Using the Large-Number-of-Colors Limit of QCD to Find the Relative Sizes of Nucleon-Nucleon Interaction Contributions to a Given Process

a.k.a.``how to walk in with a Lagrangian/potential and walk out knowing which interactions (in the standard model or beyond) are most important according to large Nc''

- Why do this?
- The elements involved in the scaling with Nc
- Example applications
- [For NNN see Phillips/Schat PRC 88 (2013) 3, 034002.]

R.P. Springer, DOE DE-FG02-05ER41368
based on upcoming ARNPS review with T. Richardson and M.Schindler
parity/large Nc collaborators: S.Nguyen, S.Pastore,T.Richardson,M.Schindler,J.Vanasse
24 Aug 2022

WHY USE NC TO ANALYZE/ORDER OPERATORS?

- Exploit “hidden” symmetry of QCD
- 1/Nc as expansion parameter
- Some things simplify as Nc becomes large
- Enhanced symmetries as Nc becomes large: SU(3) -> SU(Nc)
- Guidepost for:
  - ✴ Theories for which there is no data
  - ✴ High order calculations
  - ✴ Choosing which experiments to do first
  - ✴ Choosing which simulations to do first

WALK IN WITH:

New particle
New symmetry
New idea
⋮

bag of interactions

$$\Rightarrow$$

operator

$$\mathcal{L}_{int} = C_1 O_1 + C_2 O_2 + \cdots$$

$$V = C_1 Q_1 + C_2 Q_2 + \cdots$$

low energy coefficient

spin,isospin,momentum structure

WALK OUT WITH:

$$C_4, C_9 > C_1, C_{10} > \cdots ; \quad C_4 = C_5; \quad \cdots$$

observable=function of C_4 and C_9 at leading order in Nc

ELEMENTS NEEDED TO DETERMINE LARGE-N_C SCALING:

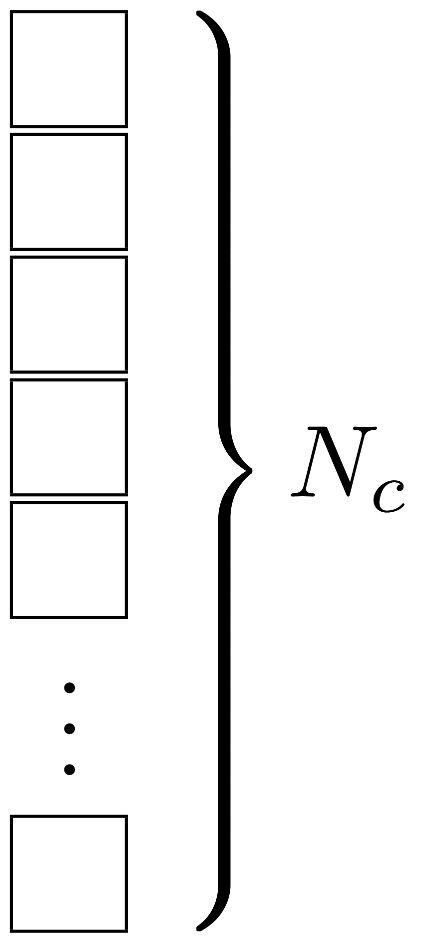
1. Begin with an overcomplete basis including all possible structures (and/or understand the impact of any Fierz reductions).
2. Determine spin-isospin scaling of single nucleon matrix elements.
3. Determine spin-isospin scaling of two-nucleon matrix elements (e.g., relevant for scattering).
4. Establish source of momenta in the operators.
5. Count the number of external pions.
6. Remember the subtraction point dependence of the low energy coefficients.
7. [Delta issues]

WHAT DOES A NUCLEON LOOK LIKE IN LARGE NC?

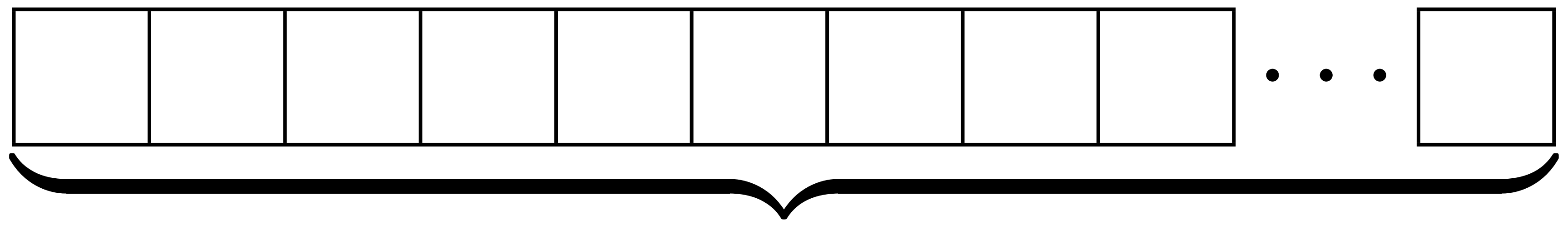
- Pauli: antisymmetric under interchange of any two quarks
- product wavefunction: $\psi_{color} \times \psi_{spatial} \times \psi_{spin} \times \psi_{isospin}$
- singlet in color degrees of freedom

$$3 \times 3 \times 3 = \cdots + 1 \Rightarrow \psi_{3colors} = \frac{1}{\sqrt{6}}(bgr - brg + grb - gbr + rbg - rgb)$$

$$\underbrace{N_c \times N_c \times N_c \cdots}_{N_c} = \cdots + 1 \Rightarrow \psi_{N_c colors} =$$



- ground state spatially symmetric
- symmetric representation in spin-isospin



N_c "bosonic" quarks

SPIN UP PROTON MADE OF NC QUARKS

$N_c = 3$

3 spin-1/2 quarks make a total spin 1/2 or 3/2 baryon

orthogonal to $J_-|\uparrow\uparrow\uparrow\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\right)$

and symmetric under interchange of any two quarks

$$\frac{1}{\sqrt{6}}[uud]\,[2\uparrow\uparrow\downarrow - (\uparrow\downarrow + \downarrow\uparrow)\uparrow]$$ (distributed, symmetrize)

$$\frac{1}{\sqrt{18}}\Big([uud]\,[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow] + [duu]\,[2\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow] + [udu]\,[2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow]\Big)$$

$$= \frac{1}{\sqrt{18}}\Big(2\,[u\uparrow u\uparrow d\downarrow]_S - [u\uparrow u\downarrow d\uparrow]_S\Big)$$

see, e.g., Manohar hep-ph/9802419;
X. Ji https://www.physics.umd.edu/courses/Phys741/xji/chapter3.pdf

SPIN UP PROTON MADE OF Nc QUARKS

$$N_c = 3 \qquad 2\left[u\uparrow u\uparrow d\downarrow\right] - \left[u\uparrow u\downarrow d\uparrow\right]$$

