Liquid theory and fundamental bounds
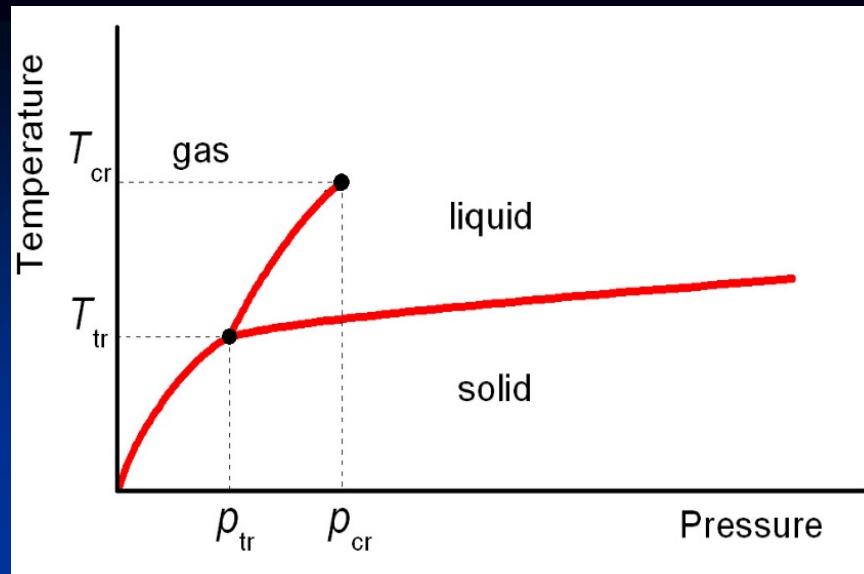
Kostya Trachenko

Queen Mary University of London

1. Liquids and supercritical fluids
- problem of theoretical description
- phonons in liquids and liquid energy

2. Fundamental bounds:
viscosity, thermal conductivity, speed of sound

3. Viscosity of quark-gluon plasma

General outlook on liquids

gases:

fluidity
viscosity
…

**duality of liquids
gas-like? solid-like?
(hydrodynamic vs elastic
description)**

solids:

density
bulk modulus
heat capacity

Review paper: Trachenko and Brazhkin, Rep Prog Phys 2016

Energy and heat capacity of matter

Gases, solids

$E_{gas} = 3NT/2$

$E_{solid} = 3NT$
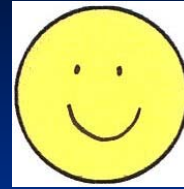
Liquids

$E_{liquid} = ?$

A Granato JNCS 2002

Landau&Lifshitz, *Statistical Physics*:
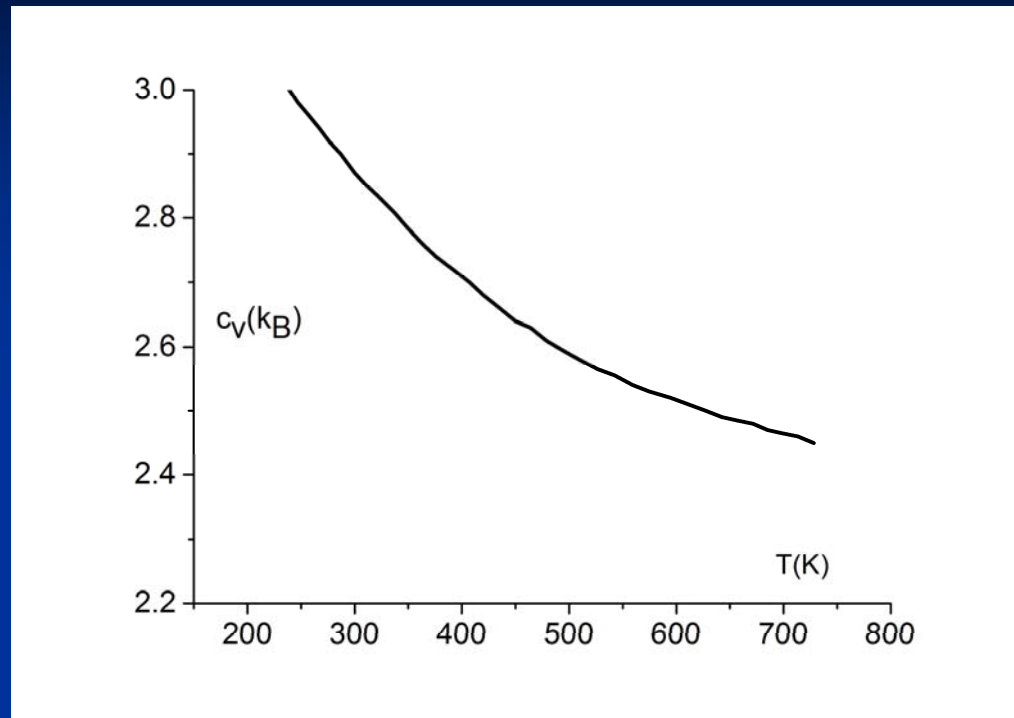"The absence of smallness of particle vibrations...
Interactions in a liquid are both strong and system-specific
Liquid energy can not be calculated in general form"

$$E = \frac{3}{2}NT + \frac{N\rho}{2}\int U(r)g(r)dV$$

Landau school: "liquids have no small parameter"

Experiments: liquid Hg

- Van Der Waals, hard-spheres models of liquids etc: ideal-gas $c_v = 3k_B/2$

- Can liquid c_v be understood on the basis of collective modes as in solids?
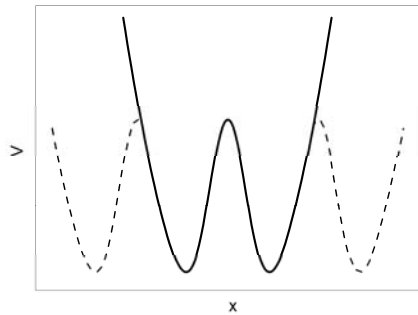
Failure of first-principles description

Solids:
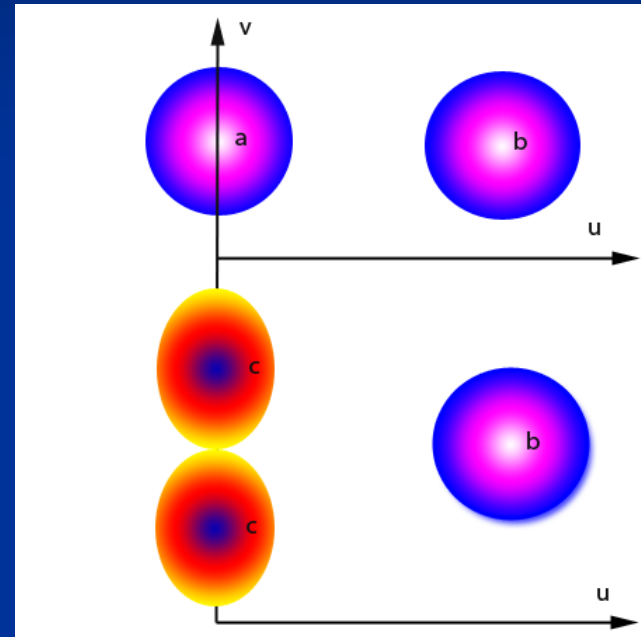
$$U = U_0 + \frac{1}{2} \sum k_{ij} x_i x_j$$

$$\sum m_i \ddot{x}_i + k_{ij} x_i = 0$$

Liquids:

$$\ddot{x}_1 + \alpha x_1 + \varepsilon(x_1 - x_2) - \beta x_1^3 = 0$$

$$\ddot{x}_2 + \alpha x_2 + \varepsilon(x_1 - x_2) - \beta x_2^3 = 0$$
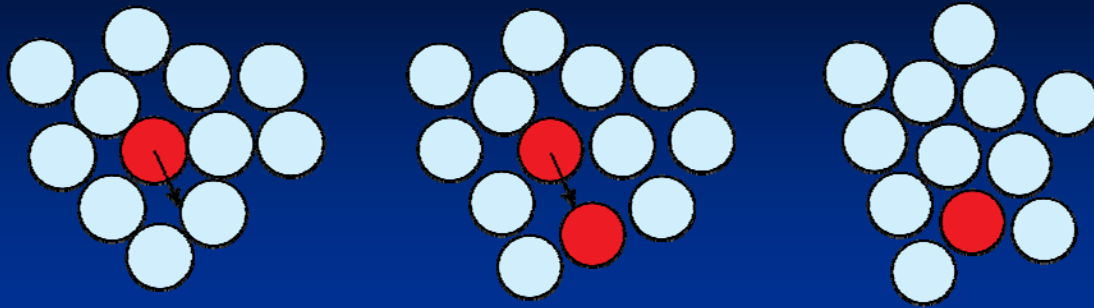

$N=3$ – already complicated. $N=4$ – only qualitative picture

Number of bifurcations, new stationary points and collective modes around
them increases exponentially with N.
The problem becomes exponentially complex and intractable for large N

Frenkel reduction

τ - time between particle jumps

Reduction: assuming equivalent conditions for bifurcations and stationary points of all generations in a homogeneous system on average $U=8\pi Gr\Delta r^2$ and $\tau \sim \exp(U/T)$ after each bifurcation

$t<\tau$: solid

$t>\tau$: liquid

$\omega>\omega_F=1/\tau$: liquid supports solid-like collective modes.

$\omega<\omega_F=1/\tau$: hydrodynamic modes

Prediction: liquid supports two transverse modes with $\omega>\omega_F$ and one longitudinal mode (propagating in two different regimes)

Continuity of the Solid and the Liquid States

By Prof. J. Frenkel, Physico-Technical Institute, Leningrad

IT was believed until quite recently that liquids were in all respects—save their density—more similar to gases than to solid bodies. This belief was strengthened by the van der Waals' theory of the continuity between the liquid and the gaseous states, the sharp transition between them (corresponding to the boiling or condensation point) being replaced by the hook-like portion of the (p–v) curve, representing unstable intermediary states.

Recently, a number of new facts have corroborated the view that liquids—at least in the vicinity of the melting point—are much more similar to solids than to gases, not only with regard to density, but also with respect to the character of the *heat motion* and *structure*. As I pointed out ten years ago[1], the heat motion in simple liquids consists of vibrations about an equilibrium position which after an average time $\tau = \tau_0 e^{U/kT}$ (τ_0 is vibration period, T is temperature, U is activation energy) is shifted through a distance δ comparable with the interatomic distances. Under the influence of alternating forces with a period τ' much larger than τ, the liquid behaves in the ordinary way, displaying a viscosity coefficient $\eta = N\tau$, where N is a constant which can be defined as the rigidity modulus displayed by the same 'liquid' when subject to alternating forces with a period τ' much smaller than τ (amorphous solids are rightly denoted as supercooled liquids with a very large 'relaxation time' τ).

