UnNuclear Physics: Conformal Symmetry in Nuclear Reactions

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Igor Shovkovy's Theoretical Physics Colloquium
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References

- H. Hammer and D.T. Son, arXiv:2103.12610 (PNAS 2021)
- T. Schäfer and G. Baym, arXiv:2109.06924 (PNAS 2021)

Summary

- Conformal invariance
- Nonrelativistic conformal invariance
- Fermion at unitarity
- Unnuclear physics
- Nuclear reaction with final-state neutrons

Role of symmetry in physics

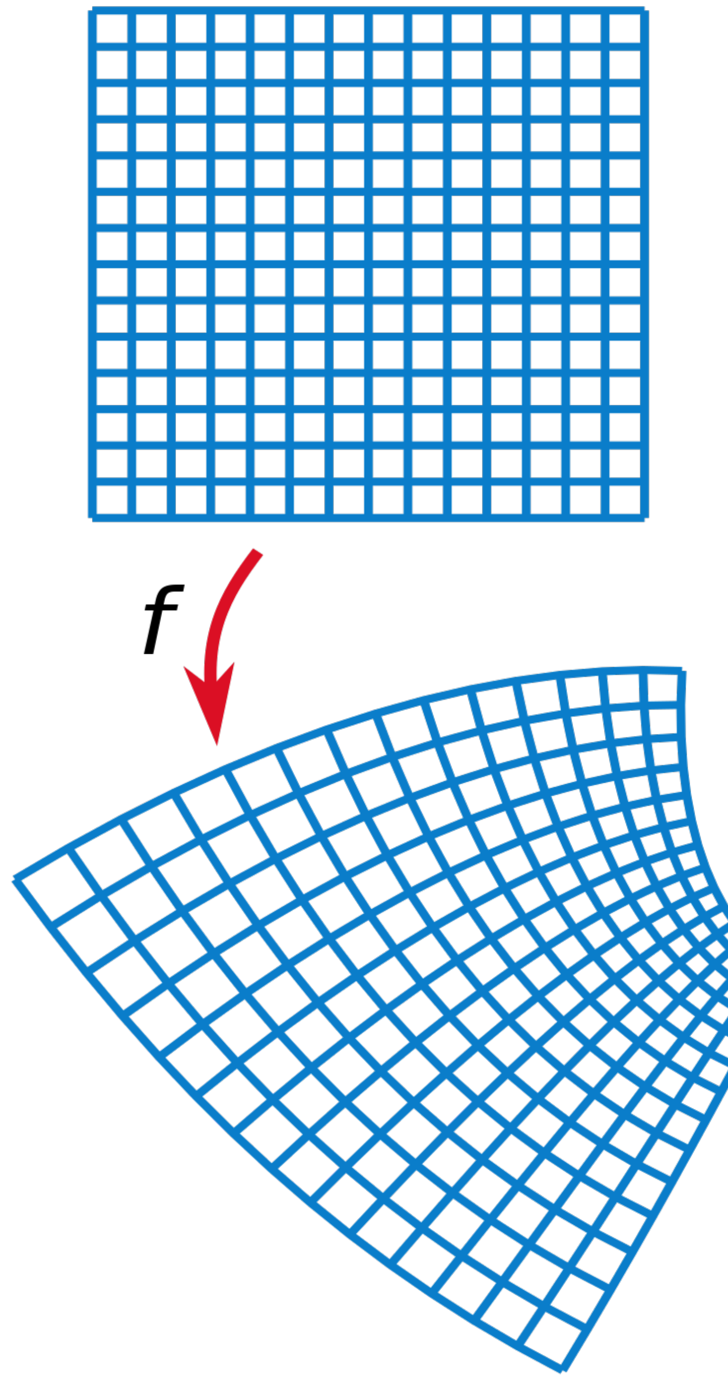
- Symmetries play a very important role in physics
- In particular, spacetime symmetry is key to understanding of elementary particles

Poincaré symmetry

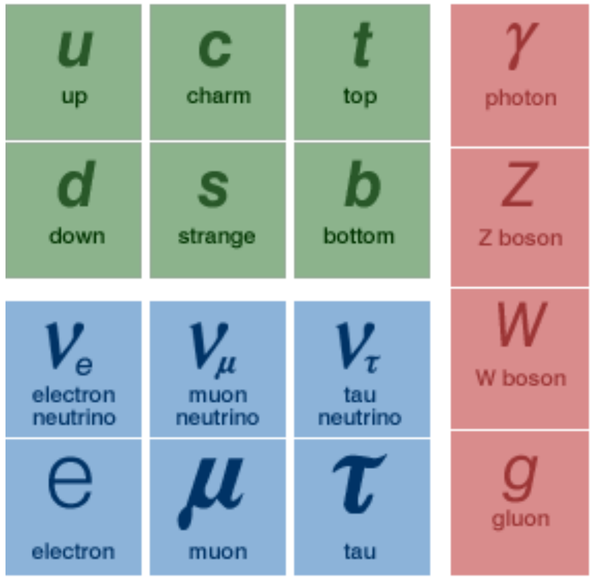
- 1 time and 3 spatial translations: 4 P^μ $\quad x^\mu \to x^\mu + a^\mu$
- 3 rotations + 3 boosts $M^{\mu\nu}$
- Elementary particles: irreducible representations of the Poincaré group, characterized by Casimirs
  - mass and spin when $m \neq 0$
  - when $m = 0$: helicity instead of spin

Conformal symmetry

- An extension of Poincaré group: conformal symmetry
- All transformations that preserve angle
- include: dilatation $x^\mu \to \lambda x^\mu$
- and 4 "proper conformal transformations"
- Field theory with this symmetry: conformal field theory
- applications in theoretical physics including phase transitions



CFT in particle physics?



