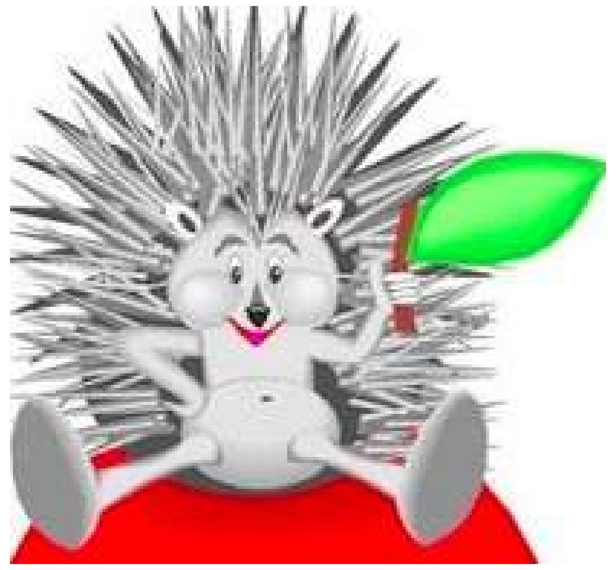
The semiclassical ensembles of instanton-dyons describe deconfinement and chiral phase transitions in the usual and deformed QCD

Edward Shuryak
Center for Nuclear Theory
Stony Brook University

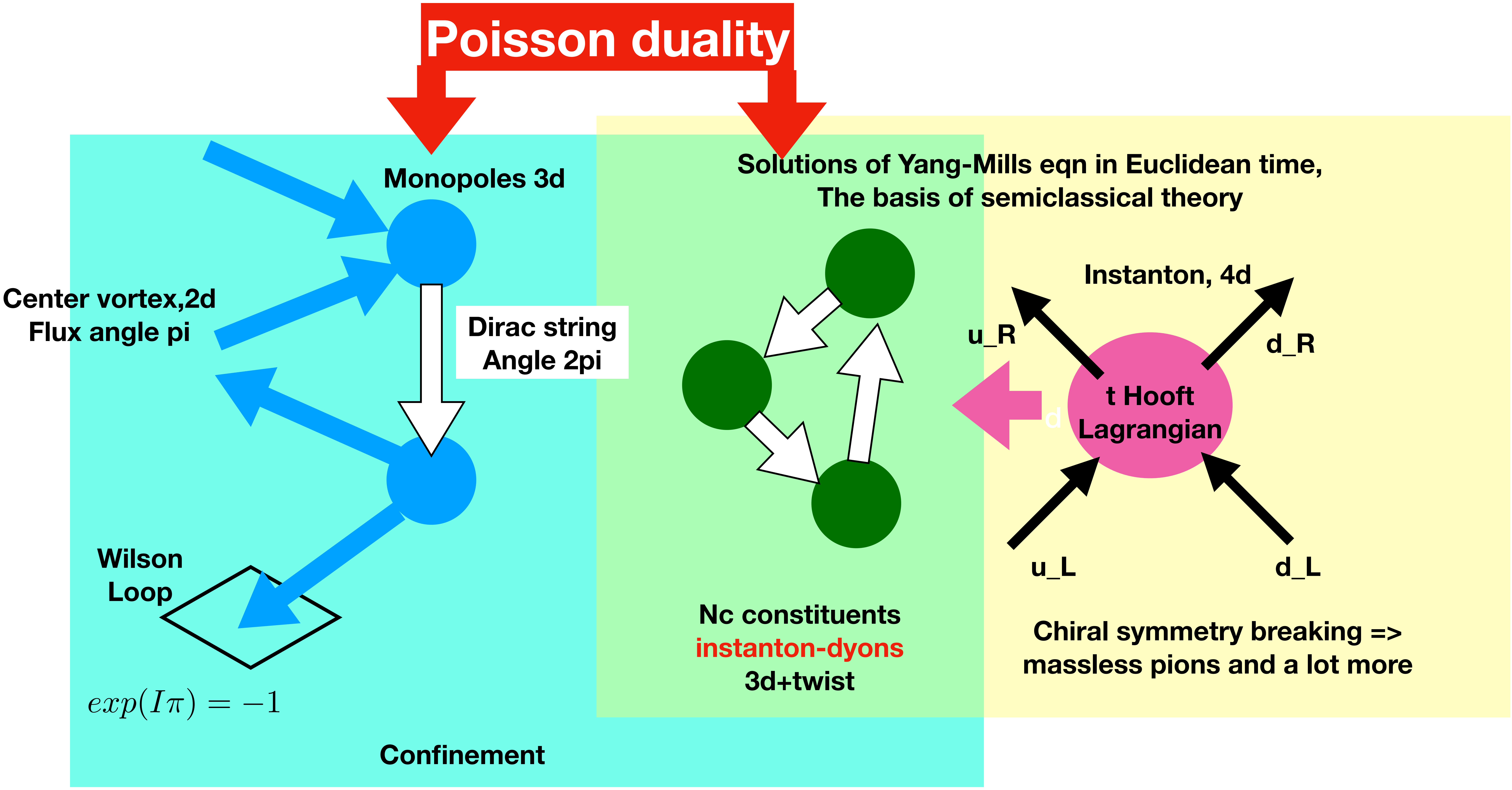
Theory coll., Sept.8, 2021

collaborators
Dallas DeMartini
Rassmus Larsen
Sayantan Sharma



outline

- map of gauge topology
- VEV of Polyakov line and the instanton-dyons
- Dirac zero and quasizero fermionic states on the lattice
- Deconfinement transition,
- QCD deformation via Polyakov line operators
- Chiral symmetry breaking
- QCD deformation via quark periodicity phases
- Poisson duality Instanton-dyons <=> Monopoles



Poisson duality

Monopoles 3d

Center vortex,2d
Flux angle pi

Dirac string
Angle 2pi

Wilson
Loop

$exp(I\pi) = -1$

Confinement

Solutions of Yang-Mills eqn in Euclidean time,
The basis of semiclassical theory

Nc constituents
instanton-dyons
3d+twist

Instanton, 4d

u_R

d_R

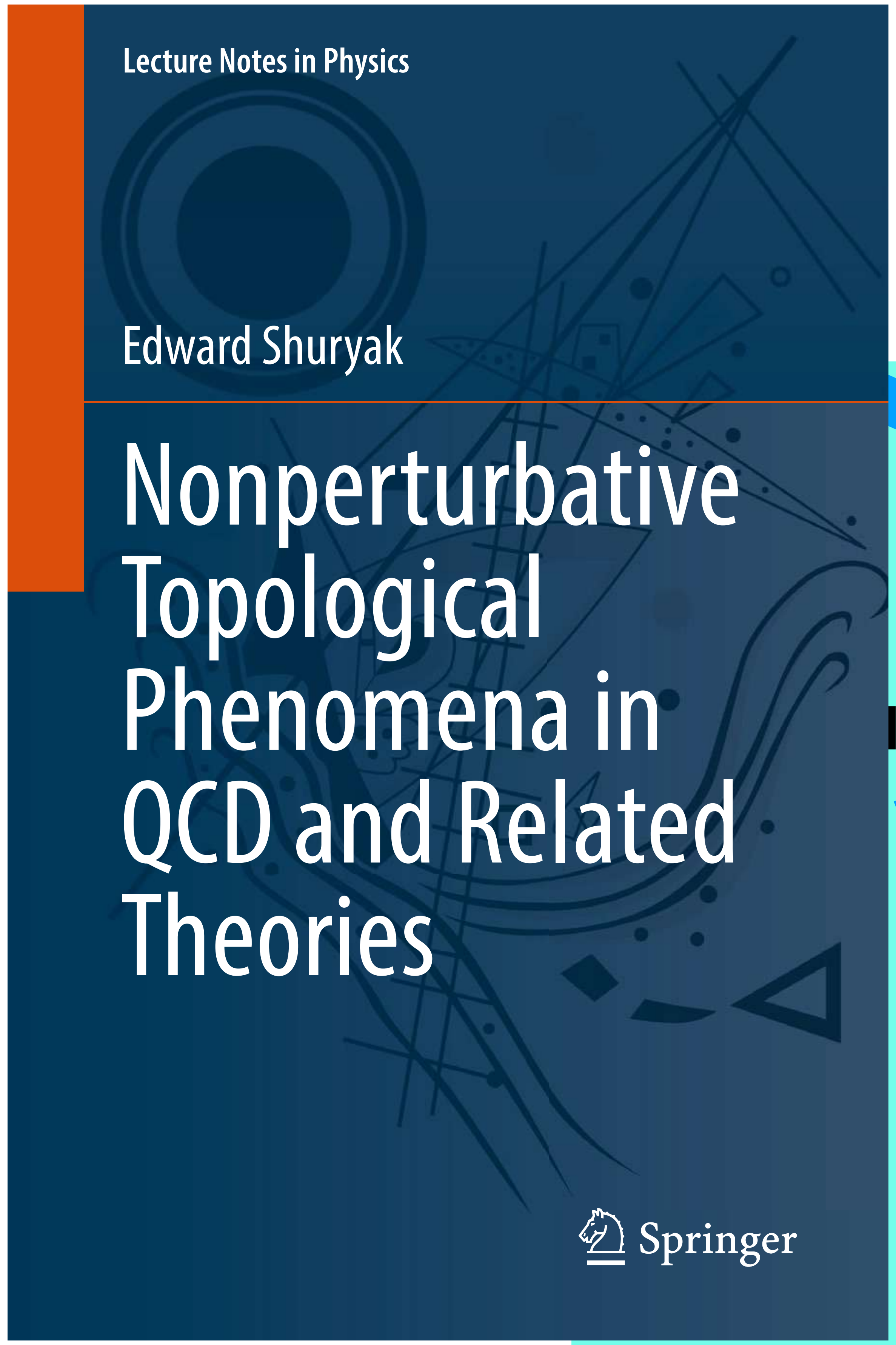
d

t Hooft
Lagrangian

u_L

d_L

Chiral symmetry breaking =>
massless pions and a lot more

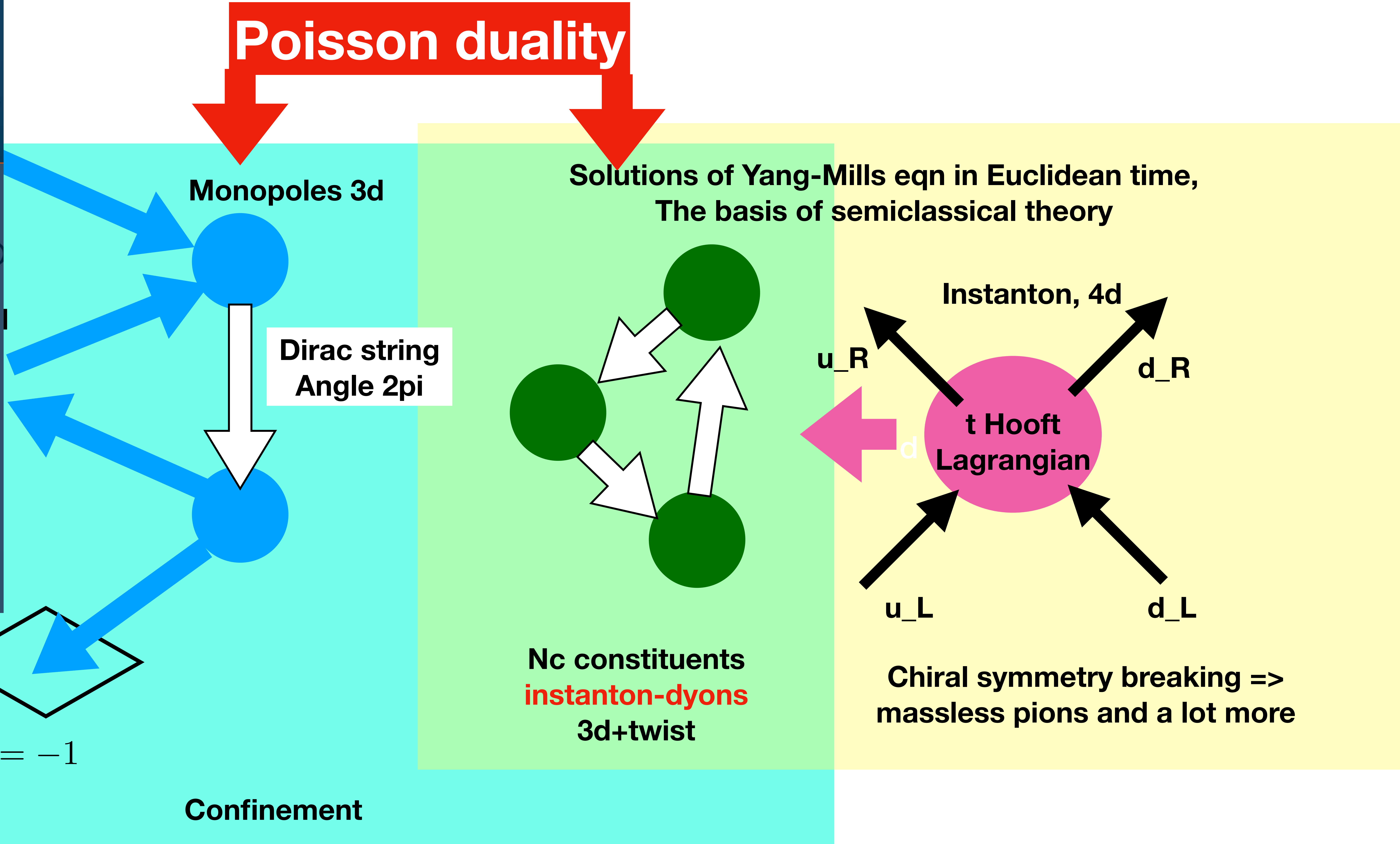


Lecture Notes in Physics

Edward Shuryak

Nonperturbative Topological Phenomena in QCD and Related Theories

Springer



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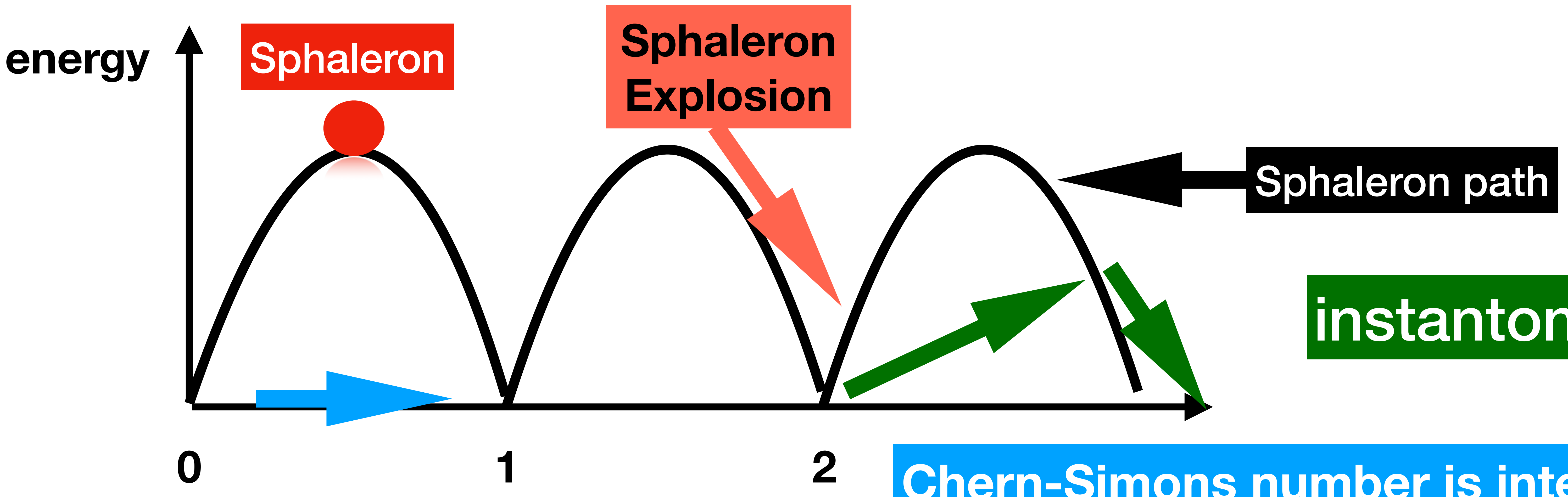
Instantons

Terminology of the topological landscape



instanton-sphaleron process

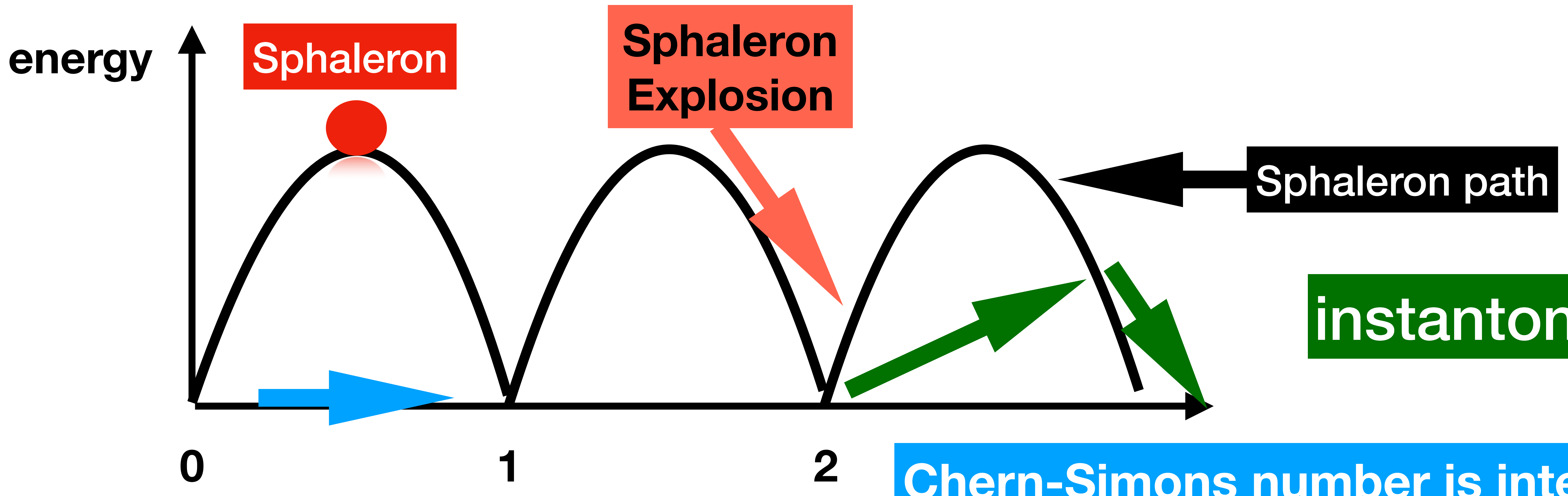
Chern-Simons number is integer at the “valleys”

Instanton is Tunneling at Zero energy

Sphaleron is static purely magnetic object
The name in Greek means
“ready to fall” (Klinkhamer and Manton)

Sphaleron path consists of configurations
Which are minima in all directions in Hilbert space
except one
Like streams going from mountain tops
to the bottom of the valley

Terminology of the topological landscape



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We do have analytic results for
All of them
In pure gauge theory
Which is not widely known

historic introduction

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961). doi:10.1103/PhysRev.122.345

NJL introduced the chiral symmetry and G large enough to break it spontaneously “constituent quark mass” like a gap in superconductors

NJL model:

$$G[\vec{\pi}^2+\sigma^2]$$

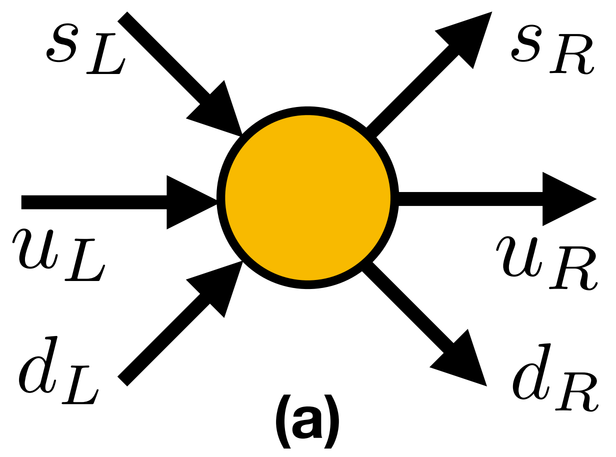
$$\vec{\pi}=(\bar{q}\vec{\tau}\gamma_5 q)$$

$$\sigma=(\bar{q}q)$$

$$G(|p|>\Lambda)=0$$

gauge topology, tunneling
Instantons: BPST and t’ Hooft, 1975-76
new effective Lagrangian
it violates U(1) chiral symmetry
Turning left-handed to right handed

Instanton liquid model (ES 1982) instead of G and Lambda of NJL another two parameters their values are such that chiral symmetry gets broken



$$\vec{\delta}=(\bar{q}\vec{\tau}q)$$

$$\eta'=(\bar{q}\gamma_5 q)$$

$$G[\vec{\pi}^2+\sigma^2-\vec{\delta}^2-\eta'^2]$$

$$n_{inst}\approx 1\,fm^{-4}$$

$$\rho\approx 1/3\,fm$$

repulsion

Interacting instanton liquid model 1990s summed all orders of ’t Hooft vertex calculated correlation functions good description of chiral symmetry breaking no confinement

Instantons in the QCD VACUUM and HADRONS

"Instanton liquid model" , Shuryak, 1981
n=1/fm^4, rho=1/3 fm => chiral symmetry breaking

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hep-ph/0008048.

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A snapshot of lattice G-dual G

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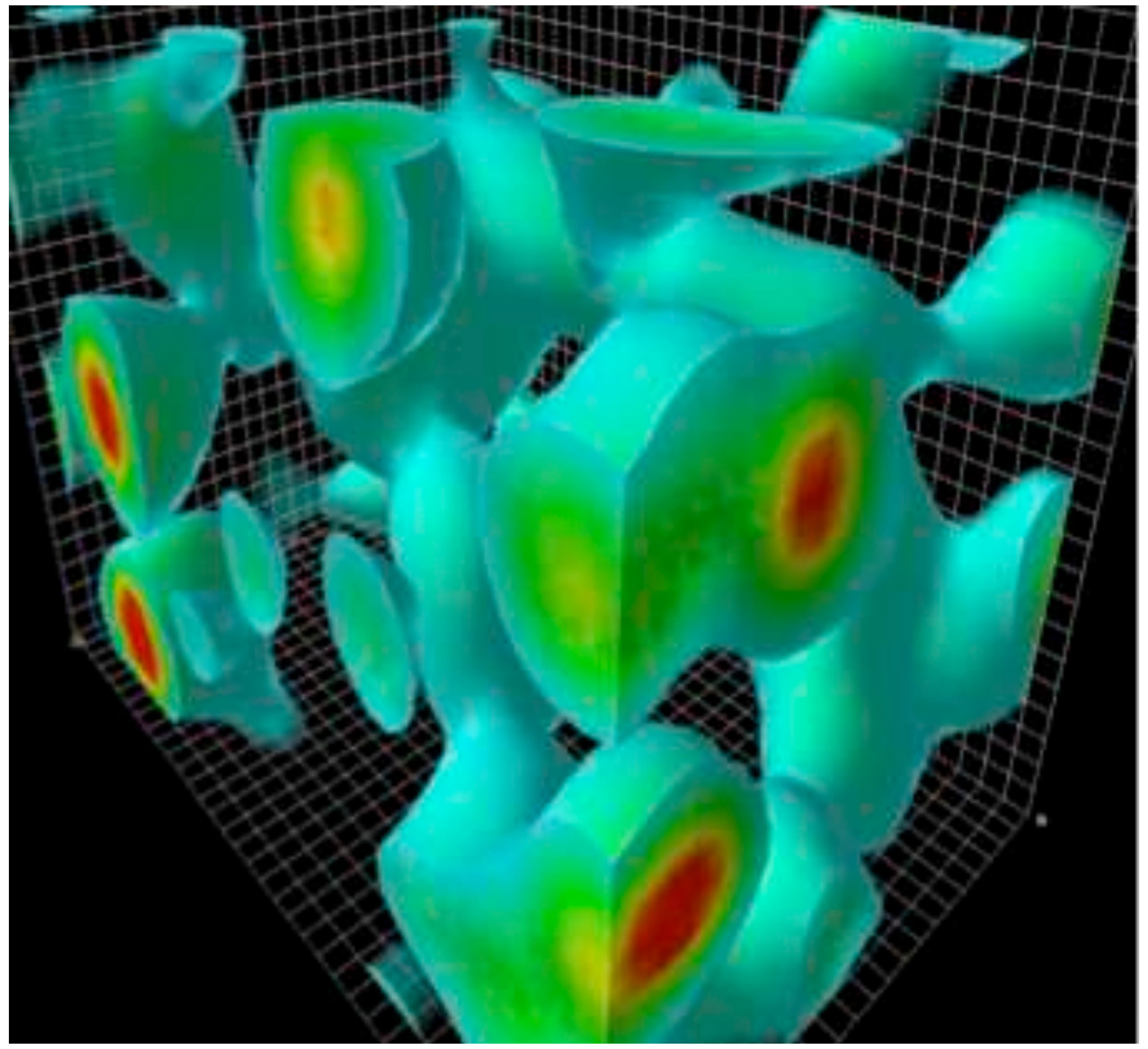

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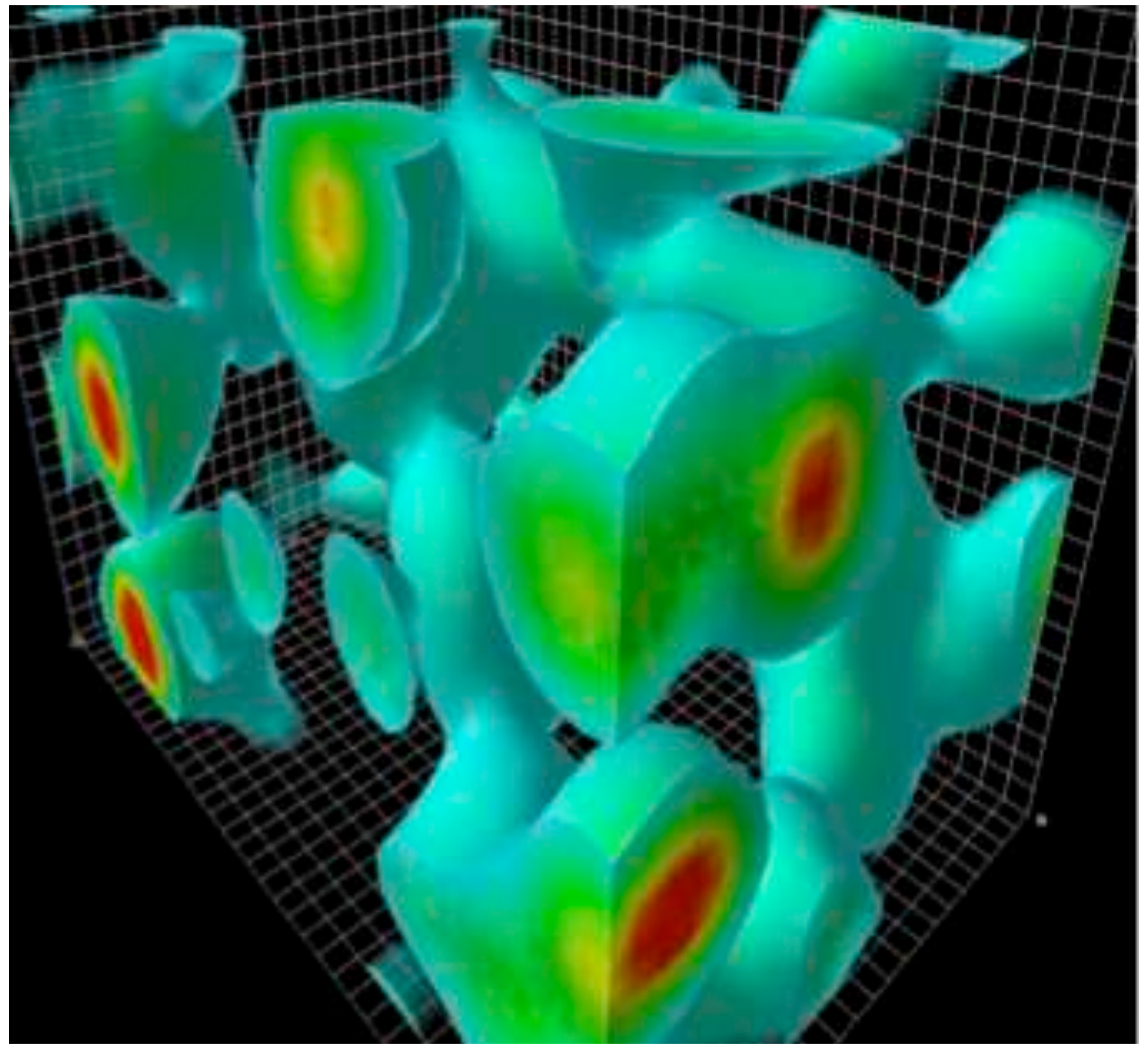
Multiple correlation functions



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(but not Deltas)
Color superconductivity 1998

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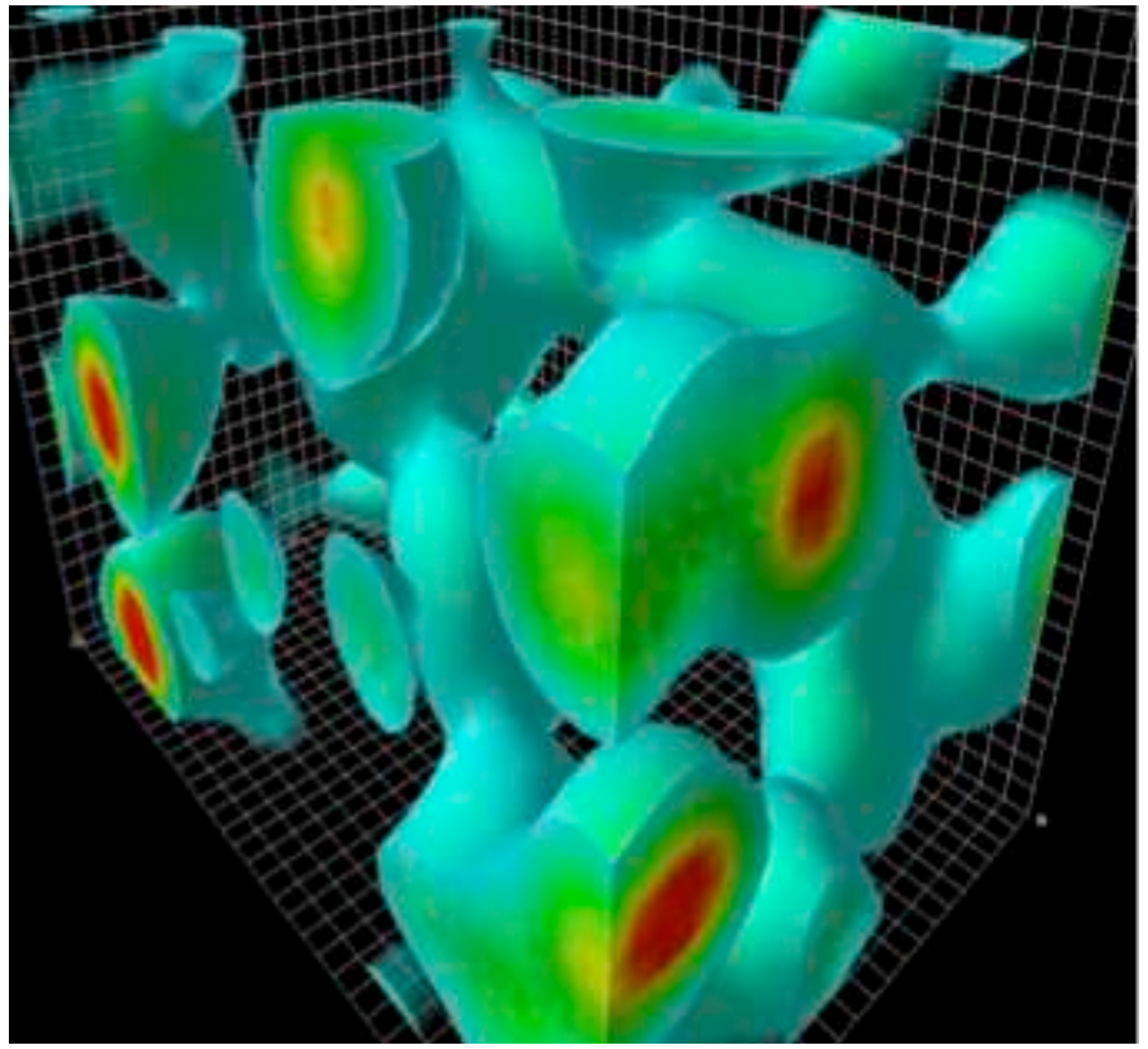

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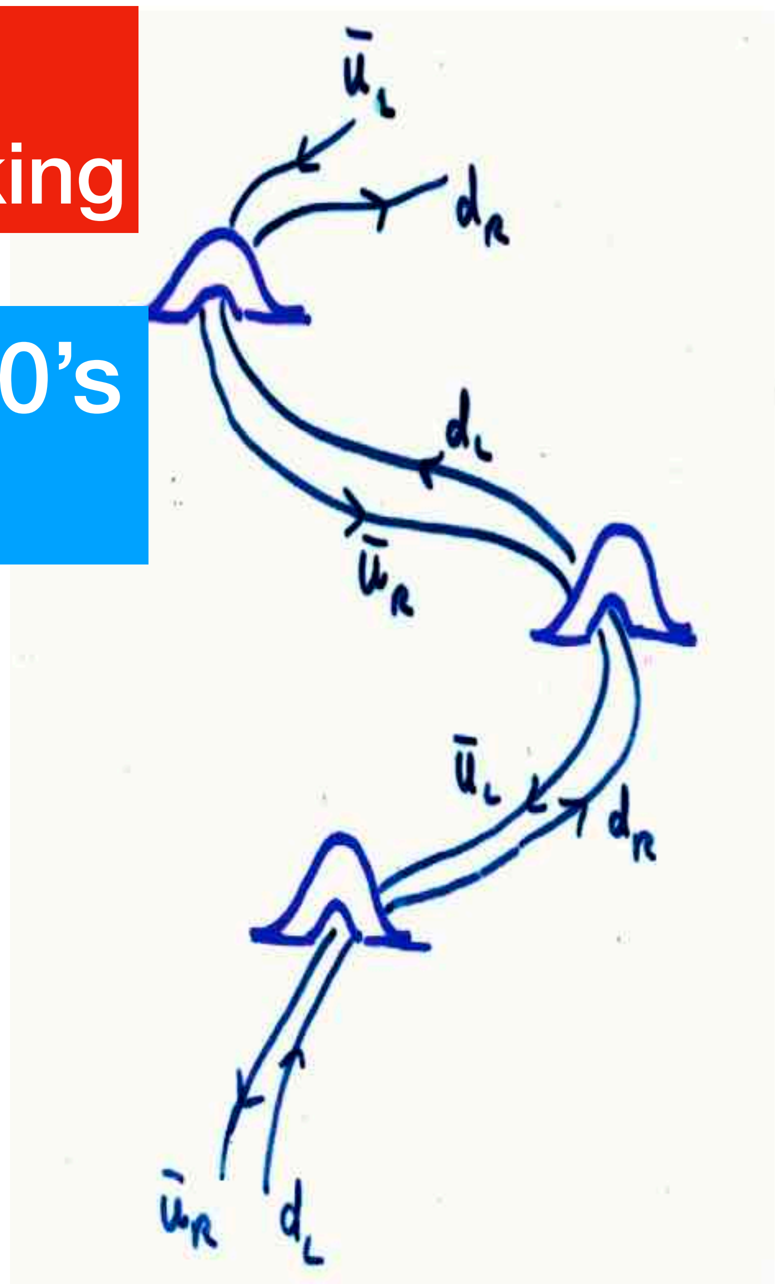

