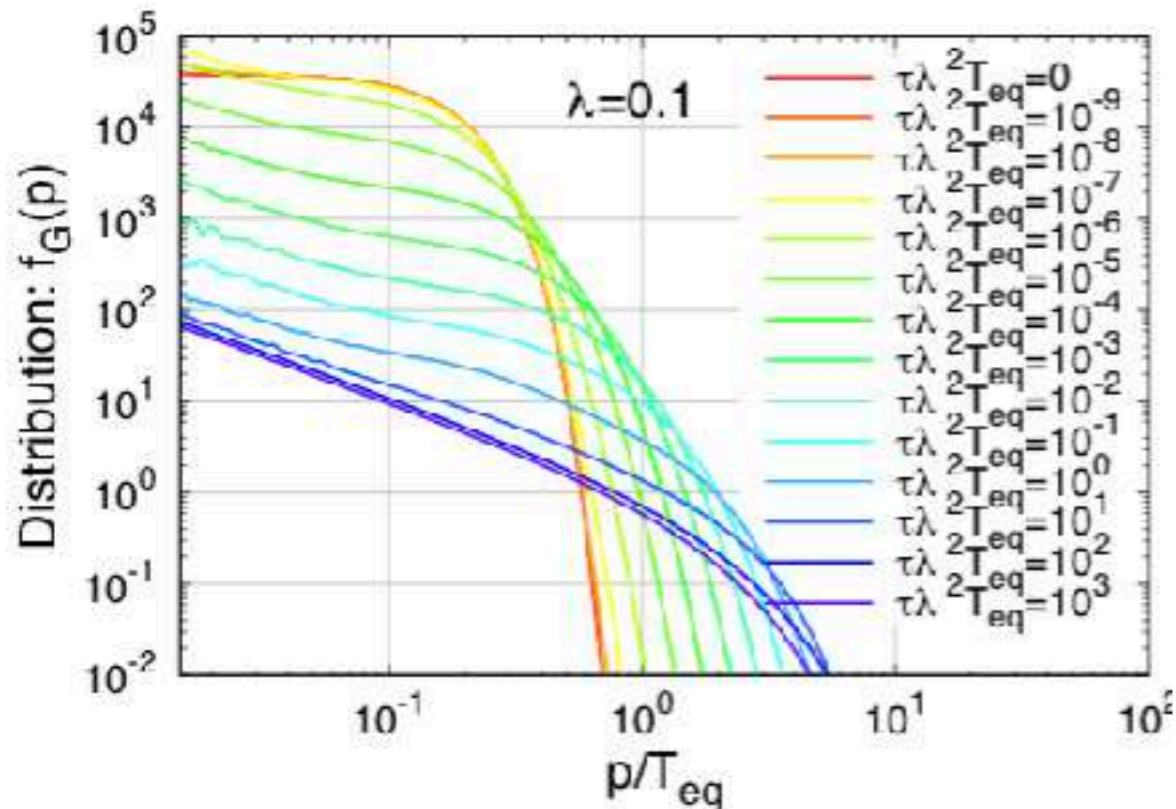
Under-occupied QCD plasma



Energy carried by small number of high energy dof's  $\langle p \rangle \gg T$  (e.g. high-energy jets)

# Over-occupied QCD plasmas

Enhanced scattering rates at early times lead to effective memory loss even before system thermalizes



X. Du, SS, in preparation

Self-similar evolution of gluon distribution  $f_G(t,p)$  associated with energy transport to UV

$$f_g(t, p) = t^\alpha f_g^S(t^\beta p)$$

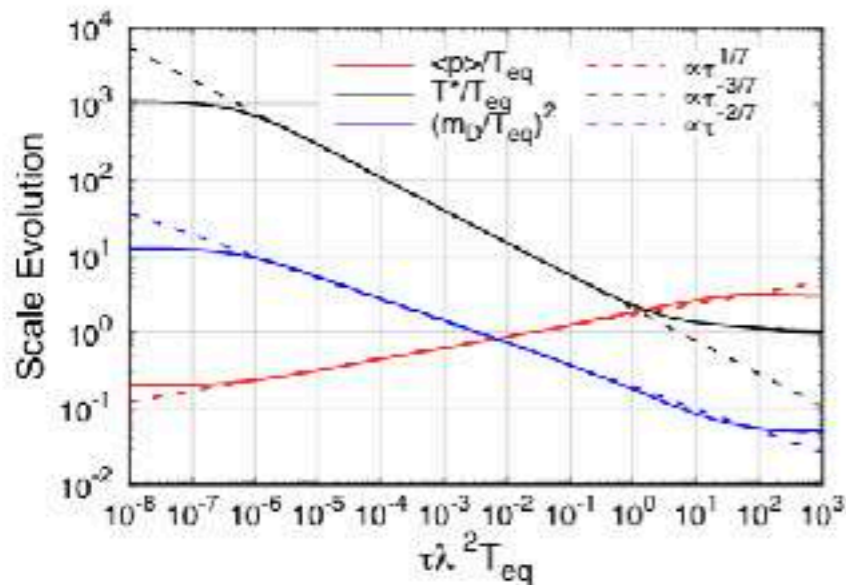
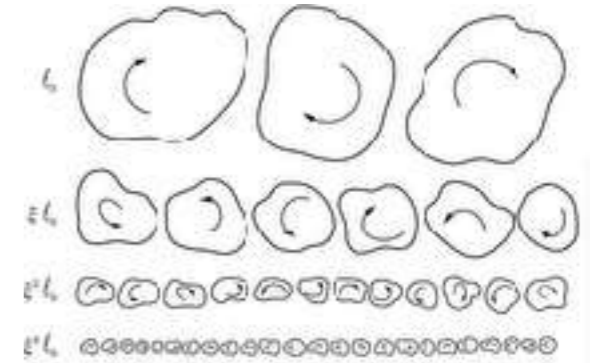
SS *Phys.Rev.D* 86 (2012); Abrao York, Kurkela, Lu, Moore *Phys.Rev.D* 89 (2014) 7  
 Berges, Boguslavski, SS, Venugopalan *Phys.Rev.D* 89 (2014) 11;  
 Berges, Mazeliauskas *Phys.Rev.Lett.* 122 (2019)

Quarks are sub-dominant and simply follow gluon distribution

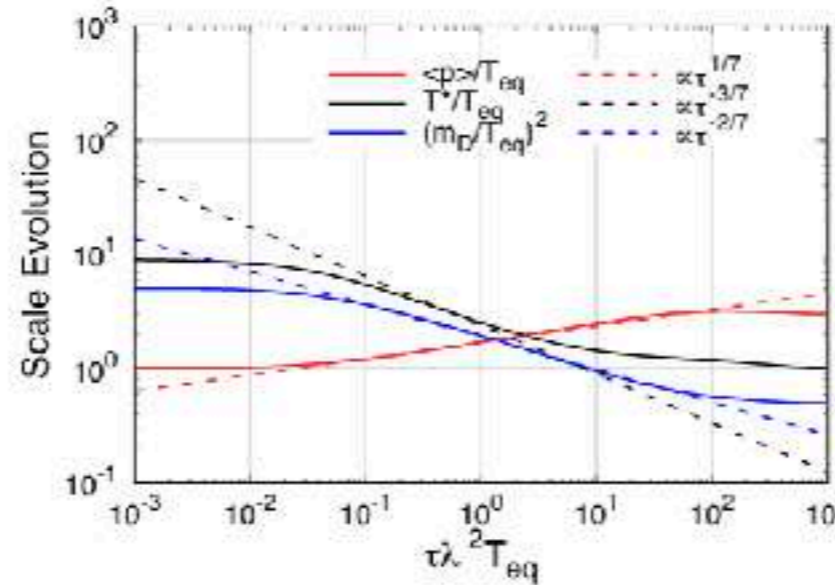
$$f_q(t, p) = f_q^S(t^\beta p)$$

# Over-occupied QCD plasmas

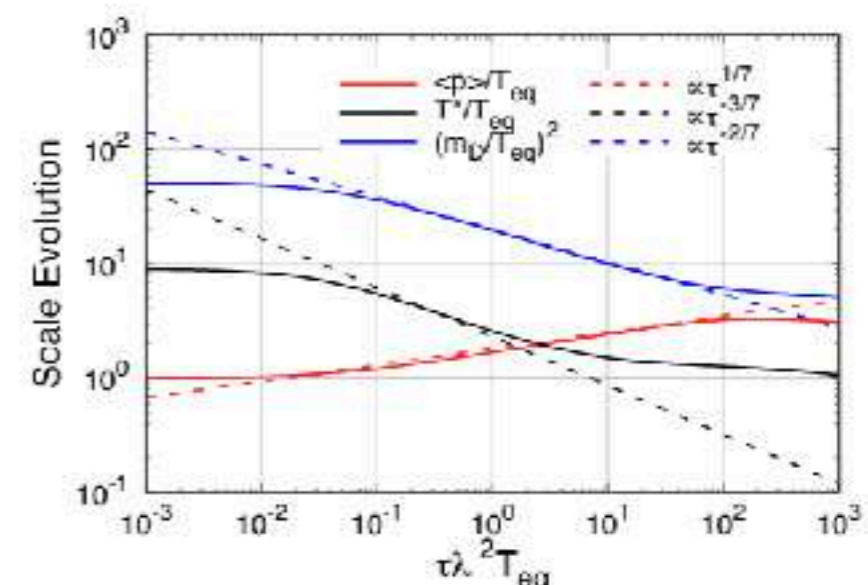
Self-similar scaling behavior associated with energy transport towards UV is analogous to direct energy cascade in decaying turbulence



$\lambda = 0.1$



$\lambda = 1$



$\lambda = 10$

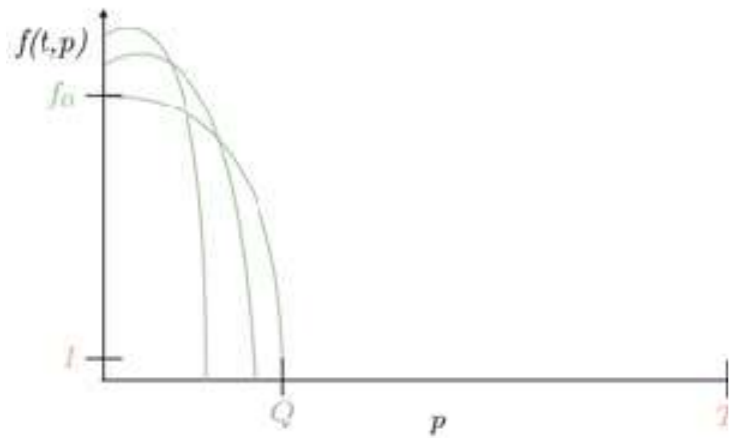
Same scaling behavior persists even for moderately large values of the coupling constant until energy transfer to the UV is complete and the system thermalizes

X. Du, SS, in preparation

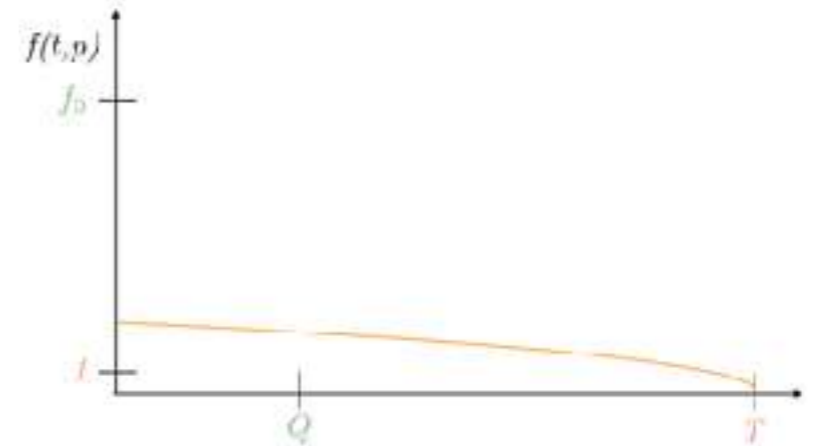
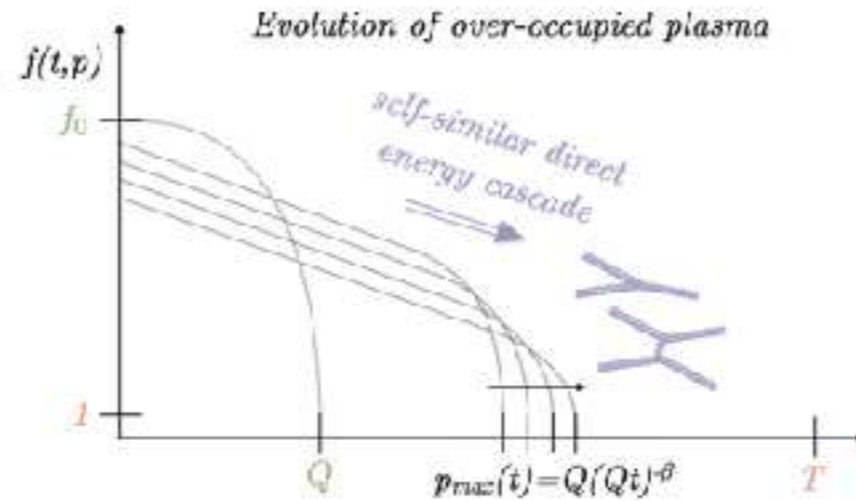
$$\alpha = -4/7 \quad \beta = -1/7$$

