Soft pions and the dynamics of the chiral phase transition

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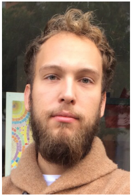
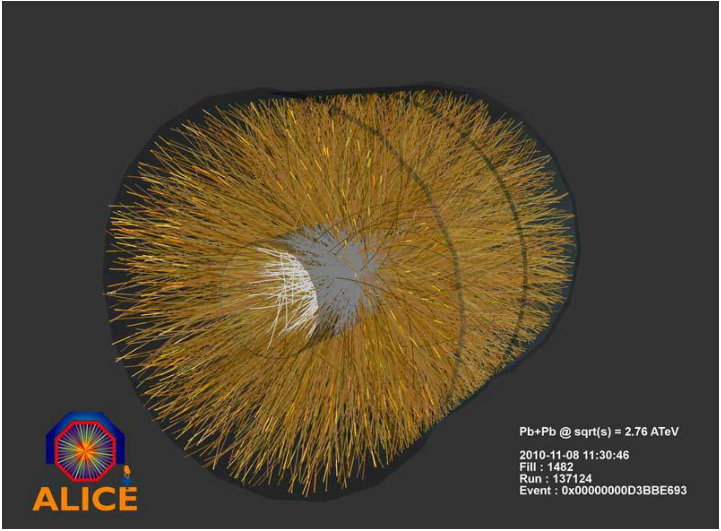
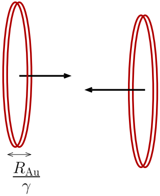
- Eduardo Grossi, Alex Soloviev, DT, Fanglida Yan: PRD, arXiv:2005.02885
- Eduardo Grossi, Alex Soloviev, DT, Fanglida Yan: PRD, arXiv:2101.10847
- Adrien Florio, Eduardo Grossi, Alex Soloviev, DT, PRD, arXiv:2111.03640



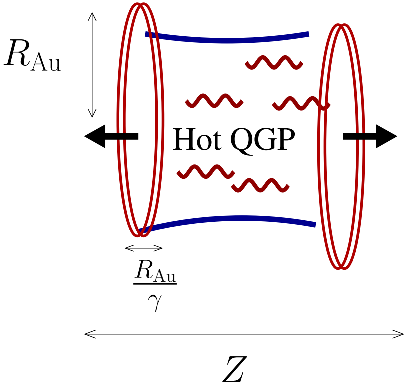


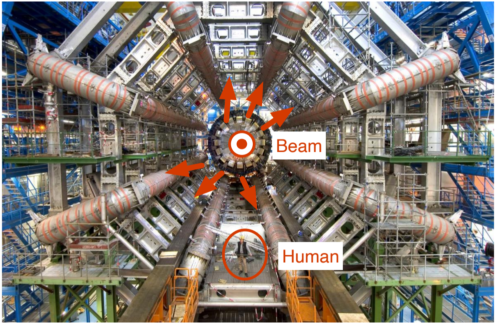
Colliding Nuclei and Creating Plasma of Quarks and Gluons (QGP)