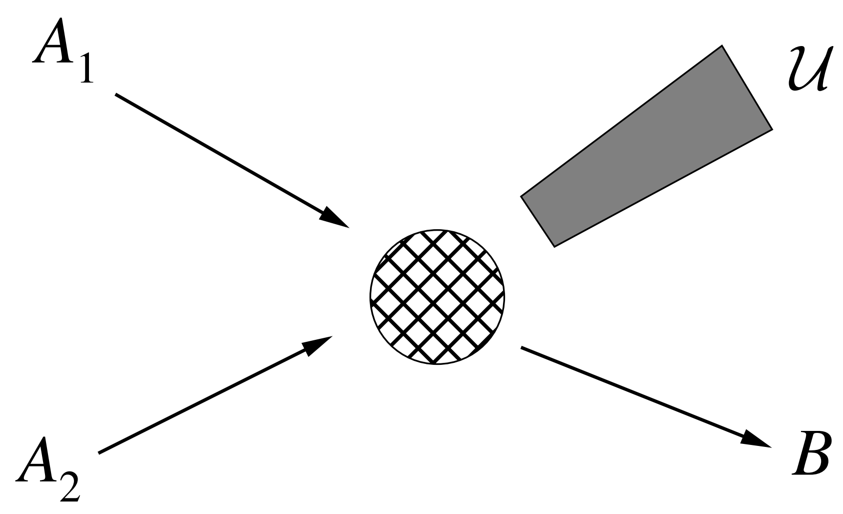
- The Standard Model is not a conformal field theory
- CFT cannot have massive particles
  - $E = \sqrt{p^2 + m^2}$ not invariant under $E \to \lambda E$, $p \to \lambda p$
- can only have massless particles or some fuzzy "stuff"

Georgi's unparticle

H. Georgi, 2007

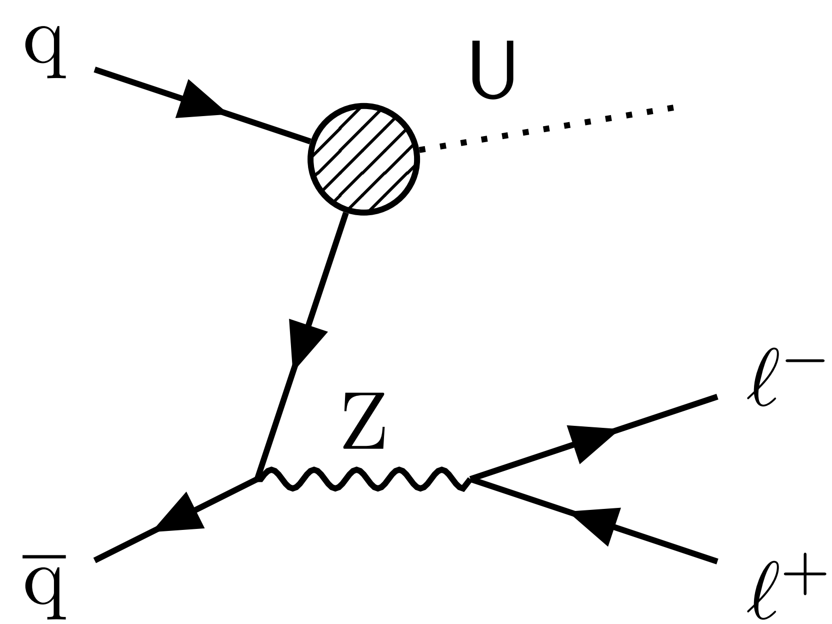
- In CFT: $\langle \mathcal{U}(x)\mathcal{U}(0)\rangle = \dfrac{c}{|x|^{2\Delta_{\mathcal{U}}}}$
- In momentum space $G_{\mathcal{U}}(p) \sim p^{2\Delta_{\mathcal{U}}-4}$
- Particle: $\Delta_\phi = 1,\ G_\phi(p) \sim p^{-2}$
- but otherwise the propagator has cuts, not poles
- Energy is not fixed when momentum is fixed: $E > pc$
- Georgi: unparticle, hypothesize that it can be a hidden sector coupled to the SM

Signal of unparticles



- Energy spectrum of B is continuous
- near end point depends on the dimension of $\mathcal{U}$:
  - $\dfrac{d\sigma}{dP^2_{\mathcal{U}}} \sim (P^2_{\mathcal{U}})^{\Delta-2}$

Search for unparticles



- CMS collaboration: no unparticle found so far at LHC

- Nonrelativistic conformal field theory is fruitful
    - Realized in nature
    - has experimentally verifiable consequences

Schrödinger symmetry

- Nonrelativistic version of conformal symmetry: "Schrödinger symmetry"
- Symmetry of the free time-dependent Schrödinger equation