Still more recently, it has been pointed out (for the first time by Pauling) that the molecules in a crystalline solid can rotate more or less freely—just as has been assumed for liquids and gases; while Debye has shown[2] that both in liquids and solids there is actually no *free* rotation, but a sequence of rotational oscillations about an equilibrium orientation which is changed abruptly from time to time[3].

a solid crystalline body, especially in the vicinity of the melting point, is no longer exactly crystalline, but contains a number of 'dissociated' atoms or ions which are irregularly distributed in the 'interstices' of the crystal lattice, and form the beginning, as it were, of the amorphous phase within the crystal. This 'dissociation' process explains diffusion in solids and the electrical conductivity of ionic crystals[4].

Thus we see that the usual conception of the existence of a fundamental difference between the solid and the liquid states is incorrect. I wish to put forward the contrary view, namely, that the *solid and liquid states are connected with each other in a continuous way, just as are the liquid and the gaseous states*. The fact of the existence of a sharp transition point between the solid and the liquid states does not in the least contradict this conception—just as the existence of a sharp boiling (or condensation) point does not contradict the continuity between the liquid and the gaseous states. It simply indicates that, just as in the latter case, a continuous transition from solid to liquid, corresponding to a gradual increase of volume and of the degree of amorphism, must go through a sequence of unstable states, characterised by the same hook-like shape of the pressure (p)–volume (v) curve, as that occurring in van der Waals' isotherms below the critical temperature. Hertzfeld and M. Göppert-Mayer[5] have indeed shown that the p–v curve for a number of crystals displays a minimum of pressure, which they interpreted as corresponding to the melting point. They were led to this interpretation—or rather misinterpretation—by the fact that solids cannot be superheated, in contradistinction from liquids. This law of analogy between the melting and the boiling process can, however, be very naturally explained by the fact that small liquid drops formed inside the solid cannot rise to the surface, as do the gas bubbles formed in a boiling

- liquids are different from solid glasses quantitatively (by τ) but not qualitatively:
Large τ ($\tau > t$) : solid glass
Small τ ($\tau < t$) : liquid

- Landau rejected (symmetry breaking&order parameter ideas)

- liquid-glass transition: remarkably, debate continues (80 years!)

Solid-like elastic modes in liquids

- $\omega > \omega_F = 1/\tau$: $\omega\tau > 1$ – strictly non-hydrodynamic solid-like modes.
  Outside hydrodynamic description
  Generalized hydrodynamics tries, but faces issues

- Can solid-like transverse modes with ω extending to ω_D exist in liquids?
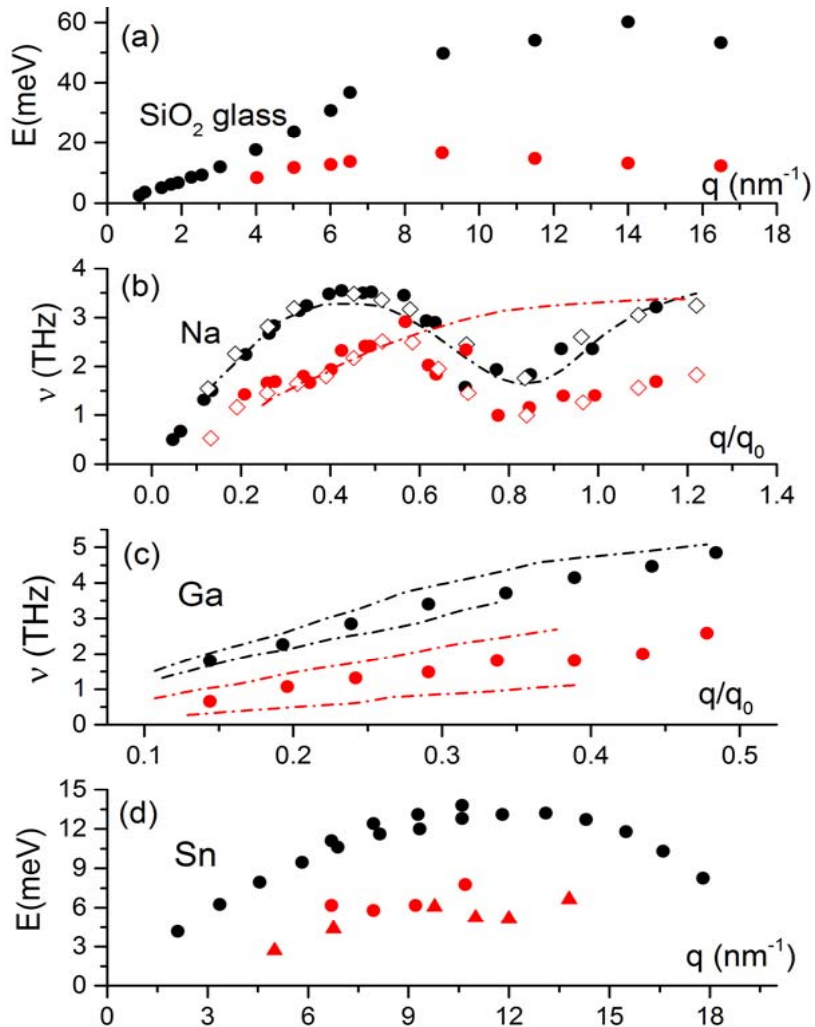
Experimental evidence for transverse collective modes

Started with viscous liquids and Brillouin scattering
B_2O_3, Grimsditch et al, PRL 1989
Glycerol, Scarponi et al, Glycerol, PRB 2004

See our ROPP 2016 review for details

Transverse collective modes from inelastic X-ray scattering

Ruzicka et al PRB 2004

G Baldi: dispersion curves in glasses and polycrystals are similar

Giordano, Monaco PNAS 2010
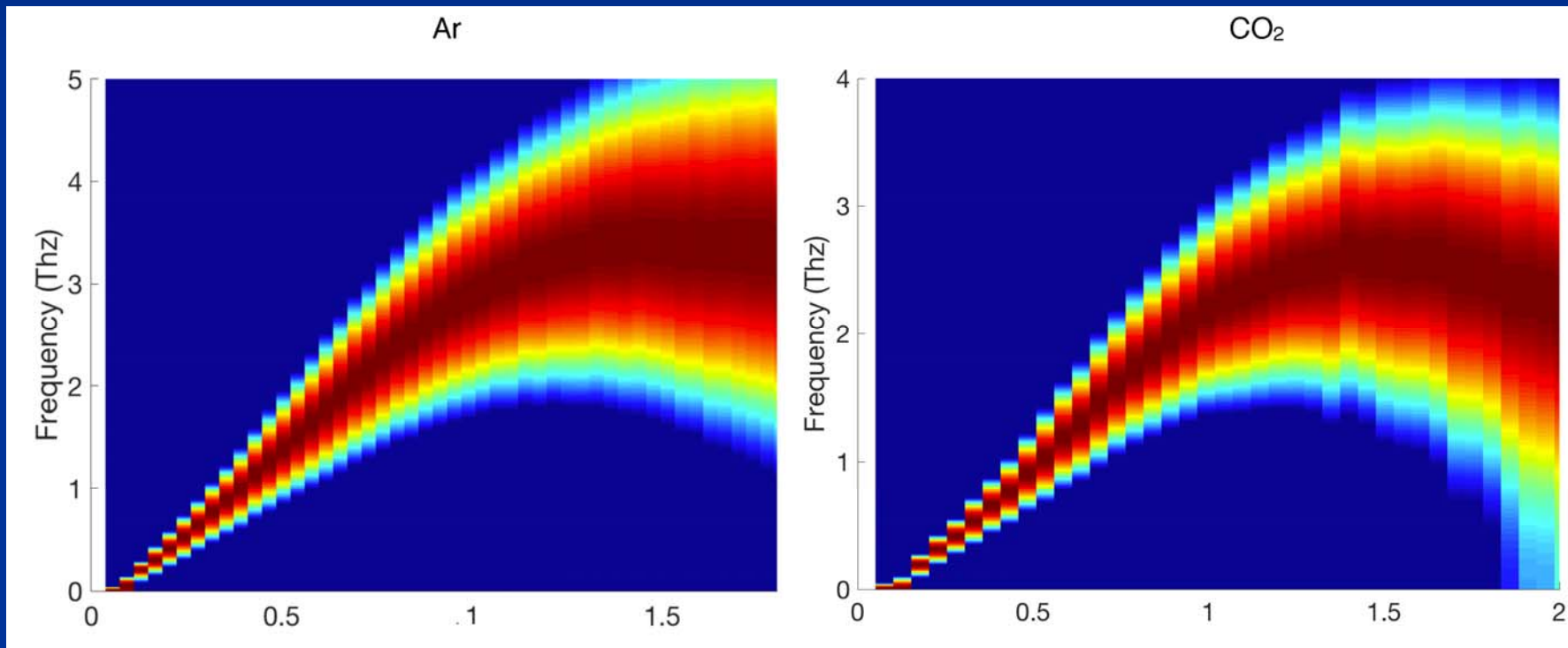Giordano, Monaco PRB 2011

Hosokawa et al JPCM 2013
Same results for liquid
Fe, Cu, Zn (JPCM 2015)

Propagation length of quasi-harmonic modes ~ 1 nm, comparable to room-T metals (Jain&McGaughey, PRB 2016)

Transverse collective modes from
MD-calculated transverse intensity maps $C(k,\omega)$

T=250 K   T=300 K

PRL 2017

More on this later

Liquid energy
(calculating the "impossible")

Disordered systems:
harmonic frequencies are well-defined (k-points are not)

in glass, consider all phonons: $E=3NT$

in liquids, consider the energy shear modes with $\omega>1/\tau$

Vibrational energy of a disordered system

$$E_{\text{liquid}} = \int E(\omega,T)g(\omega)d\omega$$

Energy per mode (even though with small prop. length) is $k_{\text{B}}T$.
Add the energy of the longitudinal mode
Add the energy of diffusing atoms

Liquid energy
(calculating the "impossible")

$$\omega_{\text{F}} = \frac{1}{\tau}$$

$$E = NT\left(3 - \left(\frac{\omega_{\text{F}}}{\omega_{\text{D}}}\right)^3\right)$$

Harmonic&classical, PRB 2008

$$E = NT\left(1 + \frac{\alpha T}{2}\right)\left(3 - \left(\frac{\omega_{\text{F}}}{\omega_{\text{D}}}\right)^3\right)$$

Anharmonic&classical, PRB 2011

$$E = NT\left(1 + \frac{\alpha T}{2}\right)\left(3D\left(\frac{\hbar\omega_{\text{D}}}{T}\right) - \left(\frac{\omega_{\text{F}}}{\omega_{\text{D}}}\right)^3 3D\left(\frac{\hbar\omega_{\text{F}}}{T}\right)\right)$$

Quantum& anharmonic, Sci Rep 2012

$$E = NT\left(3 - \left(\frac{\omega_F}{\omega_D}\right)^3\right)$$

$$\omega_F = \frac{1}{\tau} = \frac{G_\infty}{\eta}$$

VS

$$E = \frac{3}{2}NT + \frac{N\rho}{2}\int U(r)g(r)dV$$

1. Equation for liquid energy with no fitting parameters

2. Depends on ω_F (or τ, widely available) but not on system-specific correlation functions and interatomic potentials (available only for a narrow set of liquids like Ar). Energy does not depend on these in our approach (but does in the L&L argument)

3. τ can be the same for liquids with very different structure and interactions => the results are more universal than previously thought

4. Simple: this theory of a liquid is no more complicated than a theory of a solid

5. Explains experimental data

Theory and experiment:

Take experimental τ and calculate $c_v=dE/dT$ directly.

