NORDITA
The Nordic Institute for Theoretical Physics

Electron hydrodynamics in solids

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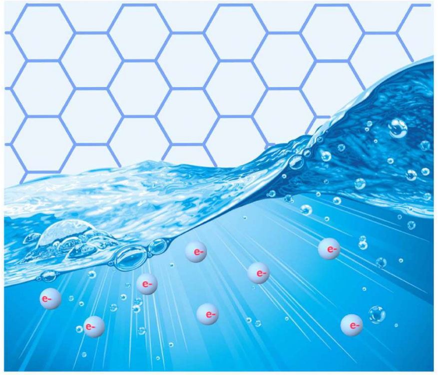
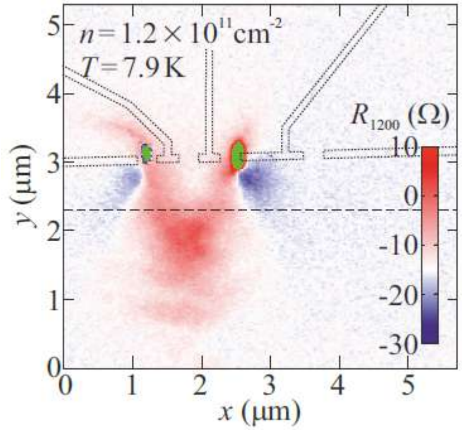
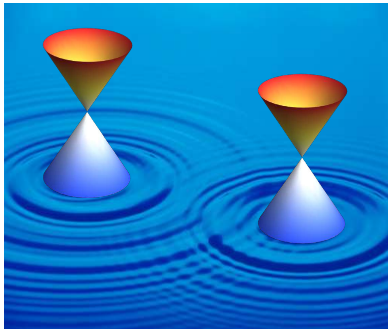
Outline

1. Basics of hydrodynamics
2. Hydrodynamics in solids and experimental observations
3. Consistent hydrodynamics in Weyl semimetals and electron flows



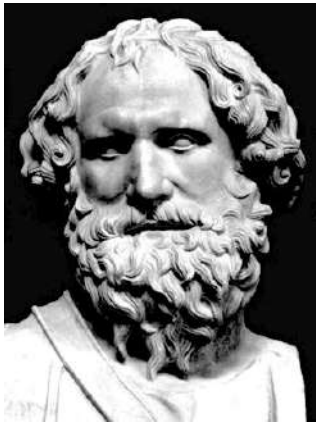




Basics of hydrodynamics

Definition of hydrodynamics

- **Hydrodynamics** is the macroscopic theory that studies the motion of various fluids (including gases).


Archimedes


Leonhard Euler


Daniel Bernoulli


Claude-Louis Navier


George Stokes

- Key variables:
  - $\mathbf{u} = \mathbf{u}(t, \mathbf{r})$ fluid velocity
  - $n = n(t, \mathbf{r})$ particle density
  - $\epsilon = \epsilon(t, \mathbf{r})$ energy density
- Hydrodynamics is based on the conservation laws: momentum, mass, and energy.

Key equations

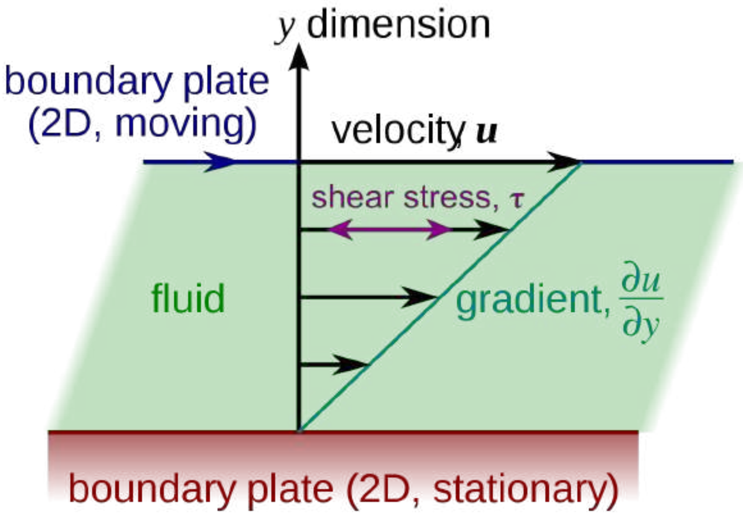
❖ **Navier-Stocks** equation:

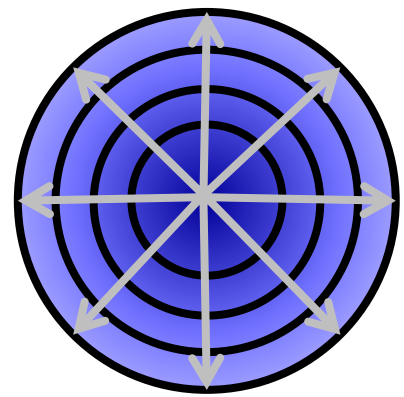
$$n\left[\partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}\right] = -\boldsymbol{\nabla}P + \eta\Delta\mathbf{u} + \left(\zeta + \frac{\eta}{3}\right)\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{u}).$$

Advection term

Diffusion term

Compressibility





η is the shear viscosity

ζ is the bulk viscosity

Key equations

❖ **Heat transfer** equation:

$$nT\left[\partial_t s + (\mathbf{u}\cdot\boldsymbol{\nabla})s\right] = \nabla_j \kappa \nabla_j T + \frac{\eta}{2}\left(\partial_j u_i + \partial_i u_j - \frac{2}{3}\delta_{ij}\partial_l u_l\right)^2 + \zeta(\boldsymbol{\nabla}\cdot\mathbf{u})^2$$

Friction terms

Thermoconductivity term

Friction terms

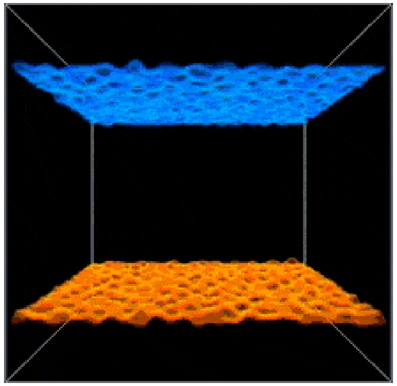
s is the entropy density

κ is the thermoconductivity

❖ **Continuity** equation:

$$\partial_t n + (\boldsymbol{\nabla}\cdot\mathbf{J}) = 0.$$

$\mathbf{J} = n\mathbf{u}$ is the particle current density



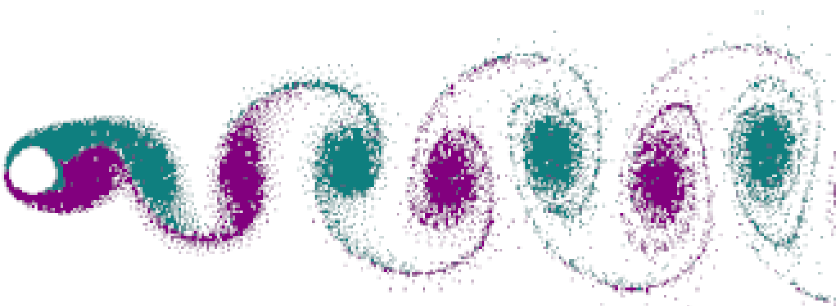
Reynolds number and turbulence

❖ Reynolds number:

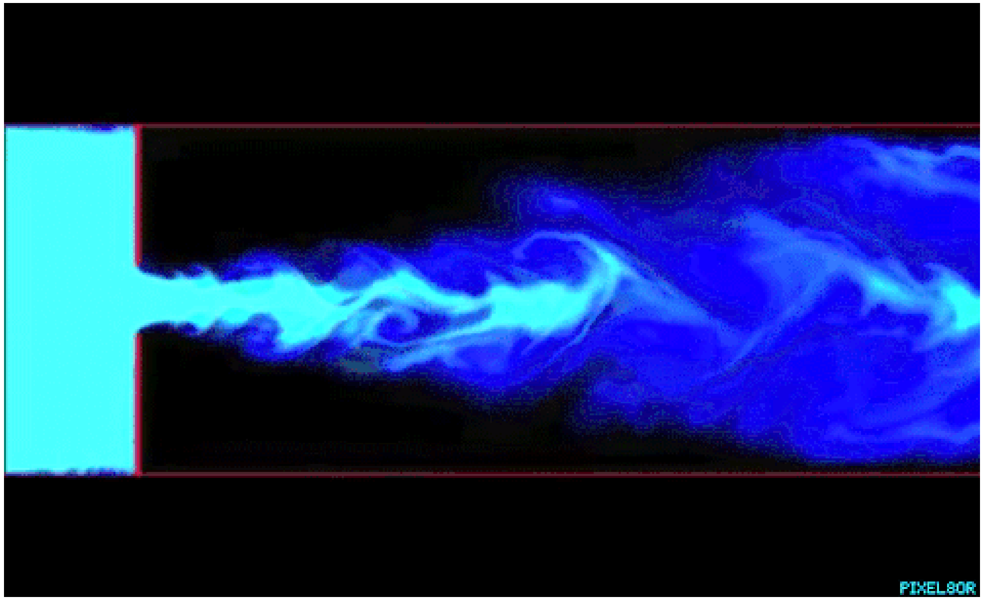
$$\mathrm{Re} = \left| \frac{n u_j \nabla_j u_i}{\eta \nabla_j \nabla_j u_i} \right| = \frac{nuL}{\eta} = \frac{uL}{\nu}$$

$\mathrm{Re} \ll 1$ laminar (layered) flow $\mathrm{Re} \gg 1$ turbulent (chaotic) flow