# Over-occupied QCD plasmas

Initial state  
far from-equilibrium  
 $Q \ll T$



Thermal equilibrium



## Thermalization time

Kurkela, Lu *Phys.Rev.Lett.* 113 (2014) 18  
SS, Teaney *Ann.Rev.Nucl.Part.Sci.* 69 (2019)

$$t_{\text{thermal}} \sim \alpha_s^{-2} f_0^{-1/4} Q^{-1} \sim \alpha_s^{-2} T^{-1}$$

Non-perturbative classical-statistical simulations give same picture but are unable to describe approach to equilibrium

Berges, Boguslavski, SS, Venugopalan *Phys.Rev.D* 89 (2014) 11;

Mace, SS, Venugopalan *Phys.Rev.D* 93 (2016) 7; Berges, Mace SS *Phys.Rev.Lett.* 118 (2017) 19

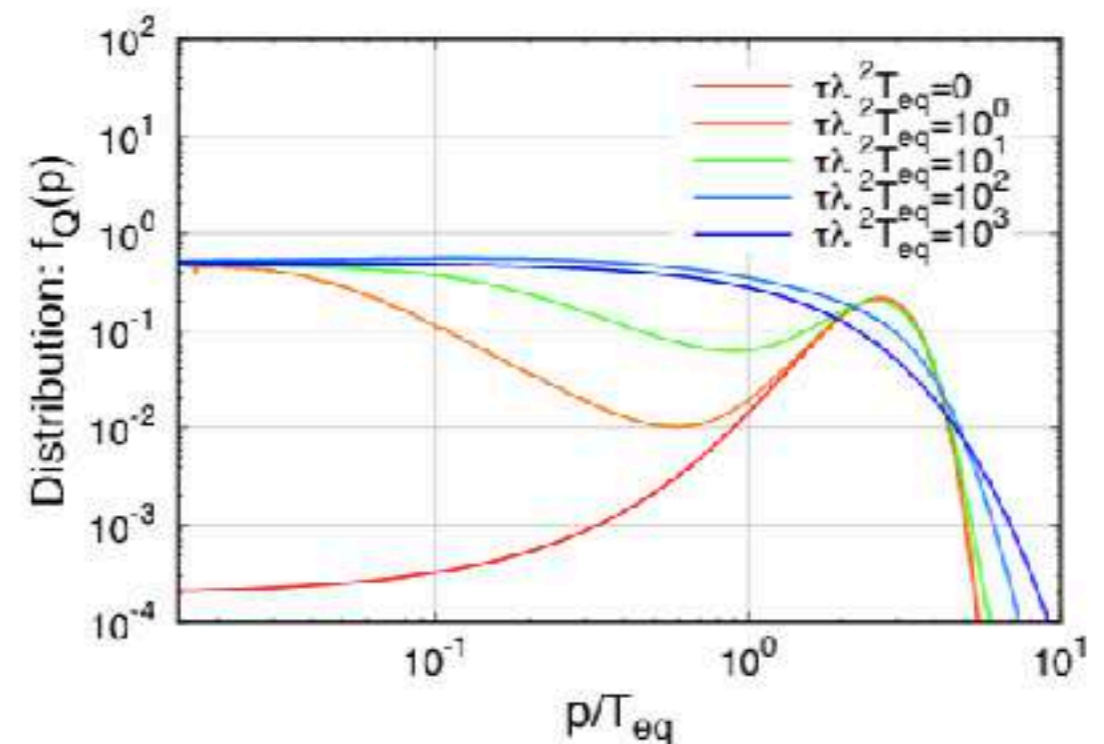
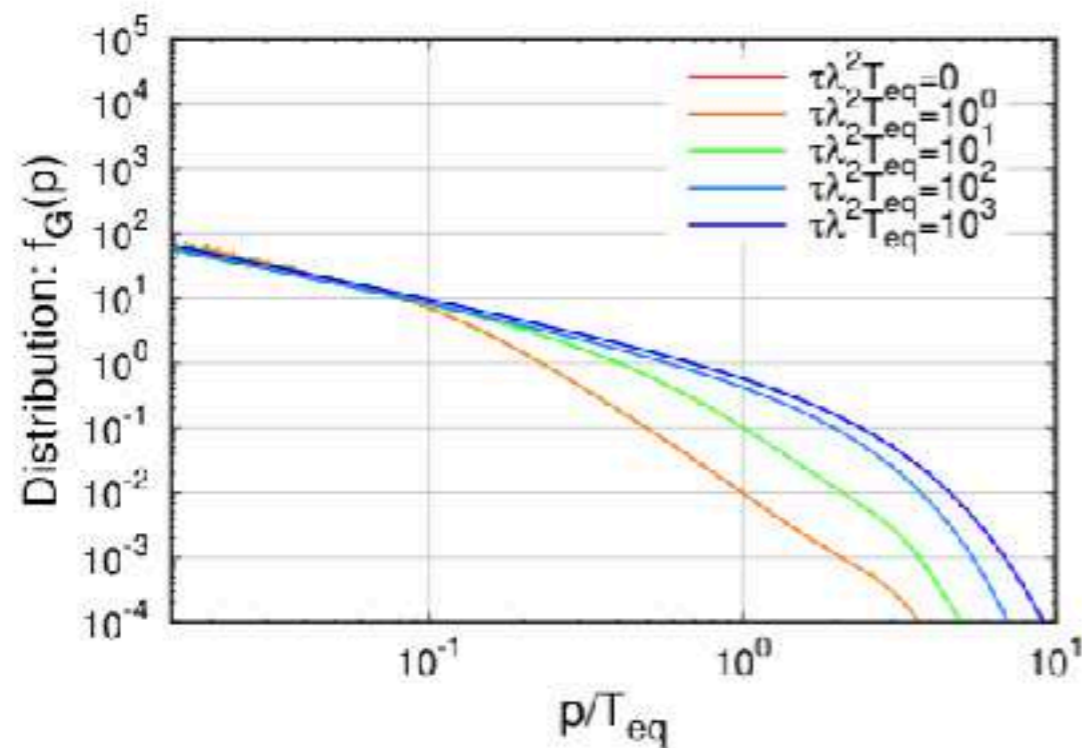
Same kind of thermalization pattern emerges in a variety of far-from-equilibrium systems incl. relativistic and non-relativistic Bose gases

Micha, Tkachev *Phys.Rev.D* 70 (2004) 043538; Berges, Boguslavski, SS Venugopalan *Phys.Rev.Lett.* 114 (2015) 6, 061601;  
Berges, Heller, Mazeliauskas, Venugopalan arXiv:2005.12299

# Under-occupied QCD plasmas

Even for relatively small scale separation evolution shows characteristic features of “bottom-up” thermalization

Baier et al. Phys.Lett.B 502 (2001)



Hard particles emit soft quark/gluon radiation

X. Du, SS, in preparation

Soft quarks/gluons thermalize and form a thermal bath with low temperature

Inverse energy cascade deposits energy of hard particles into soft-thermal bath

# Under-occupied QCD plasmas

Inverse energy cascade associated with scale invariant energy flux from hard sector  $\sim Q$  to soft sector  $\sim T$

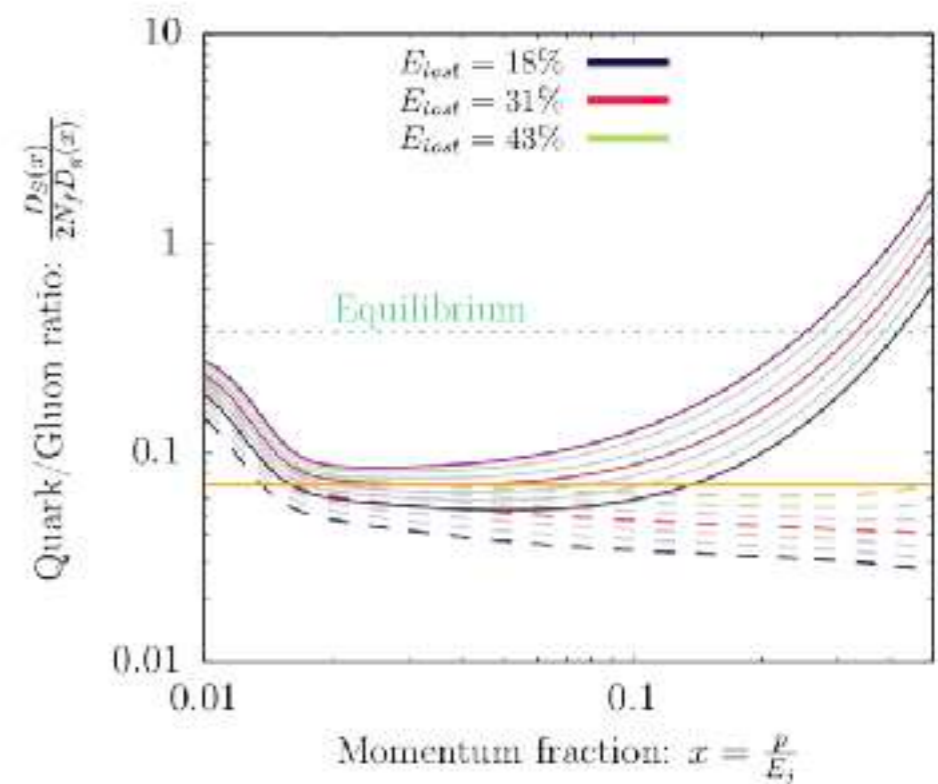
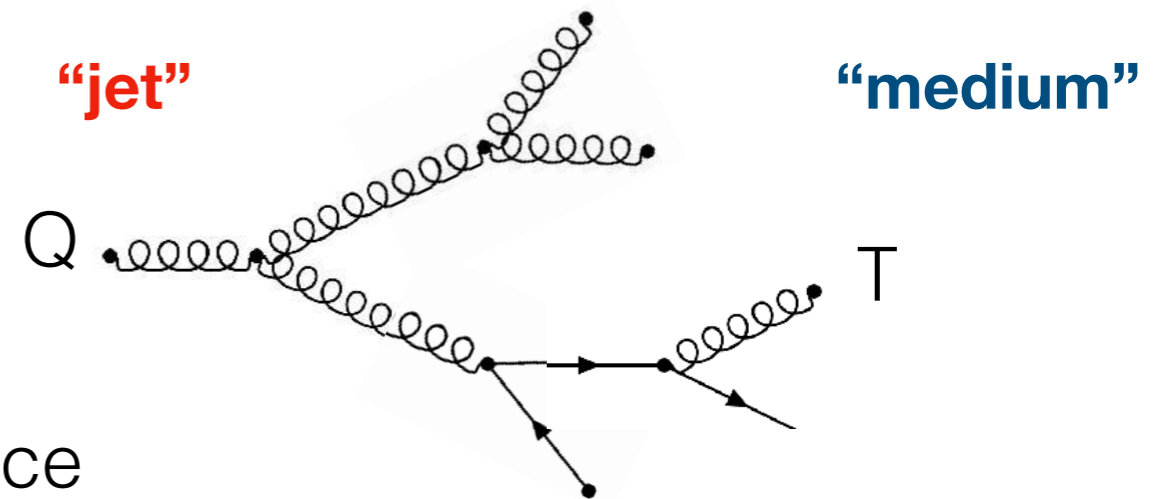
Standard features of weak wave turbulence observed for sufficiently large scale separation (e.g. high-energy Jet in thermal medium)