$$i\partial_t \psi = -\frac{\nabla^2}{2m}\psi$$

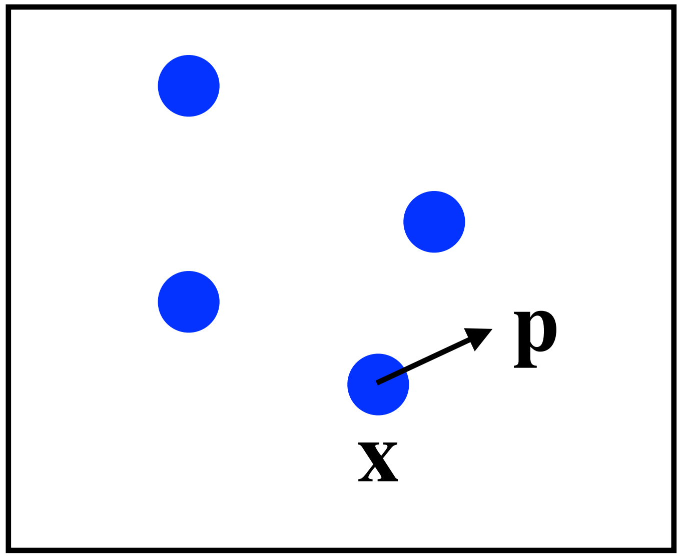
Galilean boost: $\tilde{\psi}(t,\mathbf{x}) = e^{im\mathbf{v}\cdot\mathbf{x} - \frac{i}{2}mv^2 t}\,\psi(t, \mathbf{x} - \mathbf{v}t)$

Dilatation: $\tilde{\psi}(t,\mathbf{x}) = \psi(\lambda^2 t, \lambda \mathbf{x})$

Special conformal transformation:

$$\tilde{\psi}(t,\mathbf{x}) = \frac{1}{(1+\alpha t)^{3/2}} \exp\left(\frac{i}{2}\frac{m\alpha x^2}{1+\alpha t}\right)\psi\left(\frac{t}{1+\alpha t}, \frac{\mathbf{x}}{1+\alpha t}\right)$$

Schrödinger algebra



- Free particles $(\mathbf{x}_a, \mathbf{p}_a), a = 1,2,\ldots N$
- $\mathbf{P} = \sum_a \mathbf{p}_a \qquad H = \sum_a \frac{p_a^2}{2m}$
- $\mathbf{K} = \sum m\mathbf{x}_a$ Galilean boosts
- $D = \sum \frac{1}{2}(\mathbf{x}_a \cdot \mathbf{p}_a + \mathbf{p}_a \cdot \mathbf{x}_a)$ dilatation
- $C = \frac{1}{2}m \sum x_a^2$ proper conformal transformation
- Angular momentum
- Mass $M = Nm$

Schrödinger algebra

$X \setminus Y$	P_j	K_j	D	C	H
P_i	0	$-i\delta_{ij}M$	$-iP_i$	$-iK_i$	0
K_i	$i\delta_{ij}M$	0	iK_i	0	iP_i
D	iP_j	$-iK_j$	0	$-2iC$	$2iH$
C	iK_j	0	$2iC$	0	iD
H	0	$-iP_j$	$-2iH$	$-iD$	0

$$[J_{ij}, N] = [J_{ij}, D] = [J_{ij}, C] = [J_{ij}, H] = 0,$$
$$[J_{ij}, P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i), \qquad [J_{ij}, K_k] = i(\delta_{ik}K_j - \delta_{jk}K_i),$$
$$[J_{ij}, J_{kl}] = i(\delta_{ik}J_{jl} + \delta_{jl}J_{ik} - \delta_{il}J_{jk} - \delta_{jk}J_{il}).$$

Special role of dilatation

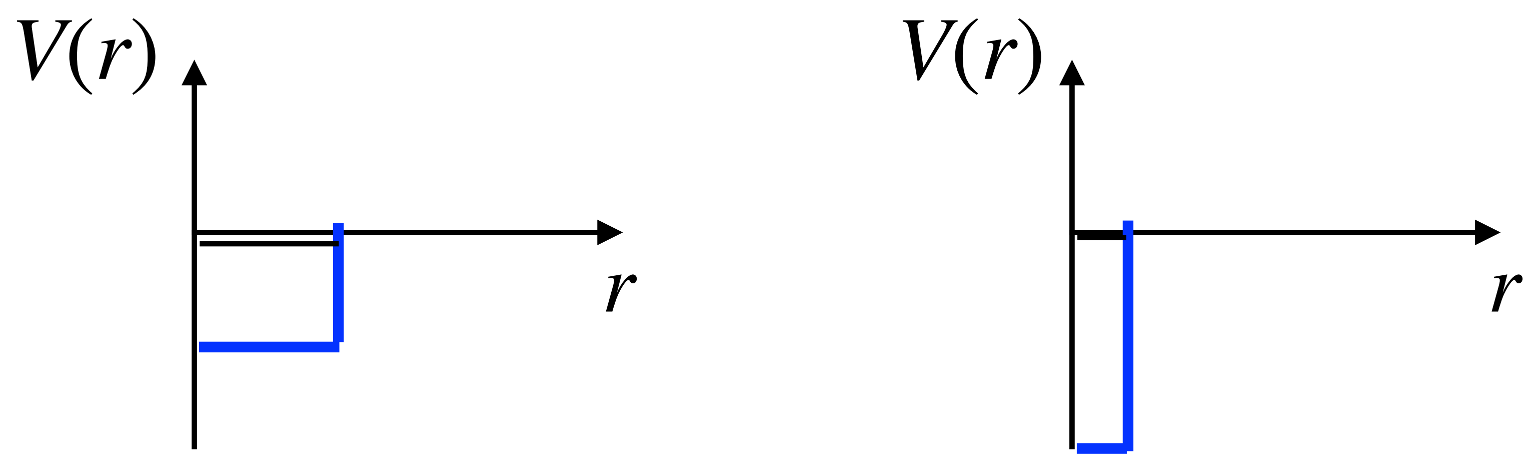
$$D = \sum \frac{1}{2}(\mathbf{x}_a \cdot \mathbf{p}_a + \mathbf{p}_a \cdot \mathbf{x}_a)$$