❖ Vortices and turbulence:



von Kármán vortex street

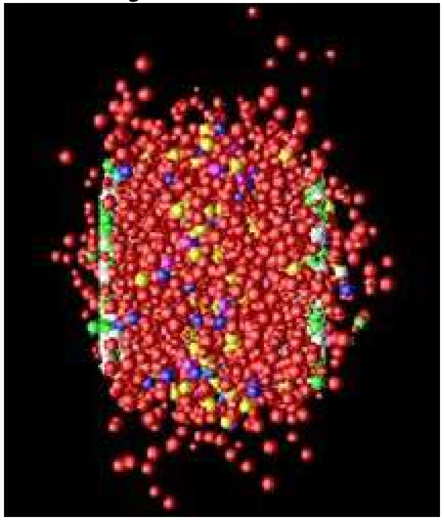


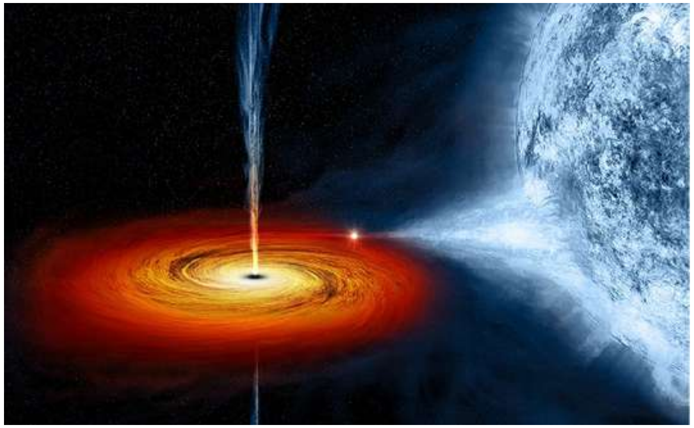
Subfields of fluid dynamics

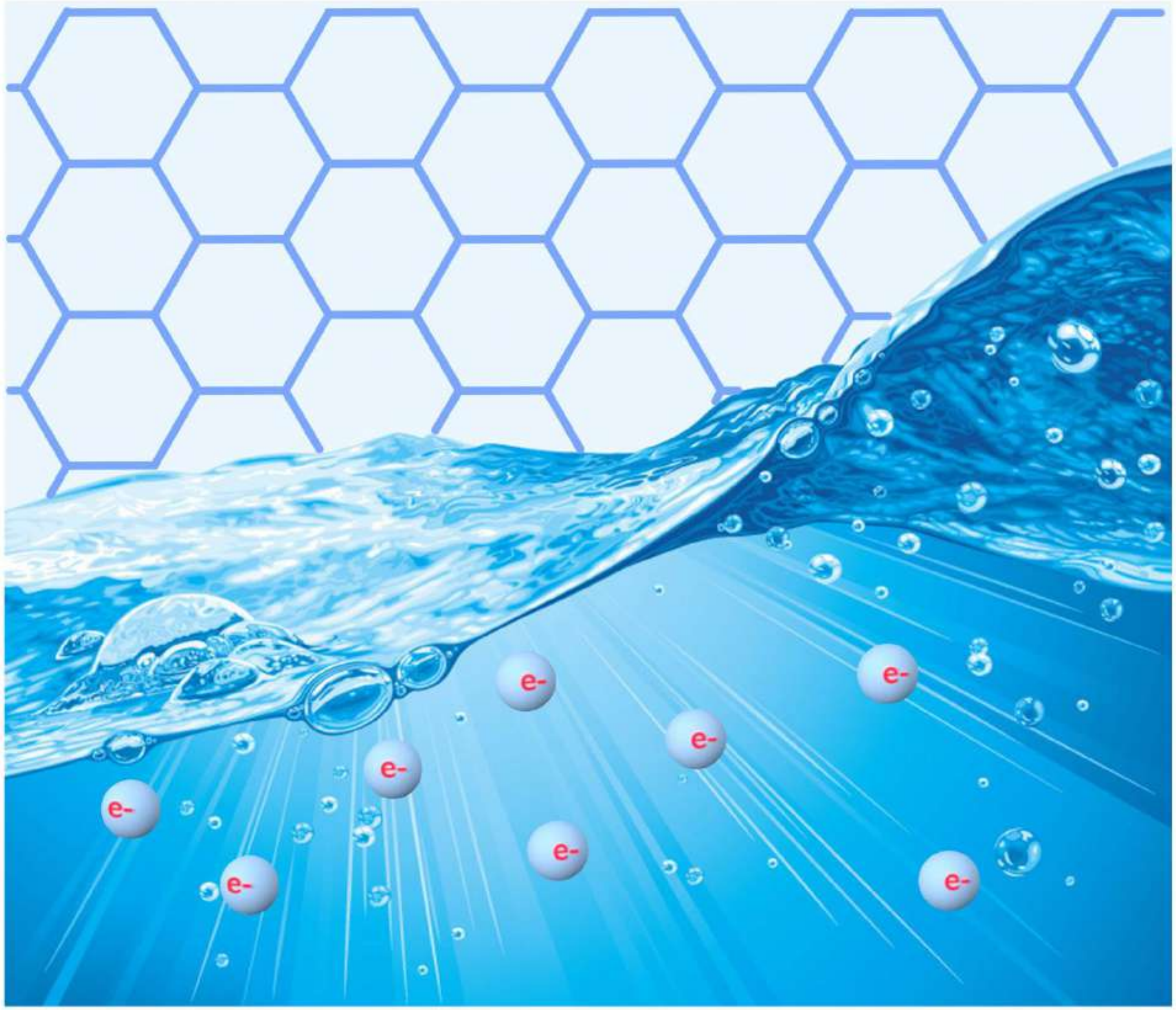
❖ The number of subfields in fluid dynamics is numerous:

- Aerodynamics
- Magneto-hydrodynamics
- Geophysical fluid dynamics and meteorology
- Hemodynamics
- etc.

❖ System scales range from fm to parsec





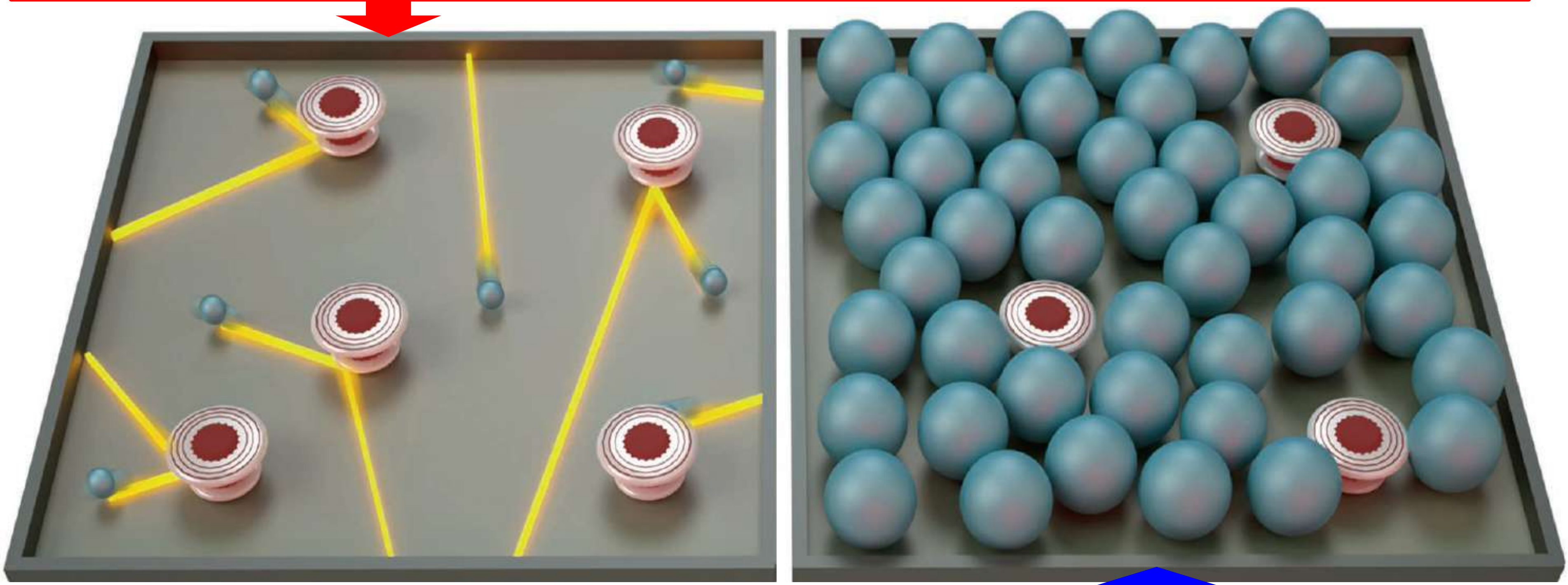


Hydrodynamics in solids and experimental observations

Two main regimes

- Two types of collisions: momentum-relaxing (l_{MR}) and momentum-conserving (l_{MC}). [R.N. Gurzhi, J. Exp. Theor. Phys. **17**, 521 (1963); Sov. Phys. Usp. **11**, 255 (1968).]

Nonhydrodynamic regimes: $l_{MR} \ll l_{MC}, L$, where L is the sample size.

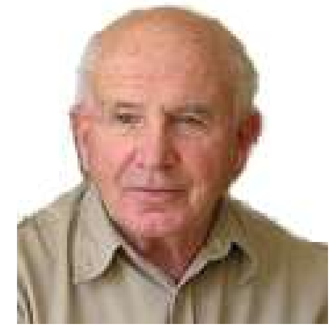


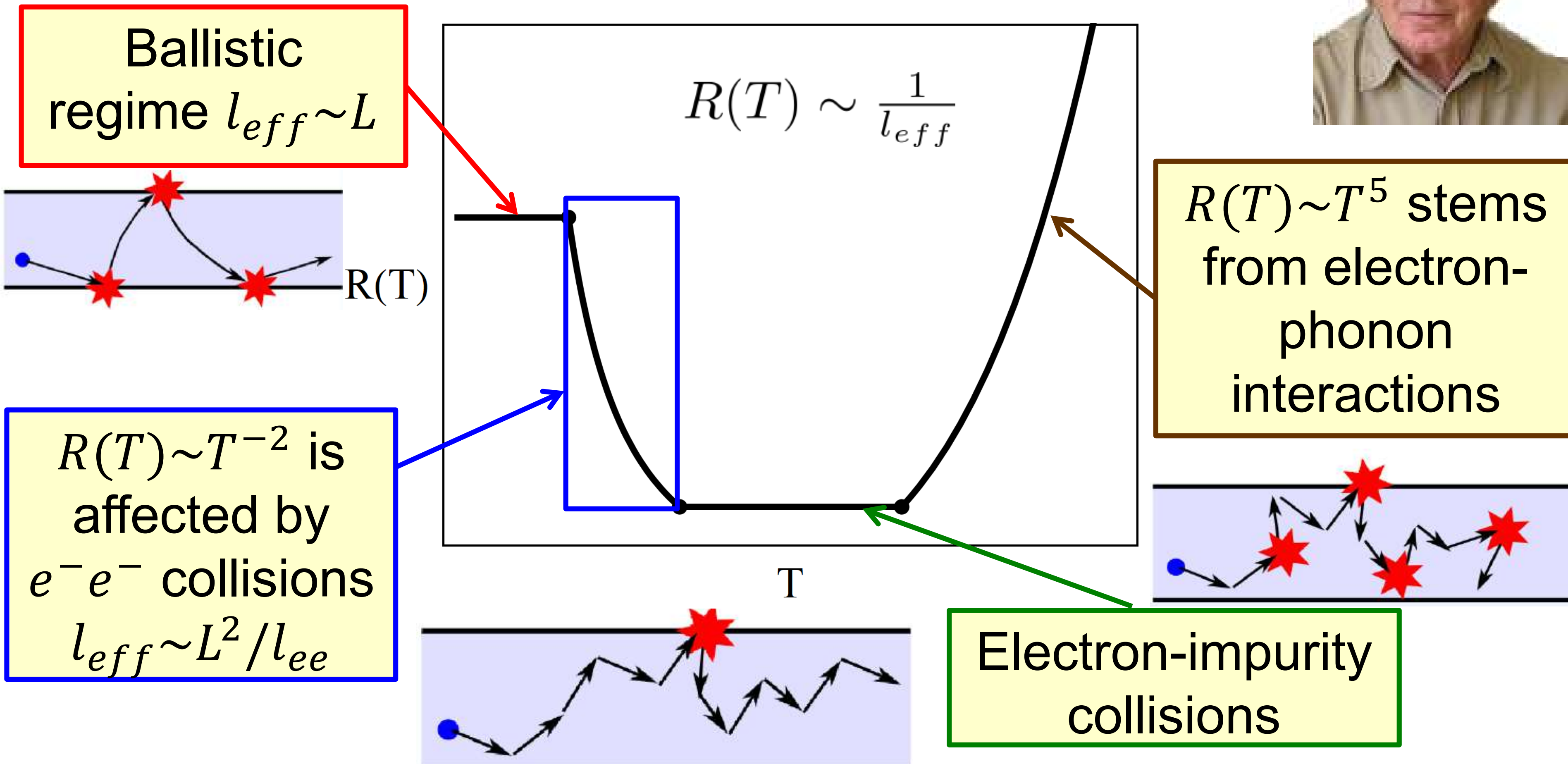
[J. Zaanen, Science **351**, 1058 (2016)]