Kolmogorov spectrum  $f_{g/q}(T \ll p \ll Q) \sim p^{-7/2}$

Baier et al. Phys.Lett.B 502 (2001),; Blaizot, Iancu, Mehtar-Tani  
 Phys.Rev.Lett. 111 (2013) 052001; Mehtar-Tani, SS JHEP 09 (2018) 144

Chemistry  $f_q(T \ll p \ll Q) / f_g(T \ll p \ll Q)$  determined by balance of  $g \rightarrow qq\bar{q}$  and  $q \rightarrow qg$  processes

Mehtar-Tani, SS JHEP 09 (2018) 144 I. Soudi, SS, arXiv:2008.04928



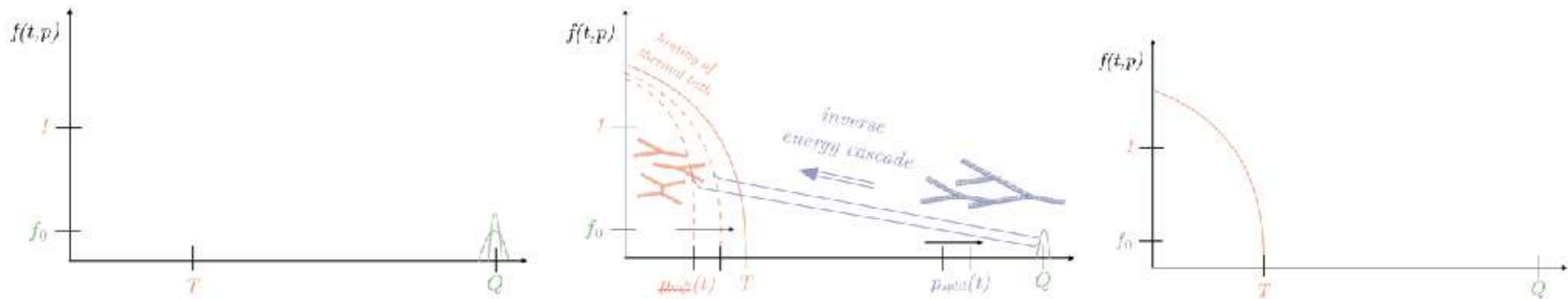
I. Soudi, SS, arXiv:2008.04928

# Under-occupied QCD plasmas

Initial state  
far from-equilibrium  
 $Q \gg T$

radiative break-up  
via inverse energy cascade

Thermal equilibrium



## Thermalization time

Kurkela, Lu Phys.Rev.Lett. 113 (2014) 18  
SS, Teaney Ann.Rev.Nucl.Part.Sci. 69 (2019)

$$t_{\text{thermal}} \sim \alpha_s^{-2} f_0^{-3/8} Q^{-1} \sim \alpha_s^{-2} T^{-1} \sqrt{\frac{Q}{T}}$$

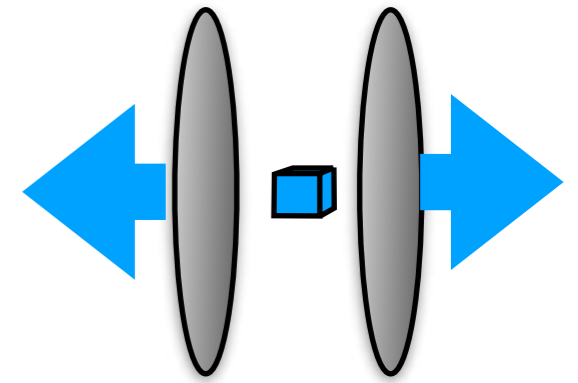
“Bottom-up” thermalization is delayed due to reduced splitting rates of high-momentum particles  $\Gamma_{\text{inel}}(Q) \sim (T/Q)^{1/2} \Gamma_{\text{eq}}$



# Expanding systems created in HICs

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Non-equilibrium QCD plasma created immediately after the collision of heavy nuclei is subject to a rapid longitudinal expansion



Since particles with large  $p_{\text{long}}$  escape quickly central region, the central QGP will be highly anisotropic with  $p_T \gg p_{\text{long}}$  and becomes increasingly dilute

Generally interested in evolution over short time scales  $\tau \sim 1 \text{ fm}/c \ll R_A$  where transverse expansion of QGP ( $\sim 1/R$ ) can be neglected

Describe non-eq. QGP locally as approximately homogenous in transverse plane and invariant under long. boosts

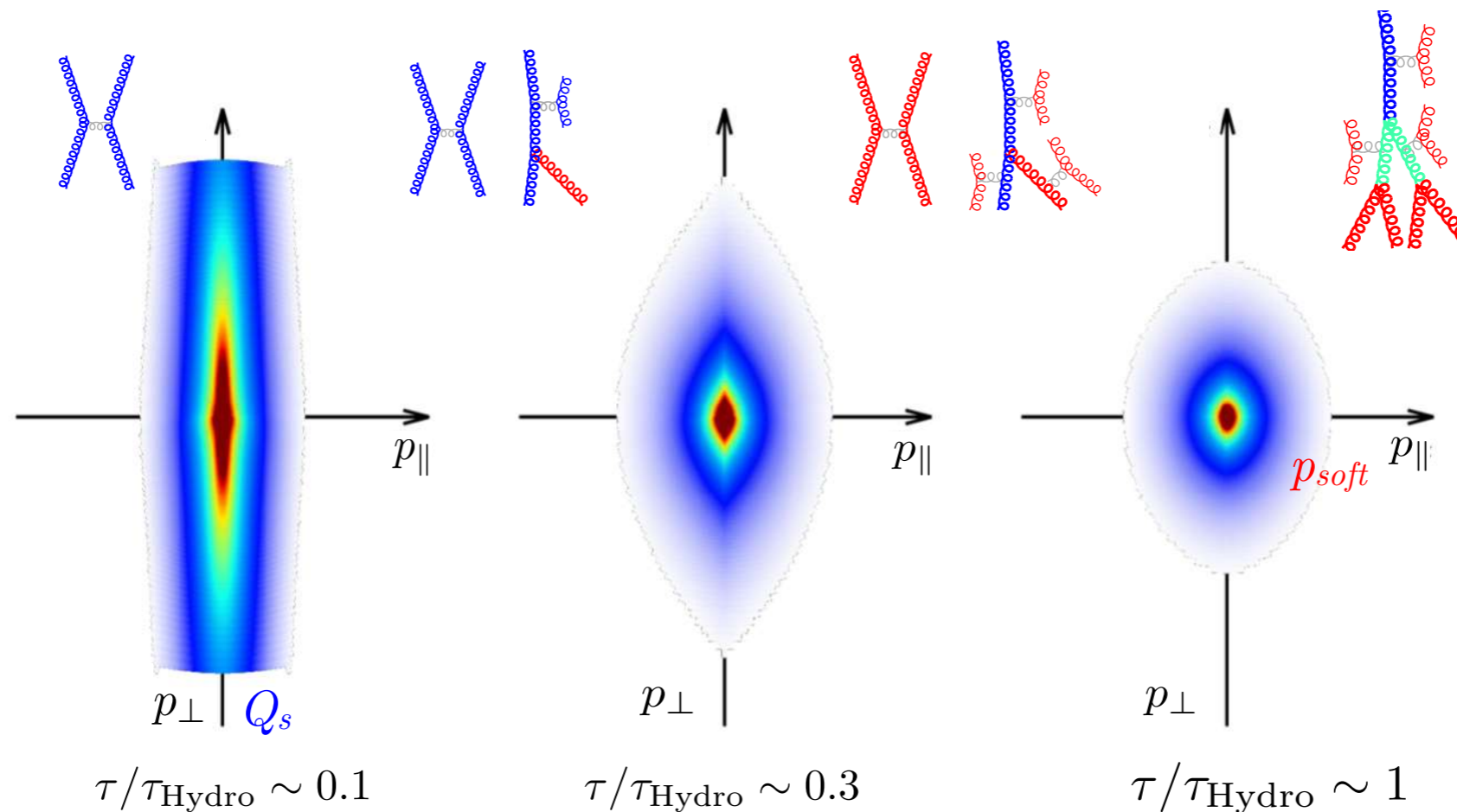
Energy momentum tensor assumes average form

$$T^{\mu\nu} = \text{diag}(e, p_T, p_T, p_L) \quad \text{with } p_L \ll p_T \text{ at early times}$$

# Microscopic evolution

## Evolution of homogenous boost invariant system in QCD kinetic theory

Kurkela, Zhu PRL 115 (2015) 182301; Keegan, Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171;  
Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302; PRC 99 (2019) no.3, 034910



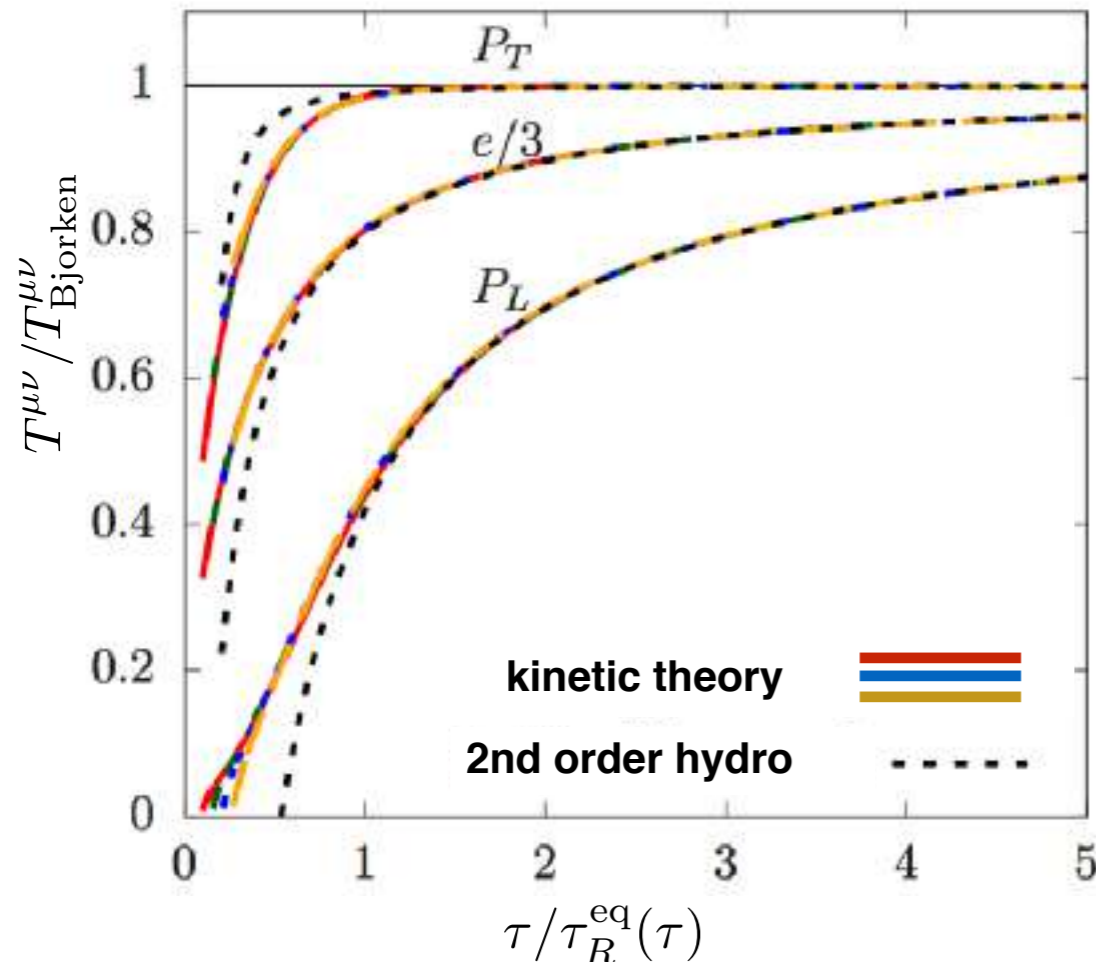
Eventually equilibration of expanding QCD plasma proceeds  
a la ``bottom-up'' via radiative break-up