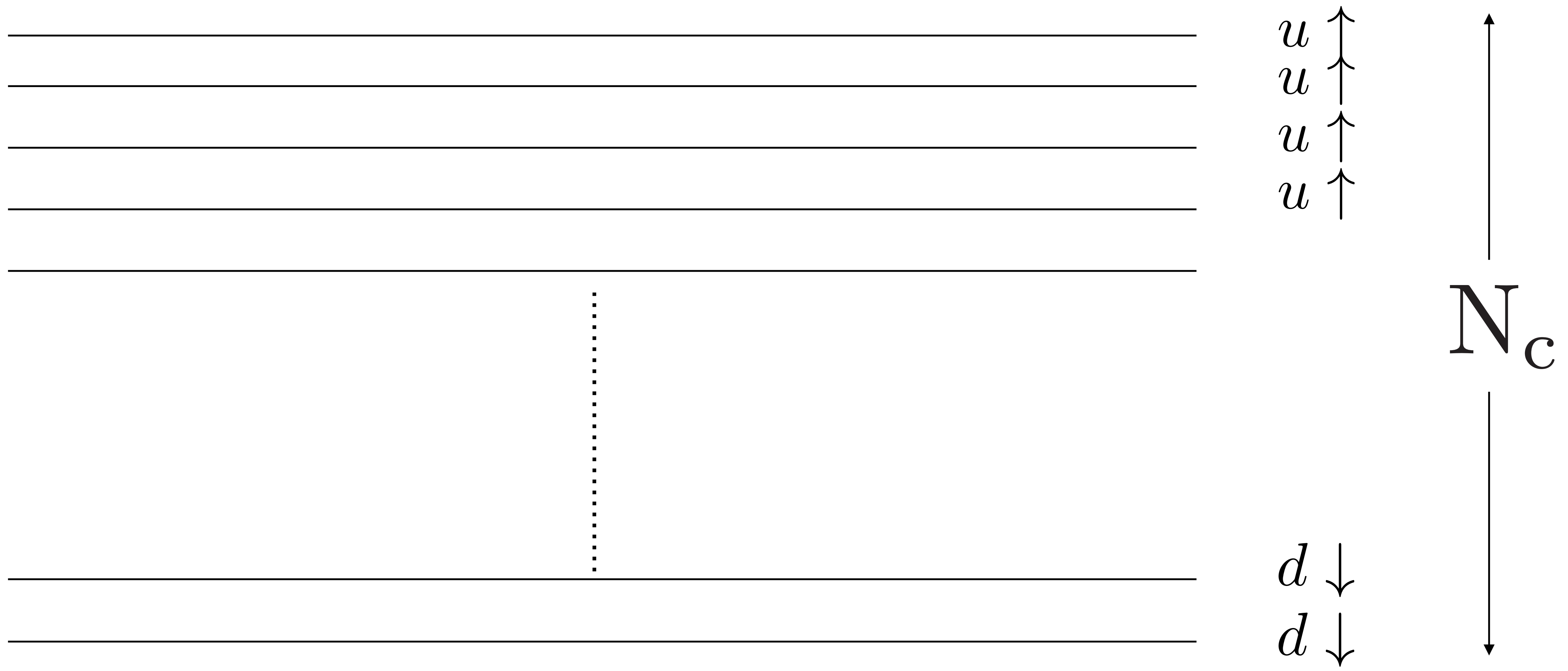
$$N_c = 5 \qquad 3\left[u\uparrow u\uparrow u\uparrow d\downarrow d\downarrow\right] - \left[u\uparrow u\uparrow u\downarrow\right]_S \left[d\uparrow d\downarrow\right]_S \cdots$$

$$N_c = 7 \qquad 4\left[u\uparrow u\uparrow u\uparrow u\uparrow d\downarrow d\downarrow d\downarrow\right] - \left[u\uparrow u\uparrow u\uparrow u\downarrow\right]_S \left[d\uparrow d\downarrow d\downarrow\right]_S \cdots$$

$$\vdots$$

$$N_c = 2m+1$$

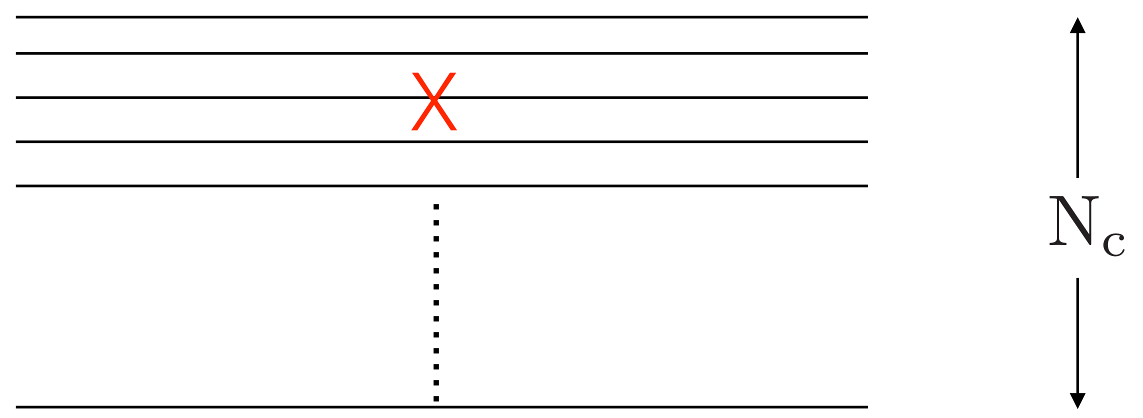
$$\Big[\underbrace{u\uparrow u\uparrow u\uparrow \cdots u\uparrow}_{2m+1}\underbrace{d\downarrow d\downarrow d\downarrow d\downarrow \cdots d\downarrow}_{m}\Big]$$