Hydrodynamic regime: $l_{MC} \ll L \ll l_{MR}$.

Gurzhi effect

- Schematic nonlinear dependence of resistance on temperature [R.N. Gurzhi, J. Exp. Theor. Phys. **17**, 521 (1963)]

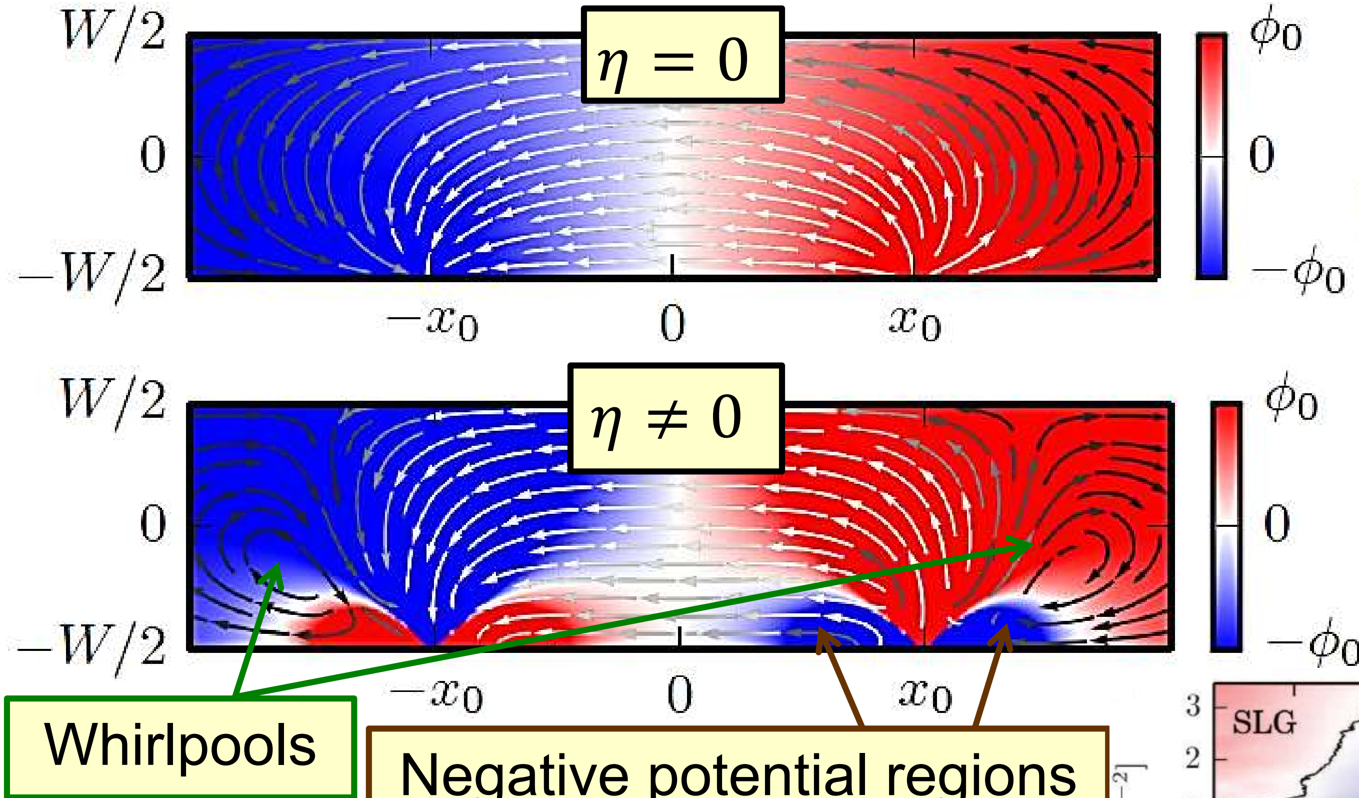




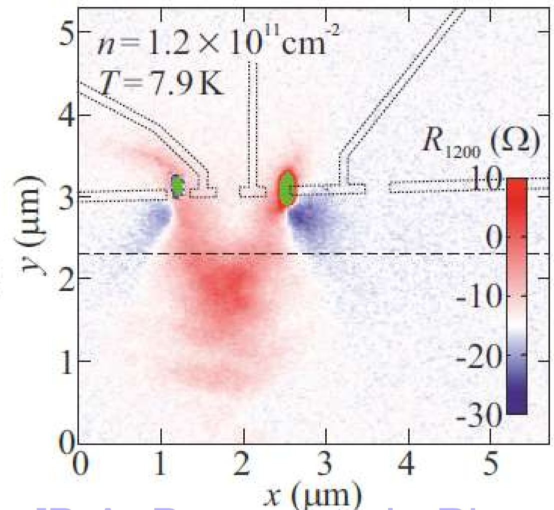
Experimental observations

- **Gurzhi effect** in 2D electron gas of (Al,Ga)As heterostructures [L.W. Molenkamp and M.J.M. de Jong, Solid-State Electron. **37**, 551 (1994); Phys. Rev. B **51**, 13389 (1995)]
- Viscous contribution to the resistance of 2D metal $PdCoO_2$ (Poiseuille flow) [P.J.W. Moll et al., Science **351**, 1061 (2016)]
- **Graphene** [Recent review: A. Lucas and K.C. Fong, Hydrodynamics of electrons in graphene, J. Phys.: Condens. Matter **30**, 053001 (2018)]
  - Negative nonlocal resistance and whirlpools [D.A. Bandurin et al., Science **351**, 1055 (2016); F.M.D. Pellegrino et al., Phys. Rev. B **94**, 155414 (2016); L. Levitov and G. Falkovich, Nat. Phys. **12**, 672 (2016)]
  - Higher than ballistic transport in constrictions [H. Guo et al., PNAS **114**, 3068 (2017); R. Krishna Kumar et al., Nat. Phys. **13**, 1182 (2017)]
  - Visualization of the Poiseuille flow via the Hall field profile [J.A. Sulpizio et al., Nature **576**, 75 (2019)]

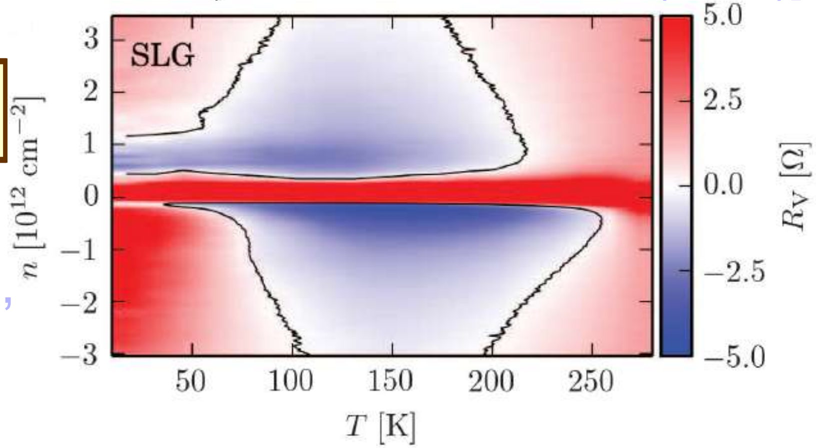
Backflows in graphene and GaAs



[I. Torre et al., Phys. Rev. B **92**, 165433 (2015)]



[B.A. Braem et al., Phys. Rev. B **98**, 241304(R) (2018)]