# Hydrodynamic behavior

Hydrodynamics describes the macroscopic evolution of energy-momentum tensor  $T^{\mu\nu}$  based on an expansion around local thermal equilibrium

$$Kn \sim \lambda_{micro}/L_{macro} \quad Re^{-1} \sim \delta T_{dissipative}^{\mu\nu}/T_{eq}^{\mu\nu}$$

Since the system is highly anisotropic at early times ( $P_L \ll P_T$ ), key question is to understand evolution of  $T^{\mu\nu}$  towards local equilibrium ( $P_L = P_T$ )



Equilibration process controlled by a single equilibrium relaxation rate

$$\tau_R^{eq}(\tau) = \frac{4\pi\eta/s}{T_{eff}(\tau)}$$

Simulations at different coupling ( $\lambda=5-20$ ) strength indicate small sensitivity to  $\alpha_s$  when compared in units of  $\tau_R^{eq}$

# Hydrodynamic behavior

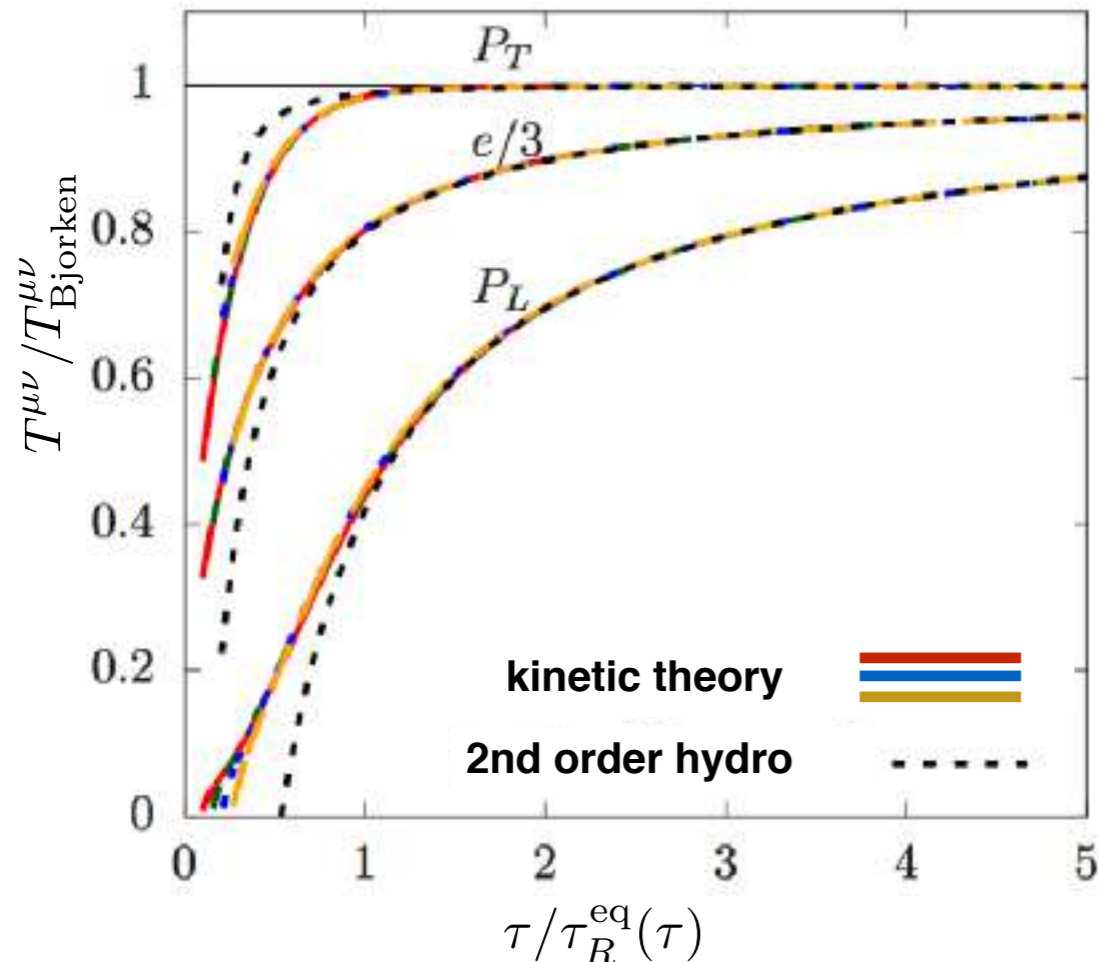
Effective description in viscous hydrodynamics becomes applicable on time scales

$$\tau_{\text{hydro}} \approx \tau_R^{\text{eq}}(\tau) \quad \tau_R^{\text{eq}}(\tau) = \frac{4\pi\eta/s}{T_{\text{eff}}(\tau)} \quad \tau_{\text{hydro}} \approx 1.1 \text{ fm} \left( \frac{4\pi(\eta/s)}{2} \right)^{\frac{3}{2}} \left( \frac{\langle \tau s \rangle}{4.1 \text{ GeV}^2} \right)^{-1/2}$$

in line with earlier phenomenological estimates

Kurkela, Zhu PRL 115 (2015) 182301

Kurkela, Mazeliauskas, Paquet, SS, Teaney 1805.01604; 1805.00961



Viscous hydrodynamics becomes applicable when

$$Kn \sim \tau_R^{\text{eq}}(\tau) / \tau \sim 1$$

$$Re^{-1} \sim 1 - T^{\mu\nu} / T_{\text{Bjorken}}^{\mu\nu} \sim 1$$

Surprising effectiveness of viscous fluid dynamics also found e.g. at strong coupling

Heller, Janik, Witaszczyk PRL 108 (2012) 201602

Romatschke PRL 120 (2018) no.1, 012301

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# Dynamics of HICs

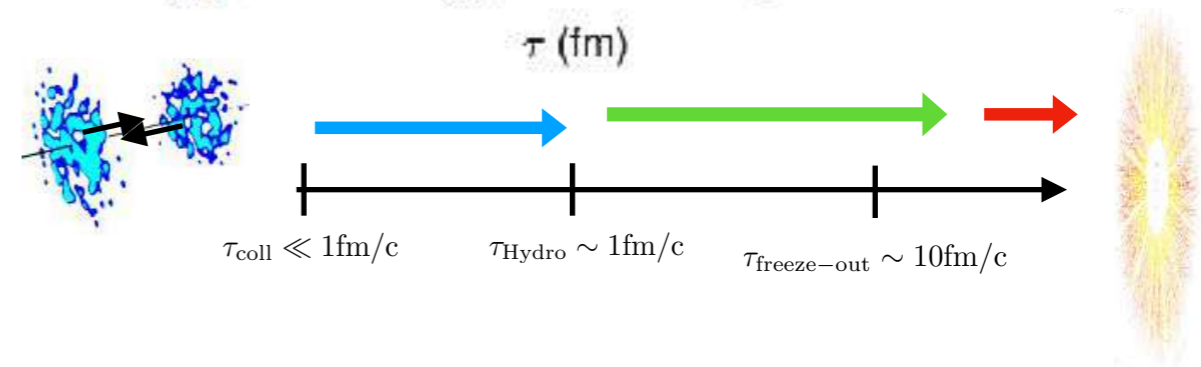
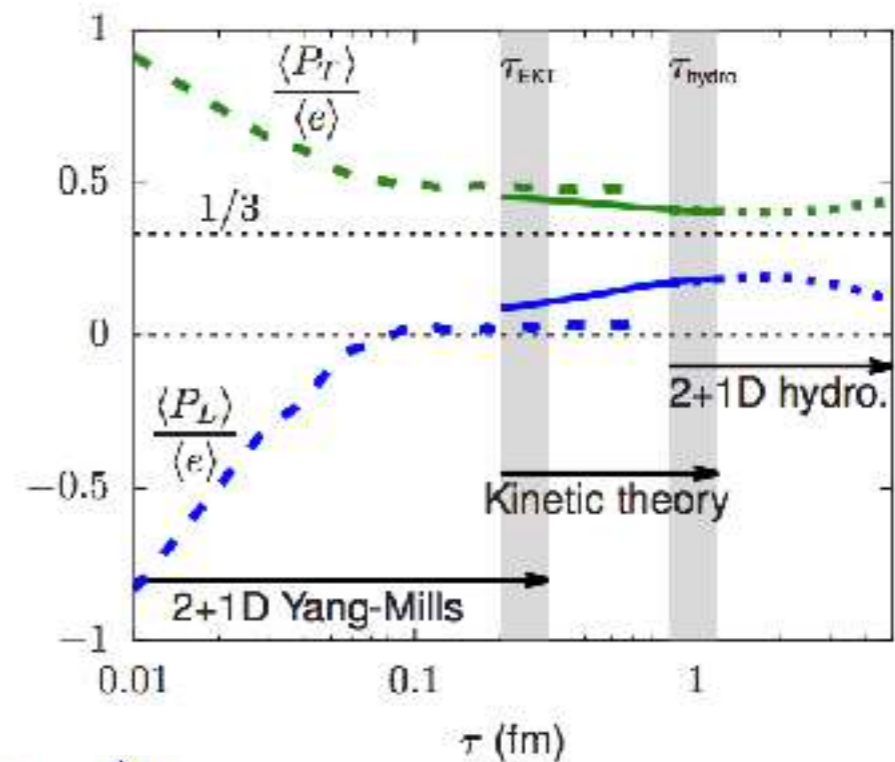
Based on progress in understanding early time dynamics & equilibration can now describe HIC from beginning to end by matching different effective descriptions of QCD

Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302;  
PRC 99 (2019) no.3, 034910

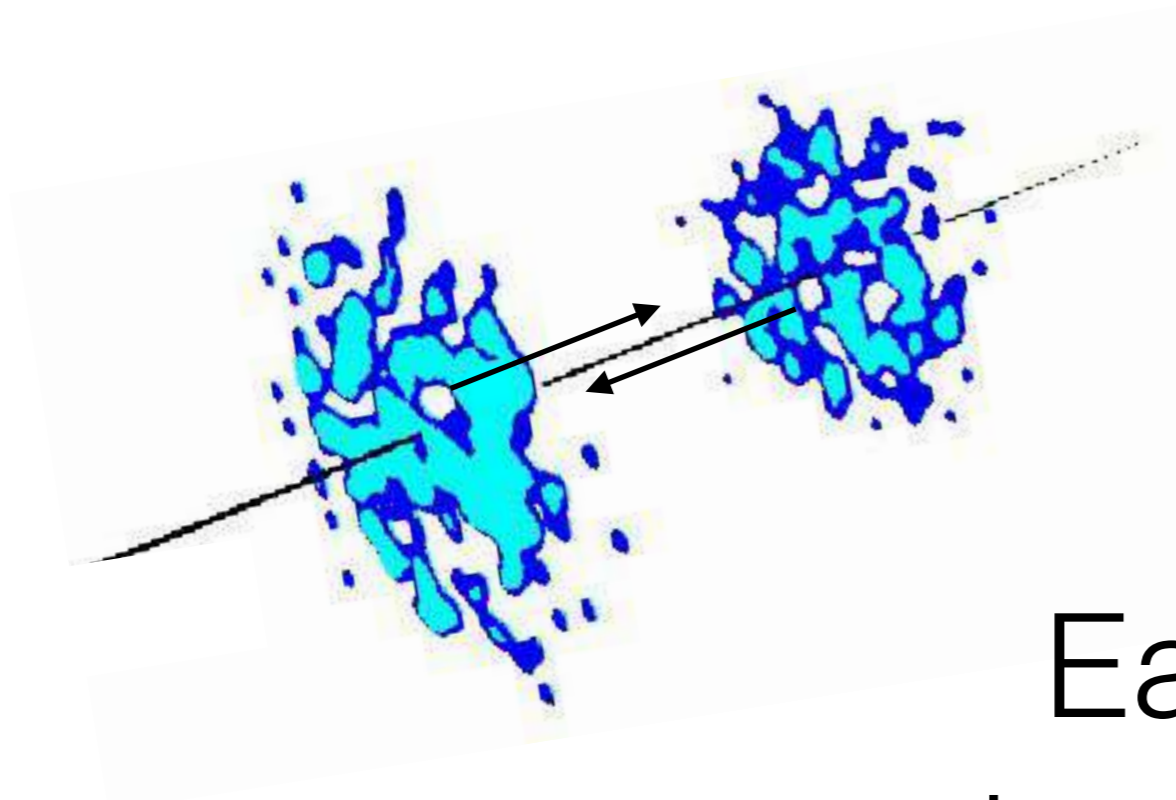
Effects of including pre-equilibrium dynamics on experimental observables generally small