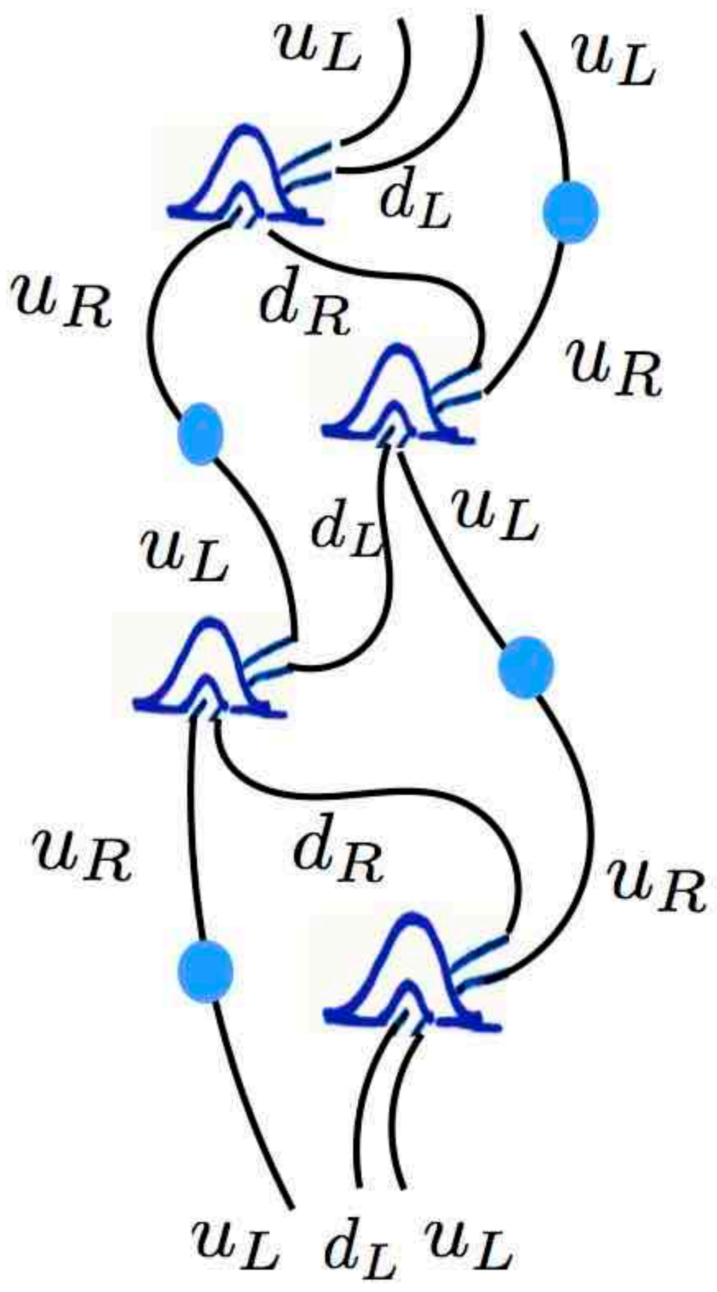

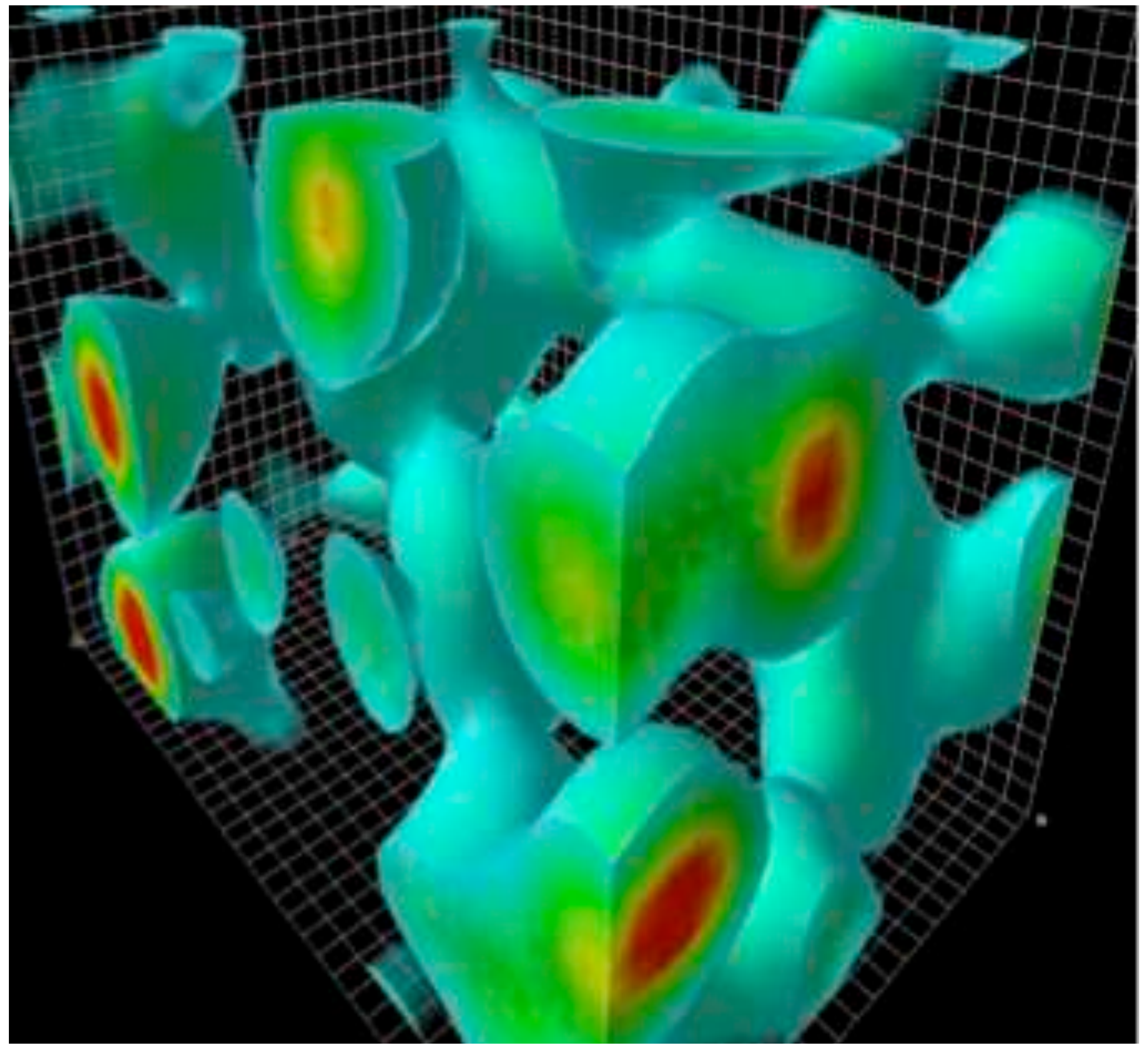

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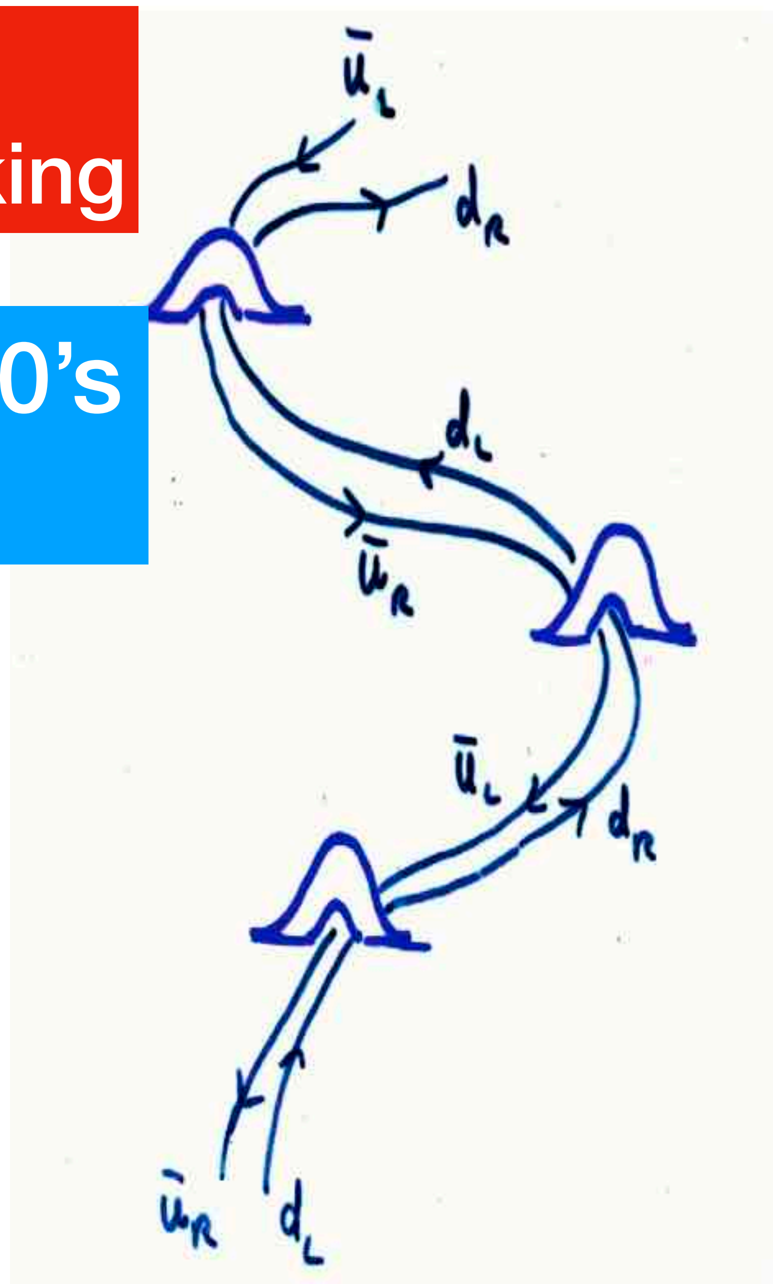

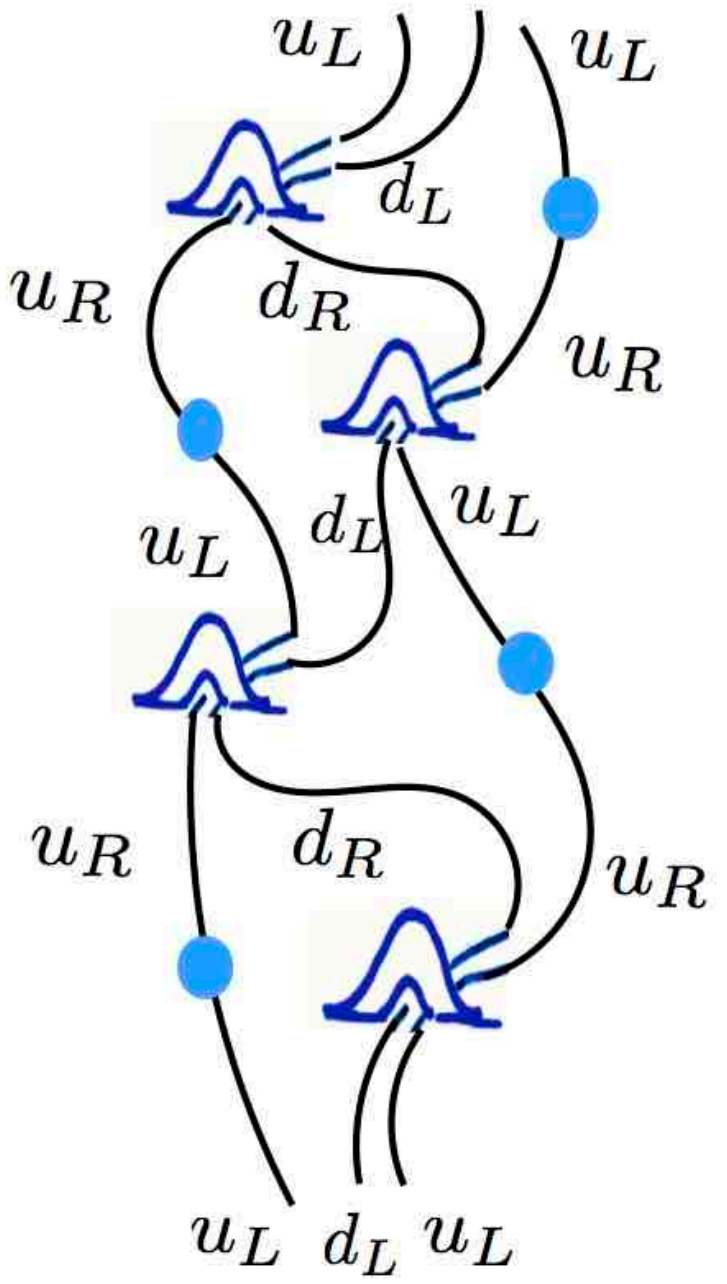

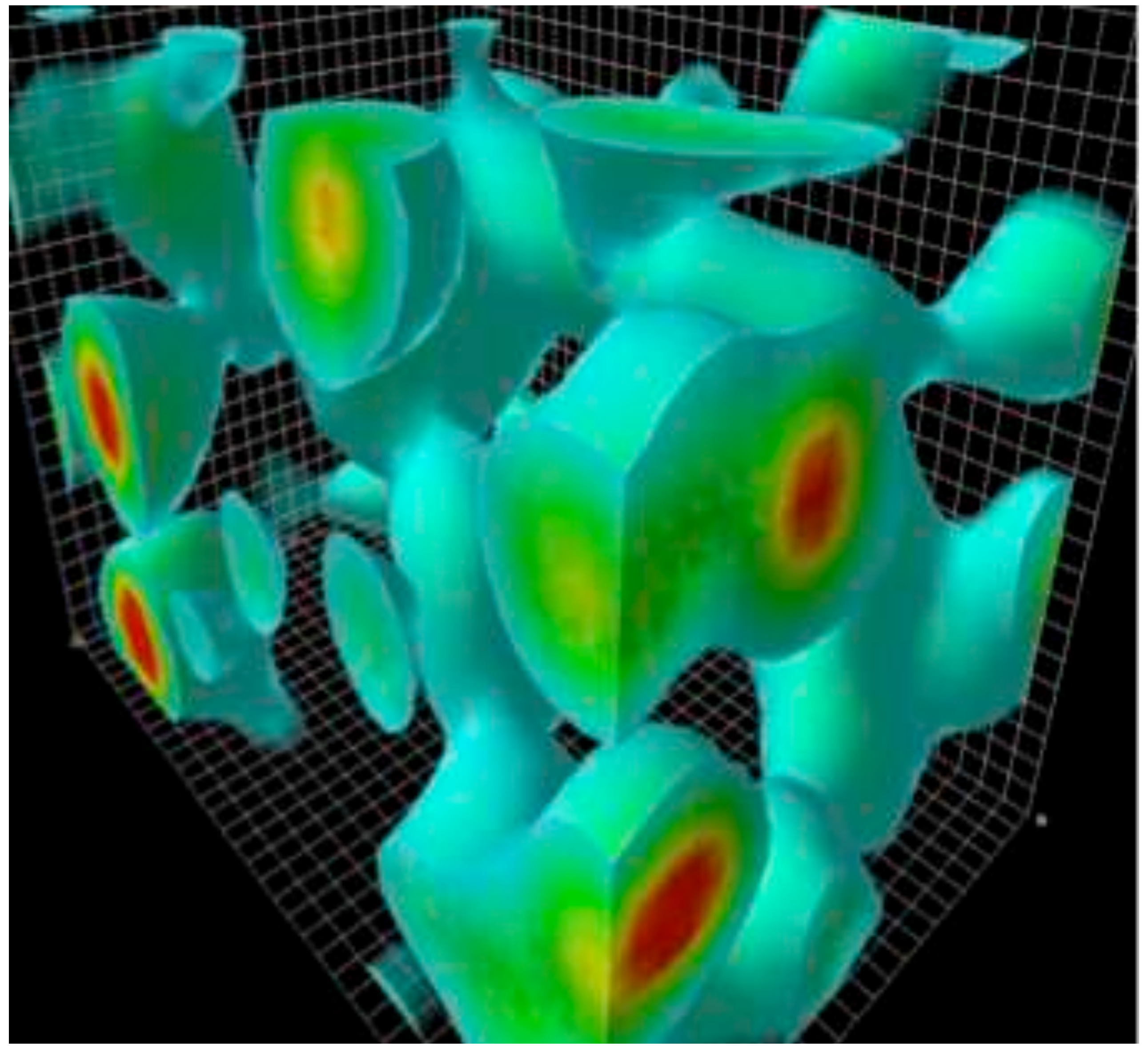

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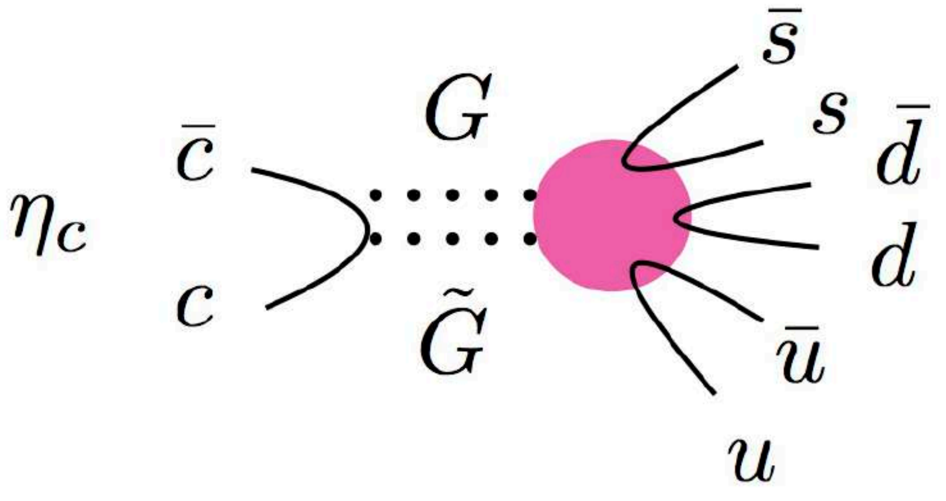

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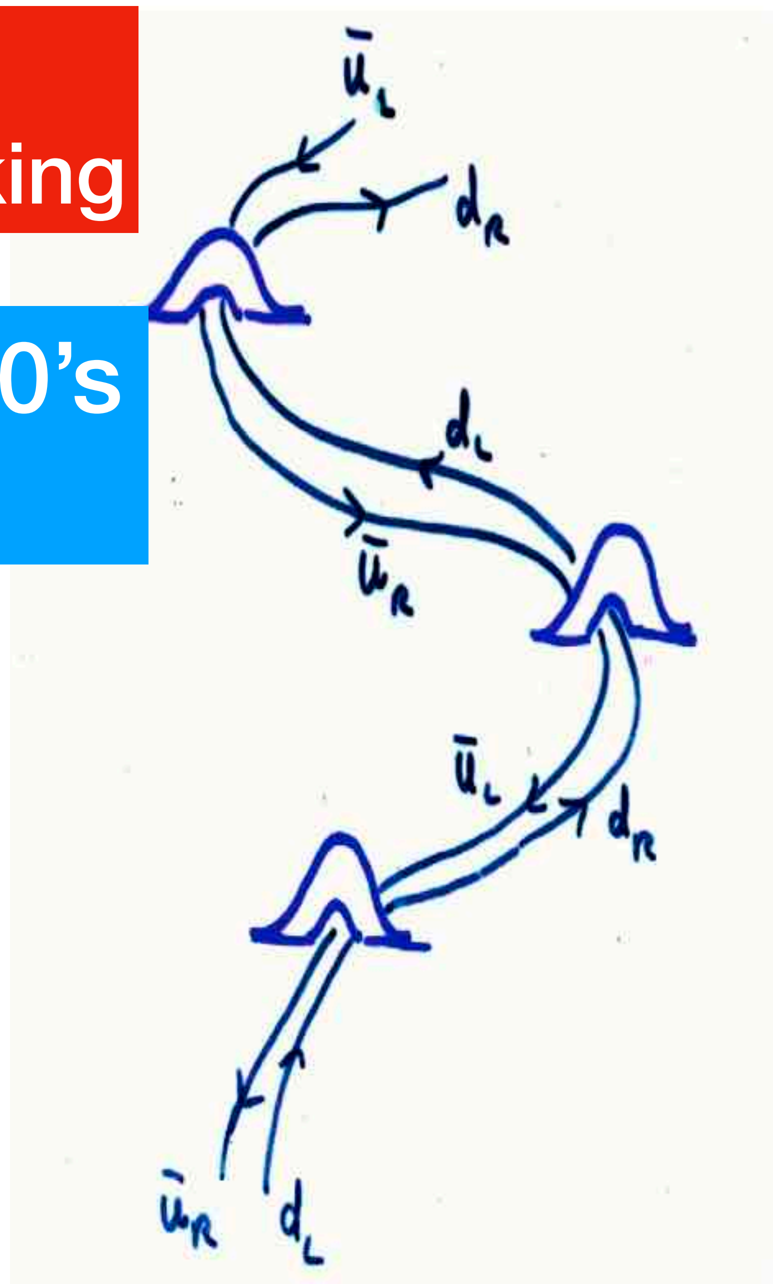

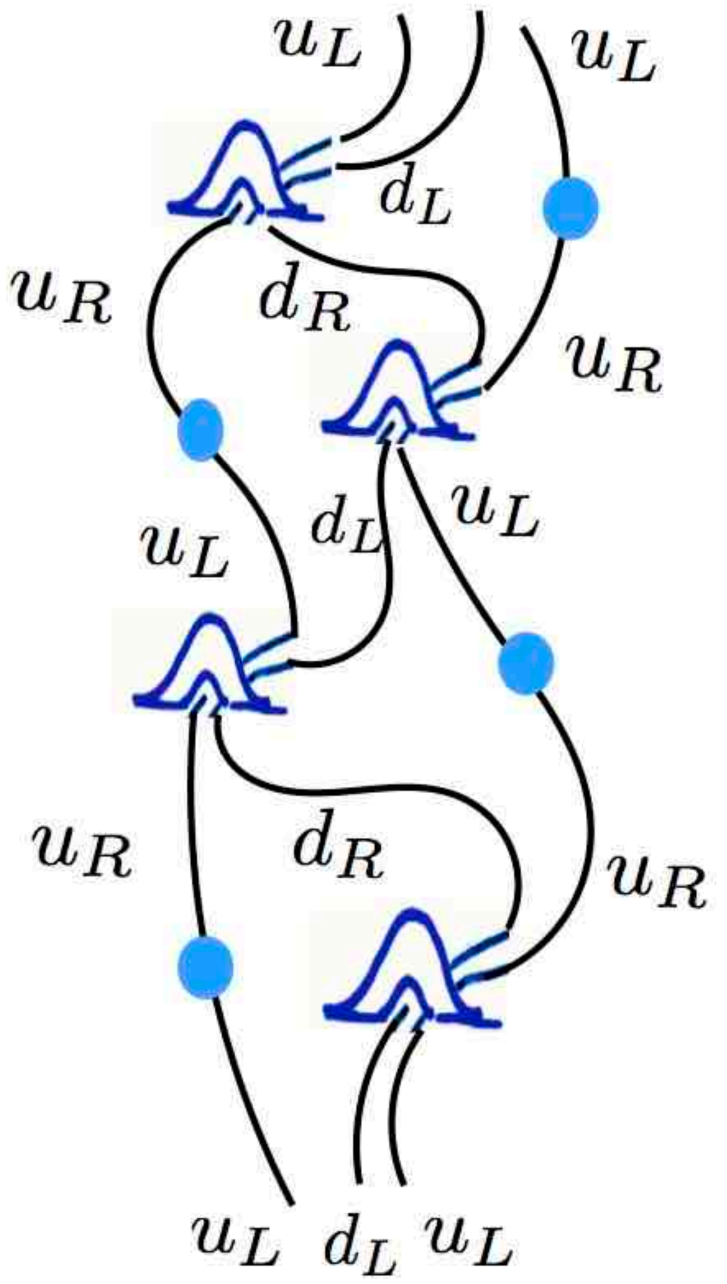

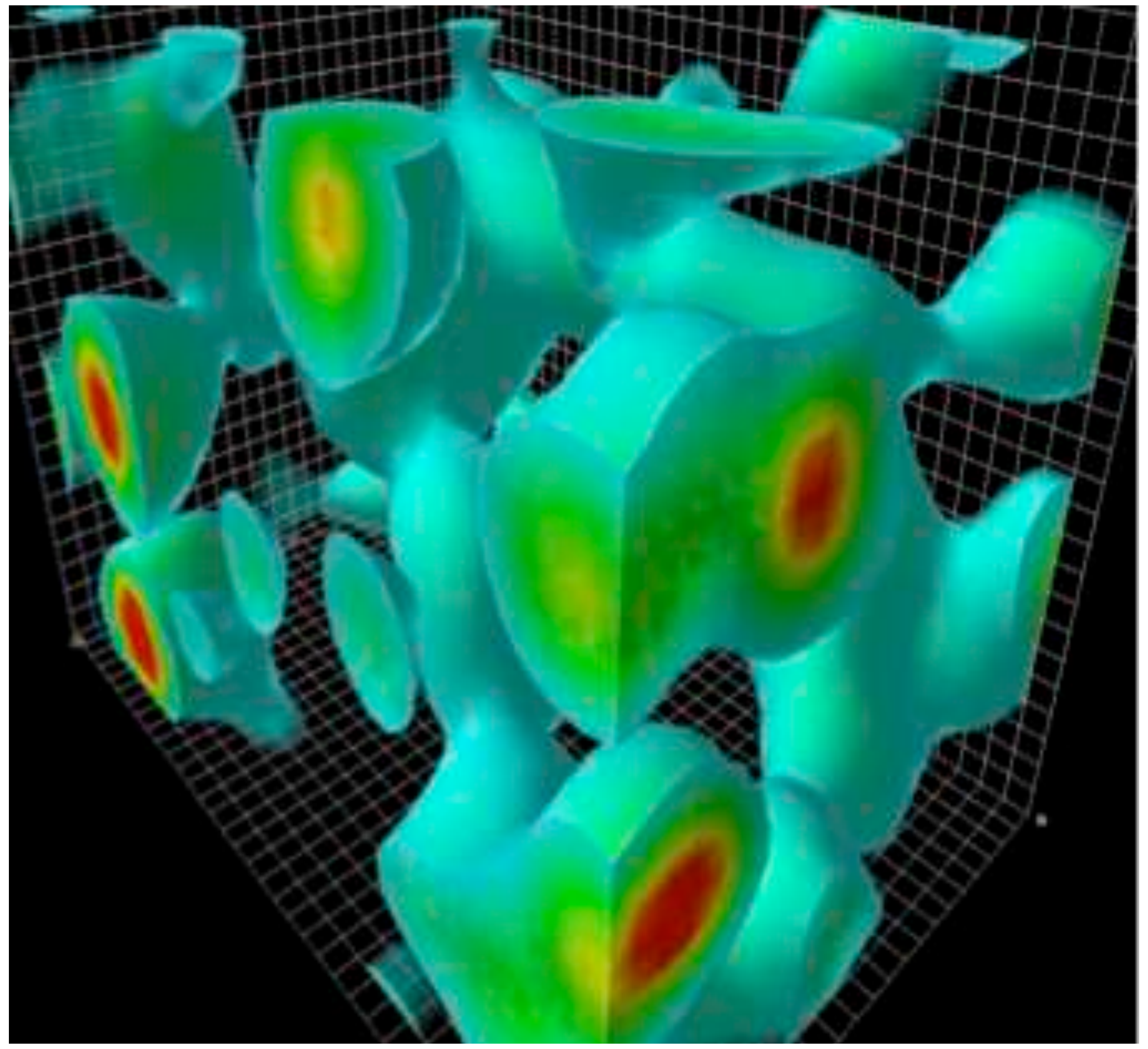
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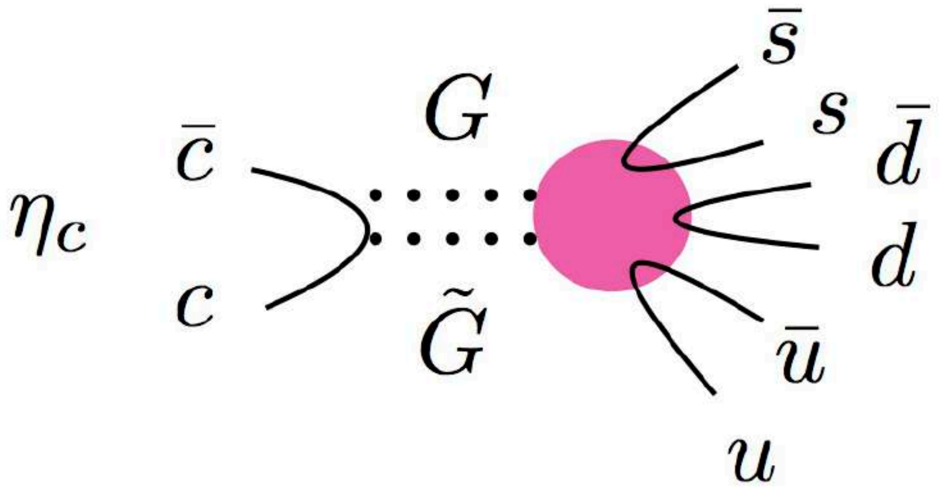

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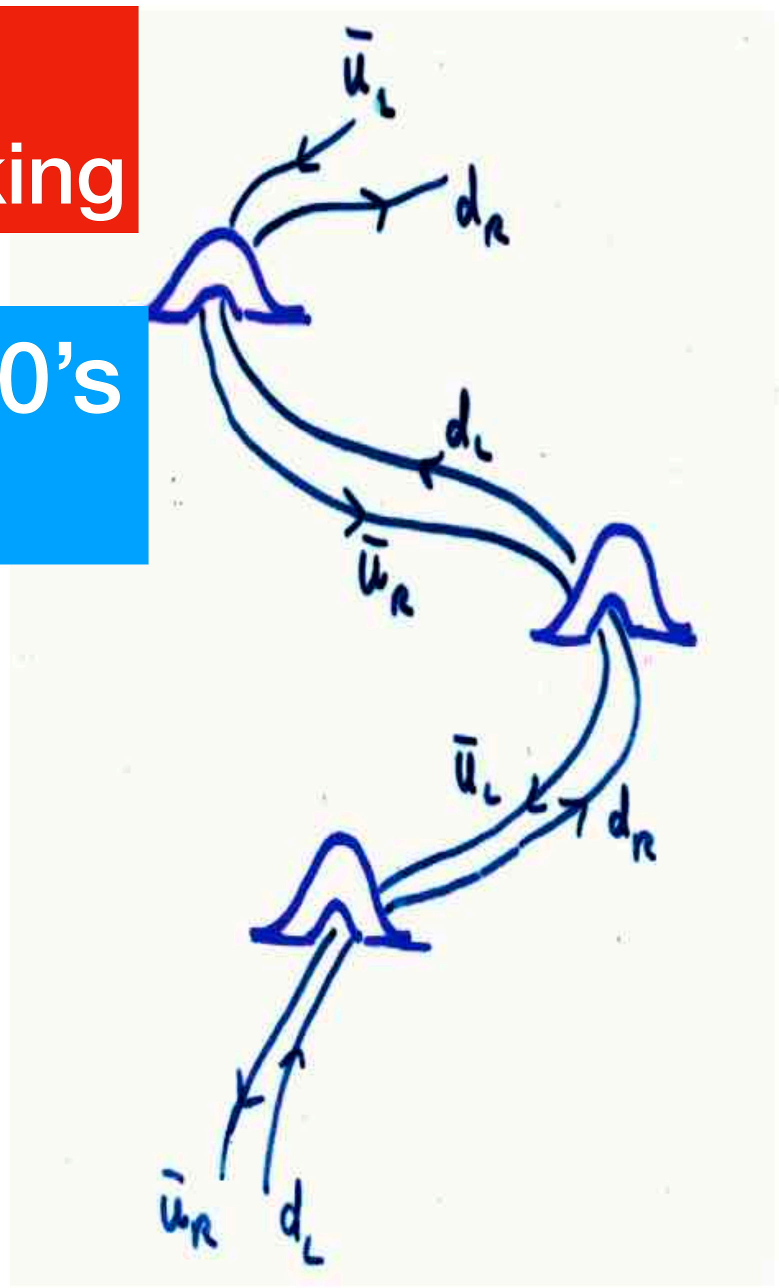

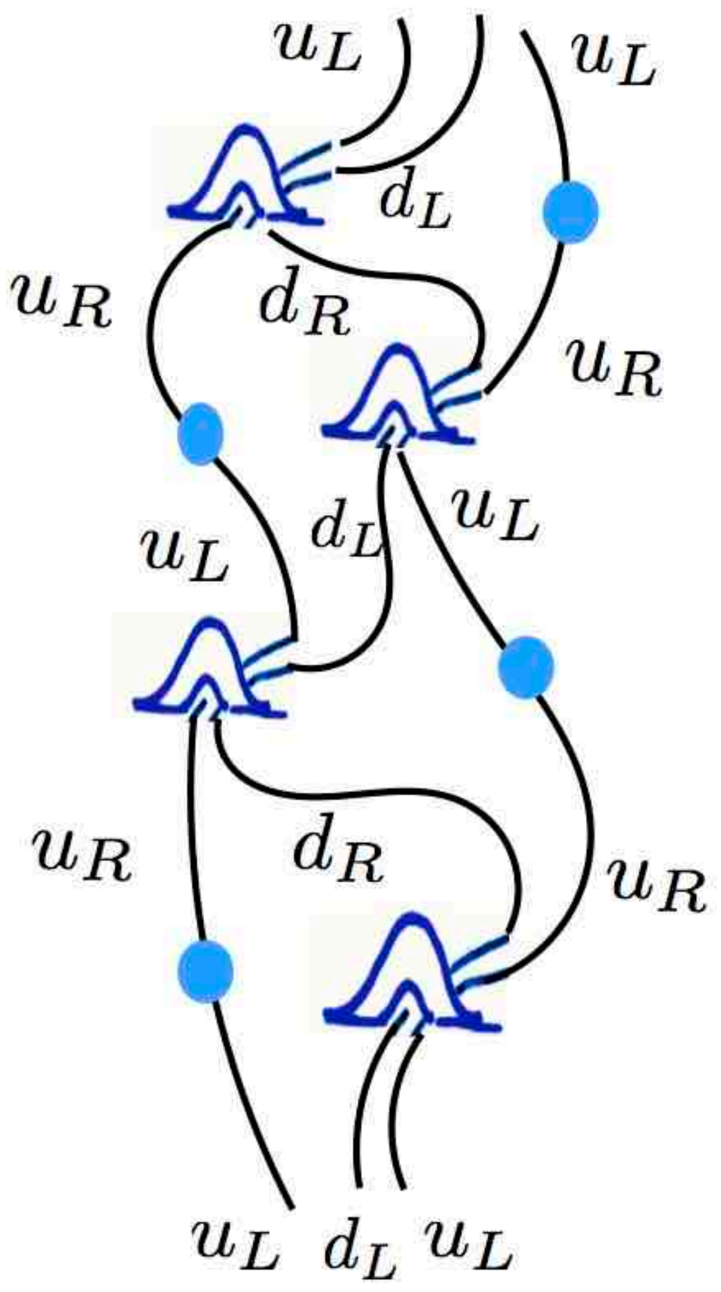

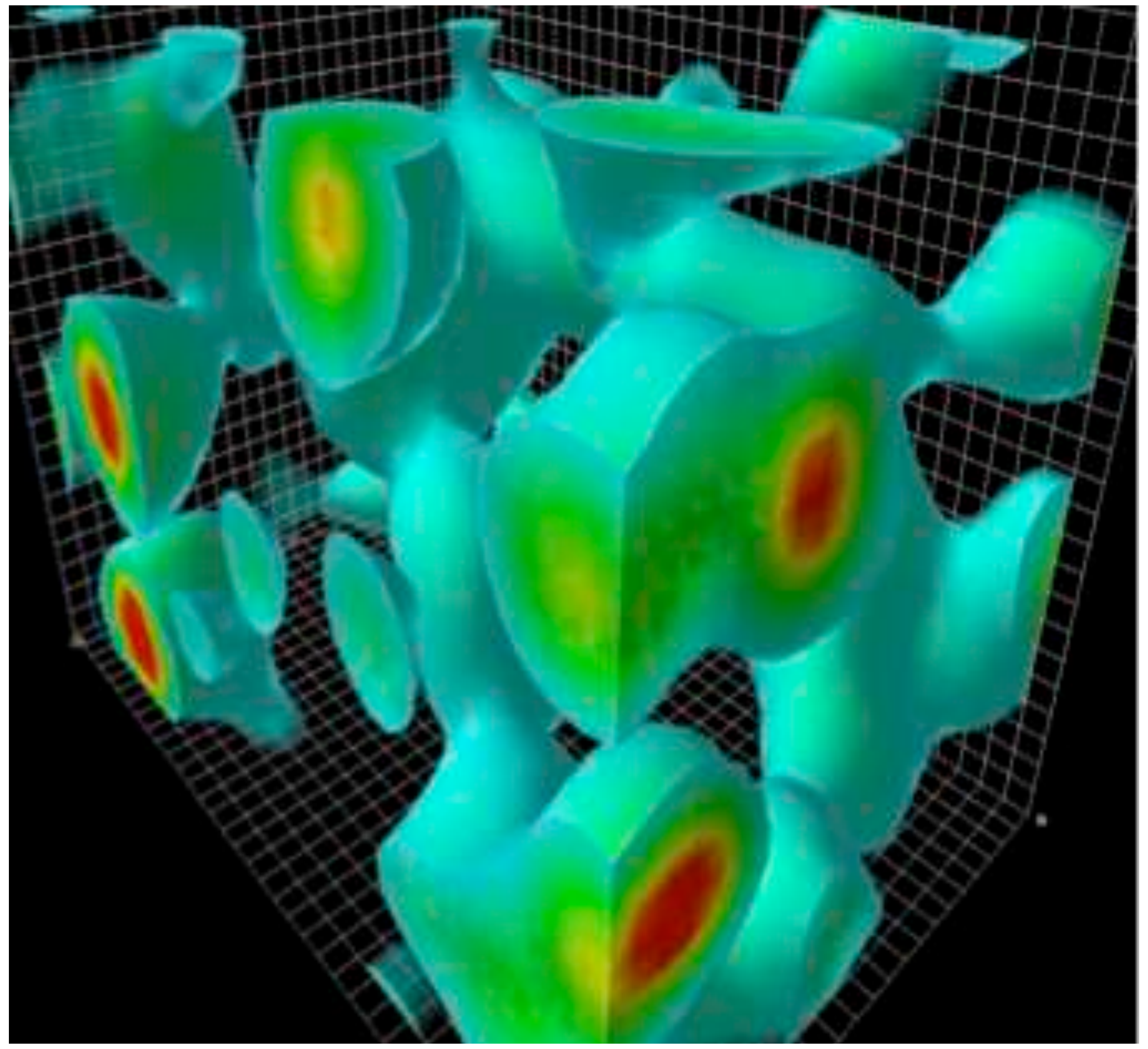

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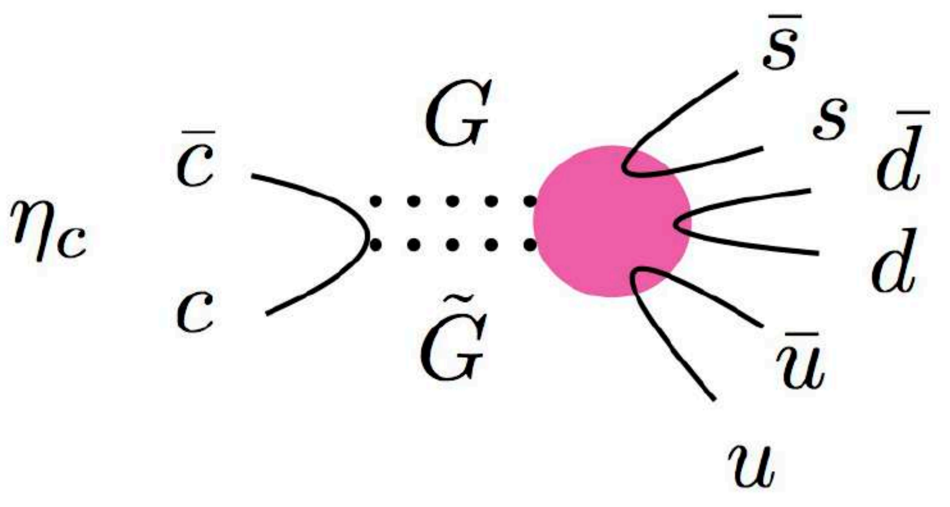

