ELECTRODYNAMICS OF HOT NUCLEAR MATTER AND OTHER CHIRAL MEDIA

Kirill Tuchin

IOWA STATE UNIVERSITY
OF SCIENCE AND TECHNOLOGY

Arizona S.U. Theoretical Physics Colloquium (remote)

April 28, 2021

OUTLINE

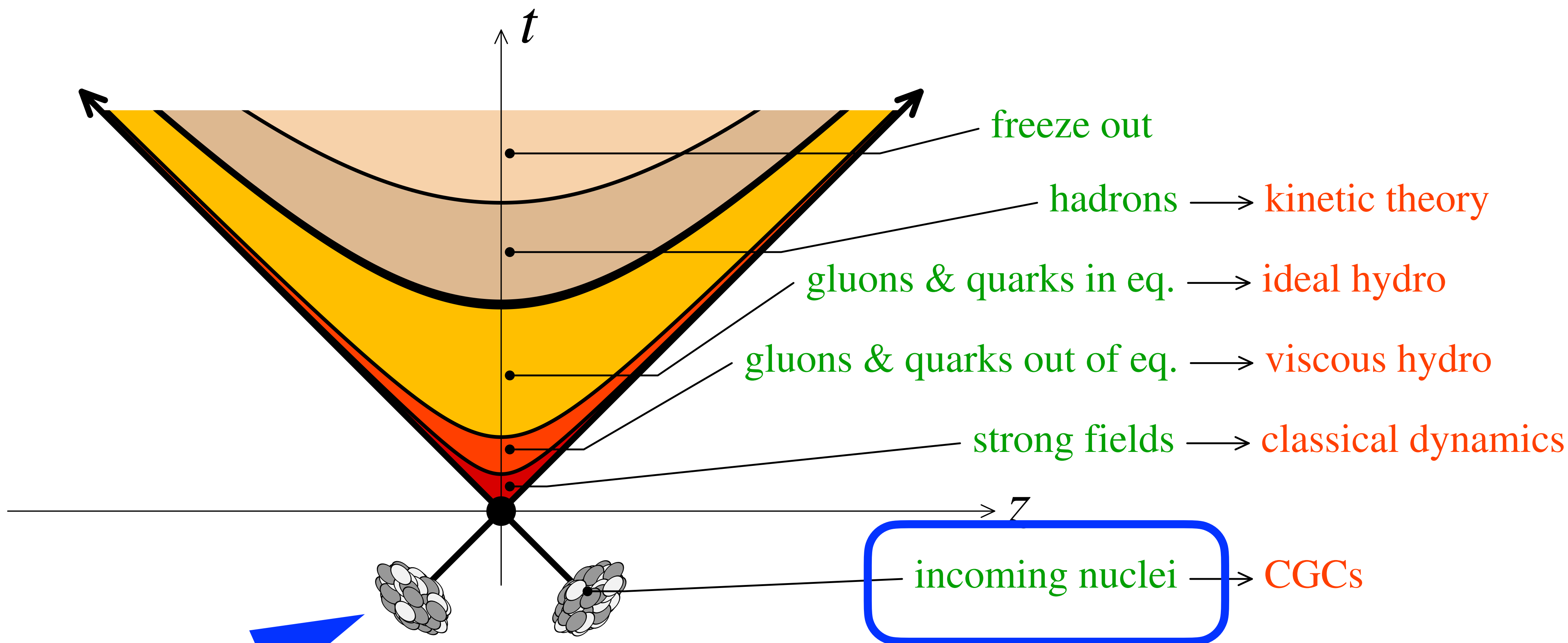
EM fields in relativistic heavy-ion collisions:

- External classical field generated by colliding ions
- Quantum excitations inherent to QGP

◉ Electrodynamics in the presence of the chiral anomaly

Generation of intense EM field in relativistic heavy-ion collisions

MACROSCOPIC EM FIELD IN HEAVY-ION COLLISIONS



Ions about to collide

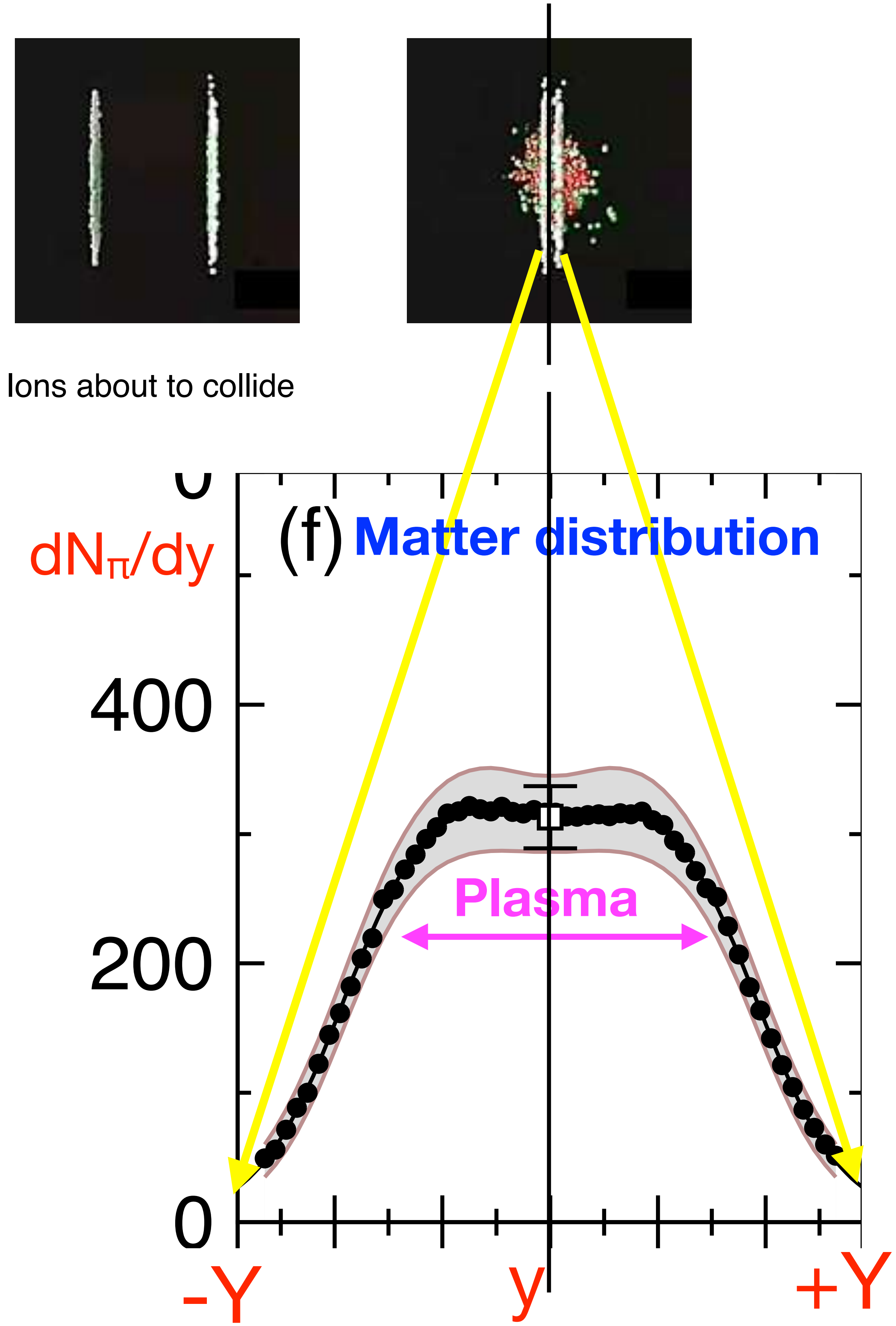
EM field of each ion is a boosted Coulomb field

$$\boldsymbol{B}_1 = \frac{\gamma e v \hat{\boldsymbol{\phi}}}{4\pi} \frac{b}{(b^2 + \gamma^2(vt - z)^2)^{3/2}}$$

$Z_{Au} = 79$, $b \sim R = 7\ fm$, $\gamma = 100 \Rightarrow$ $eB = (200\ MeV)^2 \approx m_\pi^2$

$$B \sim 10^{18}\mathrm{G}$$

EM FIELD IN QUARK-GLUON PLASMA



It is too "expensive" to transfer net baryon and electric charge to the central plateau region.

$$\frac{dN_{\rm val}}{dy} \sim e^{-\Delta_R(Y-y)} + e^{-\Delta_R(Y+y)}$$

$$\Delta_R \approx 0.47$$

Number of valence quarks (μ_B) at y=0 decreases with energy: "baryon stopping".

⇒ The contribution of the "stopped" baryons is exponentially (in y) small

EM field = sum of two boosted Coulomb fields of each ion

EM FIELD IN QUARK-GLUON PLASMA

EM field in plasma: need to solve Magneto-Hydrodynamics.

EM is weakly coupled to the plasma ⇒ 0th approximation:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J}, \qquad \nabla \cdot \boldsymbol{E} = \rho = 0.$$

$$\boldsymbol{J} = \sigma\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right) + \boldsymbol{J}_{\mathrm{ext}},$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \frac{1}{\sigma}\left(\nabla^2 \boldsymbol{B} - \frac{\partial^2 \boldsymbol{B}}{\partial t^2} + \nabla \times \boldsymbol{J}_{\mathrm{ext}}\right),$$

$$\frac{\partial \boldsymbol{E}}{\partial t} + \frac{\partial \boldsymbol{v}}{\partial t} \times \boldsymbol{B} = \boldsymbol{v} \times (\nabla \times \boldsymbol{E}) + \frac{1}{\sigma}\left(\nabla^2 \boldsymbol{E} - \frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \frac{\boldsymbol{J}_{\mathrm{ext}}}{\partial t}\right),$$

Lattice calculations for T~Tc: σ=5.8 MeV, very small compared to the typical QCD scale of 200 MeV.

So let's neglect the medium effect on the EM field altogether… ?

EM FIELD IN QUARK-GLUON PLASMA

Is electrical conductivity indeed that small?

Consider a single valence charge e moving with velocity v:

$$\nabla \cdot \boldsymbol{B} = 0\,, \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}\,,$$

$$\nabla \cdot \boldsymbol{D} = e\delta(z - vt)\delta(\boldsymbol{b})\,, \qquad \nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \sigma \boldsymbol{E} + ev\hat{\boldsymbol{z}}\delta(z - vt)\delta(\boldsymbol{b})$$

In momentum space:

$$\boldsymbol{H}_{\omega\boldsymbol{k}} = -2\pi i e v \frac{\boldsymbol{k} \times \hat{\boldsymbol{z}}}{\omega^2\tilde{\epsilon}\mu - \boldsymbol{k}^2}\delta(\omega - k_z v)\,, \qquad \boldsymbol{E}_{\omega\boldsymbol{k}} = -2\pi i e \frac{\omega\mu v\hat{\boldsymbol{z}} - \boldsymbol{k}/\epsilon}{\omega^2\tilde{\epsilon}\mu - \boldsymbol{k}^2}\delta(\omega - k_z v)$$

where $\tilde{\epsilon} = \epsilon + i\sigma/\omega$

Time dependence of electromagnetic field is determined by singularities in the complex ω-plane with finite imaginary part. Take for simplicity ε=μ=1 (neglect the polarization and magnetization response of QGP).

EM FIELD IN QUARK-GLUON PLASMA

$$\boldsymbol{B}(t,\boldsymbol{r}) = \frac{e}{2\pi\sigma}\hat{\boldsymbol{\phi}}\int_0^\infty \frac{J_1(k_\perp b)k_\perp^2}{\sqrt{1+\frac{4k_\perp^2}{\gamma^2\sigma^2}}}\exp\left\{\frac{1}{2}\sigma\gamma^2 x_-\left(1-\sqrt{1+\frac{4k_\perp^2}{\gamma^2\sigma^2}}\right)\right\}dk_\perp\,, \qquad x_- = t - z/v$$

Relevant parameter $\lambda = \gamma\sigma b$

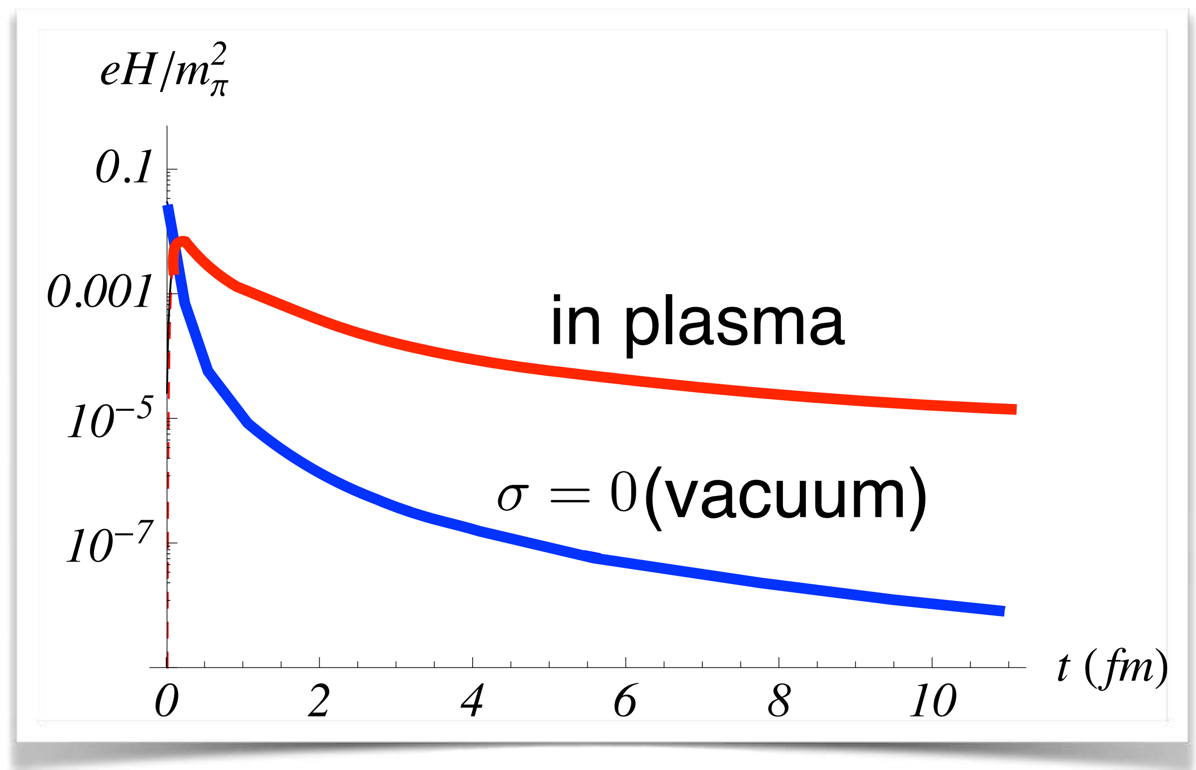
If $\lambda \ll 1$ plasma has no effect on the field: $\boldsymbol{B} = \frac{e\gamma}{4\pi}\frac{vb\hat{\boldsymbol{\phi}}}{(b^2+\gamma^2v^2x_-^2)^{3/2}}$

$$\lambda > 1 \qquad B_\phi = \frac{e}{2\pi}\frac{b\sigma}{4x_-^2}e^{-\frac{b^2\sigma}{4x_-}}$$

In practice: γ=100, σ≈5.8 MeV, b=7 fm: λ=19

Due to the relativistic time-dilation, the characteristic time scale for the medium response is

$1/(\sigma\gamma)$



Intense EM field coexists with the QGP

EM FIELD IN QUARK-GLUON PLASMA

Gürsoy, Kharzeev, Marcus, Rajagopal, Shen, 2018

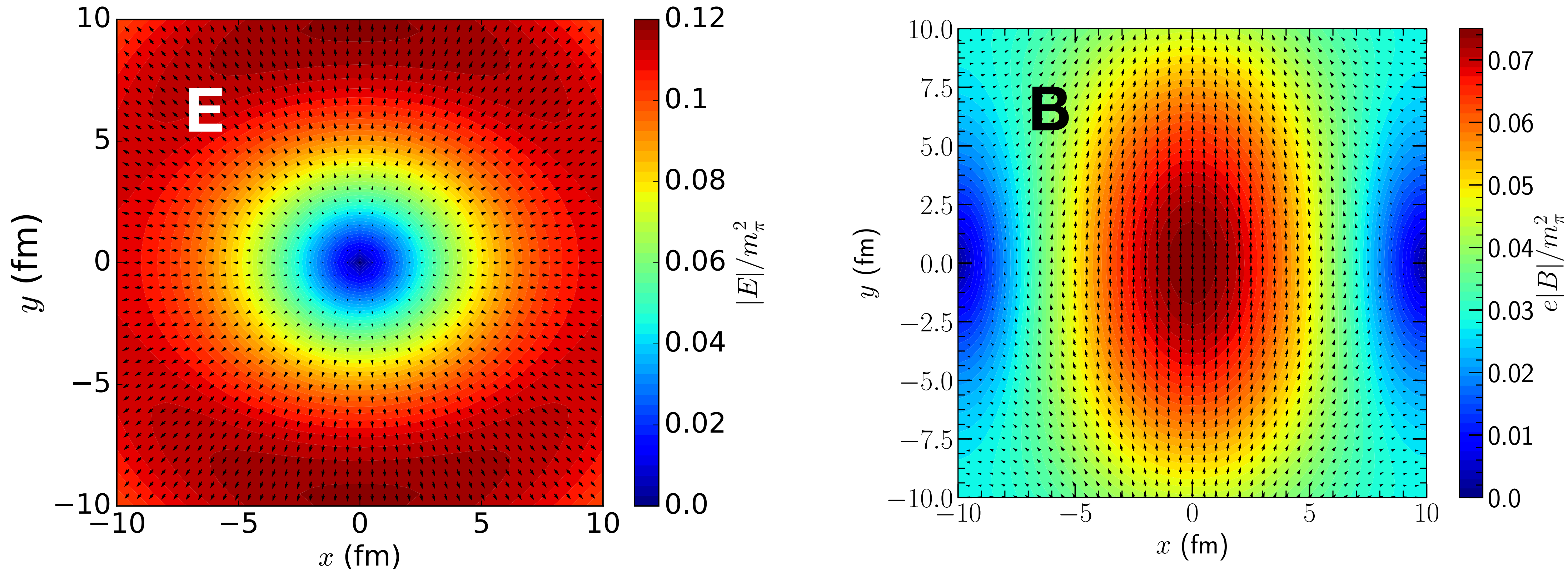


FIG. 2. The electric (left) and magnetic (right) fields in the transverse plane at $z = 0$ in the lab frame at a proper time $\tau = 1$ fm/c after a Pb+Pb collision with 20-30% centrality (corresponding to impact parameters in the range 6.24 fm $< b <$ 9.05 fm) and with a collision energy $\sqrt{s} = 2.76$ ATeV.

INITIAL VALUE PROBLEM FOR EM FIELD IN QGP

$$-\nabla^2 \boldsymbol{A}_2 + \partial_t^2 \boldsymbol{A}_2 + \sigma\partial_t \boldsymbol{A}_2 - \sigma \boldsymbol{u} \times (\boldsymbol{\nabla} \times \boldsymbol{A}_2) = \boldsymbol{j}\,;$$

Stewart, KT (2015)

Matching conditions:

$$A_2^\mu(\boldsymbol{r}, t_0) = A_1^\mu(\boldsymbol{r}, t_0) \equiv \mathcal{A}^\mu(\boldsymbol{r})\,,$$

$$\partial_t A_2^\mu(\boldsymbol{r}, t)\big|_{t=t_0} = \partial_t A_1^\mu(\boldsymbol{r}, t)\big|_{t=t_0} \equiv \mathcal{V}^\mu(\boldsymbol{r})$$

Solution:

$$\begin{aligned} A_2^\mu(\boldsymbol{r}, t) = & \int_\tau^{t_0+} dt' \int d^3r' j^\mu(\boldsymbol{r}', t') G_2(\boldsymbol{r}, t|\boldsymbol{r}', t') \\ & + \int d^3r' \left[\sigma\mathcal{A}^\mu(\boldsymbol{r}') + \mathcal{V}^\mu(\boldsymbol{r}')\right] G_2(\boldsymbol{r}, t|\boldsymbol{r}' \\ & - \int d^3r' \mathcal{A}^\mu(\boldsymbol{r}')\partial_{t'} G_2(\boldsymbol{r}, t|\boldsymbol{r}', t')\big|_{t'=t_0} \end{aligned}$$

$$\begin{aligned} G_2(\boldsymbol{r}, t|\boldsymbol{r}', t') = & \frac{1}{4\pi} e^{-\frac{1}{2}\sigma(t-t')} \frac{\delta(t - t' - R)}{R} \theta(t - t') \\ & + \frac{1}{4\pi} e^{-\frac{1}{2}\sigma(t-t')} \frac{\sigma/2}{\sqrt{(t-t')^2 - R^2}} I_1\left(\frac{\sigma}{2}\sqrt{(t-t')^2 - R^2}\right) \theta(t - t' - R)\theta(t - t') \end{aligned}$$