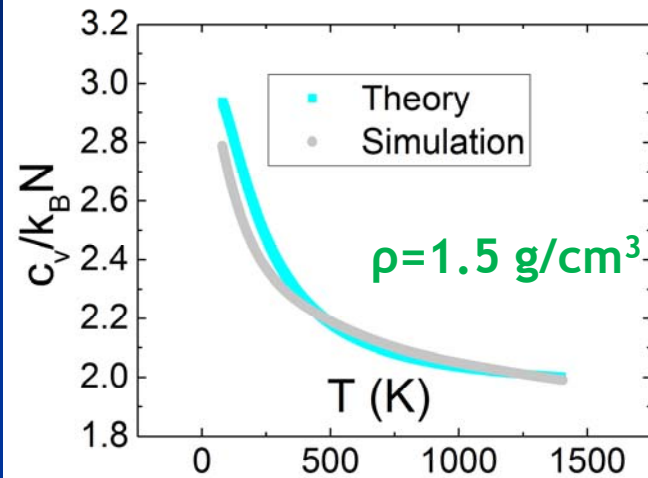**Experimental SUPERCRITICAL data are from NIST**
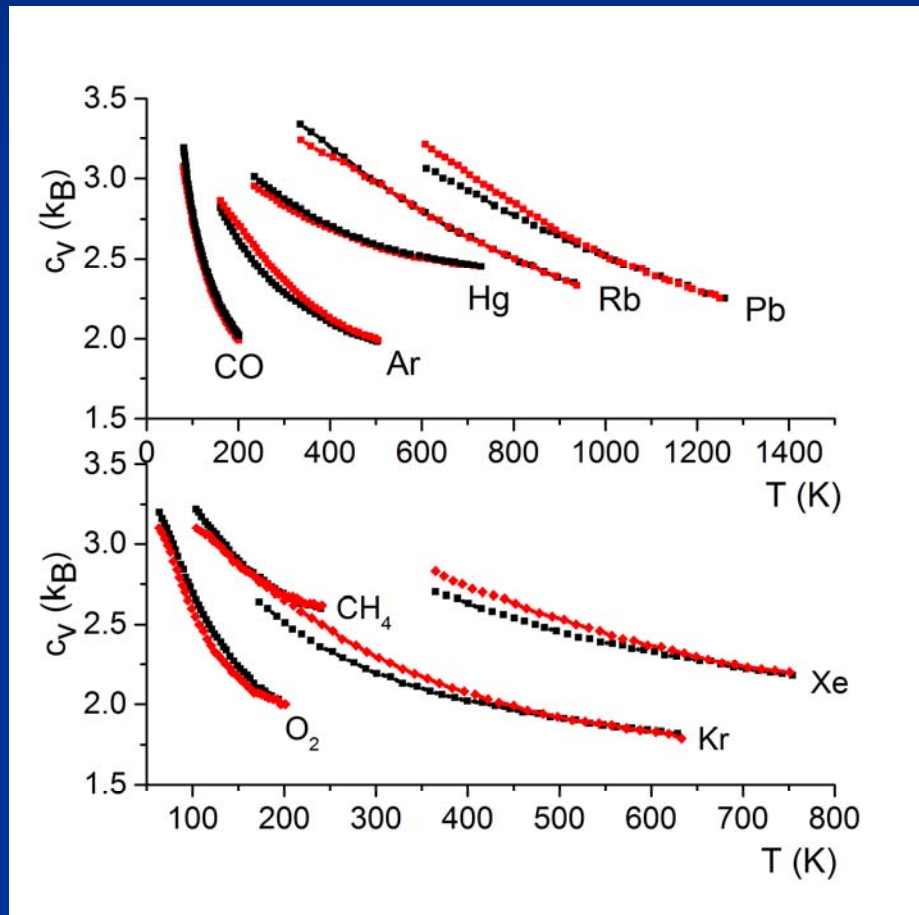
Experimental decrease of c_v is due to progressive loss of transverse modes with temperature
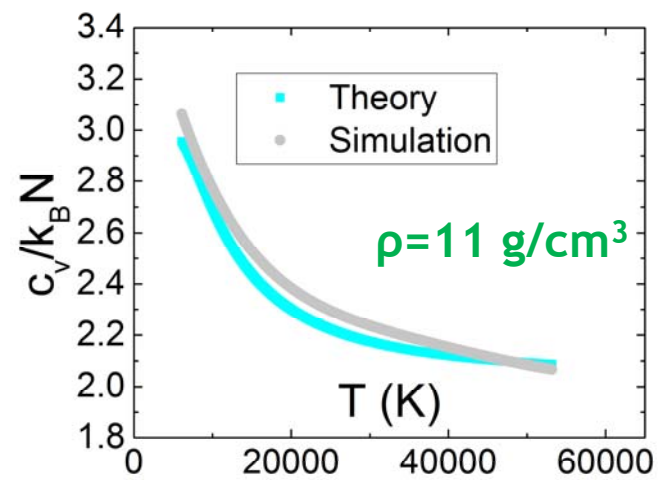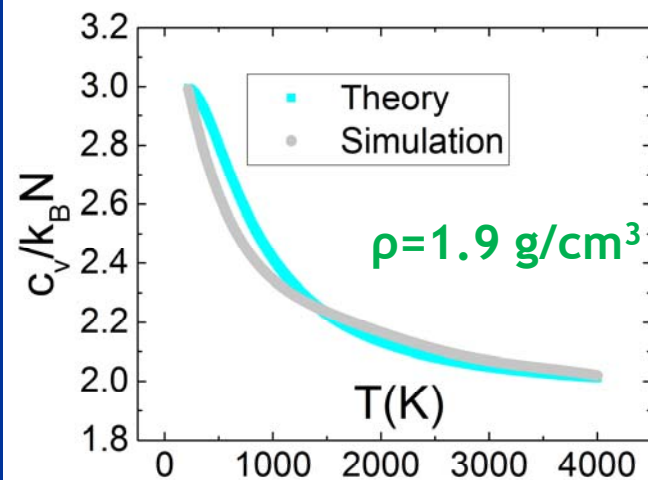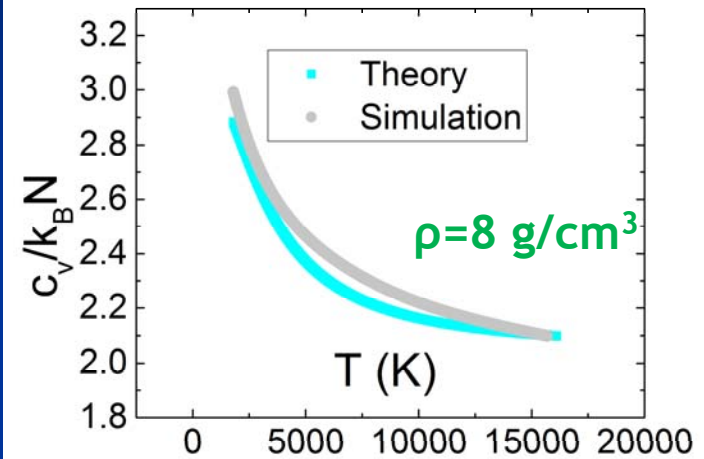
Theory and MD simulations: calculate τ directly (PRE 2017)

Ar

Fe

Can a theory of liquid heat capacity, in fact, exist?

- Good agreement for 21 metallic, noble, molecular and network hydrogen-bonded liquids in a wide range of temperature and pressure

- No fitting parameters used. Parameters correspond to physically sensible $G_\infty \sim$ GPa, $\tau_D \sim 0.1$ ps (τ_D is fixed at their solid experimental values)

- Despite many pessimistic statements, liquids emerge as exciting systems amenable to theoretical understanding in a consistent picture

Back to low temperature: *k*-gap in the transverse spectrum

$$\frac{ds}{dt} = \frac{P}{\eta} + \frac{1}{G}\frac{dP}{dt} = \left(\frac{1}{\eta} + \frac{1}{G}\frac{d}{dt}\right)P =$$

$$\frac{1}{\eta}\left(1 + \tau\frac{d}{dt}\right)P \quad (\text{where } \tau = \eta/G)$$

$$\nabla^2 v = \frac{1}{\eta}\left(\rho\frac{dv}{dt} + \nabla p\right)$$

$$\eta\nabla^2 v = \left(1 + \tau\frac{d}{dt}\right)\left(\rho\frac{dv}{dt} + \nabla p\right)$$

$$\eta\frac{\partial^2 v}{\partial x^2} = \rho\tau\frac{\partial^2 v}{\partial t^2} + \rho\frac{dv}{dt}$$

$$c^2\frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2} + \frac{1}{\tau}\frac{dv}{dt}$$

$$v\infty\exp\left(-\frac{t}{2\tau}\right)\exp(i\omega t)$$

$$\omega = \sqrt{c^2 k^2 - \frac{1}{\tau^2}}$$

Rep. Prog. Phys. 2016

$$k > k_{gap} = \frac{1}{c\tau}$$

K-gap!
(rather than frequency gap $\omega = 1/\tau$,
frequency gap $=0$)

Physical interpretation of the gap:

The gap is related to the length scale in liquids $d_{el} = c\tau$
over which shear stress is relaxed (shear wave propagation length) $d_{el} = c\tau$

Observe the gap in MD simulations

LJ and CO_2 empirical potentials, calculate transverse current
correlation function $C(k,t)=<J_x(-k,t)J(k,0)>$, $J=\Sigma[\boldsymbol{k}\boldsymbol{v}_i]\exp(-ikr_i(t))$.