Controlled extraction of QGP transport properties without large uncertainties from early times

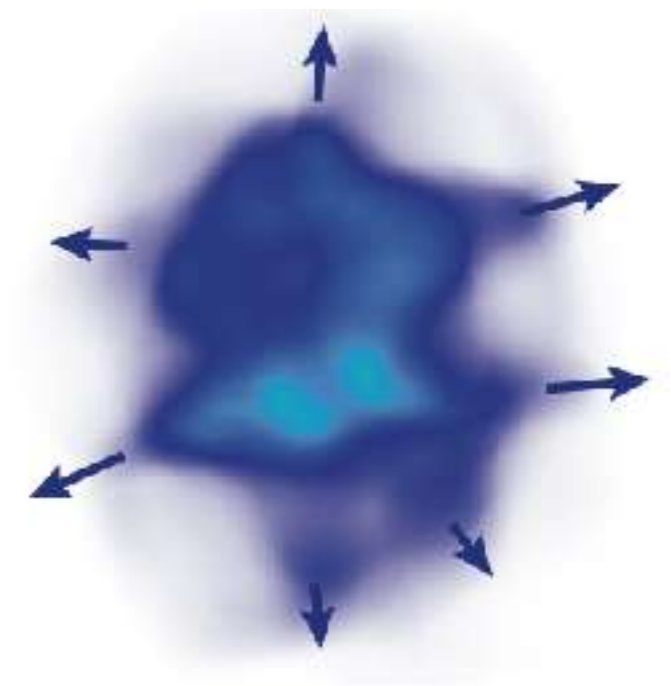
Difficult to gain experimental access to early time non-equilibrium dynamics in A+A collisions



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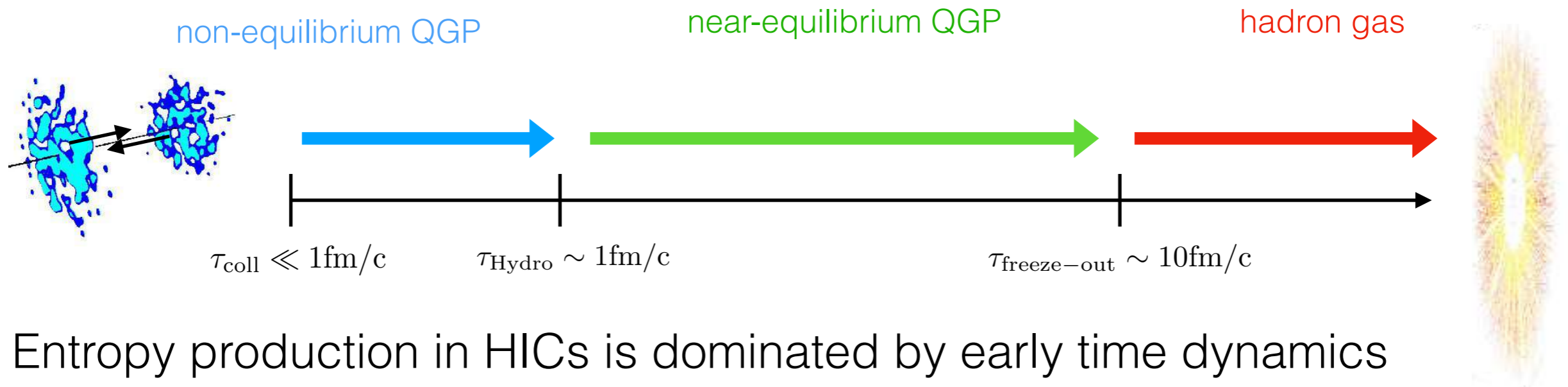
**BOOM!**



Early time dynamics & entropy production in HIC

# Sensitivity to Initial state

Entropy production occurs only when system is significantly out-of-equilibrium



Entropy production in HICs is dominated by early time dynamics and directly accessible by measurement of  $dN_{\text{ch}}/d\eta$

Schematically:

$$\left\langle \frac{dE_{\perp}}{d\eta} \right\rangle_{\tau_{\text{coll}}} \xrightarrow{\text{initial entropy production}} \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\text{Hydro}}} \xrightarrow{\text{(nearly) isentropic expansion}} \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\text{freeze-out}}} \approx \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\text{Hydro}}} \xrightarrow{\text{freeze-out}} \left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle \approx \left\langle \frac{S}{N_{\text{ch}}} \right\rangle \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\text{freeze-out}}}$$

Based on insights from non-equilibrium studies, can now make relation between  $dE/d\eta$  and  $dN_{\text{ch}}/d\eta$  explicit

# Early time dynamics & entropy production

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Non-equilibrium initial state better characterized in terms of energy density ( $e$ ), better to use entropy density ( $s$ ) only once system is close to equilibrium

Energy momentum tensor assumes average form  $T^{\mu\nu} = \text{diag}(e, p_T, p_T, p_L)$

Evolution of energy density is governed by conservation equation

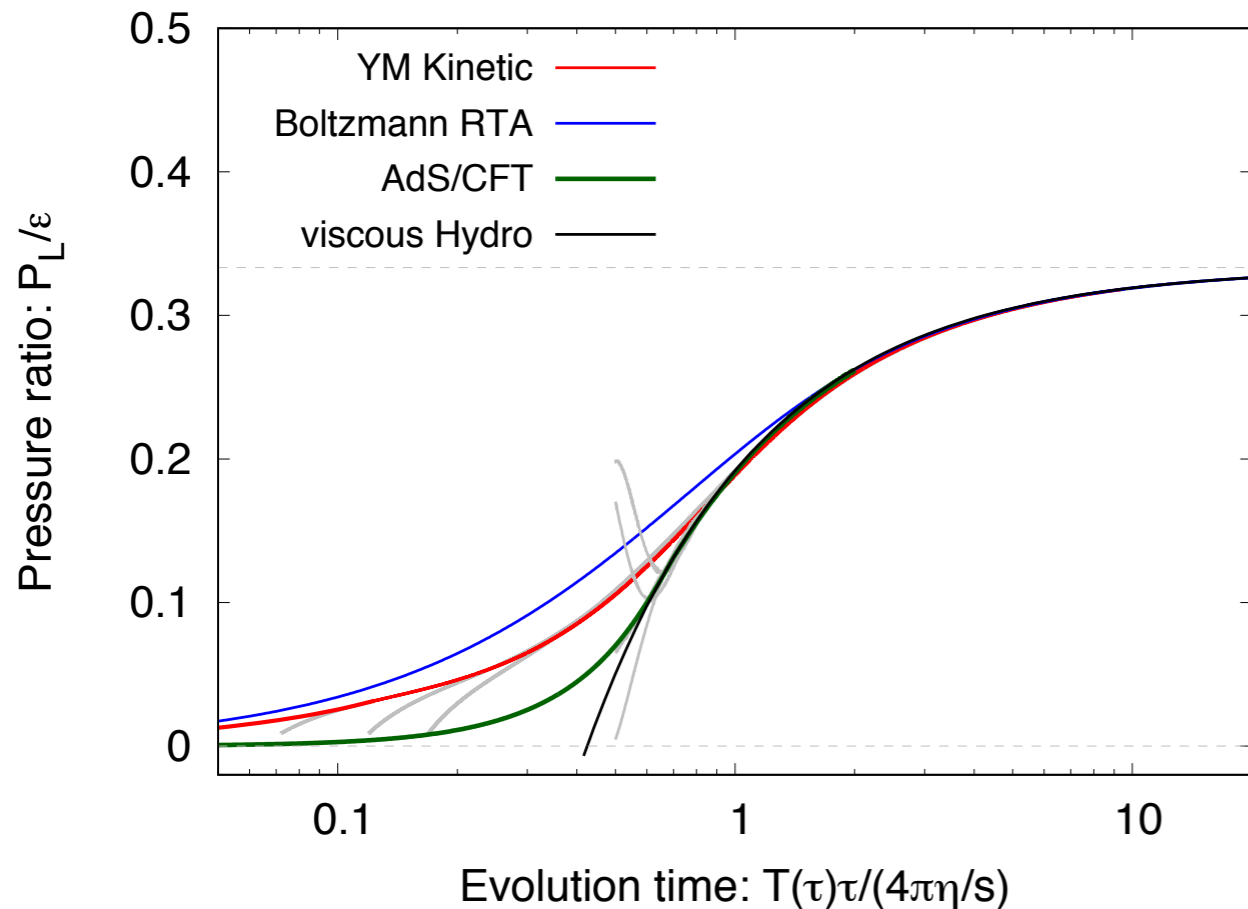
$$\partial_\tau \epsilon = -\frac{\epsilon}{\tau} - \frac{P_L}{\tau}$$

need equation of state/constitutive relation  $P_L(e, \tau)$  to obtain evolution

Generally no constitutive equations out-of-equilibrium and  $P_L/e$  depends on microscopics



# Hydrodynamic attractors



Non-equilibrium evolution towards hydrodynamics described by “hydrodynamic attractor”

Heller, Spalinski PRL 115 (2015) no.7, 072501

Effective constitutive relations for far-from-equilibrium systems

$$\frac{P_L}{\epsilon} = f(\epsilon, \tau) = f\left(\tilde{w} = \frac{T(\tau)\tau}{4\pi\eta/s}\right)$$

Despite clear microscopic differences the macroscopic features of the evolution are remarkably similar when compared in meaningful fashion

Universality at early times (free-streaming)  
& late times (visc. hydrodynamics)

# Entropy production in HICs

Effective constitutive equation allows to obtain evolution of energy density from

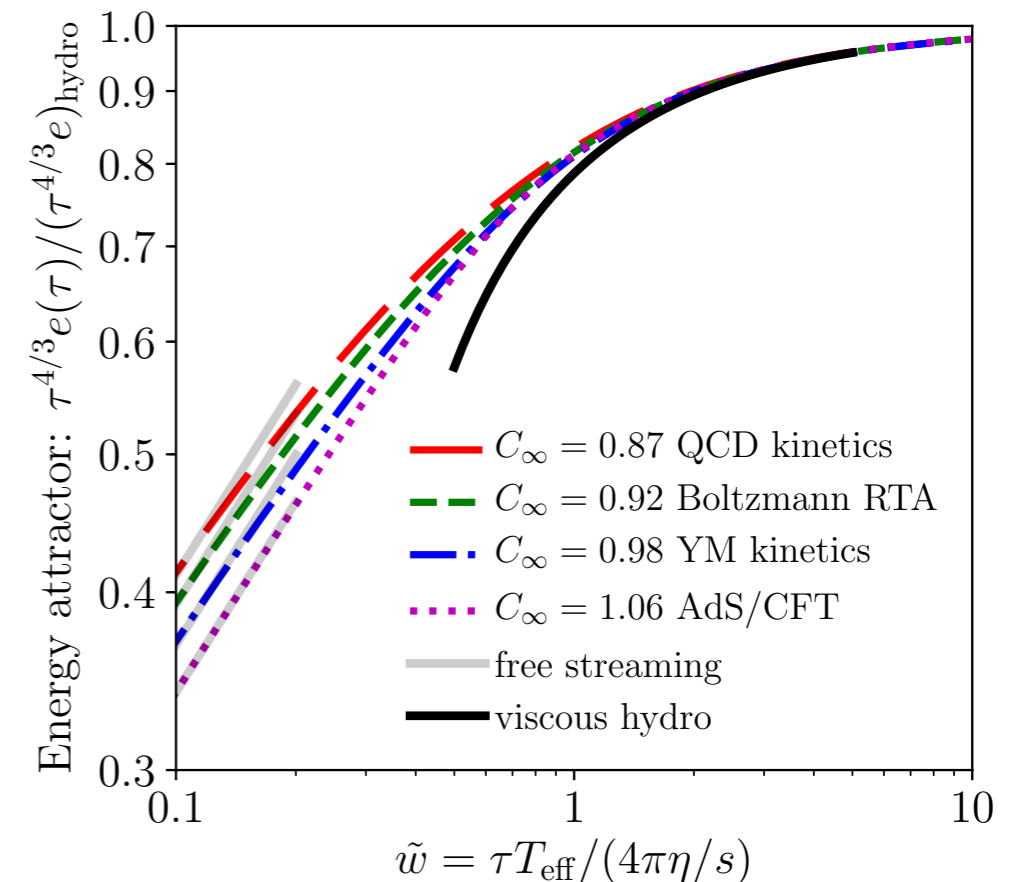
$$e(\tilde{w}_\tau) = e(\tilde{w}_0) \exp \left( - \int_{\tilde{w}_0}^{\tilde{w}_\tau} \frac{d\tilde{w}}{\tilde{w}} \frac{1 + P_L(\tilde{w})/e(\tilde{w})}{\frac{3}{4} - \frac{1}{4} P_L(\tilde{w})/e(\tilde{w})} \right)$$