- Dilatation operator rescales coordinates and momenta:
  - $p \to \lambda p, x \to \lambda^{-1} x$ $\quad [D, P_i] = iP_i$
  - $H = p^2/2m \to \lambda^2 H$ $\quad [D, H] = 2iH$

Beyond free theory

- Is the Schrödinger symmetry good only for non-interacting theory?
- Most general Hamiltonian: not scale-invariant
  $[D, H] \neq 2iH$
- There exists a way to have the symmetry in interacting theory: unitarity regime

Unitarity regime

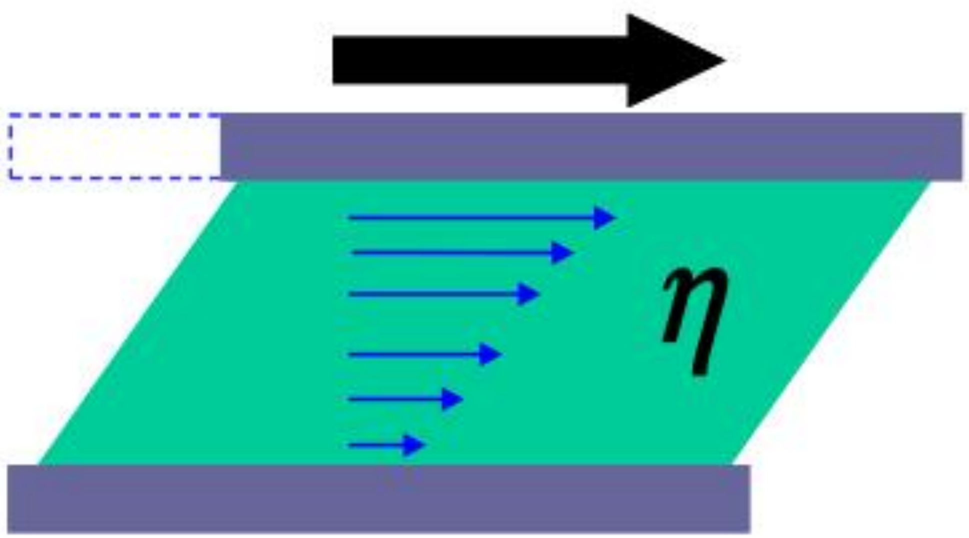


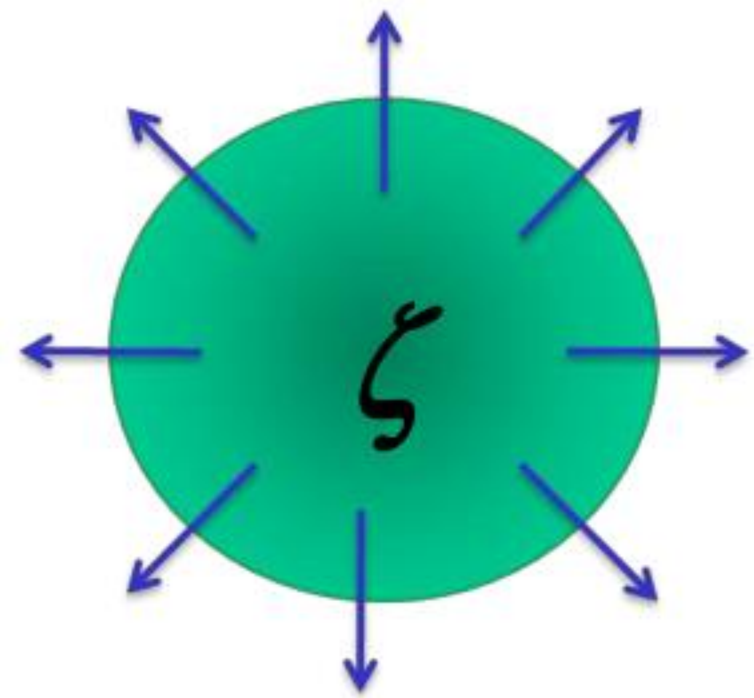
- Take a potential of a certain shape (e.g., square well)
- shrink the range, adjusting the depth so that there is one bound state at threshold
- In the language of scattering theory: infinite scattering length, zero range
- Interaction has no energy/length scale: Hamiltonian is scale invariant $[D, H] = 2iH$

Properties of unitary gas

- A gas of spin-1/2 particles with fine-tuned to unitarity
- Can be realized with trapped cold atoms Feshbach resonance
- Scale invariance: physical quantities can be figured out by scaling arguments
- Example: Bertsch parameter ξ $(T = 0)$
  - $\dfrac{E}{N} = \xi \dfrac{3}{5} \varepsilon_F, \quad \varepsilon_F = \dfrac{1}{2m}(3\pi^2 n)^{2/3}$

Shear and bulk viscosities





- Scaling: $\eta, \zeta = \hbar n f_{\eta,\zeta}\left(\frac{T}{\varepsilon_F}\right)$
- Conformal invariance: $\zeta = 0$

Nonrelativistic CFT

Y. Nishida, DTS, 2007

- One can build up the formalism of nonrelativistic conformal field theory in analogy with the relativistic theory
- Many notions can be extended
  - primary operators
  - operator-state correspondence