~120,000 atoms in MD simulations

Rather than using a fitting function, we average over 20 different runs

Fourier transforms of $C(k,t)$ gives intensity maps (structure factor)
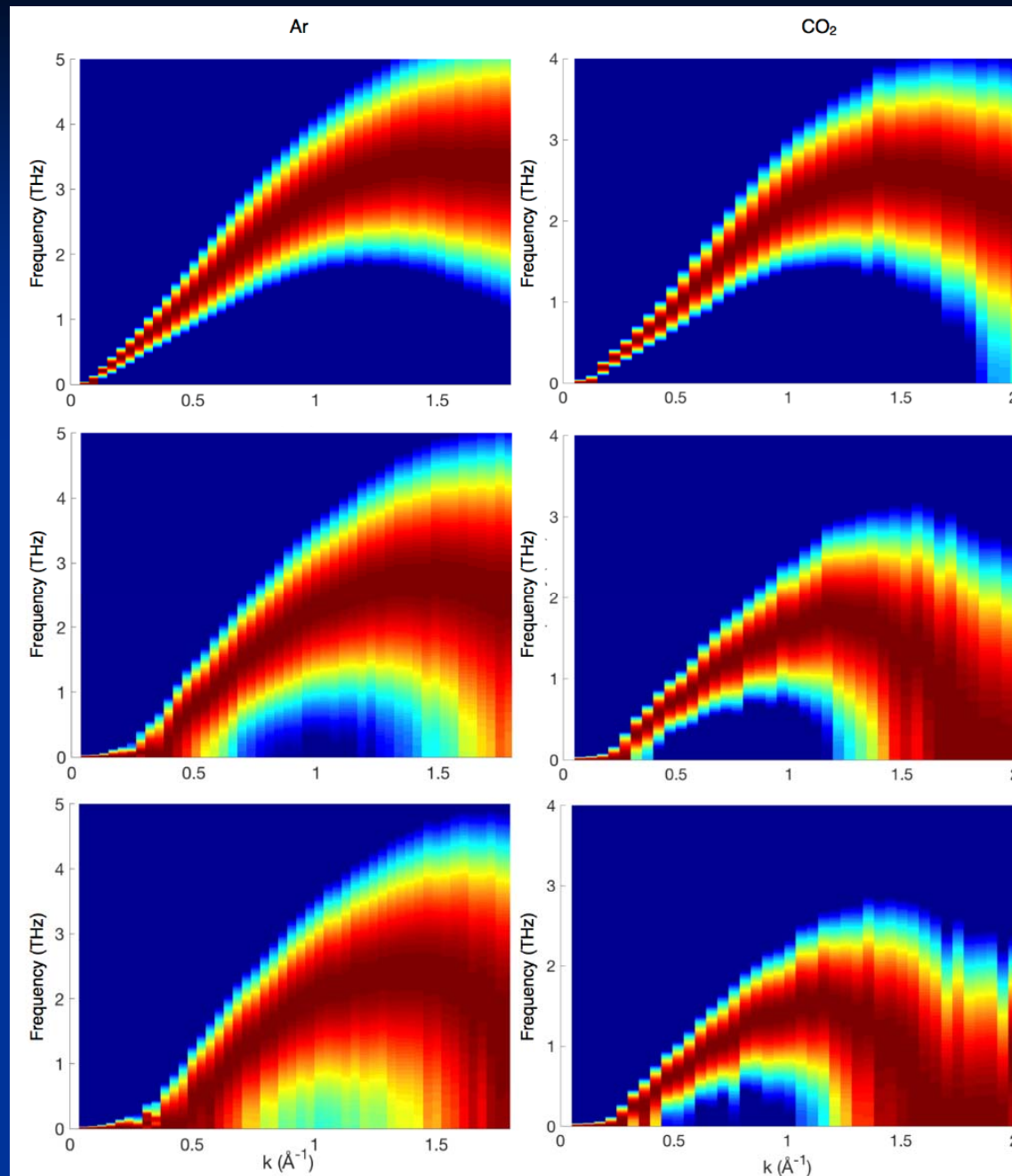
Intensity maps $C(k,\omega)$

$$k > k_{gap} = \frac{1}{c\tau}$$
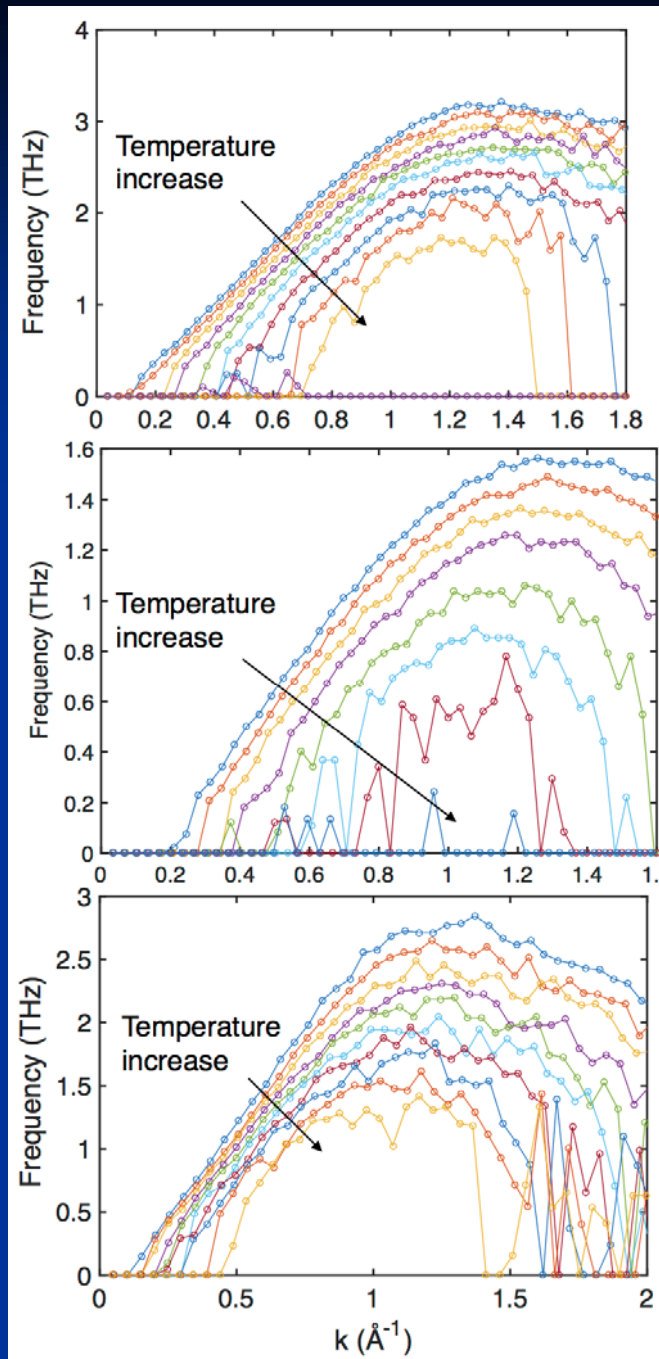

T=250 K

T=300 K

T=350 K

T=400 K

T=450 K

T=500 K

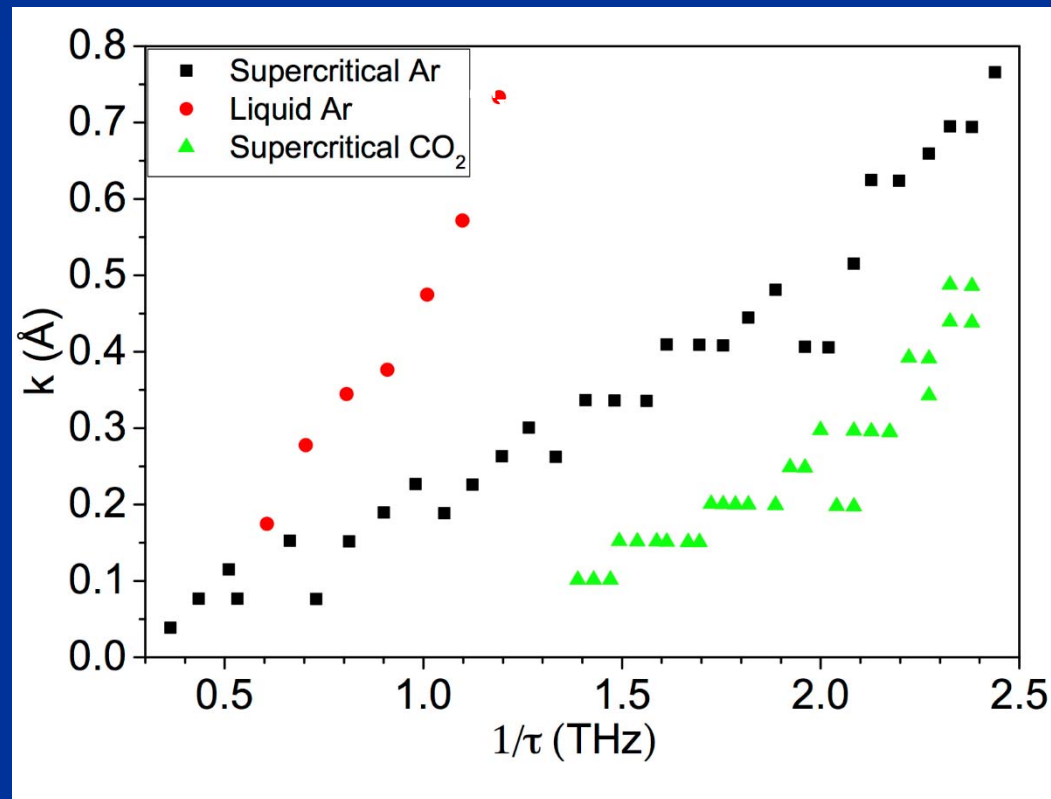# Dispersion curves

$$k > k_{gap} = \frac{1}{c\tau}$$

PRL 2017

Supercritical Ar

Subcritical Ar
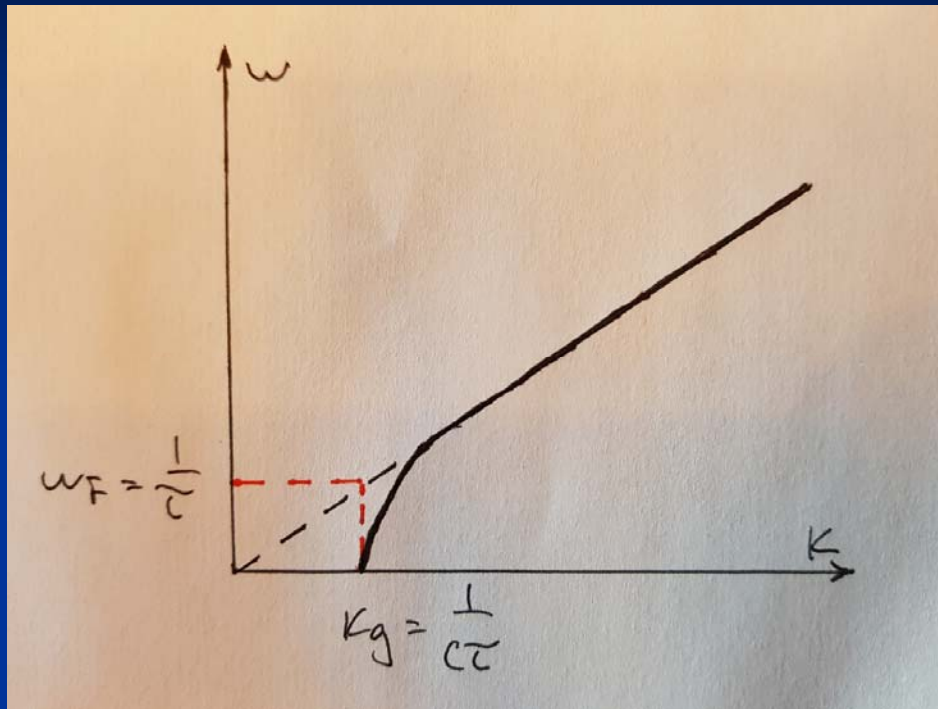
Supercritical CO_2

$$k > k_{gap} = \frac{1}{c\tau}$$

τ is calculated as structural relaxation time
from the decay of the intermediate scattering function