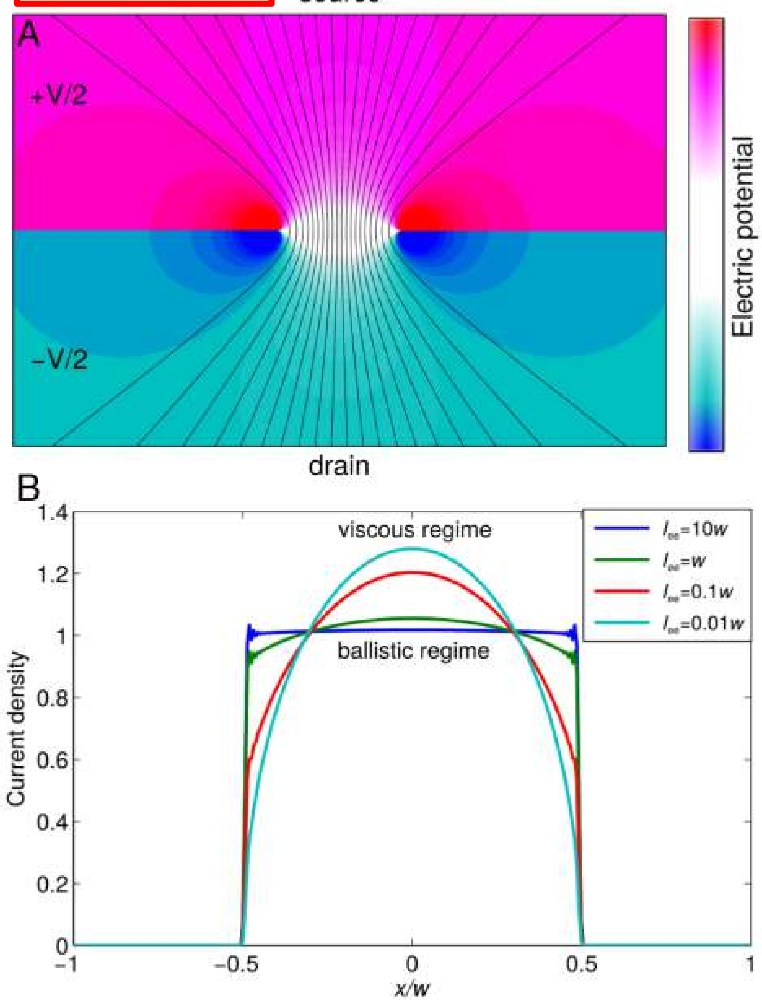


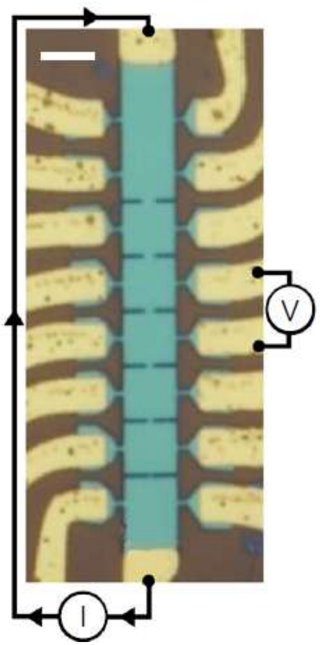
[D.A. Bandurin et al., Science **351**, 1055 (2016)]

Electron flow through a constriction

Theory

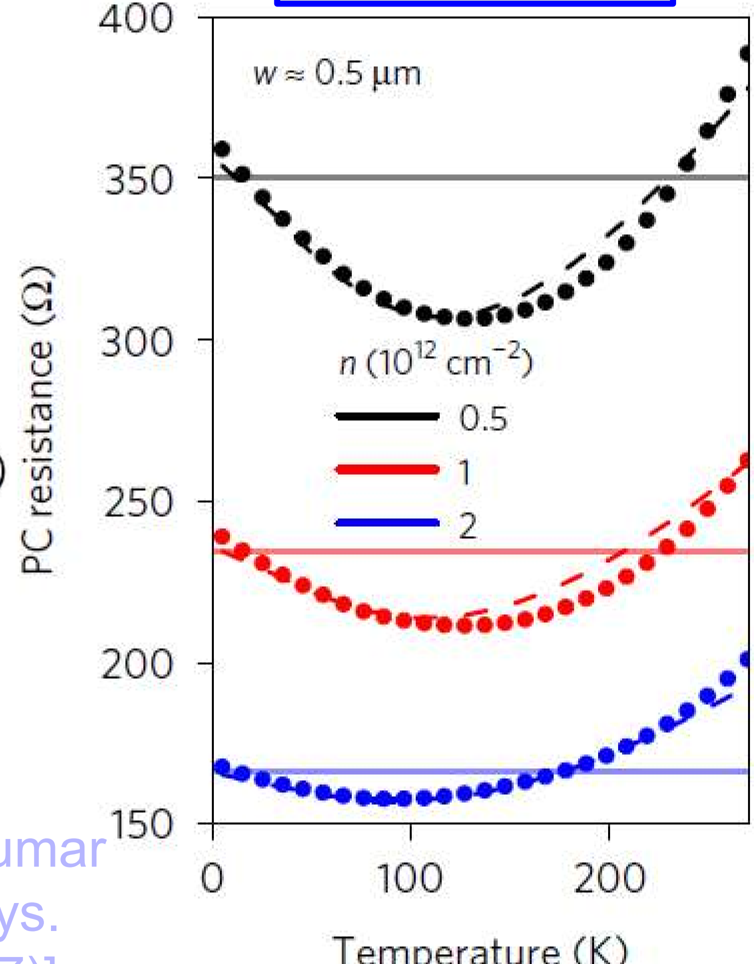
[H. Guo et al., PNAS USA **114**, 3068 (2017)]





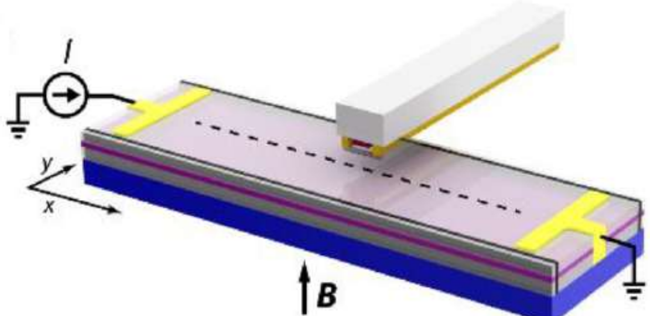
[R. Krishna Kumar et al., Nat. Phys. **13**, 1182 (2017)]

Experiment



Visualizing electron flow: Hall voltage

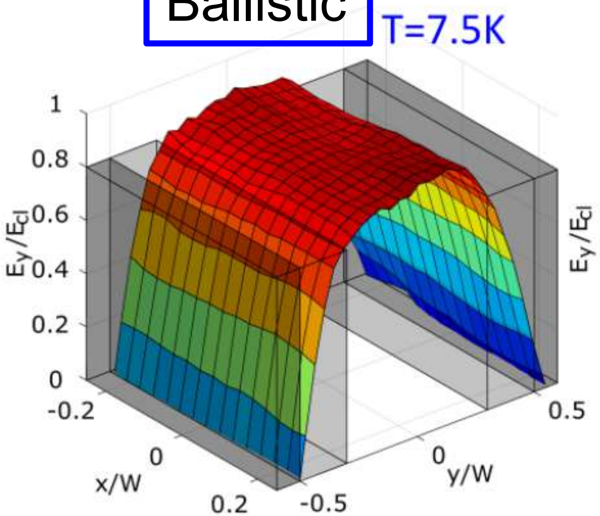
[J.A. Sulpizio, L. Ella, A. Rozen et al., Nature **576**, 75 (2019)]