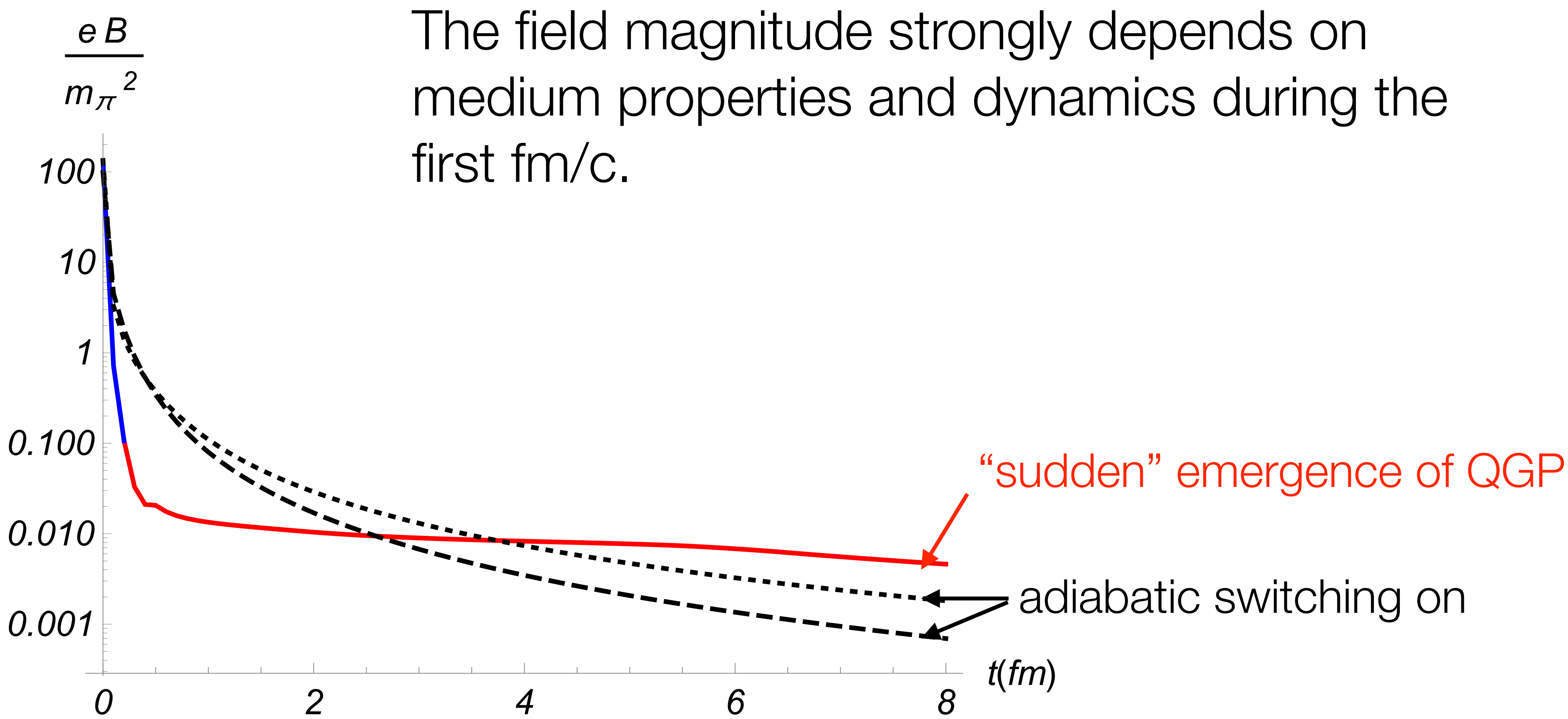
the original pulse

wake produced by the induced currents

proportional to σ²
→ small

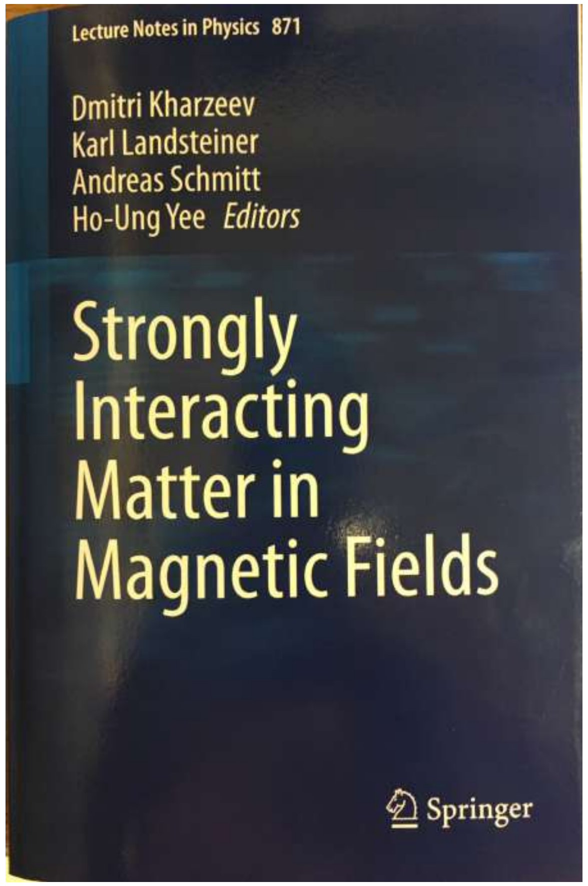
EM FIELD AT LATER TIME

The field magnitude strongly depends on medium properties and dynamics during the first fm/c.

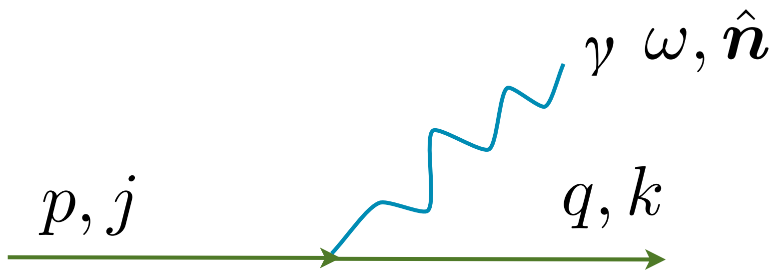


Stewart, KT (to appear)

Effects induced by EM field in Quark-Gluon Plasma



SYNCHROTRON RADIATION



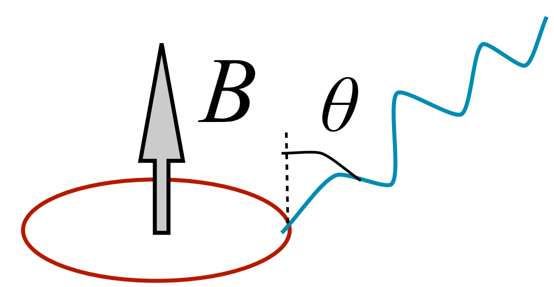
$$\varepsilon_j = \sqrt{m^2 + p^2 + 2je_f B}\,, \quad \varepsilon_k = \sqrt{m^2 + q^2 + 2ke_f B}$$

j,k are the quantum numbers of the fermion Landau orbits.

Spacing between the Landau levels ~ ***eB/ε***, while their thermal width ~ ***T***. When ***eB/ε≳T*** it is essential to account for quantization of fermion spectra.

Magnetic field does no work, thus energy is conserved. Magnetic Lorentz force has no component along the *B*-direction:

$$\varepsilon_j = \omega + \varepsilon_k\,, \quad p = q + \omega\cos\theta$$



Angular distribution of the power spectrum:

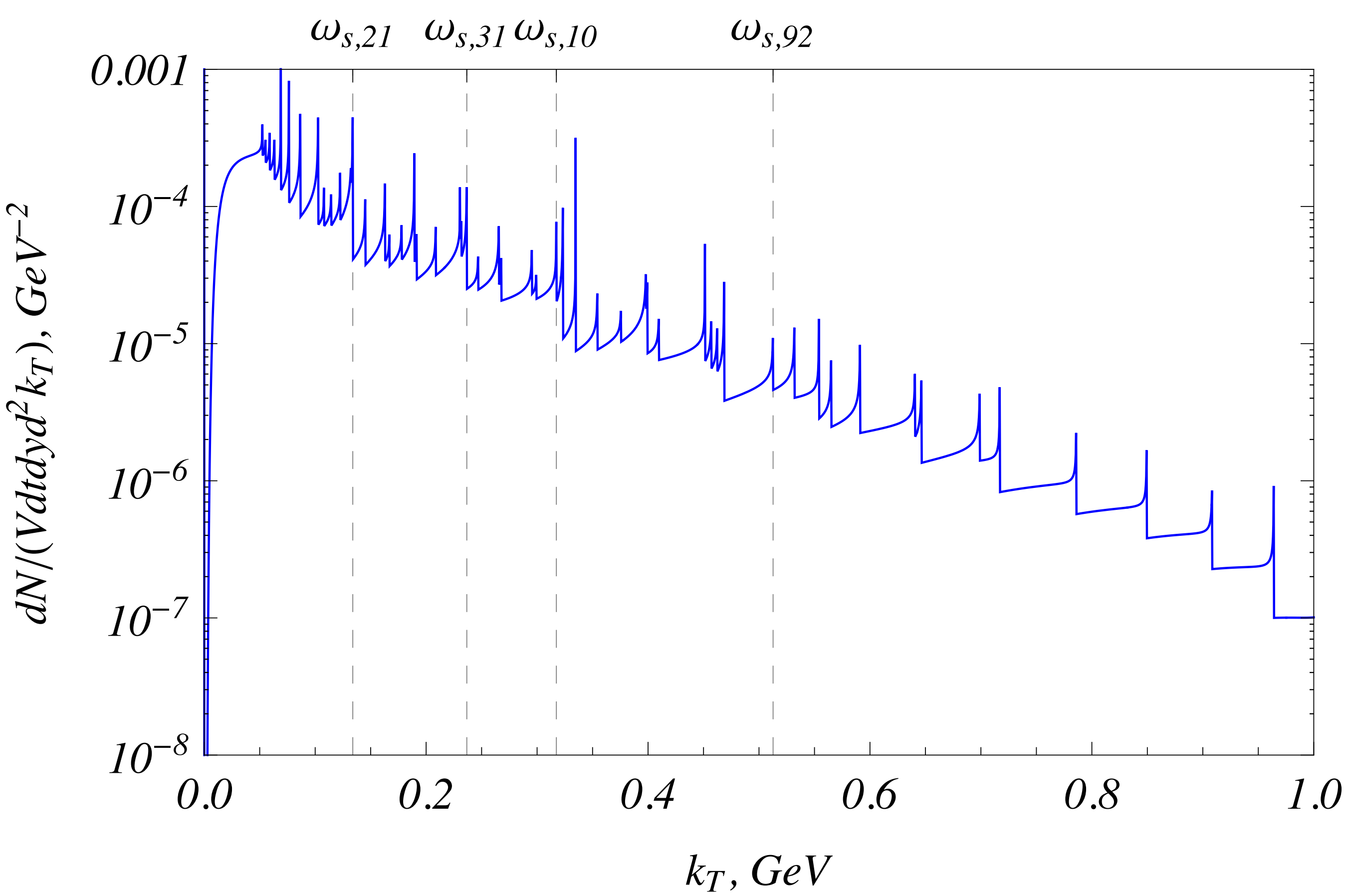
$$\frac{dI^j}{d\omega d\Omega} = \sum_f \frac{z_f^2 \alpha}{\pi} \omega^2 \sum_{k=0}^{j} \Gamma_{jk} \left\{ |\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2 \right\} \delta(\omega - \varepsilon_j + \varepsilon_k)$$

Matrix elements are well-known functions of Laguerre polynomials.

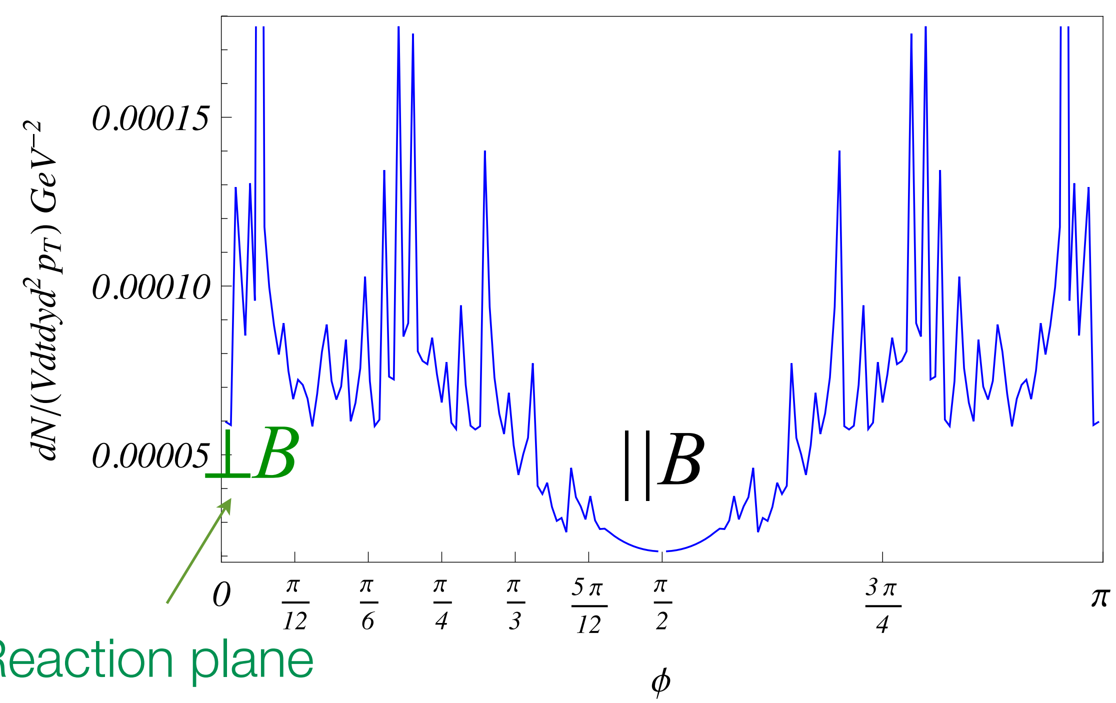
Sokolov, Ternov (1968) and others

SYNCHROTRON RADIATION

Contribution to the total photon spectrum



Azimuthal asymmetry of photons in magnetic field

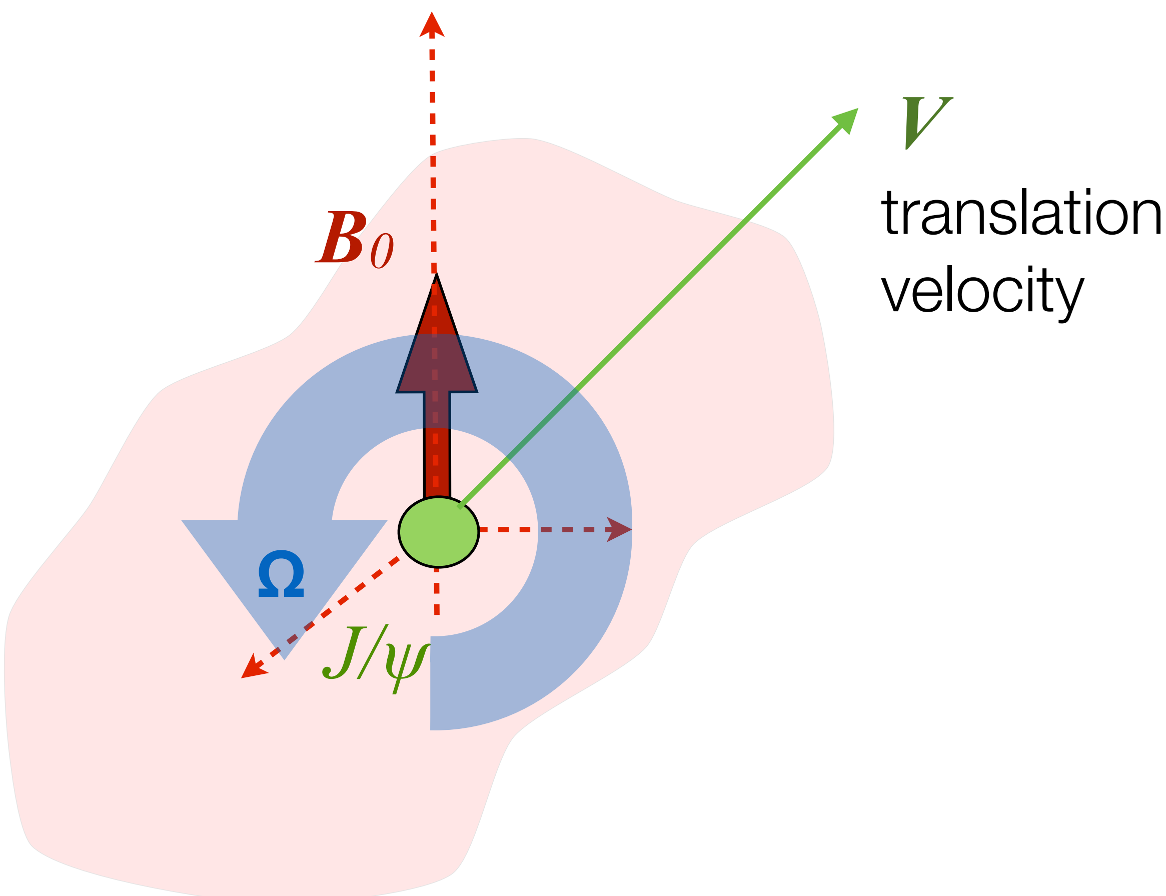


$$\frac{dN}{d\phi} = C\left(1 + \frac{8}{7}\cos(2\phi) + \frac{1}{5}\cos(4\phi) + \ldots\right)$$

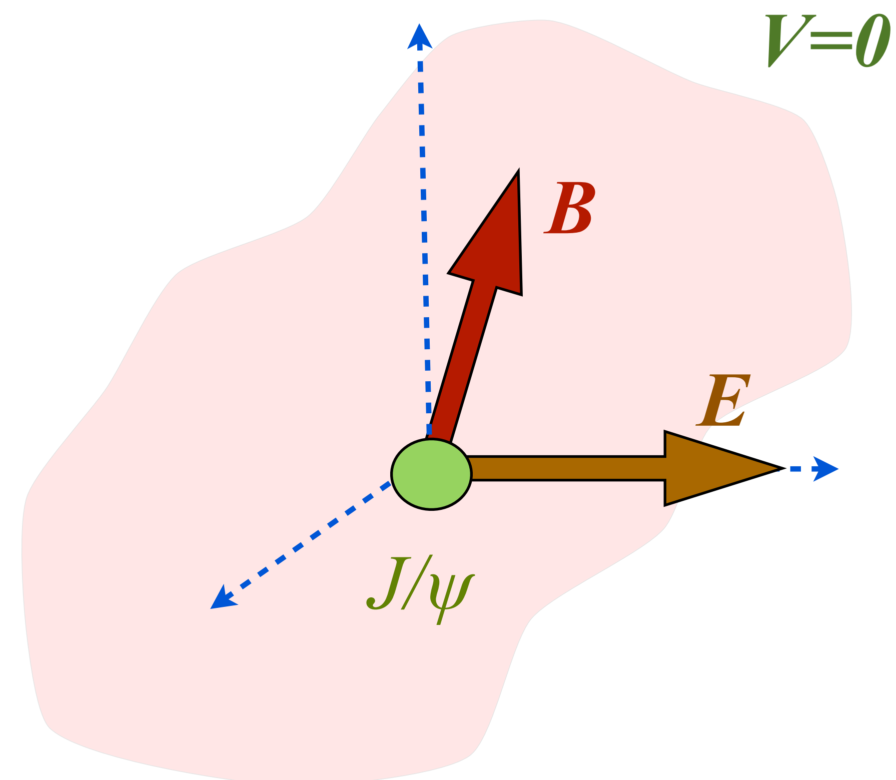
$2v_2$

γ-ray bursts by collapsing stars

HADRON DISSOCIATION



Lab frame: center-of-mass
frame of a heavy-ion collision



hadron rest frame

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

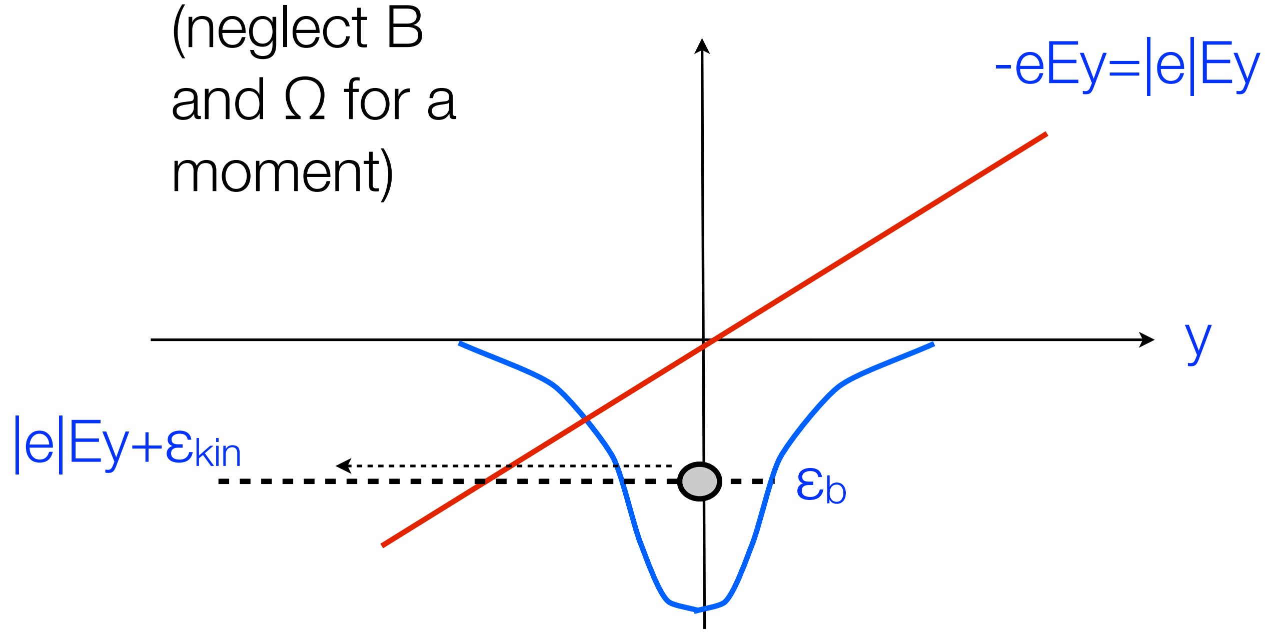
$$ds^2 = [c^2 - \Omega^2(x^2+y^2)]dt^2 - dx^2 - dy^2 - dz^2$$
$$+2\Omega y dx dt - 2\Omega x dy dt\,.$$

Electric field breaks the bound state

QUALITATIVE PICTURE OF DISSOCIATION

J/ψ rest frame. There is finite quantum probability for the anti-quark (e<0) to tunnel through the potential barrier and go to y→-∞.