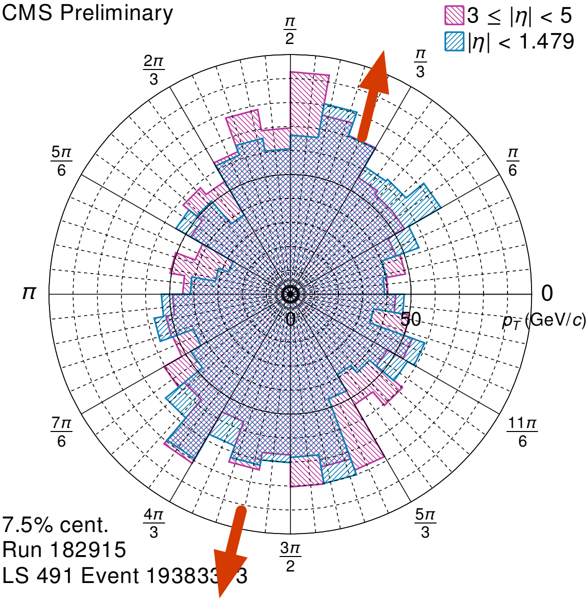
The QGP is Born

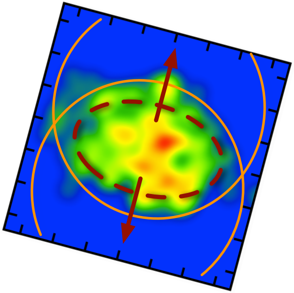




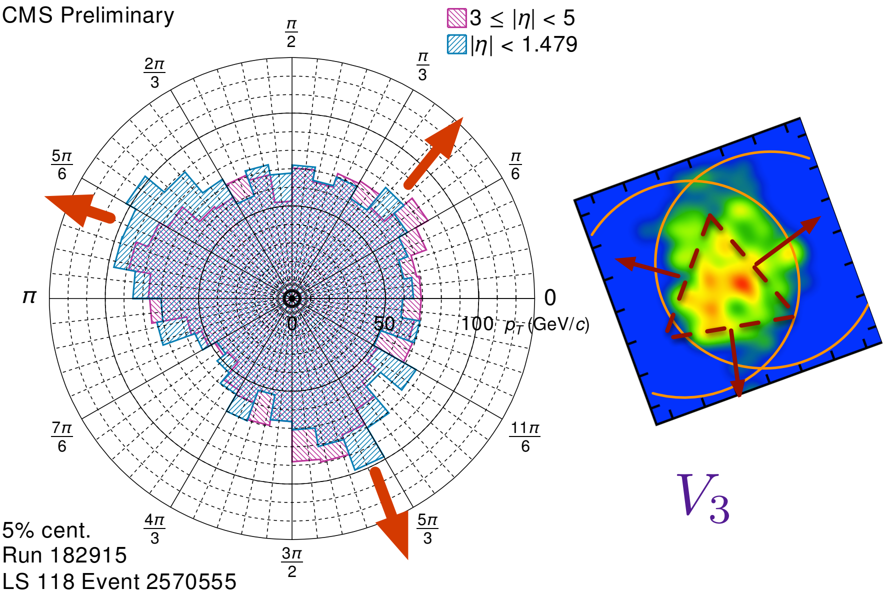
The nuclei *pass through* each other leaving QGP expanding rapidly

Measuring the hydrodynamics of the plasma





V_2


CMS Preliminary
3 ≤ |η| < 5
|η| < 1.479
π/2
2π/3
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5π/6
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pT (GeV/c)
7π/6
11π/6
4π/3
5π/3
3π/2
5% cent.
Run 182915
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V3

Amazing Success: the “Standard” Hydro Model

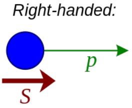
1. $V_1 \ldots V_6$
2. Momentum dependence $V_n(p)$
3. Probabilities $P(|V_n|^2)$
4. Covariances between harmonics: $\langle V_2 V_3 V_5^* \rangle$
5. Full covariance matrix: $\langle V_2(p_1) V_2^*(p_2) \rangle$

Uses the equation of state from lattice QCD and

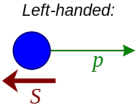
$$\partial_\mu T^{\mu\nu} = 0$$

but we want more...

QCD and Chiral Symmetry



u_R, d_R



u_L, d_L

$$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = U_L \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

and ditto for right

QCD is (almost) symmetric between, left and right, and up and down:

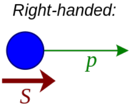
$$\mathcal{L}_{QCD} = \sum_{q=u,d} \bar{q}_L (i\not{D}) q_L + \bar{q}_R (i\not{D}) q_R \; - \underbrace{m_q \left(\bar{q}_L q_R + \bar{q}_R q_L \right)}_{\text{small}}$$

Then one would expect four approx. conservation laws, u_L, d_L, u_R, d_R:

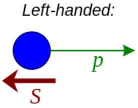
$$n_B : \quad (u_L + d_L) + (u_R + d_R) \qquad \text{Baryon number}$$

$$n_{anom} : \quad (u_L - u_R) + (d_L - d_R) \qquad \text{Anomalous: not consv.}$$

QCD and Chiral Symmetry



u_R, d_R



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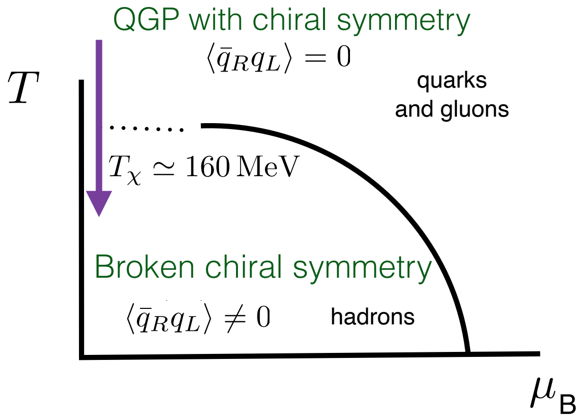
Then one would expect four approx. conservation laws, u_L, d_L, u_R, d_R:

$\vec{n}_V$:	$(u_L + u_R) - (d_L + d_R)$	Isovector charge
$\vec{n}_A$:	$(u_L - u_R) - (d_L - d_R)$	Isoaxial vect. charge

Pisarski, Wilczek



For two massless quarks the chiral symmetry group is

$$SU_L(2) \times SU_R(2) \simeq O(4)$$

This is broken, and the transition is 2nd order.

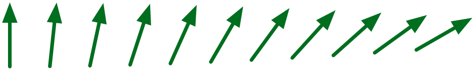
The mass smooths the transition to a crossover, like a magnetic field in the Ising model

Chiral symmetry plays no role in the "Standard Hydro Model" ...