Fermions at unitarity as a NRCFT

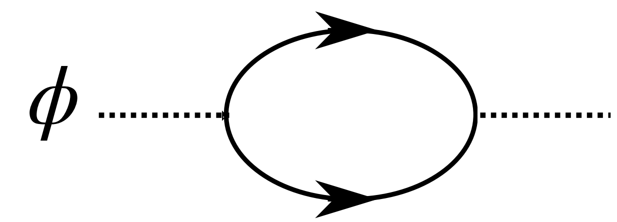
- $L = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - c_0\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$ $\Delta[\psi] = \frac{3}{2}$
- Introducing auxiliary field ϕ
- $L = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger\phi - \phi^\dagger\psi_\downarrow\psi_\uparrow + \frac{\phi^\dagger\phi}{c_0}$
  - Propagator of ϕ

$$G_\phi(\omega, \mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}$$

$$\Delta[\phi] = 2 \neq 2 \times \frac{3}{2}$$

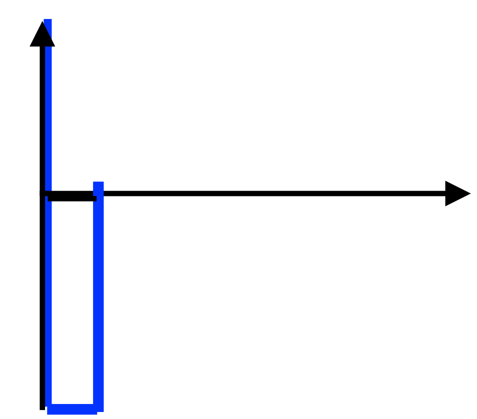
Renormalization

- $G_\phi^{-1}(\omega, \mathbf{p}) = c_0^{-1}$ + one-loop integral



- $= c_0^{-1} + \Lambda + \left(\frac{p^2}{4m} - \omega\right)^{1/2}$
- Unitarity: fine-tuning so that $c_0 + \Lambda = 0$
- (scattering length: $c_0 + \Lambda = \frac{1}{a}$)
- Physically: fine-tune the attractive short-range potential to have a bound state at threshold

$$G_\phi(\omega, \mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}$$

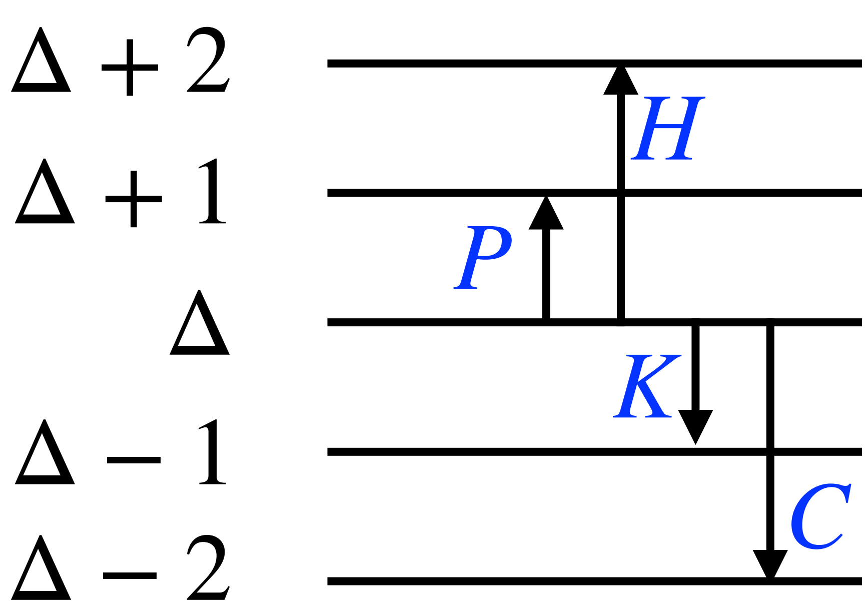


Local operators

- Local operators are classified by mass and dimension
  - $[M, O(x)] = -M_O O(x)$
  - $[D, O(0)] = i\Delta_O O(0)$
- Commuting with P and H increases the dimension by 1 and 2, commuting with K and C by −1 and −2
- Representation theory for operators with $M \neq 0$ is simple

Raising and lowering dimensions

- Operators with $M \neq 0$ are organized in towers
- Dimension raised by P and H, lowered by K and C
- Primary operators: lowest in a ladder
  $[K, O(0)] = [C, O(0)] = 0$



Operator-state correspondence

- Dimension of a primary operator = energy of a state in a harmonic potential

- Example: in free theory $[\psi] = \frac{d}{2}$, ground state of 1 particle in harmonic potential: $E = \frac{d}{2}\hbar\omega$

Two-point function of a primary operator

- $G_{\mathscr{U}}(t,\mathbf{x}) = -i\langle T\mathscr{U}(t,\mathbf{x})\mathscr{U}^\dagger(0,\mathbf{0})\rangle = C\frac{\theta(t)}{(it)^\Delta}\exp\left(\frac{iMx^2}{2t}\right)$
- $G_{\mathscr{U}}(\omega,\mathbf{p}) \sim \left(\frac{p^2}{2M} - \omega\right)^{\Delta-\frac{5}{2}}$

$\omega - \frac{p^2}{2M}$ is the energy in the CM frame

Operator dimensions for fermions at unitarity

- Dimensions of operators: either by field theory or quantum mechanical calculation in a harmonic trap
- Lowest-dimension operators

N	S	L	O	Δ
2	0	0	$\psi_\uparrow\psi_\downarrow$	2
3	1/2	1	$\psi_\downarrow\psi_\uparrow\boldsymbol{\nabla}\psi_\uparrow$	4.273
3	1/2	0	$\psi_\downarrow\boldsymbol{\nabla}\psi_\uparrow\cdot\boldsymbol{\nabla}\psi_\uparrow$	4.666
4	0	0	$\psi_\downarrow\psi_\uparrow\boldsymbol{\nabla}\psi_\downarrow\cdot\boldsymbol{\nabla}\psi_\uparrow$	5.0–5.1