(neglect B and Ω for a moment)



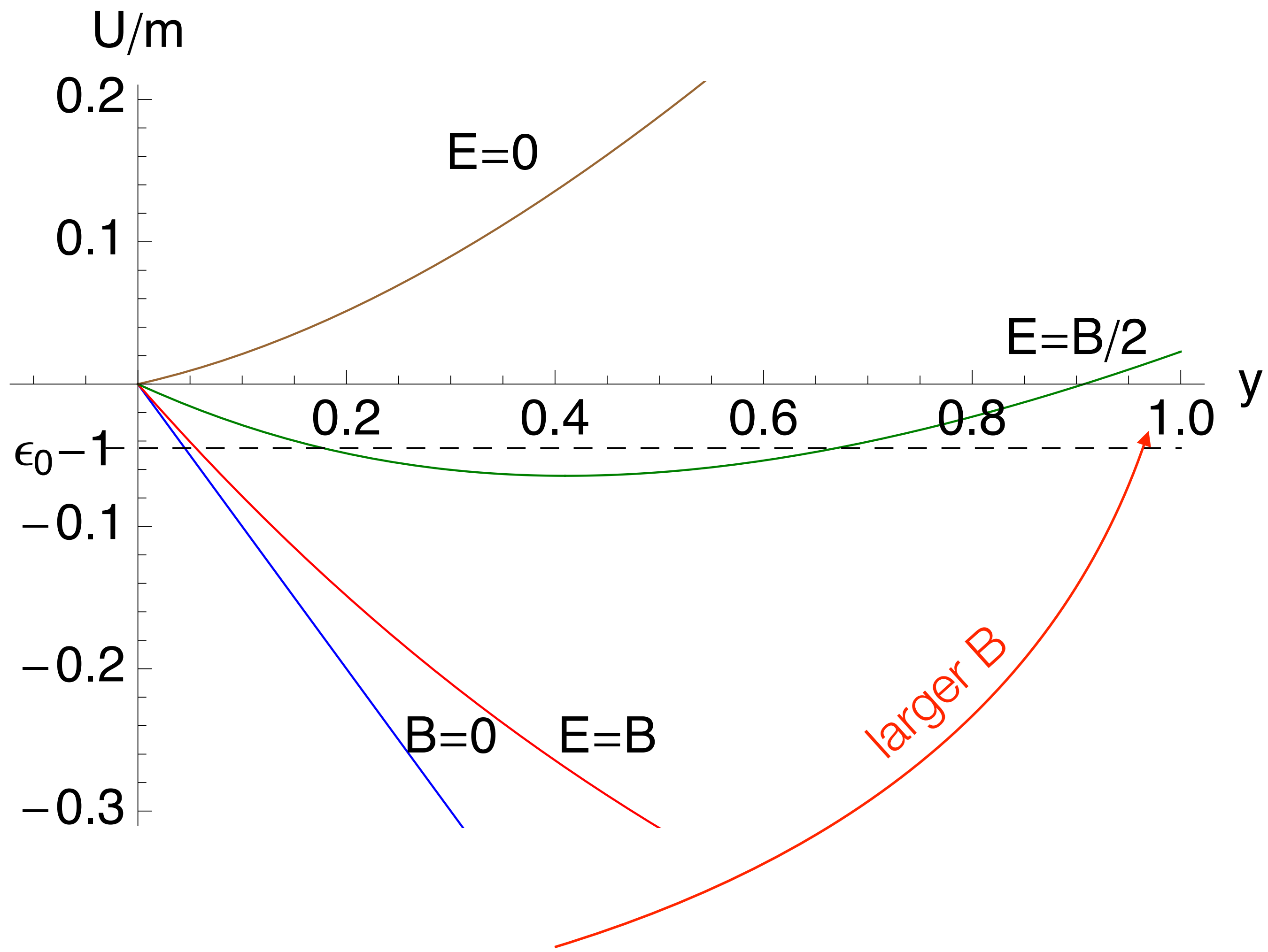
Dissociation rate can be calculated in the WKB approximation as a tunneling rate of quark thru the potential barrier.

$$w = \exp\left\{-\frac{2}{3}\frac{(2\varepsilon_b m)^{3/2}}{meE}\right\}$$

Keldysh (1965)

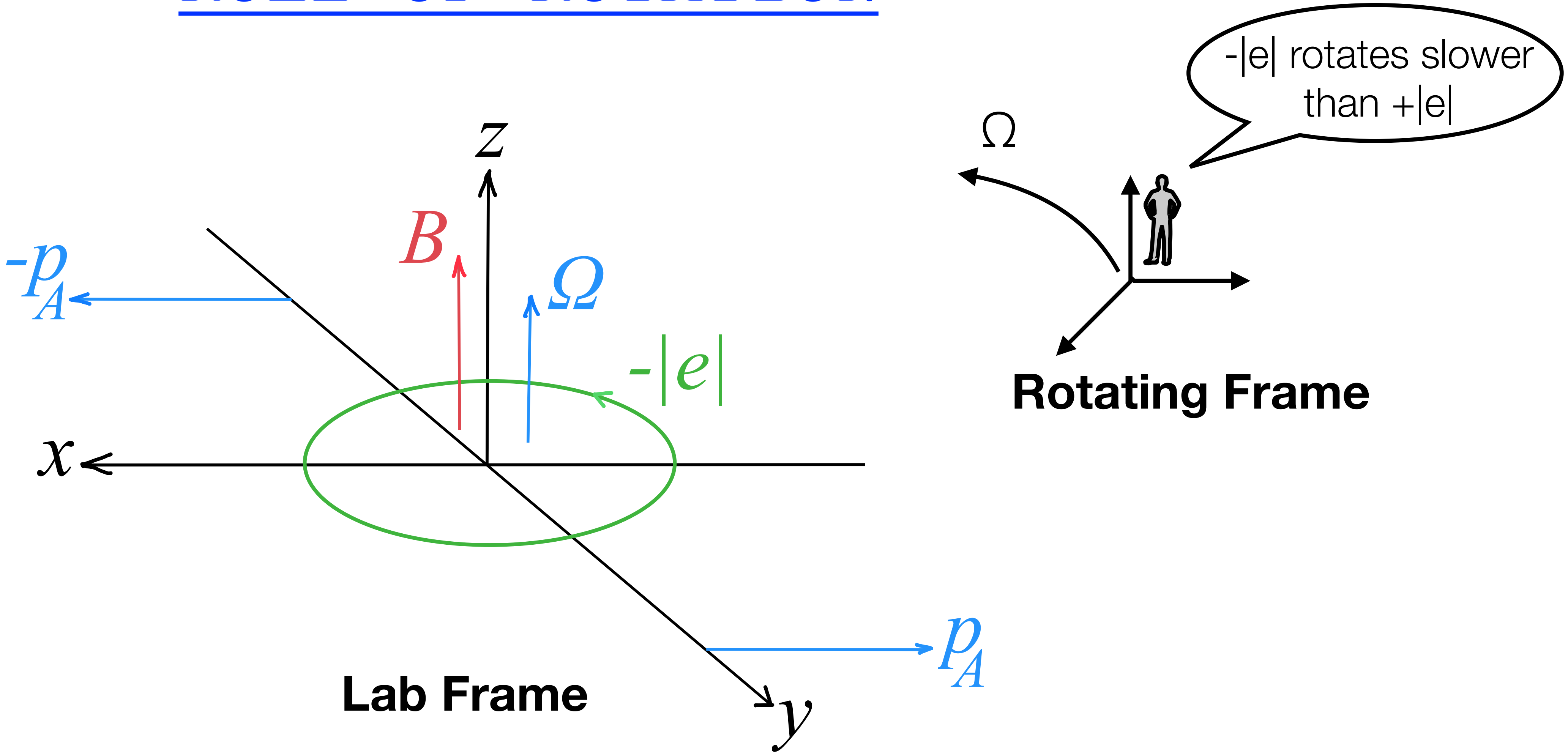
ROLE OF MAGNETIC FIELD

$$\varepsilon_0 = \sqrt{m^2 + (\boldsymbol{p} - e\boldsymbol{A})^2} + e\varphi = \sqrt{m^2 + (p_x + eBy)^2 + p_y^2 + p_z^2} - eEy$$



The larger B, the smaller is the dissociation probability

ROLE OF ROTATION



The effective synchrotron frequency of the negative charge appears to be smaller than the effective synchrotron frequency of the positive one.

The magnetic field decreases the charge of escaping $\Rightarrow$

The dissociation probability is larger for negative charges

IMAGINARY TIME METHOD

Imaginary Time Method: the quasi-classical transition probability is V.S. Popov et al.

$$w = \exp(-2\mathrm{Im}W) \qquad\qquad W = \int_{t_0}^{0} (L + \varepsilon_0)dt - \boldsymbol{p}\cdot\boldsymbol{r}|_{t=0}$$

where the action is computed along the extremal classically forbidden trajectory.

Assumptions: (i) motion of the CoM is negligible in the hadron rest frame (e.g. the hadron is made up of a heavy and a light quarks); (ii) The binding potential is short range, (iii) motion of the light quark is non-relativistic; (iv) B and Ω are constant.

Extremal trajectory:

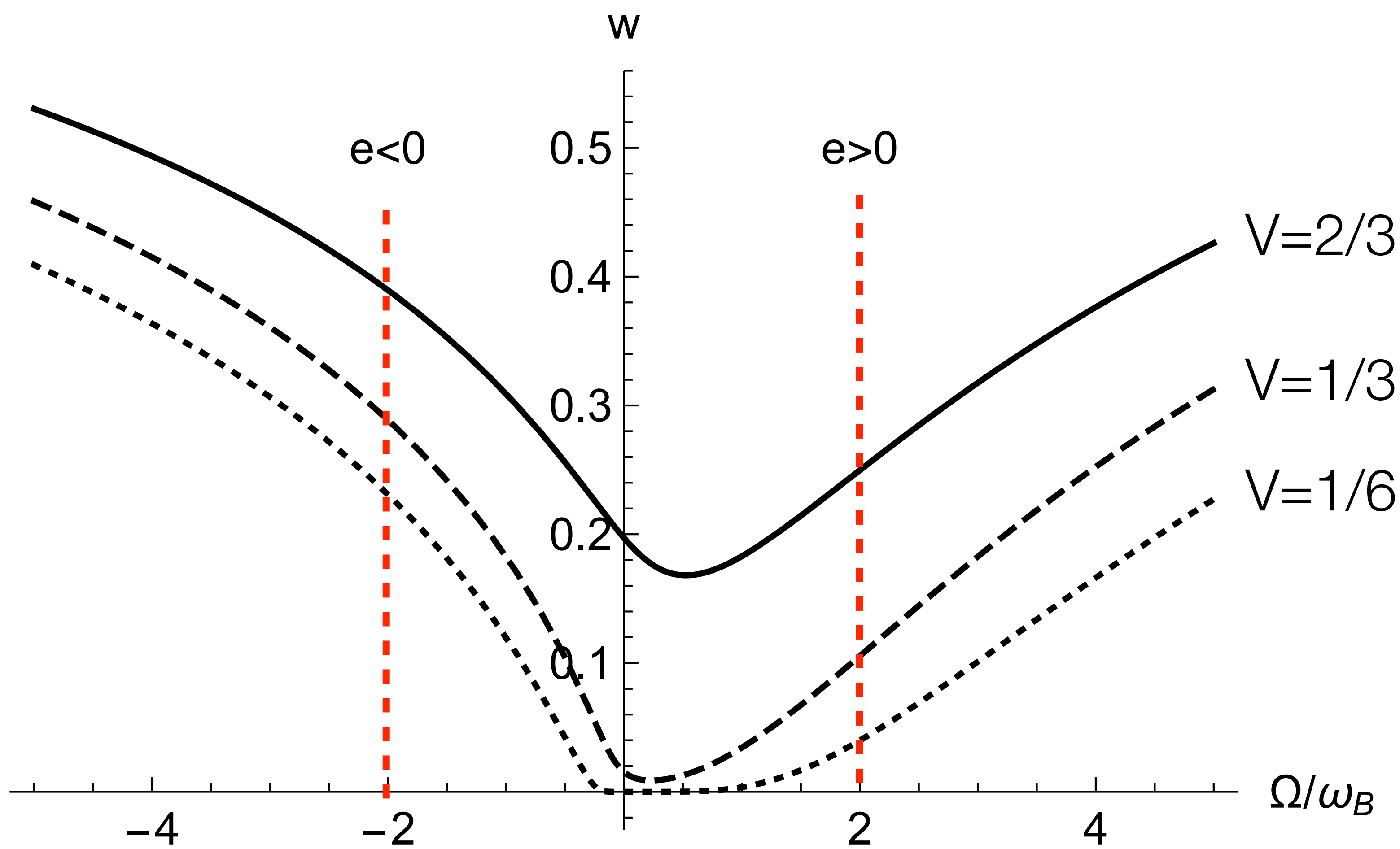
$$x(\tau) = \frac{i\omega_E}{\Omega^2 \sinh[(\omega_+ - \omega_-)\tau_0]}\{\sinh(\omega_-\tau_0)\sinh(\omega_+\tau) - \sinh(\omega_+\tau_0)\sinh(\omega_-\tau)\}\ ,$$

$$y(\tau) = \frac{\omega_E}{\Omega^2}\left\{-\frac{\sinh(\omega_-\tau_0)\cosh(\omega_+\tau)}{\sinh[(\omega_+ - \omega_-)\tau_0]} + \frac{\sinh(\omega_+\tau_0)\cosh(\omega_-\tau)}{\sinh[(\omega_+ - \omega_-)\tau_0]} - 1\right\}\ ,$$

$$\gamma^2 = \frac{\omega_B^2}{\Omega^4\sinh^2[(\omega_+ - \omega_-)\tau_0]}\{2\omega_+\omega_-\sinh(\omega_-\tau_0)\sinh(\omega_+\tau_0)\cosh[(\omega_+ - \omega_-)\tau_0]$$
$$-\omega_+^2\sinh^2(\omega_-\tau_0) - \omega_-^2\sinh^2(\omega_+\tau_0)\}\ ,$$

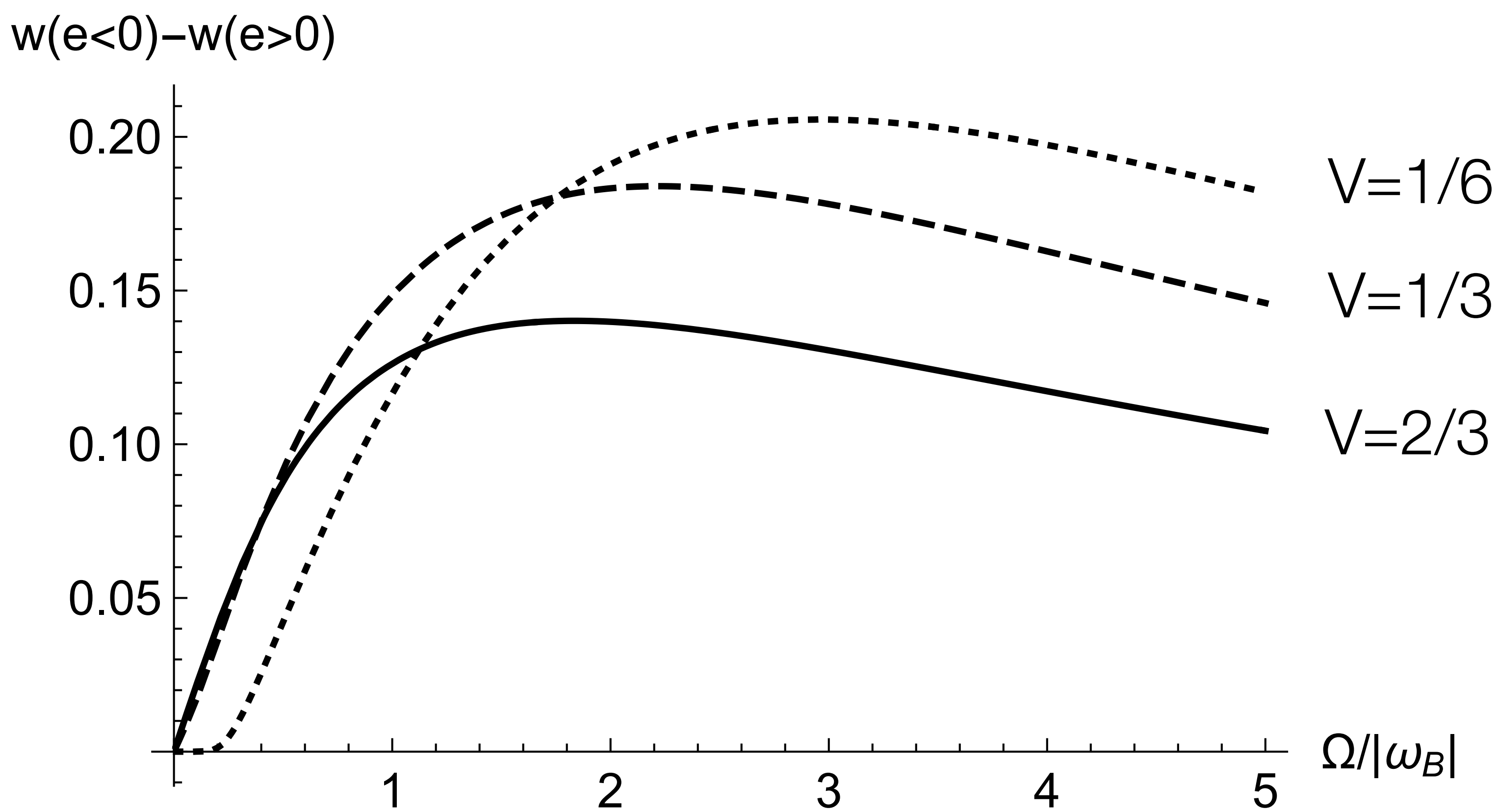
$$\omega_E = \frac{eE}{m}\,, \qquad \omega_B = \frac{eB}{m} \qquad \omega_\pm = \Omega + \frac{\omega_B}{2} \pm \sqrt{\Omega\omega_B + \frac{\omega_B^2}{4}} \qquad \gamma = \sqrt{\frac{2\varepsilon_b}{m}}\frac{B}{E}$$

HADRON DISSOCIATION



- Dissociation probability increases with hadron velocity V
- The centrifugal force increases the dissociation probability.
- At the same B and Ω the negative charge has larger probability to run away.

CHARGE ASYMMETRY



- $w(D^-) > w(D^+)$, i.e. there are more D^+ than D^- in the final spectrum.

There are many other effects of B on QGP

Chiral anomaly and Chiral Magnetic Effect

CHIRAL ANOMALY

Chiral symmetry of nuclear matter

$$U_L(N_f) \times U_R(N_f) \simeq \boxed{SU_L(N_f) \times SU_R(N_f)} \times \boxed{U_B(1)} \times \boxed{U_A(1)}$$

Broken spontaneously

Baryon symmetry (exact)

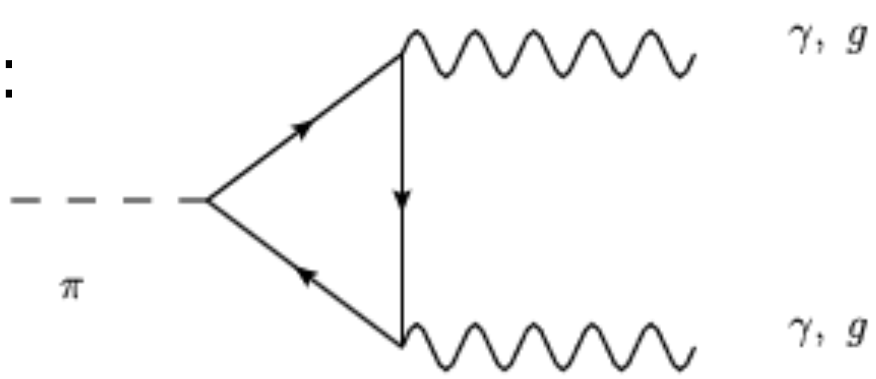
Axial symmetry (broken by anomaly)

Axial symmetry $\psi \to e^{i\gamma_5 \theta}\psi$ broken by quantum fluctuations!

$$\partial_\mu J_5^\mu = c_A \vec{E} \cdot \vec{B} \quad \Rightarrow \quad \frac{d(N_R - N_L)}{dt} = c_A \int \vec{E} \cdot \vec{B} d^3x$$

Magnetic helicity

E.g.:

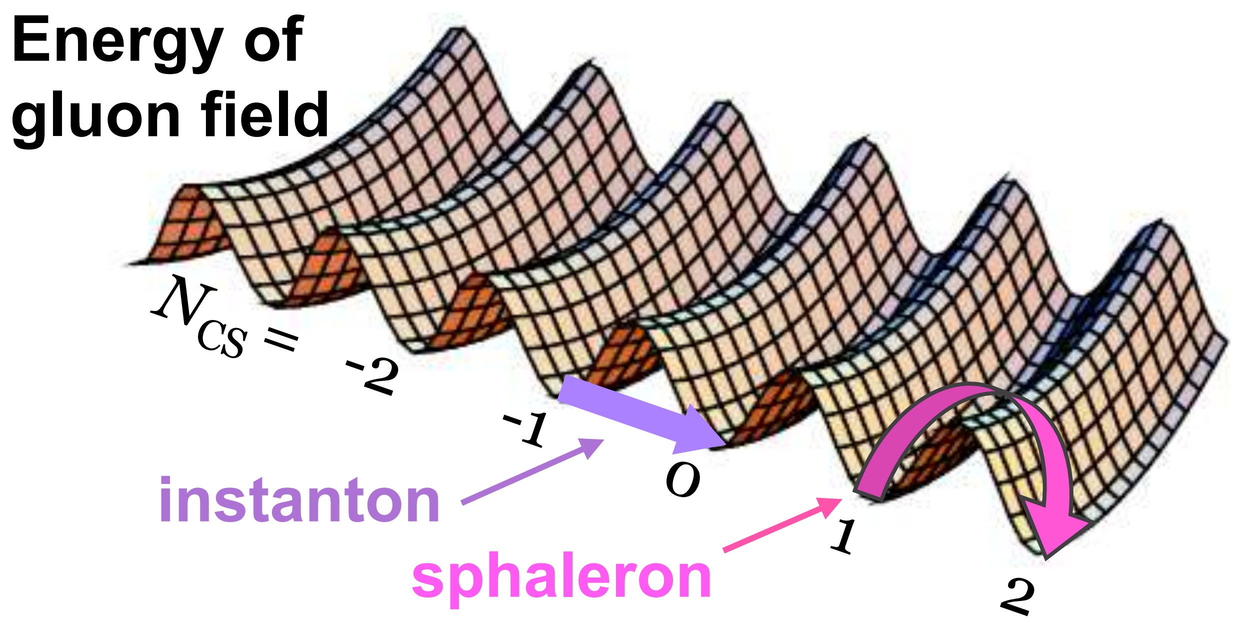


anomaly coefficient c_A
is topologically protected

TOPOLOGY OF THE QCD VACUUM

QCD vacuum is a superposition of states with different topology, characterized by the topological charge density

$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}(x)$$



Transitions between such states creates local imbalance of chirality.

The transition rate per unit volume is exponentially suppressed at low temperatures, but increases at high temperatures as $\Gamma_{\text{sph}} \sim (\alpha_s N_c)^5 T^4$

The topological domains with finite q may be as large as few fm.