Why is Frenkel-gap approximation so successful for liquid thermodynamics?

$$u \sim \exp(-t/\tau)\exp(i\omega t)$$

$$\omega = \sqrt{c^2 k^2 - \frac{1}{\tau^2}}$$

The frequency corresponding to $k_g = 1/(c\tau)$ is $\omega = 1/\tau$

Frenkel approximation neglects the low-ω crossover to $\omega = ck$.

This works because:

1. *Propagating* modes are above k_g, so there is a frequency gap for propagating modes.

2. The density of states of modes around k_g is $g(\omega) \sim \dfrac{1}{dw/dk} \sim 0$ from

Back to liquid thermodynamics:

This is how we did it on the basis of frequency gap $\omega_F = \dfrac{1}{\tau}$ in the Debye model :
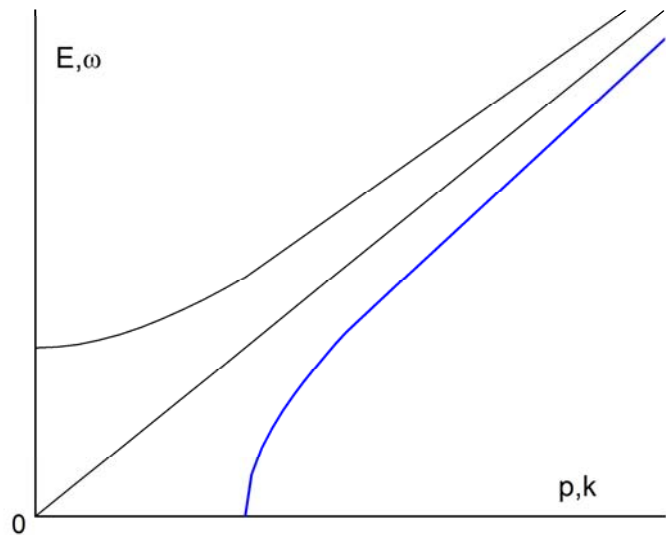
$$E_t = \int_{\omega_F}^{\omega_D} k_B T \, \frac{6N}{\omega_D^3} \, \omega^2 d\omega = 2Nk_B T \left(1 - \left(\frac{\omega_F}{\omega_D} \right)^3 \right)$$

Calculation in k - space :

$$E_t = \int_{k_{gap}}^{k_D} k_B T \, \frac{6N}{k_D^3} \, k^2 dk = 2Nk_B T \left(1 - \left(\frac{\omega_F}{\omega_D} \right)^3 \right)$$

where $\omega_D = ck_D$

From the point of view of thermodynamics,
liquids behave as if they have a frequency gap

$$\omega = \sqrt{c^2 k^2 - \frac{1}{\tau^2}}$$

What Lagrangian or field theory gives this?
(thought to be impossible)

Need *two* fields Trachenko, PRE 2017):

$$L = \frac{\partial \varphi_1}{\partial t}\frac{\partial \varphi_2}{\partial t} - c^2 \frac{\partial \varphi_1}{\partial x}\frac{\partial \varphi_2}{\partial x} + \frac{1}{2\tau}\left(\varphi_1 \frac{\partial \varphi_2}{\partial t} - \varphi_2 \frac{\partial \varphi_1}{\partial t}\right)$$

$\varphi_1 = \varphi_0 \exp(-t/\tau) \cos(kx-\omega t)$
$\varphi_2 = \varphi_0 \exp(t/\tau) \cos(kx-\omega t)$

$$\omega = \sqrt{c^2 k^2 - \frac{1}{\tau^2}}$$

(the energy is constant and positive)

A longer paper is: Baggioli et al, Phys. Rev. D 2020

Recent book by J Proctor:
The Liquid and Supercritical Fluid States of Matter

talks about liquids and their understanding on the basis of our theory

- **Gapped momentum states:**
review paper Baggioli et al, Physics Reports 2020

- **Similarities between liquids and holographic models:**
k-gap, viscoelastic behaviour etc

Baggioli & Trachenko, Phys. Rev. D, JHEP 2019

Higher temperature: Supercritical fluids

Decaffination (non-toxic) of coffee and tea, CO_2

Purification and extraction of unwanted solvents, CO_2

Supercritical H_2 in Jupiter, Saturn, exoplanets, brown dwarfs

supercritical fluids: green and effective

plants to treat toxic and hazardous wastes: "supercritical H_2O oxidation" as alternative to incineration and landfills
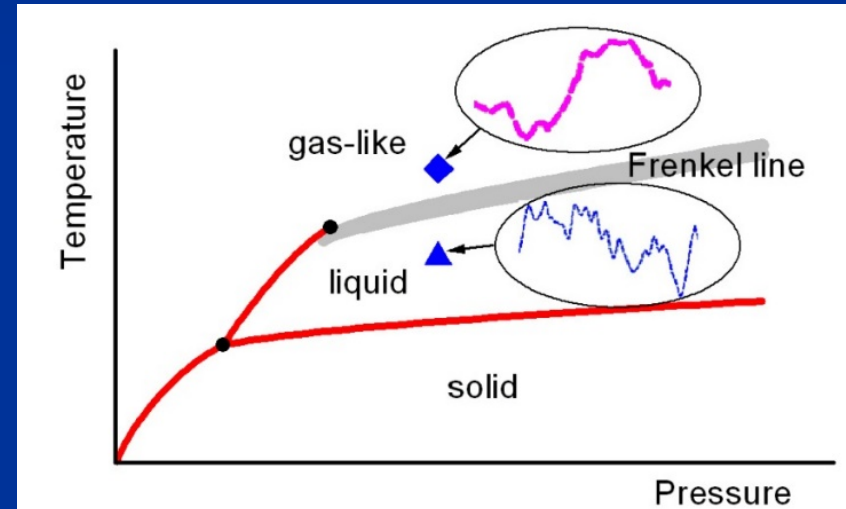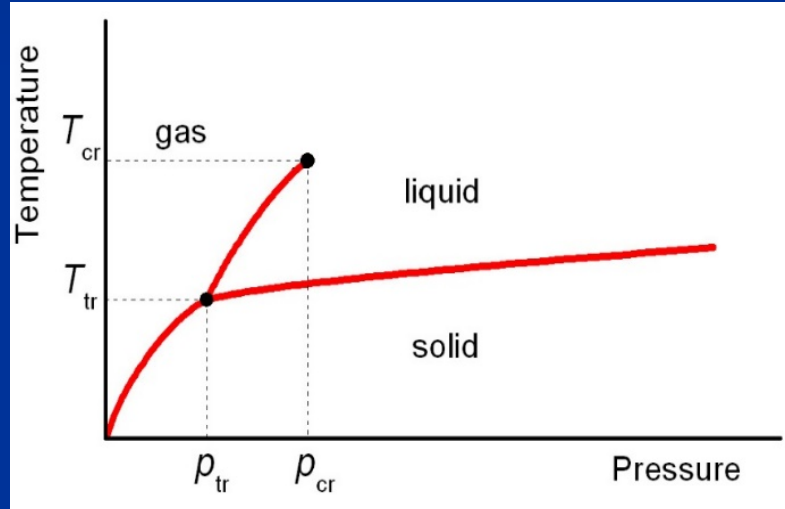

Extraction and removing of actinides in nuclear waste, CO_2

Higher temperature: Frenkel line

$$E = NT \left(3 - \left(\frac{\omega_F}{\omega_D} \right)^3 \right)$$

What happens when $\omega_F -> \omega_F$ and $c_v = 2$?

Crossover at $c_v = 2$ at the Frenkel line in the supercritical region
Nature Comm. 2013

The Frenkel line separates two different dynamical states
2012-2013: PRL 2013, Physics Today 2012

Recent experimental evidence for the crossover at the Frenkel line

1. supercritical Ne, X-ray scattering. Prescher et al, PRB 2017

2. supercritical N_2, Proctor JPC Lett 2018

3. supercritical CH_4 and C_2H_6, X-ray scattering and Raman. Smith et al, PRE 2017, Proctor et al JPC 2018

4. supercritical CO_2, neutron scattering. Marinakis et al, PRE 2020

5. Earlier experiments in supercritical water.

Review on condmat

Collective modes at the Frenkel line
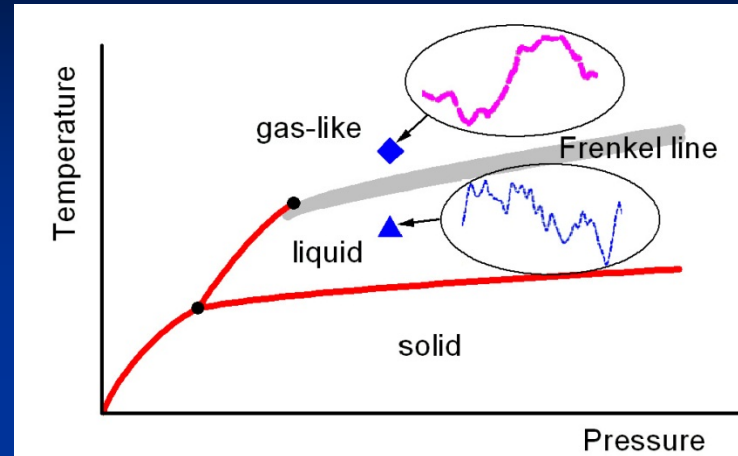
The Frenkel line corresponds to $\tau \approx \tau_D$