NUCLEONS IN LARGE NC

cannot calculate the nonperturbative matrix elements but can determine how they scale with Nc

$$|proton\uparrow\rangle = (\uparrow\downarrow\,\uparrow\downarrow\,\uparrow\downarrow\cdots\uparrow)\otimes(ud\;ud\;ud\;ud\cdots u)$$



operator insertion X

$nucleon\ N = n, p$ $\quad$ $bosonic\ quark$

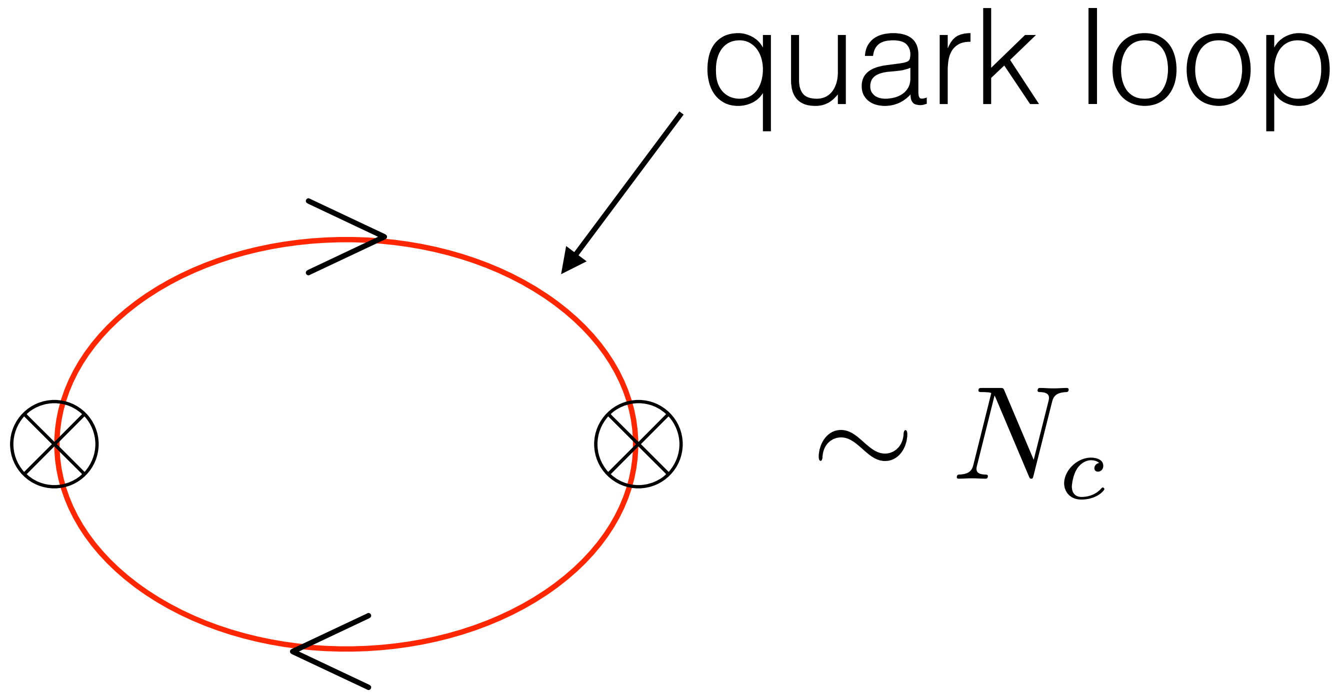
$$\langle N|q^\dagger \mathbb{1} q|N\rangle \lesssim \mathrm{N_c}\ ;\quad \langle N|q^\dagger \sigma_i\tau_a q|N\rangle \lesssim \mathrm{N_c}\ ;\quad \langle N|q^\dagger \tau_a q|N\rangle \lesssim 1\ ;\quad \langle N|q^\dagger \sigma_i q|N\rangle \lesssim 1$$

Dashen, Jenkins, Manohar PRD 49 4713 (1994); 51 2849(E) (1995); 51 3697 (1995); Manohar “Large N QCD” hep-ph/9802419.

More Nc counting:

loop diagrams need to be finite as N_c becomes large

$$-\left(\frac{11}{3}N_c - \frac{2}{3}N_F\right)\frac{g^3}{16\pi^2} + \cdots \qquad \Rightarrow \qquad g \to \frac{g}{\sqrt{N_c}}$$



$$\sim N_c$$

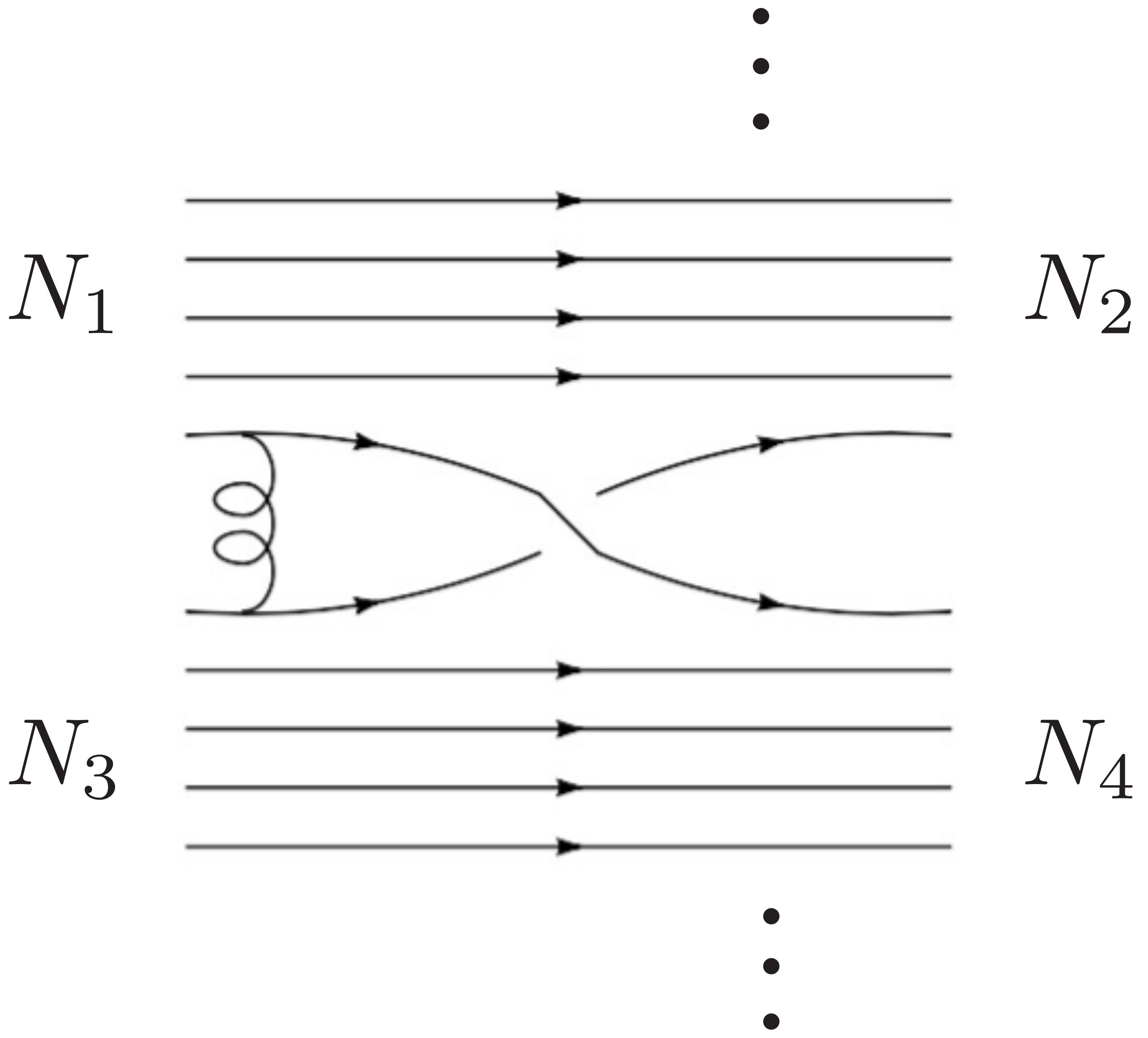
$$\langle 0|q^\dagger \sigma_i \tau_a q|\pi\rangle \sim \sqrt{N_c}$$