Zhitnitsky et al

QED WITH CHIRAL ANOMALY

Sikivie (84), Wilczek (87), Carroll et al (90)

$$\mathcal{L}_{\mathrm{MCS}} = \mathcal{L}_{\mathrm{QED}} + c_A \theta(x) \vec{E} \cdot \vec{B}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - c\, \boldsymbol{\nabla}\theta \cdot \boldsymbol{B} \,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B} \,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + \boxed{c(\partial_t \theta\, \boldsymbol{B}} + \boxed{\boldsymbol{\nabla}\theta \times \boldsymbol{E}}) \,,$$

The anomalous current

$$\boldsymbol{j} = \sigma_\chi \boldsymbol{B}$$

Fukushima, Kharzeev, Warringa (2008)

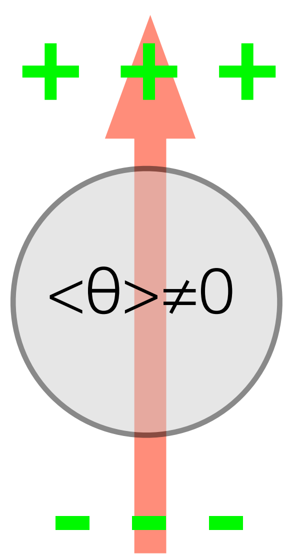
P-odd, T-odd

P-even, T-odd

Anomalous Hall Effect

Chiral magnetic effect

Kharzeev, McLerran, Warringa (2008)



External magnetic field drives the charge separation.

Breaks Parity!

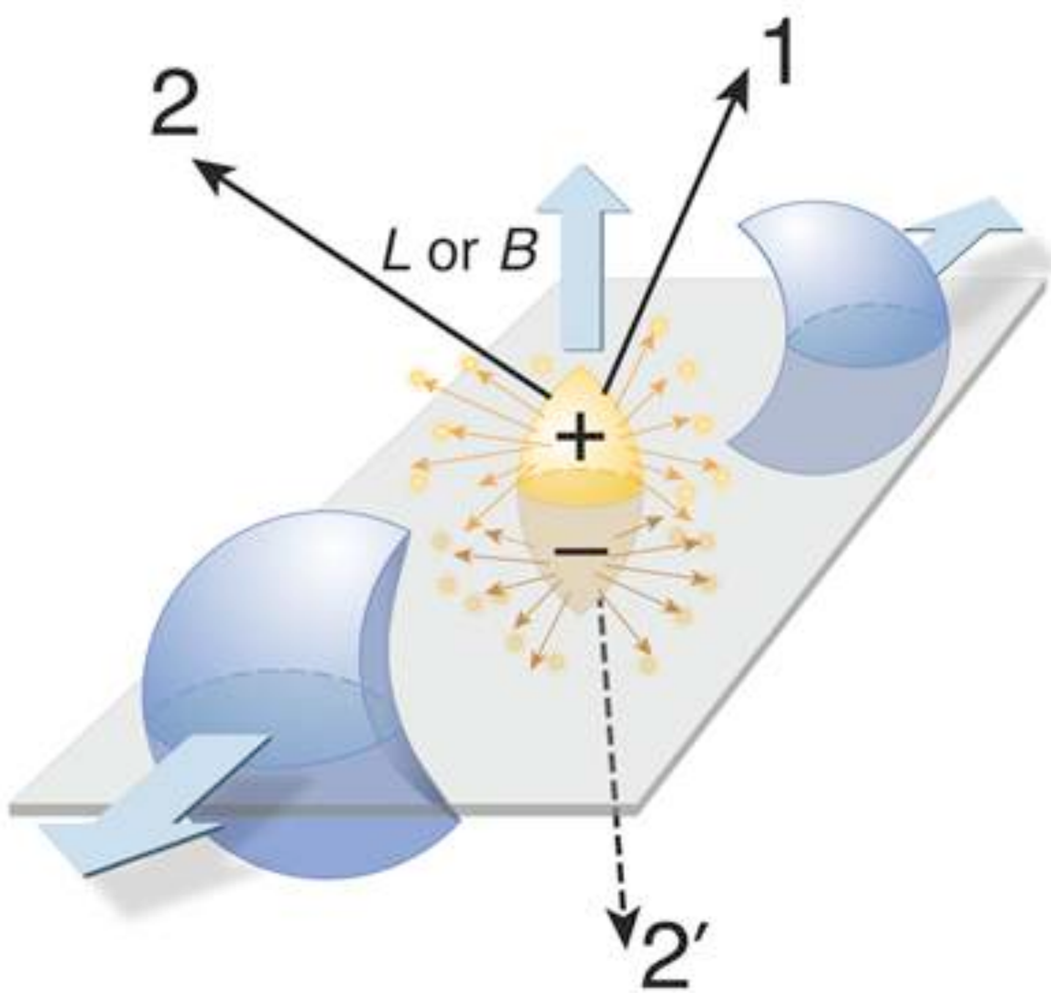
Often used notations: $\sigma_\chi = c_A \dot{\theta}$ $\quad$ $\boldsymbol{b} = c_A \boldsymbol{\nabla}\theta.$

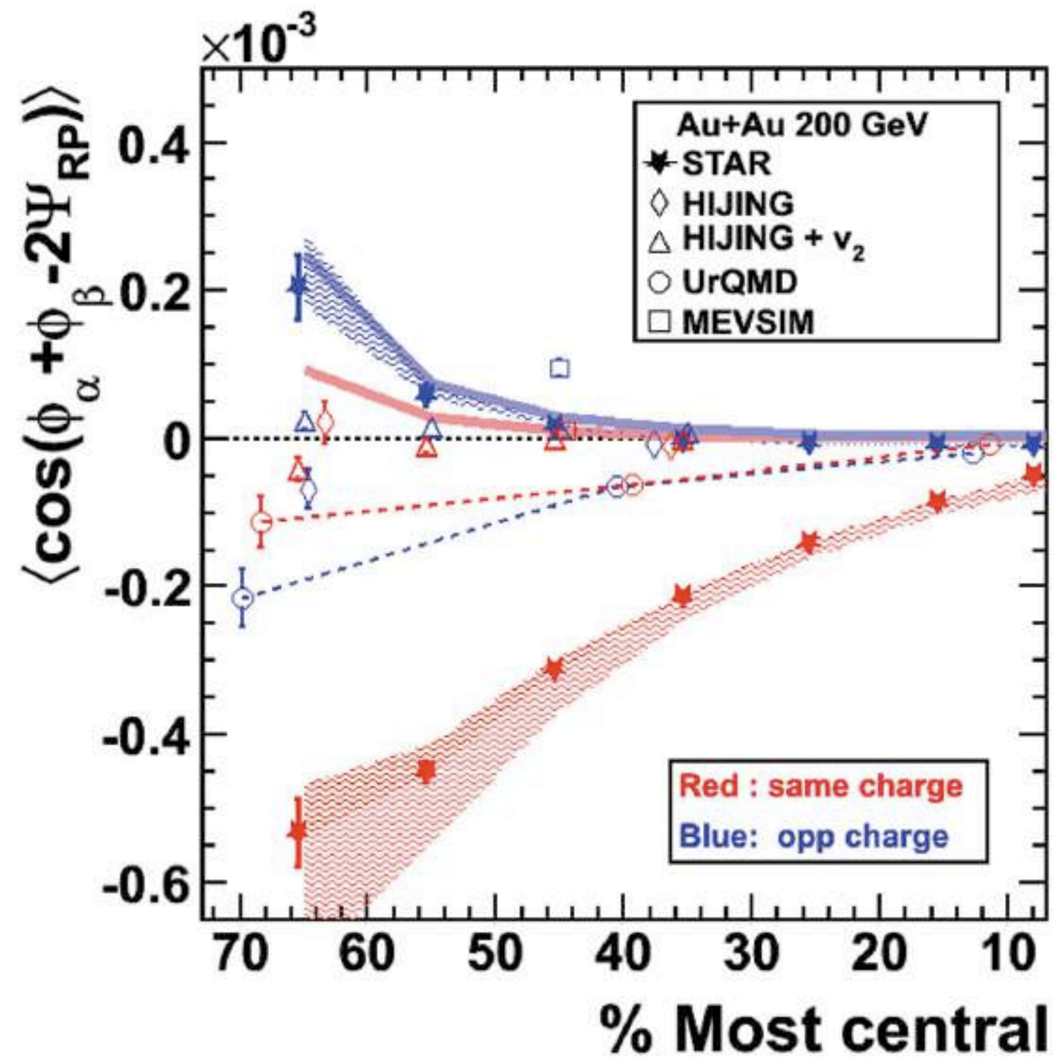
"chiral (magnetic) conductivity" $\quad$ axial chemical potential μ_5

Selected for a Viewpoint in *Physics*
PRL **103,** 251601 (2009) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2009

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

(STAR Collaboration)

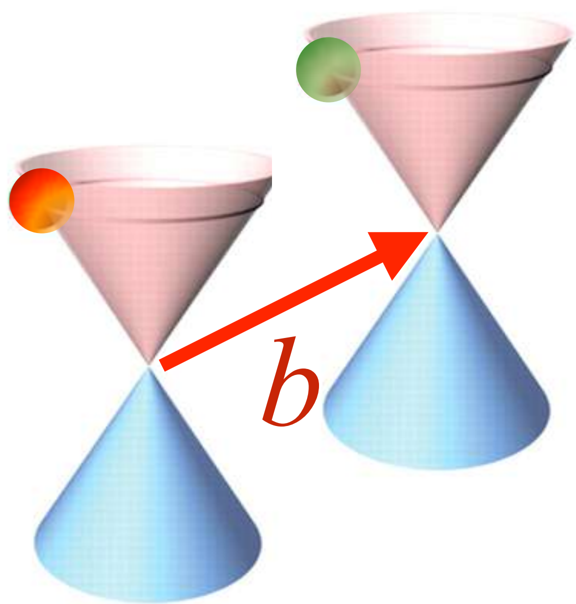




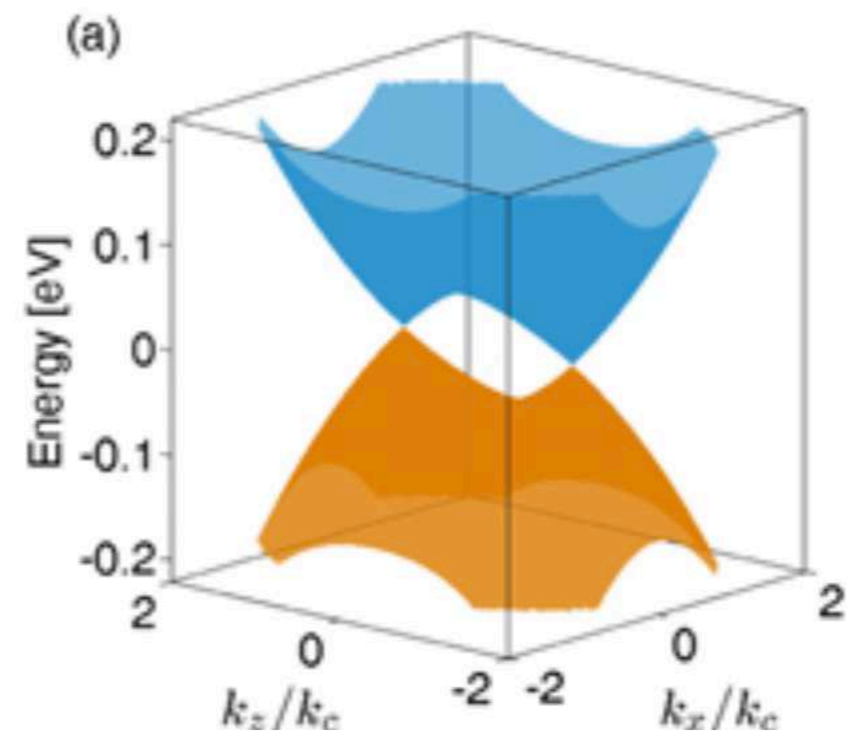
CHIRAL MAGNETIC EFFECT IN CMP

In Weyl semimetals and non-uniform QGP: field θ is time-independent and

$$\boldsymbol{\nabla}\theta = \boldsymbol{b}/c_A \approx \text{const.}$$



TaAs
NbAs
NbP
TaP



nature physics
LETTERS
PUBLISHED ONLINE: 8 FEBRUARY 2016 | DOI: 10.1038/NPHYS3648

Chiral magnetic effect in $ZrTe_5$

Qiang Li[1*], Dmitri E. Kharzeev[2,3*], Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1*]

arXiv:1412.6543 [cond-mat.str-el]

Chiral Magnetic Instability of EM field

CME EFFECT AND EM FIELD

Far away from any sources, Maxwell equations in momentum space read

$$\boldsymbol{k}\cdot\boldsymbol{B}_{\omega,\boldsymbol{k}} = 0\,,$$

$$\epsilon\boldsymbol{k}\cdot\boldsymbol{E}_{\omega,\boldsymbol{k}} = 0\,,$$

$$\boldsymbol{k}\times\boldsymbol{E}_{\omega,\boldsymbol{k}} = \omega\boldsymbol{B}_{\omega,\boldsymbol{k}}\,,$$

$$\boldsymbol{k}\times\boldsymbol{B}_{\omega,\boldsymbol{k}} = -\omega\epsilon\boldsymbol{E}_{\omega,\boldsymbol{k}} - i\sigma_\chi\boldsymbol{B}_{\omega,\boldsymbol{k}}$$

Electromagnetic waves have the dispersion relation

$$[\omega(\omega+i\sigma)-\boldsymbol{k}^2]^2 = \sigma_\chi^2\boldsymbol{k}^2$$

Four solutions: $\omega_{\lambda_1,\lambda_2} = -\frac{i\sigma}{2} + \lambda_1\sqrt{k^2+\lambda_2\sigma_\chi k - \sigma^2/4}$ $\qquad \lambda_1,\lambda_2 = \pm 1$

All four poles lie in the lower-half plane of complex ω except when $k < \sigma_\chi$

In this case $\mathrm{Im}\,\omega_\pm > 0$ indicating a solution exponentially growing with time:

$B \sim e^{\mathrm{Im}\,\omega_\pm t}$ $\Rightarrow$ ***instability***

CHIRAL MAGNETIC INSTABILITY

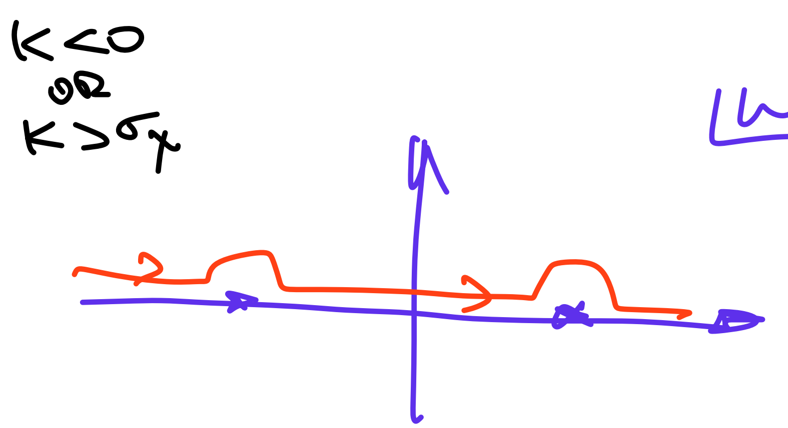
Limits on a Lorentz- and parity-violating modification of electrodynamics

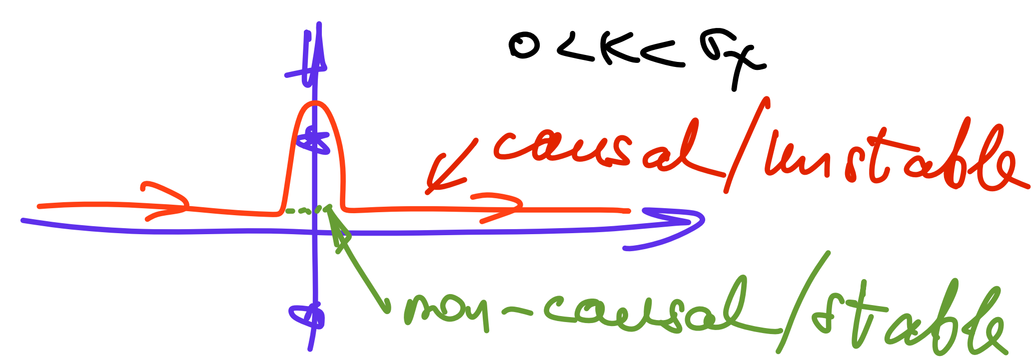
Sean M. Carroll and George B. Field
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138

Roman Jackiw*
Department of Physics, Columbia University, New York, New York 10027
(Received 5 September 1989)

Photon propagator $G^{ij}(t,\mathbf{r})=[(\delta^{ij}-\partial_i\partial_j/\nabla^2)\Box+m\epsilon^{ijk}\partial_k]g(t,\mathbf{r})$

$$g(t,r)=\frac{2\sin\frac{m}{2}r}{\pi m r}\left[\theta(t)\int_0^\infty dz\frac{\cos r\left[\frac{m^2}{4}+z^2\right]^{1/2}}{\left[\frac{m^2}{4}+z^2\right]^{1/2}}\sin tz-\frac{1}{2}\int_0^{m/2}dz\frac{\cos r\left[\frac{m^2}{4}-z^2\right]^{1/2}}{\left[\frac{m^2}{4}-z^2\right]^{1/2}}e^{-|t|z}\right].\qquad m=\sigma_\chi$$





A very partial list of other references: Redlich, Wijewardhana(1985), Rubakov (1986), Joyce, Shaposhnikov (1987), Adam, Klinkhamer (2001), Boyarsky at al (2012), Sadofyev, Zakharov et al (2013), Akamatsu,Yamamoto(2013), Hirono, Kharzev, Yin (2015), Manuel, Torres-Rincon(2015), Buividovich, Ulybyshev (2016), Kaplan, Reddy, Sen (2016).

TAMING INSTABILITY: HELICITY CONSERVATION

Chiral anomaly $\partial_\mu j^\mu_A = c_A \boldsymbol{E}\cdot\boldsymbol{B}$

Hirono, Kharzev, Yin (2015)

In a homogeneous medium $\boldsymbol{\nabla}\cdot\boldsymbol{j}_A = \sigma_\chi \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0 \quad \Rightarrow \quad \dot{n}_A = c_A \boldsymbol{E}\cdot\boldsymbol{B}$

Averaging over volume $\partial_t \langle n_A \rangle = \frac{c_A}{V}\int \boldsymbol{E}\cdot\boldsymbol{B}\, d^3x = -\frac{c_A}{2V}\partial_t \mathcal{H}_{\rm em}$

Integrate over time $\Rightarrow$ $$\boxed{\frac{2V}{c_A}\langle n_A\rangle + \mathcal{H}_{\rm em} = \mathcal{H}_{\rm tot}}$$

$\mathcal{H}_{\rm em}$: Magnetic helicity

Magnetic helicity depends on $\sigma_\chi(t)$ rather than on $\langle n_A\rangle$ $\Rightarrow$ need equation of state

In hot medium $\langle n_A \rangle = \chi\,\mu_5$ (recall: $\sigma_\chi = c_A\mu_5$)

Equation of state $\sigma_\chi(t) = \frac{c_A}{\chi}\langle n_A(t)\rangle$

EVOLUTION OF CHIRAL CONDUCTIVITY

$$-\nabla^2 \boldsymbol{A} = -\partial_t^2 \boldsymbol{A} + \boldsymbol{j} + \sigma_\chi(t) \boldsymbol{\nabla} \times \boldsymbol{A}$$

Expand in the circularly polarized plane wave basis

$$\boldsymbol{A} = \sum_{\boldsymbol{k},\lambda} [a_{\boldsymbol{k}\lambda}(t) \boldsymbol{W}_{\boldsymbol{k}'\lambda'}(\boldsymbol{x}) + a^*_{\boldsymbol{k}'\lambda'}(t) \boldsymbol{W}^*_{\boldsymbol{k}'\lambda'}(\boldsymbol{x})]$$

λ =helicity

The fastest growing mode:
(adiabatic approx.)

$$a_0(t) = e^{\gamma(t)/2} \qquad \gamma(t) = \int_0^t \left[\sqrt{\sigma^2 + \sigma_\chi^2(t')} - \sigma\right] dt'$$

Helicity conservation

$$\frac{\sigma_\chi(t)}{\alpha} = 1 - \frac{\mathcal{H}_{\rm em}(t)}{\mathcal{H}_{\rm tot}}$$

Helicity flows between the magnetic field and medium. Total helicity is conserved.