Once QGP is close to equilibrium can use thermodynamic relations  $s=(e+p)/T$  and EOS once QGP to calculate entropy

$$(s\tau)_{\text{hydro}} = \frac{4}{3} C_\infty^{3/4} \left( 4\pi \frac{\eta}{s} \right)^{1/3} \left( \frac{\pi^2}{30} \nu_{\text{eff}} \right)^{1/3} (e\tau)_0^{2/3}$$

Since overall entropy is approx. conserved during hydro expansion charged particle multiplicity at freeze-out directly determined

$$\frac{dN_{\text{ch}}}{d\eta} \approx A_\perp (s\tau)_{\text{hydro}} \frac{N_{\text{ch}}}{S}$$

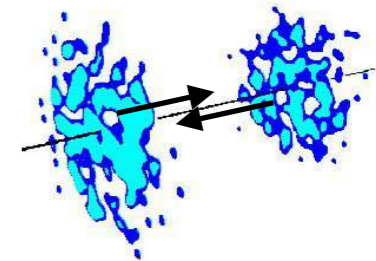


# Entropy production in HICs

Based on macroscopic considerations one can establish one-to-one correspondence between initial state energy density and charged particle multiplicity including all relevant pre-factors



$$\frac{dN_{\text{ch}}}{d\eta} \approx \frac{4}{3} \left( \frac{N_{\text{ch}}}{S} \right) A_{\perp} C_{\infty}^{3/4} \left( 4\pi \frac{\eta}{s} \right)^{1/3} \left( \frac{\pi^2}{30} \nu_{\text{eff}} \right)^{1/3} (\epsilon\tau)_0^{2/3}$$



Sensitivities/Uncertainties:

**Equilibrium properties:**  $N_{\text{ch}}/S \sim 7.5$ ,  $\nu_{\text{eff}} \sim 40$  approximately known

**Non-equilibrium/transport properties:**

$C_{\infty} \sim 0.95 \pm 0.15$  surprisingly well constraint

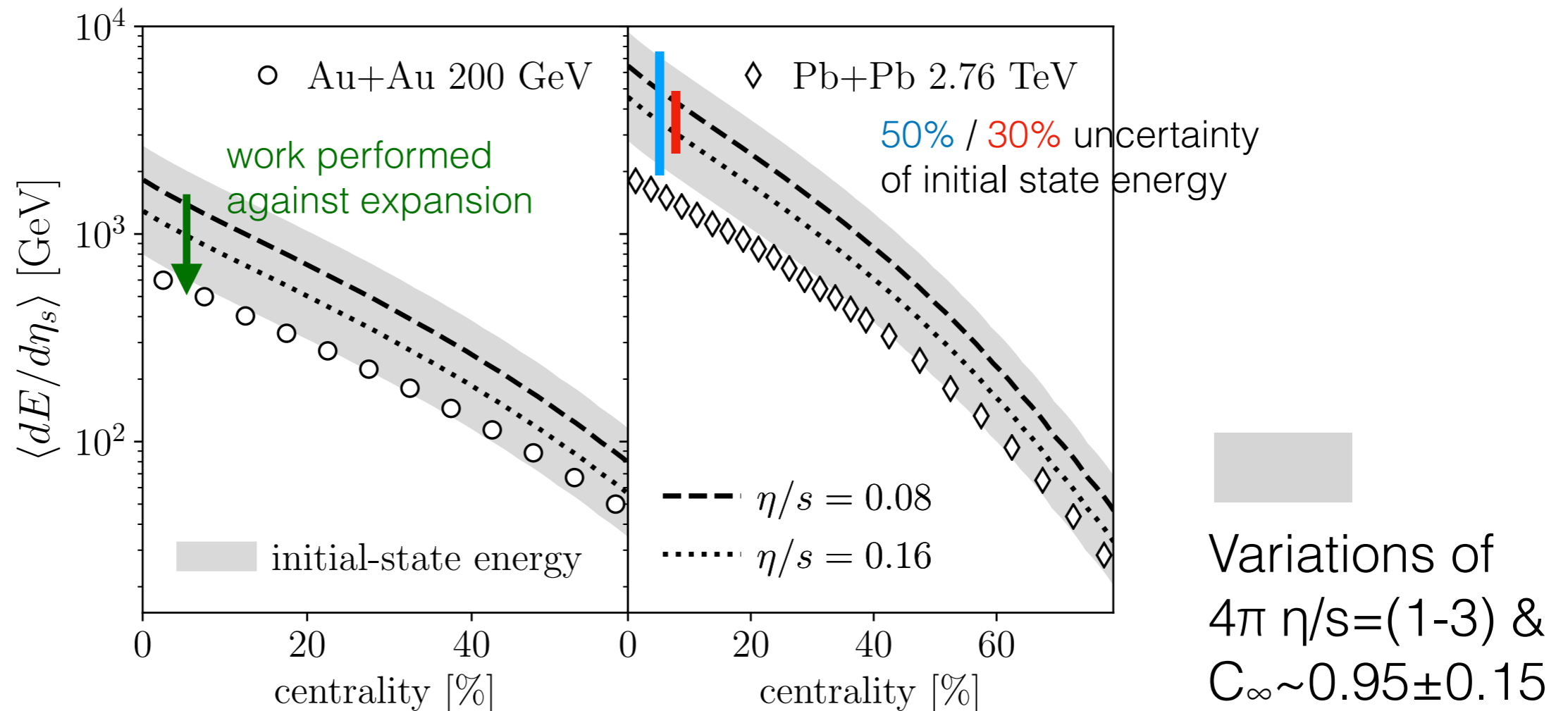
$4\pi \eta/s \sim (1-3)$  not well constraint in relevant temperature range ( $T \sim 4T_c$ )

**Initial state energy density:**

$(\epsilon\tau)_0$  significant uncertainties from small-x TMDs  
and perturbative corrections

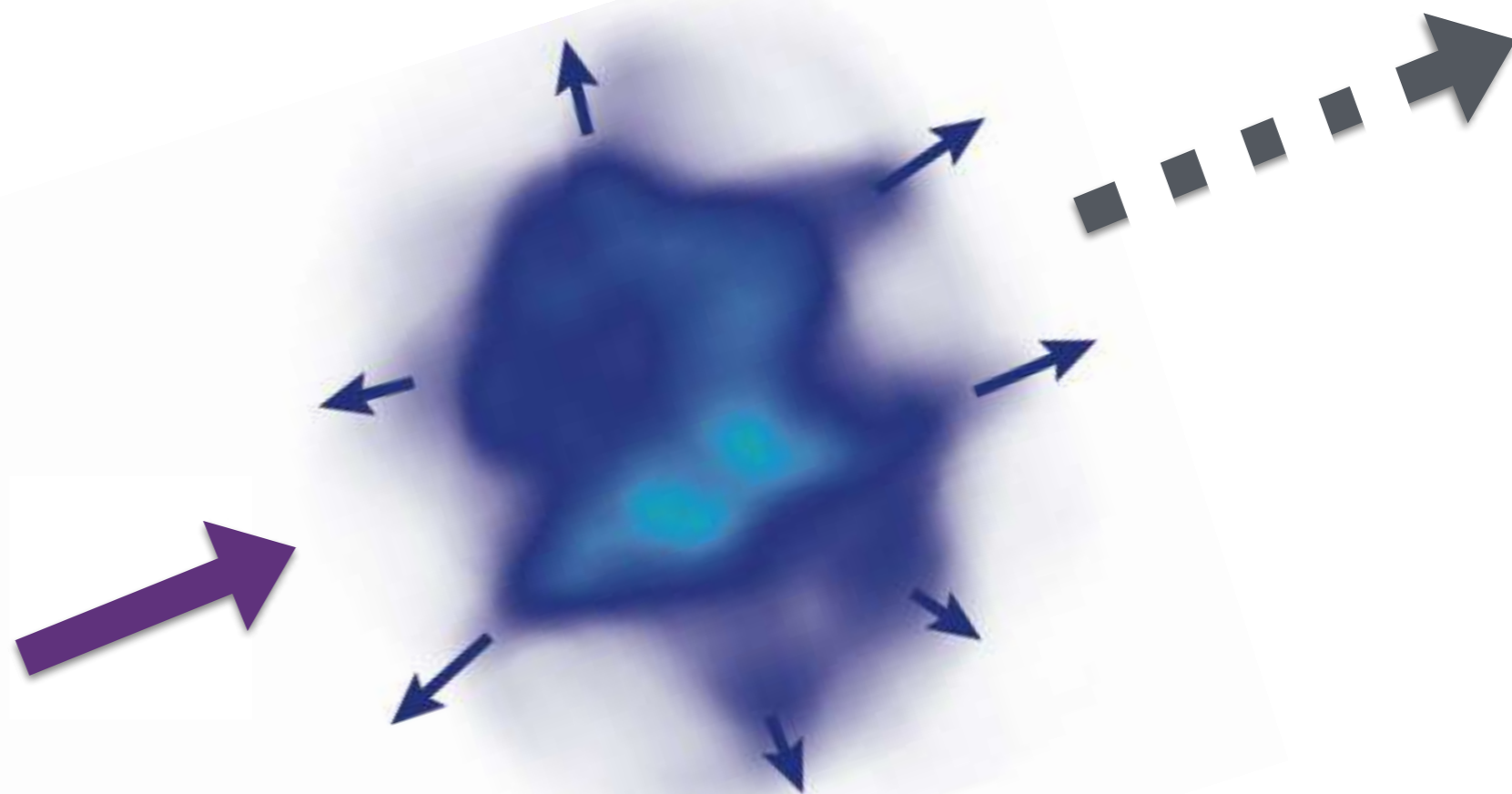
# Initial state in HICs

Extraction of initial state energy  $dE/d\eta$  from final measured multiplicity



Sensitivity to  $\eta/s$  and  $(e\tau)_0$  can be exploited to obtain combined constraints on initial state & transport properties

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Conclusions & Outlook

Significant progress in understanding non-equilibrium QCD dynamics wrt onset of hydrodynamic behavior & equilibration of QGP

- Established thermalization of patterns of QCD(-like) plasmas
- Viscous hydrodynamics  $\neq$  local thermal equilibrium

-> Implications for non-equilibrium plasmas in early universe?