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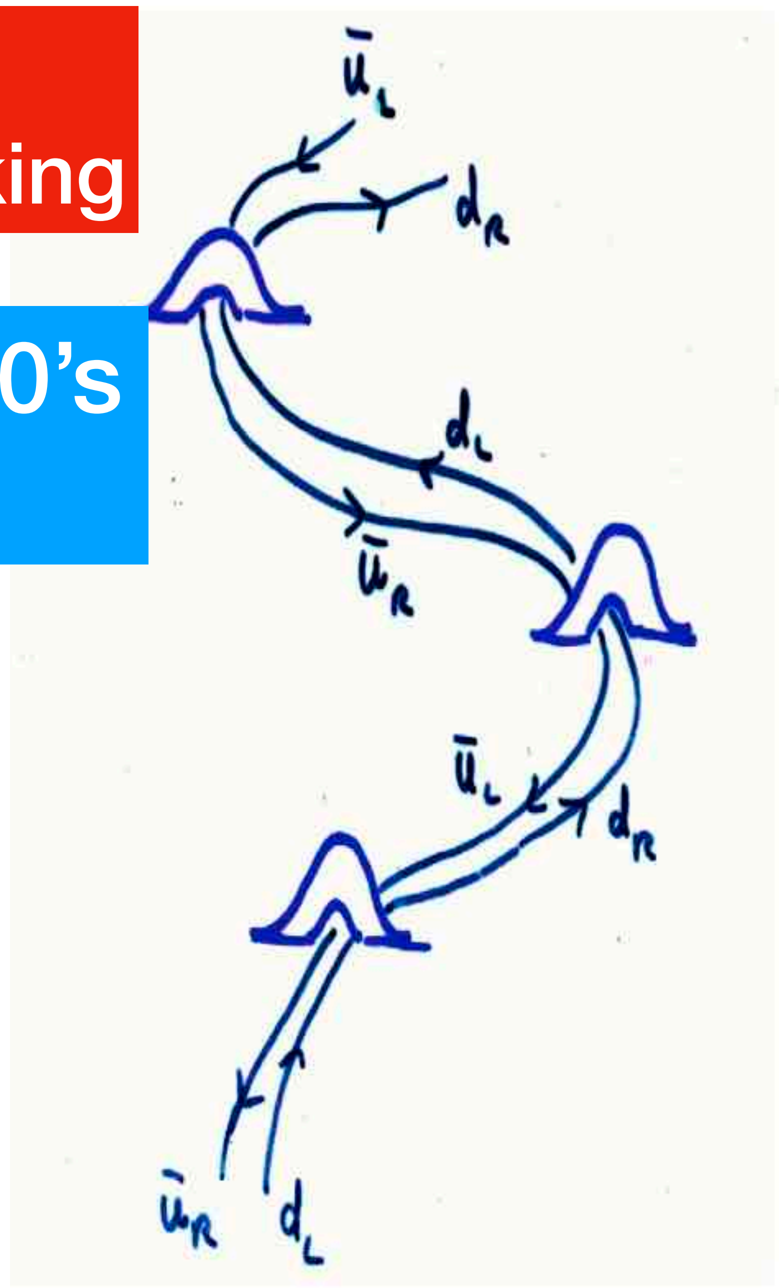

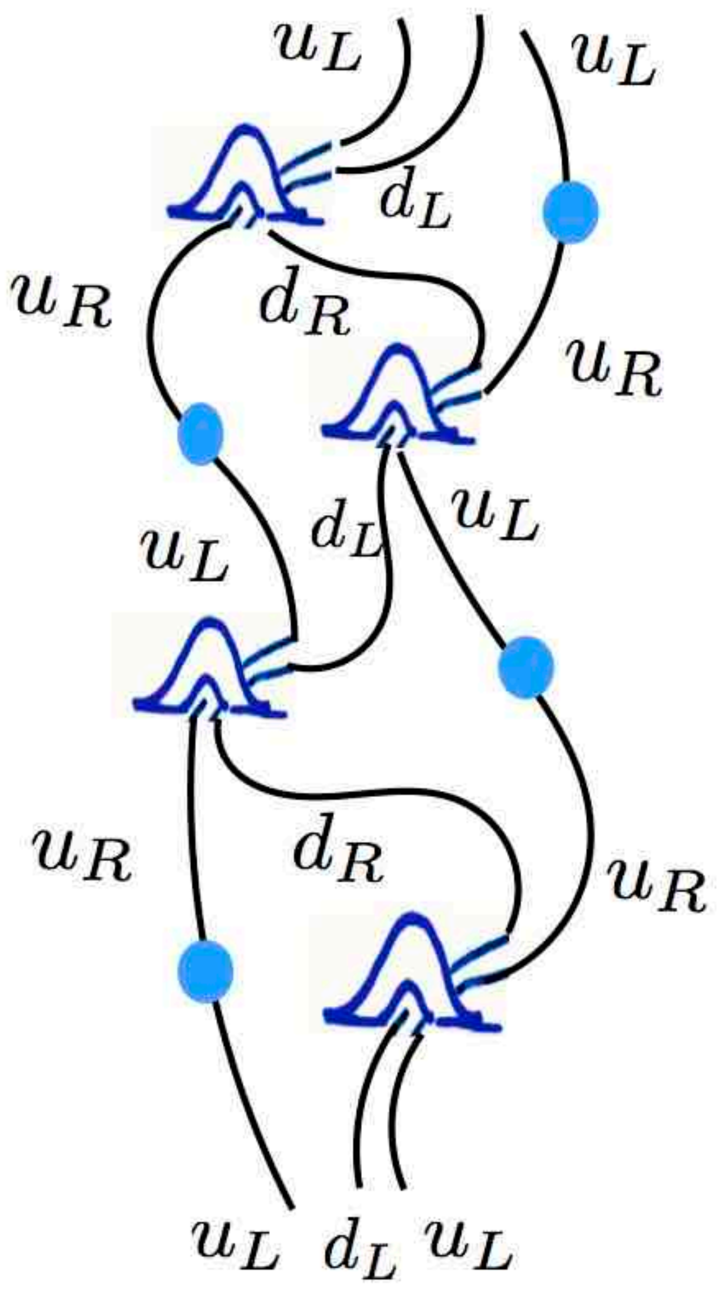

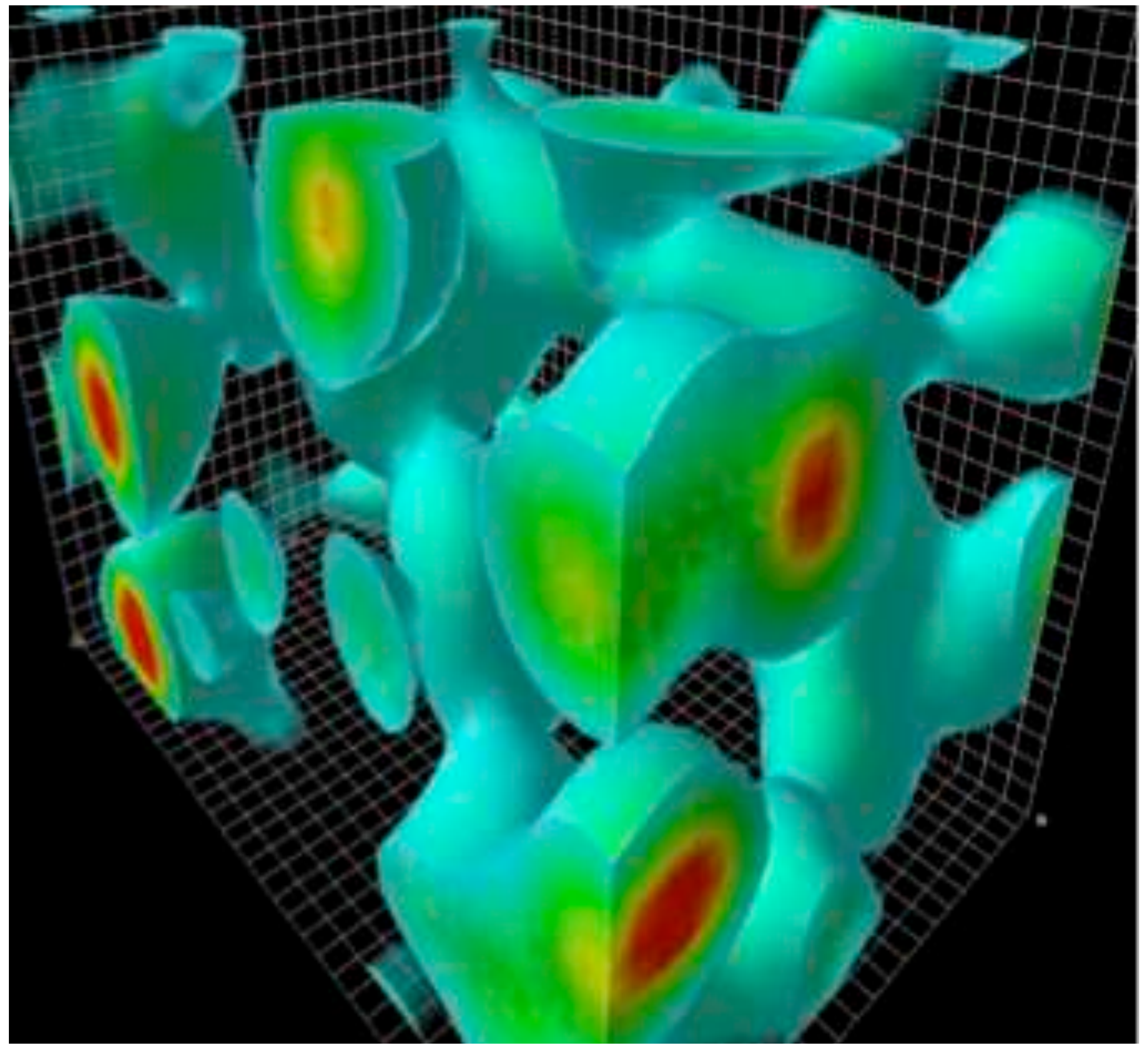

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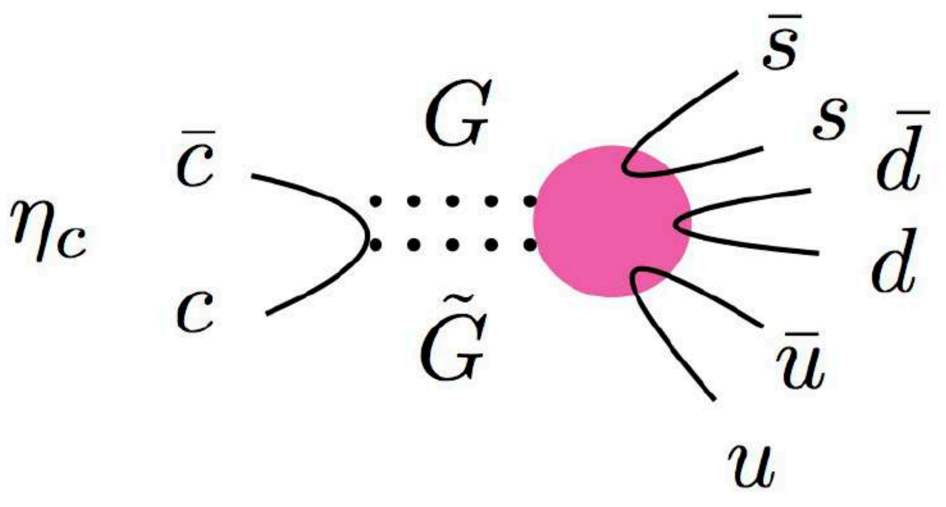

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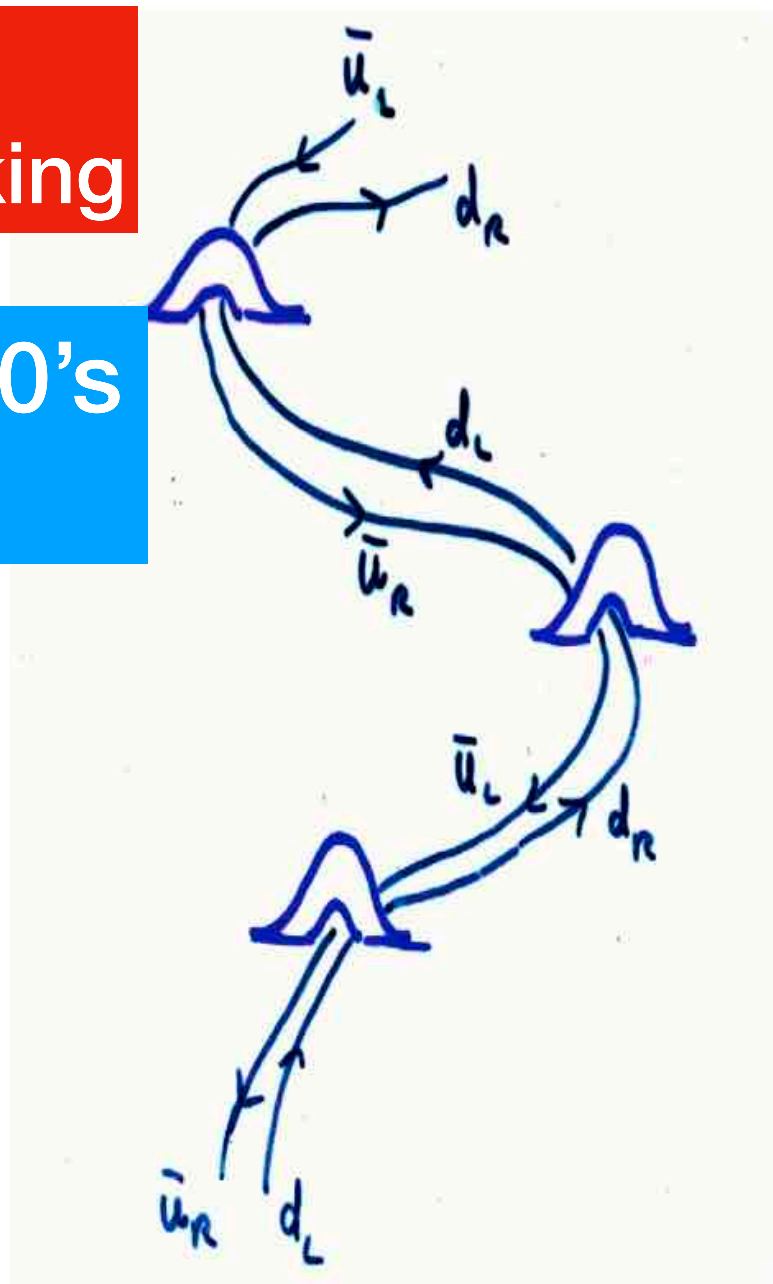

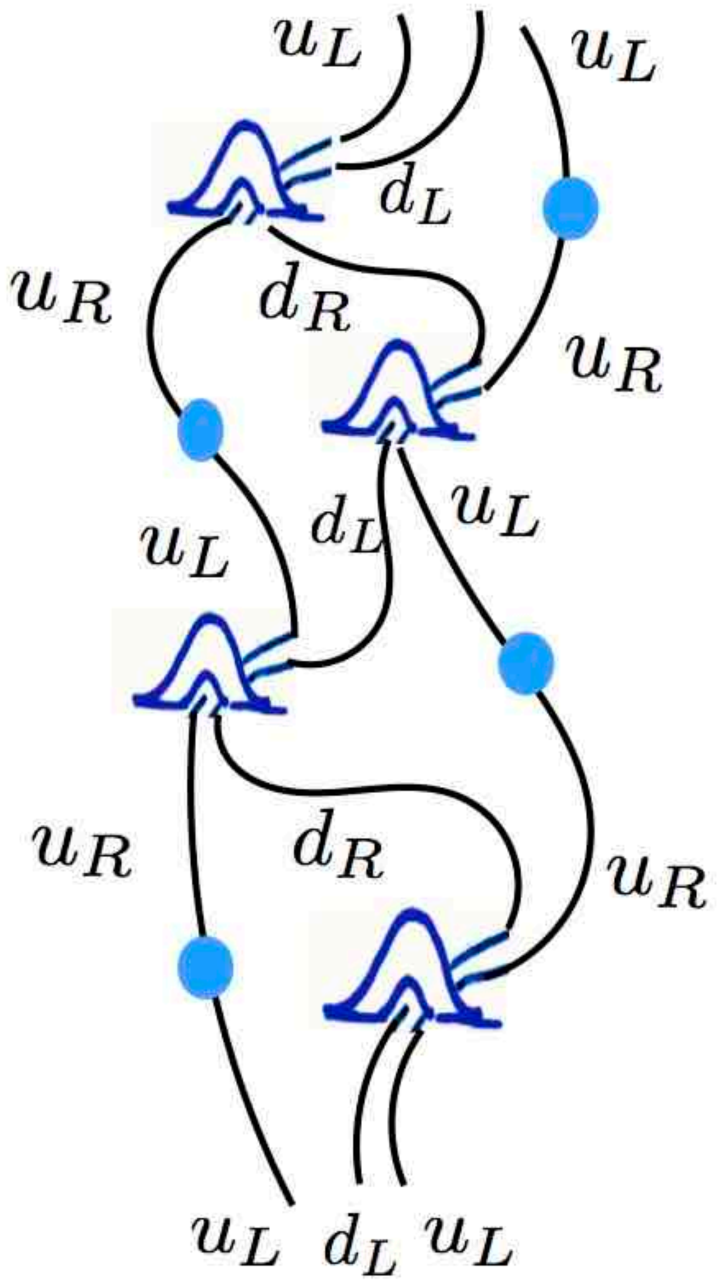

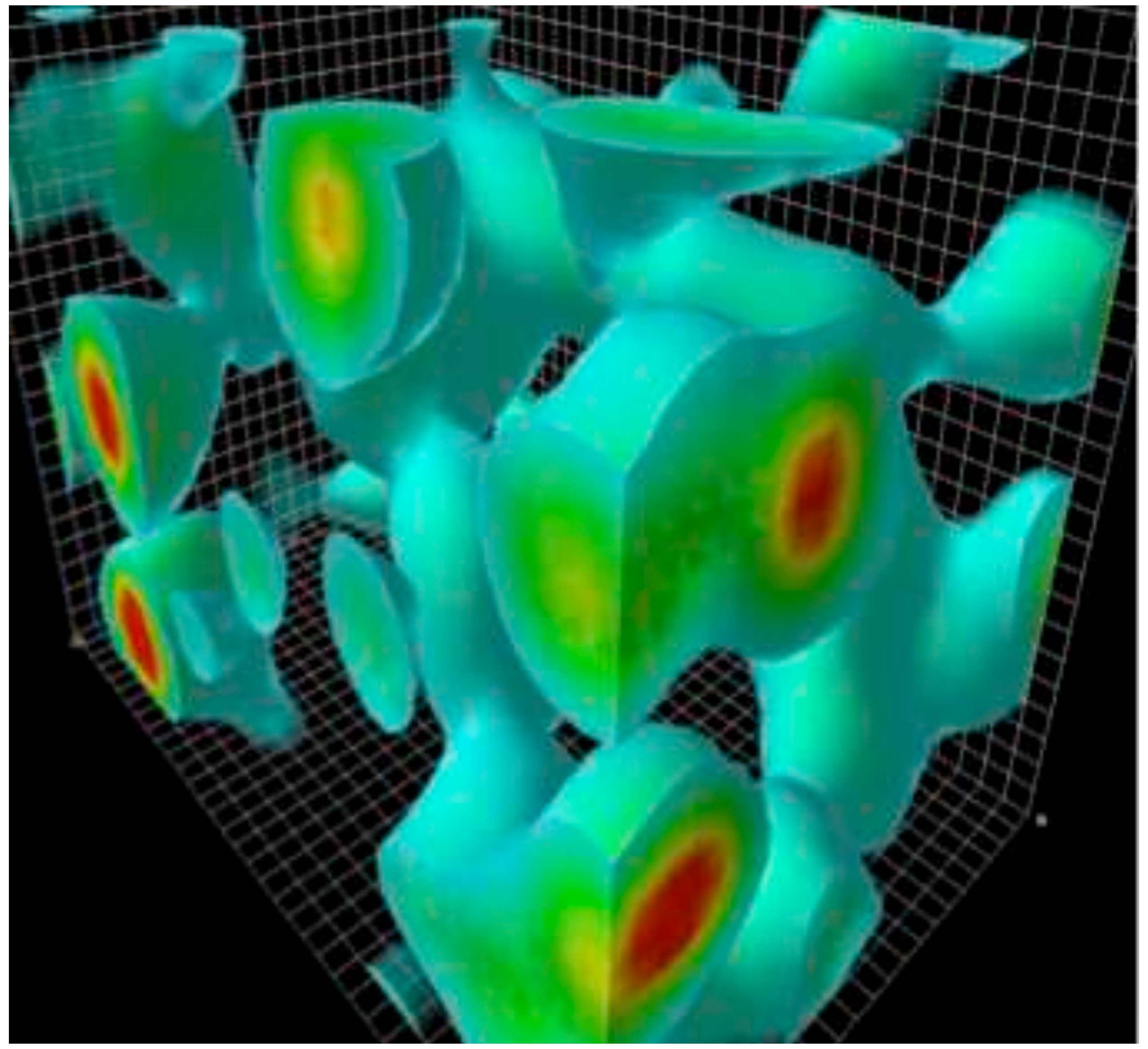

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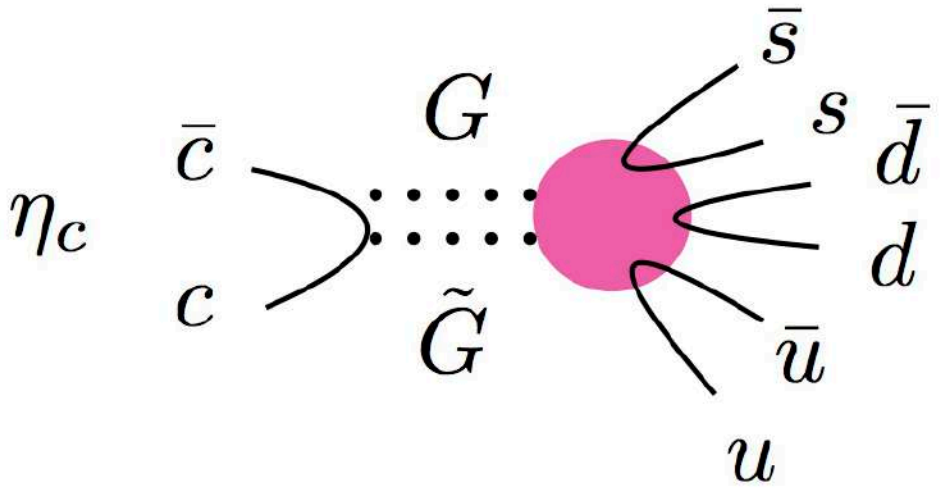

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Not seen in the control group
The J/psi decays

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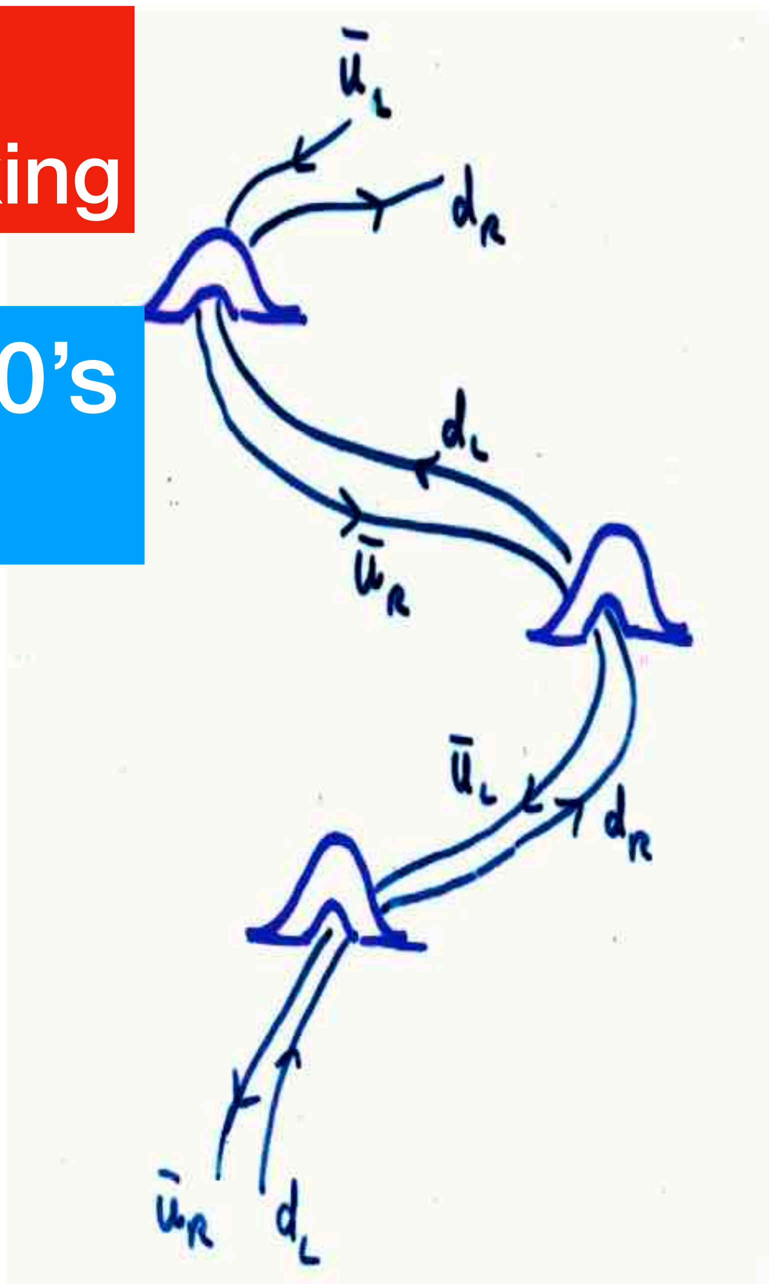

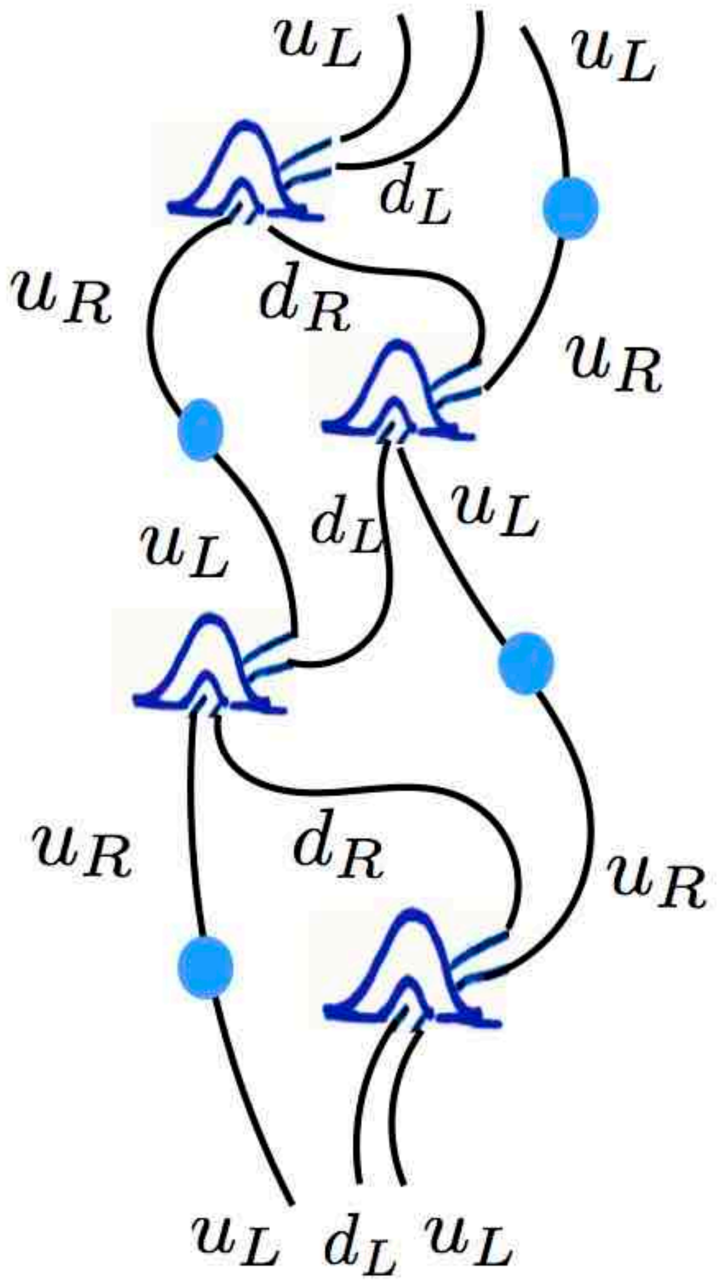

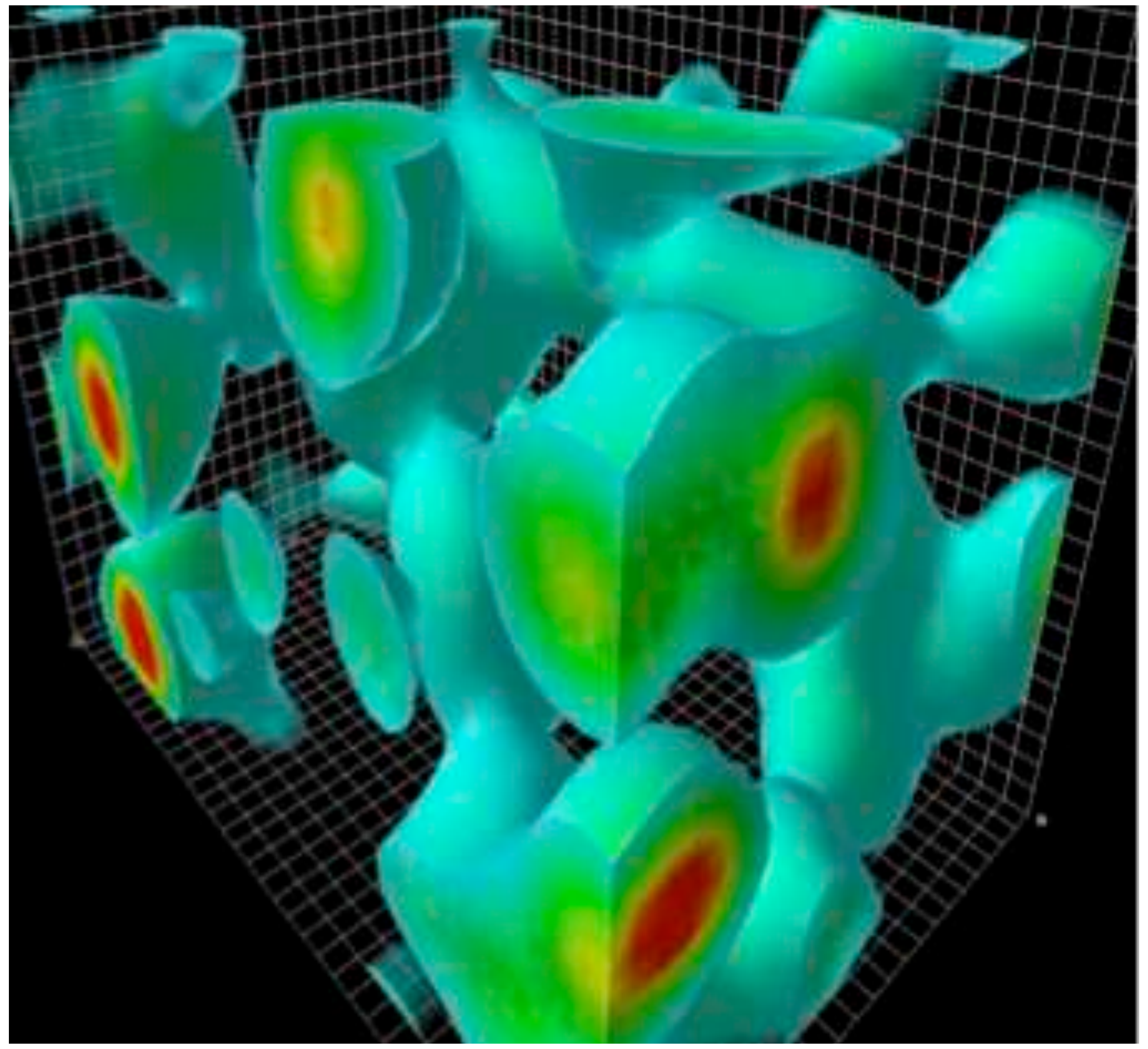

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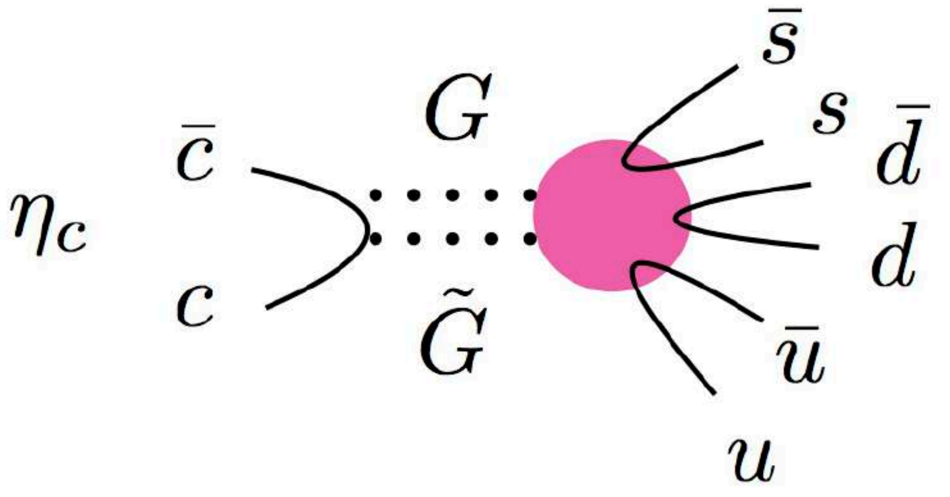