$$\Rightarrow f_\pi \sim \sqrt{N_c}$$

pion decay constant

NN Scattering in large Nc

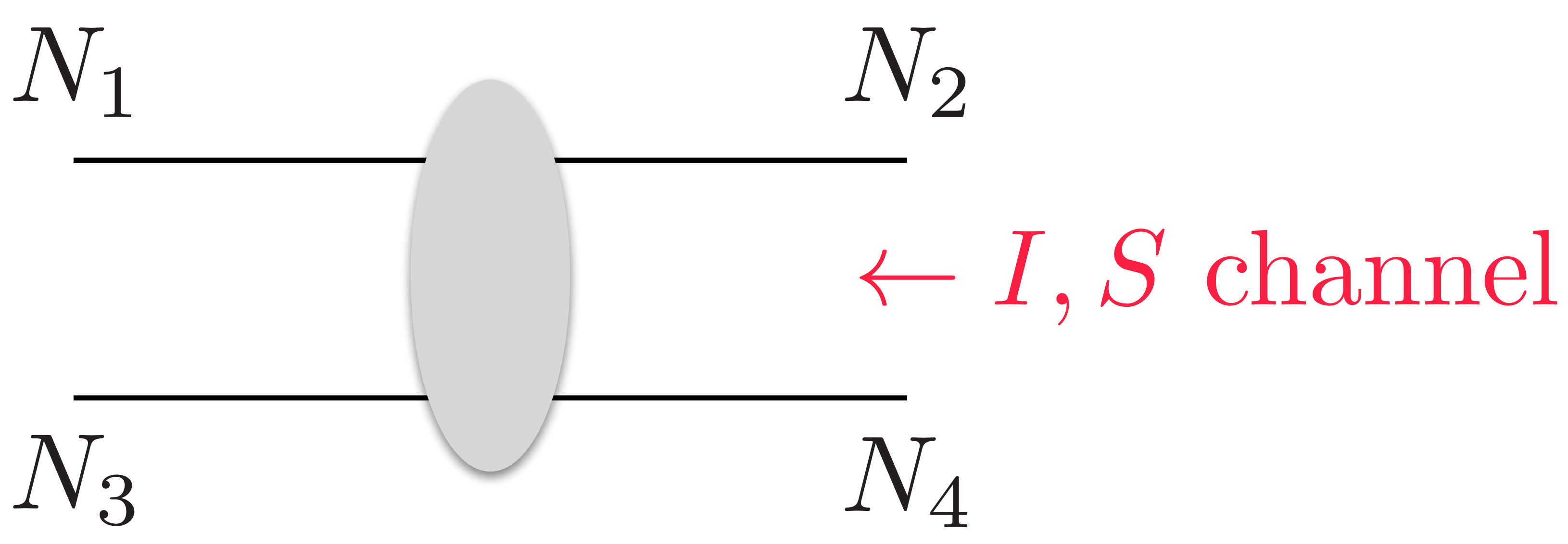
Kaplan and Savage, PLB365(1996)244;
Kaplan and Manohar, PRC 56 (1997) 76;
Mehen, Stewart, and Wise, PRL83 (1999)931;
Banarjee, Cohen, Gelman PRC 65 034011 (2002)



$$\sim \frac{g}{\sqrt{N_c}} \frac{g}{\sqrt{N_c}} N_c^2 \sim N_c$$

$$p \sim N_c^0$$

$$N_c \langle N_2 | \frac{\mathcal{O}_{IS}^{(n)}}{N_c^n} | N_1 \rangle \langle N_4 | \frac{\overline{\mathcal{O}}_{IS}^{(n')}}{N_c^{n'}} | N_3 \rangle$$



$$\langle N' | \frac{\mathcal{O}_{IS}^{(n)}}{N_c^n} | N \rangle \leq \frac{1}{N_c^{|I-S|}}$$

Manohar, "Large N QCD" in Probing the Standard Model of Particle Interactions," ed. F. David and R. Gupta, 2008

MATCH TO THE LARGE-Nc HARTREE FORM

$$\psi^{\text{nucleon}}_{\text{Hartree}} = \prod_{i=1}^{N_c} \phi_i \qquad H_{\text{Hartree}} = N_c \sum_n \sum_{s,t} v_{stn} \left(\frac{S^i}{N_c}\right)^s \left(\frac{I^a}{N_c}\right)^t \left(\frac{G^{ia}}{N_c}\right)^{n-s-t}$$

$$S^i = q^\dagger \sigma^i q \qquad I^a = q^\dagger \tau^a q \qquad G^{ia} = q^\dagger \sigma^i \tau^a q$$

$$\langle N'|S|N\rangle \sim \langle N'|I|N\rangle \sim 1, \quad \langle N'|G|N\rangle \sim \langle N'|\mathbb{1}|N\rangle \sim N_c$$

EXAMPLE:

$$\mathcal{L} = C_{\text{scalar}}(N^\dagger N)(N^\dagger N) + C_{\text{tensor}}(N^\dagger \sigma^i N)(N^\dagger \sigma^i N)$$

$$\Rightarrow C_{\text{scalar}}(\mathbb{1})_1(\mathbb{1})_2 + C_{\text{tensor}}(S)_1(S)_2 \sim C_{\text{scalar}} N_c^2 + C_{\text{tensor}} N_c^0$$

$$\Rightarrow C_{\text{scalar}} \sim N_c \quad , \quad C_{\text{tensor}} \sim N_c^{-1}$$

Dashen, Jenkins, Manohar PRD 51 3697. [operator reduction…]

SOURCE OF MOMENTUM DEPENDENCE IN OPERATOR

Does it come from a relativistic correction?

$$M_N \sim \langle N | q^\dagger \mathbb{1} q | N \rangle \lesssim \mathrm{N_c}$$

$$\langle N_\gamma(\vec{p}_1{}') N_\delta(\vec{p}_2{}') | \mathcal{O} | N_\alpha(\vec{p}_1) N_\beta(\vec{p}_2) \rangle$$

velocity: $\dfrac{p}{M_N} \sim \dfrac{1}{\mathrm{N}_c}$

$$\vec{p}_\pm = \vec{p}\,' \pm \vec{p}$$

$$\vec{p}\,' = \vec{p}_1{}' - \vec{p}_2{}', \quad \vec{p} = \vec{p}_1 - \vec{p}_2$$

$$\vec{p}_- \sim 1\,, \quad \vec{p}_+ \sim \mathrm{N}_c^{-1}\,.$$

THE TROUBLE WITH FIERZING

- Spin-flavor symmetry is in the context of baryons
- Operator reductions are in the context of matrix elements
- Spin-flavor scaling is in the context of matrix elements