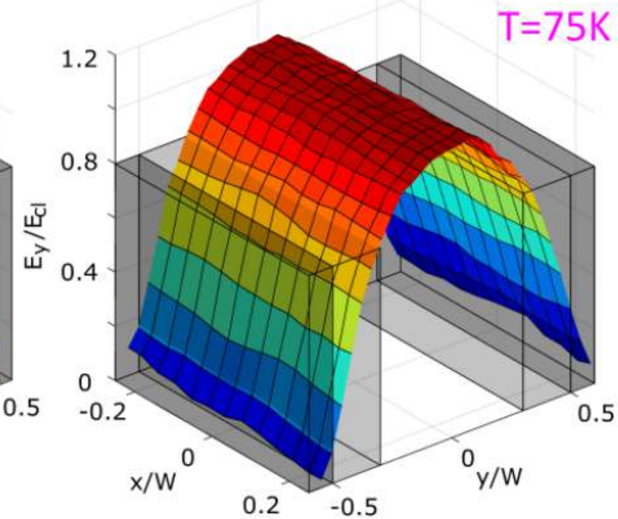
single electron transistor

$$E_y = \frac{B}{en}\left(j_x + \frac{l_{ee}^2}{2}\partial_y^2 j_x\right)$$

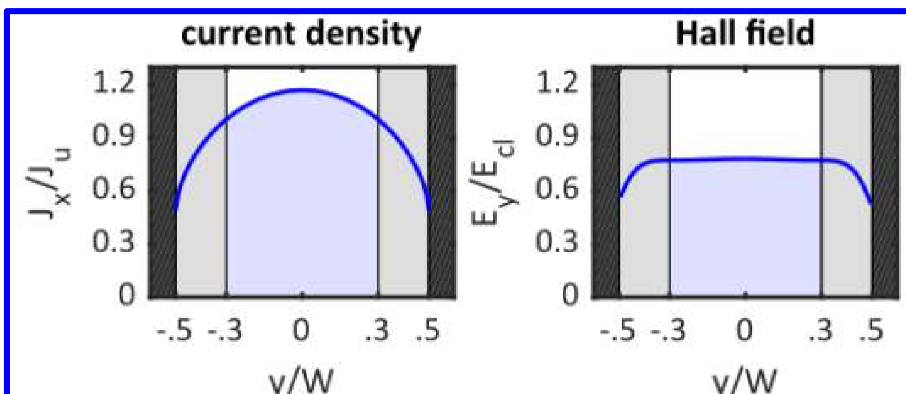
Ballistic

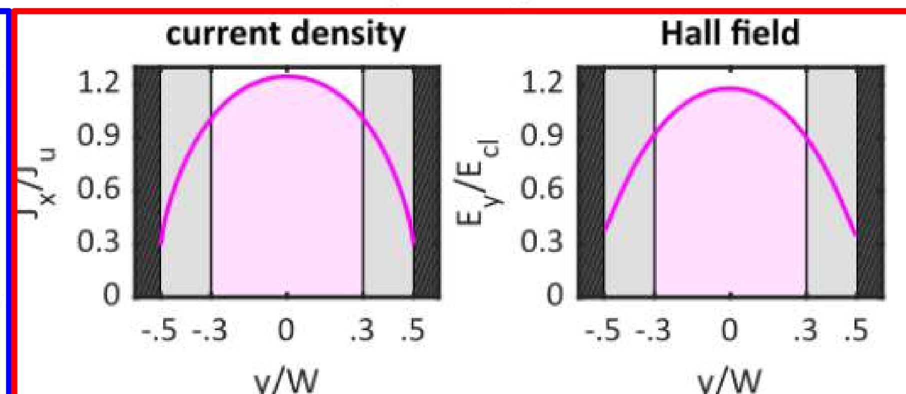


Hydrodynamic



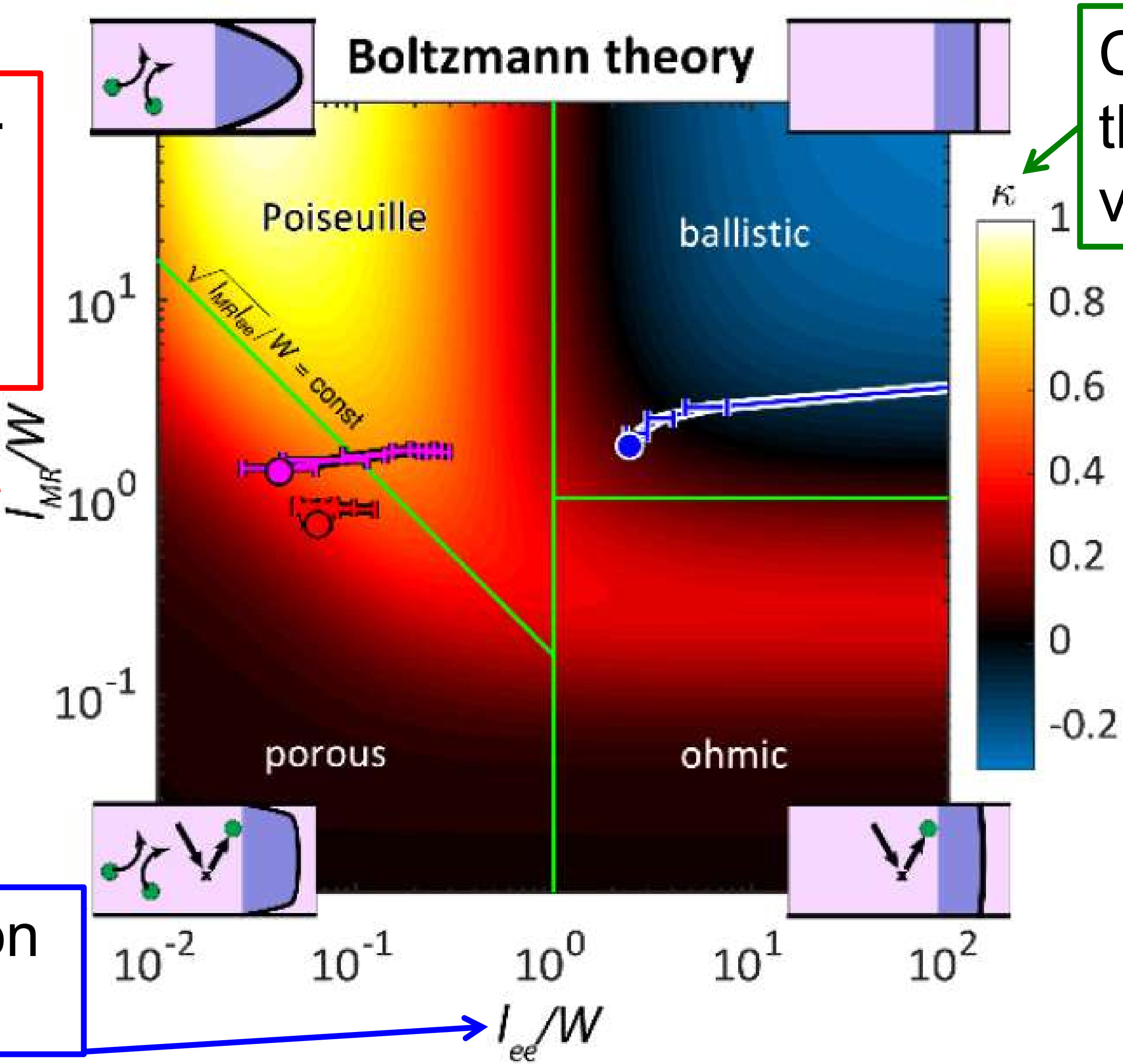
$B = 12.5\ \mathrm{mT}$ ballistic $n = -6 \times 10^{11}\ \mathrm{cm}^{-2}$ hydrodynamic





Visualizing electron flow: phase diagram

Momentum-relaxing mean-free path



Curvature of the Hall voltage

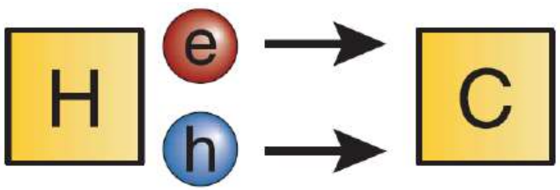
e^-e^- collision length

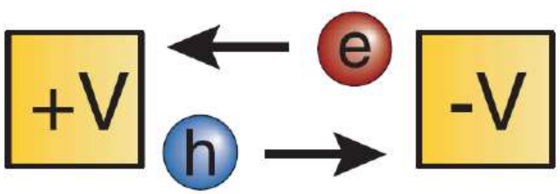
Dirac fluid and WF law violation

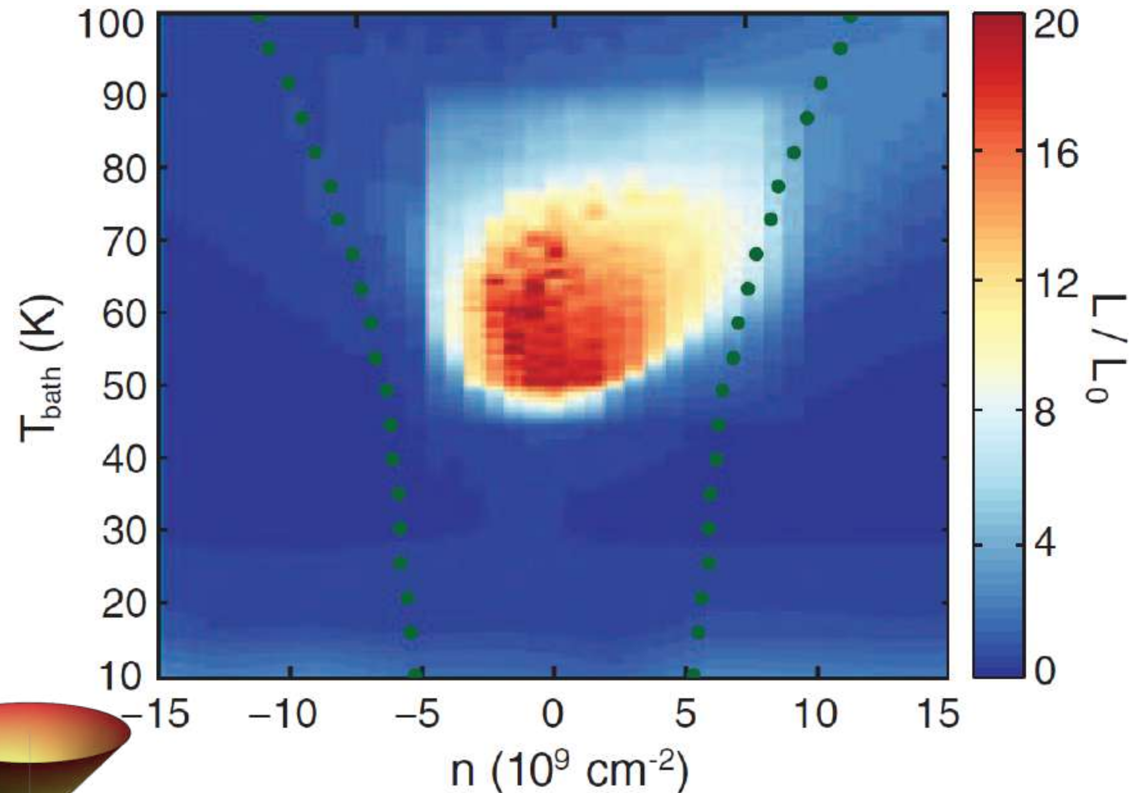
[J. Crossno, J.K. Shi, K. Wang et al., Science **351**, 1058 (2016)]

Wiedemann-Franz law:

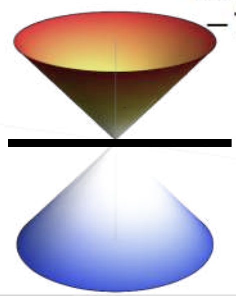
$$L \equiv \frac{\kappa_e}{\sigma T} = \frac{\pi^2}{3e^2} = L_0$$







Electric transport is sensitive to the h^+e^- collisions



$n \approx 0$

Dirac fluid regime

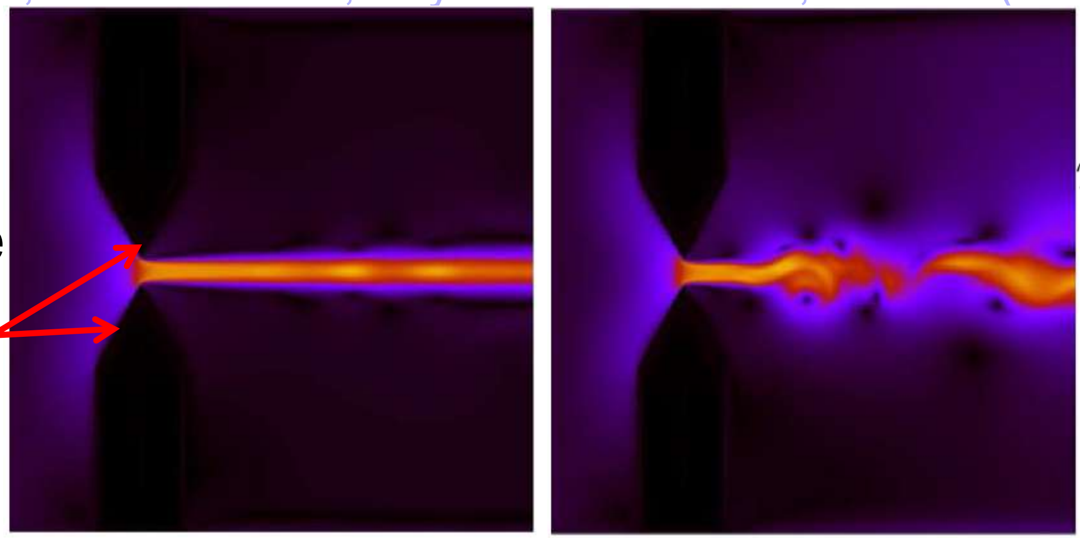
Preturbulent regimes in graphene

[M. Mendoza, H. J. Herrmann, and S. Succi, Phys. Rev. Lett. **106**, 156601 (2011); A. Gabbana, M. Polini, S. Succi et al., Phys. Rev. Lett. **121**, 236602 (2018)]