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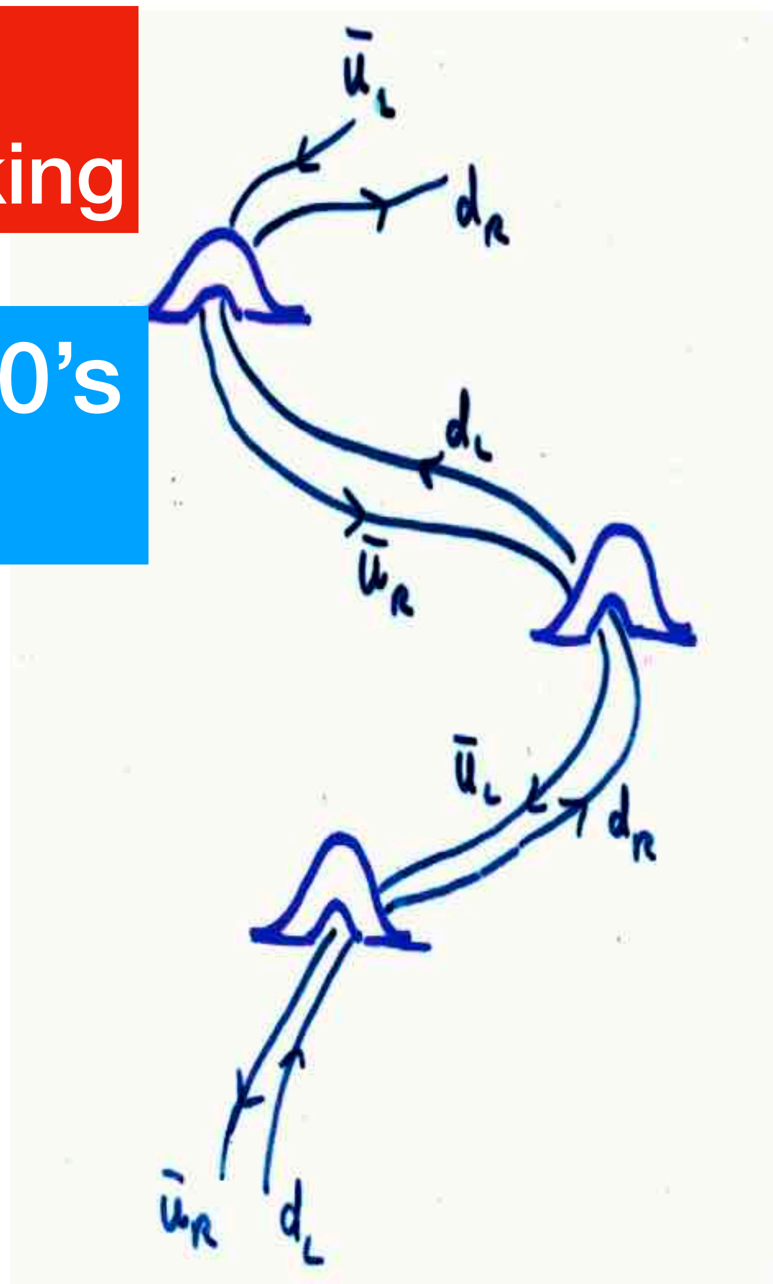

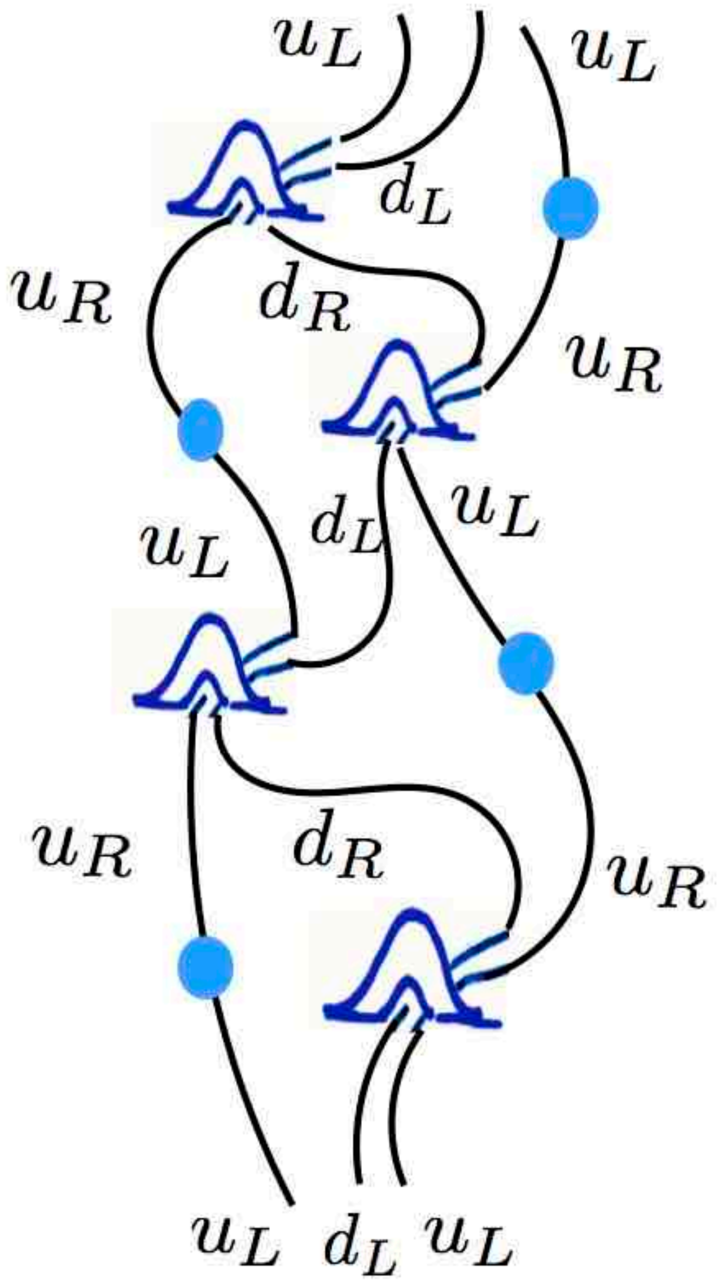

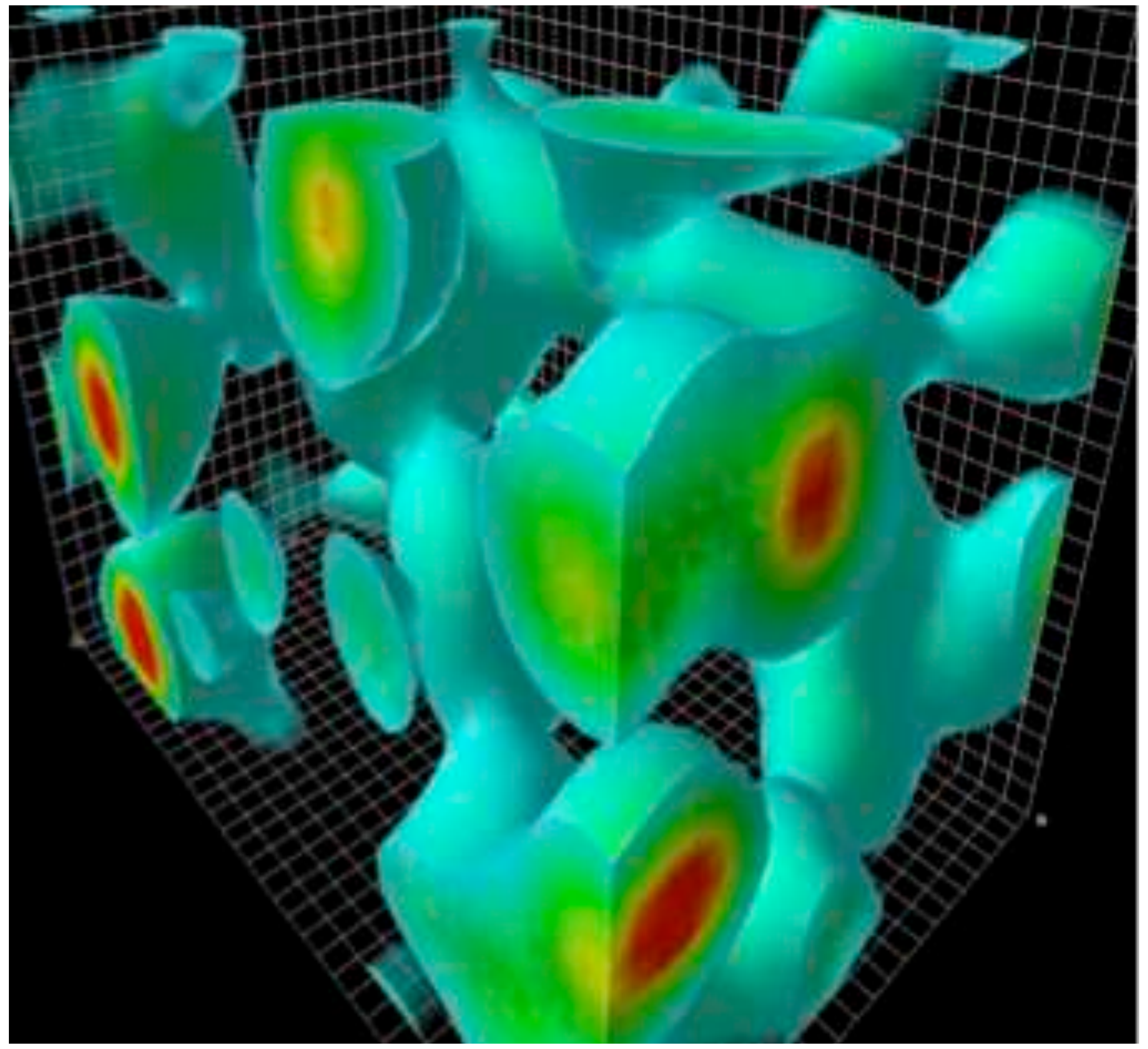

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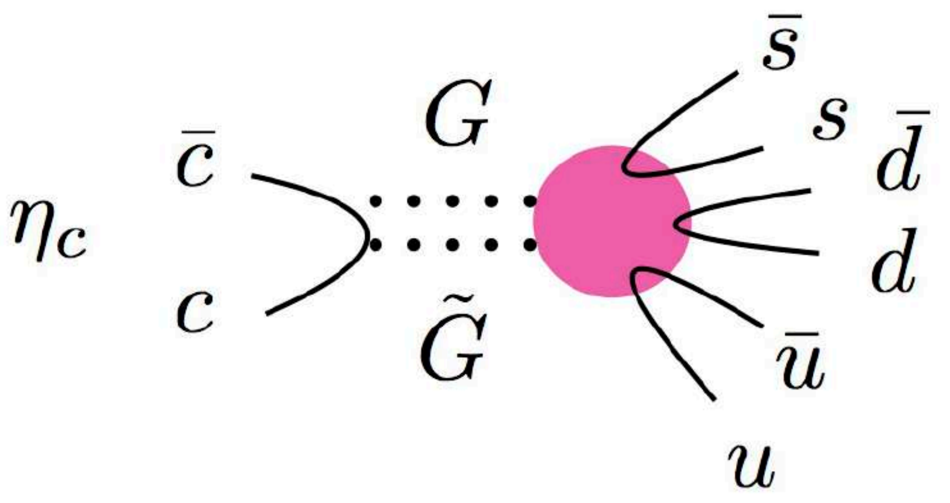
Light-front wave functions of mesons, baryons, and pentaquarks with topology-induced local four-quark interaction
ES, *Phys.Rev.D* 100 (2019) 11, 114018 • e-Print: 1908.10270



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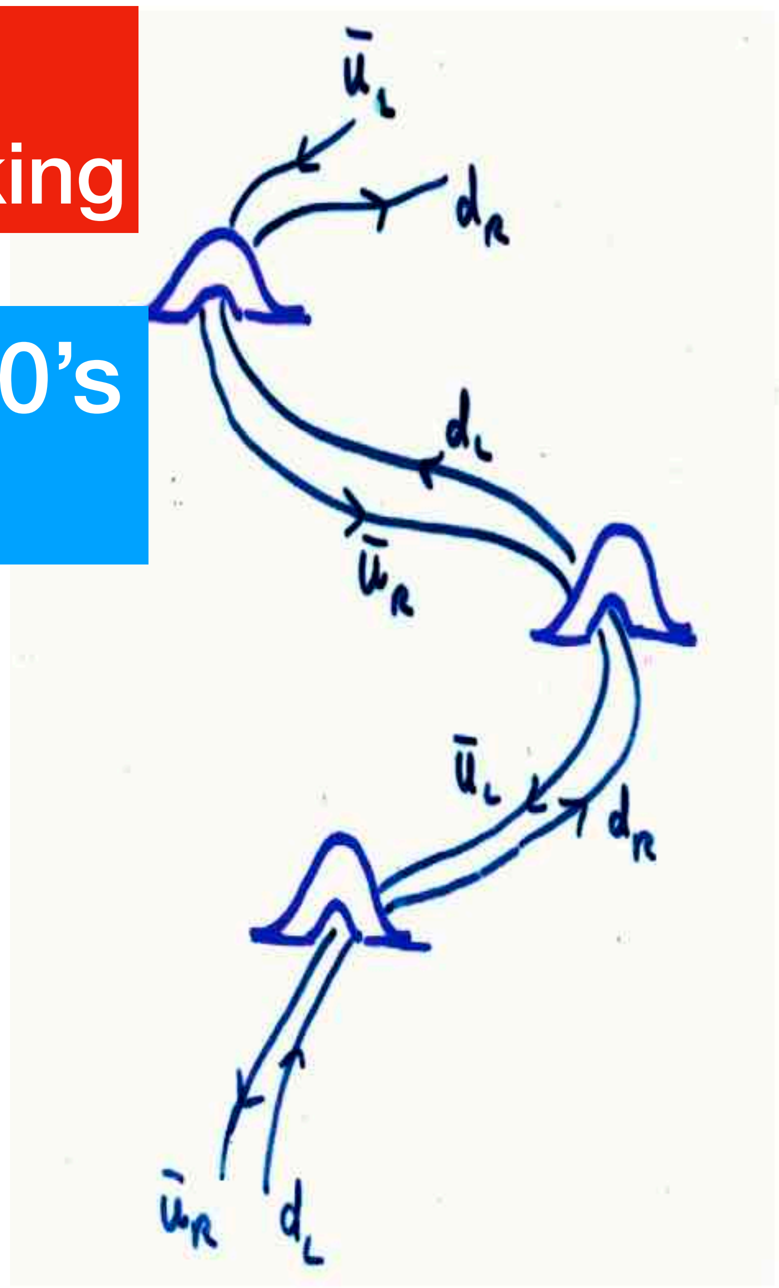

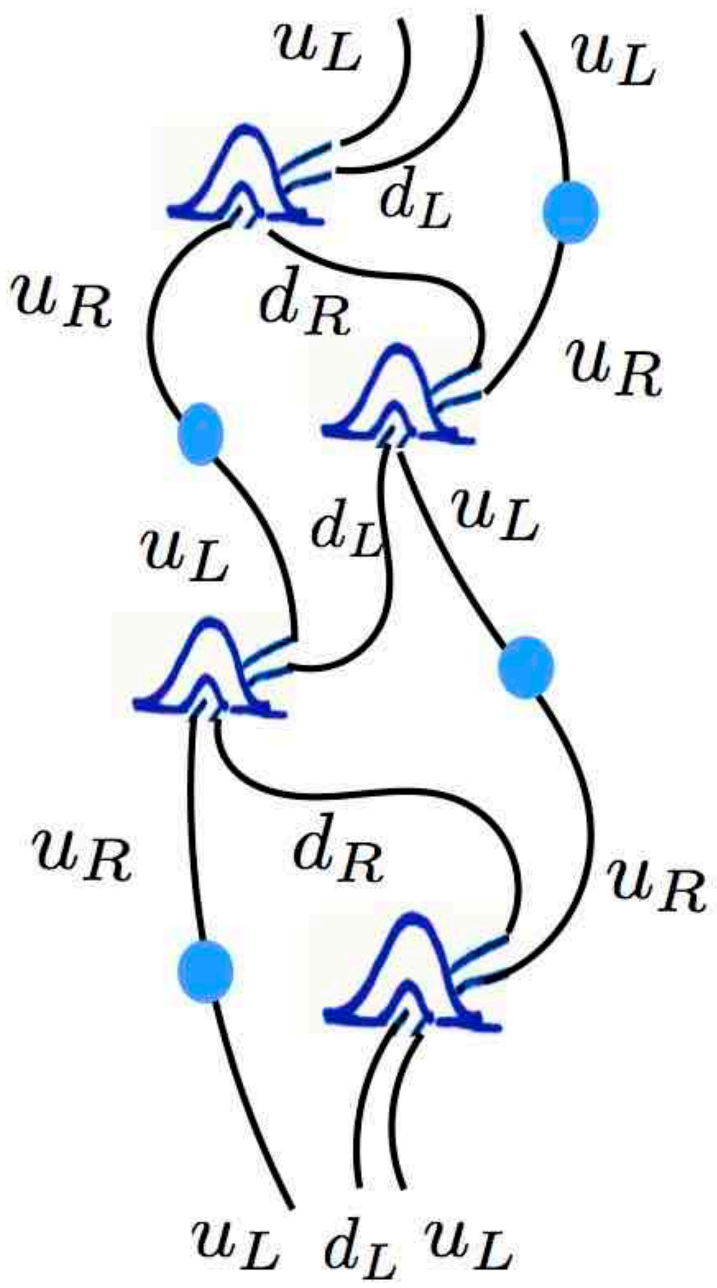

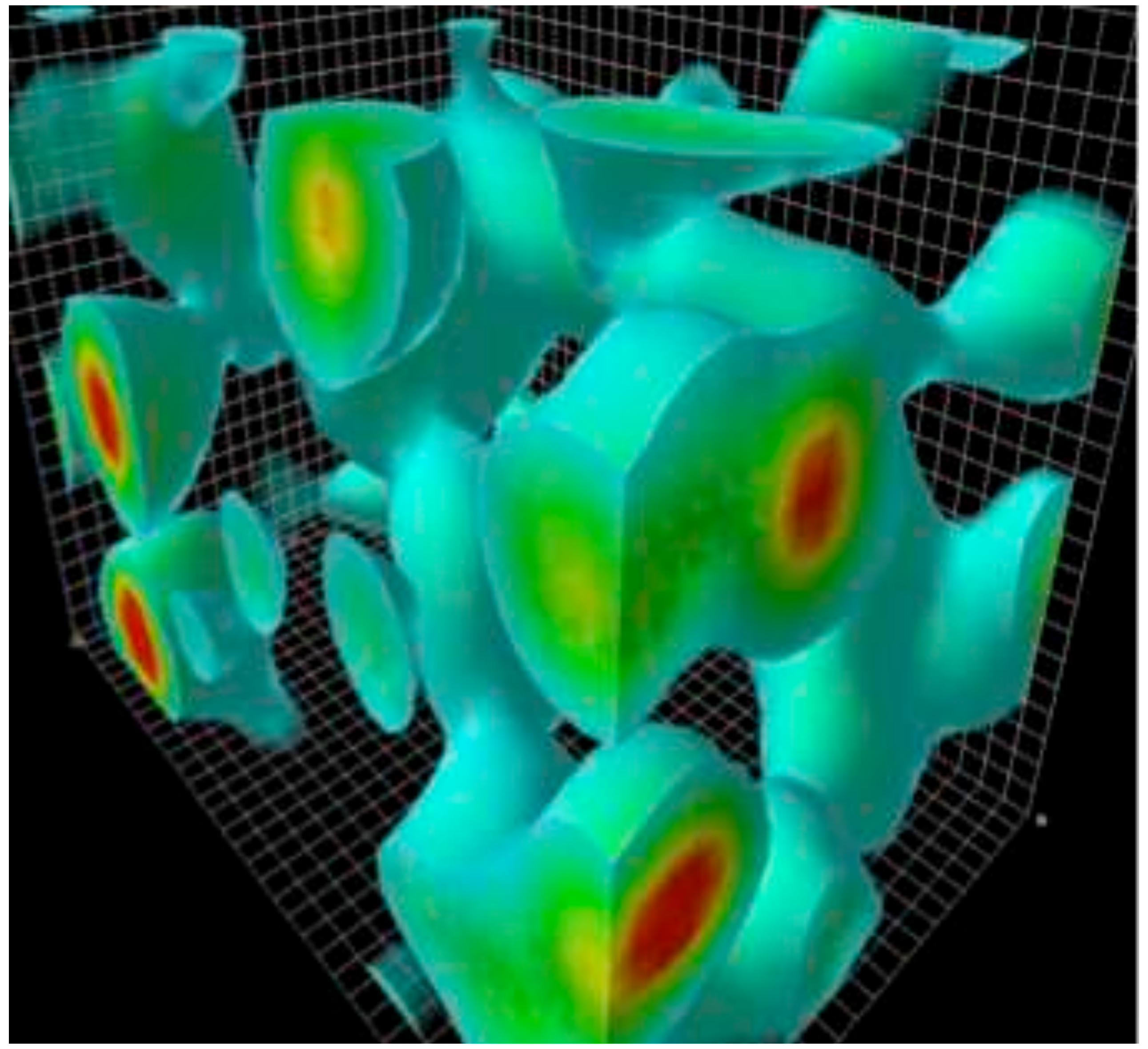

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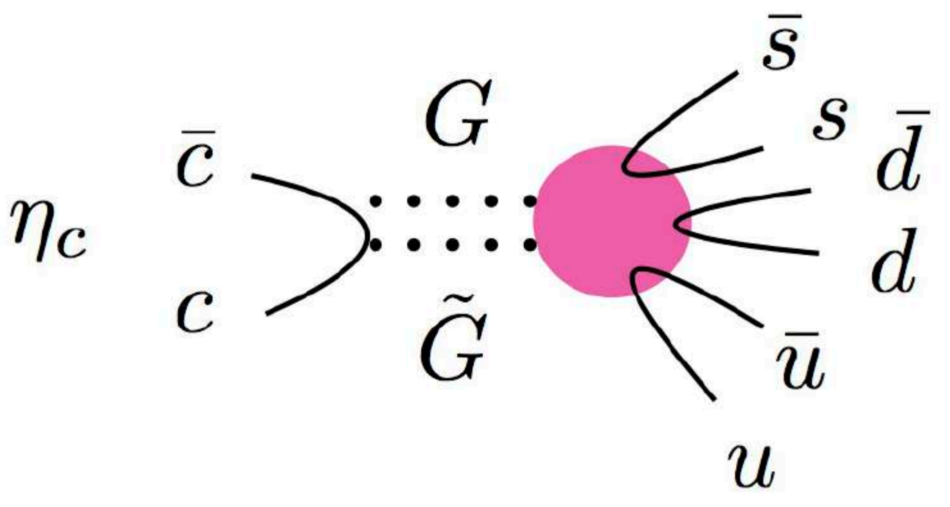
Nonperturbative quark-antiquark interactions in mesonic form factors ES, Ismail Zahed , 2008.06169



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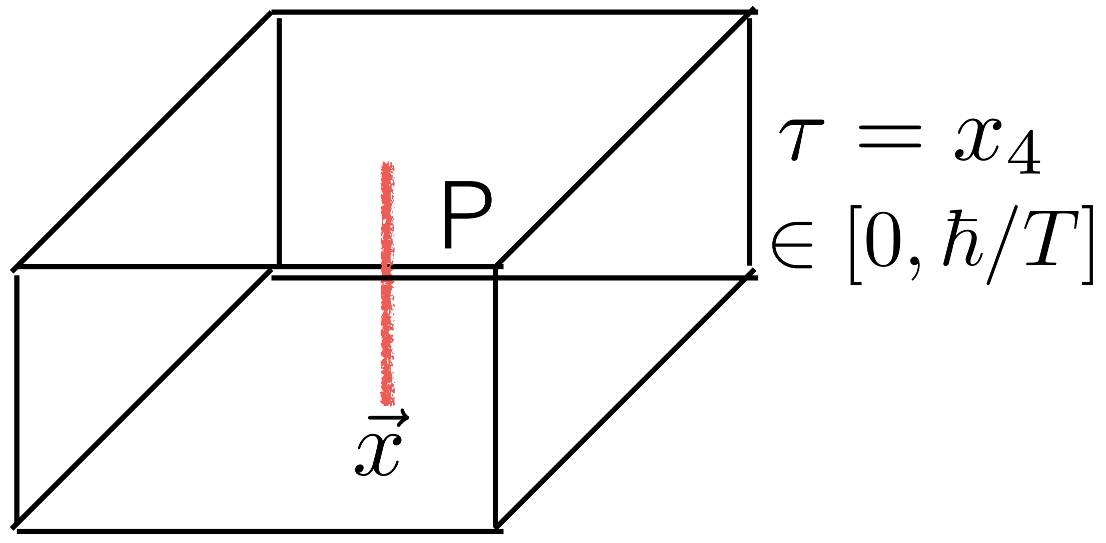
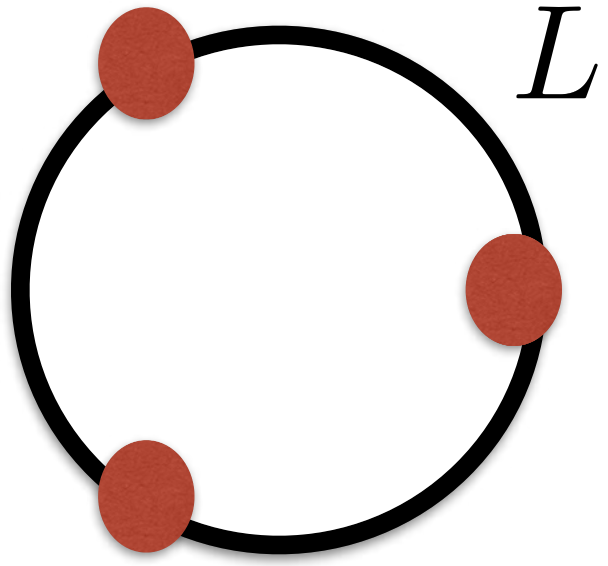
Instanton-dyons

The Polyakov line is used as order parameter for deconfinement

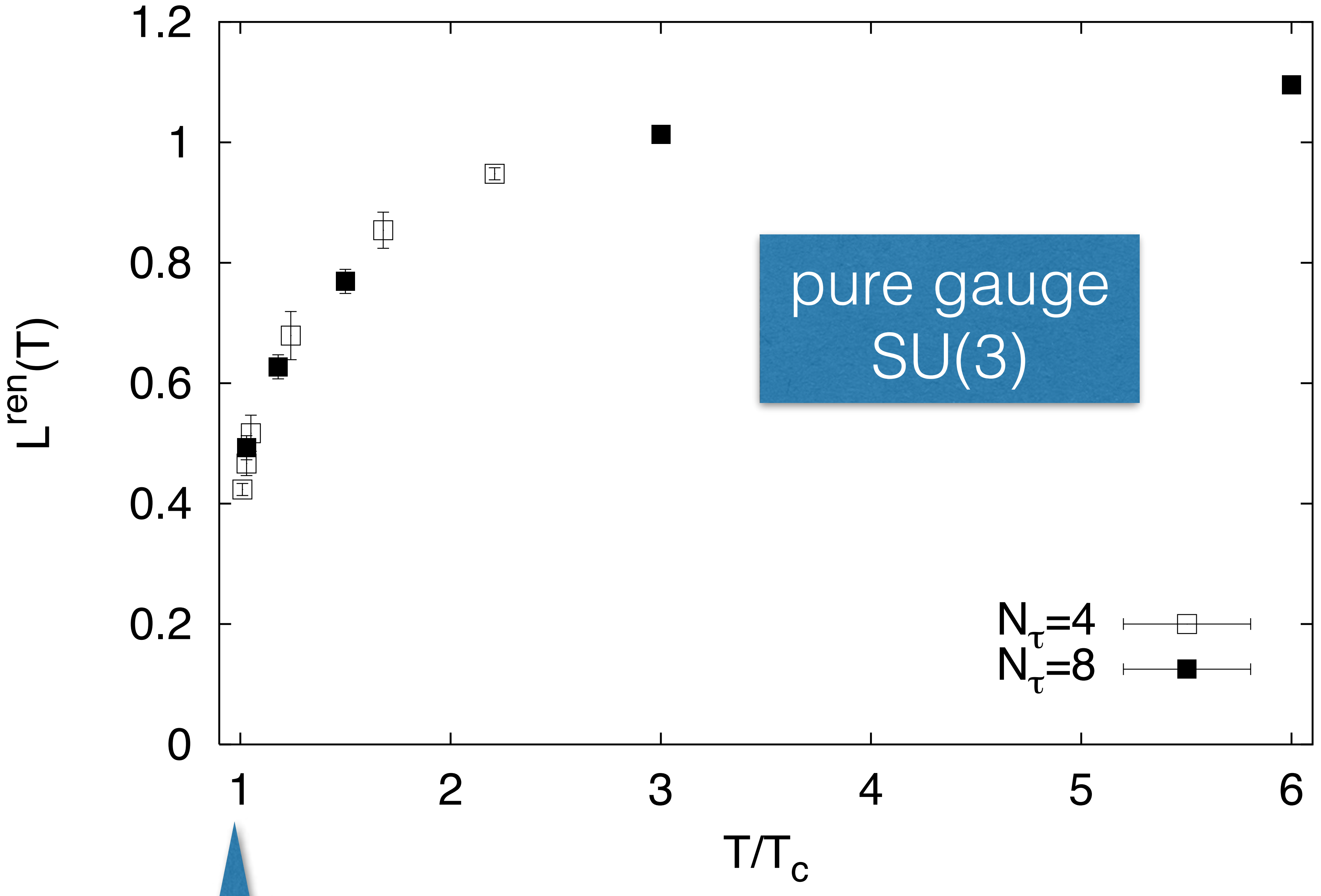




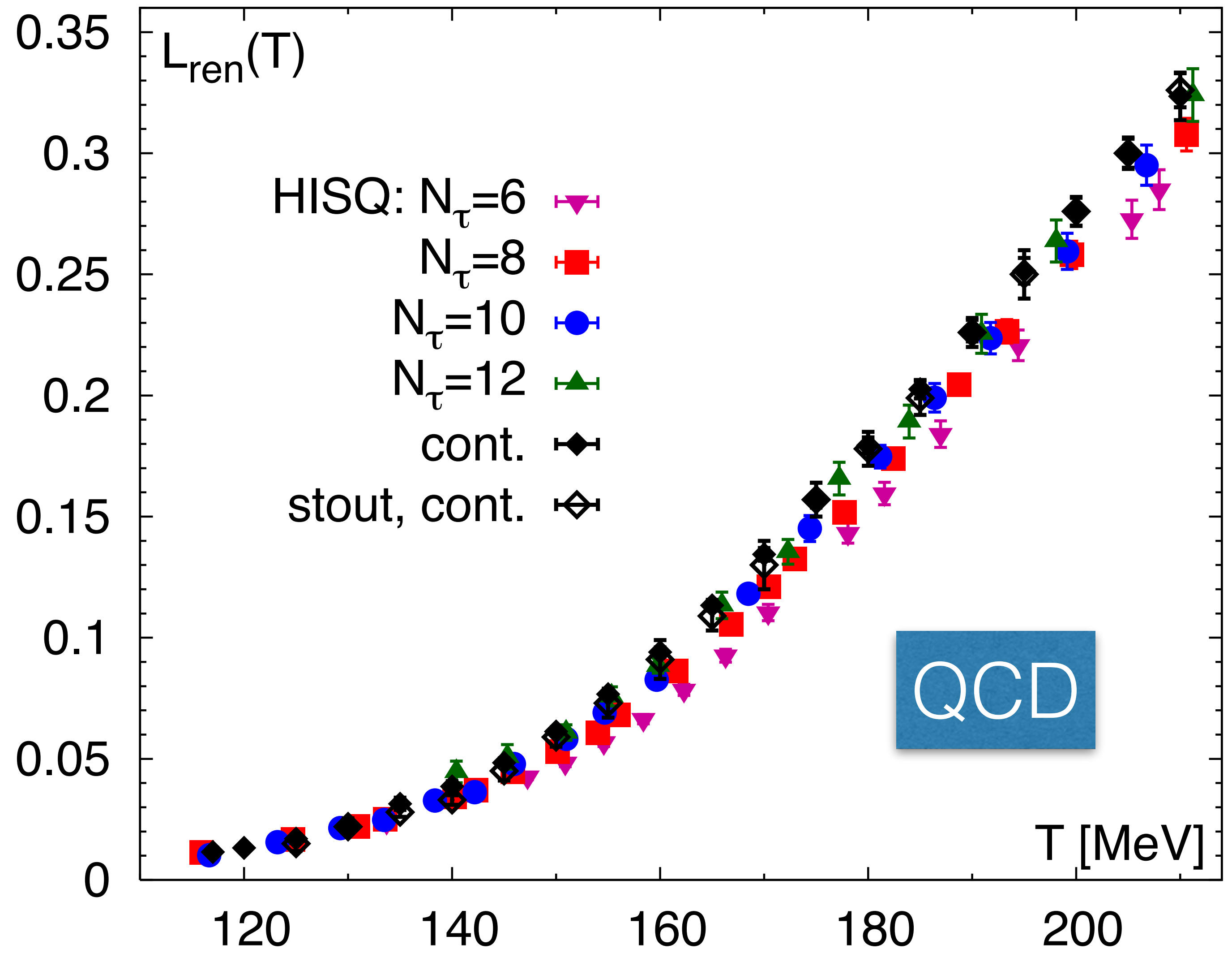
$$L = P = Pexp(i \oint A^a_\mu T^a dx_\mu)$$

$$L = diag(e^{i\mu_1}, e^{i\mu_2}, ...e^{i\mu_{Nc}})$$

$$\frac{1}{N_c} Tr(L) \sim e^{-F_q/T}$$



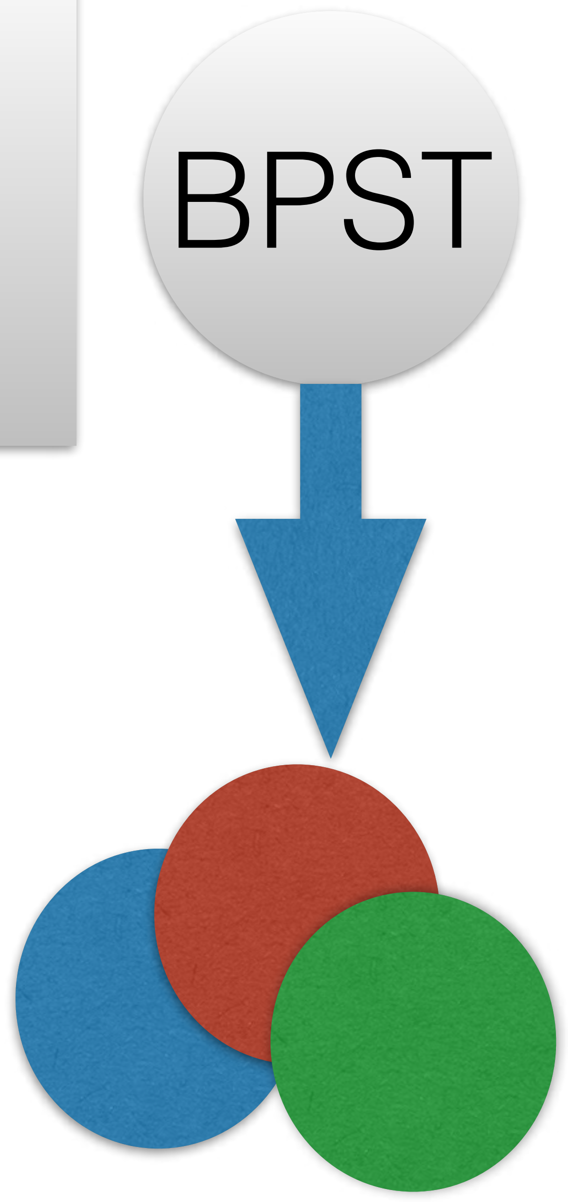
L jumps to zero
the first order transition



Kaczmarek et al 2002
Bazavov et al 2016

Pisarski ``semi-QGP" paradigm,
PNJL model

Non-zero Polyakov line splits instantons into Nc instanton-dyons (Kraan,van Baal, Lee,Lu 1998)