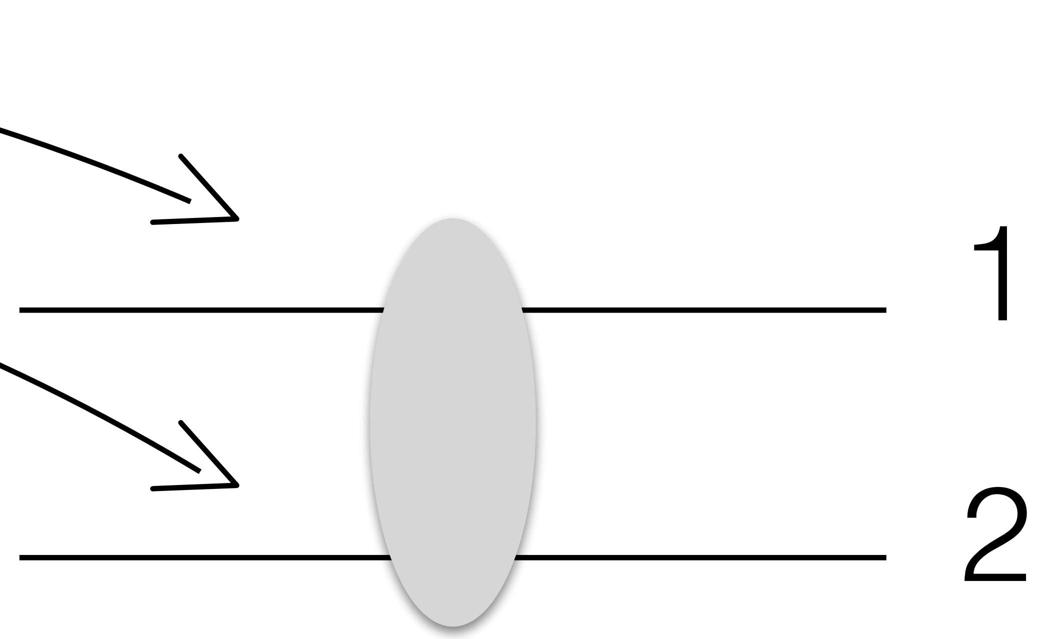
- Fierzing can exchange $\vec{p}_-$ and $\vec{p}_+$

- Fierzing can be used to eliminate isospin completely from strong interaction NN operators

LOW ENERGY 2N PARITY VIOLATION ILLUSTRATES (MOST OF) THE STEPS

lowest order S-P wave transitions: momentum will play a role

Girlanda PRC **77**, 067001 (2008)



$$
\begin{aligned}
V^{\text{minimal}}_{\text{Girlanda}} = &- \mathcal{G}_1 \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\
&- i\tilde{\mathcal{G}}_1 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \\
&- i\mathcal{G}_2 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)(\tau_1 + \tau_2)^3 \\
&- i\tilde{\mathcal{G}}_5 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
&+ \frac{i}{2}\mathcal{G}_6 \vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)(\tau_1 \times \tau_2)^3
\end{aligned}
$$

$$\Rightarrow \mathcal{G}_1 N_c^{-1}\left[(S)_1(\mathbb{1})_2 + (\mathbb{1})_1(S)^2\right] + \cdots \sim \mathcal{G}_1 N_c^{-1} N_c + \cdots$$

$$\tilde{\mathcal{G}}_5 \sim N_c$$

$$\mathcal{G}_2 \sim \mathcal{G}_6 \sim N_c^0$$

$$\mathcal{G}_1 \sim \tilde{\mathcal{G}}_1 \sim N_c^{-1}$$

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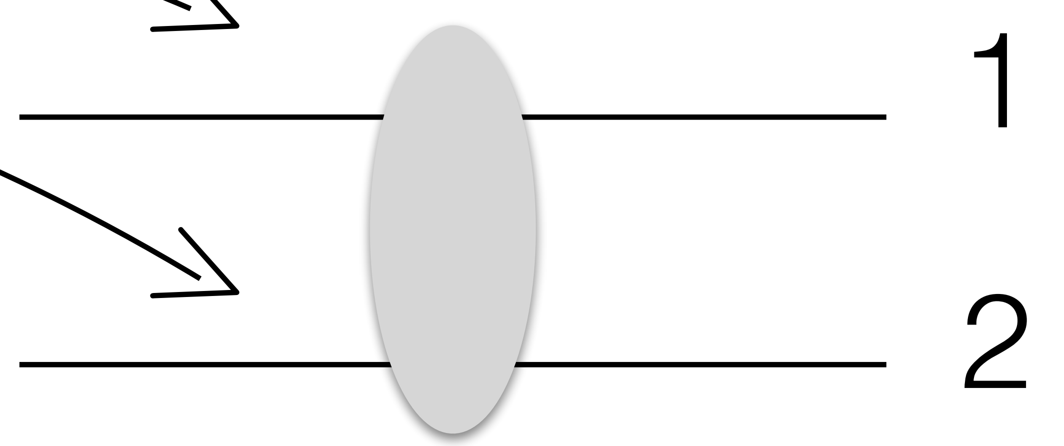
$$
\begin{aligned}
V_{\rm Girlanda}^{\rm minimal} = & - \mathcal{G}_1 \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\
& - i\tilde{\mathcal{G}}_1 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \\
& - i\mathcal{G}_2 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)(\tau_1 + \tau_2)^3 \\
& - i\tilde{\mathcal{G}}_5 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
& + \frac{i}{2}\mathcal{G}_6 \vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)(\tau_1 \times \tau_2)^3 \ .
\end{aligned}
$$



$$\Rightarrow \mathcal{G}_1 N_c^{-1} \left[(S)_1 (\mathbb{1})_2 + (\mathbb{1})_1 (S)^2 \right] + \cdots \sim \mathcal{G}_1 N_c^{-1} N_c + \cdots$$