Re=25

Preturbulence

Constriction



$$T \gg \mu$$

$$\eta \approx 0.45 \frac{T^2}{4\hbar v_F^2 \alpha^2}$$

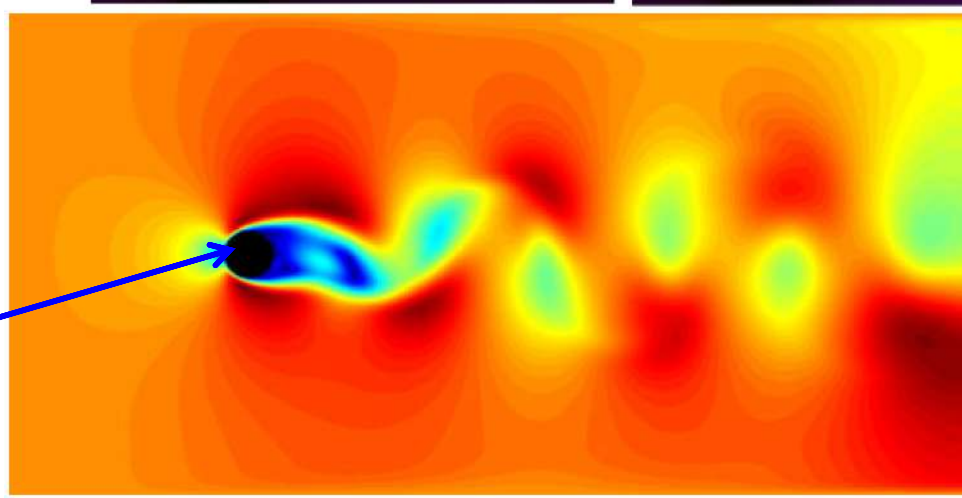
$$u \approx 10^5 \text{ m/s}$$

$$L \approx 5 \ \mu\text{m}$$

Re=100

Vortex shedding

Circular impurity



$$\nu_{\text{eff}} = v_F^2 \eta/(Ts)$$

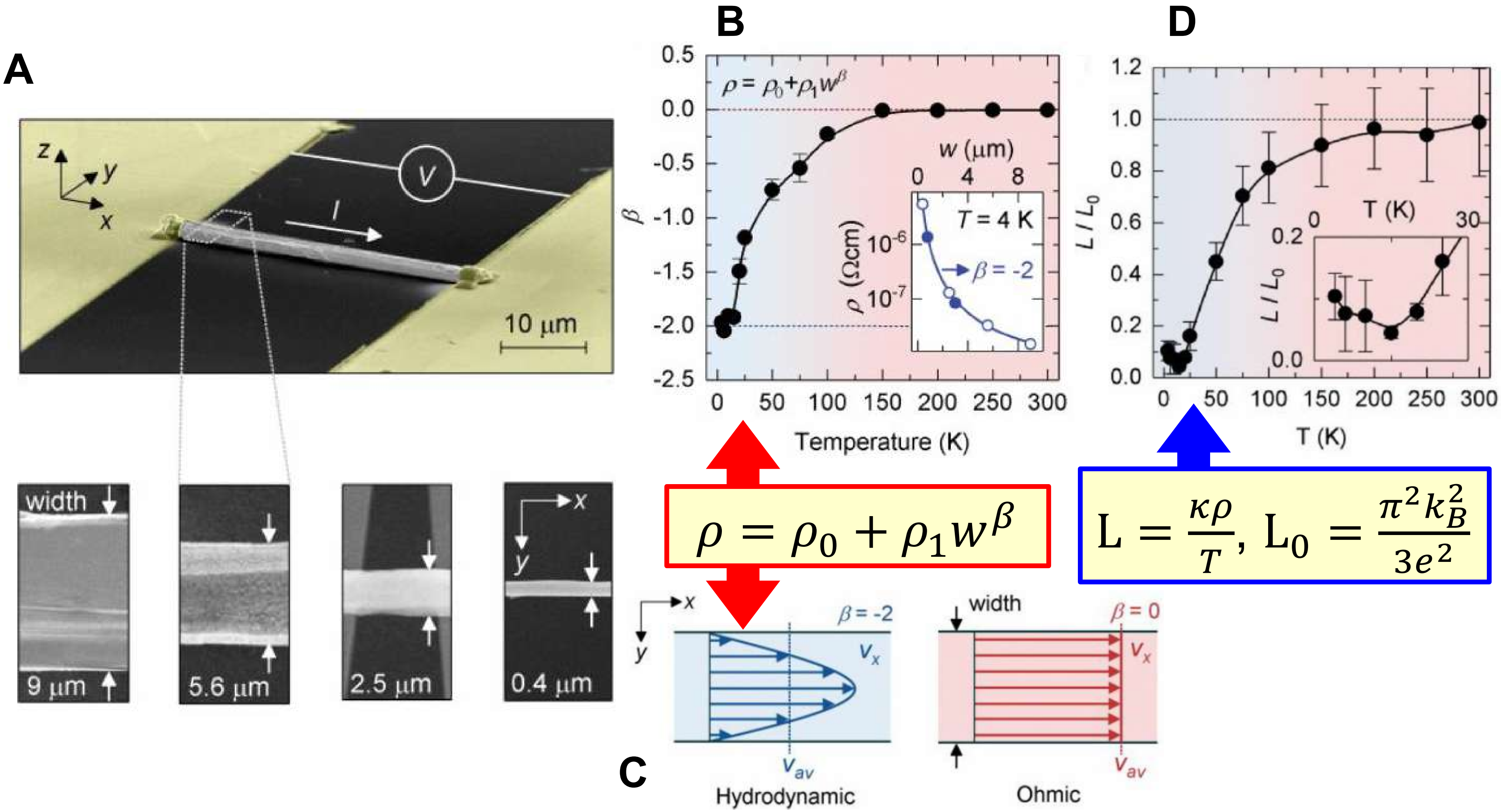
$$\approx 5 \times 10^{-3} \text{ m}^2/\text{s}$$

$$\text{Re} = \frac{sT}{v_F^2}\frac{Lu}{\eta} = \frac{uL}{\nu_{\text{eff}}},$$

$$\omega_{\text{shedding}} \sim 0.1\text{GHz}$$

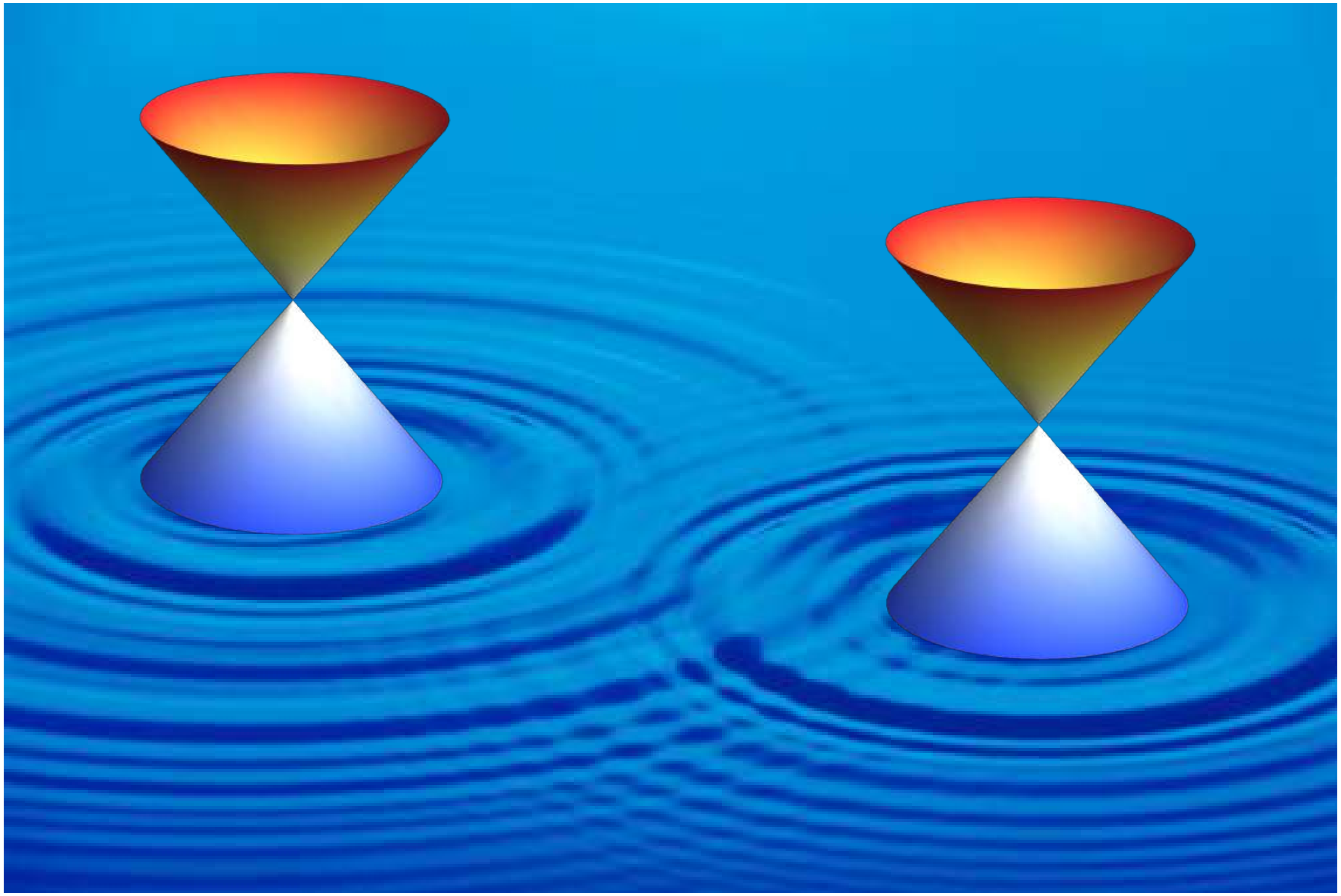
Hydrodynamics in Weyl semimetals

❖ Weyl semimetals WP_2 [J. Gooth et al., Nat. Commun. **9**, 4093 (2018)]