Our cold world: T< Tcritical

$$\langle \bar{q}_R q_L \rangle = \bar{\sigma}\, \mathbb{I}_{2\times 2}$$

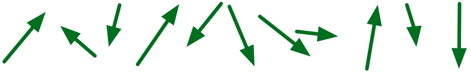
State is ordered. $\langle \bar{q}_R q_L \rangle$ is like the magnetization. $q = u, d$



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

The slow modulation of the $SU_A(2)$ phase of $\bar{q}_R q_L$ is a pion, $\vec{\pi}{=}\bar{\sigma}\vec{\varphi}$

The hot world: T> Tcritical



State is disordered: pion propagation is frustrated

This talk will describe pion propagation during the $O(4)$ phase transition

Ising Model

magnetization $\vec{M}$

magnetic field $\vec{H}$

$$\mathcal{H} = \int d^3x \, \vec{H} \cdot \vec{M}$$

QCD

$$\bar{q}_L q_R = \sigma e^{i\vec{\tau}\cdot\vec{\varphi}}$$ condensate

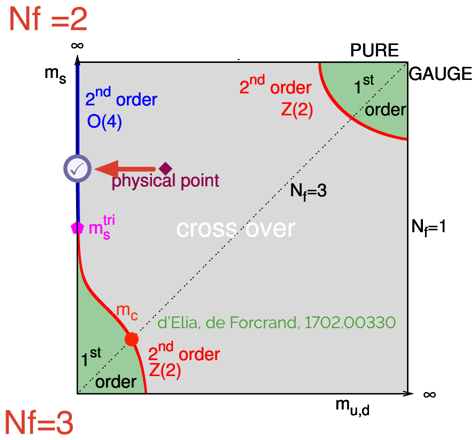
m_q or H quark mass

$$\mathcal{H} = \int d^3x \, m_q \left(\bar{q}_R q_L + \bar{q}_L q_R\right)$$

$\vec{\tau}$ are Pauli matrices for the SU(2) order parameter

Real World QCD

- There are three flavors of quarks u, d, s which are massive
  - This changes structure phase diagram
- We will assume the real world is "close" to the $O(4)$ critical point.



See review Phillipsen, 2021

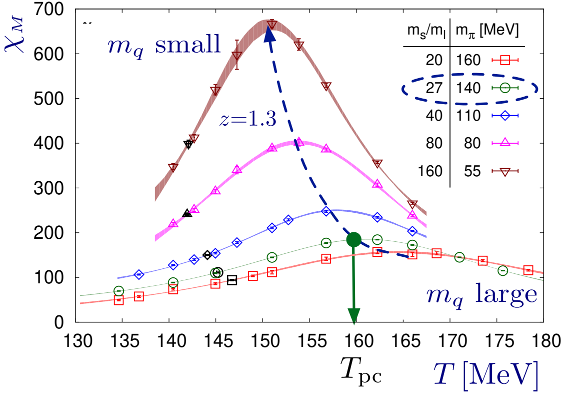
HotQCD 2019, 2020,
Cuteri, Philipsen, Sciara 2021
Kotov, Lombardo, Trunin, 2021

Strong evidence of 2nd order phase transition at physical strange mass

Real world lattice QCD and the $O(4)$ critical point: Hot QCD, PRL 2019

Fluctuations of order parameter, $\sigma \propto \bar{u}u + \bar{d}d$, vs temperature and m_q

$$\chi_M = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$



O(4) Scaling predictions

$$\chi_M = m_q^{1/\delta - 1} f_\chi(z)$$

$$z \equiv z_0 \frac{(T - T_c)}{T_c} m_q^{-1/\beta\delta}$$

The QCD lattice knows about the $O(4)$ critical point! Hydro should too!

Static Universality and the Chiral Phase Transition

- The $O(4)$ order parameter fluctuates in amplitude and phase:

$$\phi_a = (\phi_0, \phi_1, \phi_2, \phi_3) = (\sigma, \vec{\pi})$$

The quark condensate scales as

$$\bar{q}_R q_L \sim \sigma e^{i\vec{\tau}\cdot\vec{\varphi}} \simeq \sigma + i\vec{\tau}\cdot\vec{\pi}$$

- The Landau Ginzburg function for the $O(4)$ order parameter is: $\phi^2 \equiv \phi_a \phi_a$

$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2}m_0^2(T)\,\phi^2 + \frac{\lambda}{4}\,\phi^4 - \underbrace{H}_{\propto\, m_q}\,\sigma$$

- The model has a critical mass, $m_0 - m_c \propto (T - T_c)$

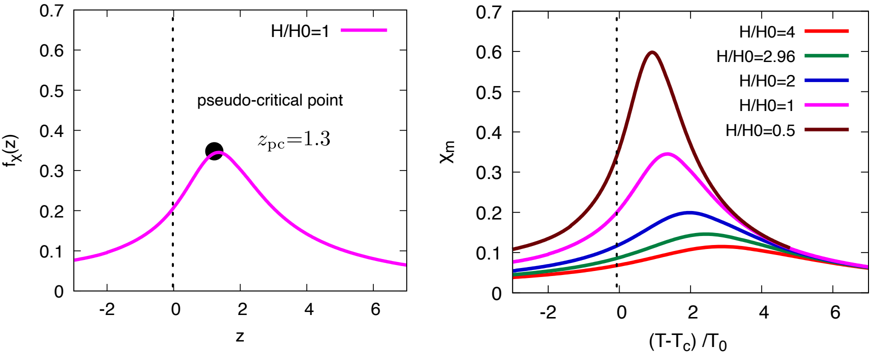
The critical model makes a definite prediction for the susceptibility:

Scaling predictions from the $O(4)$ model

Simulations at different magnetic field are related to each other

$$\chi_M = h^{1/\delta - 1} f_\chi(z) \qquad z = z_0\, t_{\rm r} h^{-1/\beta\delta}$$

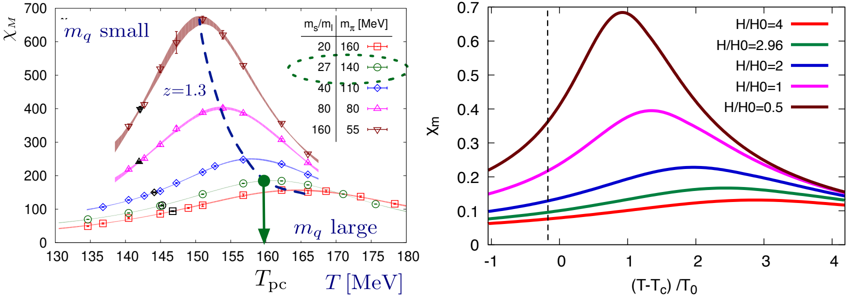
Here $h \propto H$ and $t_{\rm r} \propto (T - T_C)$ are the reduced field and temperature



numerical data from
Engels, Seniuch, Fromme, Karsch

Scaling predictions and QCD

$$\chi_M = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$



Scaling predictions reasonably describe how the peak rises and shifts.

$$\chi_M \propto m_q^{1/\delta - 1} f_\chi(z) \qquad z = z_0 \left(\frac{T - T_C}{T_C} \right) m_q^{-1/\beta\delta}$$

From Thermodynamics to Hydrodynamics

Hydrodynamics of the $O(4)$ transition:

Rajagopal and Wilczek '92, Son '99, Son and Stephanov '01, and finally us, arxiv:2101.10847.

1. The order parameter

$$\phi_a = (\sigma, \vec{\pi})$$

2. The approximately conserved charges quantities:

$$\vec{n}_V = \underbrace{\bar{\psi}\gamma^0\vec{\tau}\psi}_{\text{isovect chrg}} \qquad \textit{and} \quad \vec{n}_A = \underbrace{\bar{\psi}\gamma^0\gamma^5\vec{\tau}\psi}_{\text{isoaxial-vect chrg}}$$

which are combined into an anti-symmetric $O(4)$ tensor n_{ab}

$$n_{ab} = (\vec{n}_A, \vec{n}_V)$$

The charge n_{ab} generates $O(4)$ rotations, $\phi \to \phi_c + \frac{i}{\hbar}\theta_{ab}[n_{ab}, \phi_c]$, implying a Poisson bracket between the hydrodynamic fields:

$$\{n_{ab}(\boldsymbol{x}), \phi_c(\boldsymbol{y})\} = \epsilon_{abcd}\,\phi_d(\boldsymbol{x})\,\delta(\boldsymbol{x} - \boldsymbol{y})$$

The Landau-Ginzburg Hamiltonian for the $O(4)$ transition:

The Hamiltonian is tuned to the crit. point with $m_0^2(T) < 0$ and $H \propto m_q$:

$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2}m_0^2(T)\phi^2 + \frac{\lambda}{4}\phi^4 - H\sigma + \frac{n_{ab}^2}{4\chi_0}$$

and gives the equilibrium distribution with the correct critical EOS:

$$Z = \int D\phi \, Dn \, e^{-\mathcal{H}[\phi,n]/T_c}$$

The hydro equations of motion take the form

$$\frac{\partial \phi}{\partial t} + \{\phi, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

$$\frac{\partial n_{ab}}{\partial t} + \{n_{ab}, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

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$$\frac{\partial\phi}{\partial t} + \{\phi, \mathcal{H}\} = -\Gamma\frac{\delta\mathcal{H}}{\delta\phi_a} + \xi_a$$

$$\frac{\partial n_{ab}}{\partial t} + \{n_{ab}, \mathcal{H}\} = \underbrace{\sigma_0\nabla^2\frac{\delta\mathcal{H}}{\delta n_{ab}}}_{\text{dissipation}} + \underbrace{\nabla\cdot\xi_{ab}}_{\text{noise}}$$