$$\tilde{\mathcal{G}}_5 \sim N_c$$

$$\mathcal{G}_2 \sim \mathcal{G}_6 \sim N_c^0$$

$$\mathcal{G}_1 \sim \tilde{\mathcal{G}}_1 \sim N_c^{-1}$$

$$
\begin{aligned}
&V^{\text{nonminal}} \\
&= \mathcal{A}_1^+ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\
&+ \mathcal{A}_1^- \vec{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2) \\
&+ \mathcal{A}_2^+ \ \vec{p}_+ \cdot (\vec{\sigma}_1 \tau_1^3 - \vec{\sigma}_2 \tau_2^3) \\
&+ \frac{1}{2} \mathcal{A}_2^- \vec{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)(\tau_1 + \tau_2)^3 \\
&+ \mathcal{A}_3^+ \ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2 \\
&+ \mathcal{A}_3^- \ \vec{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2 \\
&+ \mathcal{A}_4^+ \ \vec{p}_+ \cdot (\vec{\sigma}_1 \tau_2^3 - \vec{\sigma}_2 \tau_1^3) \\
&+ \mathcal{A}_5^+ \ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
&+ \mathcal{A}_5^- \vec{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\
&- \frac{1}{2} \mathcal{A}_6^- \vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) i(\tau_1 \times \tau_2)^3
\end{aligned}
$$

cf. Zhu et al. NPA 748 435 (2005)

$$
\begin{aligned}
\mathcal{G}_1 &= -\mathcal{A}_1^+ + \mathcal{A}_3^+ - 2\mathcal{A}_3^- \\
\tilde{\mathcal{G}}_1 &= -\mathcal{A}_1^- - 2\mathcal{A}_3^+ + \mathcal{A}_3^- \\
\mathcal{G}_2 &= -\frac{1}{2}\left(\mathcal{A}_2^- + \mathcal{A}_2^+ + \mathcal{A}_4^+\right) \\
\tilde{\mathcal{G}}_5 &= -\left(\mathcal{A}_5^- + \mathcal{A}_5^+\right) \\
\mathcal{G}_6 &= -\mathcal{A}_6^- + \mathcal{A}_2^+ - \mathcal{A}_4^+
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{G}_1 &\sim N_c \\
\tilde{\mathcal{G}}_1 &\sim N_c \\
\mathcal{G}_2 &\sim N_c^0 \\
\tilde{\mathcal{G}}_5 &\sim N_c \\
\mathcal{G}_6 &\sim N_c^0
\end{aligned}
$$

Schindler,rps,Vanasse
PRC 93 2 025502 (2016)

WHAT ABOUT PIONS?

chiral perturbation theory includes pions via

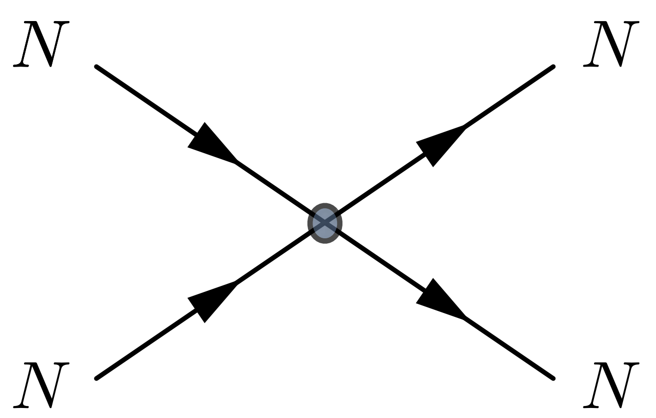
$$exp\left(\frac{i}{2f_\pi}\phi_a\tau_a\right) \qquad\qquad f_\pi \sim \sqrt{N_c}$$

pions

$$\overline{C}_2\left[\frac{1}{2}\left(1-\frac{1}{2f_\pi^2}\phi_a\phi_a\right)\left(N^\dagger N\right)\left(N^\dagger\tau^3 N\right)+\frac{1}{4f_\pi^2}\phi_3\phi_a\left(N^\dagger N\right)\left(N^\dagger\tau^a N\right)\right]$$

Richardson, Schindler, Pastore, rps PRC 103 5 055501 (2021)

HOW WELL DOES THIS WORK?



NN scattering S-wave no derivatives

partial wave basis vs large-N counting basis

$$\mathcal{L}_0^{(S)} = -C_0^{(^3S_1)}(N^T P_i N)^\dagger (N^T P_i N) - C_0^{(^1S_0)}(N^T P_a N)^\dagger (N^T P_a N) = -\frac{1}{2}C_{\rm scalar}(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_{\rm tensor}(N^\dagger \sigma_i N)(N^\dagger \sigma_i N)$$

$$P_i = \frac{1}{\sqrt{8}}\sigma_2\sigma_i\tau_2 \qquad P_a = \frac{1}{\sqrt{8}}\sigma_2\tau_2\tau_a$$

$$\xrightarrow[\rm N_c]{\rm large} -\frac{1}{2}C_{\rm scalar}(N^\dagger N)(N^\dagger N)$$