$$\rho = \rho_0 + \rho_1 w^{\beta}$$

$$\mathrm{L} = \frac{\kappa\rho}{T},\ \mathrm{L}_0 = \frac{\pi^2 k_B^2}{3e^2}$$



Consistent hydrodynamic in Weyl semimetals

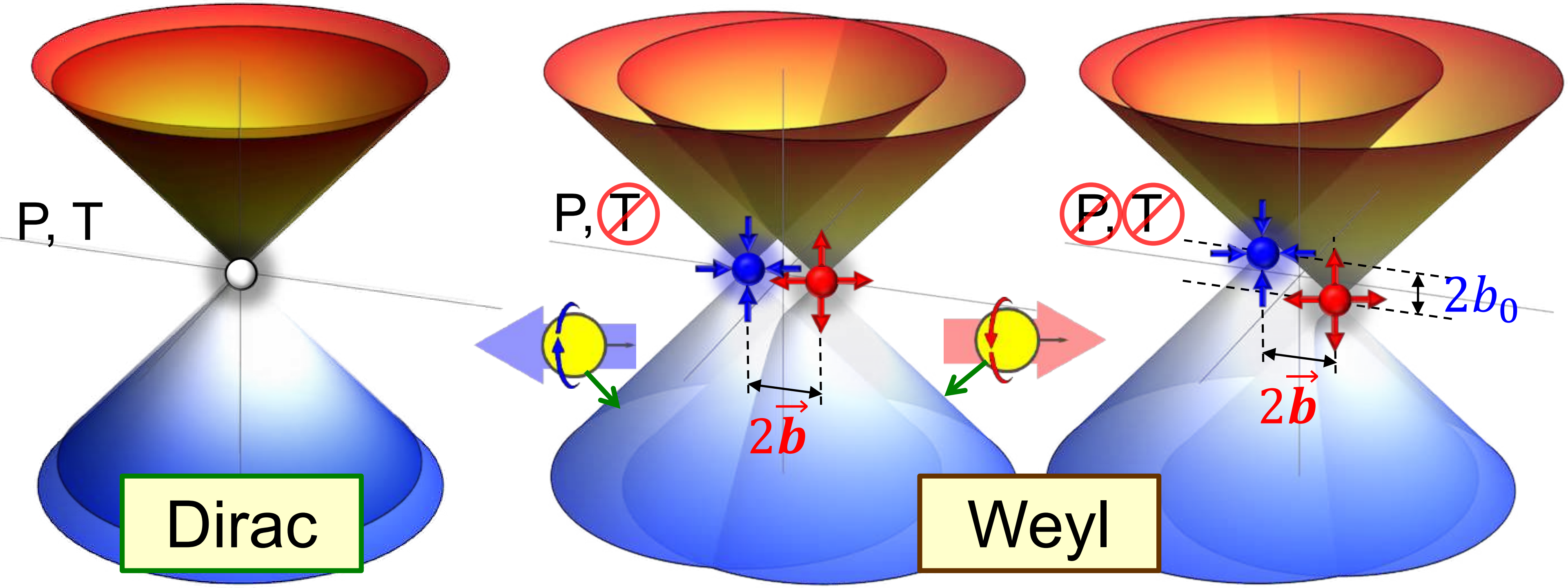
[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **97**, 121105(R) (2018); **97**, 205119 (2018); **98**, 035121 (2018)]

Low energy Weyl fermions

$$H_0(\mathbf{k}) = \begin{pmatrix} v_F \boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{b}) + b_0 & 0 \\ 0 & -v_F \boldsymbol{\sigma}\cdot(\mathbf{k}+\mathbf{b}) - b_0 \end{pmatrix}.$$

Chiral shift parameter $-\vec{b}\cdot\vec{\gamma}\,\gamma_5$

[E.V. Gorbar, V.A. Miransky, and I.A. Shovkovy, Phys. Rev. C **80**, 032801(R) (2009)]



Berry curvature

- Consider the adiabatic evolution of a system [M.V. Berry, Proc. R. Soc. A **392**, 45 (1984)]. At each time moment, the system is at its instantaneous eigenstate:

$$H = H(\mathbf{k}), \qquad H(\mathbf{k})\phi_n(\mathbf{k}) = \epsilon_n \phi_n(\mathbf{k}).$$

- For a closed trajectory in the parameter space, the wave function is:

$$\psi(t) = \boxed{e^{-i\gamma(t)}} e^{-i\int_0^t dt' \epsilon_n(\mathbf{k}(t'))} \phi_n(\mathbf{k})$$

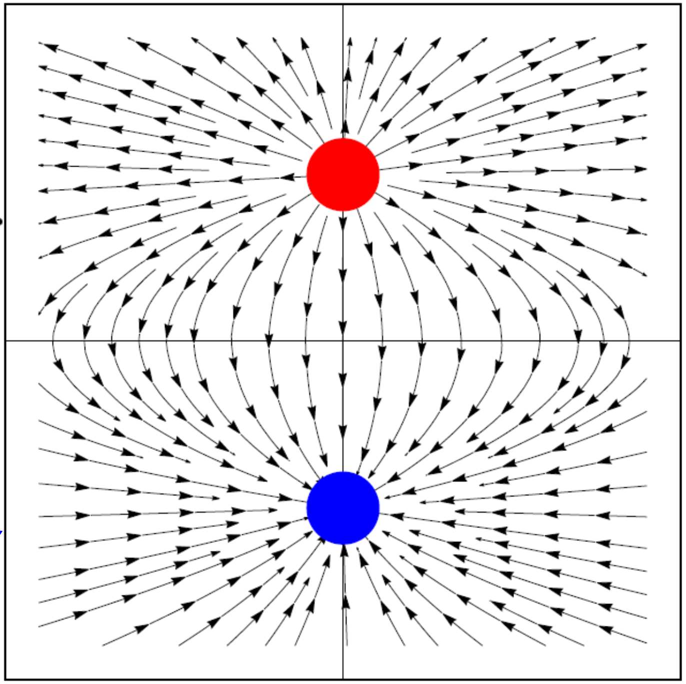
- The Berry phase and the Berry connection:

$$\gamma(t) = \oint d\mathbf{k} \mathcal{A}(\mathbf{k}), \mathcal{A}(\mathbf{k}) = -i\phi_n^\dagger(\mathbf{k}) \nabla_{\mathbf{k}} \phi_n(\mathbf{k}).$$

- The Berry curvature:

$$\boldsymbol{\Omega} = \nabla_{\mathbf{k}} \times \mathcal{A}(\mathbf{k}), \Rightarrow \boxed{\boldsymbol{\Omega} = \pm \frac{\mathbf{k}}{2\hbar k^3}}$$

The Berry curvature and its field lines for the Weyl semimetal



Chiral kinetic equation

❖ Boltzmann equation:

[D. Xiao, M.-C. Chang, and Q. Niu, R.M.P. **82**, 1959 (2010)]
[D.T. Son and N. Yamamoto, P.R.D **87**, 085016 (2013)]
[M.A. Stephanov and Y. Yin, P.R.L. **109**, 162001 (2012)]

$$\left[1-\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]\partial_t f_\lambda+\left\{-e\tilde{\mathbf{E}}_\lambda-\frac{e}{c}\left[\mathbf{v}_\mathbf{p}\times\mathbf{B}_\lambda\right]+\frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right\}\cdot\partial_\mathbf{p} f_\lambda$$

$$+\left\{\mathbf{v}_\mathbf{p}-e\left[\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda\right]-\frac{e}{c}(\mathbf{v}_\mathbf{p}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right\}\cdot\boldsymbol{\nabla} f_\lambda=I_{\rm coll}(f_\lambda),$$

Collision integral

Anomalous velocity

$$\boldsymbol{\Omega}_\lambda=\lambda\hbar\frac{\mathbf{p}}{2p^3}$$

$$I\propto-\frac{f_\lambda-f_\lambda^{(0)}}{\tau}$$

where $\tilde{\mathbf{E}}_\lambda=\mathbf{E}_\lambda+(1/e)\boldsymbol{\nabla}\epsilon_\mathbf{p}, \mathbf{v}_\mathbf{p}=\partial_\mathbf{p}\epsilon_\mathbf{p},\ \epsilon_\mathbf{p}=v_F p\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right].$

Distribution function:

$$f_\lambda=\frac{1}{e^{[\epsilon_\mathbf{p}-(\mathbf{u}\cdot\mathbf{p})-\lambda\hbar(\mathbf{p}\cdot\boldsymbol{\omega})-\mu_\lambda]/T}+1}.$$

Fluid velocity

Vorticity $\boldsymbol{\omega}=[\boldsymbol{\nabla}\times\boldsymbol{u}]/2$

Euler (Navier-Stokes) equation

❖ Euler (inviscid) equation for the charged electron liquid:

$$\frac{1}{v_F}\partial_t\left[\frac{w\mathbf{u}}{v_F}+\sigma^{(\epsilon,\mathrm{B})}\mathbf{B}+\frac{\hbar\boldsymbol{\omega}n_5}{2}\right]+\boldsymbol{\nabla}P+\mathcal{O}\left(\boldsymbol{\nabla}\right)$$

$$=-en\mathbf{E}+\frac{1}{c}\left[\mathbf{B}\times\left(en\mathbf{u}-\frac{\sigma^{(V)}\boldsymbol{\omega}}{3}\right)\right]+\frac{\sigma^{(\mathrm{B})}\mathbf{u}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}+\frac{5c\sigma^{(\epsilon,u)}\left(\mathbf{E}\cdot\mathbf{B}\right)\boldsymbol{\omega}}{v_F}$$

$$-\frac{w\mathbf{u}}{v_F^2\tau}-\frac{\hbar\boldsymbol{\omega}n_5}{2v_F\tau}.$$

Electrostatic and Lorentz forces

Anomalous terms

Dissipative terms

where $w=\epsilon+P,\ \sigma^{(\mathrm{B})}\propto\mu_5,\ \sigma^{(\epsilon,\mathrm{u})}=\mathcal{O}(1),\ \sigma^{(\epsilon,\mathrm{B})}\propto\mu\mu_5.$

❖ Viscosity terms with shear η and bulk ζ viscosities:

$$-\eta\Delta\mathbf{u}-\left(\zeta+\frac{\eta}{3}\right)\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\mathbf{u}\right).$$

Energy conservation relation

❖ Energy conservation equation:

$$\partial_t \epsilon + (\boldsymbol{\nabla} \cdot \mathbf{u}) w + \mathcal{O}(\boldsymbol{\nabla}) = -\mathbf{E} \cdot \left(en\mathbf{u} - \sigma^{(\mathrm{B})}\mathbf{B} - \frac{\sigma^{(\mathrm{V})}\boldsymbol{\omega}}{3} \right) .$$

Hydrodynamic term

CME current

CVE contribution

❖ Viscosity and thermoconductivity terms:

$$-\eta(\mathbf{u}\Delta\mathbf{u}) - \left(\zeta + \frac{\eta}{3}\right)(\mathbf{u} \cdot \boldsymbol{\nabla})(\boldsymbol{\nabla} \cdot \mathbf{u}) - \kappa \boldsymbol{\nabla} \cdot \left(\boldsymbol{\nabla} T - \frac{T}{w} \boldsymbol{\nabla} P \right) .$$

Electric and chiral currents

❖ Currents:

$$\mathbf{J} \simeq -en\mathbf{u} + \sigma\mathbf{E} + \kappa_e \boldsymbol{\nabla} T + \frac{\sigma_5}{e}\boldsymbol{\nabla}\mu_5 + \sigma^{(V)}\boldsymbol{\omega} + \sigma^{(B)}\mathbf{B} + \frac{[\boldsymbol{\nabla}\times\boldsymbol{\omega}]\,\sigma^{(\epsilon,V)}}{2} + \frac{e^2}{2\pi^2\hbar^2 c} b_0 \mathbf{B} - \frac{e^2}{2\pi^2\hbar}[\mathbf{b}\times\mathbf{E}],$$

$$\mathbf{J}_5 \simeq -en_5\mathbf{u} + \sigma_5\mathbf{E} + \kappa_{e,5}\boldsymbol{\nabla} T + \frac{\sigma}{e}\boldsymbol{\nabla}\mu_5 + \sigma_5^{(V)}\boldsymbol{\omega} + \sigma_5^{(B)}\mathbf{B} + \frac{[\boldsymbol{\nabla}\times\boldsymbol{\omega}]\,\sigma_5^{(\epsilon,V)}}{2}.$$