Fraction of the total helicity in plasma

Fraction of the total helicity in the field

Vector potential is normalized so that

$$\mathcal{H}_{\rm em}(0) = 1$$

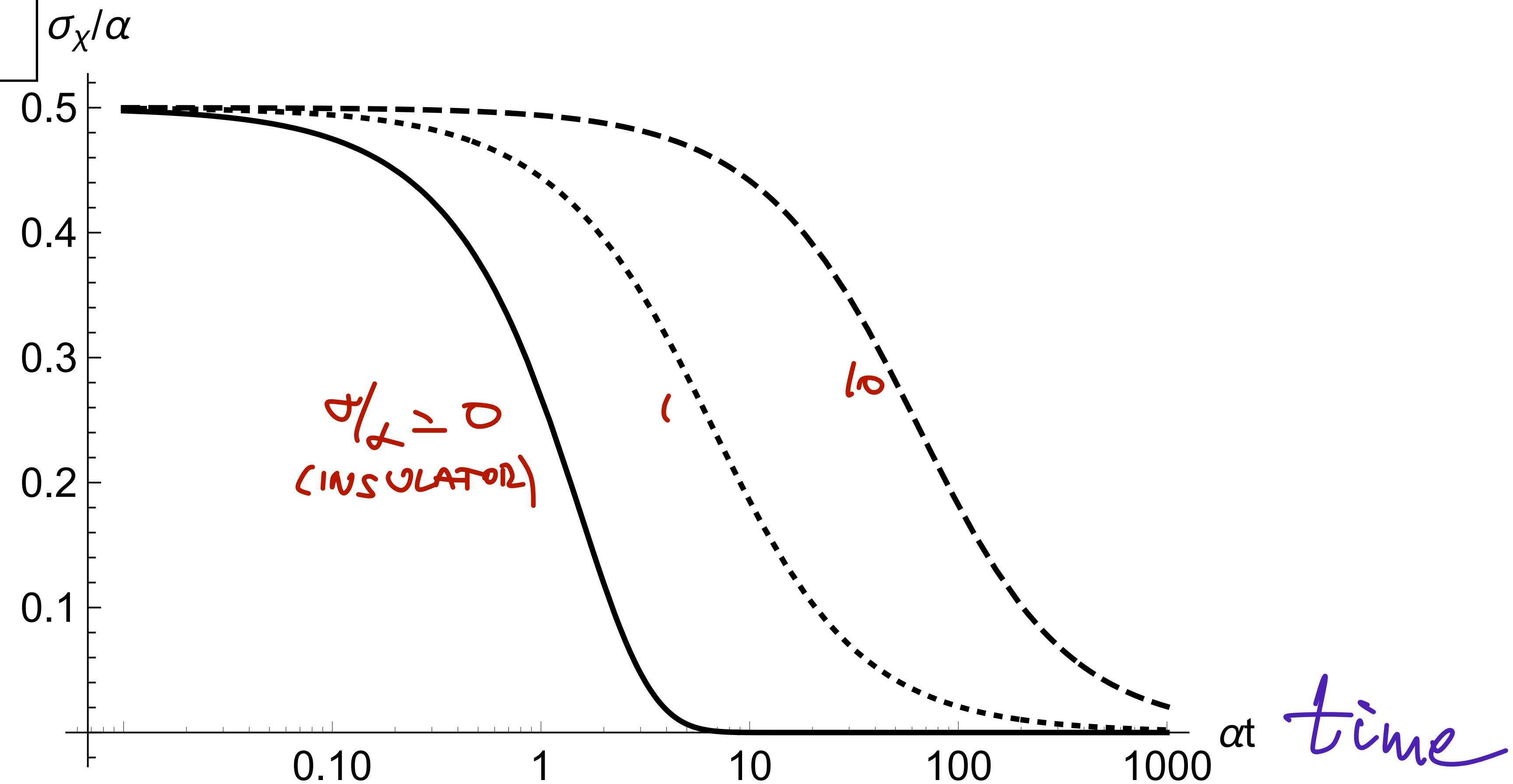
$$\dot{\sigma}_\chi = -\left(\sqrt{\sigma^2 + \sigma_\chi^2} - \sigma\right)(\alpha - \sigma_\chi)$$

Characteristic energy scale

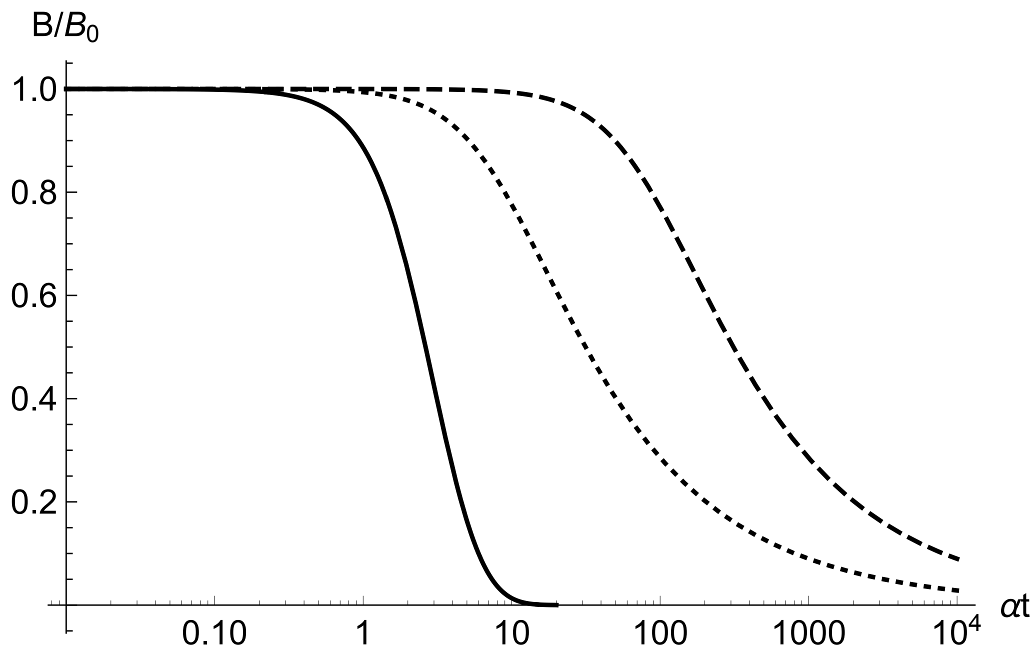
$$\alpha = \mathcal{H}_{\rm tot} c_A^2 / (2V\chi)$$

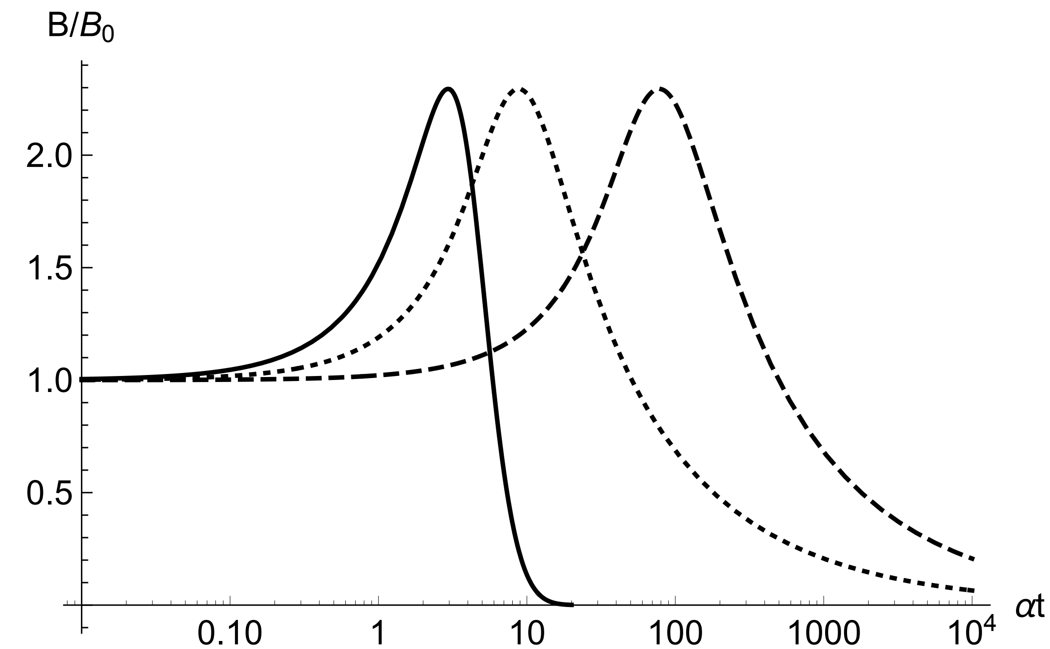
EVOLUTION OF CHIRAL CONDUCTIVITY

Fraction of the total helicity in plasma



EVOLUTION OF CHIRAL CONDUCTIVITY





50 % OF INITIAL HELICITY IS IN MEDIUM

95 % OF INITIAL HELICITY IS IN MEDIUM

Magnetic field develops a maximum only if the initial helicity in medium >50%

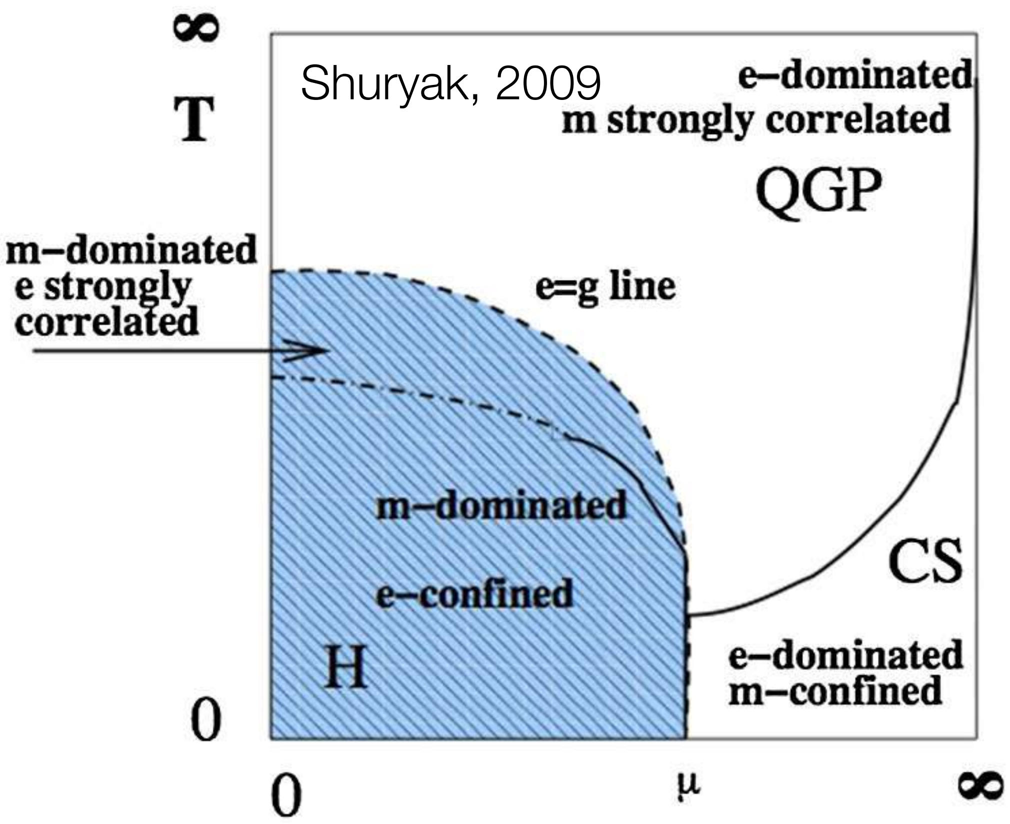
MAGNETIC MONOPOLES AND THE INSTABILITY

Motivation:

Magnetic monopoles at T=0: dual superconductor, color confinement.

The condensate may not melt away at T_c

$\Rightarrow$ Important part of QGP dynamics



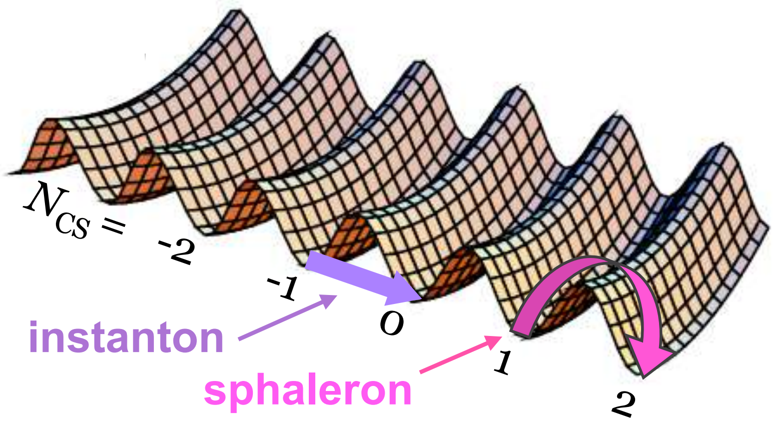
Chiral evolution with magnetic monopoles: always ends up in a superconducting state.

Li, KT, 2018

TIME-VARIATION OF TOPOLOGICAL CHARGE

❖ Early times $t \ll \tau_c \quad \Rightarrow \quad \sigma_\chi$ is adiabatic

where $\tau_c \sim 1/(g^4 T)$ sphaleron transition time



❖ By the time t=τ_c the chiral instability becomes $\exp(\sigma_\chi \tau_c)$

Since $\sigma_\chi \sim e^2 \mu_5 \quad \Rightarrow \quad \sigma_\chi \tau_c \ll 1 \quad \Rightarrow$

the topological charge changes by the time the instability fully develops.

❖ How does magnetic field evolve at $t \gg \tau_c$?

Model: the chiral conductivity is a stochastic process with

$$\langle \sigma_\chi \rangle = 0 \qquad \Sigma_\chi = \sqrt{\langle \sigma_\chi^2 \rangle} = c_A \mu_5 \qquad \langle \sigma_\chi(t) \sigma_\chi(t-\tau) \rangle \neq 0 \quad \text{when} \quad t < \tau_c$$

HARMONIC OSCILLATOR WITH RANDOM FREQUENCY

Magnetic field amplitude in medium with fluctuating topological charge is harmonic oscillator with random frequency

$$x = a_{\boldsymbol{k}\lambda} e^{\sigma t/2} \qquad \Rightarrow \qquad \boxed{\ddot{x}(t) + \omega^2[1 + \alpha\xi(t)]x(t) = 0}$$

$$\omega^2 = k^2 - \frac{\sigma^2}{4}\,, \qquad \alpha = -\frac{\lambda k}{\omega^2}\Sigma_\chi\,, \qquad \xi = \frac{\sigma_\chi}{\Sigma_\chi}\,, \qquad \Sigma_\chi = \sqrt{\langle \sigma_\chi^2 \rangle}$$

λ =helicity

Harmonic oscillator with random frequency belongs to the class of linear stochastic equations

$$\frac{du(t')}{dt'} = [A_0 + \alpha\xi(t')B]u(t') \qquad t' = \omega t$$

It can be converted to an ordinary integro-differential equation at $t \gg \tau_c$

$$\frac{d\,\langle u(t')\rangle}{dt'} = \left\{ A_0 + \alpha^2 \int_0^\infty \langle \xi(t')\xi(t' - \tau')\rangle \, B e^{A_0\tau'} B e^{-A_0\tau'} d\tau' \right\} \langle u(t')\rangle$$

Van Kampen (1975)

provided that $\alpha \ll 1$ i.e. the fluctuating term is a perturbation.

EVOLUTION OF AVERAGE AMPLITUDE

Equation for the first moments $u = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$ $\qquad A_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}$

$$\frac{d^2 \langle x \rangle}{dt'^2} + \frac{1}{2}\alpha^2 c_2 \frac{d \langle x \rangle}{dt'} + \left(1 - \frac{1}{2}\alpha^2 c_1\right) \langle x \rangle = 0$$

$$c_1 = \int_0^\infty \langle \xi(t')\xi(t'-\tau') \rangle \sin(2\tau') d\tau' ,$$
$$c_2 = \int_0^\infty \langle \xi(t')\xi(t'-\tau') \rangle \, [1 - \cos(2\tau')] d\tau'$$

Evolution of average amplitude $\langle a_{\boldsymbol{k}\lambda}(t) \rangle_\pm = \exp\left\{ \pm i\omega t - \frac{\alpha^2}{4}(c_2 \pm i c_1)\omega t - \frac{1}{2}\sigma t \right\} .$

It can be shown that for any conductivity all <a> modes are decreasing with time.

$\Rightarrow$ **no instability** at $t \gg \tau_c$

EVOLUTION OF AVERAGE ENERGY

Ornstein-Uhlenbeck process (for illustration): $\langle \xi(t)\xi(t-\tau)\rangle = e^{-\tau/\tau_c}$

$$c_1 = \frac{2(\omega\tau_c)^2}{1+4(\omega\tau_c)^2}\,, \qquad c_2 = \frac{4(\omega\tau_c)^3}{1+4(\omega\tau_c)^2}\,, \qquad c_3 = \frac{\left[2+4(\omega\tau_c)^2\right](\omega\tau_c)}{1+4(\omega\tau_c)^2}$$

Good conductor: $\sigma \sim T/e^2$ $\Rightarrow$ average energy is always stable (due to dissipation)

Poor conductor: $\sigma \sim e^2 T$ e.g. QGP near T_c

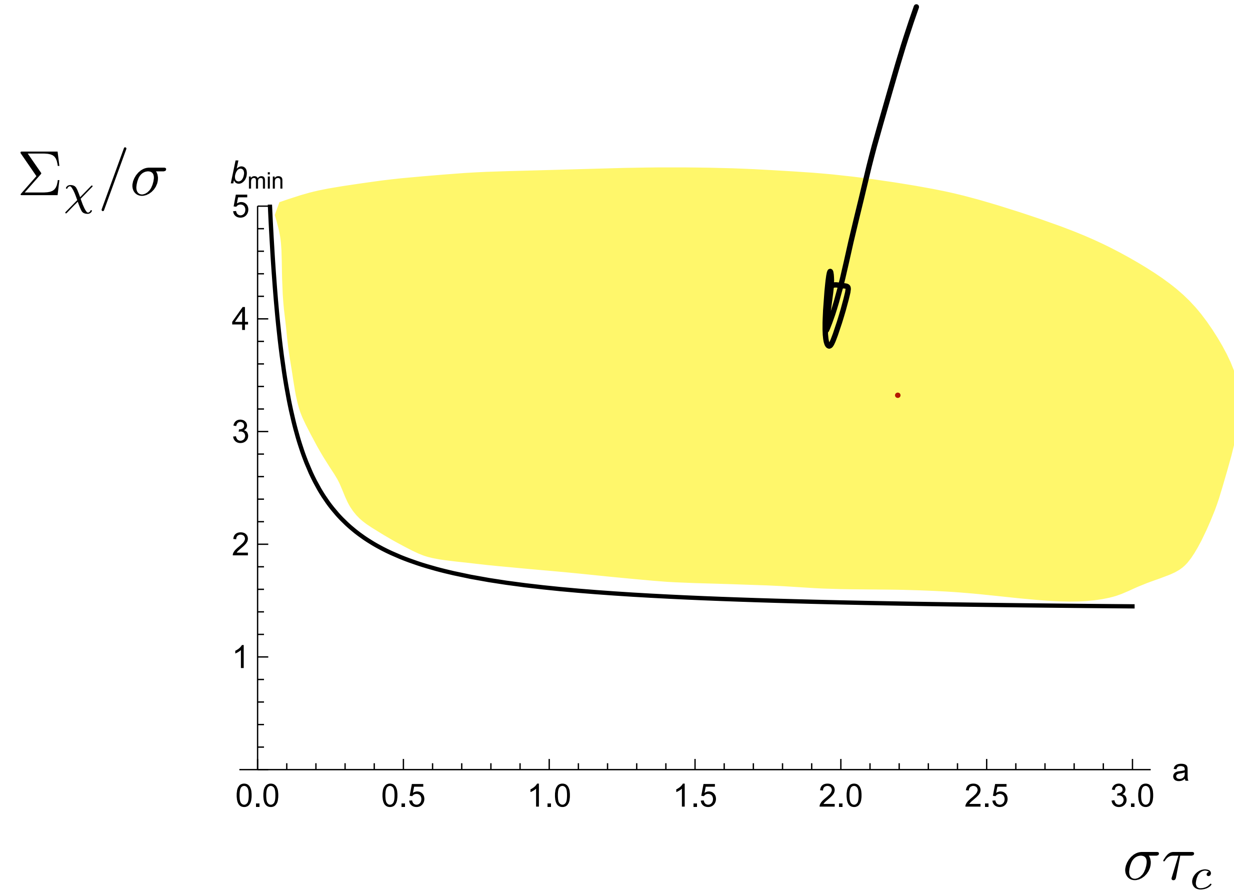
The unstable modes of average energy:

$$\langle \mathcal{E}_{\boldsymbol{k}\lambda}\rangle = \frac{k}{2}u_0 \exp\left\{\frac{\Sigma_\chi^2}{2k}\frac{2\omega\tau_c}{1+4\omega^2\tau_c^2}t - \sigma t\right\}$$

Does not depend on helicity λ

- Average energy is unstable in poor conductors (such as QGP) if $\sigma < g^4 T$ and $\Sigma_\chi \gg \sigma$.
- The magnetic helicity of R and L modes increases exponentially. However, their sum vanishes. Thus, the helicity conservation cannot tame the instability at later times as is does at early times. The instability is not chiral!