Explained mismatch of quark condensate in SUSY QCD

V.Khoze (jr) et al 2001

Explained confinement by back reaction to free energy

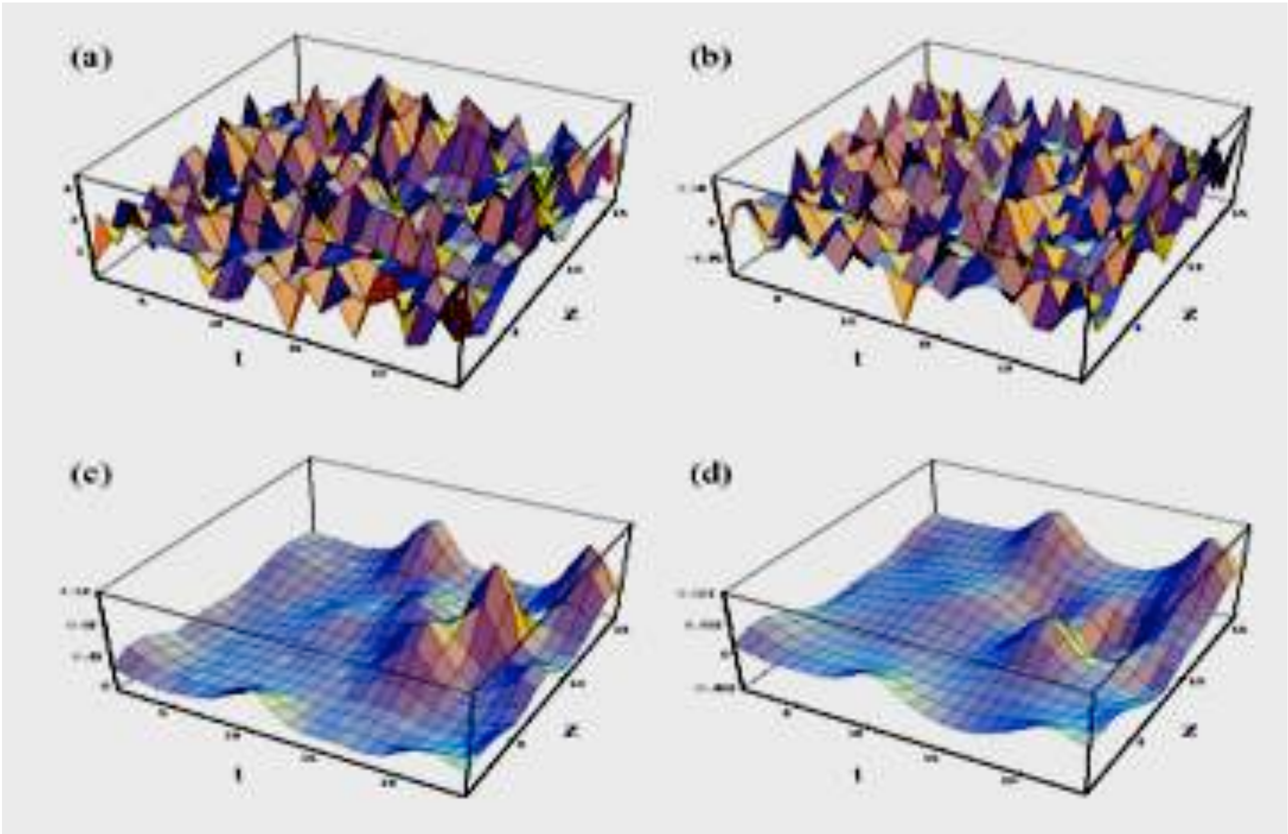
D.Diakonov 2012, Larsen+ES,Liu,Zahed+ES 2016

Explain chiral symmetry breaking in QCD and in setting with modified fermion periodicities

R.Larsen+ES 2017, Unsal et al 2017

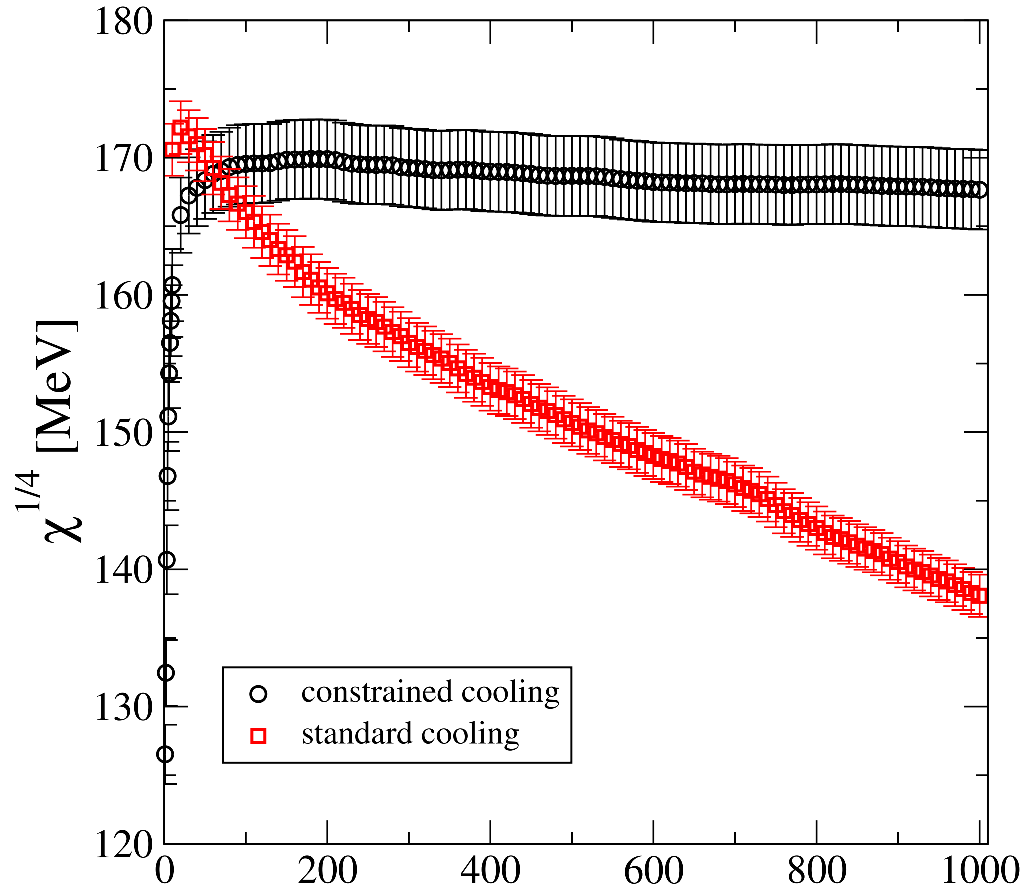

Pierre van Baal

“action cooling” is known to eliminate gluons and lead to instantons

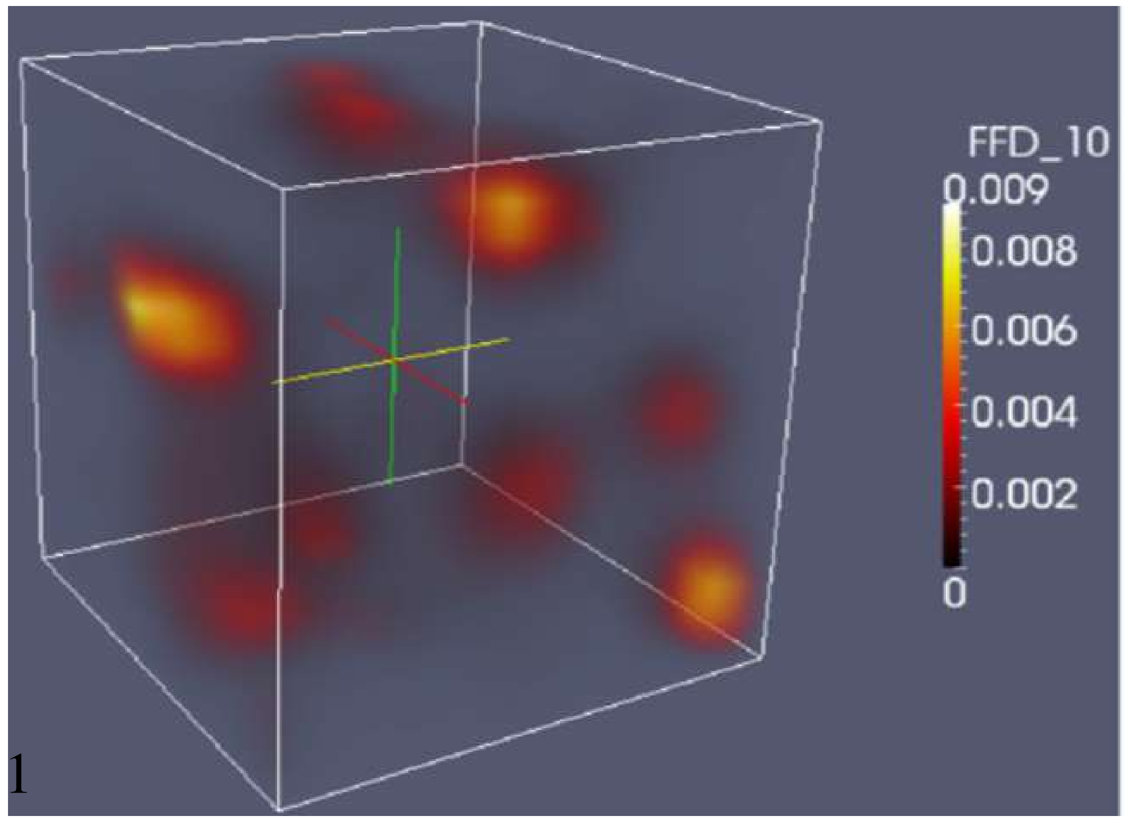


Negele et al, 97

perhaps dyons were first observed in “constrained cooling” preserving local L



Langfeld and Ilgenfritz, 2011



while the total top.charge of the box is always integer, local bumps are not! They are all (anti)selfdual But top charge and actions Were not integers!

a lot of work on finding instanton-dyons was done by C.Gatringer et al, Ilgenfritz et al

QCD with near-real quark masses, at T slightly above Tc

- *Phys.Lett.B* 794 (2019) 14-18 • e-Print: 1811.07914 [hep-lat]
- *Phys.Rev.D* 102 (2020) 3, 034501 • e-Print: 1912.09141

*** correlations with local Polyakov loop, in progress**

- Rasmus N. Larsen, Sayantan Sharma, Edward Shuryak

the cleanness case: domain wall fermions Q=1 configurations Nt=8,Nx=32, T/Tc=1,1.08

excellent agreement of the shape with analytic formulae

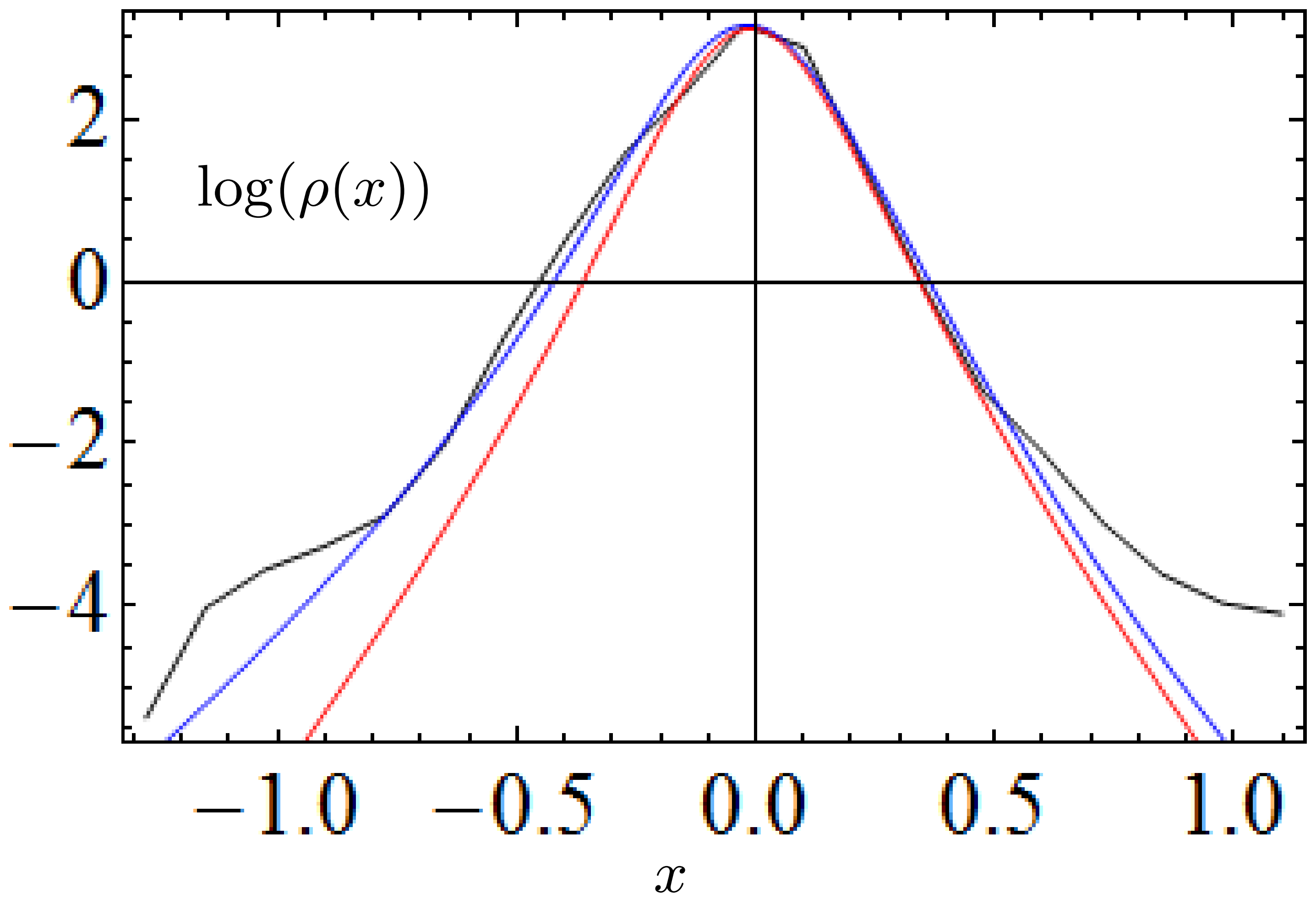
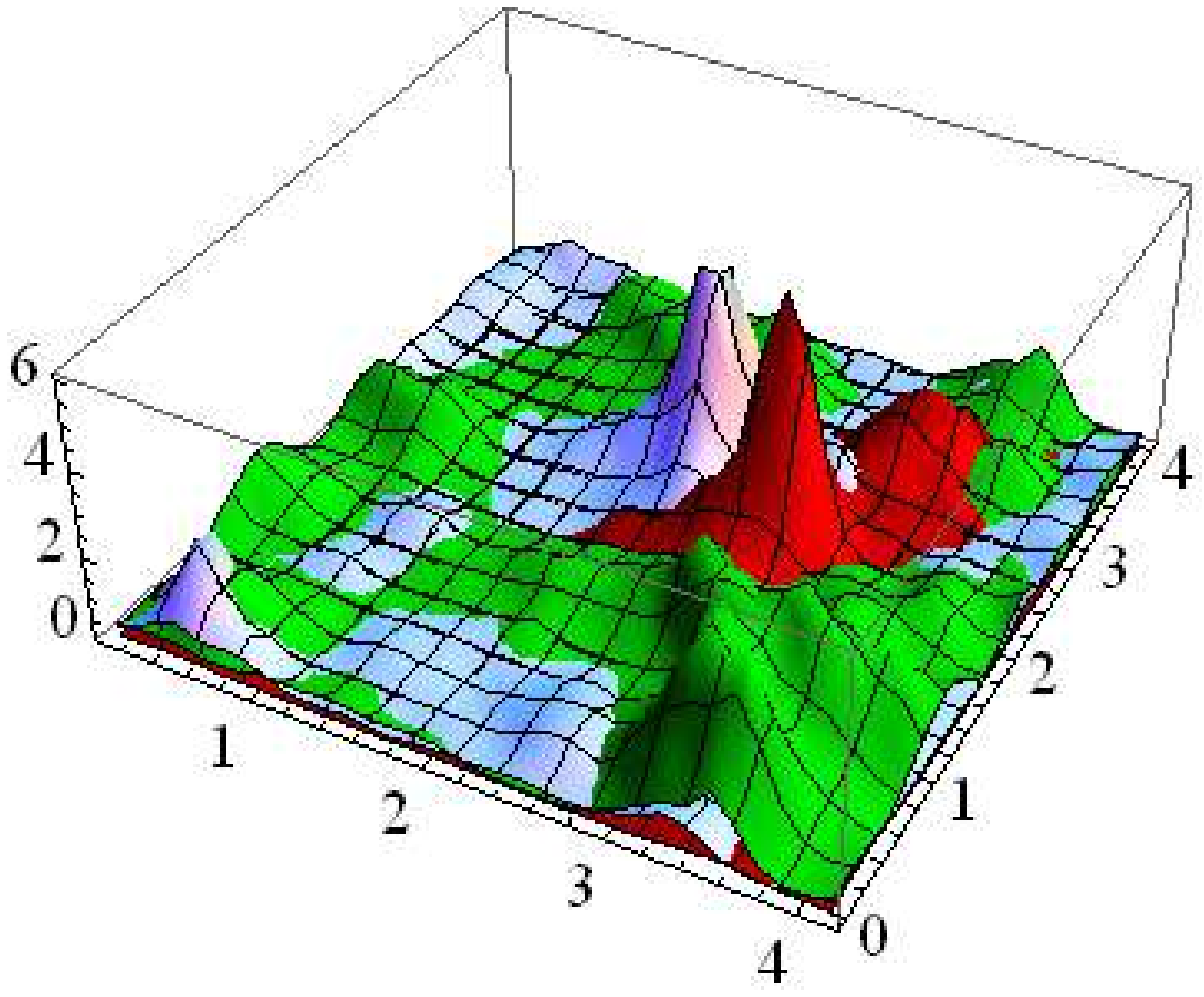


FIG. 8: $\log(\rho(x))$ of the zero mode of conf. 2960 at $\phi = \pi$ (black) and the log of the analytic formula for $P = 0.4$ and $P = 1$ though the maximum. $T = 1.08 T_c$. Red peak only has been scaled to fit in height, while blue peak uses the found normalization.



extracting the shape of the fermonic zero mode and modyfying the phase one can find all 3 dyons

FIG. 17: $\rho(x,y)$ of the zero mode of conf. 2660 at $T = T_c$. $\phi = \pi$(red), $\phi = \pi/3$(blue), $\phi = -\pi/3$(green). Peak height has been scaled to be similar to that of $\phi = \pi$.

We found that their fields interfere with each other the interaction between them Is in excellent agreement with van Baal analytic formulae

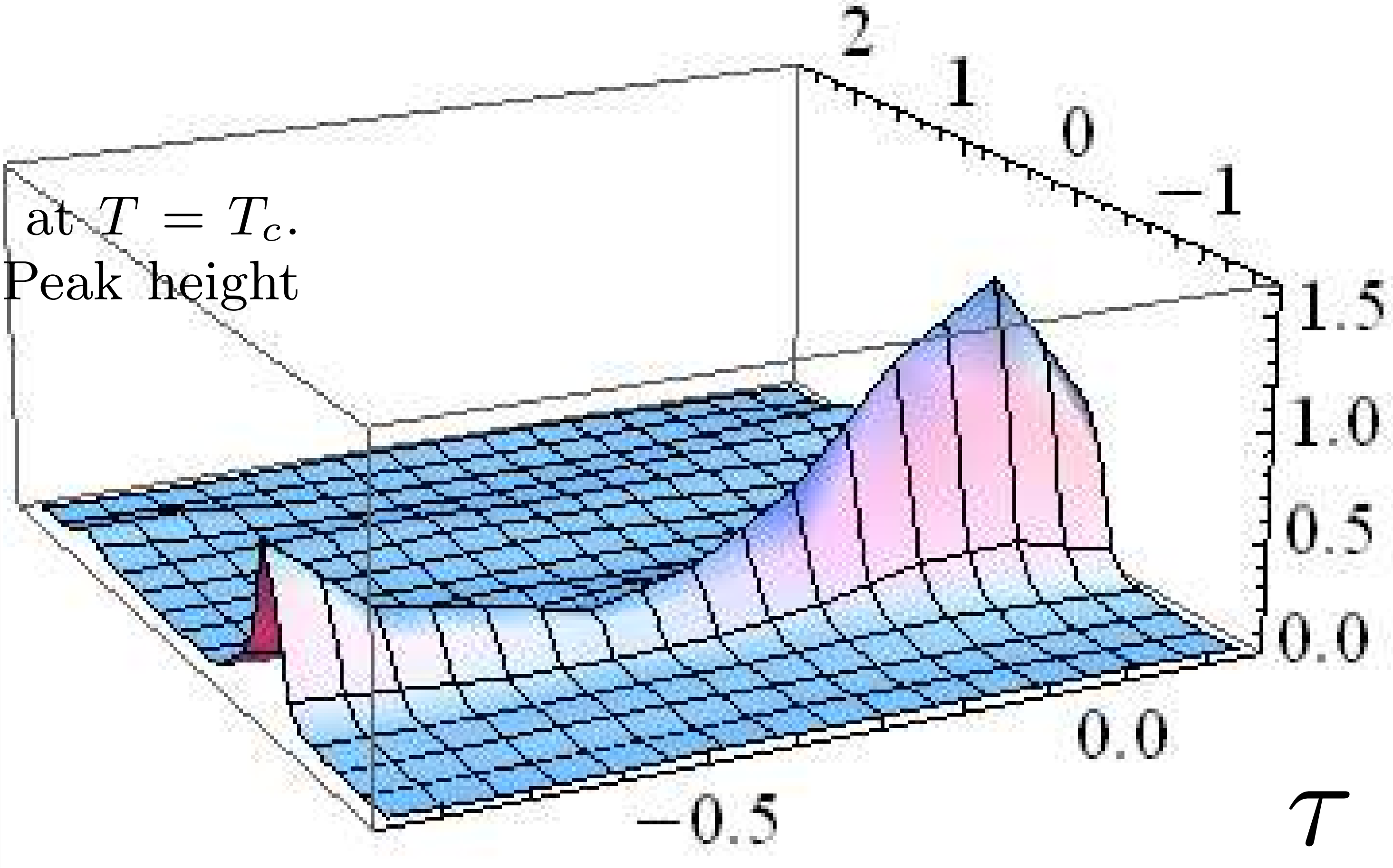


FIG. 3: $\rho(x,t)$ of the zero mode of conf. 2000 at $\phi = \pi/3$. $T = T_c$.

Ensemble of instanton-dyons

$$general\, SU(N_c)$$

the SU(2) case is the simplest

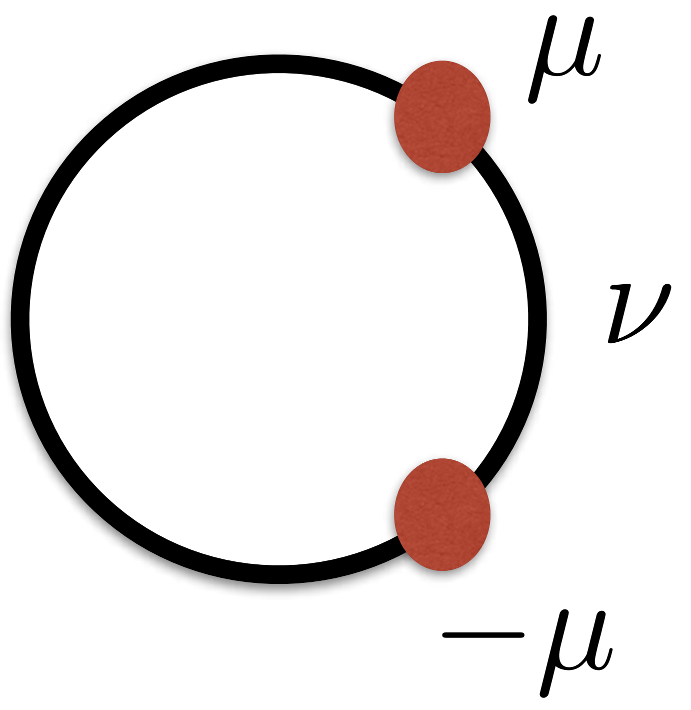
$$A_4 = 2\pi T diag(\mu_1, \mu_2, ...\mu_{N_c})$$

$$\nu_m = \mu_{m+1} - \mu_m$$

$$\sum_i \nu_i = 1$$

L type

$$\bar{\nu} = 1 - \nu$$



M type

$$S_i = \nu_i \frac{8\pi^2}{g^2} = \nu_i(\frac{11N_c}{3} - \frac{2N_f}{3})log(T/\Lambda)$$

$$A_4^a = \mp n_a v \Phi(vr)$$

$$A_i^a = \epsilon_{aij} n_j \frac{1 - R(vr)}{r}$$

together they make one instanton
instanton-dyons
=selfdual BPS mono

$$\vec{E} = \vec{B}$$

In SU(2) there are 4 types of dyons,
Electric and magnetic charges = +1,-1

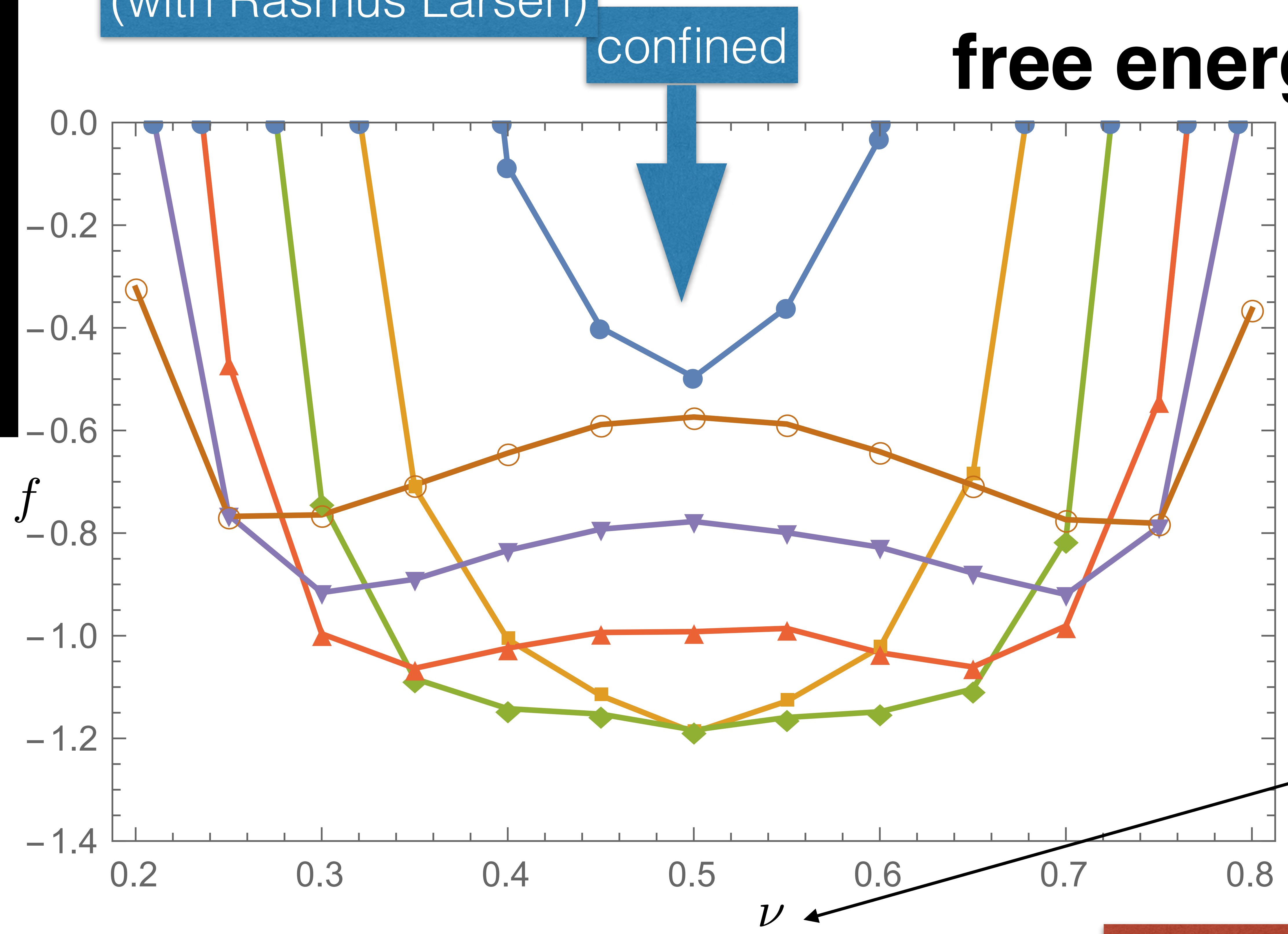
$$M, \bar{M}, L, \bar{L}$$

all one needs to do is to study their ensemble
interactions are Coulomb + one loop corrections

without dyons, there is GPY effective action which disfavors confinement its minimum is at nu=0

(with Rasmus Larsen)

free energy vs holonomy



$$\langle A_4^3 \rangle = v\frac{\tau^3}{2} = 2\pi T\nu\frac{\tau^3}{2}$$

$$< P >= cos(\pi\nu) \to 0$$

$$if\ \nu = 1/2$$

holonomy

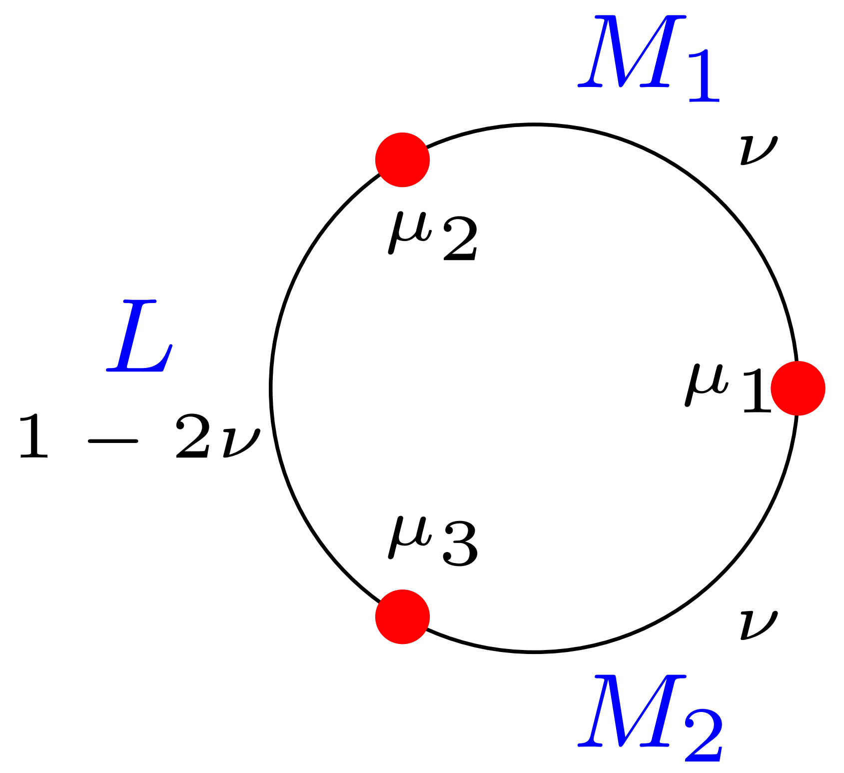
$$\nu = 0\ is\ the\ trivial\ case$$

$$\nu = 1/2\ confining$$

So, as a function of the dyon density the potential changes its shape and confinement takes place

In SU(2) pure gauge theory the deconfinement transition in the dyon ensemble is second order

red dots move to the right at higher T



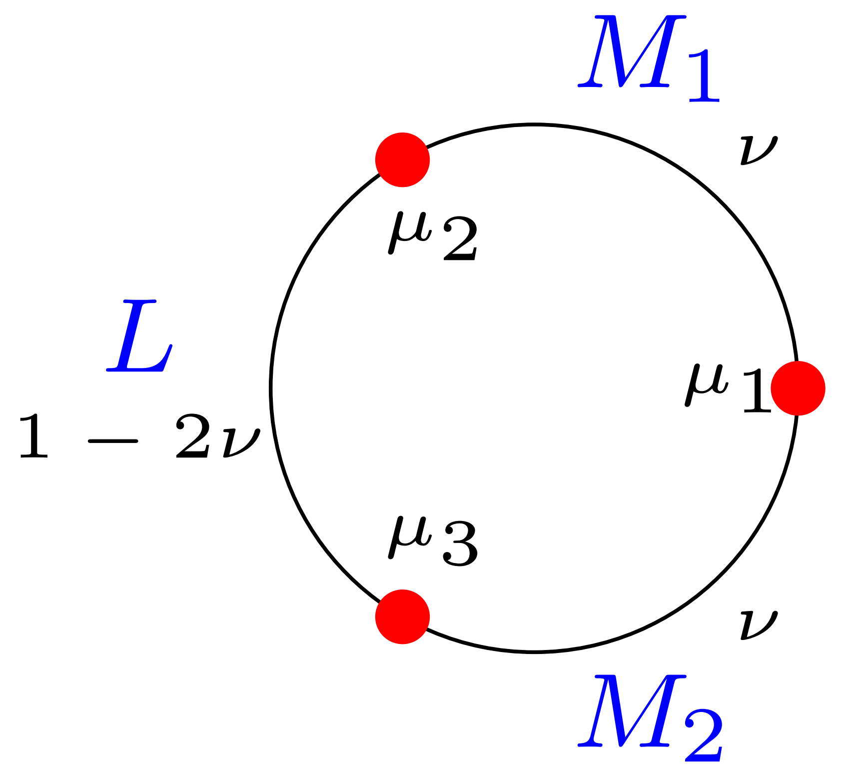
Deconfinement Phase Transition in the $SU(3)$ Instanton-dyon Ensemble