The equations and the simulations:

see also Schlichting, Smekal

We have a charge diffusion equation coupled to order parameter:

$$\partial_t n_{ab} + \underbrace{\nabla \cdot (\nabla \phi_{[a}\phi_{b]})}_{\text{poisson bracket}} + H_{[a}\phi_{b]} = \underbrace{D_0 \nabla^2 n_{ab}}_{\text{diffusion}} + \underbrace{\nabla \cdot \xi_{ab}}_{\text{noise}}$$

and a rotation of the order parameter induced by the charge:

$$\partial_t \phi_a + \underbrace{\frac{n_{ab}}{\chi_0}\phi_b}_{\text{poisson bracket}} = \underbrace{\Gamma_0 \frac{\delta H}{\delta \phi_a}}_{\text{dissipation}} + \underbrace{\xi_a}_{\text{noise}}$$

Numerical scheme based operator splitting:

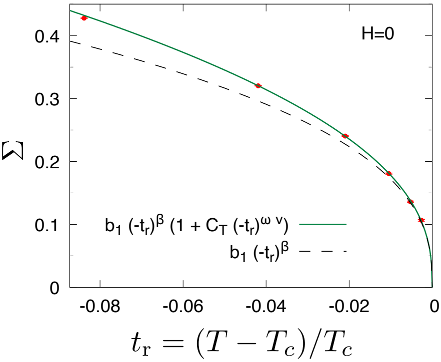
1. Evolve the Hamiltonian evolution with a position Verlet type stepper
2. Treat the dissipative Langevin steps as Metropolis-Hastings updates

Preliminaries: Statics

$$M_a(t) \equiv \frac{1}{V}\sum_{\boldsymbol{x}} \phi_a(t,\boldsymbol{x}) \equiv \text{Order Parameter}$$

with time average:

$$\bar{\sigma} = \langle M_0(t) \rangle \qquad \text{and} \qquad \Sigma = \lim_{H\to 0}\lim_{V\to\infty} \bar{\sigma}$$



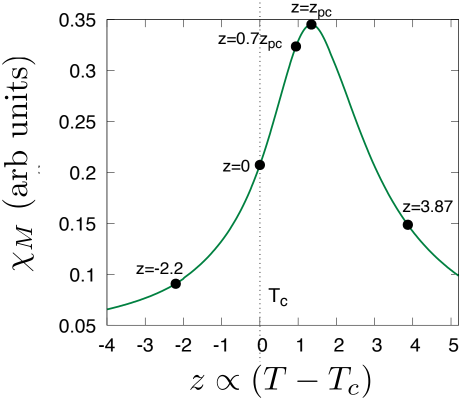
Scan the phase transition:

Engels, Fromme, Seniuch, Karsch; Hasenbuch

After measuring order parameter, susceptibility, etc

$$\bar{\sigma} = h^{1/\delta} f_G(z) \qquad z = t_{\rm r} h^{-1/\beta\delta}$$

we have fixed the scaling parameters, $h = H/H_0$, and $t_{\rm r} = (m_0^2 - m_c^2)/\mathfrak{m}^2$



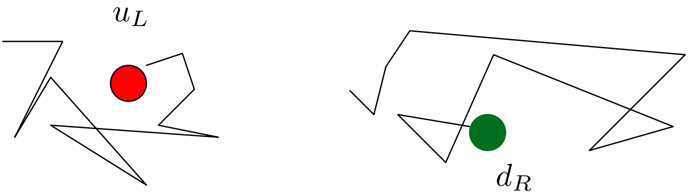
$$\chi_M = \frac{\partial \bar{\sigma}}{\partial H}$$

now scan the transition

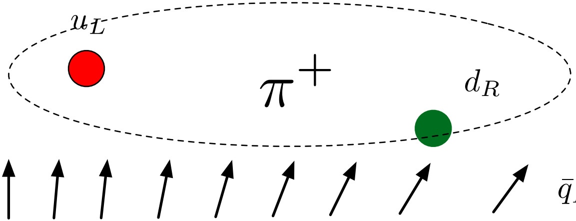
“Artists” conception of the phase transition dynamics

High Temperature: Diffusion of axial charge

$$n_A = u_L - d_R$$



Low Temperature: pion propagation

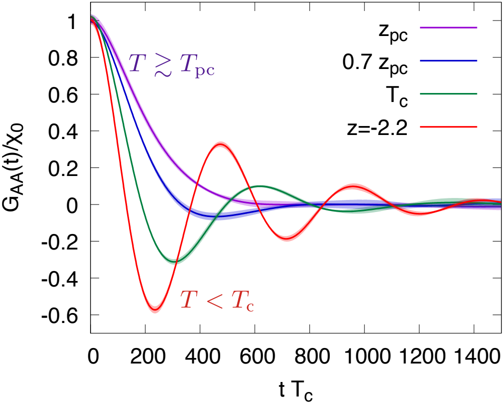


$$\bar{q}_R q_L = \bar{\sigma} e^{i\vec{\tau}\cdot\vec{\varphi}}$$

The phase transition and axial charge correlations:

$$G_{AA}(t) = \int \mathrm{d}^3x \; \langle \vec{n}_A(t, \boldsymbol{x}) \cdot \vec{n}_A(0, \mathbf{0}) \rangle$$

See a change in the dynamics across $T_{\rm pc}$:

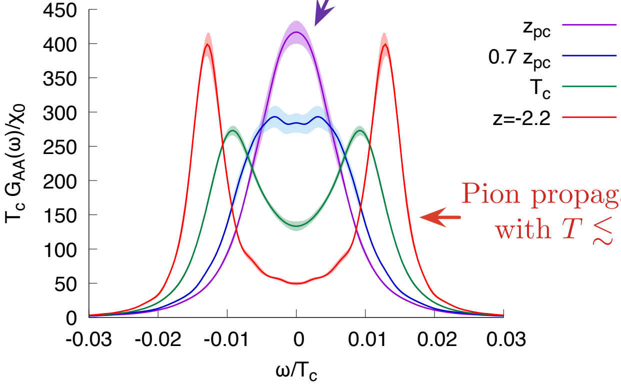


Let's take a fourier transform and analyze the transition

Features of the phase transition in the axial charge correlations:

$$G_{AA}(\omega) = \int \mathrm{d}t\, \mathrm{d}^3x\, e^{i\omega t}\ \langle \vec{n}_A(t,\boldsymbol{x}) \cdot \vec{n}_A(0,\mathbf{0}) \rangle$$

Diffusion of quarks with $T \gtrsim T_{\rm pc}$



Pion propagation with $T \lesssim T_c$

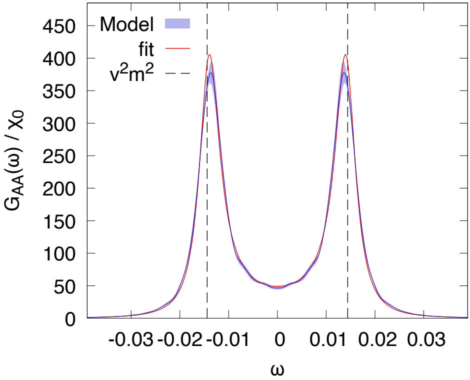
Can see the transition from diffusion of quarks to propagation of pions!

Quantitative analysis of a pion EFT well below T_c, $z = -2.2$:

The predicted pole position m_p^2 of pion waves is given by static quantities:

$$m_p^2 = v^2 m^2 = \frac{H\bar{\sigma}}{\chi_0}$$

This is the finite temperature Gell-Mann Oakes Rener relation:



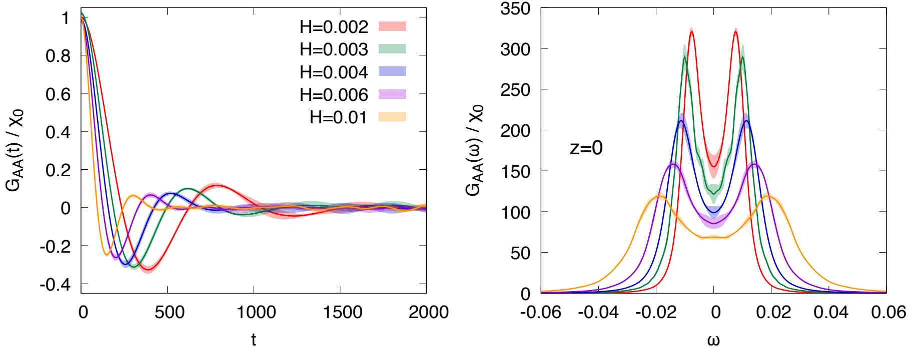
These are static inputs to fit:

$$v^2 = \frac{f^2}{\chi_0}$$

$$m^2 = \text{screening mass}$$

Scaling of simulations at T_c:

At $T = T_c$, we varied the magnetic field, finding the response functions:



See a scaling behavior of the real time correlations, with quark mass, which tunes the correlation length

Dynamical critical exponent of the $O(4)$ transition:

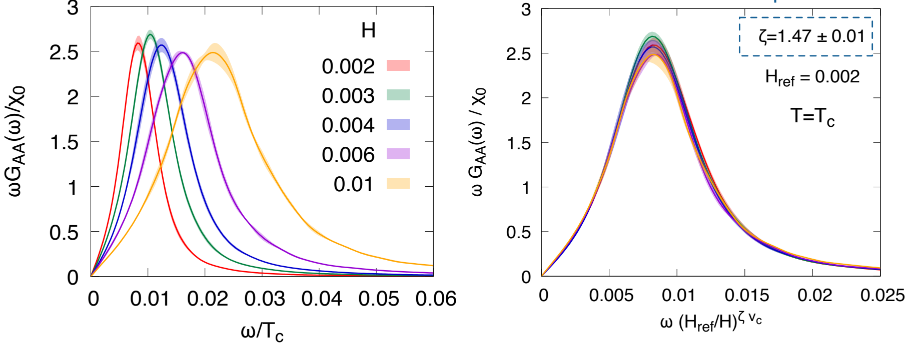
Rajagopal, Wilczek

The relaxation time and correlations *scale* with the correlation length ξ:

$$\omega\, G_{AA}(\omega, \xi) = \underbrace{f(\omega\, \tau_R)}_{\text{universal fcn}} \qquad \text{with} \qquad \underbrace{\tau_R \propto \xi^{\zeta}}_{\text{relaxation time}}$$