INSTABILITY REGION



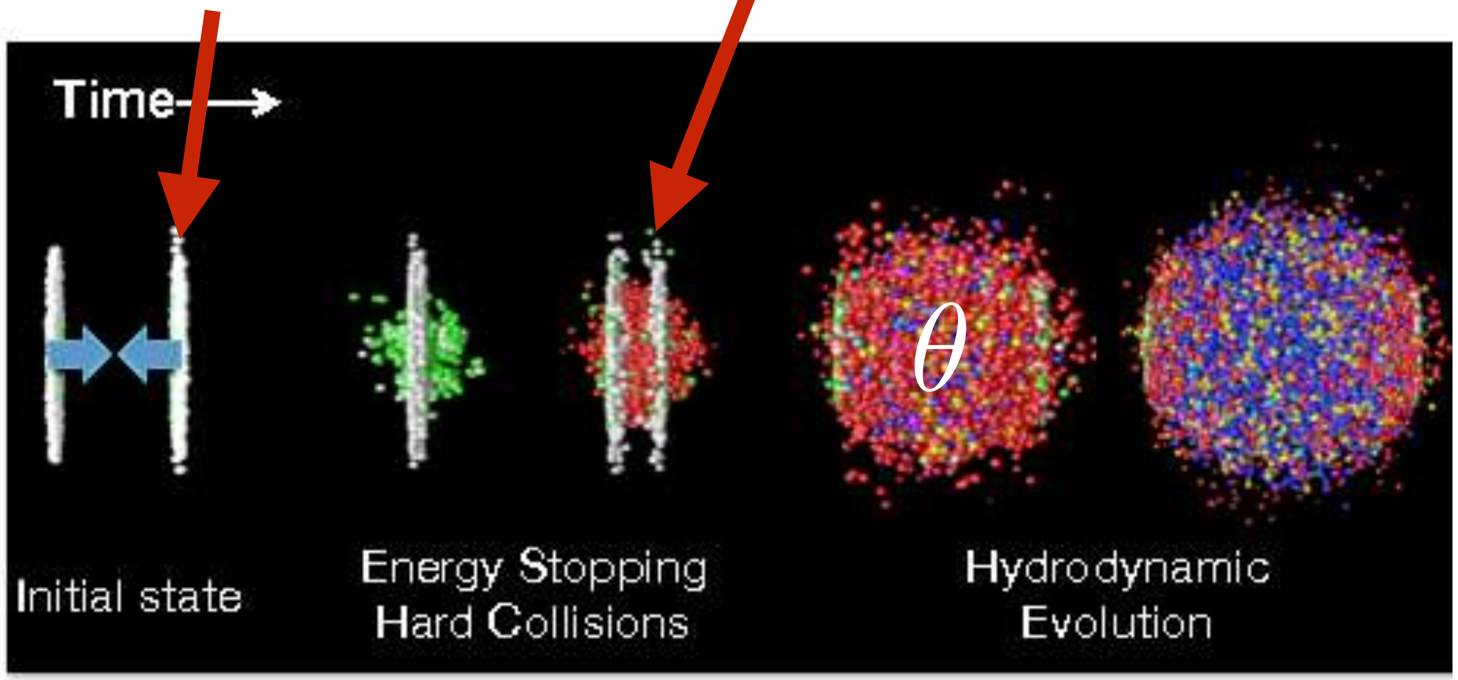
Example: QGP $\quad \sigma \approx 5\,\mathrm{MeV} \quad \tau_c \approx 5\,\mathrm{fm} \quad \Rightarrow$ Instability occurs if $\Sigma_\chi = 15\,\mathrm{MeV}$

Unstable helicity modes: $\quad \langle \mathcal{H}_{\boldsymbol{k}\lambda} \rangle = \dfrac{\lambda u_0}{2} e^{\nu_0 k t - \sigma t} \quad \Rightarrow \quad \langle \mathcal{H} \rangle = 0$

Generation of intense EM field in relativistic heavy-ion collisions at finite θ

EM FIELD OF VALENCE CHARGES AT FINITE θ

valence electric charges



EM FIELD OF A POINT CHARGE AT EARLY TIME

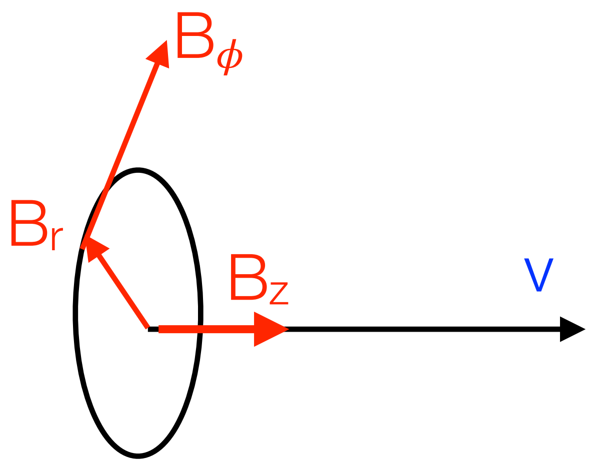
Maxwell-Chern-Simons equations

$$\boldsymbol{\nabla}\times\boldsymbol{B}=\partial_t\boldsymbol{D}+\sigma_\chi\boldsymbol{B}+qv\hat{\boldsymbol{z}}\delta(z-vt)\delta(\boldsymbol{b})\,,$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{D}=q\delta(z-vt)\delta(\boldsymbol{b})\,,$$

$$\boldsymbol{\nabla}\times\boldsymbol{E}=-\partial_t\boldsymbol{B}\,,$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}=0\,,$$



Can be solved for constant chiral conductivity

$$B_{\phi\omega}(\boldsymbol{r})=\int\frac{d^2k_\perp}{(2\pi)^2}\frac{q\,k\,e^{i\omega z/v+i\boldsymbol{k}_\perp\cdot\boldsymbol{b}}}{[k_\perp^2+\omega^2(1/v^2-\epsilon)]^2-(\sigma_\chi k)^2}\times\left\{[k_\perp^2+\omega^2(1/v^2-\epsilon)]\frac{-ik_\perp}{k}\cos\theta+\sigma_\chi k\frac{-k_zk_\perp}{k^2}\sin\theta\right\}$$

High energy approximation:

$$B_\phi=\frac{eb}{8\pi x_-^2}e^{-\frac{b^2\sigma}{4x_-}}\left[\sigma\cos\left(\frac{b^2\sigma_\chi}{4x_-}\right)+\sigma_\chi\sin\left(\frac{b^2\sigma_\chi}{4x_-}\right)\right]$$

OSCILLATIONS OF EM FIELD AT EARLY TIMES

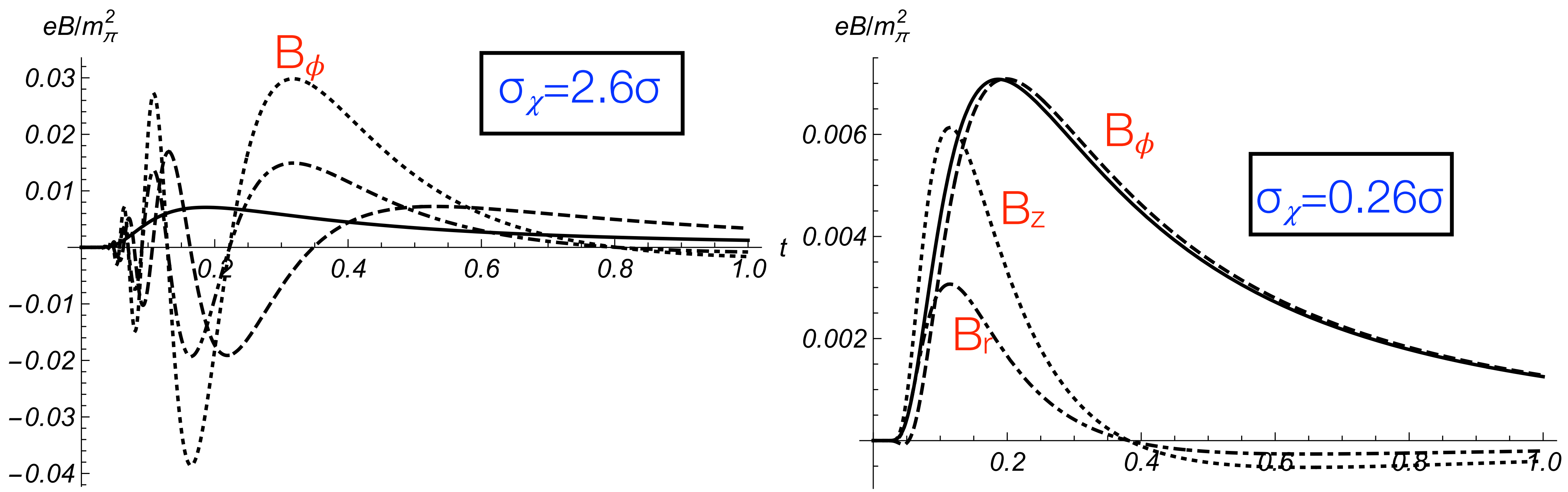


FIG. 2: Magnetic field of a point charge as a function of time t at $z = 0$. (Free space contribution is not shown). Electrical conductivity $\sigma = 5.8$ MeV. Solid line on both panels corresponds to $B = B_\phi$ at $\sigma_\chi = 0$. Broken lines correspond to B_ϕ (dashed), B_r (dashed-dotted) and B_z (dotted) with $\sigma_\chi = 15$ MeV on the left panel and $\sigma_\chi = 1.5$ MeV on the right panel. Note that the vertical scale on the two panels is different.

EM AT LATER TIMES

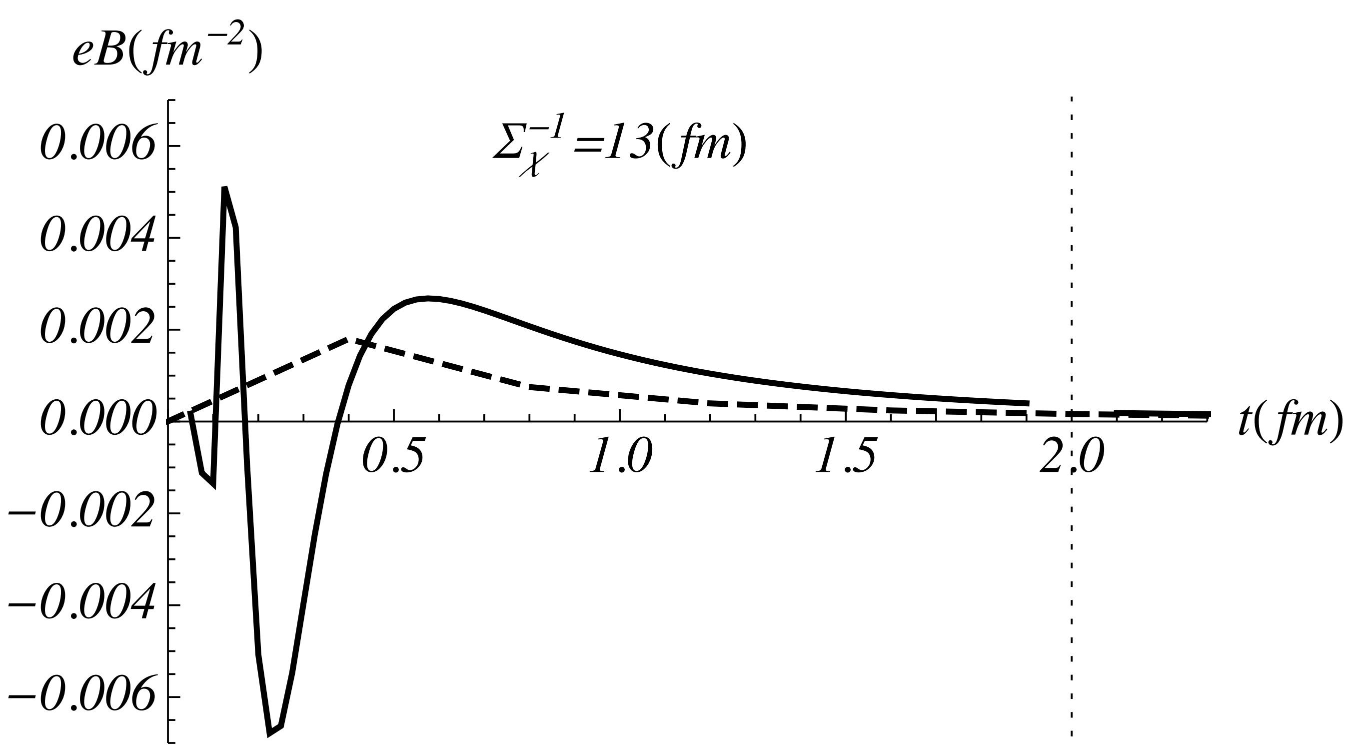
At later times needs to sum over fluctuations of the topological charge $\xi(t)$

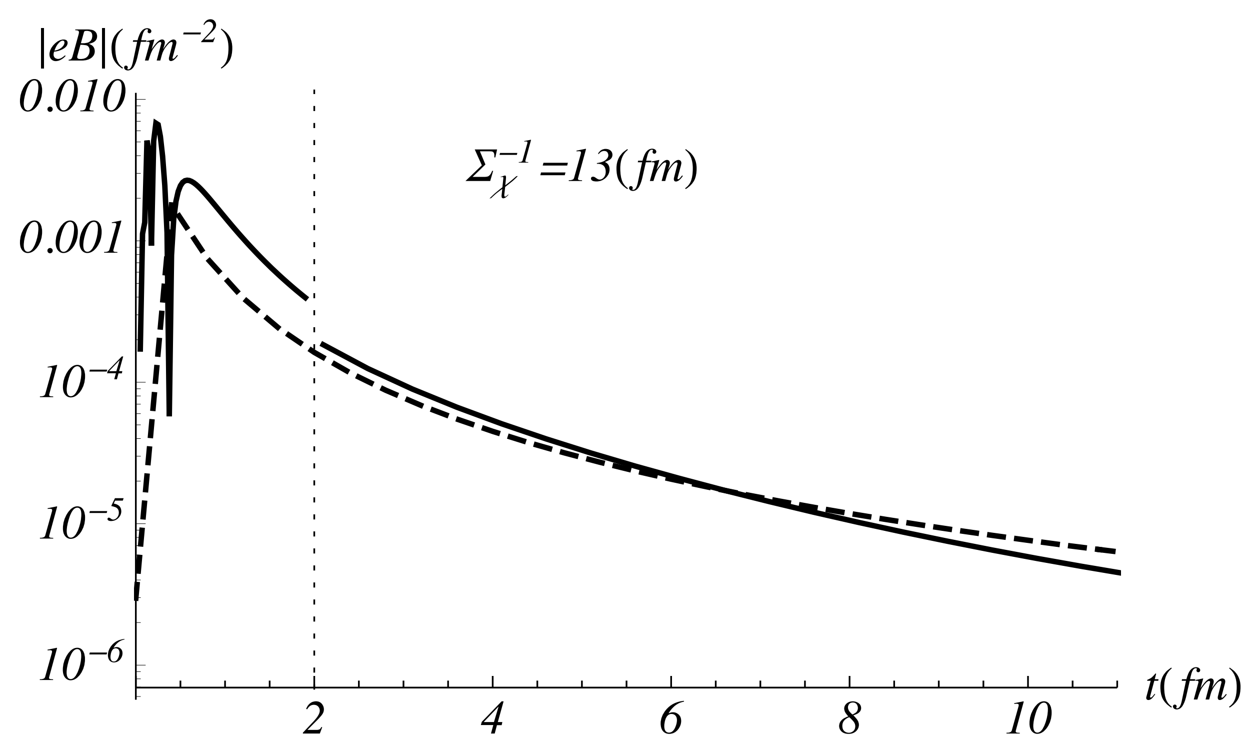
$$\ddot{x}(t) + \omega^2[1 + \alpha\xi(t)]x(t) = \lambda k J_{\lambda\boldsymbol{k}}(t) e^{\sigma t/2}$$

where $\boldsymbol{B}_{\lambda\boldsymbol{k}} = x_{\lambda\boldsymbol{k}}(t)\boldsymbol{\epsilon}_{\lambda\boldsymbol{k}} e^{-\sigma t/2}$
$= \Phi_{\lambda\boldsymbol{k}}\boldsymbol{\epsilon}_{\lambda\boldsymbol{k}}$

Solution for the average amplitude

$$\langle \Phi_{\lambda\boldsymbol{k}}(t)\rangle = \frac{qv\hat{\boldsymbol{z}}\cdot\boldsymbol{\epsilon}^*_{\lambda\boldsymbol{k}}\lambda k(1+\alpha^2 c_0)e^{-ik_z vt}}{k^2 - (k_z v)^2 - i\sigma k_z v + \alpha^2 Q(\omega)}$$

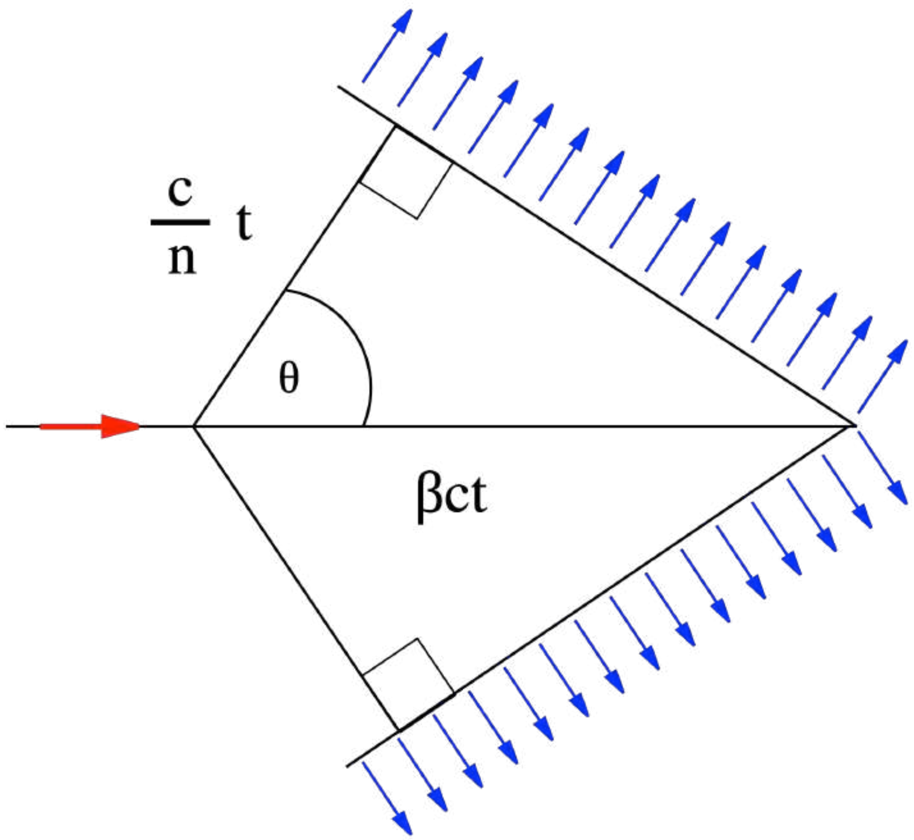


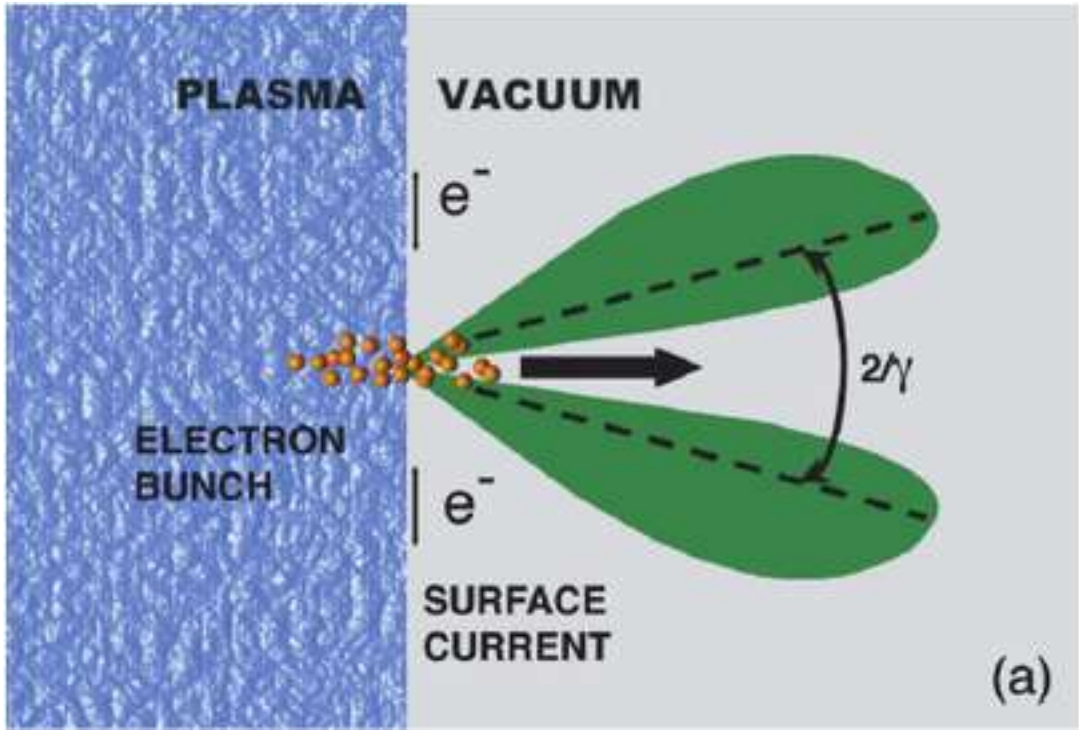


Dashed line: $\langle\sigma_\chi^2\rangle = 0$

Fast particles in chiral media: chiral Cherenkov radiation

PARTICLE RADIATION IN MATTER: CHERENKOV AND TRANSITION RADIATION





Classical Cherenkov radiation is emitted by a charged particle that moves faster than the phase velocity of light: $vn>1$

$$\cos\theta = \frac{1}{\beta\sqrt{\epsilon}} = \frac{1}{\beta n}$$

Classical transition radiation is emitted by a charged particle that moves through inhomogeneous matter.

33.7. Cherenkov and transition radiation [33,77,78]

A charged particle radiates if its velocity is greater than the local phase velocity of light (Cherenkov radiation) or if it crosses suddenly from one medium to another with different optical properties (transition radiation). Neither process is important for energy loss, but both are used in high-energy and cosmic-ray physics detectors.

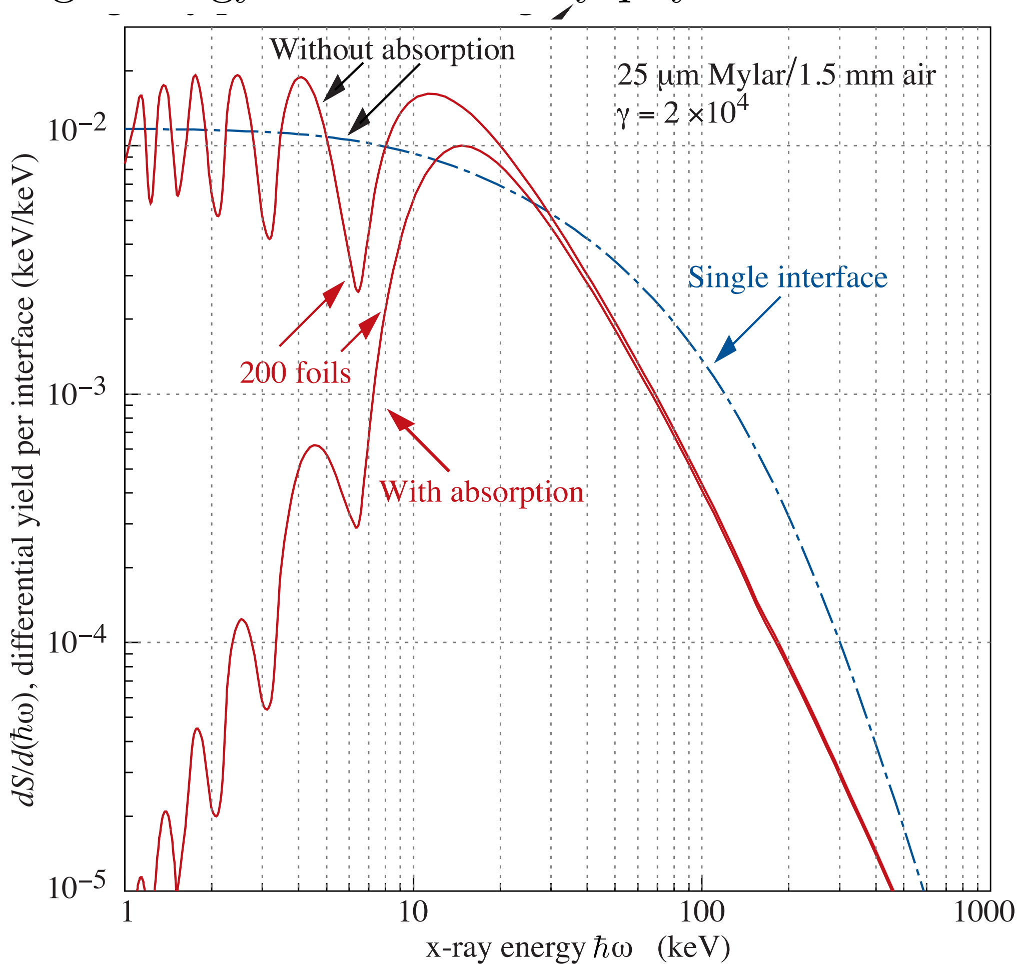


Figure 33.27: X-ray photon energy spectra for a radiator consisting of 200 25 μm thick foils of Mylar with 1.5 mm spacing in air (solid lines) and for a single surface (dashed line). Curves are shown with and without absorption. Adapted from Ref. 88.