Dallas DeMartini and Edward Shuryak
Center for Nuclear Theory, Department of Physics and Astronomy,
Stony Brook University, Stony Brook NY 11794-3800, USA

arXiv:2102.11321v1 [hep-ph] 22 Feb 2021

critical:
jump in
holonomy

red dots move to the right at higher T



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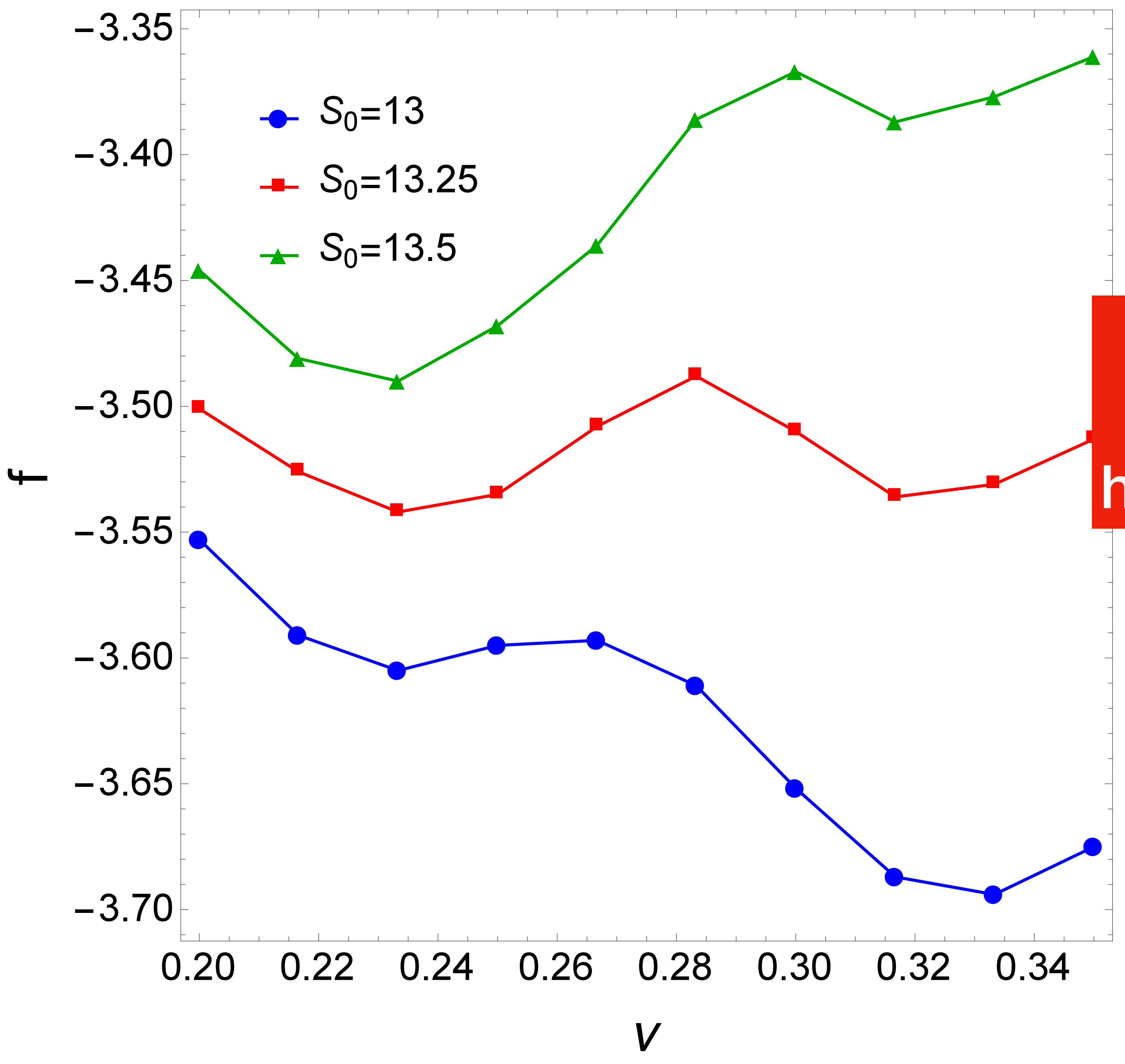
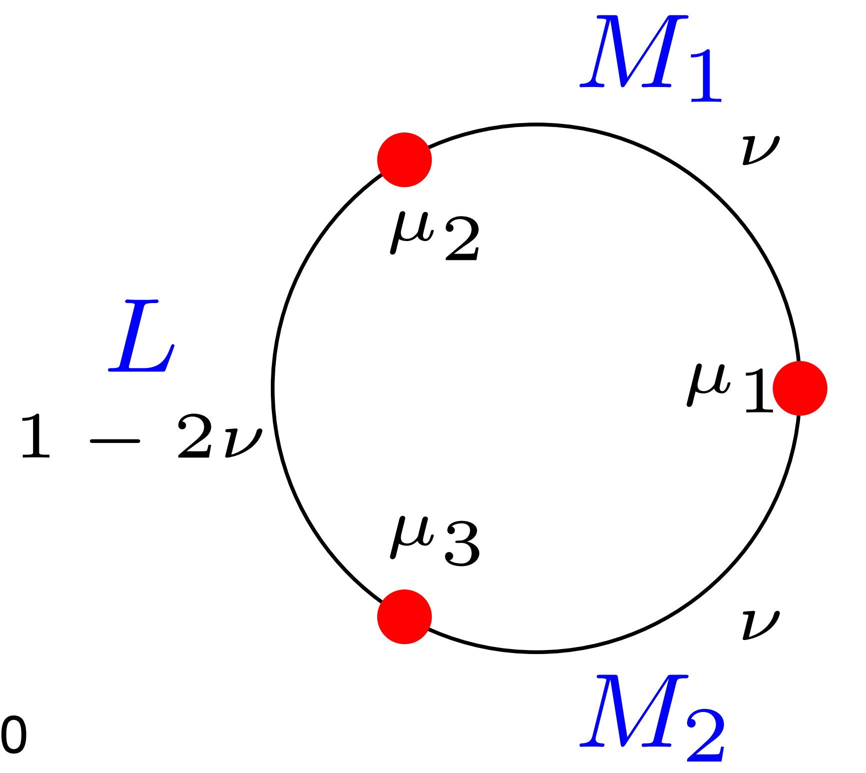


critical: jump in holonomy

FIG. 4. (Color online) Holonomy dependence of the minimum free energy density near the phase transition. Error bars not

red dots move to the right at higher T



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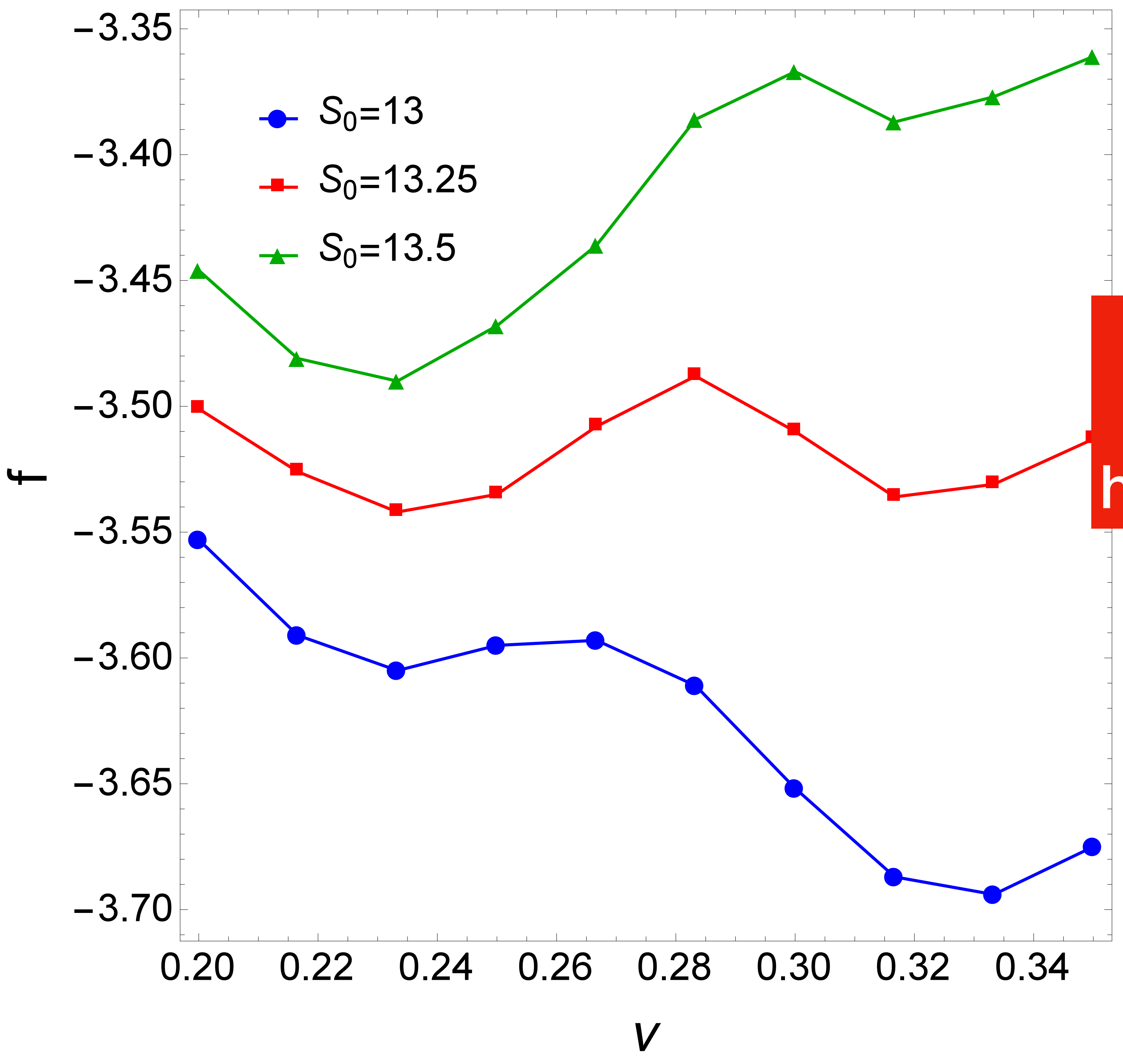


critical: jump in holonomy

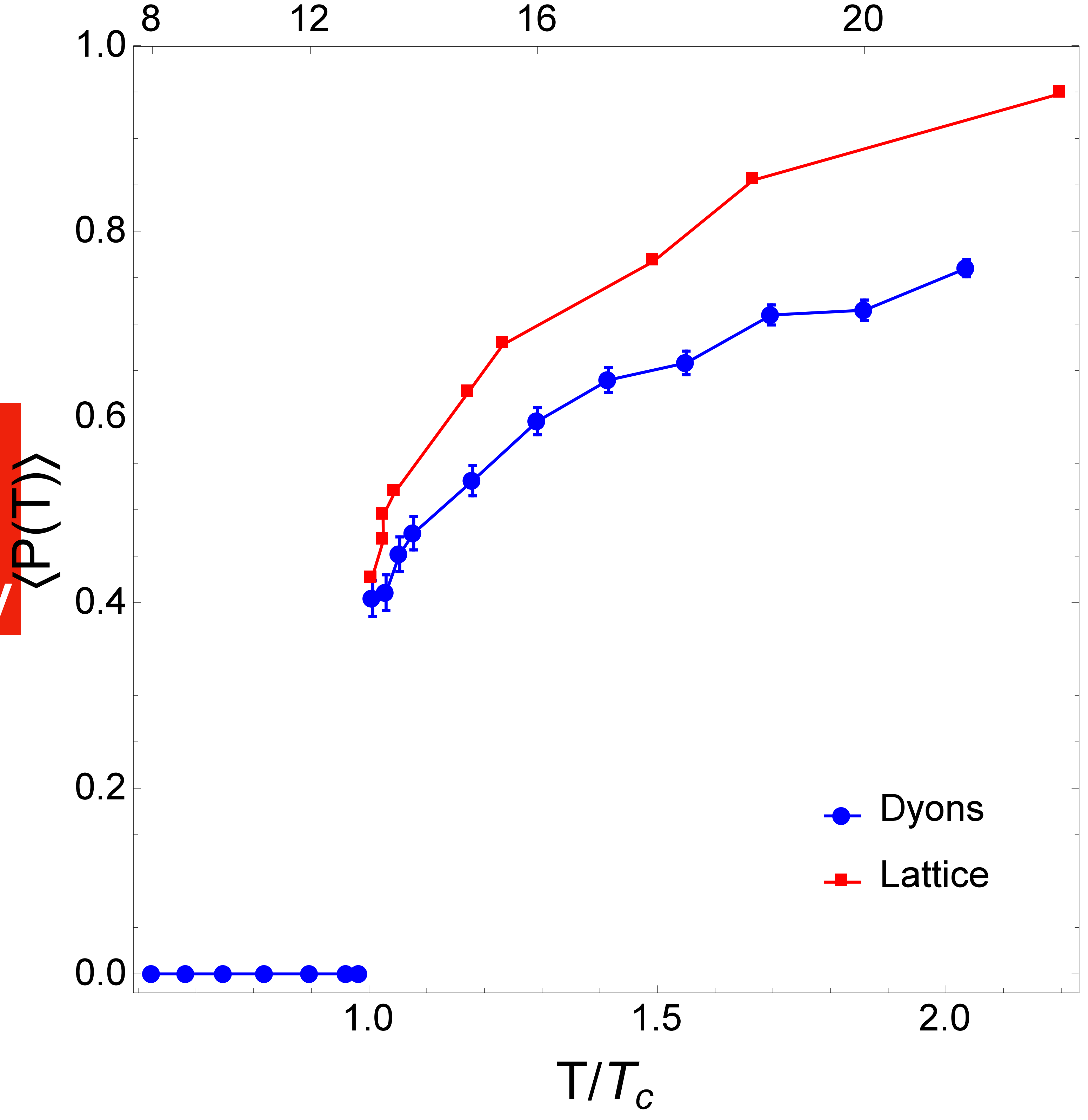
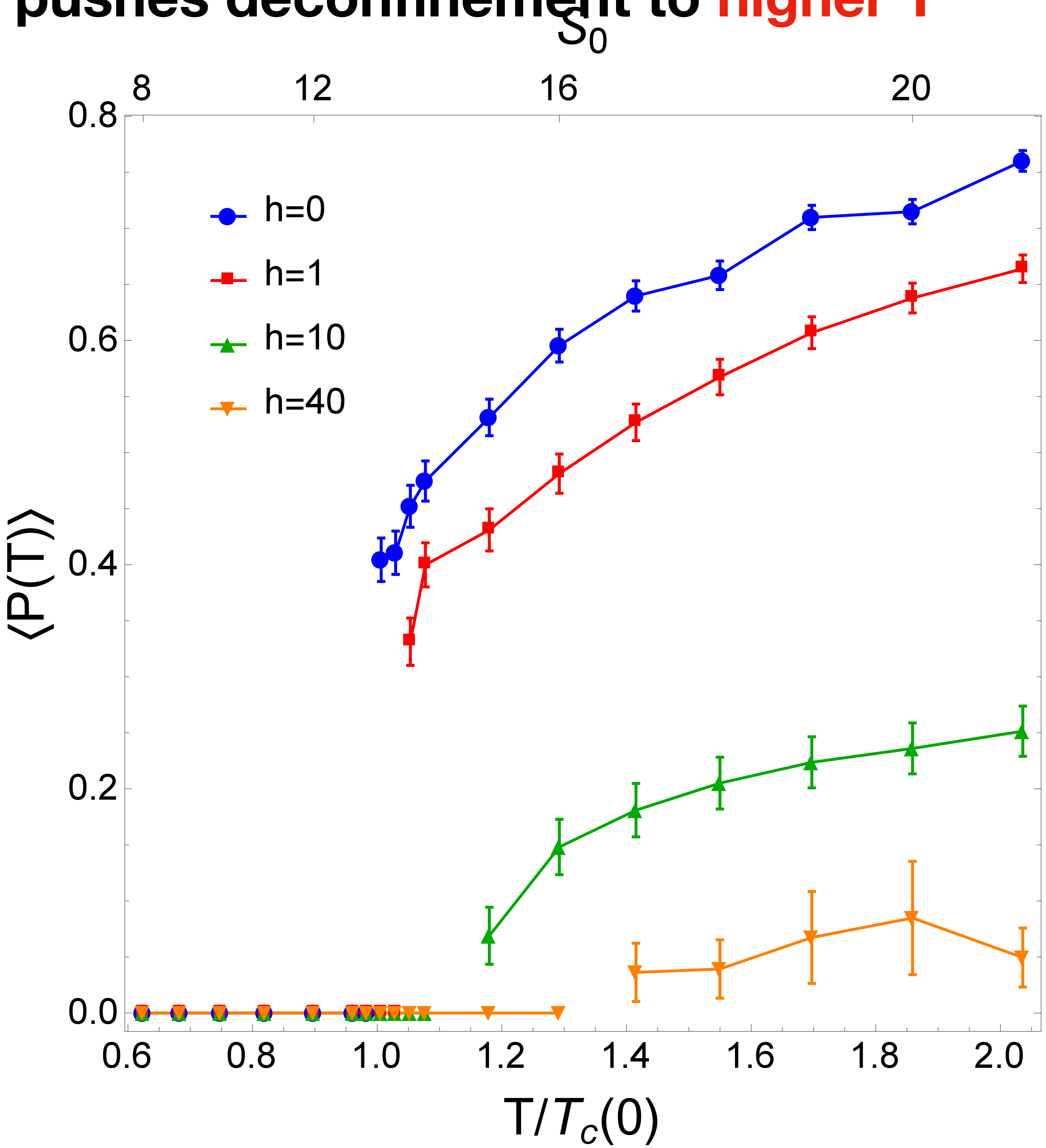


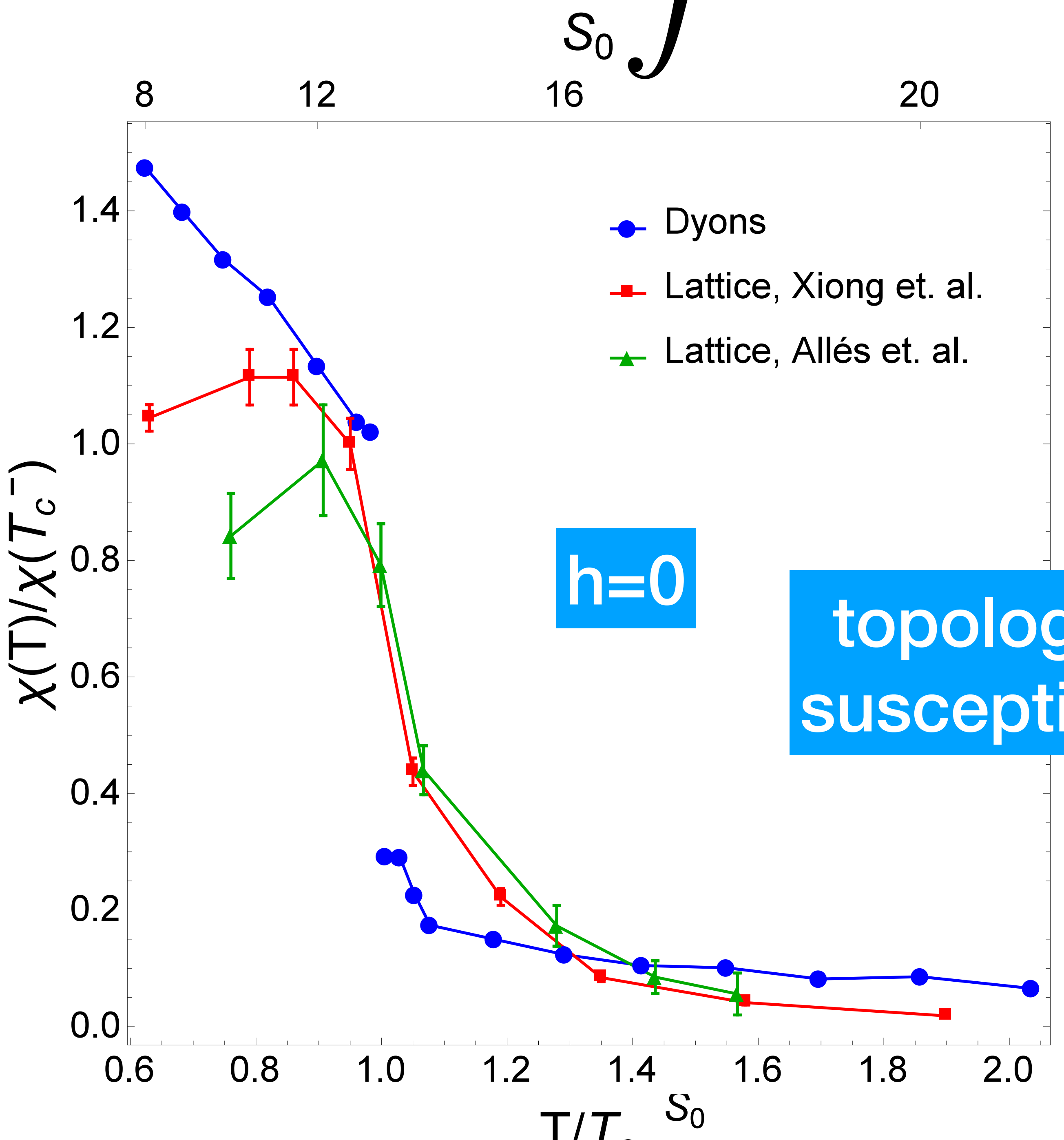
FIG. 4. (Color online) Holonomy dependence of the minimum free energy density near the phase transition. Error bars not

FIG. 6. (Color online) Temperature dependence of the average Polyakov loop of the dyon ensemble. Lattice data taken from Ref. [21] and shown without error bars. Error on lattice

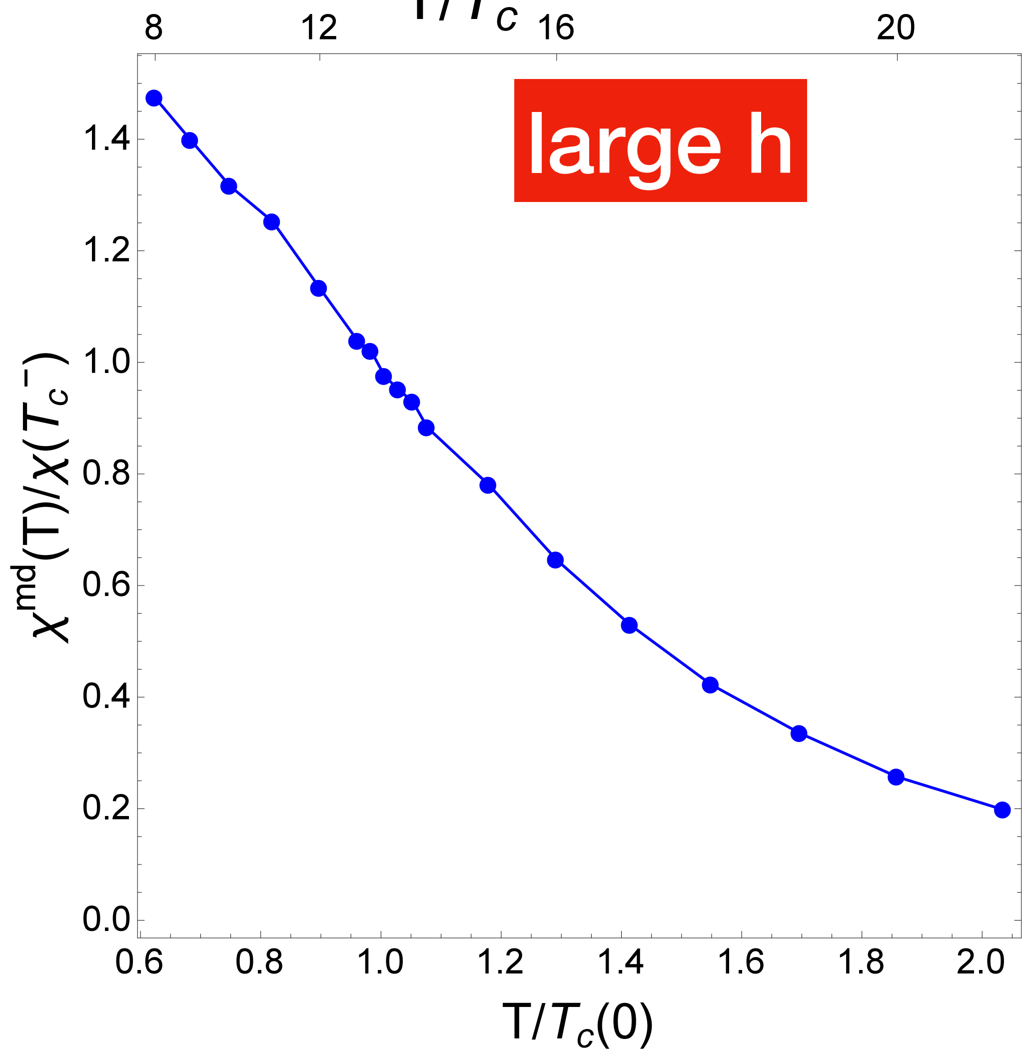
GAUGE THEORY deformation by powers of P in the action pushes deconfinement to higher T

$$\Delta S_{def} = h \int d^3x |P(\vec{x})|^2$$





topological susceptibility



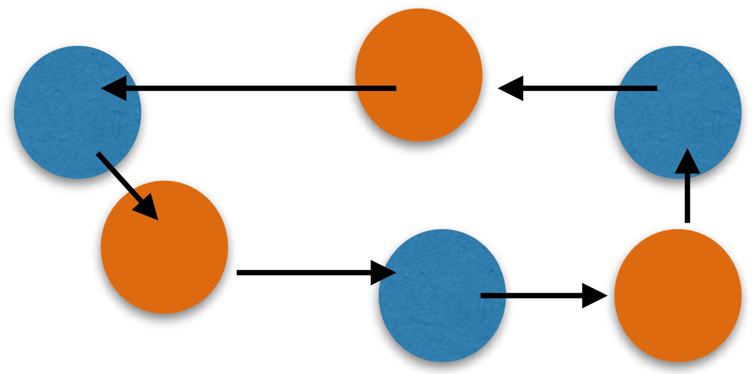
Instanton-dyon Ensemble with two Dynamical Quarks: the Chiral Symmetry Breaking

Rasmus Larsen and Edward Shuryak
Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794-3800, USA

This is the second paper of the series aimed at understanding of the ensemble of the instanton-dyons, now with two flavors of light dynamical quarks. The partition function is appended by the fermionic factor, $(detT)^{N_f}$ and Dirac eigenvalue spectra at small values are derived from the numerical simulation of 64 dyons. Those spectra show clear chiral symmetry breaking pattern at high dyon density. Within current accuracy, the confinement and chiral transitions occur at very similar densities.

arXiv:1511.02237v1 [hep-ph] 6 Nov 2015

$$|<\bar{\psi}\psi>| \;=\; \pi\rho(\lambda)_{\lambda\to 0, m\to 0, V\to\infty}$$



collectivized zero mode zone

dip near zero is a finite size effect

high density broken chiral sym

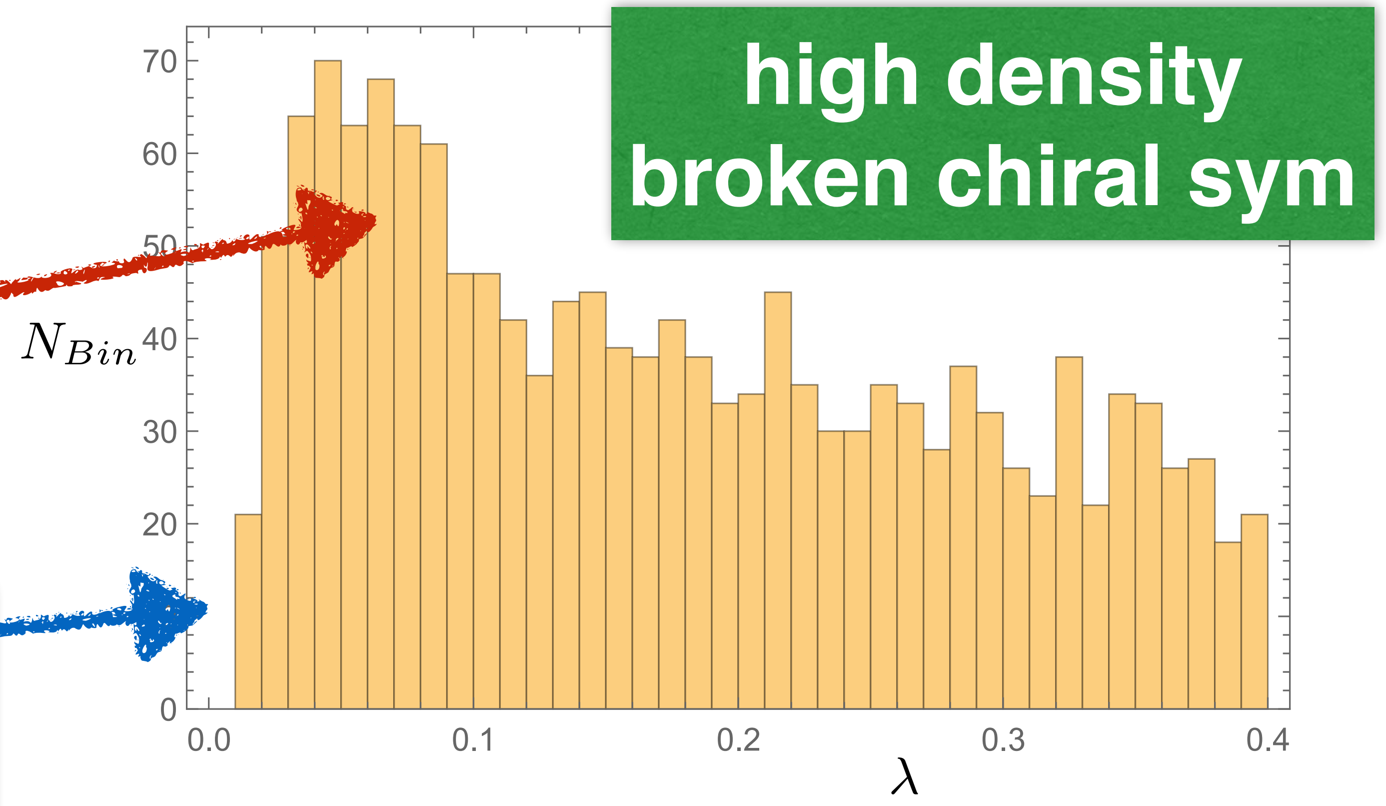


FIG. 1: Eigenvalue distribution for $n_M = n_L = 0.47$, $N_F = 2$ massless fermions.

low density unbroken chiral sum

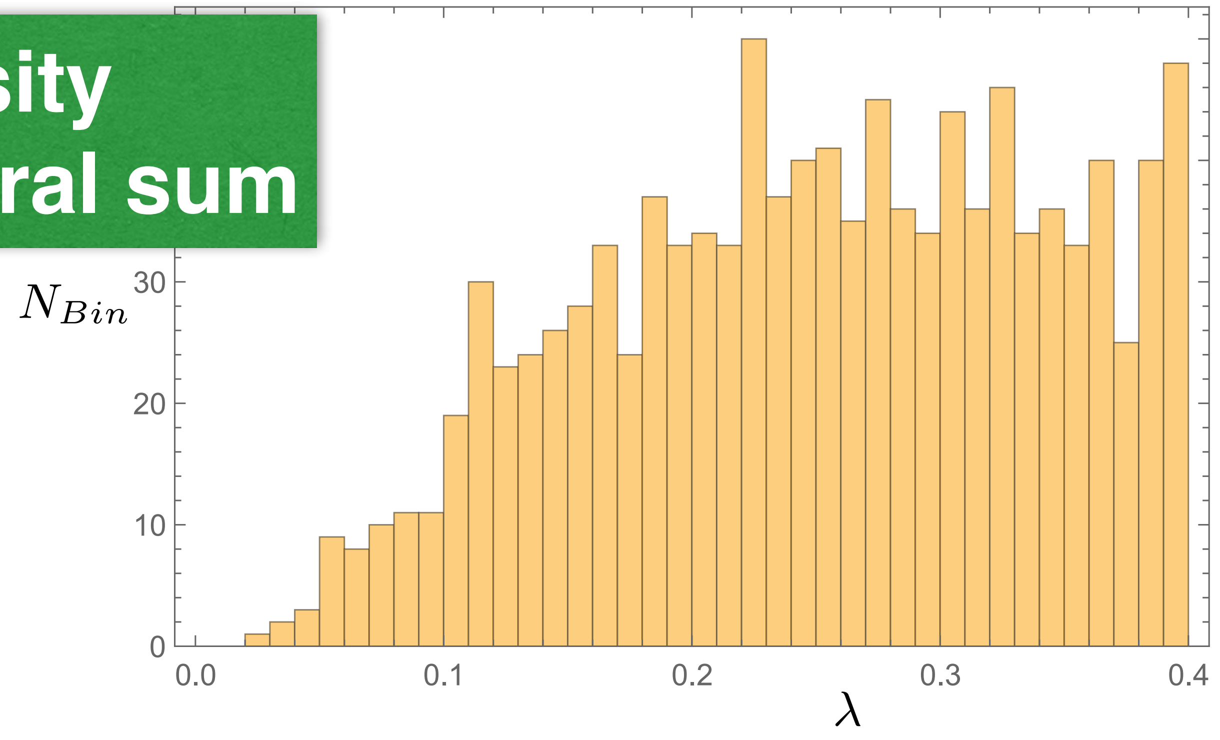


FIG. 2: Eigenvalue distribution for $n_M = n_L = 0.08$, $N_F = 2$ massless fermions.

extracting condensate
is far from trivial…

Chiral Symmetry Breaking and Confinement from an Interacting Ensemble of Instanton-dyons in Two-flavor Massless QCD

Dallas DeMartini and Edward Shuryak
Center for Nuclear Theory, Department of Physics and Astronomy,
Stony Brook University, Stony Brook NY 11794-3800, USA

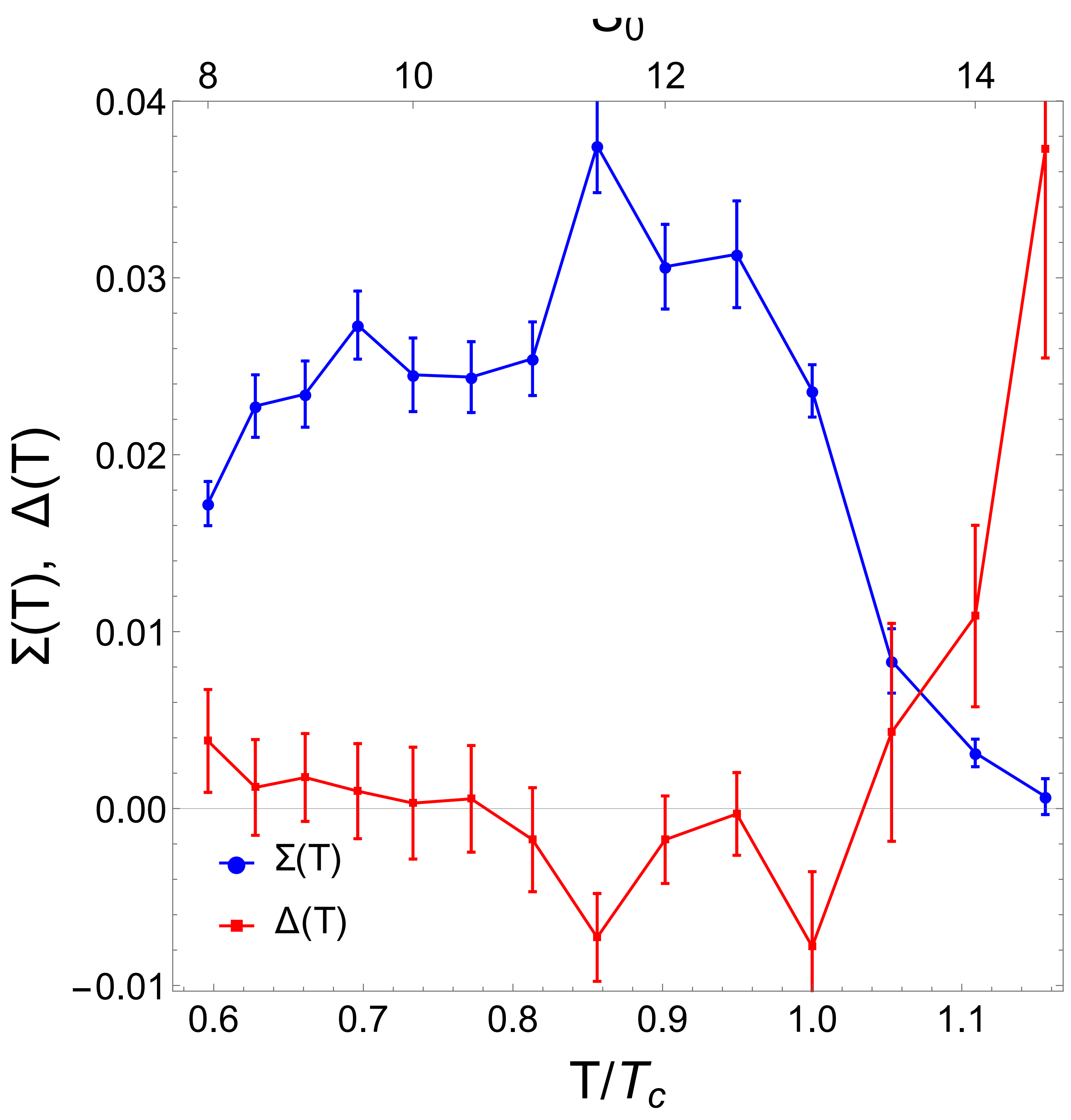


FIG. 8. (Color online) [Preliminary] The chiral quark condensate $\Sigma(T)$ and the eigenvalue gap $\Delta(T)$ as functions of the temperature.