$N\ (l)$	$\mathcal{O}_N^{(l)}$	$\Delta_{\mathcal{O}}$	$E/\hbar\omega$ in $d=3$
$2\ \ (l=0)$	$\psi_\uparrow\psi_\downarrow$	2	2 [30]
$3\ \ (l=0)$	$\psi_\uparrow\psi_\downarrow(\partial_t\psi_\uparrow)$	$5+O(\bar{\epsilon}^2)$	4.66622 [26]
$3\ \ (l=1)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow)$	$4+O(\bar{\epsilon}^2)$	4.27272 [26]
$4\ \ (l=0)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)$	$6-\bar{\epsilon}+(\bar{\epsilon}^2)$	≈ 5.028 [33]
$5\ \ (l=0)$	$(*)$	$9-\frac{11\pm\sqrt{105}}{16}\bar{\epsilon}+O(\bar{\epsilon}^2)$	≈ 8.03 [33]
$5\ \ (l=1)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)\boldsymbol{\partial}\psi_\uparrow$	$8-\bar{\epsilon}+O(\bar{\epsilon}^2)$	≈ 7.53 [33]
$6\ \ (l=0)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)^2$	$10-2\bar{\epsilon}+(\bar{\epsilon}^2)$	≈ 8.48 [33]

$(*) = a\,\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)\partial^2\psi_\uparrow + b\,\psi_\uparrow\partial_i\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)\partial_i\psi_\uparrow + c\,\psi_\uparrow\psi_\downarrow((\partial_i\boldsymbol{\partial}\psi_\uparrow)\cdot\boldsymbol{\partial}\psi_\downarrow)\partial_i\psi_\uparrow - d\,\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)i\partial_t\psi_\uparrow$ with $(a,b,c,d)=(\pm 19\sqrt{3}-5\sqrt{35}, \mp 16\sqrt{3}, -6\sqrt{35}\mp 6\sqrt{3}, 16\sqrt{35})$.

Y. Nishida, DTS, arXiv:1004.3597

“UnNuclear physics”

A nonrelativistic version of unparticle physics

field in NRCFT: “unnucleus”

H.-W. Hammer and DTS, 2103.12610

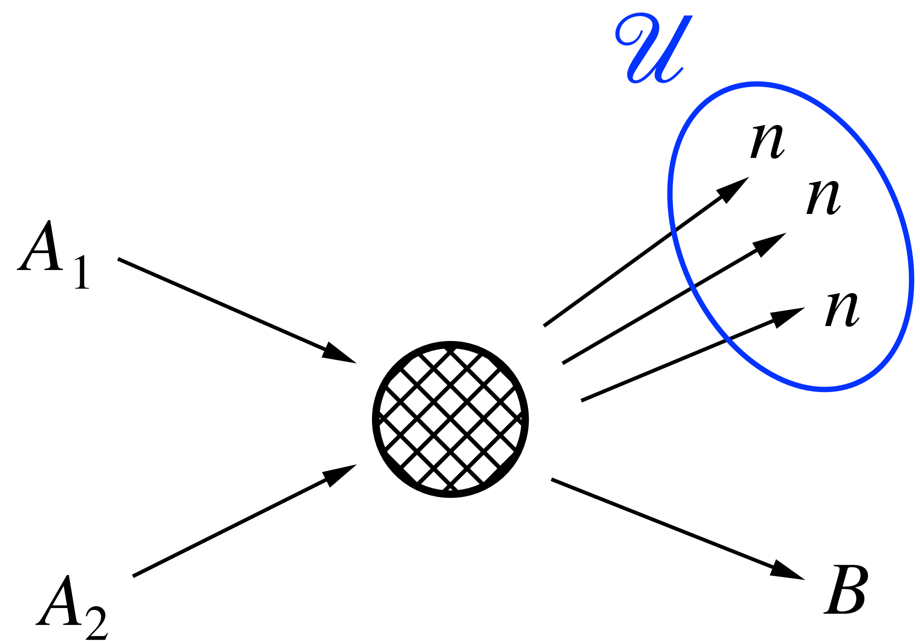
Few-neutron systems as unnuclei

- Neutrons have anomalously large scattering length: $a_{nn} \approx -19$ fm
- vs effective range $r_0 \approx 2.8$ fm
- In a wide range of energy is neutrons are fermions at unitarity

Nuclear reactions

- Many nuclear reactions with emissions of neutrons:
  - $^3H + {}^3H \rightarrow {}^4He + 2n$
  - $^7Li + {}^7Li \rightarrow {}^{11}C + 3n$
  - $^4He + {}^8He \rightarrow {}^8Be + 4n$
- Final-state neutrons can be considered as forming an "unnucleus" - a field in NRCFT
  - Regime of validity: kinetic energy of neutrons in their c.o.m. frame between $\hbar^2/ma^2 \sim 0.1$ MeV $\hbar^2/mr_0^2 \sim 5$ MeV