Studies have reached a level where they can connect properties of initial state with experimental measurements and useful phenomenological results can be obtained

KoMPoST: Eff. macroscopic description of non-Eq QCD in heavy-ion collisions including transverse dynamics

Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302; PRC 99 (2019) no.3, 034910

Establishing link between initial state and finale state important to connect Heavy-Ion Physics to Physics at EIC & Hadron Colliders

Small-x gluon (G)TMDs determine initial state energy density in A+A as well as (forward) di-jet production in p+p/A or e+p/A

So far investigations have mostly focused on bulk equilibration in A+A collisions

Electro-magnetic & hard probes, small systems (p+p/A), non-equilibrium effects in jet-medium interactions provide promising avenues to further explore non-equilibrium QCD in heavy-ion collisions

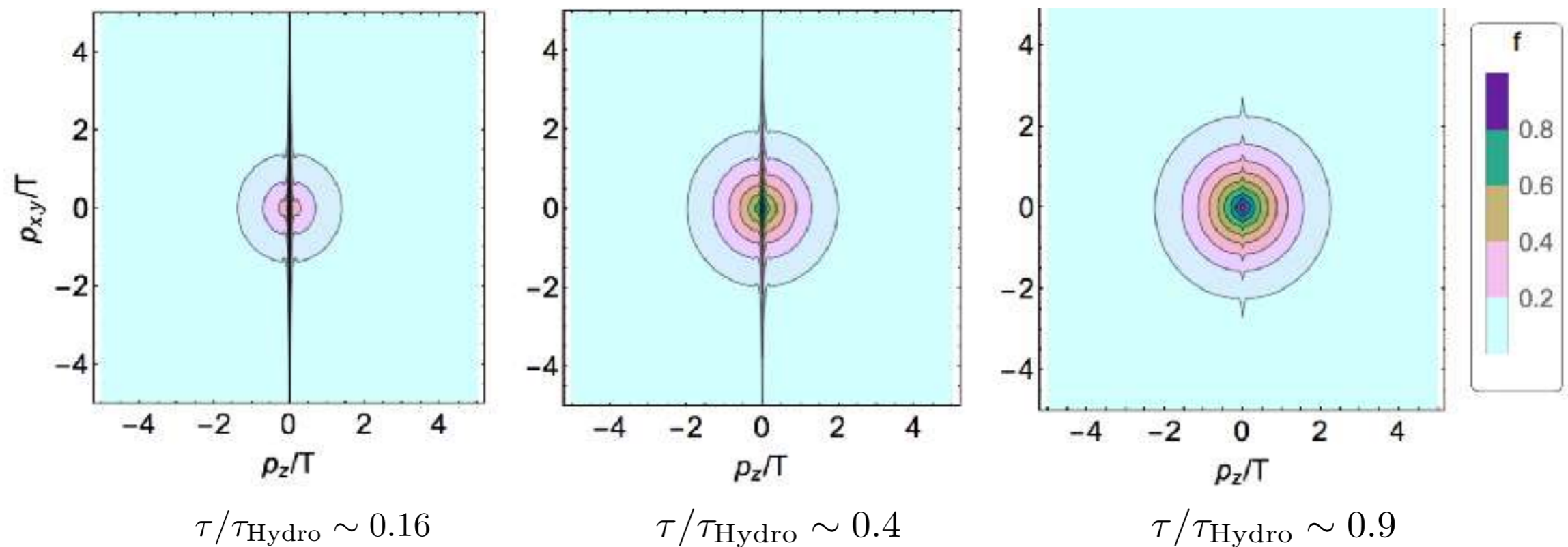
Backup

# Non-equilibrium towards hydrodynamics

## Evolution of homogenous boost invariant system in Boltzmann RTA

M. Strickland JHEP 1812 (2018) 128

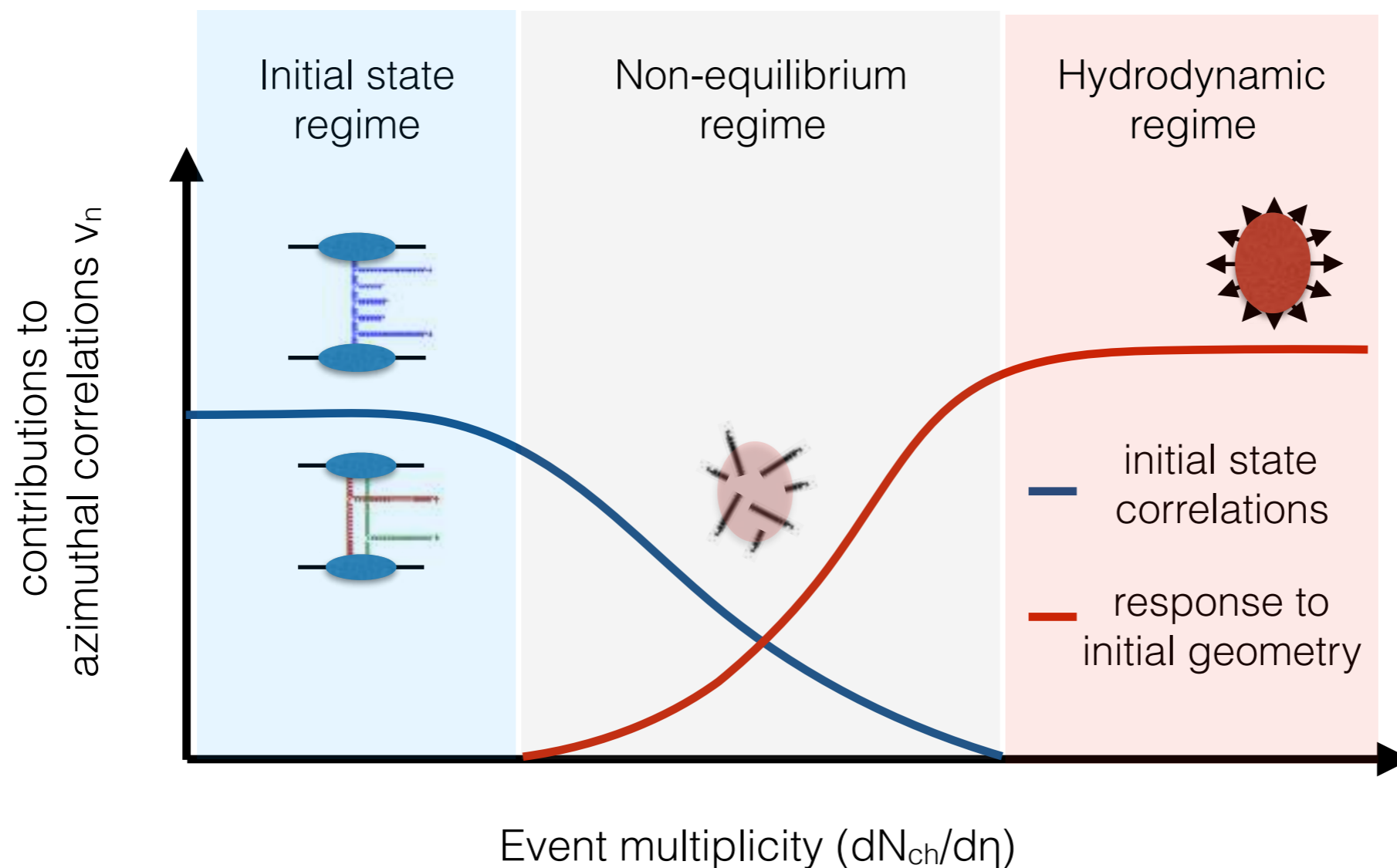
$$p^\mu \partial_\mu f(x, p) = -\frac{p^\mu u_\mu(x)}{\tau_R(x)} \left[ f(x, p) - f_{\text{eq}}(x, p) \right]$$





# Small systems (p/d/He3+p/A)

Shorter life-time of the system ( $\sim R$ ) increases sensitivity to initial state and non-equilibrium dynamics



Some additional control parameters ( $p_T, \dots$ ) besides multiplicity  $dN_{ch}/d\eta$

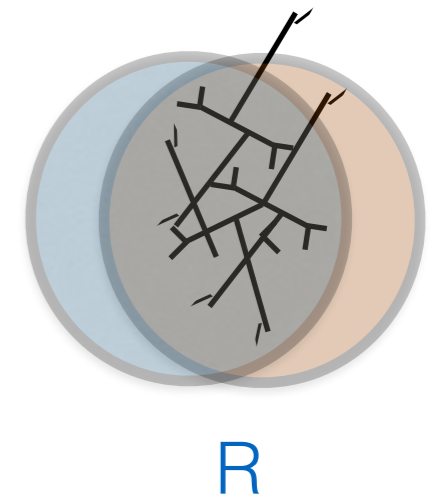
# Hydrodynamic QGP in small systems?

Non-equilibrium dynamics of QGP controlled by the ratio of hydrodynamization time ( $\tau_{\text{Hydro}}$ ) and system size ( $R$ )

$\tau_{\text{Hydro}} \gg R$ : insufficient time to achieve equilibrated QGP

$\tau_{\text{Hydro}} \ll R$ : long lived hydrodynamic QGP phase

Based on estimates of the hydrodynamization time critical multiplicity can be estimated



$$\frac{\tau_{\text{Hydro}}}{R} \simeq \left( \frac{4\pi(\eta/s)}{2} \right)^{\frac{3}{2}} \left( \frac{dN_{\text{ch}}/d\eta}{63} \right)^{-\frac{1}{2}} \Leftrightarrow \frac{dN_{\text{ch}}}{d\eta} \Big|_{\text{crit}} = 63 \left( \frac{\eta/s}{2/4\pi} \right)^3 \left( \frac{\tau_{\text{Hydro}}}{R} \right)^{-2}$$

where  $\frac{dN_{\text{ch}}}{d\eta} \Big|_{\text{min. bias}}^{p+p \ 7\text{TeV}} \sim 6$  ,  $\frac{dN_{\text{ch}}}{d\eta} \Big|_{\text{min. bias}}^{p+Pb \ 5.02\text{TeV}} \sim 16$  ,  $\frac{dN_{\text{ch}}}{d\eta} \Big|_{0-5\%}^{Pb+Pb \ 2.76\text{TeV}} \sim 1600$

Experimental results in small systems mostly fall into the regime  $\tau_{\text{Hydro}}/R \sim 1$  dominated by non-equilibrium phase

# Pre-flow ( $T^{\tau i}$ ) & Viscous corrections ( $T^{ij}$ )

Gradients in  $x_{\tau}$  induce off-diagonal components of  $T^{\mu\nu}$

**Universal pre-flow:** early expansion of matter insensitive to microscopic details

Pratt, Vredevoogd PRC79 (2009) 044915

Kurkela, Mazeliauskas, Paquet, SS, Teaney PRC 99 (2019) no.3, 034910

$$\frac{T^{\tau i}(\tau, \mathbf{x})}{T^{\tau\tau}(\tau, \mathbf{x})} \approx -\frac{(\tau - \tau_0)}{2} \frac{\partial^i T^{\tau\tau}(\tau_0, \mathbf{x})}{T^{\tau\tau}(\tau, \mathbf{x})}$$

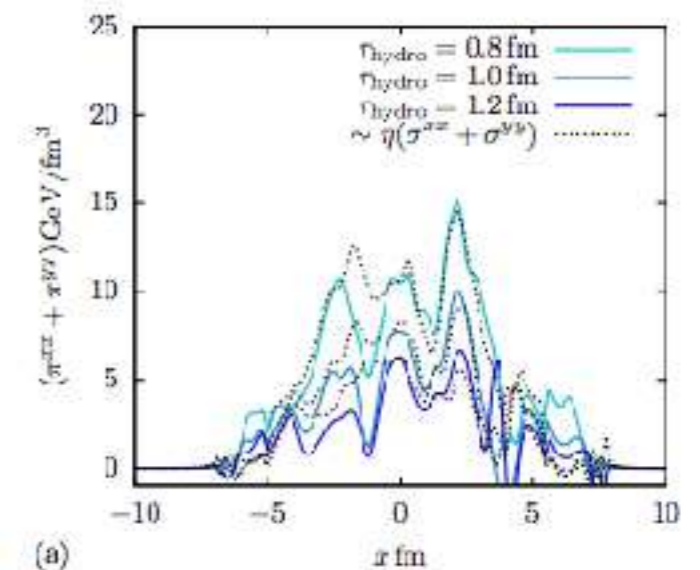
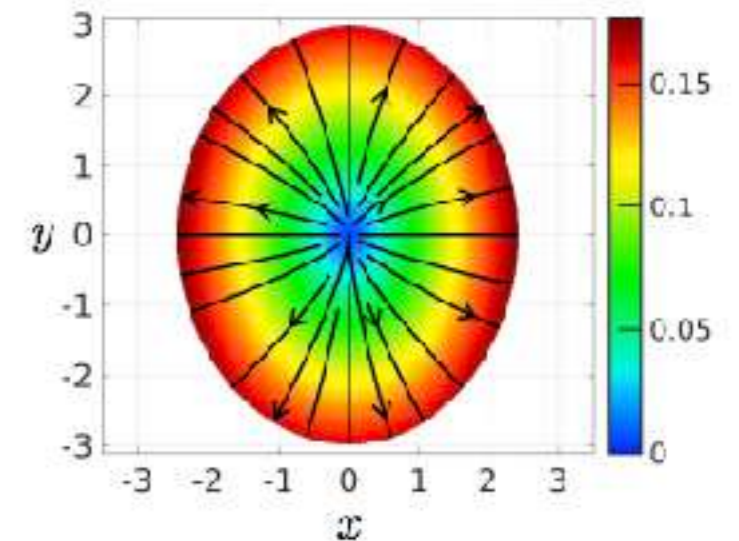
not indicative of onset of hydrodynamic flow