Large N result

$$=> \quad C_0^{(^3S_1)} = C_0^{(^1S_0)}$$

expect 30% corrections

$$a^{(^3S_1)} \sim \frac{1}{36\ \text{MeV}}$$

$$a^{(^1S_0)} \sim -\frac{1}{8\ \text{MeV}}$$

$$C_0^{(S)}(\mu) = \frac{4\pi}{M}\frac{1}{\frac{1}{a^{(S)}} - \mu}$$

scale dependence

requires

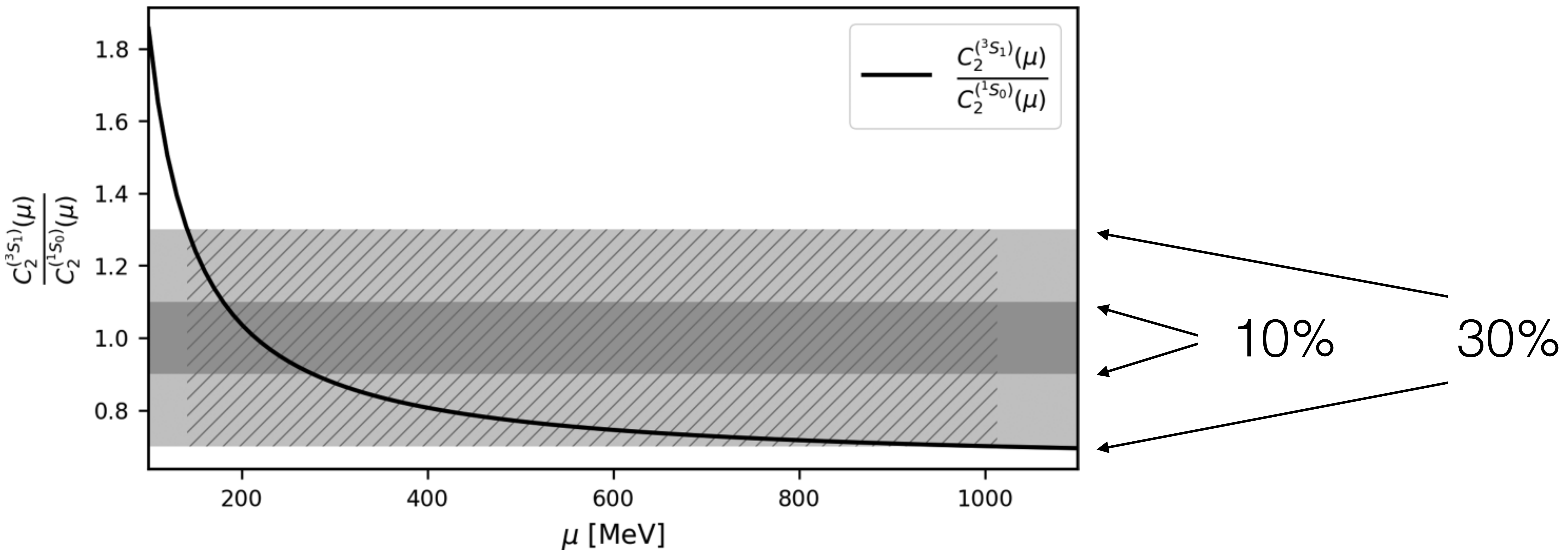
$\mu > 140$ MeV

Kaplan and Savage, PLB 365 (1996) 244; Mehen, Stewart, Wise PRL 83 (1999) 931

Schindler, Singh, rps PRC 98 4 044001 (2018)

Two-derivative terms: S-wave

$$\frac{C_2^{(^3S_1)}}{C_2^{(^1S_0)}} = \frac{r^{(^3S_1)}}{r^{(^1S_0)}} \frac{(\mu - 1/a^{(^1S_0)})^2}{(\mu - 1/a^{(^3S_1)})^2}$$