Important change is the inability to support rigidity (solid-like shear modes) at ANY frequency: rigidity is lost completely due to $k > k_{gap} = \dfrac{1}{c\,\tau}$

the supercritical system crosses over from **"rigid" liquid to "non-rigid" gas-like fluid**

Heat capacity above the Frenkel line

- Mean-free path concept used for dilute gases only. But at high energy(temperature) should also apply to dense systems such as supercritical fluids

- Calculate the energy of the longitudinal mode with wavelength $\lambda > l$

- Take l from experimental η as $\eta = 1/3 \; v \varrho l$

$$E = \frac{3}{2} NT + \frac{1}{2} NT \frac{a^3}{l^3}$$

Good agreement with experimental data of c_v with no fitting parameters (Nature Comm. 2013)

General outlook on liquids

hydrodynamic?   solid-like elastic?

duality of liquids

- Historically: hydrodynamic approach (Navier-Stokes, Langevin etc) and its extrapolations to generalized hydrodynamics (sometimes criticised)
- Less so: solid-like approach

Trachenko, PRE 2017: The same equations for liquid modes follow if we
(1) generalize hydrodynamic equations to include solid-like elastic response or
(2) generalize solid-state elasticity equations to include hydrodynamic flow

Symmetry of liquid description

Minimal quantum viscosity

Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics

P. K. Kovtun,[1] D. T. Son,[2] and A. O. Starinets[3]

[1]Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA
[2]Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA
[3]Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada

The ratio of shear viscosity to volume density of entropy can be used to characterize how close a given fluid is to being perfect. Using string theory methods, we show that this ratio is equal to a universal value of $\hbar/4\pi k_B$ for a large class of strongly interacting quantum field theories whose dual description involves black holes in anti–de Sitter space. We provide evidence that this value may serve as a lower bound for a wide class of systems, thus suggesting that black hole horizons are dual to the most ideal fluids.
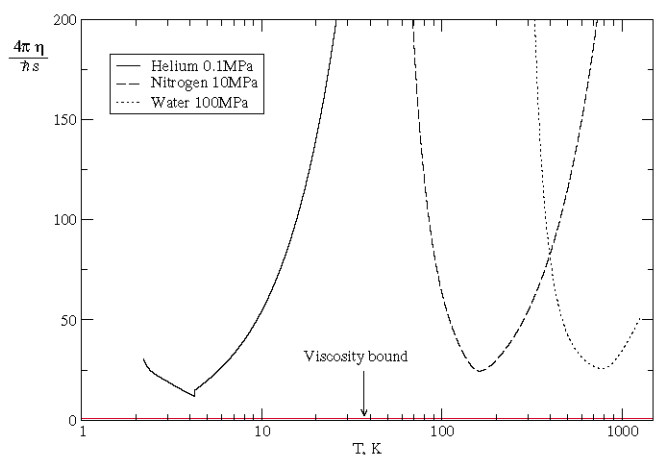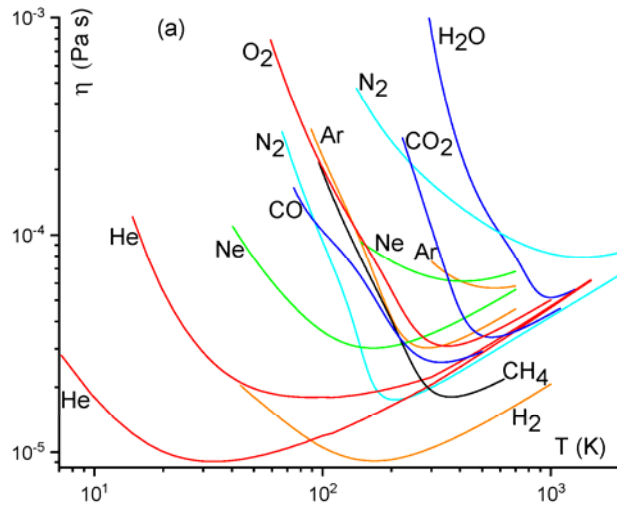
FIG. 2 (color online). The viscosity-entropy ratio for some common substances: helium, nitrogen and water. The ratio is always substantially larger than its value in theories with gravity duals, represented by the horizontal line marked "viscosity bound."

$\eta / s > \hbar/(4\pi)$

KSS theory: viscosity has no minima (contrary to liquids)

Minimal quantum viscosity

Low T: $\eta = \eta_0 exp(\frac{U}{T})$

High T: $\eta = \frac{1}{3}\rho v L$

Generally, η is impossible to predict.
But we can do this at the minimum (FL):

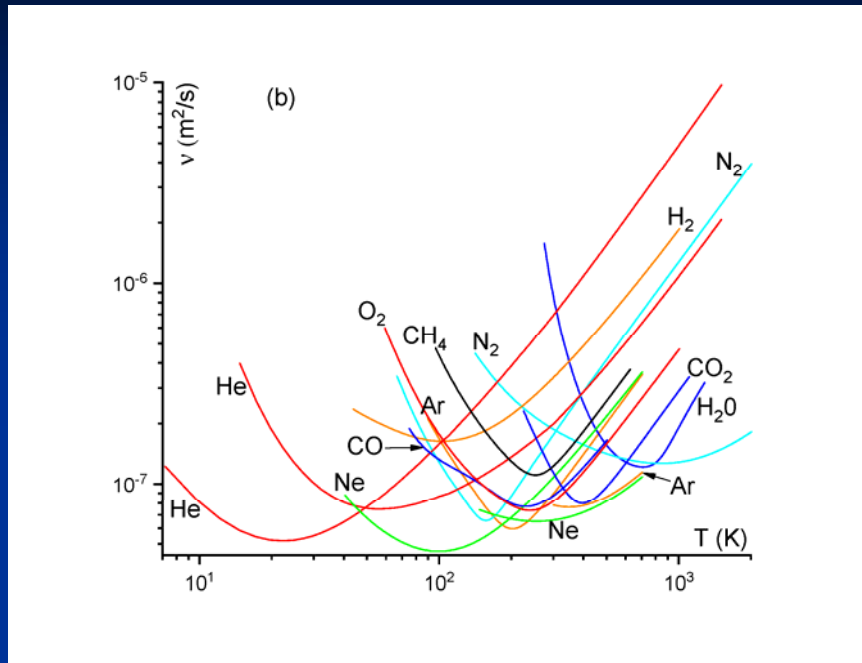At the minimum/crossover, $L=a$, $v=\dfrac{a}{\tau_D}=\dfrac{1}{2\pi}\omega_D a$, $\rho = \dfrac{m}{a^3}$

$\eta_{min} = \dfrac{1}{2\pi}\dfrac{m\omega_D}{a}$ (same from $\eta=G\tau_D$)

a=3-6 Å, $\dfrac{\omega_D}{2\pi}$=1 THz, m=2-40 ->
η_{min}=10^{-5}-10^{-4} Pa s

$\nu_{min} = \dfrac{1}{2\pi}\omega_D a^2$

Minimal quantum viscosity: how small can ν get?

$$\nu_{min} = \frac{1}{2\pi}\,\omega_D a^2$$

$$\frac{\hbar\omega_D}{E} = \left(\frac{m_e}{m_p}\right)^{\frac{1}{2}}$$

$$E_R = \frac{m_e e^4}{32\pi^2\varepsilon_0^2\hbar^2}$$

$$a_B = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$$

$$\nu_{min} = \frac{1}{4\pi}\frac{\hbar}{\sqrt{m_e m}}$$

(charge cancels out, depends on masses only)

Trachenko & Brazhkin, Science Adv. 2020

In (0.5-3) agreement with experiments.
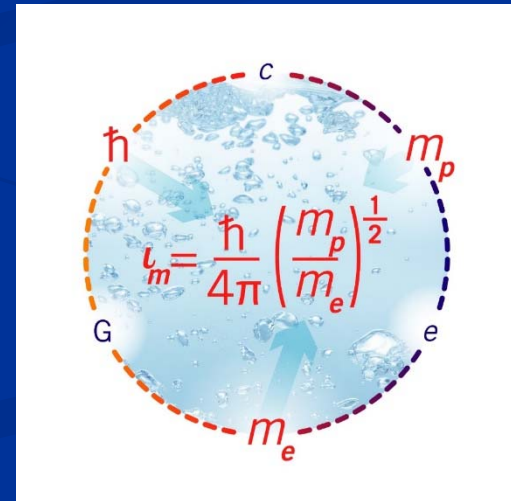
Minimal quantum viscosity: how small can ν get?

$$\nu_{\min} = \frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m}}$$

"Fundamental kinematic viscosity" (for H): $\nu_{\min} = \frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m_p}} = 10^{-7} \, \text{m}^2/\text{s}$

Introduce "elementary viscosity" $\iota = \eta a^3 = \nu m = \frac{\hbar}{4\pi} \sqrt{\frac{m}{m_e}}$

ι has absolute minimum

$$\iota_{\text{m}} = \frac{\hbar}{4\pi} \sqrt{\frac{m_p}{m_e}} \approx \hbar$$

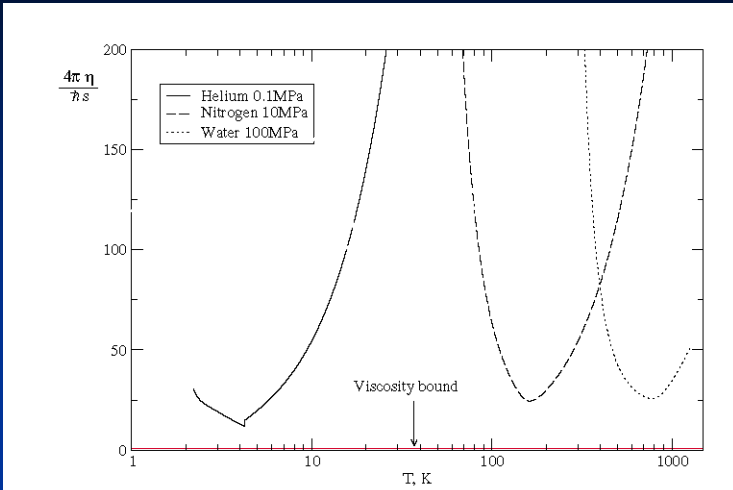Trachenko & Brazhkin, Science Adv. 2020

FIG. 2 (color online). The viscosity-entropy ratio for some common substances: helium, nitrogen and water. The ratio is always substantially larger than its value in theories with gravity duals, represented by the horizontal line marked "viscosity bound."
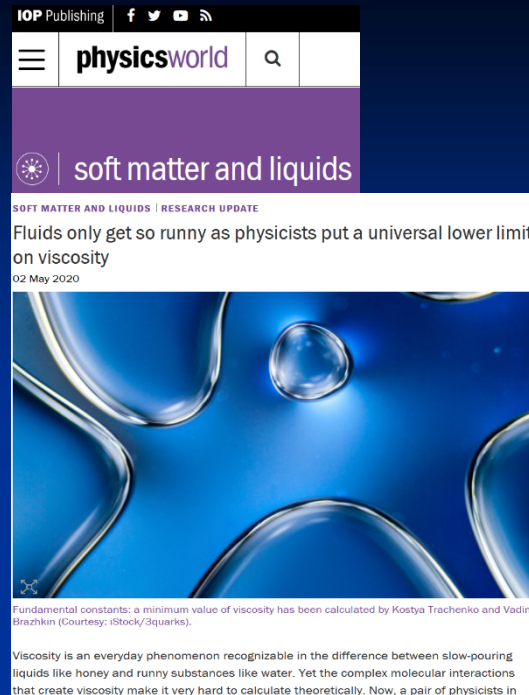
$\eta / s = \hbar/(4\pi)$

why the difference with real liquids?