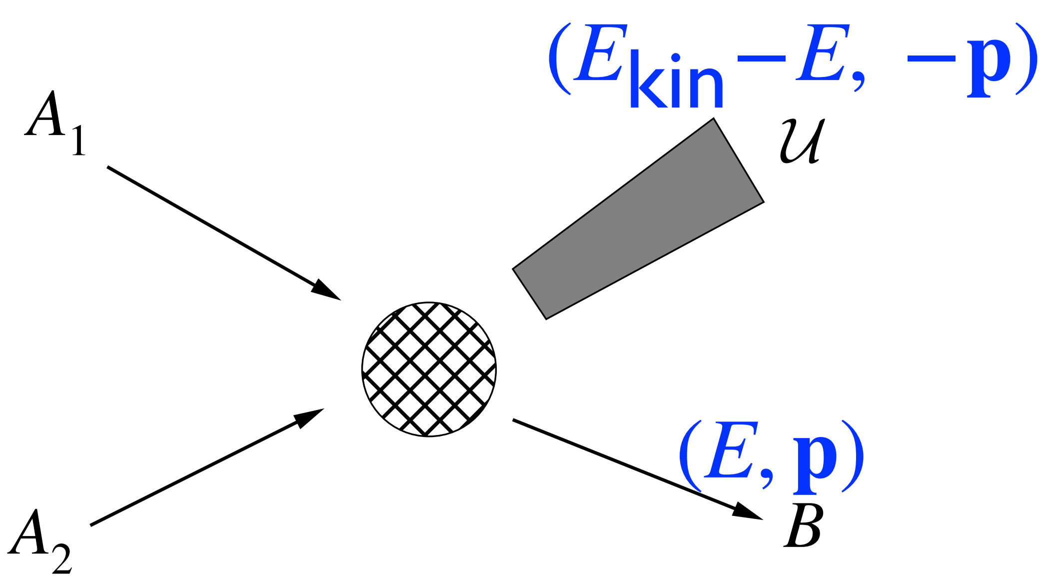
Few-neutron systems as unnuclei



Factorization: $$\frac{d\sigma}{dE} \sim |\mathscr{M}|^2 \sqrt{E_B} \times \mathrm{Im}\, G_{\mathscr{U}}(E_{\mathscr{U}}, \mathbf{p})$$

primary reaction has larger energy than final-state interaction

Rates of processes involving an unnucleus



$$E_{\rm kin} = E + E_{\mathscr{U}}$$

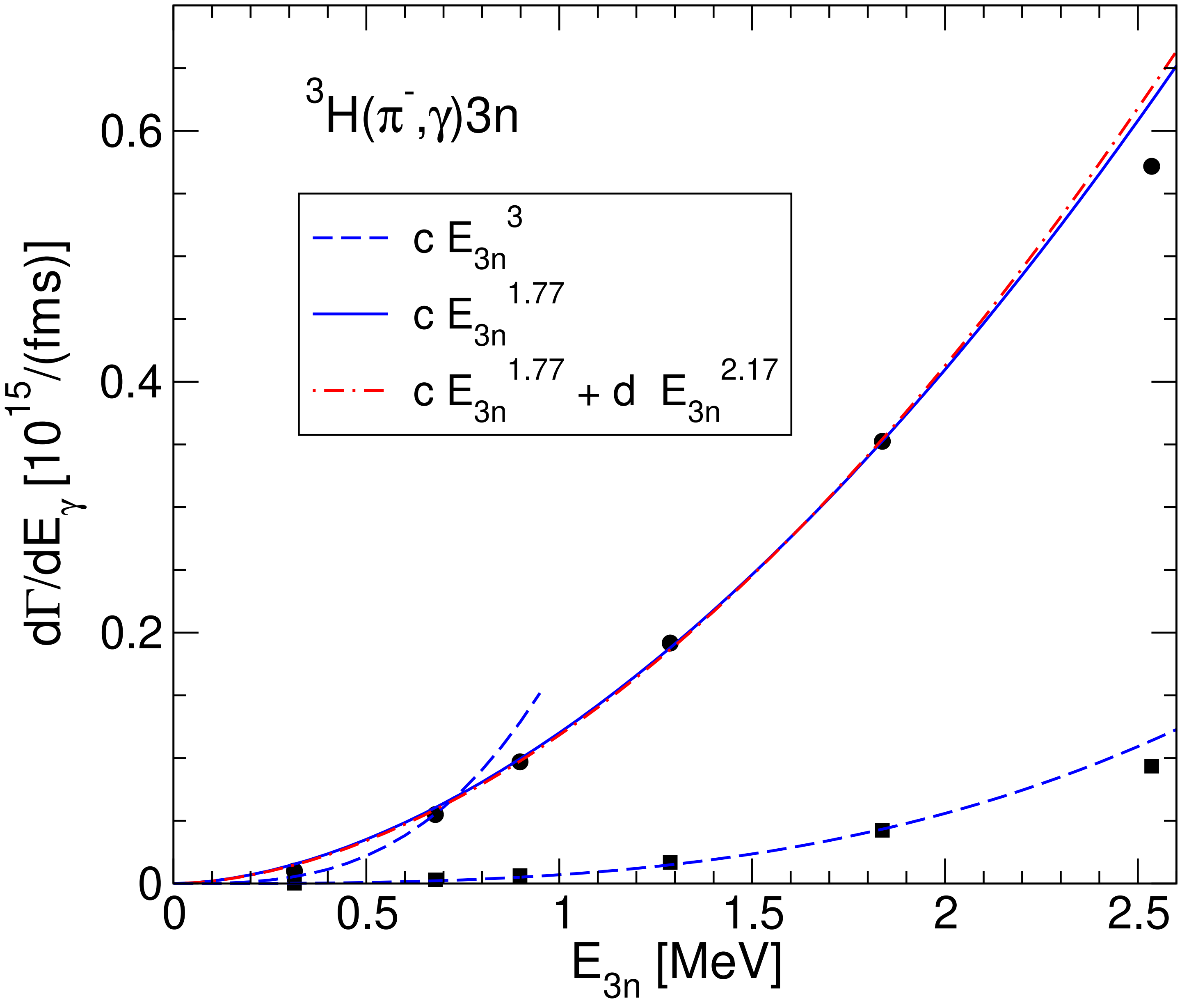
- $$\frac{d\sigma}{dE} \sim |\mathscr{M}|^2 \sqrt{E} \times \underbrace{\operatorname{Im} G_{\mathscr{U}}(E_{\rm kin} - E, \mathbf{p})} \;\rightarrow\; \left(E_{\rm kin} - E - \frac{p^2}{2M_{\mathscr{U}}}\right)^{\Delta - \frac{5}{2}}$$