**Viscous corrections:** long wavelength components satisfy hydrodynamic constitutive equations approximately at  $\tau = \tau_{\text{Hydro}}$

$$\pi^{\mu\nu}(\tau_{\text{Hydro}}, \mathbf{x}) \approx \pi_{\text{NS}}^{\mu\nu}$$

Pre-flow package KoMPoST provide unified description of pre-equilibrium evolution of  $T^{\mu\nu}$  based on linear response to gradients

P. Chesler JHEP 1603 (2016) 146



# KoMPoST

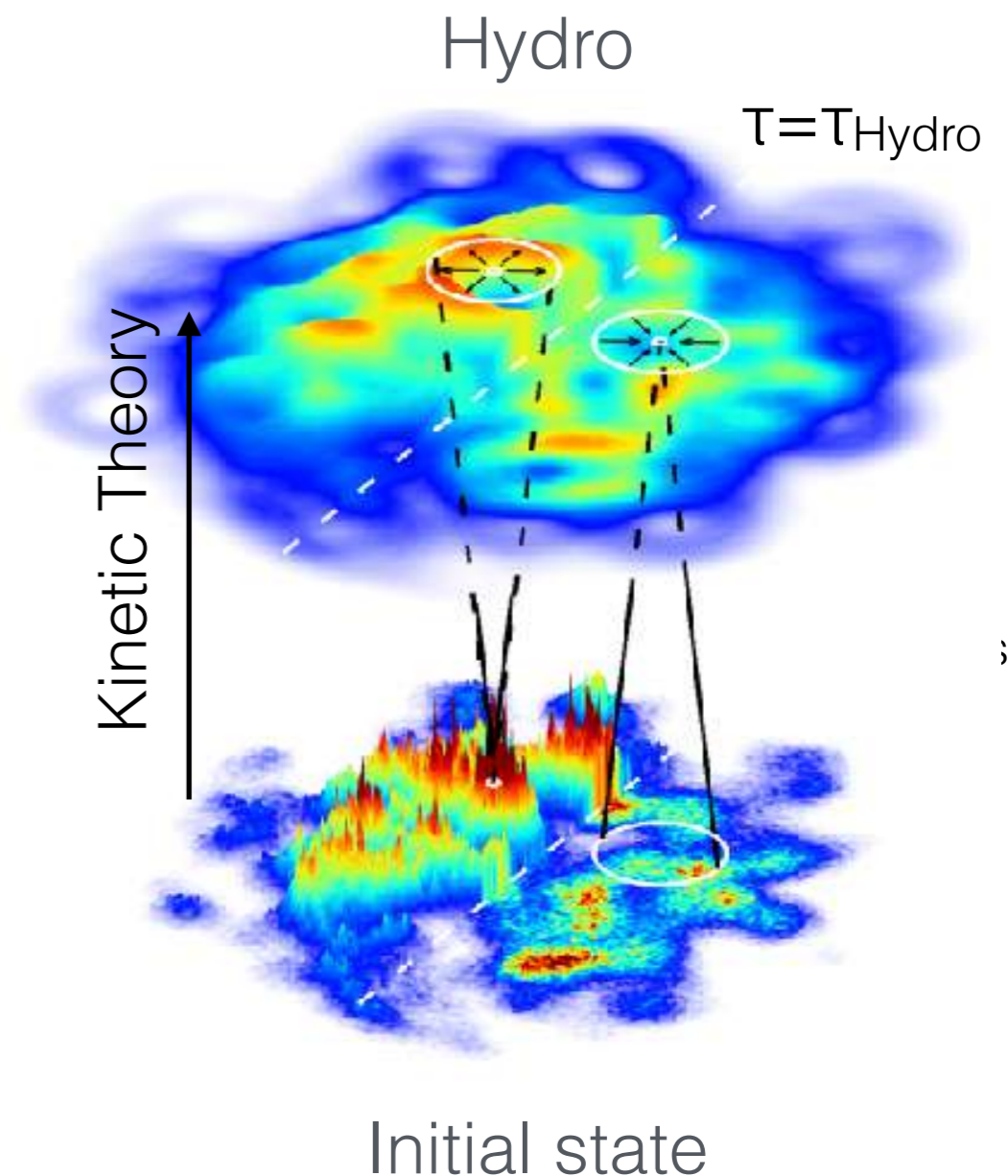
## Macroscopic description of pre-equilibrium dynamics

Exploit memory loss to use macroscopic degrees of freedom  $T^{\mu\nu}$  for description of pre-equilibrium dynamics

Separation of scales between evolution time  $\tau_{\text{Hydro}}$  and typical size scale of gradients  $R_A$

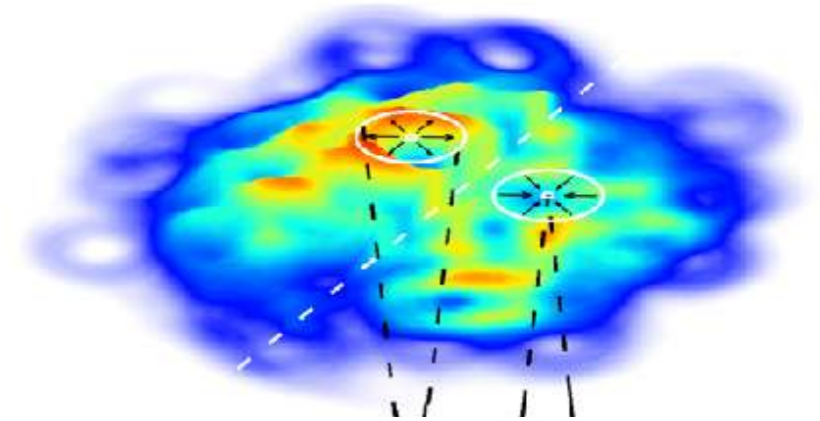
Decomposing  $T^{\mu\nu}(x)$  into a local average  $T^{\mu\nu}_{\text{BG}}(x)$  and fluctuations  $\delta T^{\mu\nu}(x)$ , are small on scales  $c(\tau_{\text{Hydro}} - \tau_0)$ , they can be treated in linear response theory

Keegan, Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171  
Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961



# Non-equilibrium linear response

Energy-momentum tensor at  $\tau_{\text{Hydro}}$  can be reconstructed directly from initial conditions

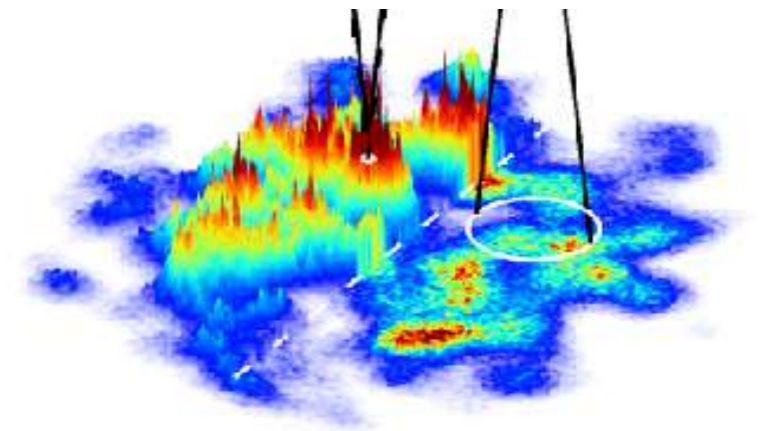


$$T^{\mu\nu}(\tau, \mathbf{x}) = T_{BG}^{\mu\nu}(\tau) + \int_{\odot} G_{\alpha\beta}^{\mu\nu}(\tau, \tau_0, \mathbf{x}, \mathbf{x}_0) \delta T^{\alpha\beta}(\tau_0, \mathbf{x}_0)$$

non-equilibrium evolution  
of (local) average background

non-equilibrium Greens function  
of energy-momentum tensor

Instead of event-by-event Monte-Carlo, effective kinetic theory simulations performed only once to compute evolution of background  $T_{BG}^{\mu\nu}$  and Greens functions  $G_{\alpha\beta}^{\mu\nu}$

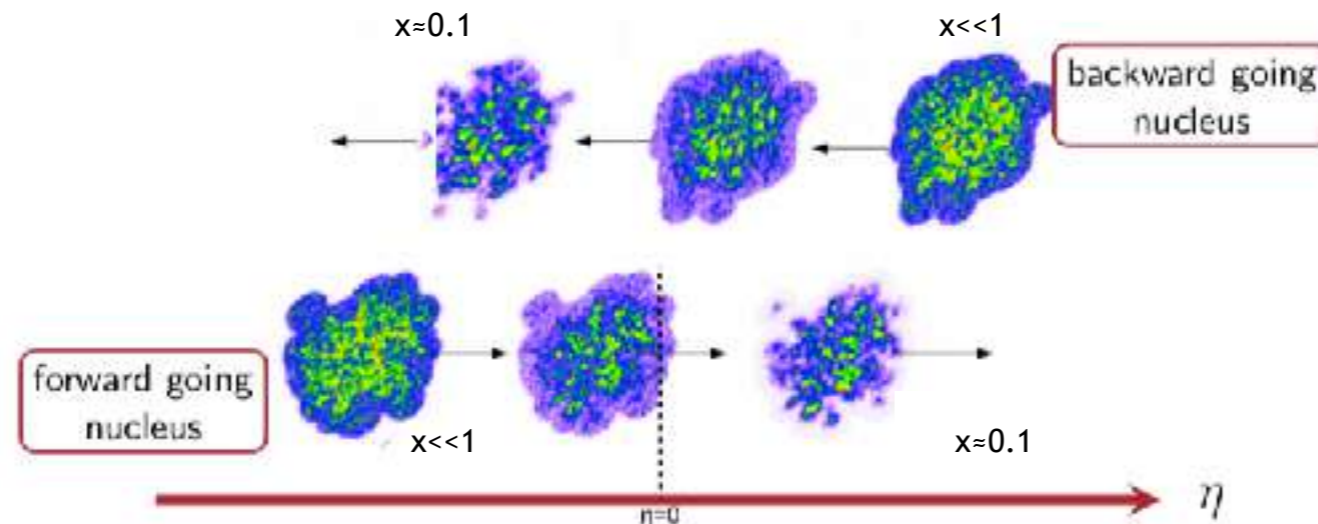


# Longitudinal dynamics ( $\eta$ )

Boost invariance ( $\eta \sim 0$ ) on average is reasonable assumption for symmetric high-energy collisions

Different models for long. fluctuations based on different degrees of freedom **partons, nucleons, strings, ...**

High-energy: Energy deposition at different  $\eta \sim y$  simultaneous probes gluon distribution at different  $x$



SS Schenke PRC 94 (2016) no.4, 044907, McDonald, Jeon, NPA 982 (2019) 239-242, Ipp, Mueller PLB 771 (2017) 74-79

Space-time dynamics of baryon stopping at forward rapidities can also be addressed within small- $x$  framework

McLerran, SS, Sen PRD99 (2019) no.7, 074009