$\langle\bar{q}q\rangle$

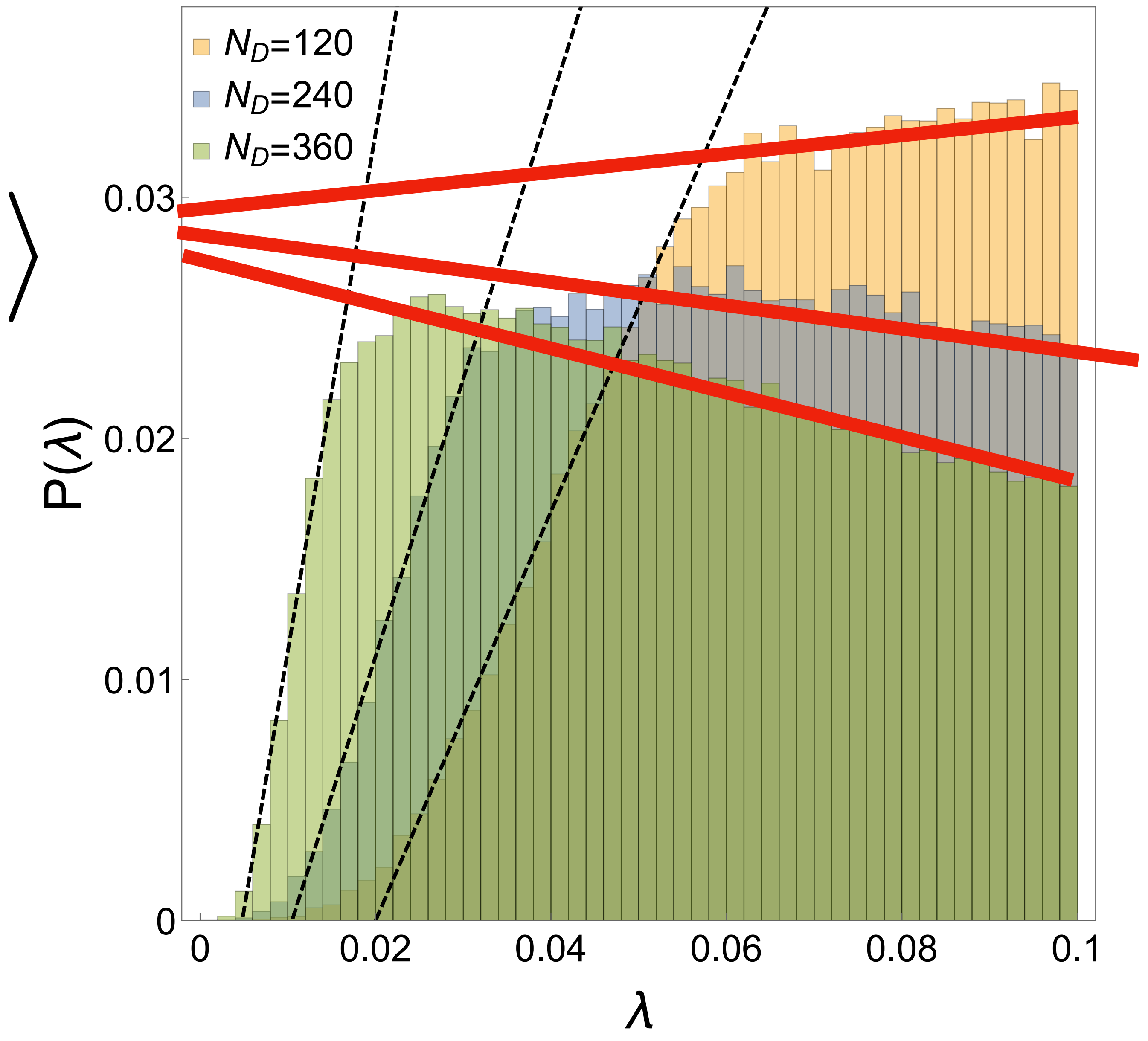


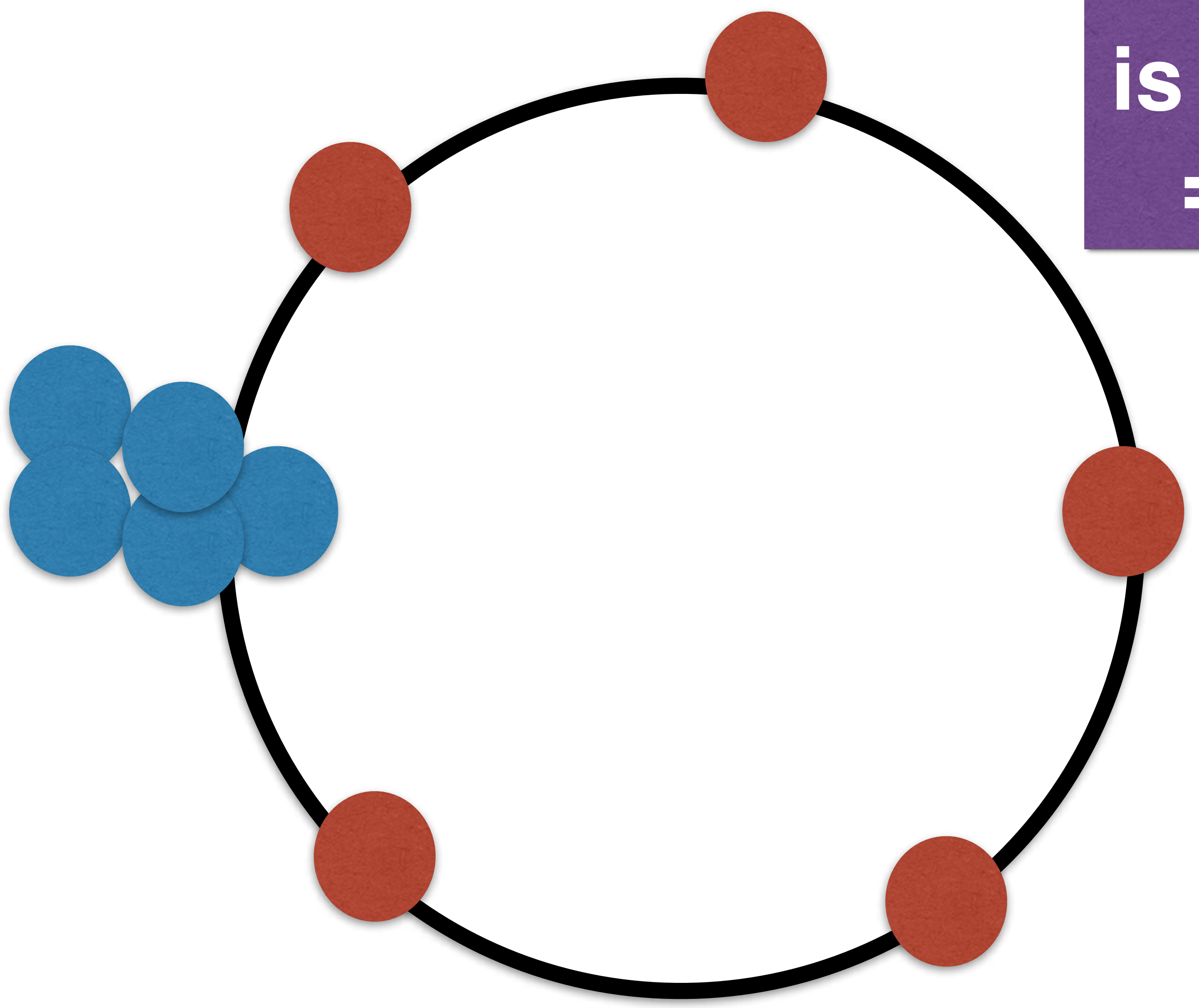
FIG. 7. (Color online) Eigenvalue distributions at $S_0 = 8.5$ for three different ensemble sizes. Dashed lines represent fits to the approximately-linear portion of the distribution near zero. The eigenvalue gaps are given by the x-intercepts of the fits. Note that the relative normalization of the distributions does not affect results.

Casher-Banks quark condensate is obtained by linear extrapolation to 0

the gap scales as 1/V and is therefore a purely finite volume effect

quarks with variable periodicity condition
(over the Matsubara time)

Ordinary Nc=Nf=5 QCD

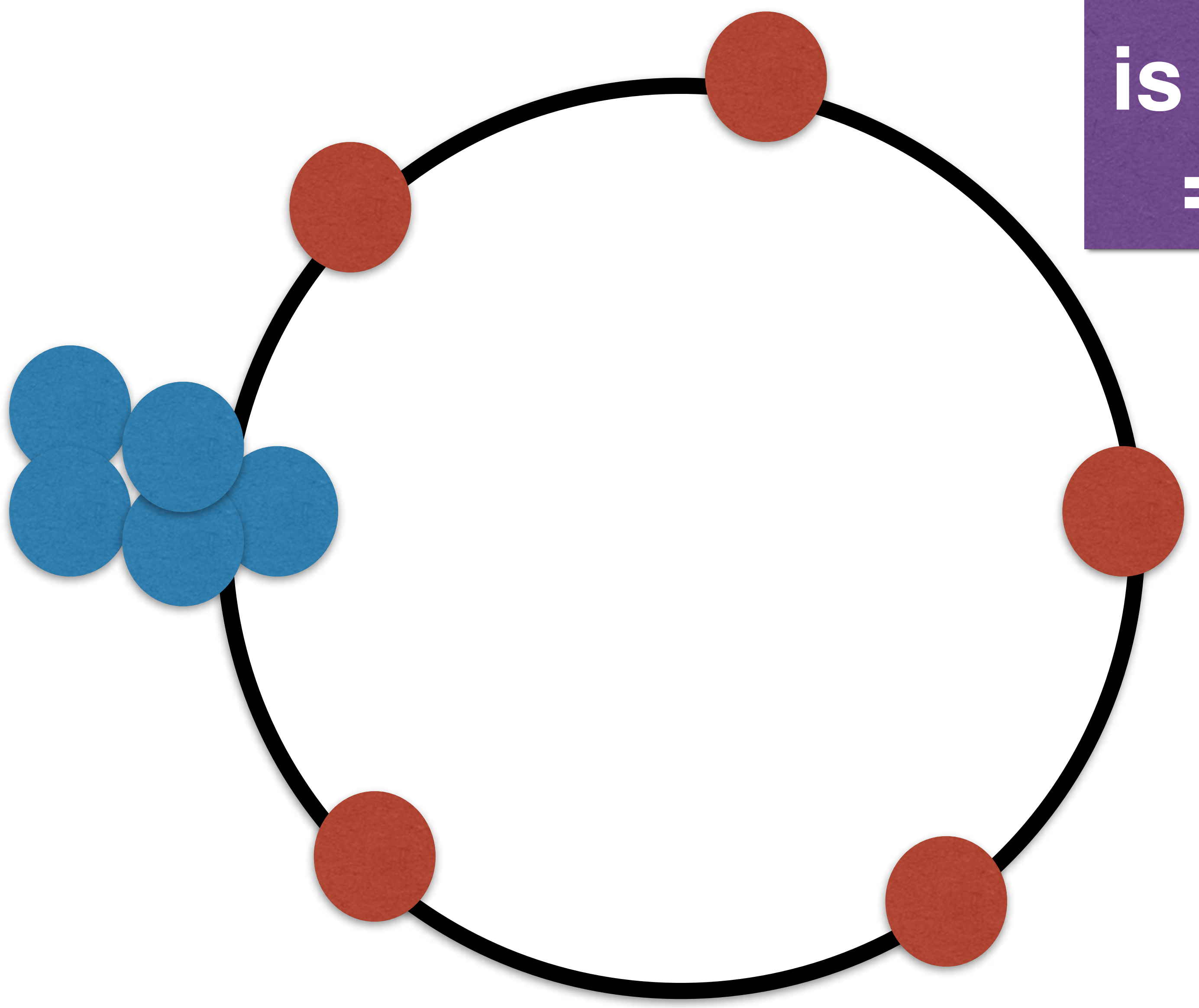


P without a trace
is a diagonal unitary matrix
=> Nc phases (red dots)

quark periodicity
phases => Nf blue dots
are in this case all =pi
quarks are fermions

as a consequence,
out of 5 types of instanton-dyons
only one L has normalizable zero modes

Ordinary Nc=Nf=5 QCD



P without a trace is a diagonal unitary matrix => Nc phases (red dots)

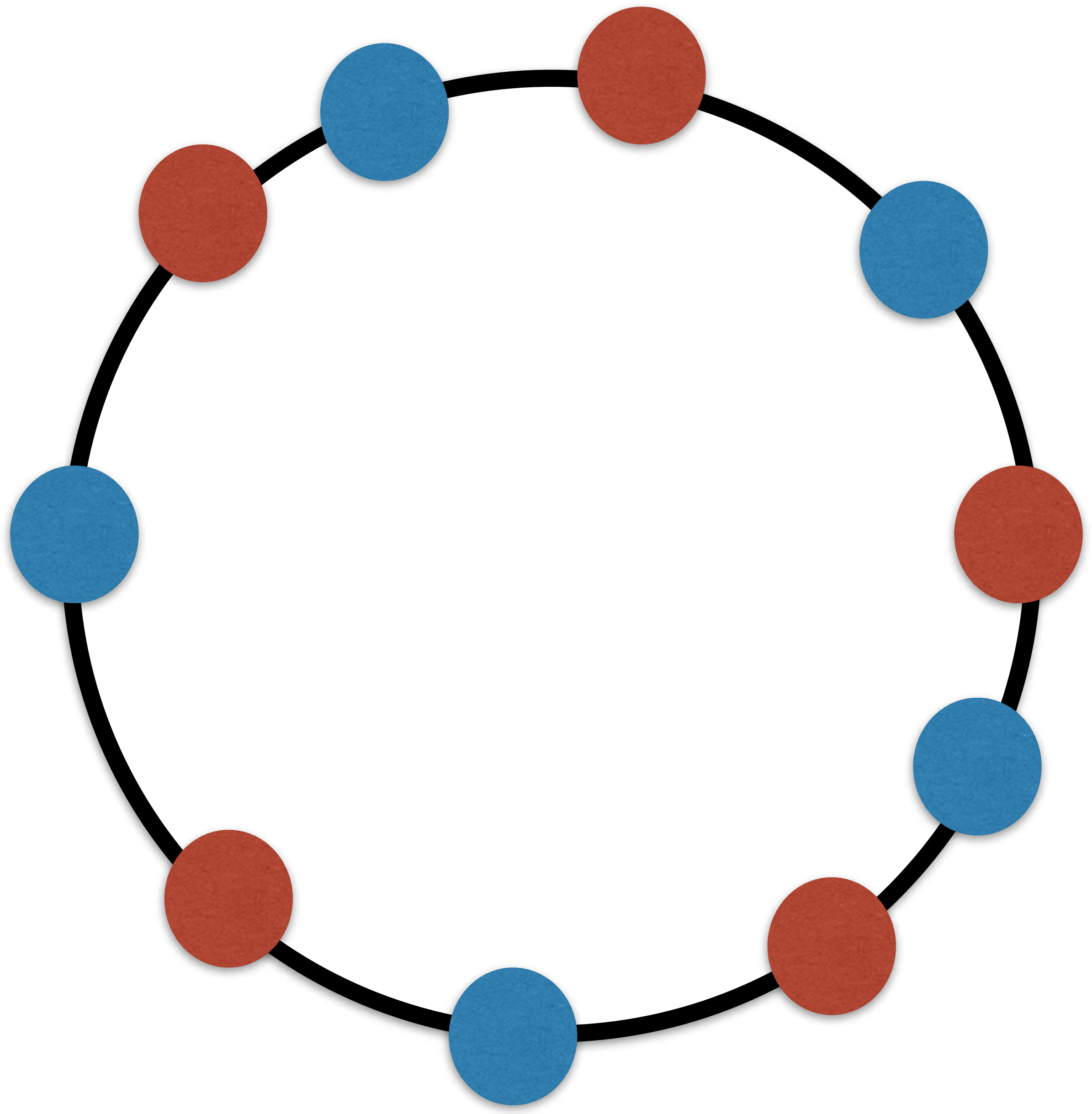
quark periodicity phases => Nf blue dots are in this case all =pi quarks are fermions

as a consequence, out of 5 types of instanton-dyons only one L has normalizable zero modes

But one can deform QCD moving fermion phases (blue dots) as we like!

still Nc=Nf=5 but with “most democratic” arrangement ZN-symmetric QCD

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki and M. Yahiro, J. Phys. G 39, 085010 (2012).



quark periodicity phases => Nf blue dots are in this case flavor-dependent

In this case each dyon type has one zero mode with some quark (flavor) =>Nc independent topological ZMZ’s!

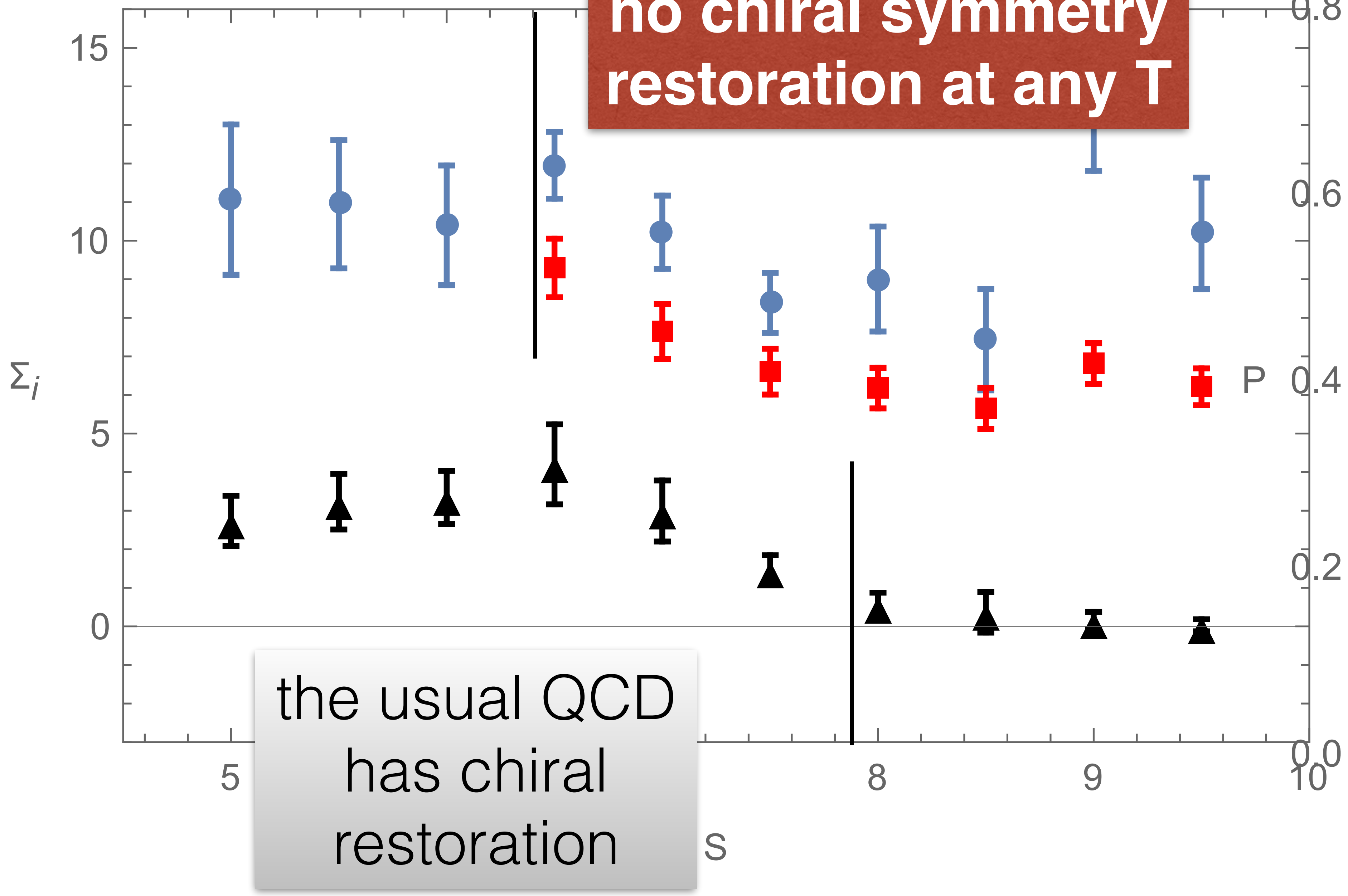
Second deformation: QCD2 and Z2QCD are dramatically different!

symmetric phase

u=>M

d=>L $< \bar{u}u > \neq < \bar{d}d >$

no chiral symmetry restoration at any T



the usual QCD has chiral restoration

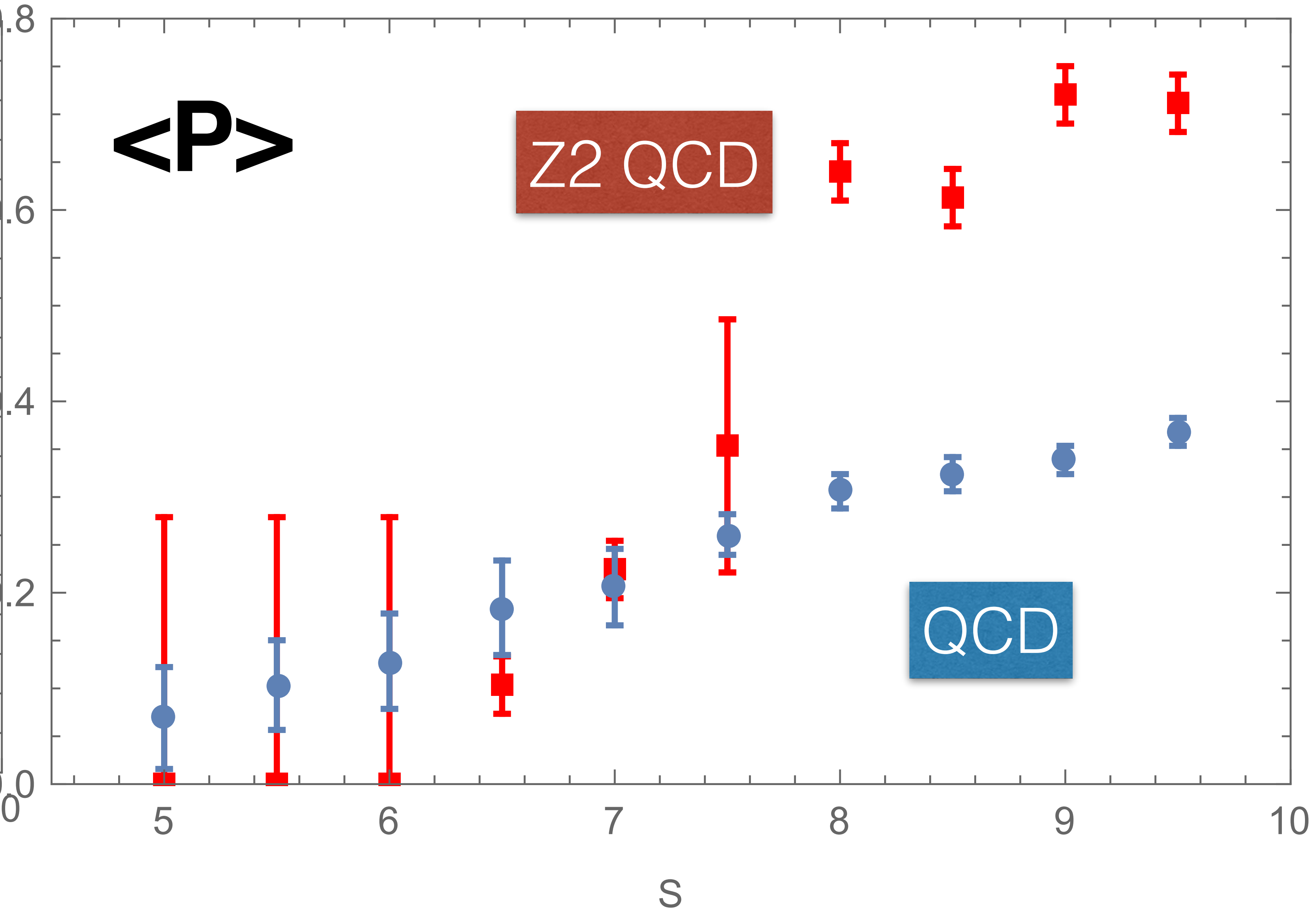


FIG. 6: Chiral condensate generated by u quarks and L dyons (red squares) and d quarks interacting with M dyons (blue circles) as a function of action S, for the Z_2-symmetric model. For comparison we also show the results from II for the usual QCD-like model with $N_c = N_f = 2$ by black triangles.

confining phase gets much more robust: strong first order mixed phase (flat F) is observed at medium densities

note the condensate is much larger for Z2?

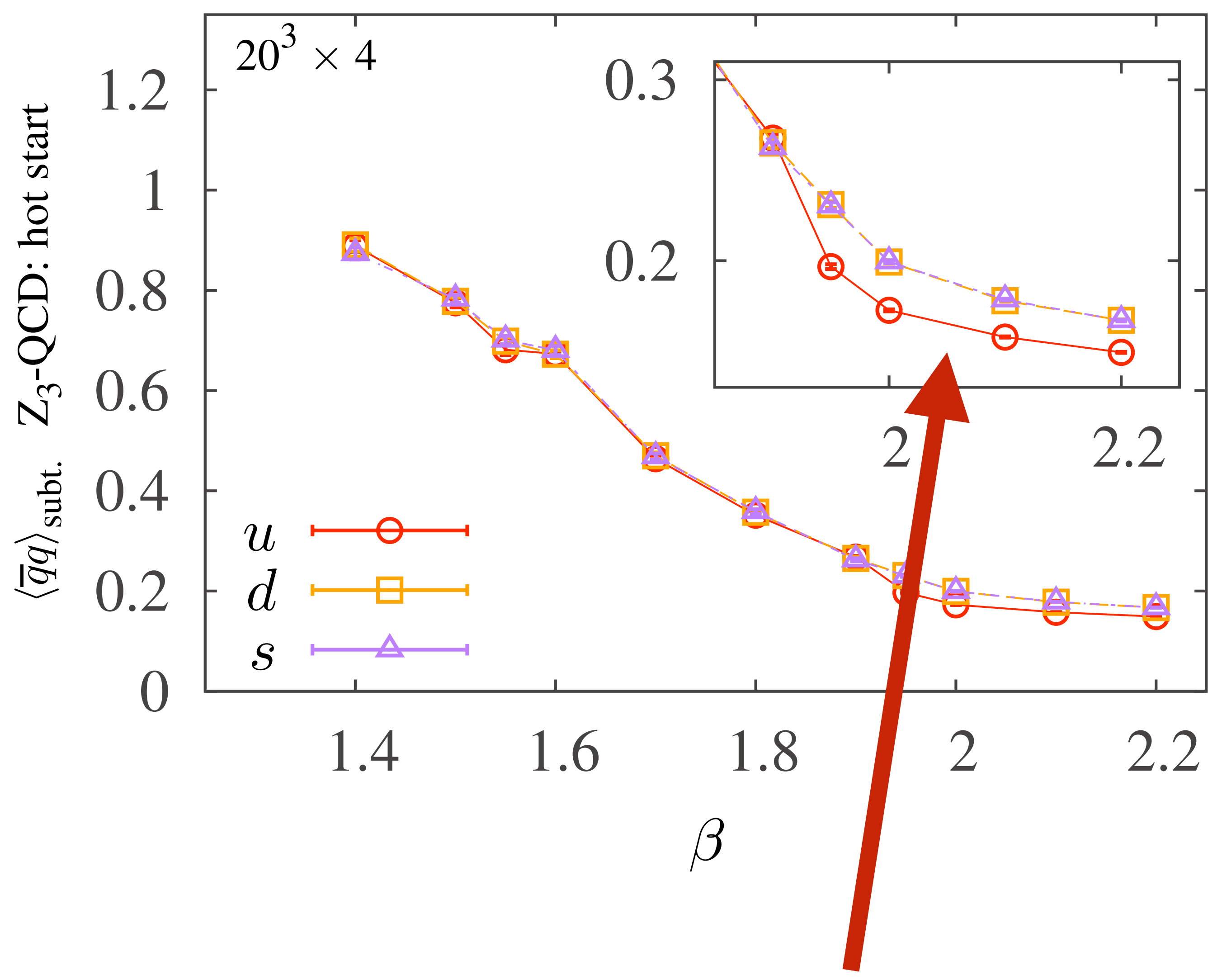
lattice study of Z3 QCD

Lattice study on QCD-like theory with exact center symmetry

Takumi Iritani*
Yukawa Institute for Theoretical Physics, Kyoto 606-8502, Japan

Etsuko Itou†
High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan

Tatsuhiro Misumi‡
Department of Mathematical Science, Akita University,



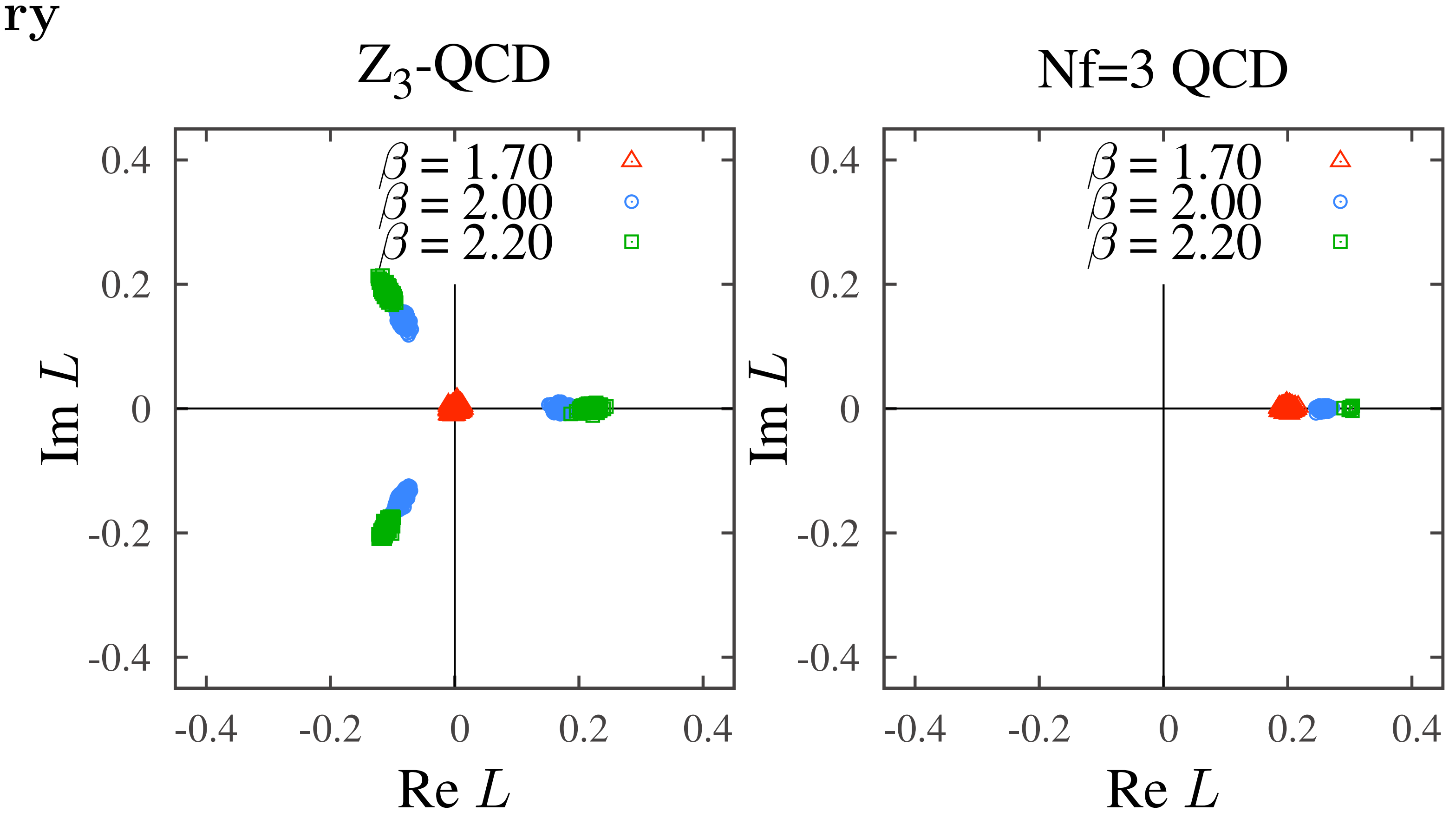
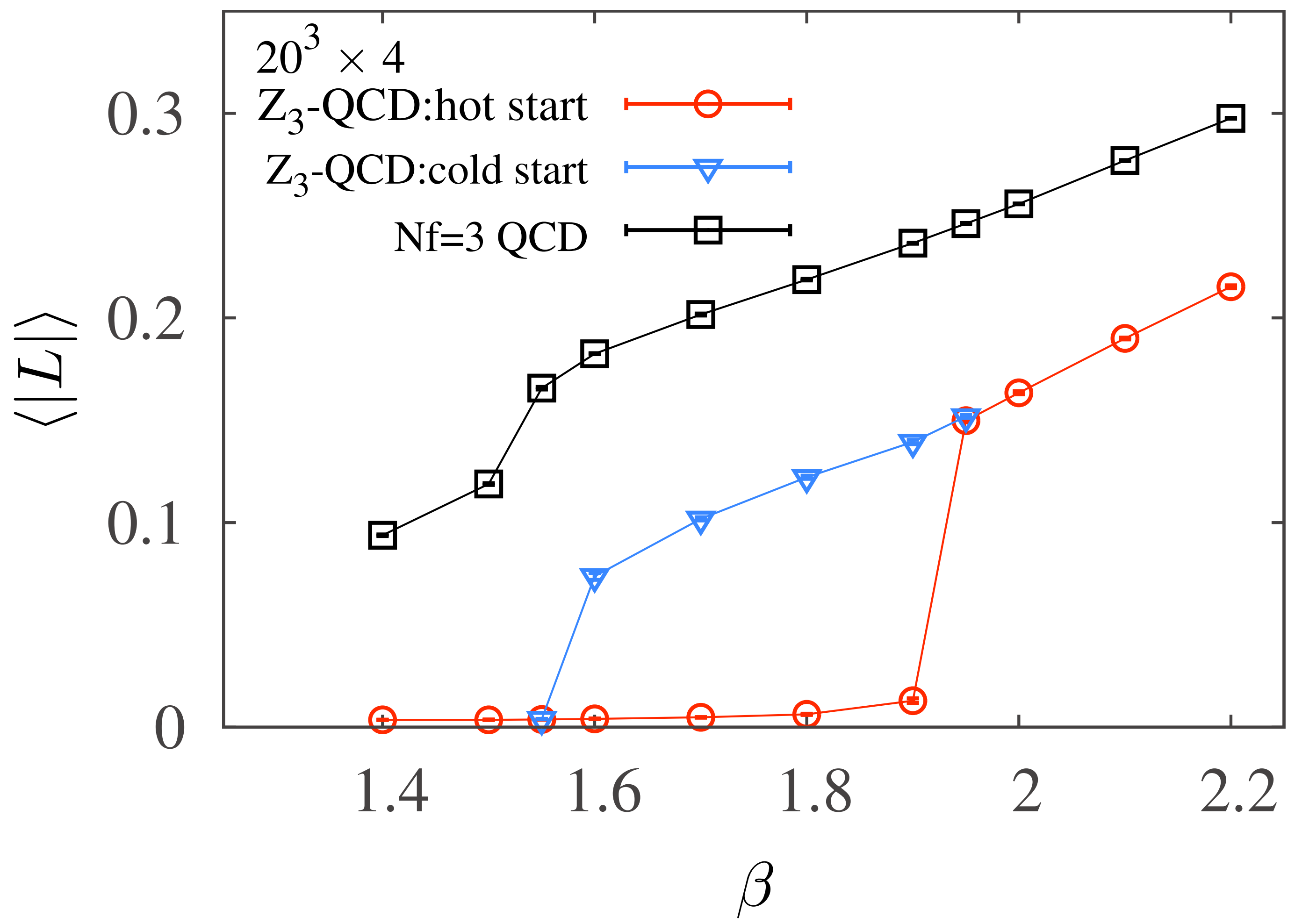


FIG. 1: Polyakov loop distribution plot in Z_3-QCD (left) and the standard three-flavor QCD (right). Based on $16^3 \times 4$ lattice for $\beta = 1.70, 2.00, 2.20$ with the same values of κ in both panels.



lattice study of Z3 QCD

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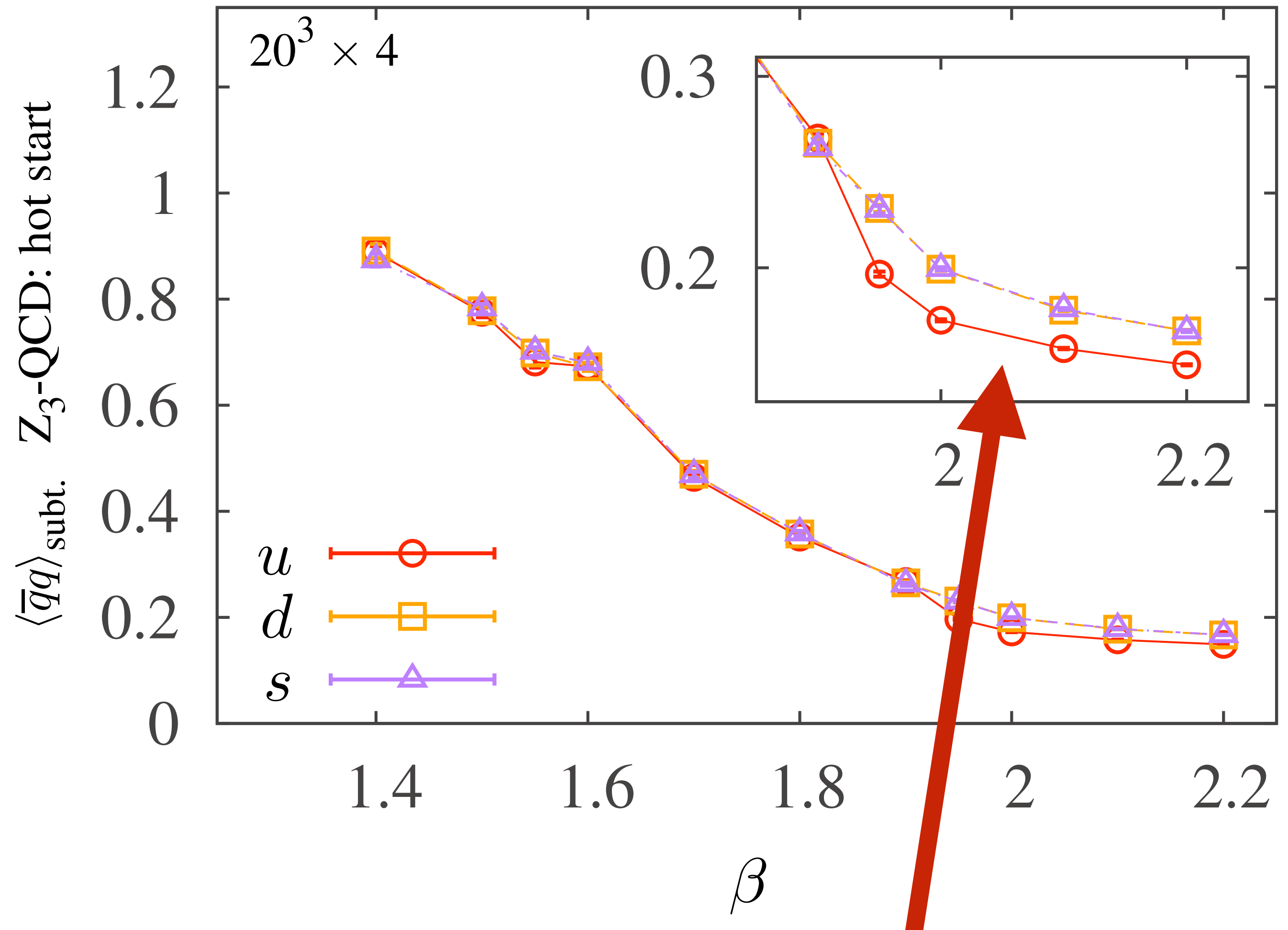
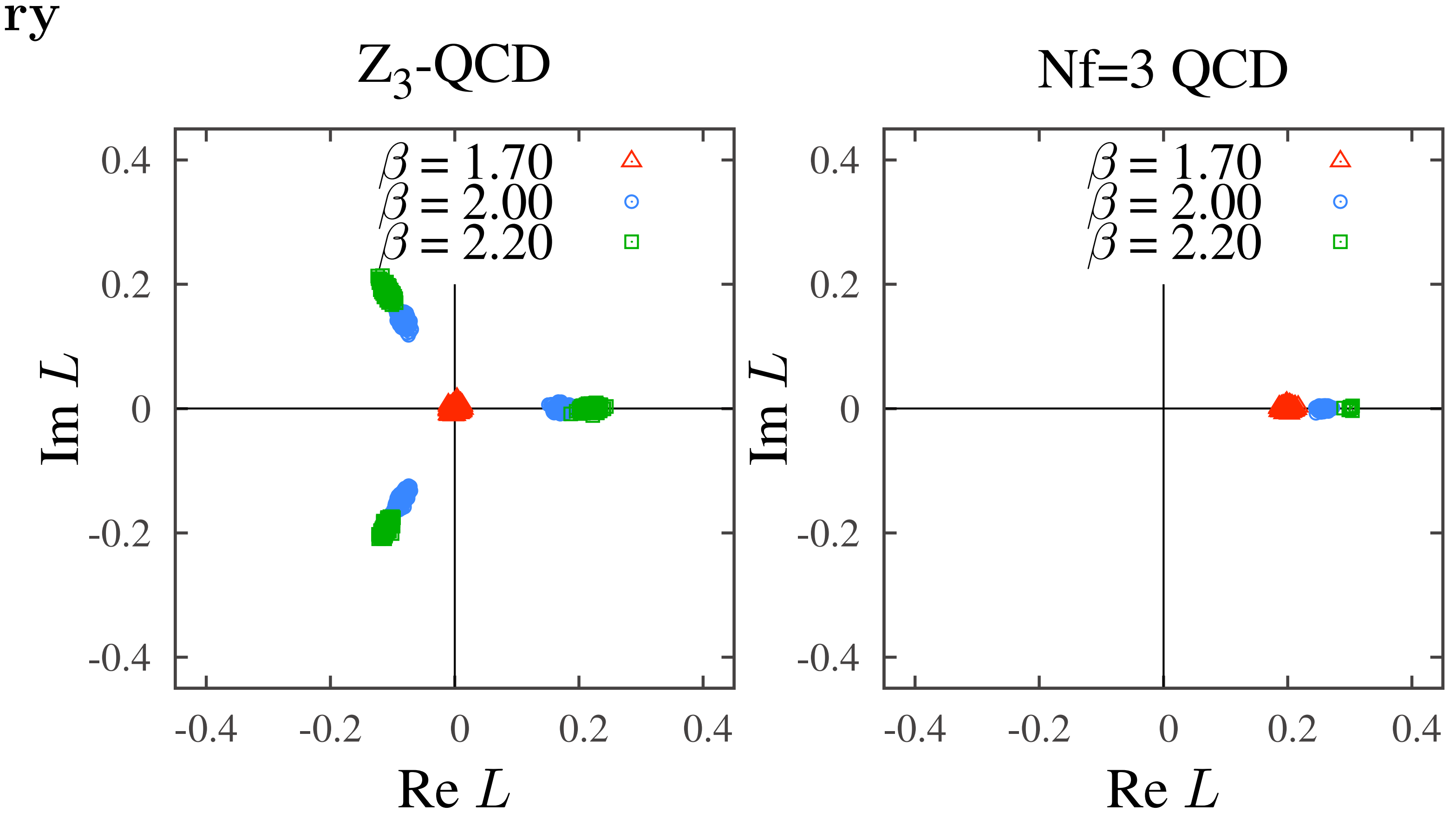
Tatsuhiro Misumi‡

Department of Mathematical Science, Akita University,



explanation: three flavors of quarks interact with three different ``liquids'' of M1,M2,L instanton-dyons!