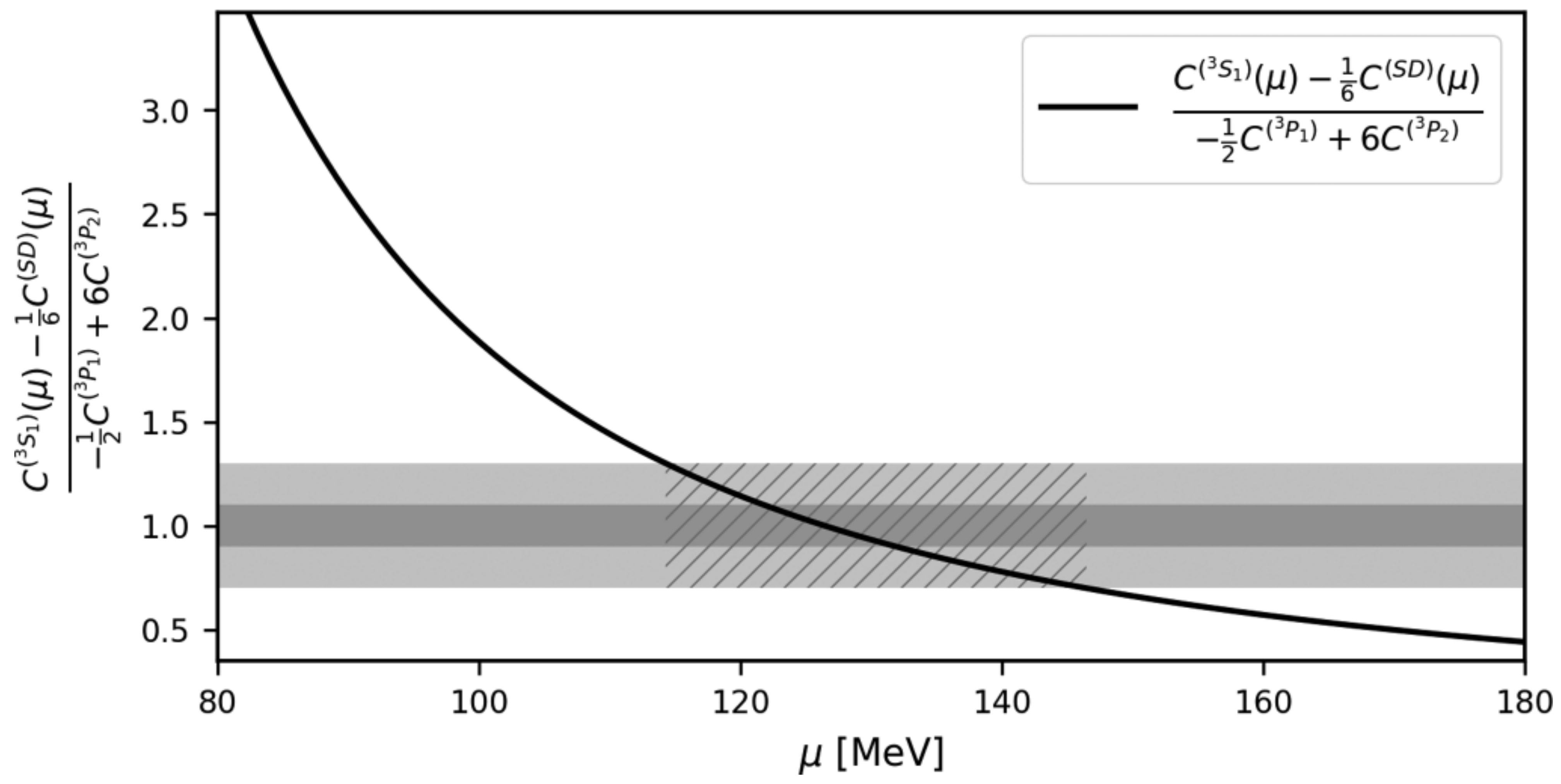


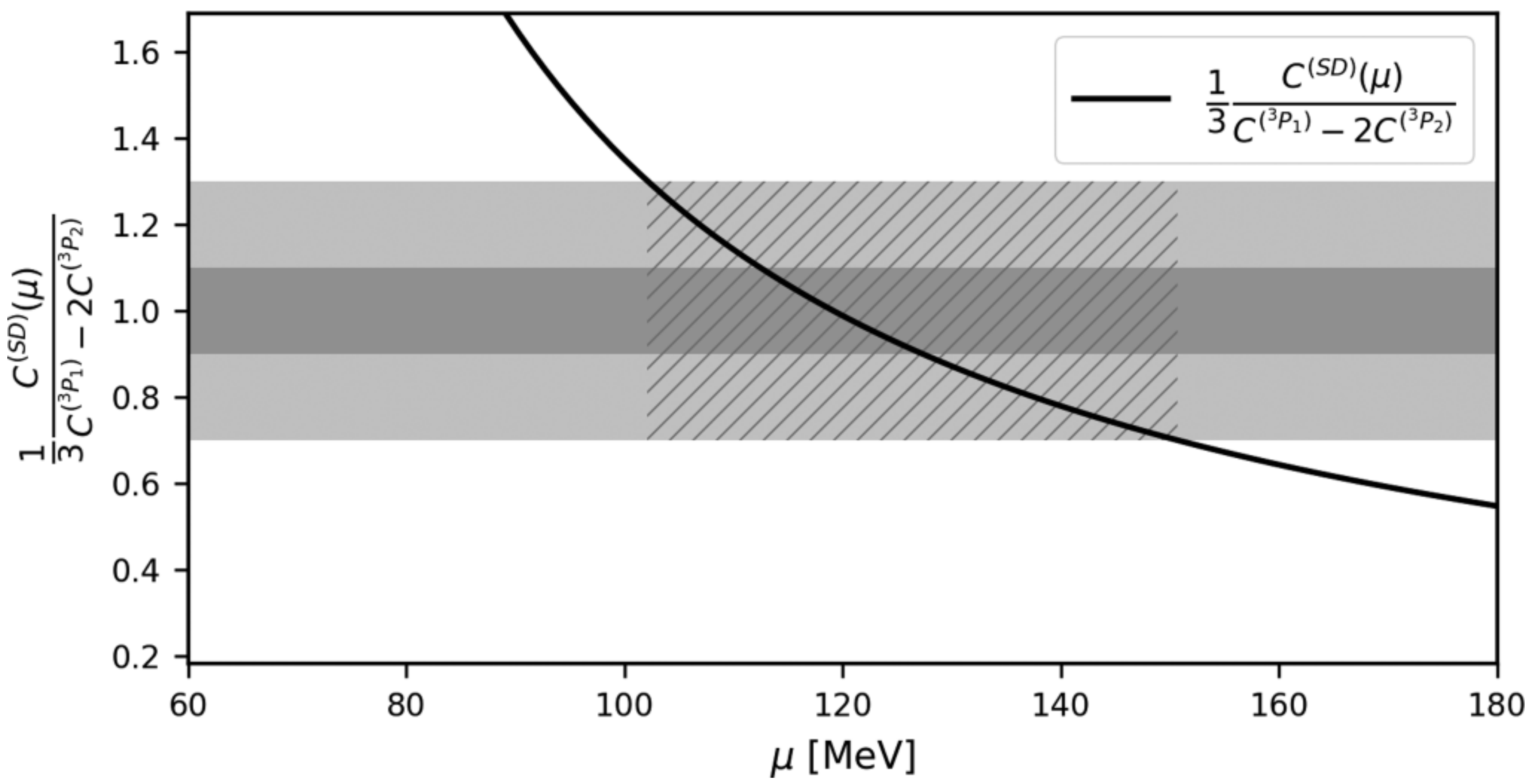
$$a^{(^1S_0)} = -23.7148(43)\,\mathrm{fm} \qquad r^{(^1S_0)} = 2.750(18)\,\mathrm{fm}$$

$$a^{(^3S_1)} = 5.4112(15)\,\mathrm{fm} \qquad r^{(^3S_1)} = 1.7436(19)\,\mathrm{fm}$$

Two-derivative terms, cont

$$\left.\frac{C^{(^3P_0)} - \frac{4}{3}C^{(^3P_2)}}{-C^{(^3P_1)} + 2C^{(^3P_2)}}\right|_{\text{LO-in-}N_c} \underset{\text{thy}}{= 1} \qquad \underset{\text{exp}}{= 0.82 \pm 0.12}$$





$$C^{(^3P_0)} = 6.6\,\text{fm}^4 \qquad C^{(^3P_1)} = -6.0\,\text{fm}^4$$

$$C^{(^3P_2)} = 0.57\,\text{fm}^4 \qquad C^{(^1P_1)} = -22\,\text{fm}^4$$

COMPARE TO EXISTING PV MEASUREMENTS AND LIMITS?

Experimental support for

$$\mathcal{C}^{(^3S_1-^1P_1)} = 3\,\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} \sim N_c$$

$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} \sim N_c\ [\sin^2\theta_W]$$

?

P.D.Eversheim et al.,PLB 256 (1991) 11

$$A_{\vec{p}p}:\quad 4\frac{\mathcal{C}^{(^3S_1-^1P_1)}}{\mathcal{C}_0} + 12\frac{\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}}{\mathcal{C}_0} = (-4.5\pm 0.9)\times 10^{-10}\ \mathrm{MeV}^{-1}$$

$$P_\gamma:\quad -\frac{16 M_N}{\kappa_1(1-\gamma a^{(^1\mathrm{S}_0)})}\left(\frac{\mathcal{C}^{(^3S_1-^1P_1)}}{\mathcal{C}_0}(1-\frac{5}{9}\gamma a^{(^1\mathrm{S}_0)}) - \frac{2}{3}\gamma a^{(^1\mathrm{S}_0)}\frac{\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}}{\mathcal{C}_0}\right)$$

$$= (1.8\pm 1.8)\times 10^{-7}$$

V.A.Knyaszkov et al., NPA 197 (1972) 241.

$$\Rightarrow\quad 4.0\frac{\mathcal{C}^{(^3S_1-^1P_1)}}{\mathcal{C}_0} + 3.6\frac{\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}}{\mathcal{C}_0} = (-1.8\pm 1.8)\times 10^{-10}\ \mathrm{MeV}^{-1}$$

Motivation for KITP 2018 and INT 2022 non-parity conservation workshops

What about NPDGamma?

$$A_\gamma = \frac{32}{3} \frac{M_N}{\kappa_1 \left(1 - \gamma a^{({}^1S_0)}\right)} \frac{\mathcal{C}^{({}^3S_1 - {}^3P_1)}}{\mathcal{C}_0}$$

$$\frac{\mathcal{C}^{({}^3S_1 - {}^3P_1)}}{\mathcal{C}_0^{({}^3S_1)}} \approx (1.8 \pm 0.8) \times 10^{-12}\ \mathrm{MeV}^{-1}$$

dimensionless factors can always modify things
look for trends rather than sharp predictions

$P_{\gamma}^{exp\ limit} \times f$

impact on LECs

	$[\mathcal{C}^{(^3S_1-^1P_1)}/\mathcal{C}_0] \times 10^{11}$	$[\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}/\mathcal{C}_0] \times 10^{11}$
f=1	-1.8	-3.1
f=0.5	1.5	-4.1
f=0.01	4.0	-5.0
f=0	4.7	-5.2

CONCLUSIONS

- Ordering interactions on the basis of large-Nc can be useful when no other info is available.
- Rules give the largest Nc behavior possible, but additional cancellations are always possible
- Trends rather than predictions are obtained.
- Take care not to “mix and match” theories that have different degrees of freedom and/or have subtraction point issues; low energy coefficients are not observables.
- So far, reasonable agreement in all NN cases that have been checked.
- It is always possible that random numbers will destroy this order
- Nice to have another expansion parameter to use in dual expansion
- Looking at expansions around enhanced symmetry points can be insightful