We have
$$\nu_{min} = \frac{1}{4\pi}\frac{\hbar}{\sqrt{m_e m}}$$

There is an extra factor not present in KSS

$$\frac{1}{s_m}\sqrt{\frac{m}{m_e}} \approx 16\text{-}21$$

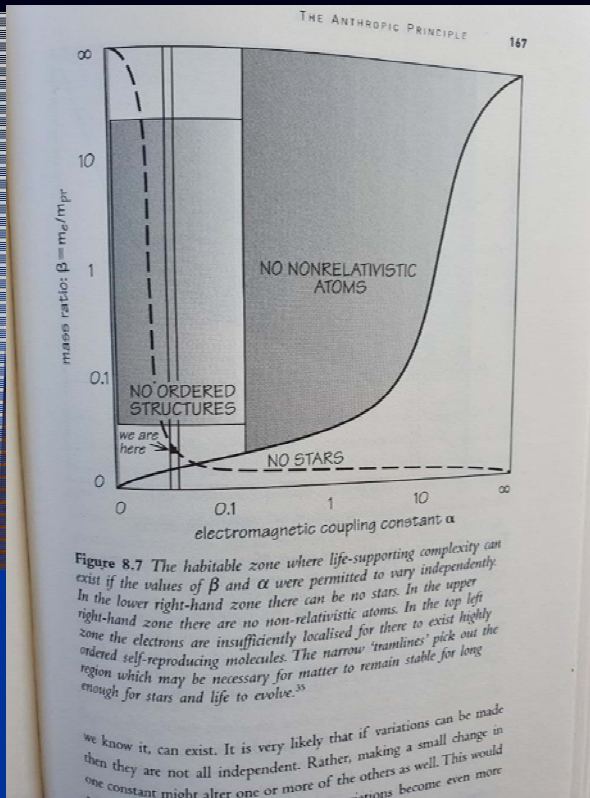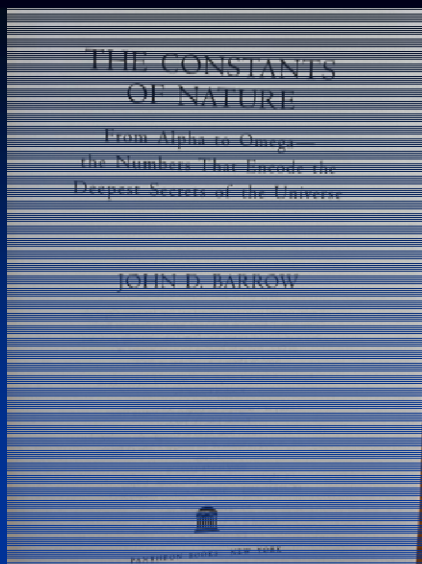Details in Trachenko & Brazhkin, Science Adv. 2020

Purcell question in the famous paper "Life at low Reynolds number" 1977:

"The viscosities have a big range *but they stop at the same place.* I don't understand that" Says that Weisskopf explained it to him.

$$\nu_{\min} = \frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m}}$$

Eddington, Dirac, Gamow etc.

Antropic principle (lately by S Weinberg)

Multiverses

Barrow: "fundamental constants are bio-friendly"

$$\nu_{\min} = \frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m}}$$

$\hbar$ sets viscosity of water and biological processes

Upper bound for the speed of sound

Two dimensionless fundamental constants:

fine-structure constant $\alpha = \dfrac{1}{137}$

proton-to-electron mass ratio $\dfrac{m_p}{m_e} = 1836$

The upper bound for the speed of sound in condensed matter (solids and liquids):
Start with $K = f^* E/a^3$ or from $\omega_D = v k_D$

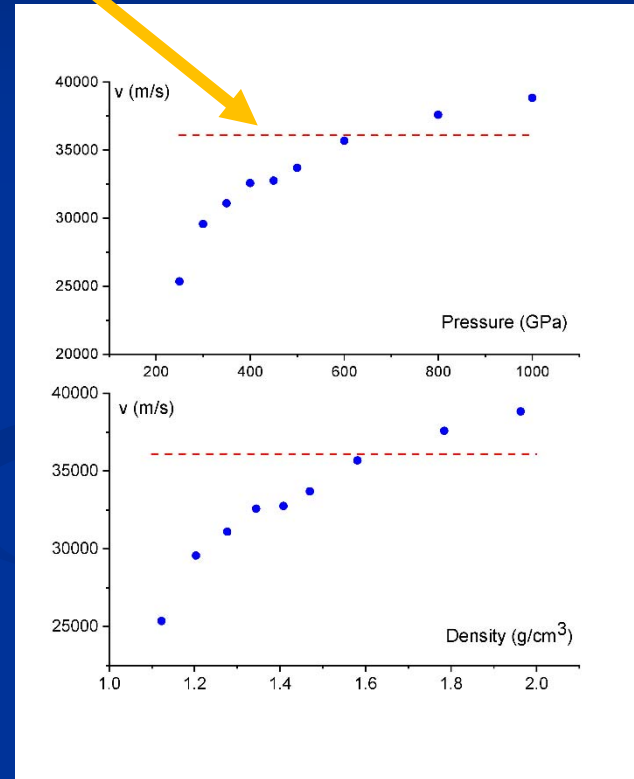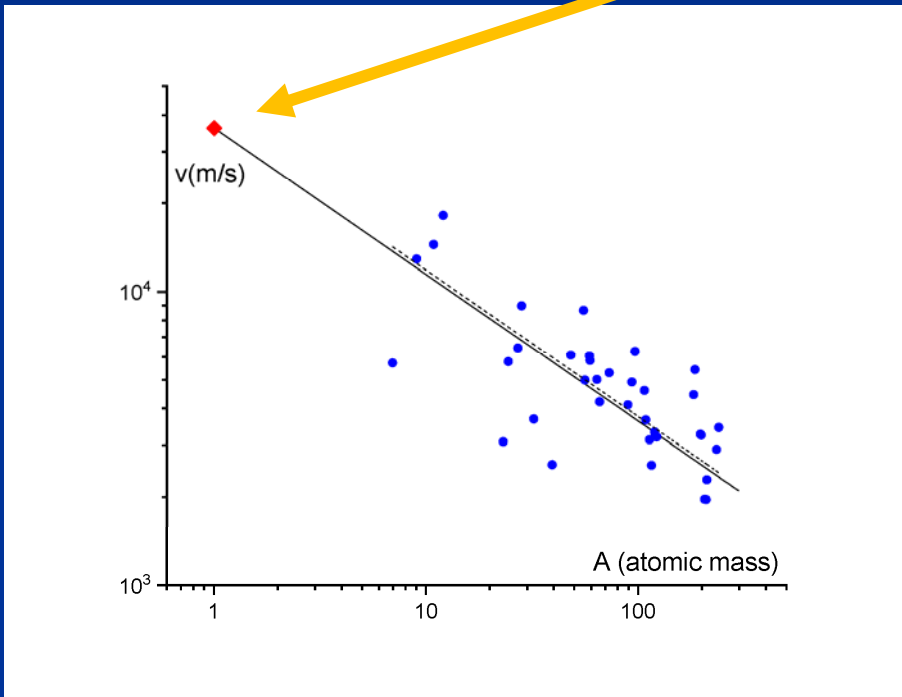$$E = E_R \Rightarrow v = \alpha \left(\frac{m_e}{2m}\right)^{\frac{1}{2}} c$$

$$\frac{v_u}{c} = \alpha \left(\frac{m_e}{2m_p}\right)^{\frac{1}{2}}$$

Trachenko, Monserrat, Pickard&Brazhkin, Science Adv 2020

Upper bound for the speed of sound

$$v = \alpha \left(\frac{m_e}{2m}\right)^{\frac{1}{2}} c$$

$m = m_p \implies v_u = 36{,}100 \text{ m/s}$
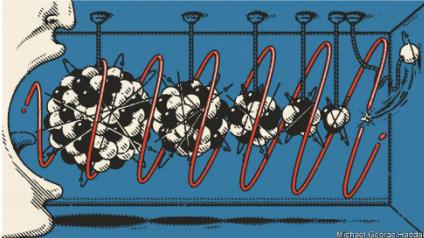

Trachenko, Monserrat, Pickard&Brazhkin, Science Adv 2020

Upper bound for the speed of sound: news like "fast"

The Economist

Science & technology
Oct 17th 2020 edition

Physics

Does sound, like light, have a maximum speed?

Quite probably, yes

WHEN IT COMES to cosmic speed limits, light gets all the attention. Its velocity in a vacuum, a tad below 300m metres per second, is an absolute upper bound on how fast anything in the universe can travel. This value, called "c" by physicists, is somehow baked into the fabric of reality as what is known as a fundamental constant.

The speed of sound, by contrast, has no obvious upper limit of its own. Find the

Scientists calculate fastest possible speed of sound is 22 miles per second — but it is only possible under conditions found in Jupiter's metallic hydrogen core

- Sound waves are travelling disturbances that move energy through mediums
- The denser the transport medium is, the faster the wave can pass through it
- Researchers set out to calculate the medium that would allow the fastest wave
- The findings could reveal other limits such as on viscosity and heat conductivity

By IAN RANDALL FOR MAILONLINE
PUBLISHED: 19:00, 9 October 2020 | UPDATED: 19:00, 9 October 2020

The fastest possible speed of sound — 22.4 miles per second, nearly twice its speed in air at the Earth's surface — has been calculated for the first time.

British researchers found that such speeds are only possible, however, when passing though metallic hydrogen, as found in the core of the **giant planet Jupiter**.