The correlation length scales as $\xi \propto H^{-\nu_c}$ and the time as $\tau_R \propto H^{-\zeta\nu_c}$:

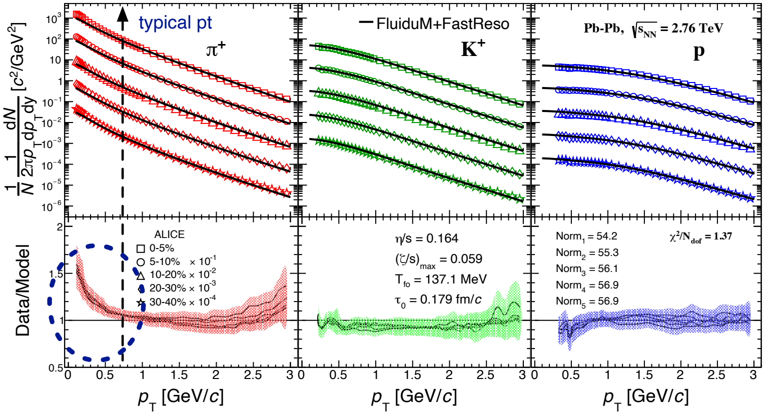
compare to 3/2



Phenomenology of Soft Pions in Data

Evidence for the chiral crossover in the heavy ion data?

A recent ordinary hydro fit from Devetak et al 1909.10485



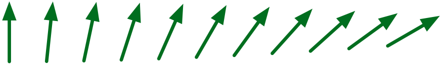
See also, Guillen&Ollitrault arXiv:2012.07898; Schee, Gürsoy, Snellings: arXiv:2010.15134

Because the pions are the Goldstones of the transition, I expect an enhancement at low p_T, relative to vanilla hydro

The Pion EFT

Son and Stephanov '01+bit from us

- Below T_C the condensate is frozen up to phase fluctuations



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

- The ideal equations of motion the phase is (with $\mu_A = n_A/\chi_0$):

$$\partial_t \varphi = \mu_A \qquad \text{Josephson Constraint}$$

while the axial charge EOM is:

$$\partial_t n_A + \nabla \cdot \boldsymbol{J}_A = f^2 m^2\, \varphi \qquad \text{Axial Current}$$

where the current is the gradient of the phase: $\boldsymbol{J}_A = f^2 \nabla \varphi$

- The pion EFT is written with $f^2 \simeq \bar{\sigma}^2$ and $f^2 m^2 = H\bar{\sigma}$

We can use the EFT to find the dispersion curve of soft pions, including dissipative corrections

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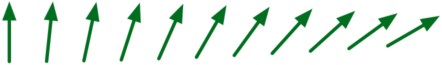

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- The ideal equations of motion the phase is (with $\mu_A = n_A/\chi_0$):

$$\partial_t \varphi = \mu_A + \mathcal{O}(\Gamma \nabla^2 \varphi) \qquad \text{Josephson Constraint}$$

while the axial charge EOM is:

$$\partial_t n_A + \nabla \cdot \boldsymbol{J}_A = f^2 m^2 \varphi + \mathcal{O}(D\nabla^2 n_A) \qquad \text{Axial Current}$$

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Long wavelength pion (superfluid) modes:

Son, Stephanov ’01 + a bit by us

- Linearizing the equations, the quasi particle energy is

$$\omega_q^2 \equiv v_0^2(q^2 + m^2) \qquad\qquad v_0^2(T) \equiv \frac{f^2}{\chi_0} \quad \Leftarrow \text{pion velocity}$$

Both v_0 and m scale with the condensate:

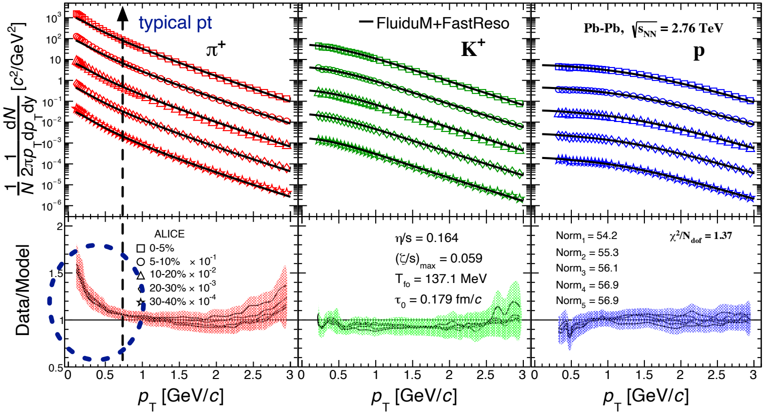
$$v_0^2 \propto \underbrace{\bar{\sigma}^2}_{\text{condensate}}$$

$$v_0^2 m^2 \propto \underbrace{\bar{\sigma}}_{\text{condensate}}$$

which vanishes at the critical point, $\bar{\sigma}(-t)^{\beta}$

Evidence for the chiral crossover in the heavy ion data?