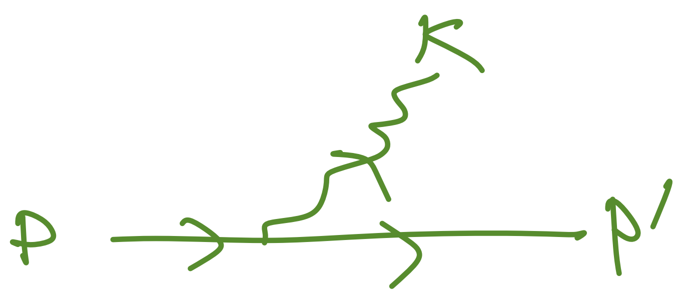
1→2 PROCESSES IN CHIRAL MATTER

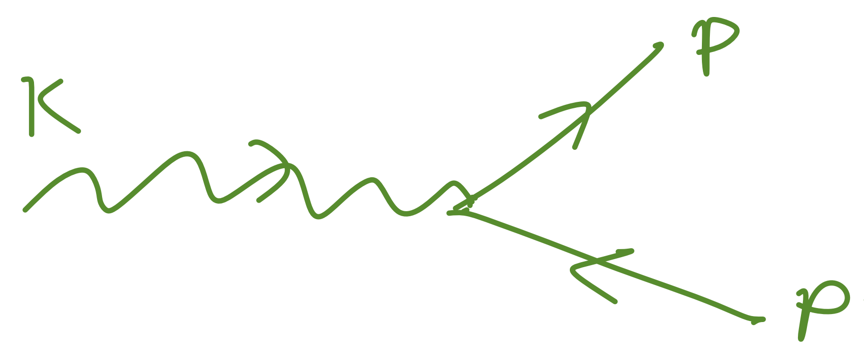
Let field θ be homogenous and weekly time-dependent $\dot{\theta} = \mathrm{const}$

In radiation gauge: $\nabla^2 \boldsymbol{A} = \partial_t^2 \boldsymbol{A} - \sigma_\chi \boldsymbol{\nabla} \times \boldsymbol{A}$

The dispersion relation $k^2 = -\lambda \sigma_\chi |\boldsymbol{k}|$ → photon becomes space- or timelike

λ =helicity





$k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon)$ forbidden in vacuum, but allowed in chiral medium

Pair production: $k^2 > 0 \Rightarrow \lambda \sigma_\chi < 0$

Photon radiation: $k^2 < 0 \Rightarrow \lambda \sigma_\chi > 0$

UR approx.: $\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}} \boldsymbol{\epsilon}_\lambda \, e^{i\omega z + i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp - i\omega t} \exp\left\{ -i \frac{1}{2\omega} \int_0^z \left[k_\perp^2 - \underbrace{\sigma_\chi(z')\omega\lambda}_{\text{"}m_\gamma^2\text{"}} \right] dz' \right\}$

A SINGLE UNIFORM INFINITE DOMAIN

$$\mathcal{M} = -eQ\bar{u}(p')\gamma^{\mu}u(p)\epsilon^{*}_{\mu} \times 4\pi\varepsilon x(1-x)\delta(q_{\perp}^{2}+\kappa_{\lambda})$$

$$\kappa_{\lambda}(z) = x^2 m^2 - (1-x)x\lambda\sigma_{\chi}\varepsilon$$

can become negative!

Chiral Cherenkov effect: photon radiation at $\vartheta \sim \sqrt{|\sigma_{\chi}|/\omega}$

Kappa is negative if $\lambda\sigma_{\chi} > 0$ and $x < x_0 = \dfrac{1}{1+m^2/(\lambda\sigma_{\chi}\varepsilon)}$

Photon radiation rate

$$\frac{dW_{+}}{dx} = \frac{\alpha Q^2}{2\varepsilon x}\left\{\sigma_{\chi}\varepsilon\left(\frac{x^2}{2}-x+1\right)-m^2 x\right\}\theta(x_0-x)$$

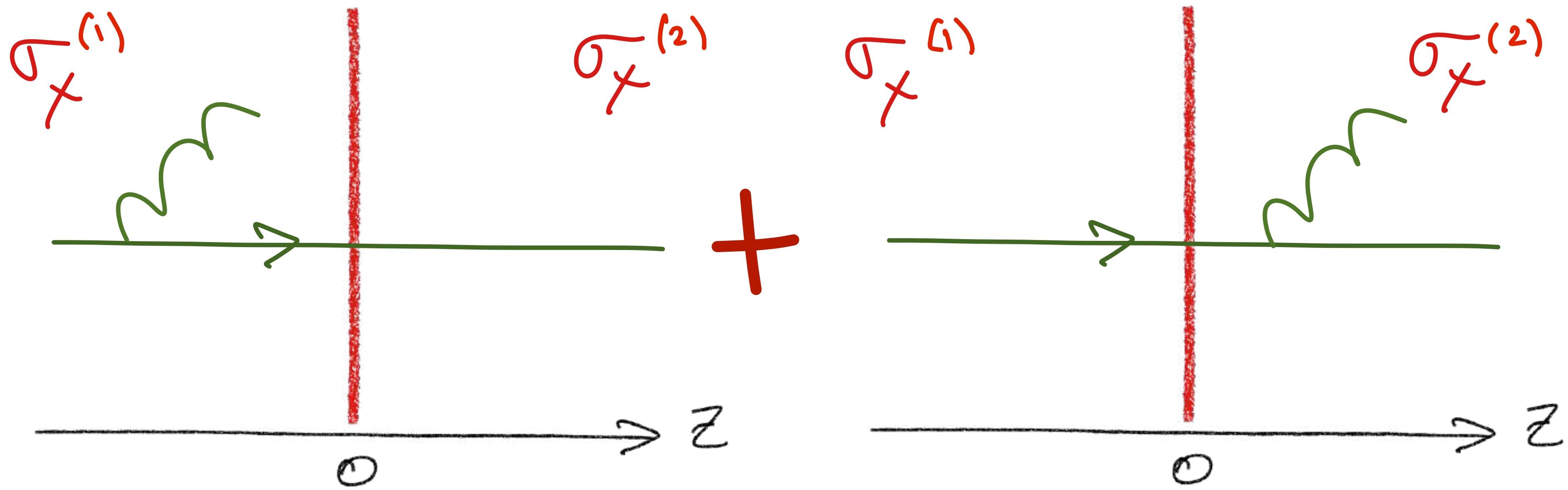
$$\frac{dW_{-}}{dx} = 0\,.$$

Vanishes as $\hbar \to 0$

Quantum anomaly!

Total rate of energy loss $\quad \dfrac{\Delta\varepsilon}{T} = \displaystyle\int_0^1 \frac{dW_{+}}{dx}x\varepsilon dx = \frac{1}{3}\alpha Q^2\sigma_{\chi}\varepsilon$

TWO SEMI-INFINITE DOMAINS



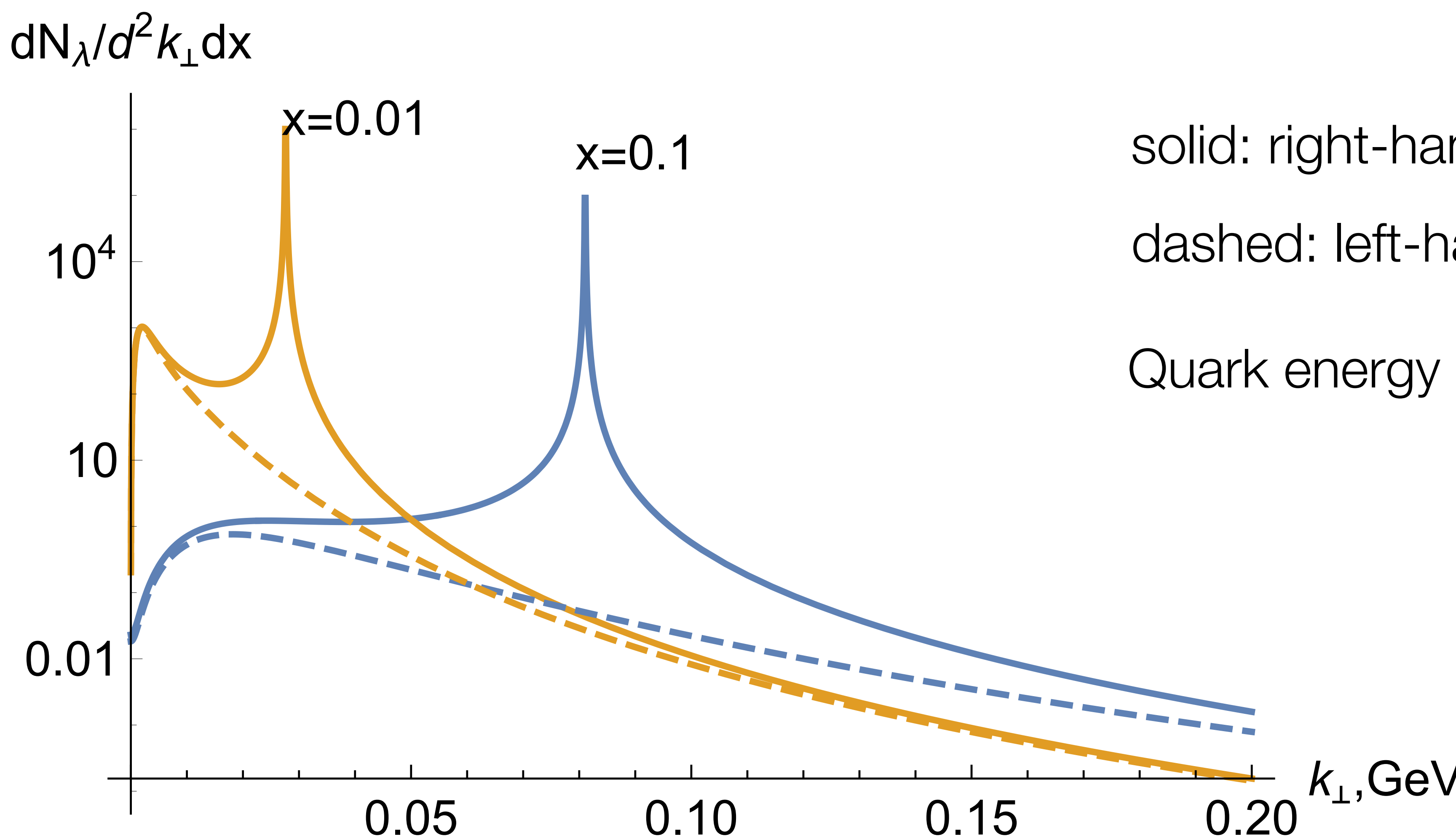
$$\frac{dN}{d^2q_\perp dx} = \frac{\alpha Q^2}{2\pi^2 x}\left\{\left(\frac{x^2}{2} - x + 1\right) q_\perp^2 + \frac{x^4 m^2}{2}\right\}\sum_\lambda \left|\frac{1}{q_\perp^2 + \kappa_\lambda^{(1)} - i\delta} - \frac{1}{q_\perp^2 + \kappa_\lambda^{(2)} + i\delta}\right|^2$$

(Transition radiation in ordinary materials corresponds to $\kappa_{\rm tr} = m^2x^2 + m_\gamma^2(1-x)$ finite at $\hbar \to 0$)

Contribution of the pole at $q_\perp^2 + \kappa_\lambda = 0$ is the chiral Cherenkov radiation.

The rest is the "chiral transition radiation"

CHERENKOV + TRANSITION RADIATION IN QGP



solid: right-hand polarization

dashed: left-hand polarization

Quark energy 10 GeV

$k_\perp \approx \theta\omega$

- Charged particles traveling through the chiral medium emit electromagnetic radiation sensitive to the chiral anomaly.

XG Huang, KT (2018)

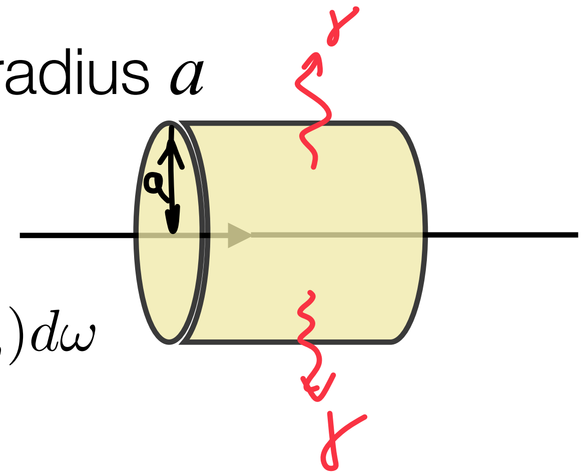
- It is circularly polarized and has resonant peaks at angles proportional to the anomaly

FERMI'S MODEL OF COLLISIONAL ENERGY LOSS

Hansen, KT (2021)

Chiral Cherenkov radiation is closely related to the collisional energy loss.

The energy loss rate = flux of the Poynting vector out of cylinder of radius $\boldsymbol{a}$ coaxial with the particle path:



$$-\frac{d\varepsilon}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a \,\mathrm{Re} \int_0^{\infty} (E_{\phi\omega} B^*_{z\omega} - E_{z\omega} B^*_{\phi\omega}) d\omega$$

The field components are known, e.g.:

$$B_{\phi\omega}(\boldsymbol{r}) = \frac{q}{2\pi} \frac{e^{i\omega z/v}}{k_1^2 - k_2^2} \sum_{\nu=1}^{2} (-1)^{\nu+1} k_\nu (k_\nu^2 - s^2) K_1(bk_\nu)$$

$$k_\nu^2 = s^2 - \frac{\sigma_\chi^2}{2} + (-1)^\nu \sigma_\chi \sqrt{\omega^2 \epsilon + \frac{\sigma_\chi^2}{4}}$$

$$B_{b\omega}(\boldsymbol{r}) = \sigma_\chi \frac{q}{2\pi} \frac{i\omega}{v} \frac{e^{i\omega z/v}}{k_1^2 - k_2^2} \sum_{\nu=1}^{2} (-1)^\nu k_\nu K_1(bk_\nu)$$

$$s^2 = \omega^2 \left(\frac{1}{v^2} - \epsilon(\omega) \right)$$

Fermi's model: $$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

Setting $\sigma_\chi = 0$ get the original Fermi's result at $a \to 0$

(small) Cherenkov radiation contribution emerges at $a \to \infty$ if $v > 1/\sqrt{\epsilon(0)}$

"CLASSICAL" CHIRAL CHERENKOV RADIATION

For simplicity consider $\omega_0 = 0$

Hansen, KT (2021)

UR limit $\gamma \gg 1$ at $a \to 0$ gives energy loss

$$-\frac{d\varepsilon}{dz} = \frac{q^2}{4\pi v^2}\left(\omega_p^2 \ln\frac{v}{a\omega_p} + \frac{1}{4}\gamma^2\sigma_\chi^2\right)$$

increases as (energy)^2 due to anomaly

Chiral Cherenkov radiation emerges at $a \to \infty$ even if $\epsilon = 1$

$$\frac{dW}{d\omega} = -\frac{d\varepsilon}{dz\omega d\omega}\bigg|_{a\to\infty} = \frac{q^2}{4\pi}\left\{\frac{1}{2}\left(1-\frac{1}{v^2}\right) + \frac{\sigma_\chi}{2\omega} + \frac{(1+v^2)\sigma_\chi^2}{8v^2\omega^2} + \dots\right\}, \quad \omega < \sigma_\chi\gamma^2$$

This classical formula coincides with the quantum calculation

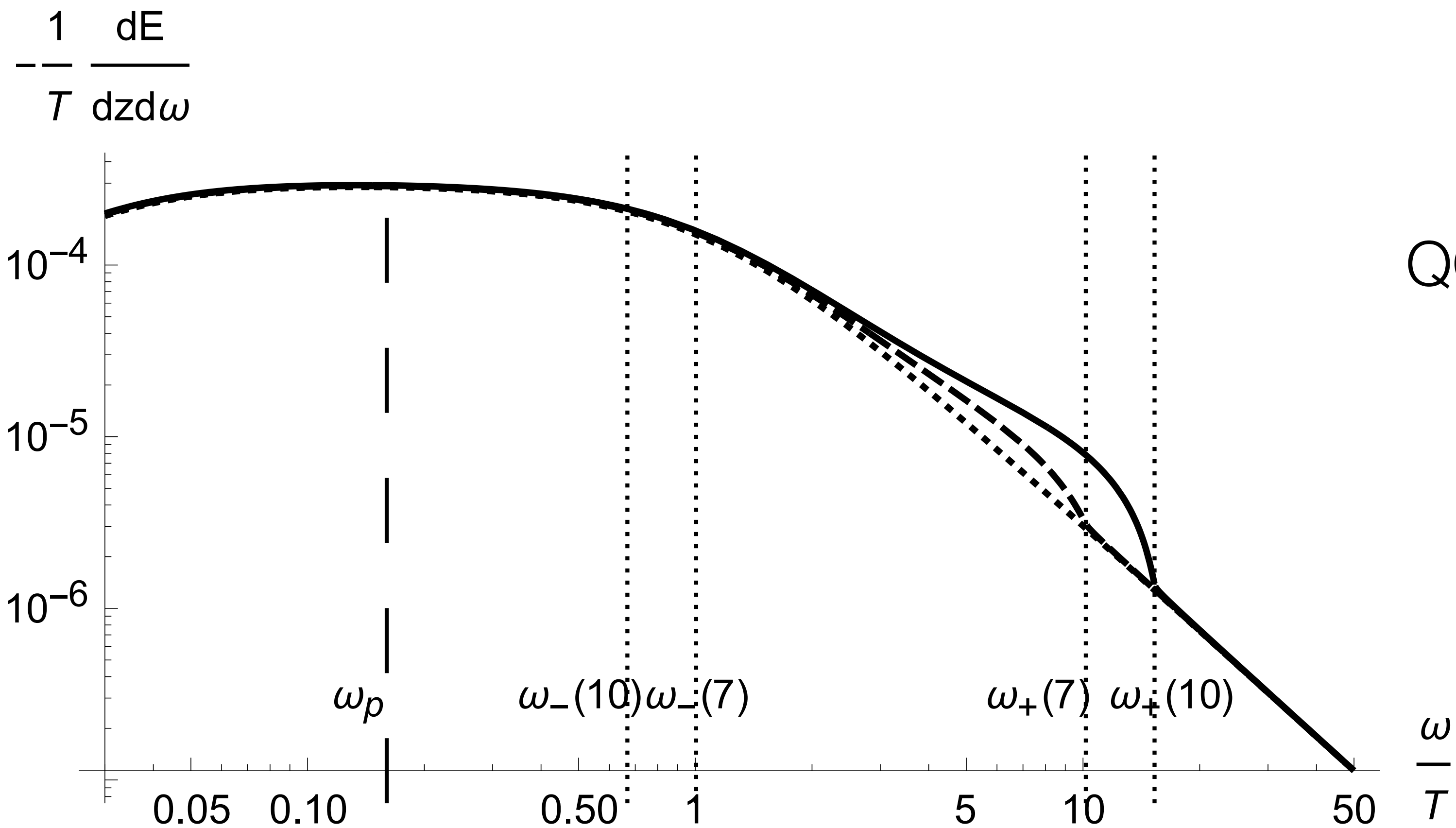
$$\frac{dW^{\text{quant}}}{d\omega} = \frac{q^2}{(4\pi)2\omega}\left\{\sigma_\chi\left(\frac{x^2}{2} - x + 1\right) - \frac{m^2}{\varepsilon}x\right\}$$

when recoil is neglected $x \ll 1$

Power of chiral Cherenkov radiation $P = \dfrac{q^2}{4\pi}\dfrac{\sigma_\chi^2\gamma^2}{4}$ (recoil reduces $\gamma^2 \to \gamma$)

AT HIGH ENERGY POWER OF CHIRAL CHERENKOV RADIATION = ENERGY LOSS.

APPLICATIONS:QGP



QCD/QED: $\times \frac{\alpha_s^2}{\alpha_{\rm em}^2}$

FIG. 1. Electromagnetic part of the collisional energy loss spectrum of a d-quark with $\gamma = 20$ in Quark-Gluon Plasma. Plasma parameters: $\omega_p = 0.16T$, $\Gamma = 1.11T$ [36], $m = T = 250$ MeV. Solid line: $\sigma_\chi = 10$ MeV, dashed line: $\sigma_\chi = 7$ MeV, dotted line: $\sigma_\chi = 0$. $\omega_\pm$ are defined in (13).