Chiral vortical effect (CVE)

Chiral magnetic effect (CME)

Chiral separation effect (CSE)

❖ Continuity relations and Maxwell's equations:

$$\partial_t\rho + (\boldsymbol{\nabla}\cdot\mathbf{J}) = 0,$$

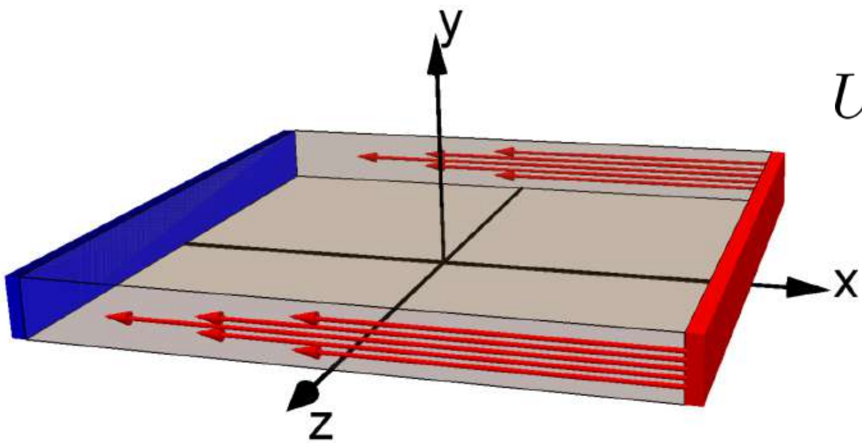
$$\partial_t\rho_5 + (\boldsymbol{\nabla}\cdot\mathbf{J}_5) = -\frac{e^3(\mathbf{E}\cdot\mathbf{B})}{2\pi^2\hbar^2 c},$$

$$\varepsilon_e \boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi(\rho + \rho_{\mathrm{b}}),$$

$$\boldsymbol{\nabla}\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$

$$\boldsymbol{\nabla}\times\mathbf{B} = \mu_{\mathrm{m}}\frac{4\pi}{c}\mathbf{J} + \varepsilon_e\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}.$$

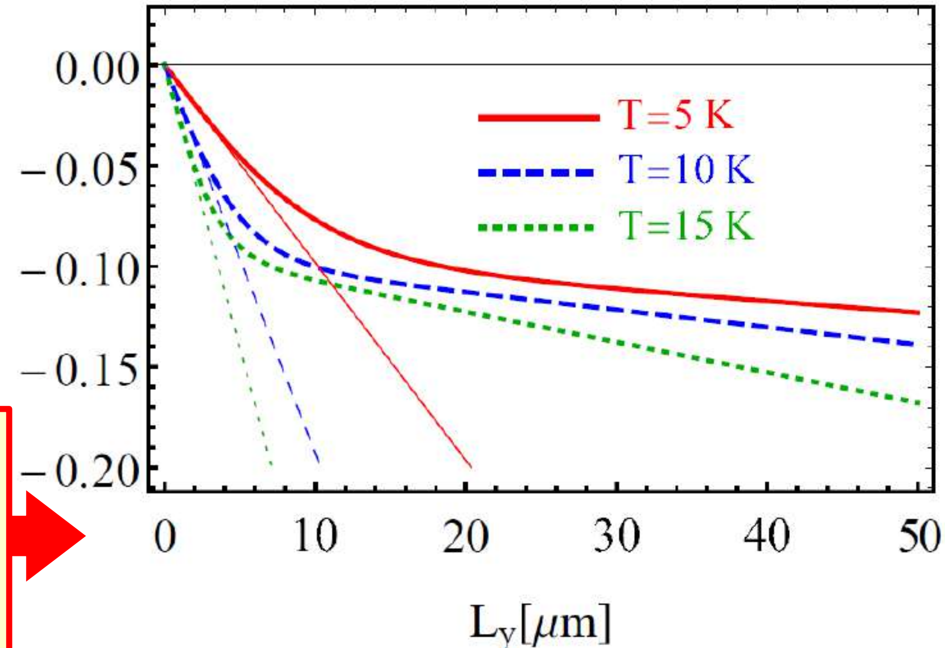
Hydrodynamic AHE voltage



$$U = -\int_0^{L_y} E_y(y)\, dy = U_{\text{hydro}} + U_{\text{CS}},$$

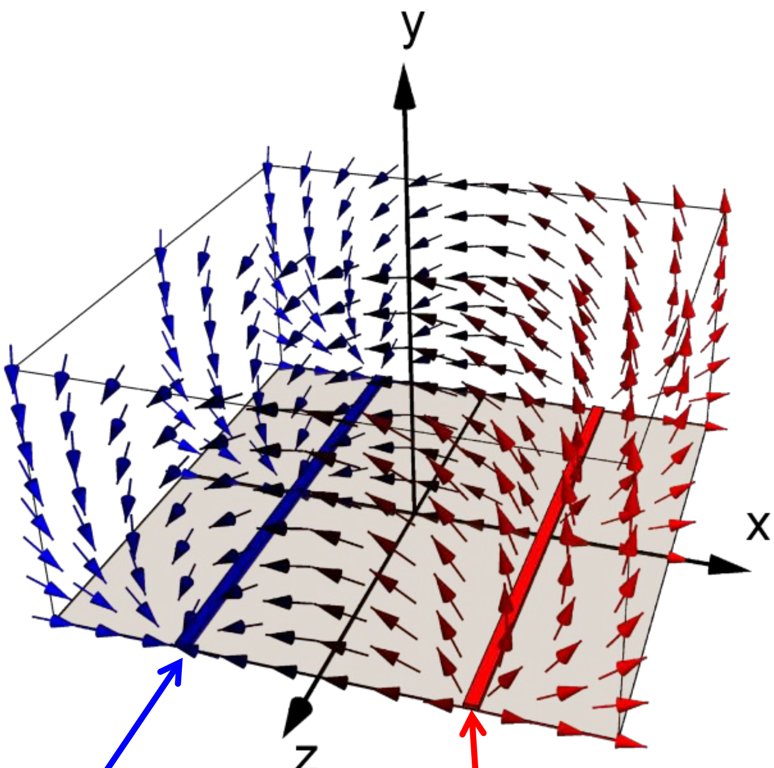
$$U_{\text{CS}} = \frac{L_y}{\sigma} \frac{e^3 b_z E_x}{2\pi^2 \hbar^2 c},$$

$$U_{\text{hydro}} = -\frac{e^3 b_z E_x}{2\pi^2 \hbar^2 c} \frac{e^2 n^2}{N \sigma^2} \times \left[L_y - \frac{2}{\lambda_y} \tanh\left(\frac{\lambda_y L_y}{2}\right)\right].$$



The step-like dependence of the AHE voltage signifies the interplay of hydrodynamic and topological effects

Nonlocal transport in semi-infinite slab



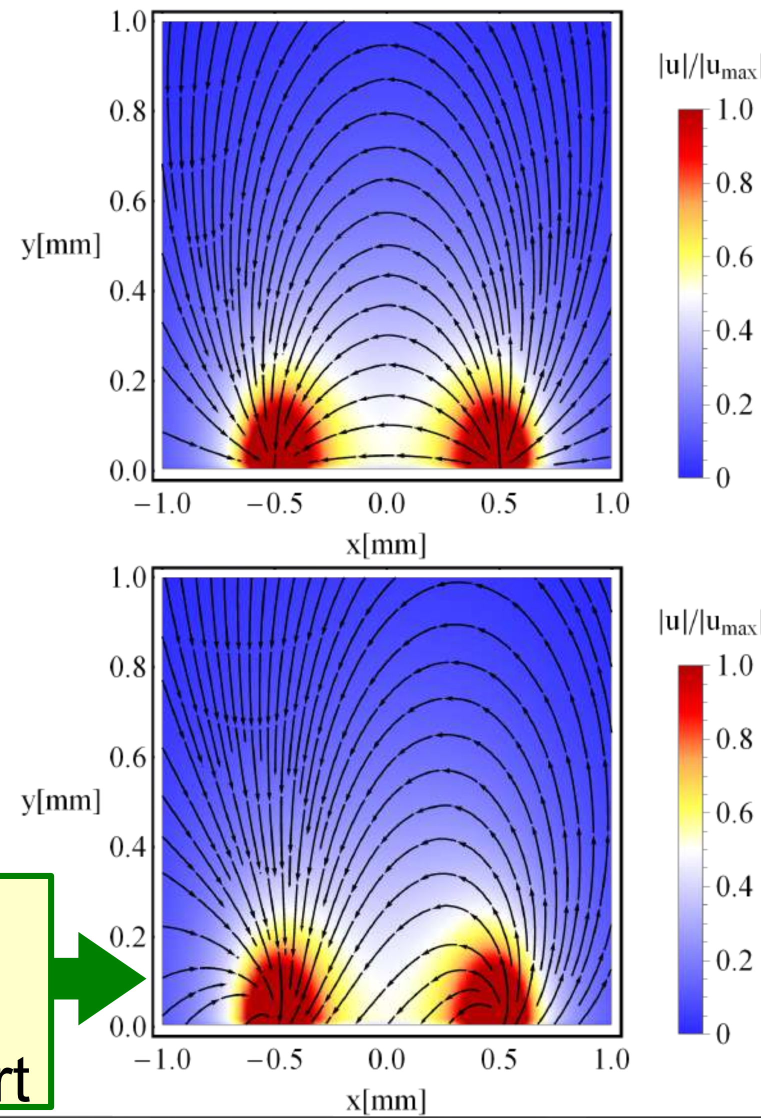
Drain

Source

Spatial asymmetry is the characteristic feature of the Chern-Simons terms in the nonlocal transport

$b = 0$

$\mathbf{b} \parallel \hat{\mathbf{z}}$



Summary

1. The **hydrodynamic regime** is possible for charge carriers in solids under certain experimentally realizable conditions.
2. Among the most interesting **hydrodynamic phenomena in solids** are the formation of vortices, the negative nonlocal resistance, the Poiseuille-like flow, the breakdown of Matthiessen's rule, etc.
3. **Consistent hydrodynamics** is needed to correctly describe topologically nontrivial chiral media such as Weyl semimetals.
4. The interplay of the Chern-Simons terms and hydrodynamic effects is manifested in the **hydrodynamic AHE**.
5. Weyl nodes separation can be also manifested in the **spatial asymmetry of the electron flow**.