A recent ordinary hydro fit from Devetak et al 1909.10485



See also, Guillen&Ollitrault arXiv:2012.07898; Schee, Gürsoy, Snellings: arXiv:2010.15134

Expect an enhancement at low p_T

$$n(\omega_q) = \frac{1}{e^{vq/T} - 1} \simeq \frac{T}{vq} \Rightarrow \infty,$$

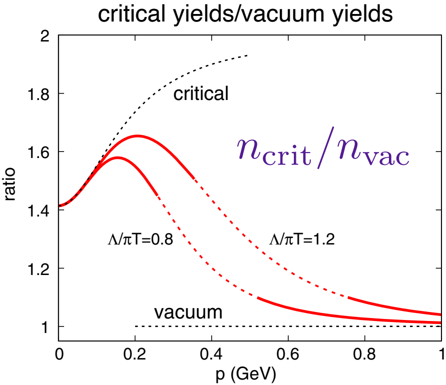
Since at T_c, the velocity $v \Rightarrow 0$!

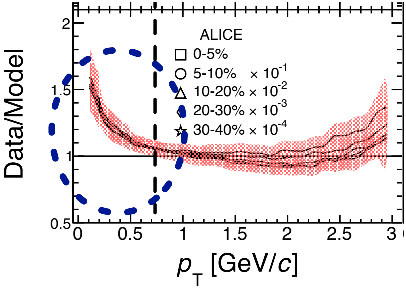
With a modified dispersion curve (relative to vacuum) the yields increase

$$n(\omega(p)) = \frac{1}{e^{\omega(p)/T} - 1} \qquad \omega^2(p) = v^2(T)(p^2 + m^2(T))$$

v goes to zero at T_c

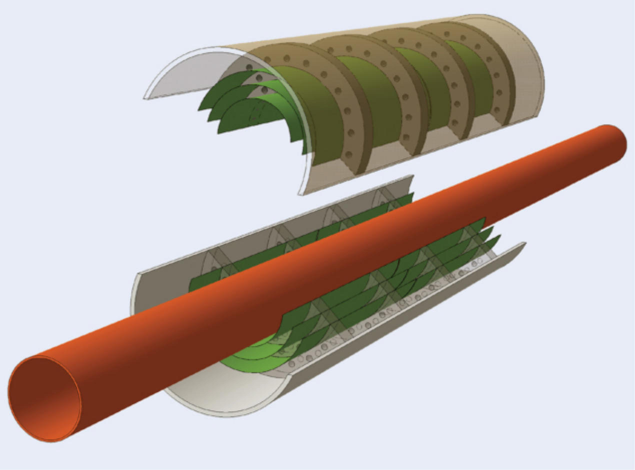
We estimated the drop in $v^2(T)$ and $v^2 m^2(T)$ from lattice data ...





Encouraging estimate which motivates additional work on critical dynamics

New Detector: ALICE ITS3



Summary and Outlook:

1. We are encouraged by estimates and current measurements.
2. We are simulating the real-time dynamics of the chiral critical point
   - The numerical method may be useful for stochastic hydro generally
3. We reproduced the expected dynamical scaling laws:

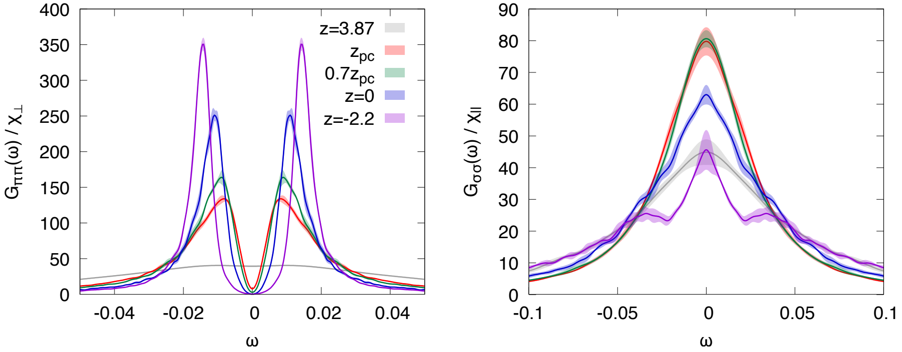
$$\tau_R \propto \xi^{\zeta} \qquad \zeta = \frac{d}{2} \simeq 1.47 \pm 0.01$$

4. The pion waves are well calibrated.
5. The next step is to study the expanding case:
   - This will predict soft pions and their correlations with expansion for heavy ion collisions

The hadronization of the pion is the (only) hadronization process that can be studied rigorously, *and only with hydrodynamics!*

Backup

Comparison of π and σ



Dynamical scaling of σ correlation functions:

$$G_{\sigma\sigma}(\omega) = \int \mathrm{d}t\, \mathrm{d}^3x\, e^{i\omega t}\, \langle \sigma(t,\boldsymbol{x}) \cdot \sigma(0,\mathbf{0}) \rangle$$