NewScientist

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Physicists have discovered the ultimate speed limit of sound

PHYSICS 9 October 2020
By Leah Crane

The universal speed limit of any kind of wave – be it electromagnetic or gravitational – travelling through a vacuum has been known since Albert Einstein developed his theory of special relativity in 1905. But the maximum speed of sound moving through a solid or a liquid has just been calculated for the first time. It is about 36 kilometres per second, more than 8000 times lower than the speed of light.

Sound waves travel through interactions between atoms, which limits their speed

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mathematical physics

MATHEMATICAL PHYSICS | RESEARCH UPDATE

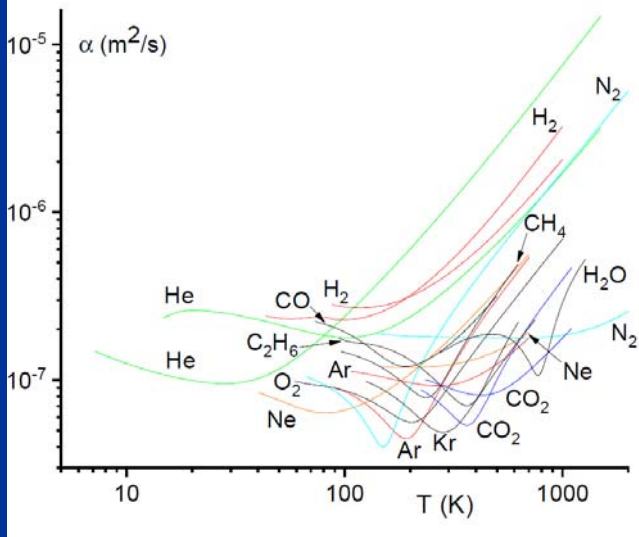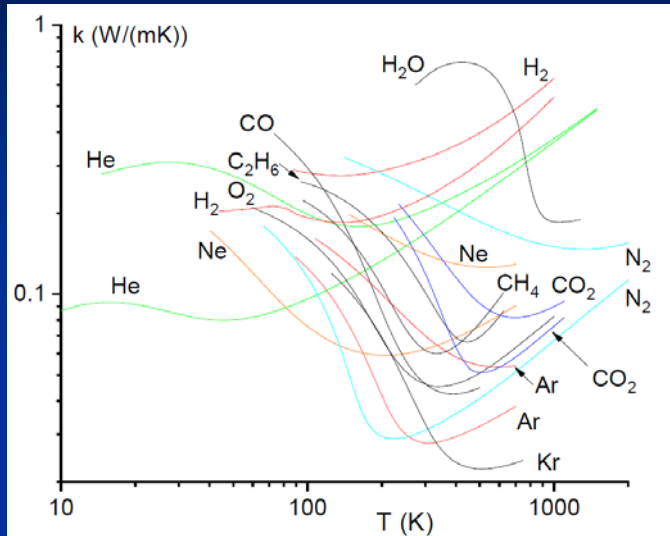## Fundamental constants set upper limit for the speed of sound

09 Oct 2020

Upper limit: a US Navy F/A-18 travelling near the speed of sound in air. The white halo comprises water droplets that have condensed from the air because of the sudden drop in pressure behind the shock cone around the aircraft . (Courtesy: John Gay/US Navy)

The upper limit on the speed of sound in solids and liquids depends on just two dimensionless quantities – the fine structure constant and the proton-to-electron mass ratio. That is the

Minimal thermal diffusivity

$$a_{\mathrm{m}} = vL = \frac{1}{2\pi}\,\omega_{\mathrm{D}}a^2 = \nu_{\min} = \frac{1}{4\pi}\frac{\hbar}{\sqrt{m_e m}}$$

	$\alpha_m^{th} = \nu_m^{th}$	α_m^{exp}	ν_m^{exp}	ν_m/α_m
Ar (20 MPa)	3.4	4.5	5.9	1.3
Ar (100 MPa)	3.4	9.3	7.7	0.8
Ne (50 MPa)	4.8	6.4	4.6	0.7
Ne (300 MPa)	4.8	11.9	6.5	0.6
He (20 MPa)	10.7	9.5	5.2	0.6
He (100 MPa)	10.7	17.9	7.5	0.4
Kr (30 MPa)	2.3	4.9	5.2	1.1
N_2 (10 MPa)	4.1	4.0	6.5	1.6
N_2 (500 MPa)	4.1	17.8	12.7	0.7
H_2 (50 MPa)	15.2	22.8	16.3	0.7
H_2 (100 MPa)	15.2	27.0	19.4	0.7
O_2 (30 MPa)	3.8	5.6	7.4	1.3
H_2O (70 MPa)	5.1	10.7	11.9	1.1
CO_2 (30 MPa)	3.2	5.4	8.0	1.5
CO_2 (90 MPa)	3.2	8.1	9.3	1.2
CH_4 (20 MPa)	5.4	7.9	11.0	1.4
C_2H_6 (20 MPa)	3.9	7.0	12.0	1.7
CO (20 MPa)	4.1	12.0	7.7	0.6

Minimal thermal diffusivity
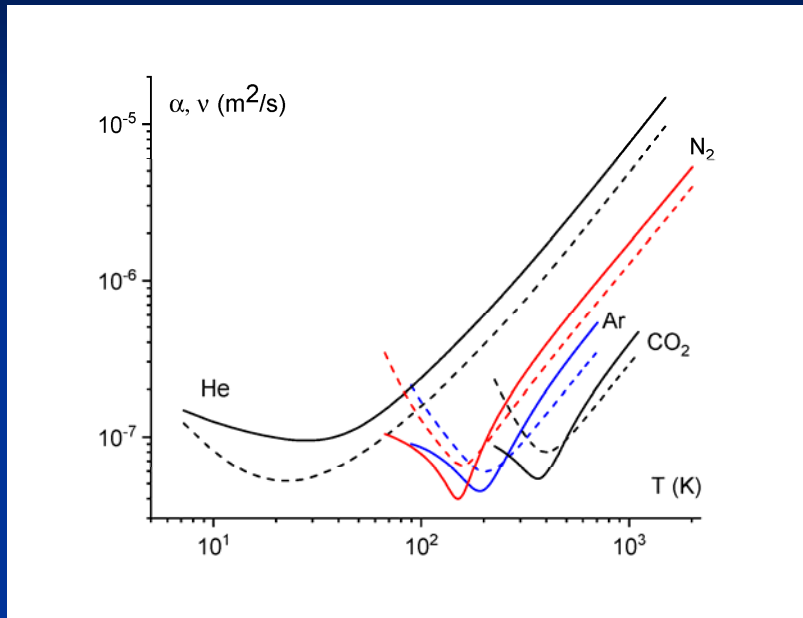
Trachenko, Baggioli, Brazhkin, Behnia   Phys Rev B 2021

Quark-gluon plasma: kinematic viscosity
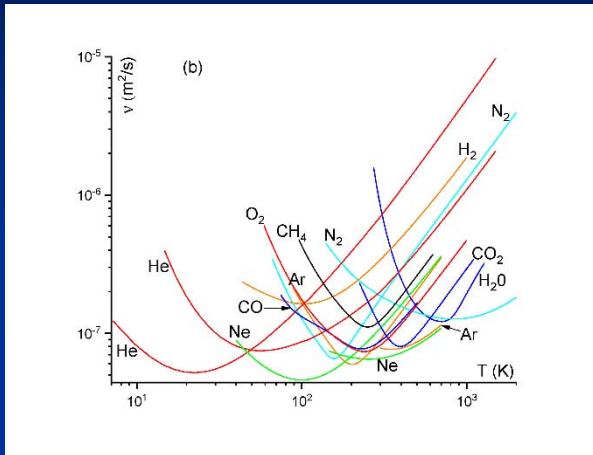
$$\nu_{\text{min}} = \frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m_p}} = 10^{-7} \, \text{m}^2/\text{s}$$

QGP: $\eta \sim 10^{11}$ Pa · s
$\varrho \, (sT) \sim 10^{18}$ kg/m^3
both non-relativistic and
relativistic (using experimental η/s)

$$\nu_{\text{QGP}} = 10^{-7} \, \text{m}^2/\text{s}$$

**(Despite very different forces and
16 orders of magnitude
larger η and ϱ)**

SciPost Physics 2021

Quark-gluon plasma: kinematic viscosity
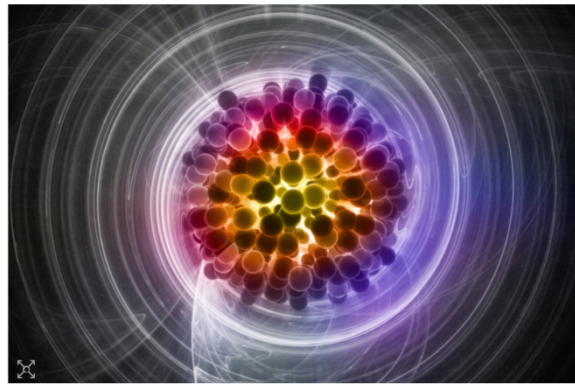

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physicsworld

PARTICLE AND NUCLEAR | RESEARCH UPDATE
Quark–gluon plasma flows like water, calculations suggest
09 Jun 2021

Similar flow despite dramatic differences in energy, temperature, charge and so on?

$$\rho \frac{dv}{dt} = \eta \frac{d^2v}{dx^2}$$

Quark-gluon plasma: kinematic viscosity

$\nu_{QGP} = 10^{-7}\ \text{m}^2/\text{s}$

Despite very different forces/energies and 16 orders of magnitude larger η and ϱ

Why so similar? Don't fully understand yet

1. For liquids, minimal ν_m does not depend on energy, charge or distance:
   $\nu_m \sim E a^2$ but $E = \hbar^2/(2ma^2)$
2. At the minimum at the Frenkel line, $\eta \approx \varrho \nu L \approx \varrho \nu a \approx p/a^2 \sim (pa \sim \hbar) = \hbar/a^3 \Rightarrow$
   $\nu_m = \hbar/m$. If $m = m_p$, $\nu_m = 10^{-7}\ \text{m}^2/\text{s}$
3. $\nu \sim D(\text{gas}) \sim a^2/\tau$. At the Frenkel line crossover, $\tau = \tau_0$ and $\nu_m = a^2/\tau_0$.

If $\tau_0 = \tau_{Pl} = \hbar/(kT)$, $\nu_m = a^2 k T_{QGP}/\hbar$. With $a = 0.5$ fm and $T \sim 10^{12}$ K, $\nu_m = 10^{-7}\ \text{m}^2/\text{s}$

In condensed-matter terms, QGP appears to be close to the dynamical crossover similar to the Frenkel line.
Why useful: QGP is dense, strongly-interacting and non-perturbative just like liquids are
$\Rightarrow$ theoretical problems
Prediction: higher temperature $\Rightarrow$ higher gas-like viscosity.

Thank you

and thanks to my co-authors and collaborators