- Near end point: $\dfrac{d\sigma}{dE} \sim (E_0 - E)^{\Delta - \frac{5}{2}}$

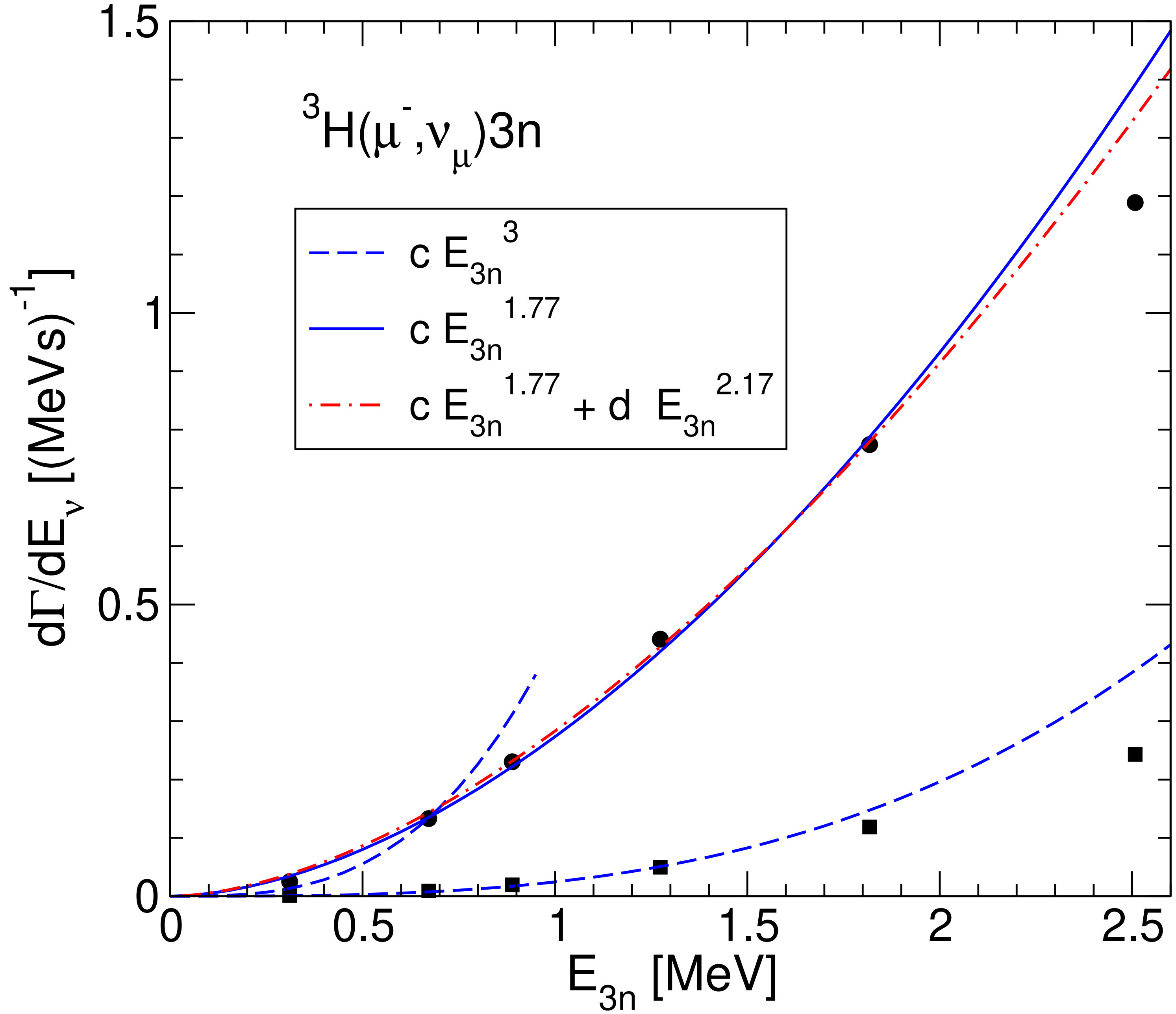
Nuclear reactions

- $^3H + {}^3H \to {}^4He + 2n$ — $\alpha = -0.5$
- $^7Li + {}^7Li \to {}^{11}C + 3n$ — $\alpha = 1.77$
- $^4He + {}^8He \to {}^8Be + 4n$ — $\alpha = 2.5 - 2.6$
- Prediction:
  - $$\frac{d\sigma}{dE} \sim (E_0 - E)^\alpha$$
- Regime of validity: kinetic energy of neutrons in their c.o.m. frame between $\hbar^2/ma^2 \sim 0.1$ MeV $\hbar^2/mr_0^2 \sim 5$ MeV

Comparison with microscopic models



$$\pi^- + {}^3\mathrm{H} \rightarrow \gamma + 3n$$



$$\mu^- + {}^3\text{H} \rightarrow \nu_\mu + 3n$$

Conclusion

- There is a nonrelativistic version of conformal field theory
- Example: fermions at unitarity
- Approximately realized by neutrons; leads to "unnuclear behavior"
- Possible extension to other systems
  X(3872) Braaten and Hammer

Thank you