The same qualitative picture in QCD (after e→g, including color factors etc.)

$$-\frac{d\varepsilon}{dz}\bigg|_{\rm anom} = \frac{g^2 C_F}{4\pi}\frac{\tilde{\sigma}_\chi \varepsilon}{3}$$

APPLICATIONS: WEYL SEMIMETAL

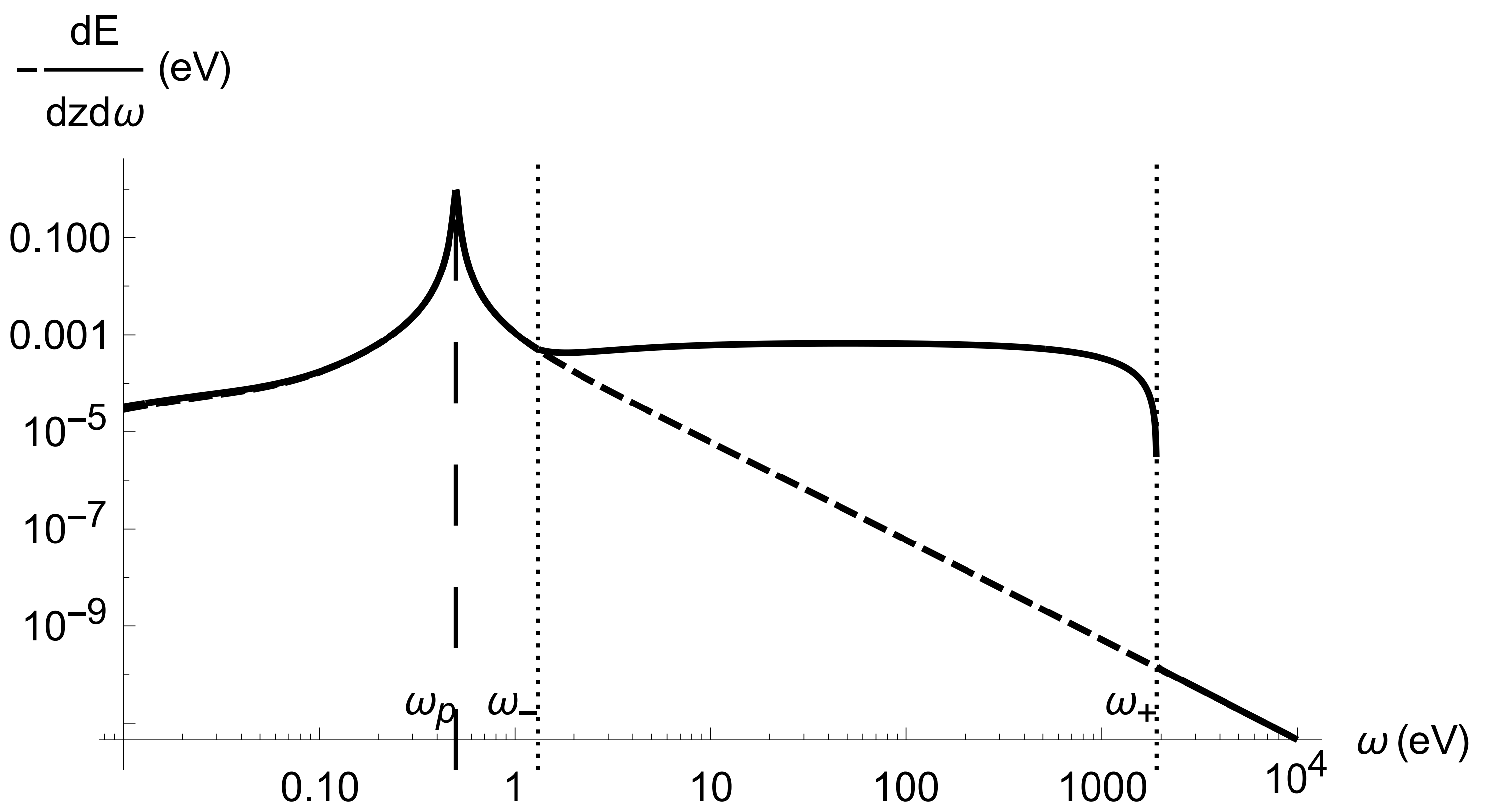


FIG. 2. Collisional energy loss spectrum of electron with $\gamma = 100$ in a semimetal with parameters $\omega_p = 0.5$ eV, $\Gamma = 0.025$ eV (so that its conductivity is 10 eV at room tempearture) [41] and $m = 0.5$ MeV. Solid line: $\sigma_\chi = 0.19$ eV [42, 43], dashed line: $\sigma_\chi = 0$. $\omega_\pm$ are defined in (13). The seeming discontinuity at $\omega = \omega_+$ is a visual artifact.

Very small recoil $\omega_M \lesssim \sigma_\chi \gamma^2 \ll \varepsilon$

CHIRAL CHERENKOV VS BETHE-HEITLER

Neglecting coherence effects: $\frac{\Delta\varepsilon^{\chi C}}{\Delta\varepsilon^{BH}} \sim \frac{\sigma_\chi}{e^2 T} \sim \frac{\mu_5}{T}$ >>1 in a TaAs at room temp.

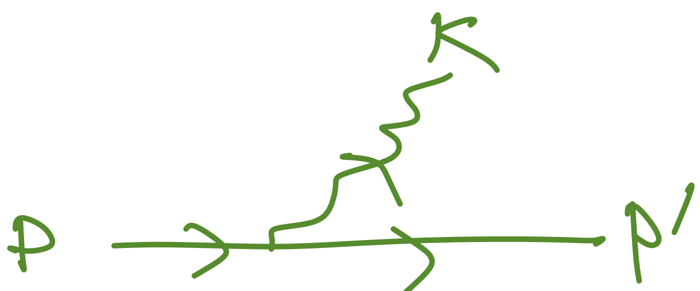
Coherence effects reduce energy dependence of BH (LPM effect) $E \to \sqrt{E}$

Contribution of the Chiral Cherenkov rapidly increases with E.

Coherence effects in Cherenkov radiation: unknown, depends on spatial distribution of topological charge density.

Electromagnetic radiation of Quark-Gluon Plasma at finite θ

PHOTON PRODUCTION BY QGP VIA THE CHIRAL ANOMALY (W/O EXTERNAL MAGNETIC FIELD)



Thermal q
and anti-q

The photon energy produced by thermal quarks is controlled by the plasma temperature
→ must take into account the plasma frequency

$$\omega_{\rm pl}^2 = \frac{m_D^2}{2} = \frac{e^2}{2}\left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)$$

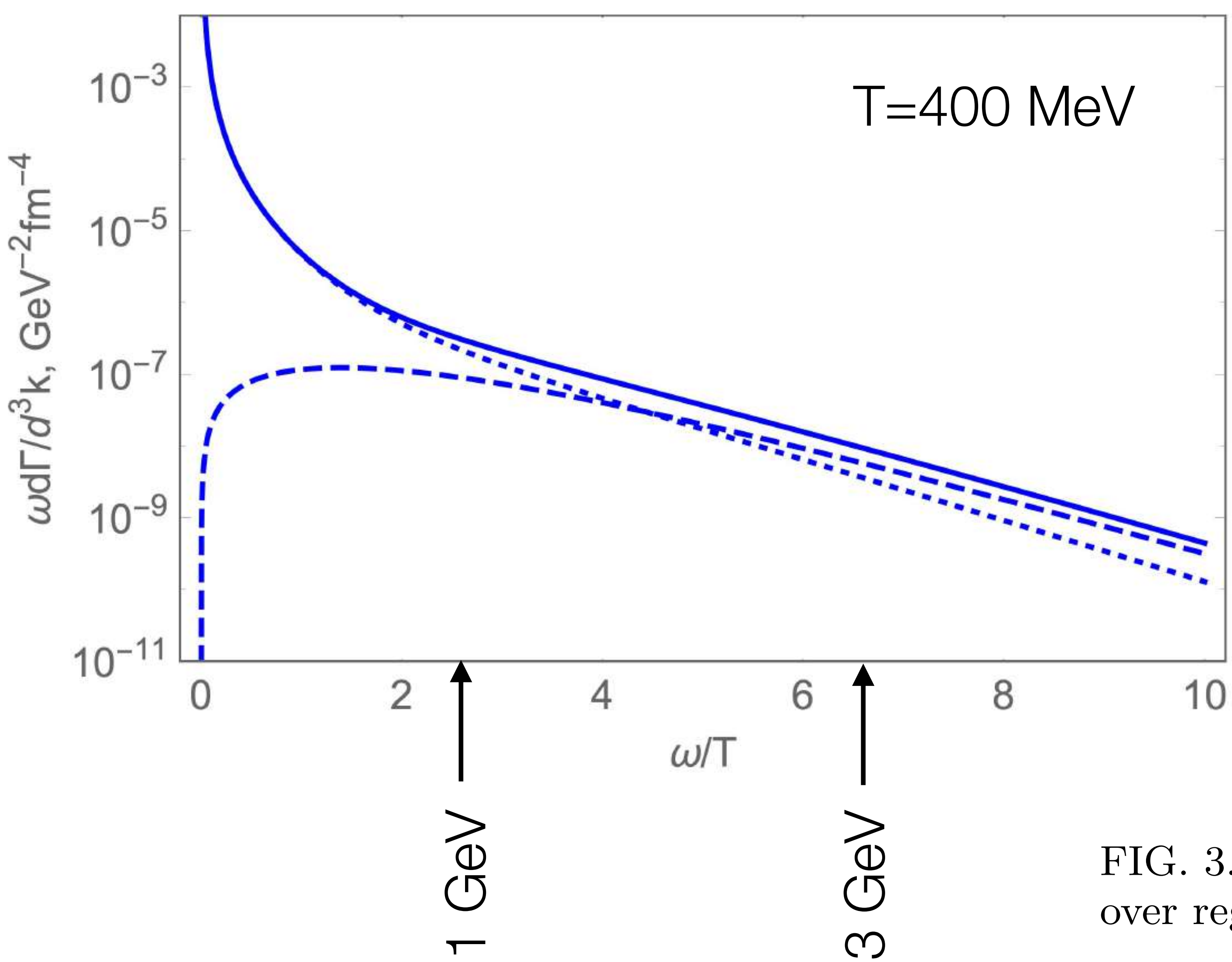
Photon mass gets two contributions: $\omega^2 - k^2 = \omega_{\rm pl}^2 + m_A^2 + \mathcal{O}(\omega - k)$

$$m_A^2 = -\lambda \sigma_\chi \omega \,, \qquad \text{or} \qquad m_A^2 = -\lambda \boldsymbol{k}\cdot\boldsymbol{b}\,,$$

Due to the topological number fluctuations $m_A \sim \sqrt{\langle \theta^2 \rangle} \sim \Gamma_{\rm sp} \sim T^4$

Thus, at high enough T $m_A \gg \omega_{\rm pl}$ → Cherenkov radiation is possible

TOTAL CHIRAL CHERENKOV RADIATION



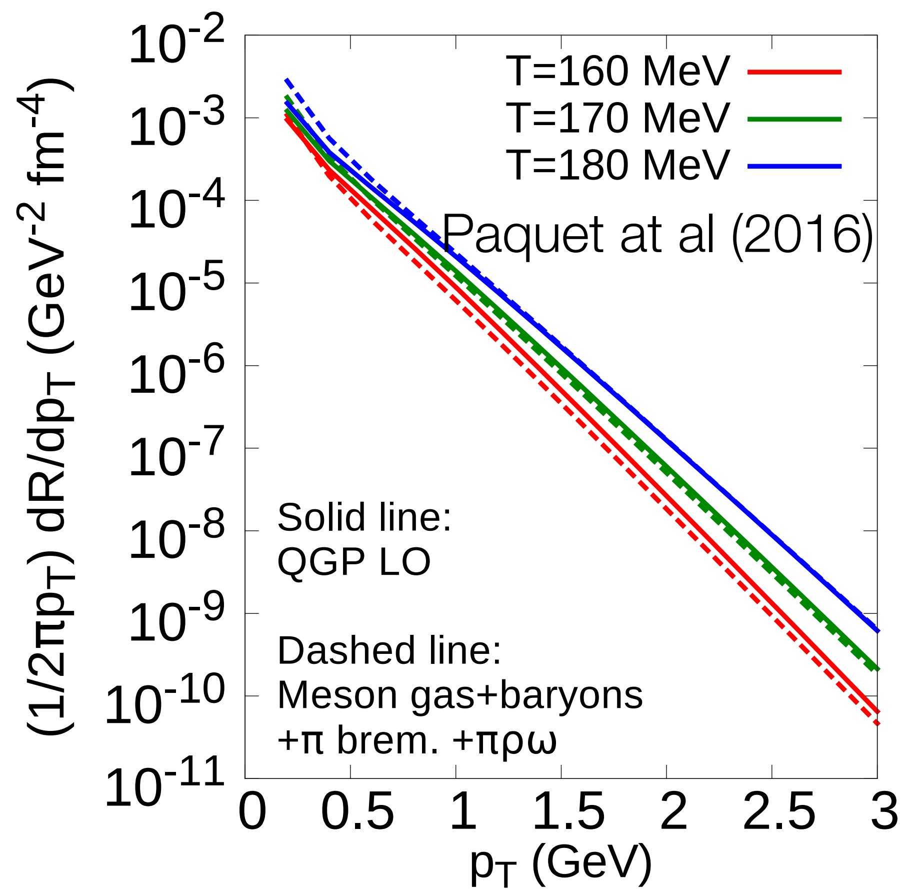
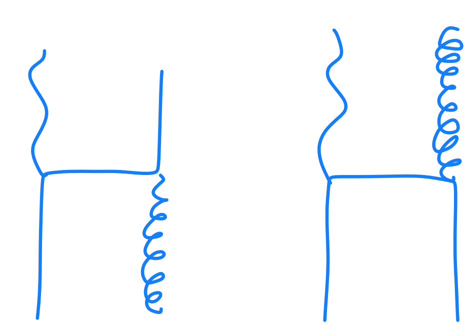


FIG. 3. Ideal QGP and hadronic photon rate near the cross-over region.

Conventional mechanisms



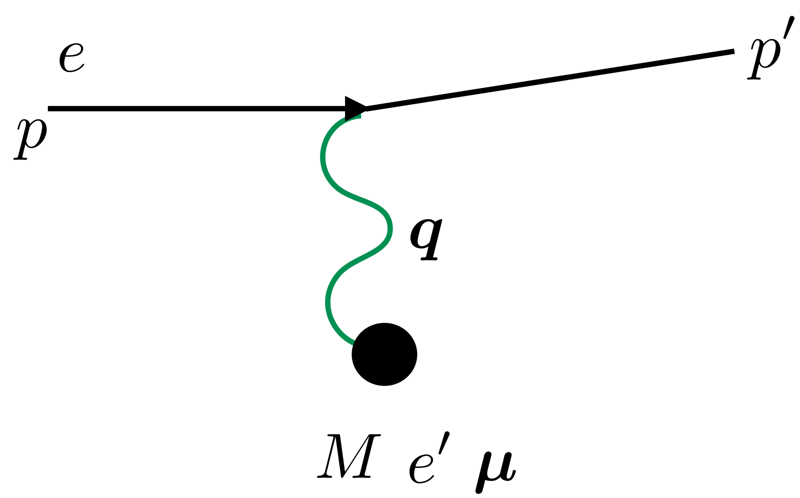
Comparable contributions

Transport anomalies

CONTRIBUTION OF ANOMALY TO TRANSPORT

Qiu, Cao, Huang
Lenhert, Potting

scattering on a heavy “ion”



Photon propagator in MCS:

$$D_{\mu\nu}(q) = -i\frac{q^2 g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma}b^\rho q^\sigma + b_\mu b_\nu}{q^4 + b^2 q^2 - (b\cdot q)^2} \qquad b^\mu = (\sigma_\chi, \boldsymbol{0})$$

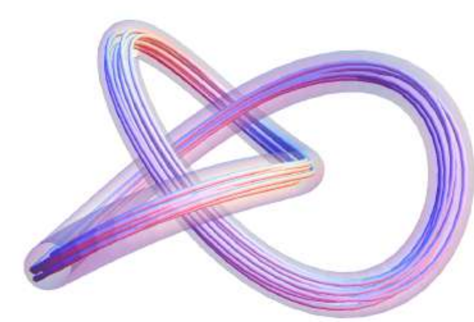
Static limit q_0=0

$q^0 \ll |\boldsymbol{q}| \quad q^0 \ll b_0$

$$D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2},$$

$$D_{0i}(\boldsymbol{q}) = D_{0i}(\boldsymbol{q}) = 0,$$

$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$



Potential induced by a stationary current

$$A^\mu(\boldsymbol{x}) = -i\int d^3x' D^{\mu\nu}(\boldsymbol{x}-\boldsymbol{x}')J_\nu(\boldsymbol{x}') = -i\int \frac{d^3q}{(2\pi)^3}e^{i\boldsymbol{q}\cdot\boldsymbol{x}}D^{\mu\nu}(\boldsymbol{q})J_\nu(\boldsymbol{q})$$

ELECTRON-ION CROSS SECTION

Current of an ion with charge e' and magnetic moment $\boldsymbol{\mu}$:

$$J^0(\boldsymbol{x}) = e'\delta(\boldsymbol{x})\,, \qquad \boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{\nabla}\times(\boldsymbol{\mu}\delta(\boldsymbol{x}))$$

The corresponding potential $A^\ell(\boldsymbol{q}) = -\frac{1}{\boldsymbol{q}^2 - b_0^2}\left[i(\boldsymbol{\mu}\times\boldsymbol{q})^\ell + \frac{b_0}{\boldsymbol{q}^2}(\boldsymbol{\mu}\cdot\boldsymbol{q}q^\ell - \boldsymbol{q}^2\mu^\ell)\right]$

In the static limit, there is only one unstable mode $|\boldsymbol{q}| = b_0$

Cross section averaged over the magnetic moment directions:

$$\left\langle \frac{d\sigma}{d\Omega'}\right\rangle = \frac{e^2}{8\pi^2}\left\{\underbrace{\frac{2E^2e'^2}{\boldsymbol{q}^4}\left(1-\frac{\boldsymbol{q}^2}{4E^2}\right)}_{\textbf{Coulomb}} + \underbrace{\frac{2\mu^2}{3(\boldsymbol{q}^2-b_0^2)^2}\left(1+\frac{b_0^2}{\boldsymbol{q}^2}\right)\left[(\boldsymbol{p}\times\boldsymbol{q})^2 + \frac{\boldsymbol{q}^4}{2}\right]}_{\textbf{Anomaly}}\right\}$$

SPIN AVERAGE CROSS-SECTION

Transport cross section $$\sigma_T = \frac{e^2}{16\pi \boldsymbol{p}^4}\Big(\underbrace{4E^2 e'^2 L}_{\textbf{Coulomb}} + \underbrace{\frac{2\mu^2}{3} 4\boldsymbol{p}^4 \mathcal{I}}_{\textbf{Anomaly}}\Big)$$

$$\mathcal{I} = 1 + \frac{1}{\epsilon}\left[2a(1+a) - \epsilon^2\right]\left(\arctan\frac{1-a}{\epsilon} + \arctan\frac{a}{\epsilon}\right) + \frac{1}{2}(1+3a)\ln\left(1 + \frac{1-2a}{a^2+\epsilon^2}\right)$$

$$a = \frac{b_0^2}{4\boldsymbol{p}^2}\,, \qquad \epsilon = \frac{\Gamma^2}{4\boldsymbol{p}^2}$$

Γ is width $\frac{\boldsymbol{q}^2}{(\boldsymbol{q}^2 - b_0^2)^2 + \Gamma^4}$

due to processes that tame the instability

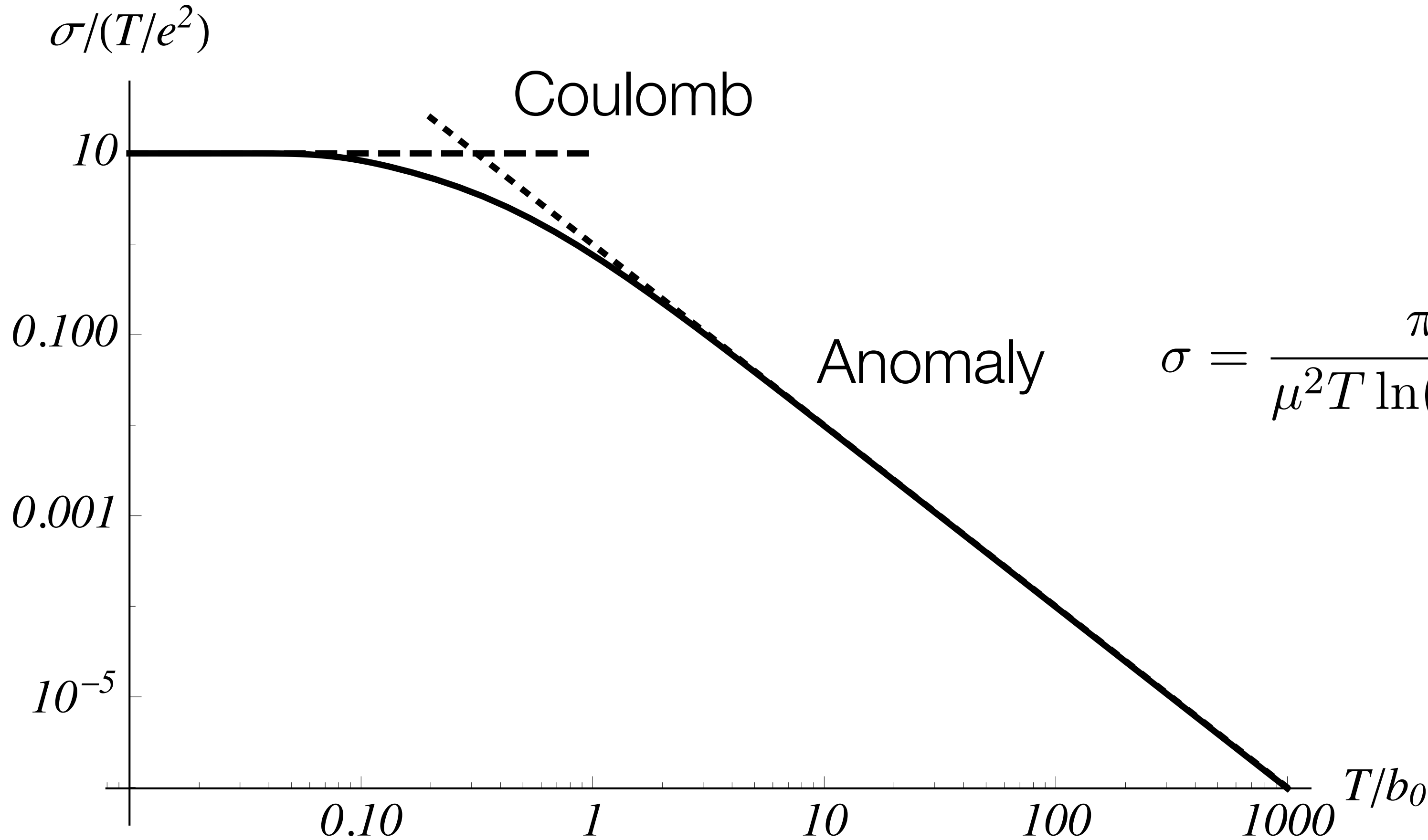
At large momenta $$\sigma_T \approx \frac{e^2\mu^2}{6\pi}\ln\frac{4\boldsymbol{p}^2}{b_0^2}$$ $\Rightarrow$ anomaly dominates Coulomb

Large σ_T $\Rightarrow$ small m.f.p. $\Rightarrow$ suppression of transport coefficients

ANOMALOUS CONTRIBUTION TO CONDUCTIVITY

Conductivity $\sigma = \frac{e^2}{3T}\int f_0 \frac{1}{n\sigma_T} d^3p$

(anomaly contribution to f_0 is neglected for simplicity)



$$\sigma = \frac{\pi}{\mu^2 T \ln(T/b_0)}$$

SUMMARY

Electrodynamics of chiral media (e.g. quark-gluon plasma, Weyl semimetals, axion dark matter, primordial magnetic fields) has many novel effects and intriguing features.

Many opportunities for ambitious experimentalists.

read more