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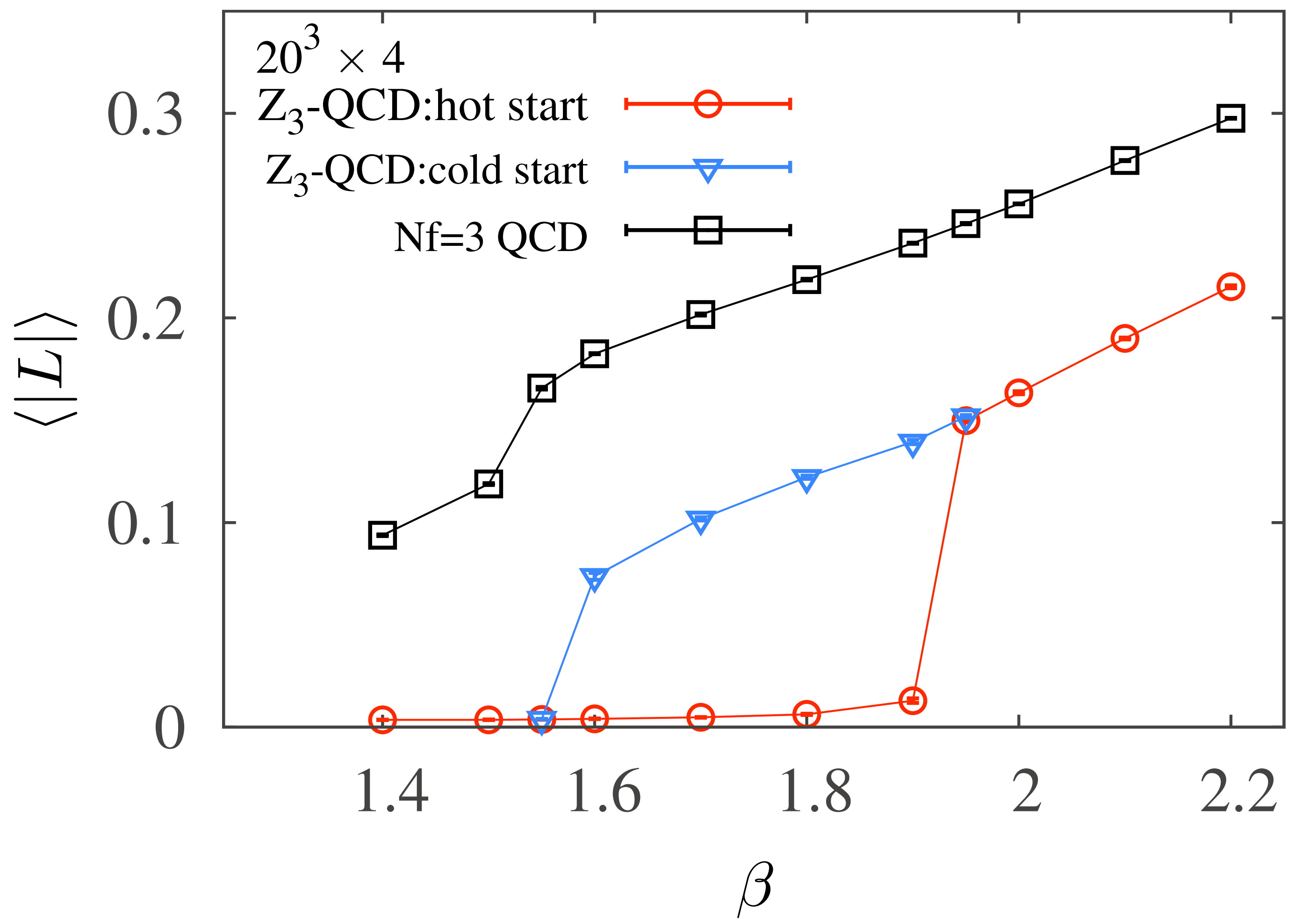


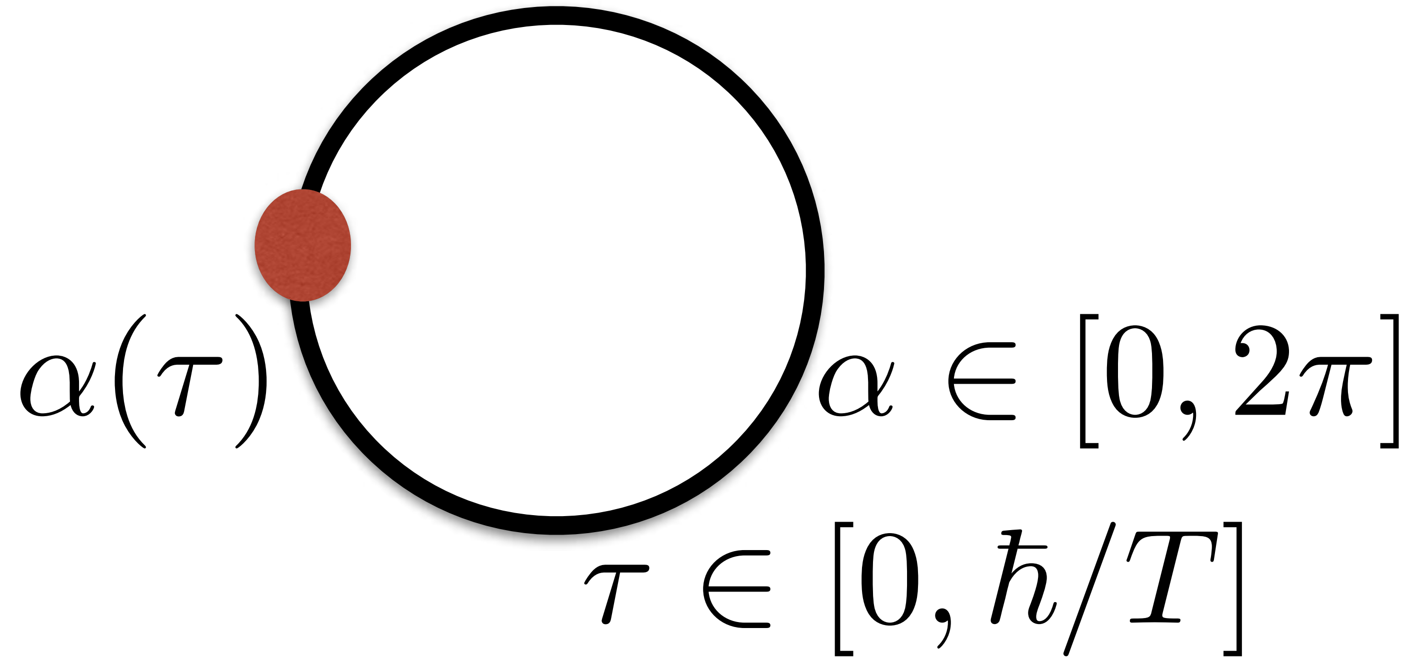
relation between monopole and instanton-dyon descriptions, the “Poisson duality”

in N=4 SYM on R3*S1, monopoles and inst-dyons give the same Z

N. Dorey and A. Parnachev, JHEP 0108, 059 (2001)
doi:10.1088/1126-6708/2001/08/059 [hep-th/0011202].

Is there any relation between the semiclassical instanton-dyons and QCD monopoles?

Adith Ramamurti,[*] Edward Shuryak,[†] and Ismail Zahed[‡]



The same phenomenon in much simpler setting:
quantum particle on a circle at finite T

A Hamiltonian vs Lagrangian approaches

$$Z_1 = \sum_{l=-\infty}^{\infty} \exp\left(- \frac{l^2}{2\Lambda T} + il\omega \right)$$

$$Z_2 = \sum_{n=-\infty}^{\infty} \sqrt{2\pi\Lambda T} \, \exp\left(- \frac{T\Lambda}{2} (2\pi n - \omega)^2 \right) .$$

moment of inertia

Aharonov-Bohm phase

Matsubara time winding number

$$\alpha_n(\tau) = 2\pi n \frac{\tau}{\beta} ,$$

based on classical paths

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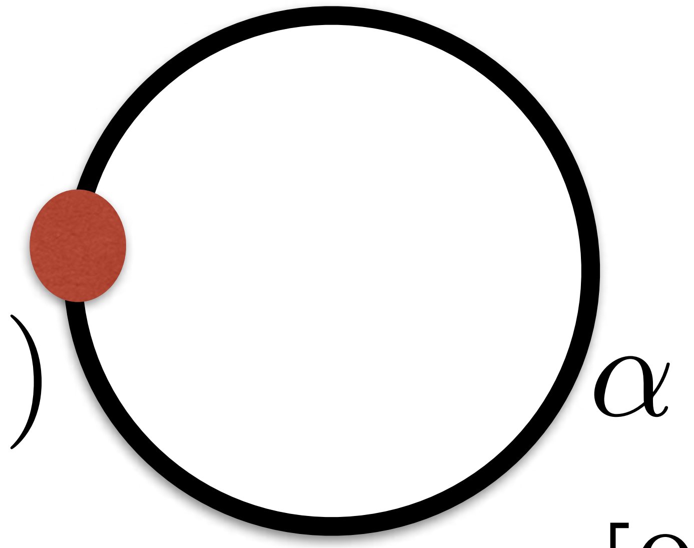

$\alpha(\tau)$ $\alpha \in [0, 2\pi]$

$\tau \in [0, \hbar/T]$

The same phenomenon in much simpler setting:
quantum particle on a circle at finite T

A Hamiltonian vs Lagrangian approaches

$$Z_1 = \sum_{l=-\infty}^{\infty} \exp\left(-\frac{l^2}{2\Lambda T} + il\omega\right) \qquad Z_2 = \sum_{n=-\infty}^{\infty} \sqrt{2\pi\Lambda T}\, \exp\left(-\frac{T\Lambda}{2}(2\pi n - \omega)^2\right).$$

moment of inertia

Aharonov-Bohm phase

Matsubara time winding number

$$\alpha_n(\tau) = 2\pi n \frac{\tau}{\beta},$$

based on classical paths

Note completely different dependence on T and holonomy omega

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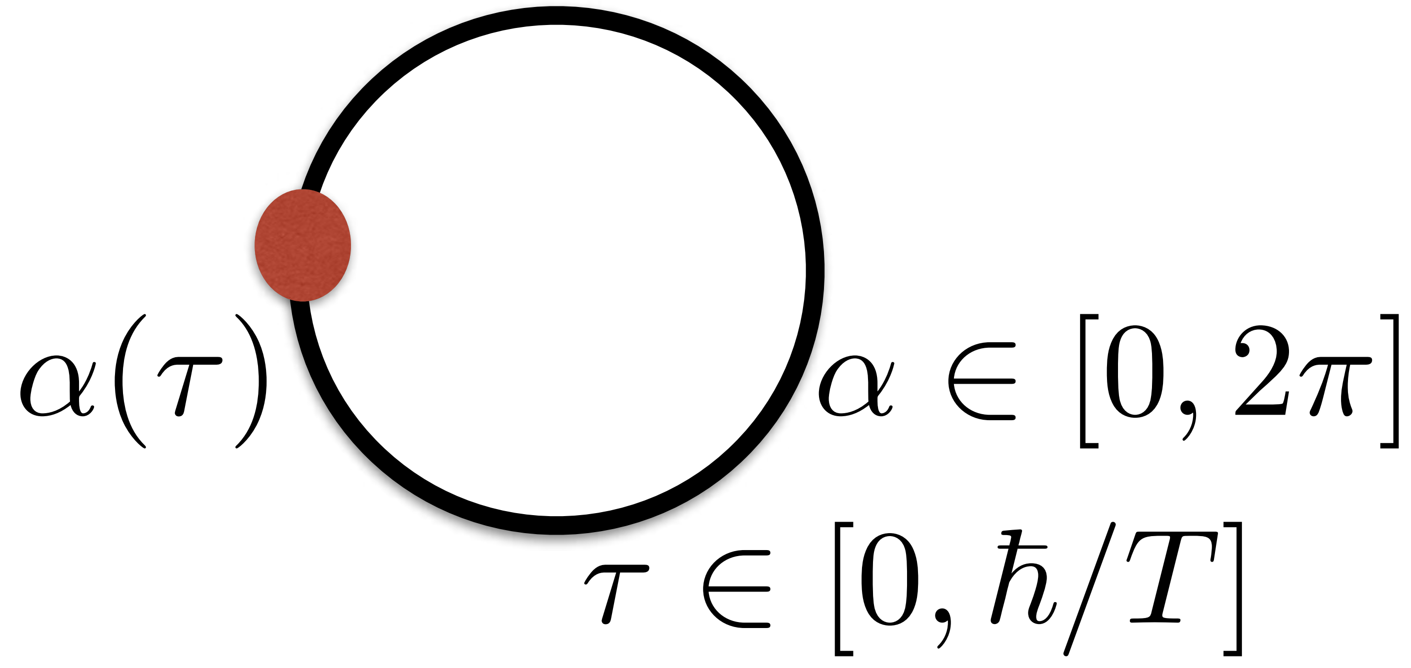

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quantum particle on a circle at finite T

A Hamiltonian vs Lagrangian approaches

$$Z_1 = \sum_{l=-\infty}^{\infty} \exp\left(-\frac{l^2}{2\Lambda T} + il\omega\right)$$

$$Z_2 = \sum_{n=-\infty}^{\infty} \sqrt{2\pi\Lambda T}\,\exp\left(-\frac{T\Lambda}{2}(2\pi n - \omega)^2\right).$$

moment of inertia

Aharonov-Bohm phase

Matsubara time winding number

$$\alpha_n(\tau) = 2\pi n \frac{\tau}{\beta}\,,$$

based on classical paths

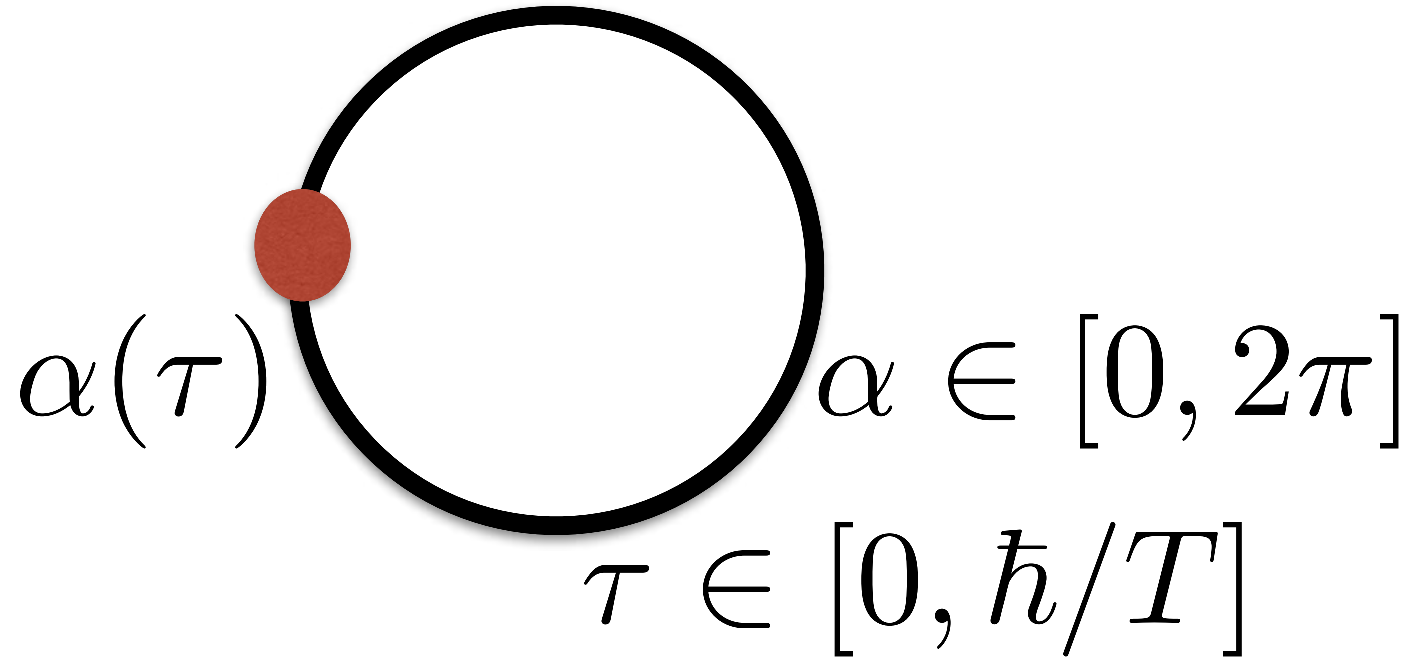
Note completely different dependence on T and holonomy omega

And yet, they are the same!
(elliptic theta function of the 3 type)

$$Z_1 = Z_2 = \theta_3\left(-\frac{\omega}{2}, \exp\left(-\frac{1}{2\Lambda T}\right)\right).$$

Is there any relation between the semiclassical instanton-dyons and QCD monopoles?

Adith Ramamurti,[*] Edward Shuryak,[†] and Ismail Zahed[‡]



arXiv:1802.10509v1 [hep-ph] 28 Feb 2018

The same phenomenon in much simpler setting:
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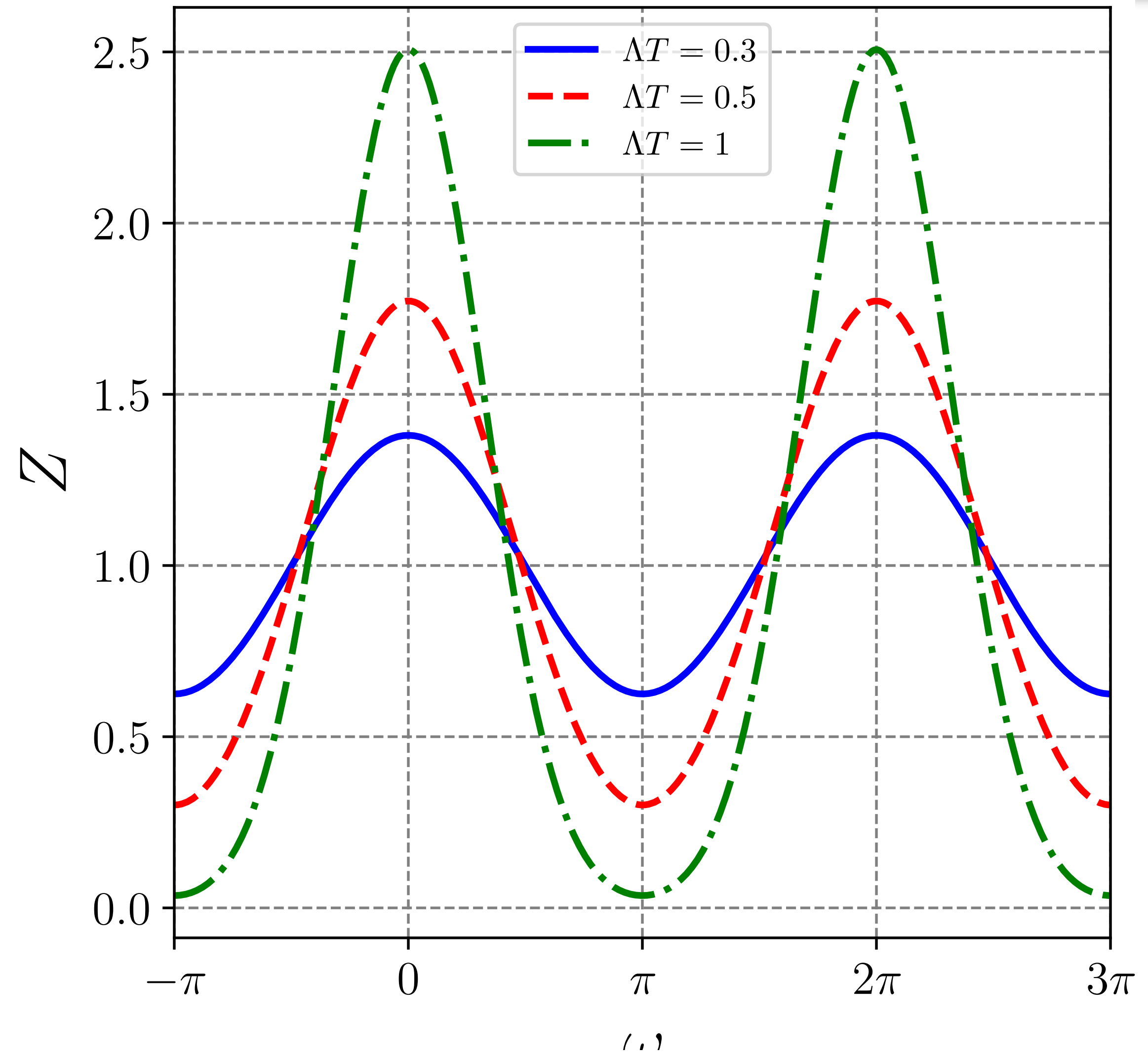

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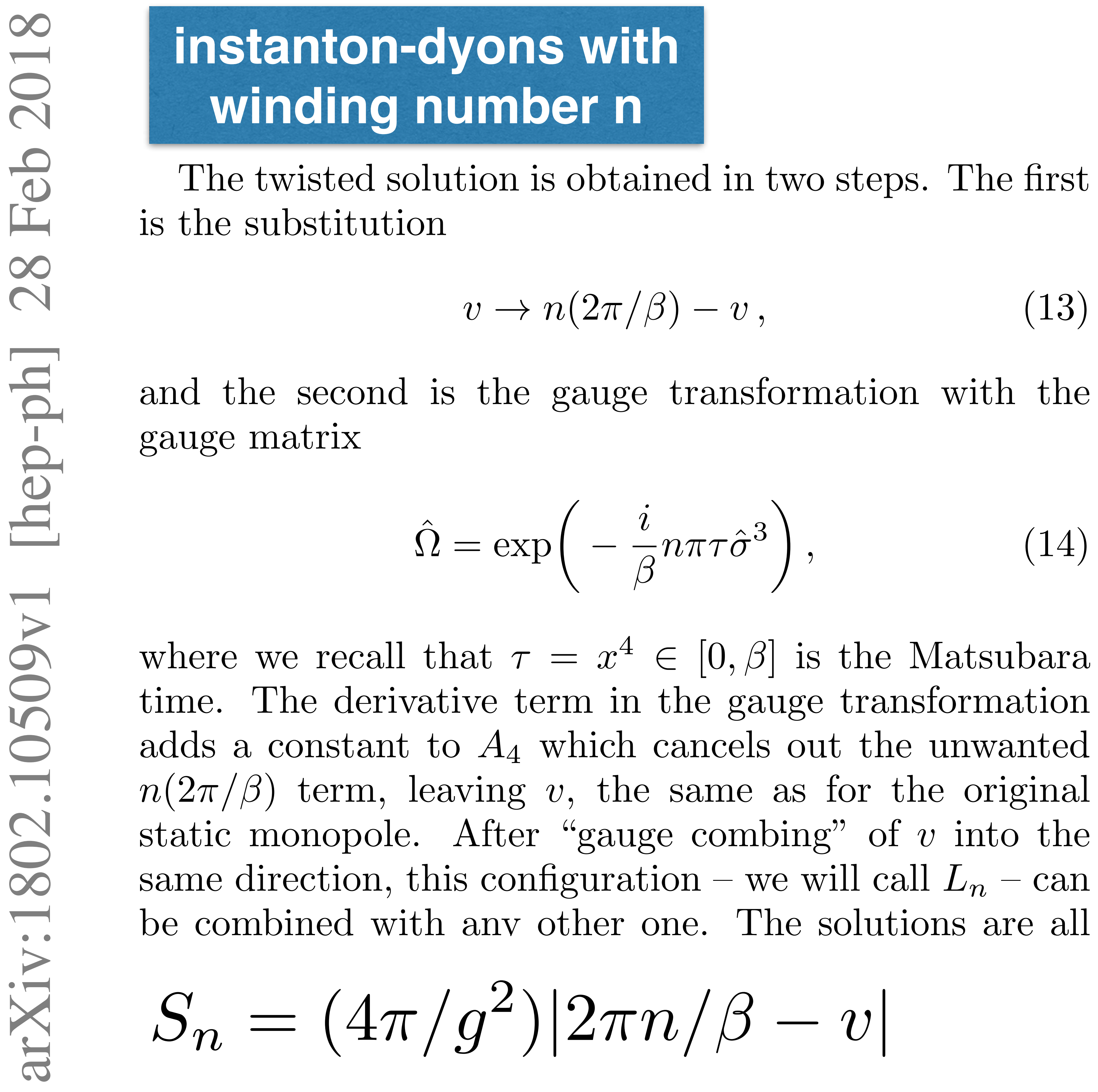
instanton-dyons with winding number n



The twisted solution is obtained in two steps. The first is the substitution

$$v \to n(2\pi/\beta) - v\,, \tag{13}$$

and the second is the gauge transformation with the gauge matrix

$$\hat{\Omega} = \exp\left(-\frac{i}{\beta} n\pi\tau\hat{\sigma}^3\right), \tag{14}$$

where we recall that $\tau = x^4 \in [0,\beta]$ is the Matsubara time. The derivative term in the gauge transformation adds a constant to A_4 which cancels out the unwanted $n(2\pi/\beta)$ term, leaving v, the same as for the original static monopole. After "gauge combing" of v into the same direction, this configuration – we will call L_n – can be combined with anv other one. The solutions are all

$$S_n = (4\pi/g^2)|2\pi n/\beta - v|$$

Found first in N=4 SYM theory, by Dorey Parnachev simple toy examplein this paper

$$\boxed{\sum_{n=-\infty}^{\infty} f(\omega + nP) = \sum_{l=-\infty}^{\infty} \frac{1}{P}\tilde{f}\left(\frac{l}{P}\right) e^{i2\pi l\omega/P}}$$

Poisson summation formula can be used to derive the monopole Z

$$Z_{\text{inst}} = \sum_n e^{-\left(\frac{4\pi}{g_0^2}\right)|2\pi n-\omega|}$$

$$Z_{\text{mono}} \sim \sum_{q=-\infty}^{\infty} e^{iq\omega - S(q)}$$

$$S(q) = \log\left(\left(\frac{4\pi}{g_0^2}\right)^2 + q^2\right)$$

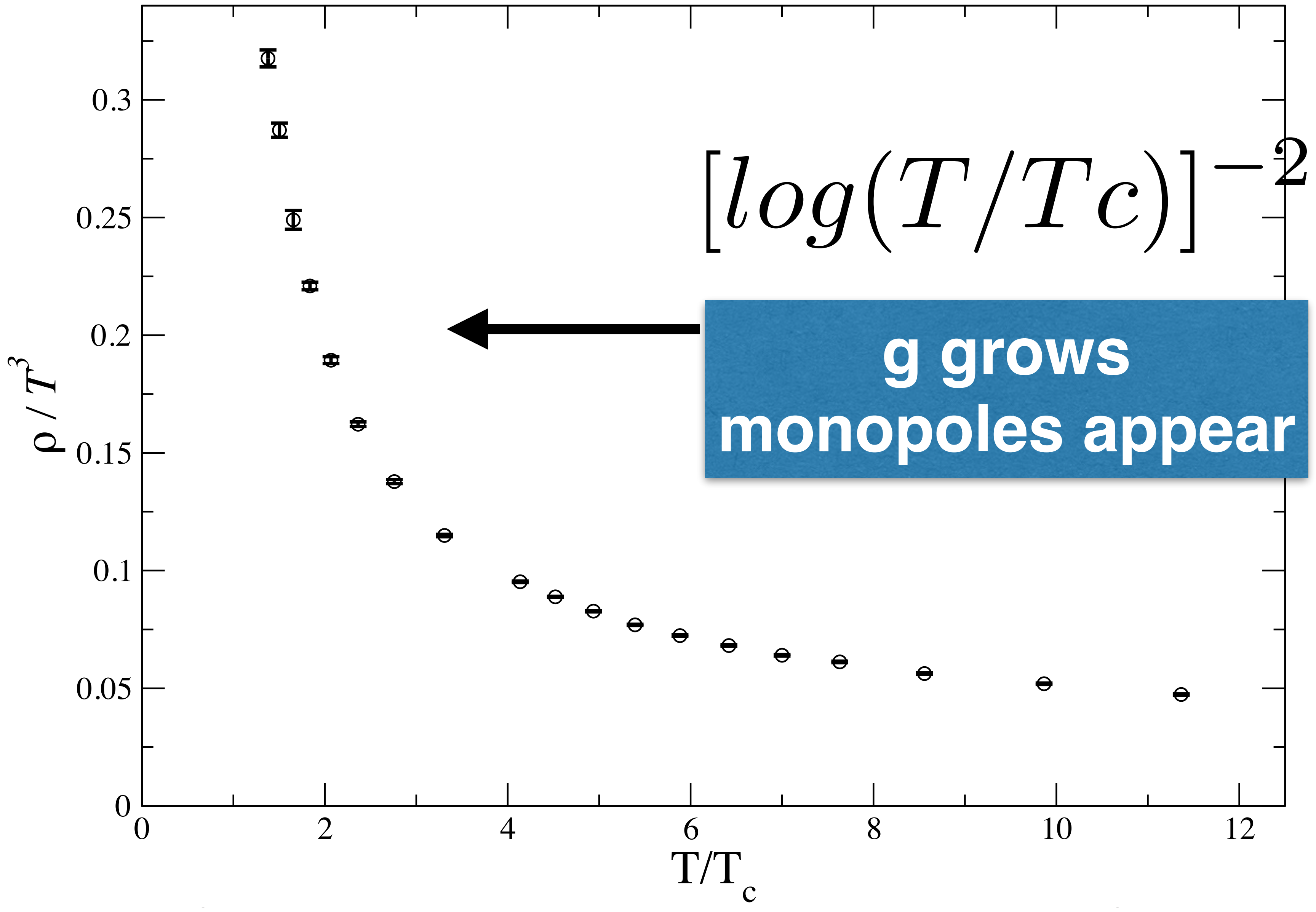
$$\approx 2\log\left(\frac{4\pi}{g_0^2}\right) + q^2\left(\frac{g_0^2}{4\pi}\right)^2 + \dots$$

q is angular momentum of rotating monopole, so it is electric charge (Zee)

Therefore we now understand why The density of monopoles depends on T as an inverse power of log(T) , not power of T =>

It is because they are not really semiclassical objects!

$$S_{mono} \sim log(const/g^2) = log\big(log(T/Tc)\big)$$



$[log(T/Tc)]^{-2}$

g grows
monopoles appear

Fig. 2.6 The normalized monopole density ρ/T^3 for the $SU(2)$ pure gauge theory as a function of the temperature, in units of the critical temperature T/T_c, above the deconfinement transition.

D'Alessandro, A. and D'Elia, M. (2008). Magnetic monopoles in the high temperature phase of Yang-Mills theories. Nucl. Phys., B799:241–254. 0711.1266.

For instantons and dyons it is different

$$\boxed{exp(-S) \sim exp(-const/g^2) = exp(-const' * log(T)) = 1/T^{power}}$$

Summary

- Semiclassical objects at finite T are instanton-dyons, fractions of instantons. Their interactions and ensembles for SU(2) and SU(3) gauge theories, with and without quarks studied
- Very cleanly they are seen in lattice configurations via Dirac zero (and quasizero) eigenmodes. Even when overlapping, the lattice shapes follow semiclassical formulae very accurately (?)
- in QCD deconfinement and chiral transitions are close, but
- can be moved by two different deformations: (1) Polyakov line suppression
- (2) changes of fermion periodicity phases
- Poisson duality for monopoles to instanton-dyons explains the monopole density(T) and why monopoles of pure gauge theories or QCD are not semiclassical